

Loss of memory of non-Gaussian initial correlations under Gaussian unitary evolution

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Loss of memory of non-Gaussian initial correlations under Gaussian unitary evolution

Evolution under quantum mechanical laws, though unitary, leads to loss of information.

In particular, evolution under noninteracting Hamiltonians eliminates memory of non-Gaussian initial correlations.

Outline

- **Introduction**

 - Position of the problem

 - Summary of results

 - Background: Quantum Quenches, Equilibration, GGE

- **Free relativistic bosonic fields**

 - Verification of validity of GGE for interacting \rightarrow noninteracting QQ

- **Free non-relativistic bosons**

 - Verification of GGE in the Lieb-Liniger QQ: $c \rightarrow 0$

 - Time decay of correlations

- **Conclusions**

Position of the Problem

- Consider an isolated quantum system initially lying in the ground state of some arbitrary Hamiltonian and evolving under another, non-interacting, Hamiltonian. What is its long time behaviour?

$$H_0|\Psi_0\rangle = 0 \quad |\Psi_0\rangle \xrightarrow{e^{-iHt}} ?$$

- Does the system **equilibrate**?
- If yes, is this equilibrium **thermal**?
- If no, what type of equilibrium is it?
- How much information about the initial state survives after a long time?

Summary of results

- At $t \rightarrow \infty$ the system exhibits **stationary** behaviour.
- *Not thermal!*
- Described instead by a **generalised Gibbs ensemble** (GGE).
- GGE is **Gaussian** (for noninteracting Hamiltonians).
- *Only* information about initial 2-point correlations survives.

Quantum Quenches

- An **instantaneous** change in parameters of the Hamiltonian of an **isolated** quantum system.

$$H_0 \longrightarrow H$$

$$H_0 |\Psi_0\rangle = 0 \quad |\Psi_0\rangle \xrightarrow{e^{-iHt}} ?$$

- Experimental implementation in condensed matter systems (gases at very low temperature).
- Theoretical study of the nature of thermalisation in quantum systems.

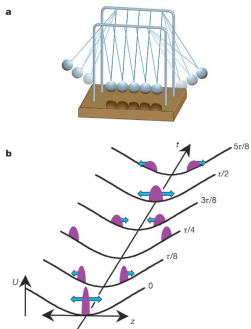


Figure: Experimental setup.

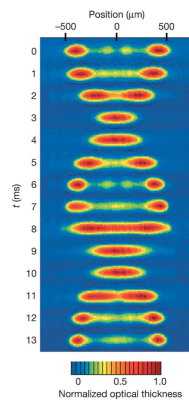


Figure: Persistent oscillations.

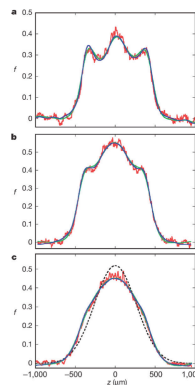
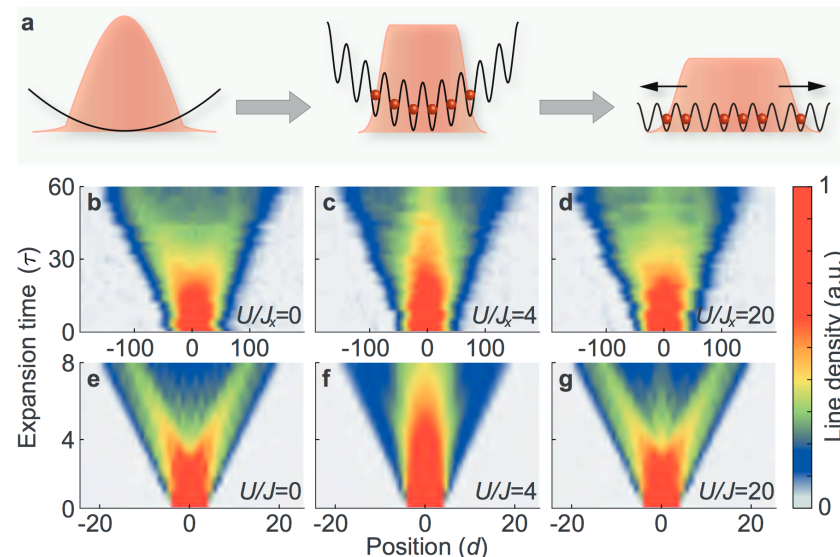
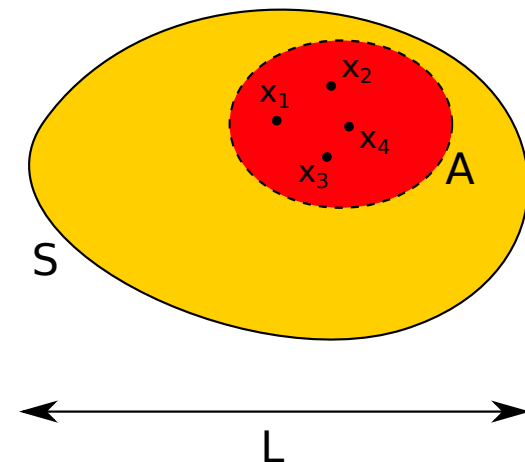
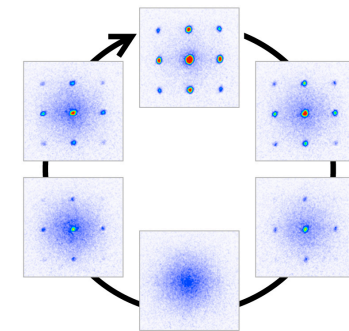
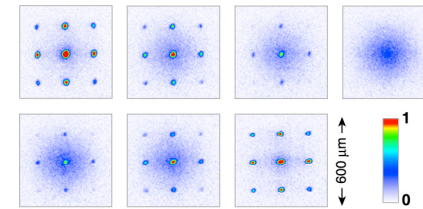


Figure: Non-thermal stationary momentum distribution for long times.



Equilibration

- How can a closed quantum system equilibrate?
- An initial **pure** state will remain pure for all times and even exhibit quantum **recurrences** (periodic or quasi-periodic evolution) with a period that increases with the system size.
 - A **special double limit** (first system size $L \rightarrow \infty$, then time $t \rightarrow \infty$) has to be considered in order for stationary behaviour to be possible.
 - Any subsystem of a large system is open, in contact with its complement, and therefore is described by a **mixed** reduced density matrix. This *can* correspond to a statistical ensemble even if the whole system remains in a pure state.
 - Equivalently the expectation value of **local** observables may be given by an ensemble average (even if the whole system remains in a pure state).



$$\langle \mathcal{O}(x_1, \dots) \rangle = \text{Tr} \{ \mathcal{O}(x_1, \dots) \rho_A \}$$

Thermalisation

- By physical intuition from classical statistical physics, a general system starting from an “arbitrary” initial state is expected to tend to thermal equilibrium i.e. it would maximise its entropy subject to the constraint of energy conservation.
- An interesting exception:
Integrable systems (i.e. 1d systems that possess an infinite number of local integrals of motion allowing their exact solvability) are not constrained only by the conservation of their energy, therefore do not thermalise.
- Examples of integrable models:
 - non-interacting bosons or fermions (trivial S-matrix ± 1)
 - Quantum Ising, XY spin $\frac{1}{2}$ chains
 - non-relativistic bosons with point-like interactions (Lieb-Liniger model)
 - sine/sinh-Gordon model
 - Heisenberg, XXZ spin chains
- **Entropy maximisation** under all additional constraints would suggest a different statistical ensemble for their long time behaviour.

Generalised Gibbs Ensemble

- Gibbs ensemble

$$\rho_{GE} \propto \exp(-\beta\mathcal{H})$$

β : Lagrange multiplier associated with the constraint of energy conservation, determined through

$$\text{Tr}(\mathcal{H} \rho_{GE}) = \langle \Psi_0 | \mathcal{H} | \Psi_0 \rangle$$

where $|\Psi_0\rangle$ is the initial state.

- For integrable systems, **generalized Gibbs ensemble**

$$\rho_{GGE} \propto \exp\left(-\sum_m \lambda_m \mathcal{I}_m\right)$$

λ_m : Lagrange multiplier associated with **local** conserved charge \mathcal{I}_m , determined through

$$\text{Tr}(\mathcal{I}_m \rho_{GGE}) = \langle \Psi_0 | \mathcal{I}_m | \Psi_0 \rangle$$

[Rigol, Dunjko, Yurovsky, Olshanii (2007)]

GGE conjecture

For long times after a quantum quench in an integrable system, local observables equilibrate to values given by the GGE.

- Verified in many cases of non-interacting models or interacting that can be mapped exactly into non-interacting ones (quantum Ising, Luttinger model, etc.), Conformal Field Theories, special cases of interacting integrable models (sinh-Gordon).

$$\rho_{GGE} \propto \exp \left(- \int dk \lambda_k n_k \right)$$

- Failure of the GGE conjecture has been reported in some cases of quenches in integrable models (Lieb-Liniger with BEC initial state or in the attractive case, XXZ spin chain with Néel initial state).
- Physical reasons underlying its validity (or failure) *not understood* yet.

Free relativistic bosonic fields

Hamiltonian and fields

- Post-quench Hamiltonian

$$H = \frac{1}{2} \sum_k \left(\tilde{\pi}_k \tilde{\pi}_{-k} + \omega_k^2 \tilde{\phi}_k \tilde{\phi}_{-k} \right)$$

- Bosonic field, conjugate momentum and creation-annihilation operators

$$\phi(x; t) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} \tilde{\phi}_k(t) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} \frac{1}{\sqrt{2\omega_k}} (a_k e^{-i\omega_k t} + a_{-k}^\dagger e^{+i\omega_k t})$$

$$\pi(x; t) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} \tilde{\pi}_k(t) = \frac{-i}{\sqrt{L}} \sum_k e^{ikx} \sqrt{\frac{\omega_k}{2}} (a_k e^{-i\omega_k t} - a_{-k}^\dagger e^{+i\omega_k t})$$

$$a_k = \sqrt{\frac{\omega_k}{2}} \tilde{\phi}_k(0) + \frac{i}{\sqrt{2\omega_k}} \tilde{\pi}_k(0)$$

$$a_{-k}^\dagger = \sqrt{\frac{\omega_k}{2}} \tilde{\phi}_k(0) - \frac{i}{\sqrt{2\omega_k}} \tilde{\pi}_k(0)$$

- We may assume a relativistic dispersion relation $\omega_k = \sqrt{k^2 + m^2}$ but our results are more generally valid.

Free relativistic bosonic fields

The 2-point function

- Time evolution
For *translationally invariant* initial states ([assumption 1](#))

$$C_t^{(2)}(x, y) = \frac{1}{L} \sum_k \frac{1}{2\omega_k} e^{ik(x-y)} \left[\langle a_k a_{-k} \rangle_0 e^{-2i\omega_k t} + \langle a_{-k}^\dagger a_{-k} \rangle_0 + \langle a_k a_k^\dagger \rangle_0 + \langle a_{-k}^\dagger a_k^\dagger \rangle_0 e^{+2i\omega_k t} \right]$$

- Large-time limit
If the post-quench Hamiltonian is *massive*, the 2-point function becomes stationary ([assumption 2](#))

$$C_\infty^{(2)}(x, y) = \frac{1}{L} \sum_k \frac{1}{2\omega_k} e^{ik(x-y)} (1 + \langle n_{-k} \rangle_0 + \langle n_k \rangle_0)$$

- GGE prediction

$$C_{\text{GGE}}^{(2)}(x, y) = \frac{1}{L} \sum_k \frac{1}{2\omega_k} e^{ik(x-y)} (1 + \langle n_{-k} \rangle_{\text{GGE}} + \langle n_k \rangle_{\text{GGE}})$$

Free relativistic bosonic fields

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Free relativistic bosonic fields

The 2-point function

- By definition of GGE

$$\langle n_k \rangle_0 = \langle n_k \rangle_{\text{GGE}}$$

so the two expressions are identical.

For the 2-point correlation function, the GGE conjecture holds trivially.

Free relativistic bosonic fields

The 4-point function

- Time evolution

$$C_t^{(4)}(x_1, x_2, x_3, x_4) = \frac{1}{L^2} \sum_{k_1, k_2, k_3, k_4} \frac{1}{4\sqrt{\prod_{i=1}^4 \omega_{k_i}}} e^{i \sum_{i=1}^4 k_i x_i} \sum_{\substack{\text{all } \{\sigma_i\} \\ \sigma_i = \pm}} \left\langle \prod_{i=1}^4 a_{-\sigma_i k_i}^{(\sigma_i)} \right\rangle_0 e^{i \sum_{i=1}^4 \sigma_i \omega_{k_i} t}$$

- Large-time limit

Under the same conditions as before, the 4-point function becomes stationary. The stationary value is given by

$$C_\infty^{(4)}(x_1, x_2, x_3, x_4) = \frac{1}{2} \frac{1}{L^2} \sum_{k,p} \frac{1}{4\omega_k \omega_p} \left(\sum_{\substack{\text{all perm.s} \\ \text{of } 1,2,3,4}} e^{ik(x_2-x_1)+ip(x_4-x_3)} \right) \left(\langle n_k n_p \rangle_0 + \langle n_k \rangle_0 + \frac{1}{4} \right)$$

- GGE prediction

$$C_{\text{GGE}}^{(4)}(x_1, x_2, x_3, x_4) = \frac{1}{2} \frac{1}{L^2} \sum_{k,p} \frac{1}{4\omega_k \omega_p} \left(\sum_{\substack{\text{all perm.s} \\ \text{of } 1,2,3,4}} e^{ik(x_2-x_1)+ip(x_4-x_3)} \right) \left(\langle n_k n_p \rangle_{\text{GGE}} + \langle n_k \rangle_{\text{GGE}} + \frac{1}{4} \right)$$

Free relativistic bosonic fields

The 4-point function

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Free relativistic bosonic fields

The 4-point function

- **Wick's theorem** is valid in the GGE, so products of occupations numbers *factorise*

$$\langle n_k n_p \rangle_{\text{GGE}} = \langle n_k \rangle_{\text{GGE}} \langle n_p \rangle_{\text{GGE}}$$

- This is *not* generally true for an arbitrary initial state.
- **Case 1: (Special)**
For a **Gaussian** initial state (e.g. ground state of a non-interacting Hamiltonian), **Wick's theorem** holds so initial values of products factorise too

$$\langle n_k n_p \rangle_0 = \langle n_k \rangle_0 \langle n_p \rangle_0$$

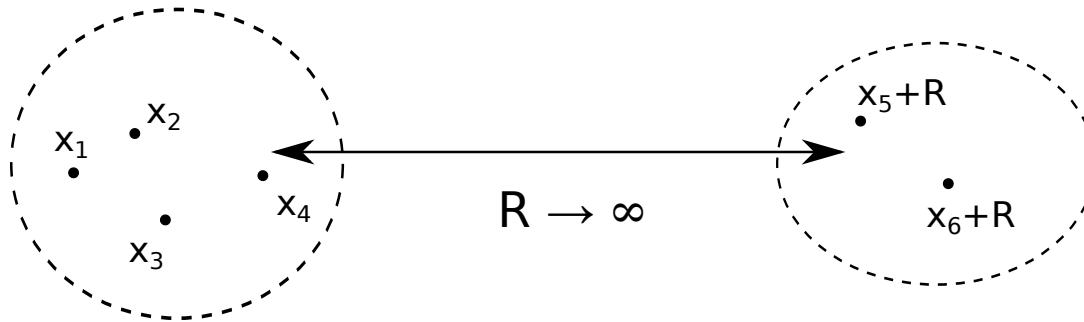
For the 4-point correlation function, the GGE conjecture is true for any quantum quench from a non-interacting pre-quench Hamiltonian, as a consequence of Wick's theorem.

Free relativistic bosonic fields

The 4-point function

- **Case 2: (General)**
For a (non-Gaussian) initial state that is the ground state of an interacting Hamiltonian, the physical principle of **cluster decomposition** applies (assumption 3).

Multi-point correlations factorise, when the distance separating a subset of points from the rest tends to infinity.

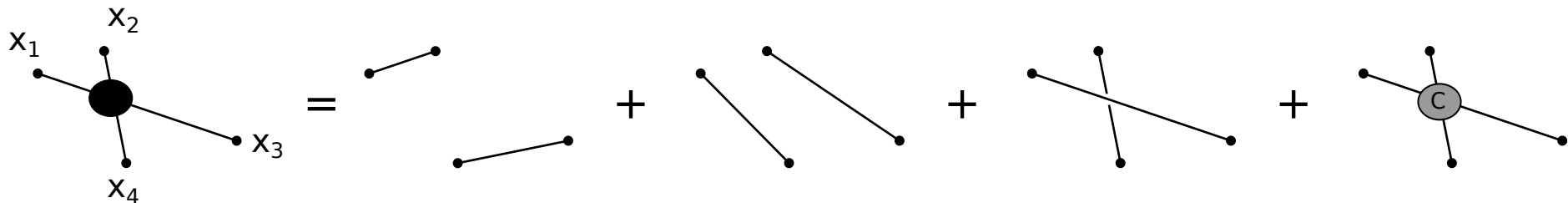


$$\lim_{R \rightarrow \infty} \left\langle \prod_i \phi(x_i) \prod_j \phi(x_j + R) \right\rangle = \left\langle \prod_i \phi(x_i) \right\rangle \left\langle \prod_j \phi(x_j) \right\rangle$$

Free relativistic bosonic fields

The 4-point function

- Express the expectation values of the charges and their products in the initial state in terms of the initial field correlations.
- Expand the initial 4-point correlations in **disconnected** and **connected** terms.



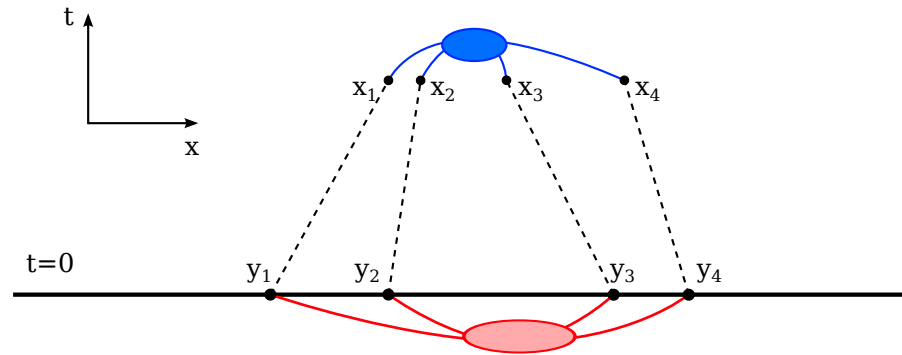
- Due to the cluster decomposition property, the connected terms contribute to the large time 4-point correlations *only* finite size corrections.
- Since the disconnected terms can be expressed in terms of 2-point initial correlations, only this information about the initial state survives at large times.
- But this information is already contained in the initial values of the charges which is already captured by the GGE.

Free relativistic bosonic fields

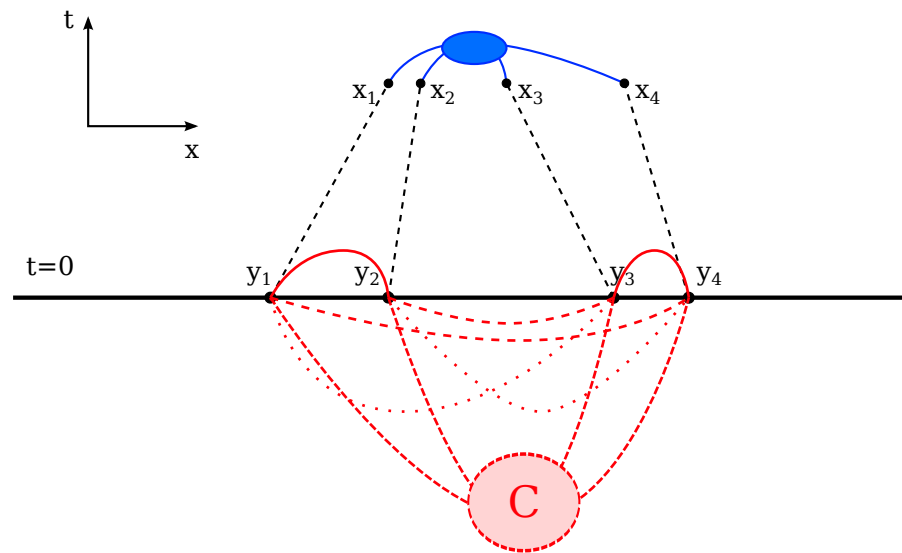
The 4-point function

For the 4-point correlation function, the GGE conjecture is true for any quantum quench from an interacting pre-quench Hamiltonian, as a consequence of the cluster decomposition property.

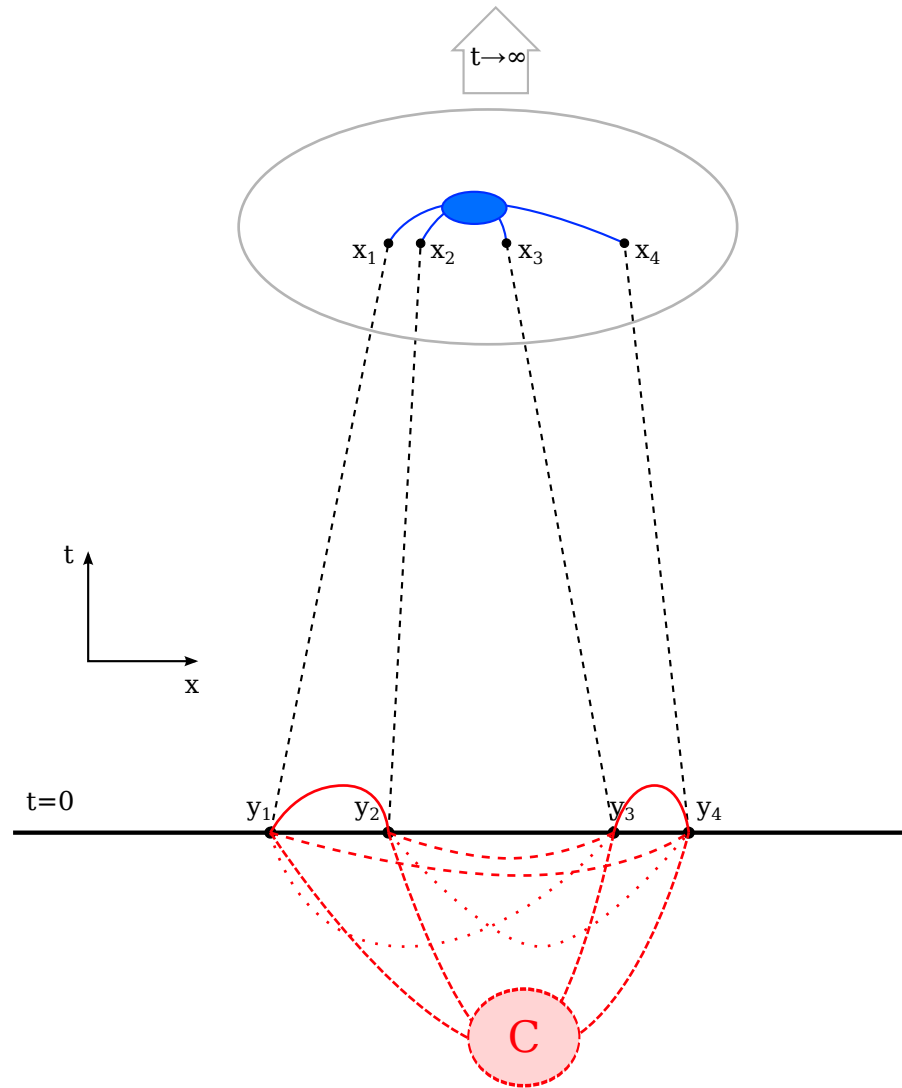
Free relativistic bosonic fields



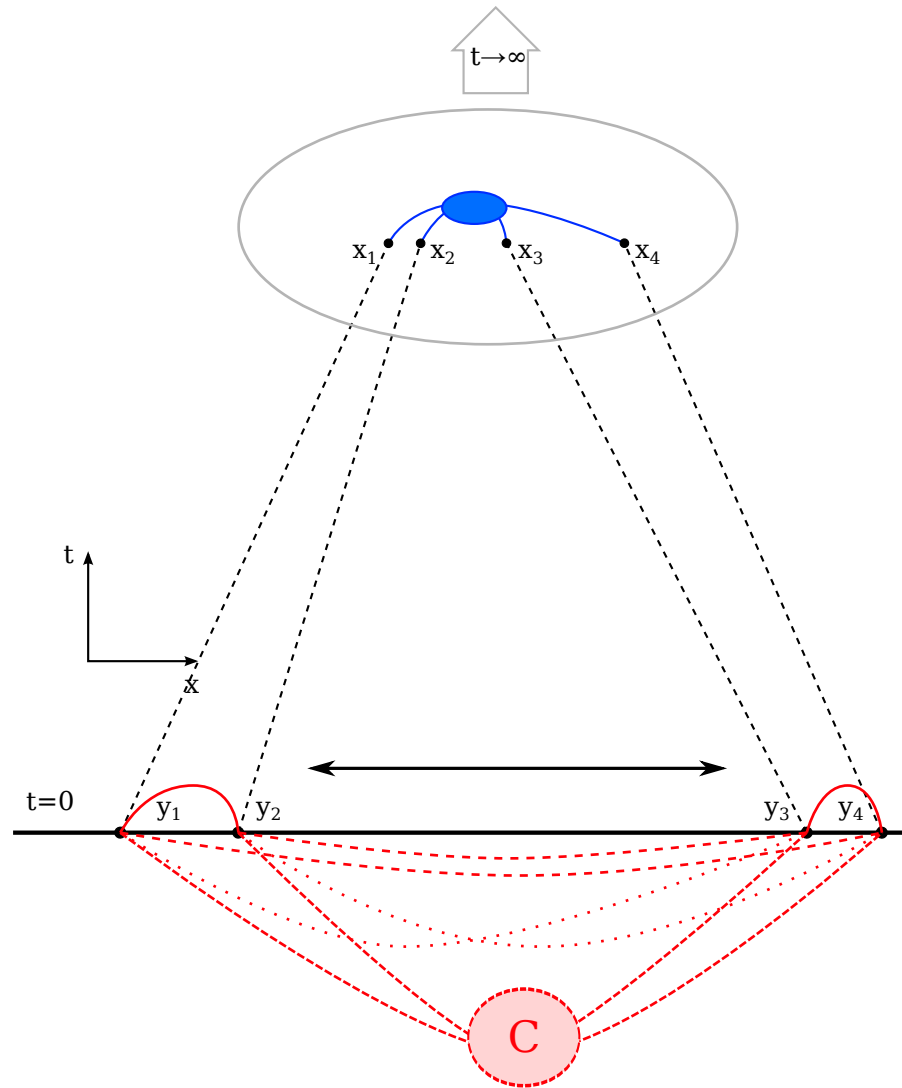
Free relativistic bosonic fields



Free relativistic bosonic fields



Free relativistic bosonic fields



Free relativistic bosonic fields

Higher order n-point functions

- Similarly for higher order n-point functions:

Due to the cluster decomposition property of the initial state, it is only the 2-point initial correlations that contribute to the large time stationary values of the n-point functions and this information is captured by the GGE since it is a Gaussian ensemble (i.e. satisfying Wick's theorem).

Lieb-Liniger model: $c > 0 \rightarrow c = 0$

- Consider a quantum quench of the interaction in the Lieb-Liniger model from $c > 0$ to $c = 0$ (free evolution of an interacting ground state).
- Problem studied numerically by: [Mossel & Caux](#) (New J. Phys. 14 (2012) 075006)
- GGE valid for g_2 function as shown by fitting and extrapolation of numerics for finite size systems.

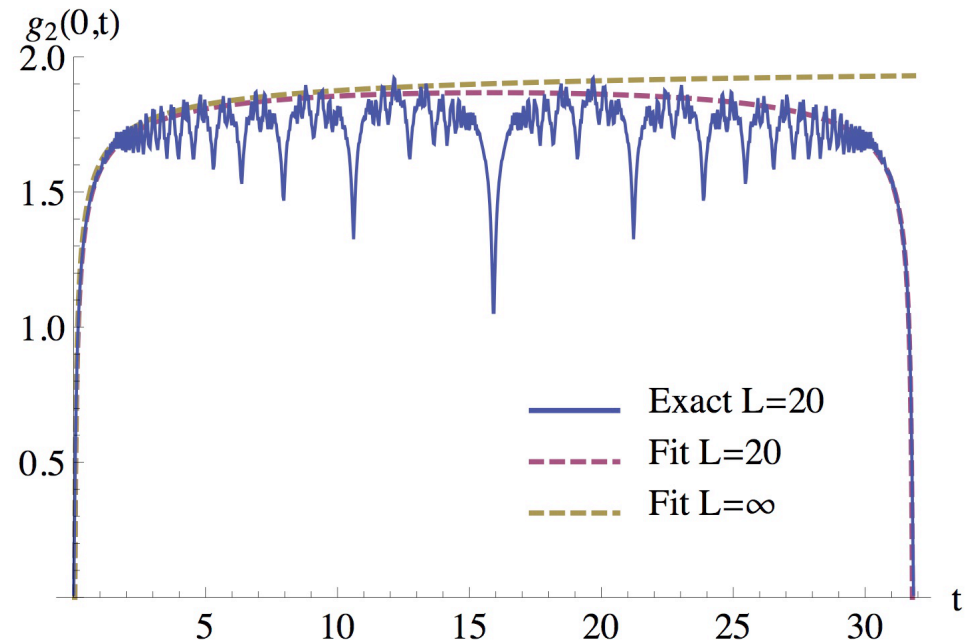


FIG. 4. Complete and partial revival of $g_2(x = 0, t)$ compared with the fits for a system of finite size and infinite size, while keeping the density fixed ($N/L = 1$).

$$\begin{aligned}
 g_2(x, t; c) &= g_{2,\text{GGE}}(x; c) + \frac{g_{2,\text{cor}}(x; c)}{|t_{\text{rev}} \sin(\pi t/t_{\text{rev}})|^{\alpha(c)}} \\
 &\rightarrow g_{2,\text{GGE}}(x; c) + \frac{g_{2,\text{cor}}(x; c)}{(\pi t)^{\alpha(c)}} \quad \text{for } L \rightarrow \infty \\
 &\rightarrow g_{2,\text{GGE}}(x; c) \quad \text{for } t \rightarrow \infty
 \end{aligned}$$

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- Problem studied numerically by: [Mossel & Caux](#) (New J. Phys. 14 (2012) 075006)
- GGE valid for g_2 function as shown by fitting and extrapolation of numerics for finite size systems.
- Corrections decay with time as a power law with **exponent** determined by initial c (monotonically decreasing function).

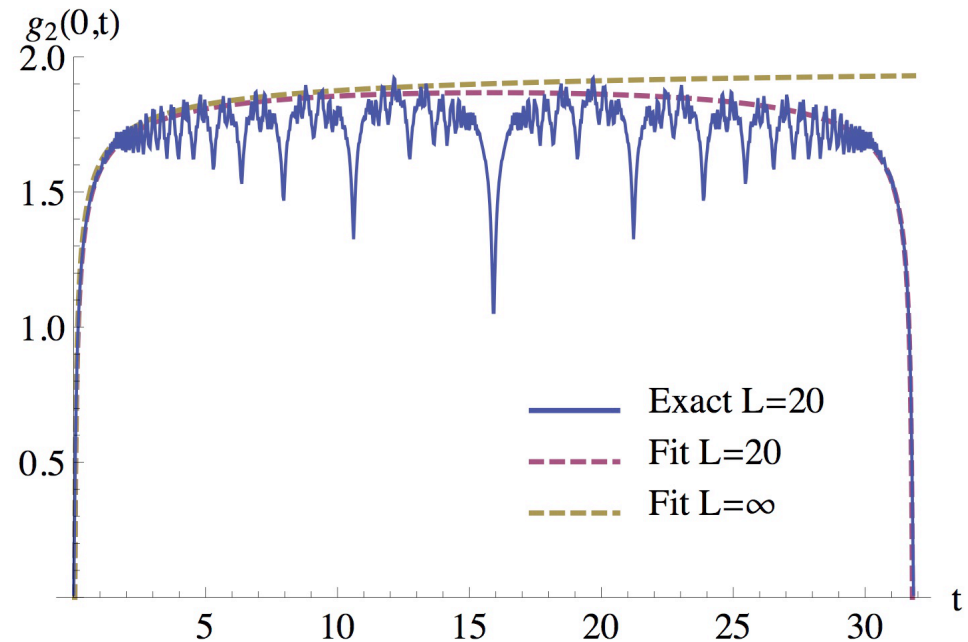


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$$\rightarrow g_{2,\text{GGE}}(x; c) \quad \text{for } t \rightarrow \infty$$

Lieb-Liniger model: $c > 0 \rightarrow c = 0$

The model

- **Lieb-Liniger** model:
System of **non-relativistic** bosons interacting through **point-like** interactions.
- Exactly solvable (integrable) by means of **Bethe Ansatz**.
Low energy physics described by **Luttinger Liquid**.

- Interacting pre-quench Hamiltonian

$$H_0 = \int_0^L dx \left(\partial_x \Psi^\dagger(x) \partial_x \Psi(x) + c \Psi^\dagger(x) \Psi^\dagger(x) \Psi(x) \Psi(x) \right)$$

- Non-interacting post-quench Hamiltonian

$$H = \int_0^L dx \partial_x \Psi^\dagger(x) \partial_x \Psi(x) = \sum_{k=-\infty}^{+\infty} k^2 \Psi_k^\dagger \Psi_k$$

- Bosonic fields in Fourier space

$$\Psi_k = \int_0^L \frac{dx}{\sqrt{L}} e^{-ikx} \Psi(x), \quad \Psi(x) = \frac{1}{\sqrt{L}} \sum_{k=-\infty}^{+\infty} e^{+ikx} \Psi_k$$

- Time evolution

$$\Psi_k(t) = e^{iHt} \Psi_k(0) e^{-iHt} = \Psi_k(0) e^{-ik^2 t}$$

Lieb-Liniger model: $c > 0 \rightarrow c = 0$

g_1 function

- Time evolution:
The g_1 function is time independent

$$g_1(x; t) = \langle \Omega | \Psi^\dagger(0; t) \Psi(x; t) | \Omega \rangle = \frac{1}{L} \sum_{k=-\infty}^{+\infty} e^{ikx} \langle n_k \rangle_0$$

- GGE prediction

$$g_{1,\text{GGE}}(x) = \langle \Psi^\dagger(0) \Psi(x) \rangle_{\text{GGE}} = \frac{1}{L} \sum_{k=-\infty}^{+\infty} e^{ikx} \langle n_k \rangle_{\text{GGE}}$$

- GGE is trivially valid for the g_1 function

$$g_{1,\text{GGE}}(x) = g_1(x; t \rightarrow \infty) = g_1(x; 0)$$

Lieb-Liniger model: $c > 0 \rightarrow c = 0$

g_2 function

- Time evolution

$$\begin{aligned} g_2(x; t) &= \langle \Omega | \Psi^\dagger(x; t) \Psi^\dagger(0; t) \Psi(x; t) \Psi(0; t) | \Omega \rangle \\ &= \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int_{-\infty}^{+\infty} dz e^{-ikz} \langle \Omega | \Psi^\dagger(x - 2kt; 0) \Psi^\dagger(z; 0) \Psi(x - 2kt + z; 0) \Psi(0; 0) | \Omega \rangle \end{aligned}$$

- Expanding initial correlations in connected and disconnected parts

$$\begin{aligned} &\langle \Omega | \Psi^\dagger(x - 2kt; 0) \Psi^\dagger(z; 0) \Psi(x - 2kt + z; 0) \Psi(0; 0) | \Omega \rangle \\ &= |g_1(z; 0)|^2 + |g_1(x - 2kt; 0)|^2 + G_{\text{conn}}^{(4)}(x - 2kt, z, x - 2kt + z, 0; 0) \end{aligned}$$

- Contribution of last term decays due to cluster decomposition.
Large time decay determined by long distance decay of initial connected correlations (interacting ground state correlations).

- The latter is given by **Luttinger Liquid** theory

$$G_{\text{conn}}^{(4)}(x - 2kt, z, x - 2kt + z, 0; 0) \sim \rho_0^2 \left(\left| \frac{1}{z^2} - \frac{1}{(x - 2kt)^2} \right|^{1/(2K)} - \frac{1}{|z|^{1/K}} - \frac{1}{|x - 2kt|^{1/K}} \right)$$

where K is the effective Luttinger parameter corresponding to initial interaction c .

Lieb-Liniger model: $c > 0 \rightarrow c = 0$

g_2 function

- Time evolution

$$\begin{aligned} g_2(x; t) &= \langle \Omega | \Psi^\dagger(x; t) \Psi^\dagger(0; t) \Psi(x; t) \Psi(0; t) | \Omega \rangle \\ &= \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int_{-\infty}^{+\infty} dz e^{-ikz} \langle \Omega | \Psi^\dagger(x - 2kt; 0) \Psi^\dagger(z; 0) \Psi(x - 2kt + z; 0) \Psi(0; 0) | \Omega \rangle \end{aligned}$$

- Expanding in connected and disconnected parts

$$\begin{aligned} &\langle \Omega | \Psi^\dagger(x - 2kt; 0) \Psi^\dagger(z; 0) \Psi(x - 2kt + z; 0) \Psi(0; 0) | \Omega \rangle \\ &= |g_1(z; 0)|^2 + |g_1(x - 2kt; 0)|^2 + G_{\text{conn}}^{(4)}(x - 2kt, z, x - 2kt + z, 0; 0) \end{aligned}$$

- Contribution of last term decays due to cluster decomposition. Large time decay determined by long distance decay of initial connected correlations (interacting ground state correlations).
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where K is the effective Luttinger parameter corresponding to initial interaction c .

Lieb-Liniger model: $c > 0 \rightarrow c = 0$

g_2 function

- Therefore the asymptotic value of g_2 function is equal to the GGE prediction

$$g_{2,GGE}(x) = \langle \Psi^\dagger(x) \Psi^\dagger(0) \Psi(x) \Psi(0) \rangle_{GGE} = (g_{1,GGE}(0))^2 + |g_{1,GGE}(x)|^2$$

while the time dependent part decays as a power law with exponent $-1/(2K)$

$$\sim A(K) \rho_0^2 t^{-1/(2K)} \rightarrow 0 \quad 1 < K < \infty$$

Conclusions

- The GGE conjecture is trivially satisfied for 2-point correlations.
- The GGE conjecture is correct for free-to-free quantum quenches, as a consequence of **Wick's theorem**.
- The GGE conjecture is also correct more generally for interacting-to-free quantum quenches, as a consequence of the physical principle of **cluster decomposition** satisfied by the initial state.
- The same is true for any other initial state or ensemble that satisfies the cluster decomposition property: excited states (*see next talk*), thermal ensembles etc.
- On the contrary, arbitrary states that do not satisfy this physical property are not expected to be described by the GGE, even if they equilibrate: memory of initial correlations of charges survives at long times.

Loss of memory of non-Gaussian initial correlations under Gaussian unitary evolution

*Thank you
for your attention*



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