Generalised Unitarity for Dimensionally Regulated Amplitudes

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Based on: **R. Fazio, P. Mastrolia, E. Mirabella, and W.T., 1404.4783 [hep-ph]**

New Frontiers of Theoretical Physics - May 28th

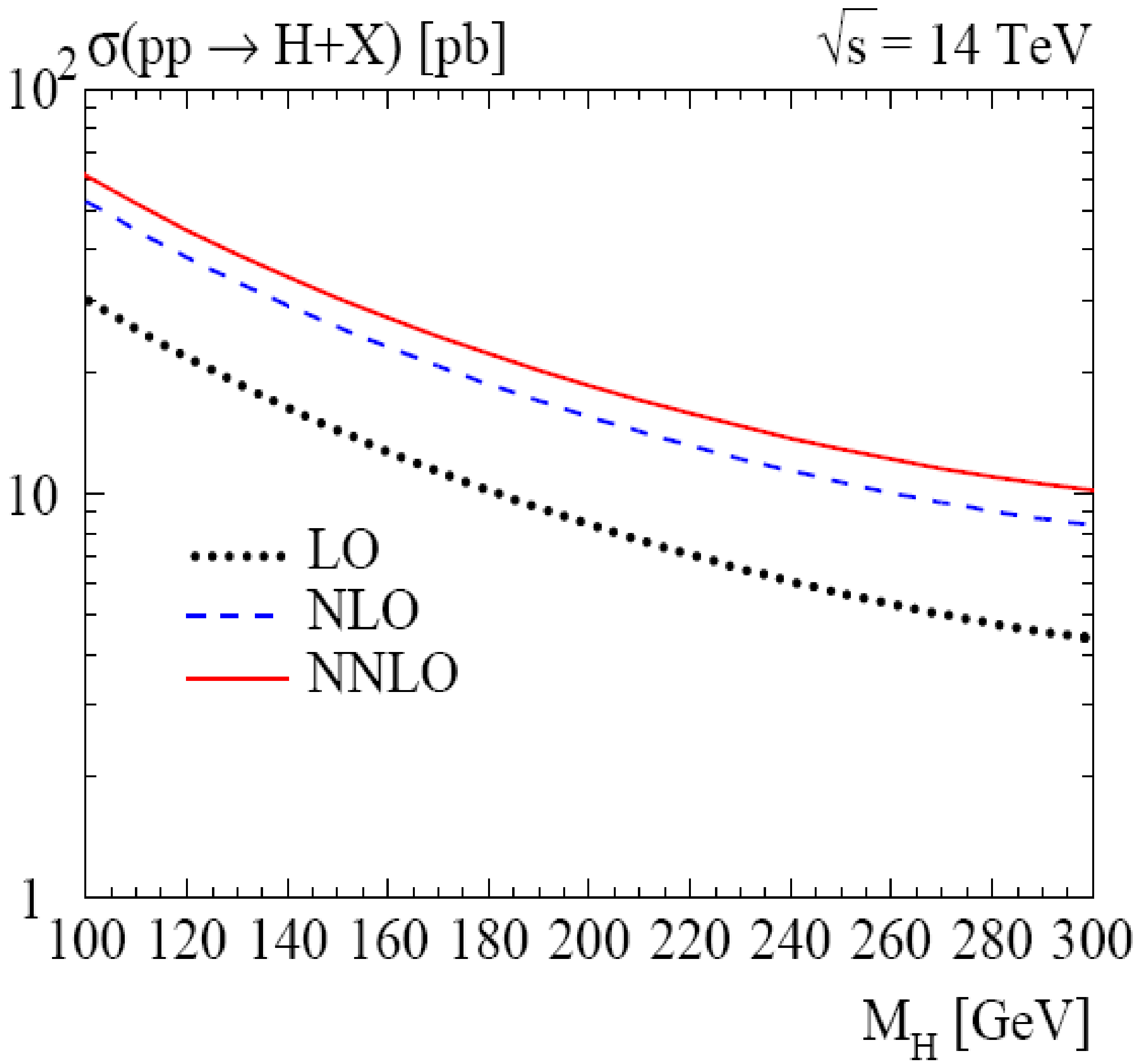
Outline

- The NLO revolution
- Color decomposition & Spinor-helicity formalism
- One-loop amplitudes
- Four Dimensional Formulation of Dimensional Regularisation
- D-dimensional Generalised Unitarity
- NLO QCD corrections to Higgs to partons
- Conclusions

What is the problem?

The on-shell methods are important in the LHC physics as a tool to compute the NLO Standard Model processes to extract new physics from the experimental results.

[Harlander, Kilgore (2002); Anastasiou, Melnikov (2002)]



- Tree-level (LO) predictions are qualitative due to the poor convergence of the truncated expansion at strong coupling.

$$\alpha_S\,(100\mathrm{GeV}) \sim 0.12$$

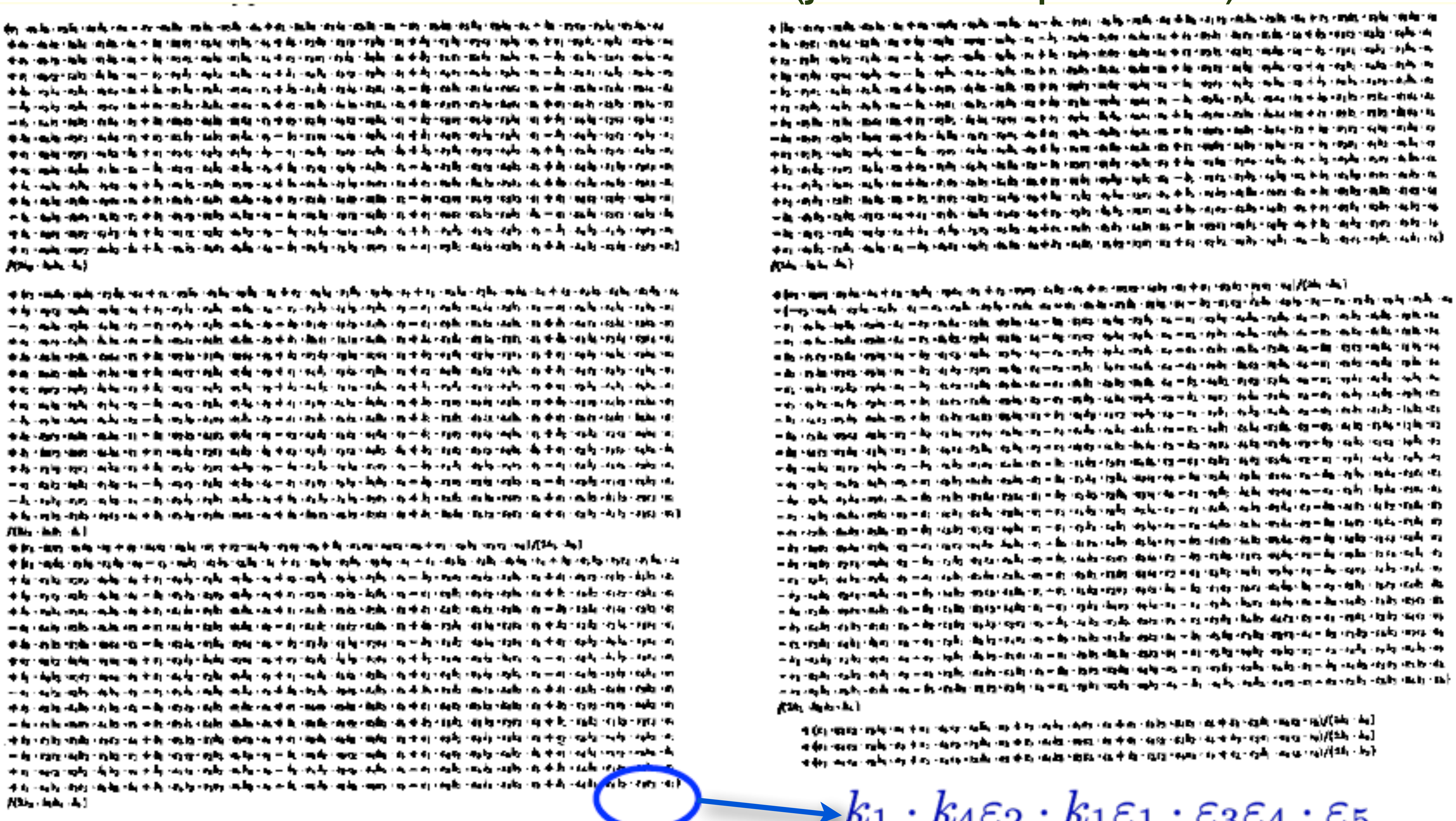
- K factors

$$K = \frac{\mathrm{NLO}}{\mathrm{LO}} \sim 30\% \div 80\%$$

What is the problem?

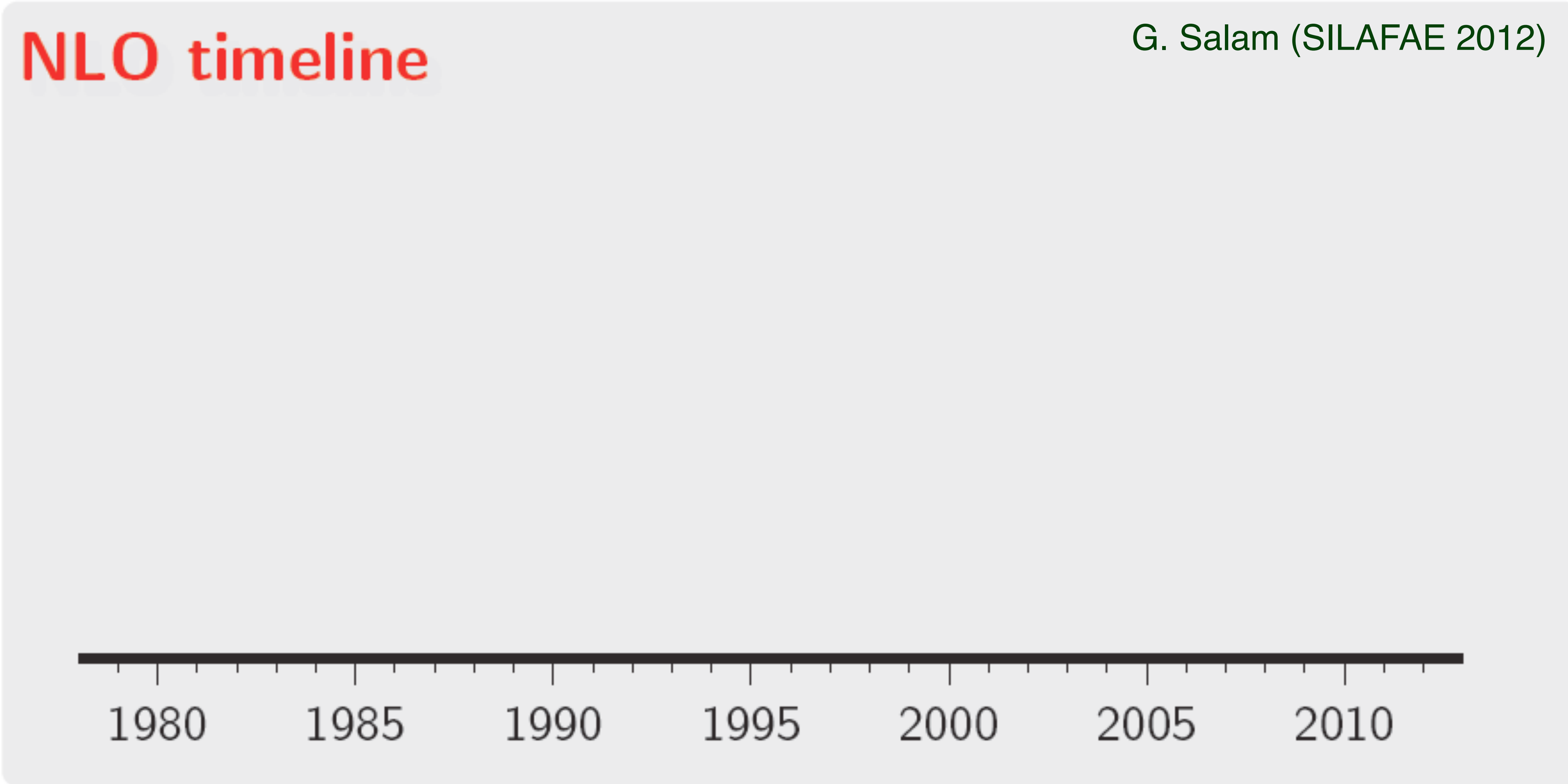
- Calculations using Feynman diagrams are redundant
- A factorial growth in the number of terms

Result of a brute force calculation (just small part of it)



$$k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_1 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5$$

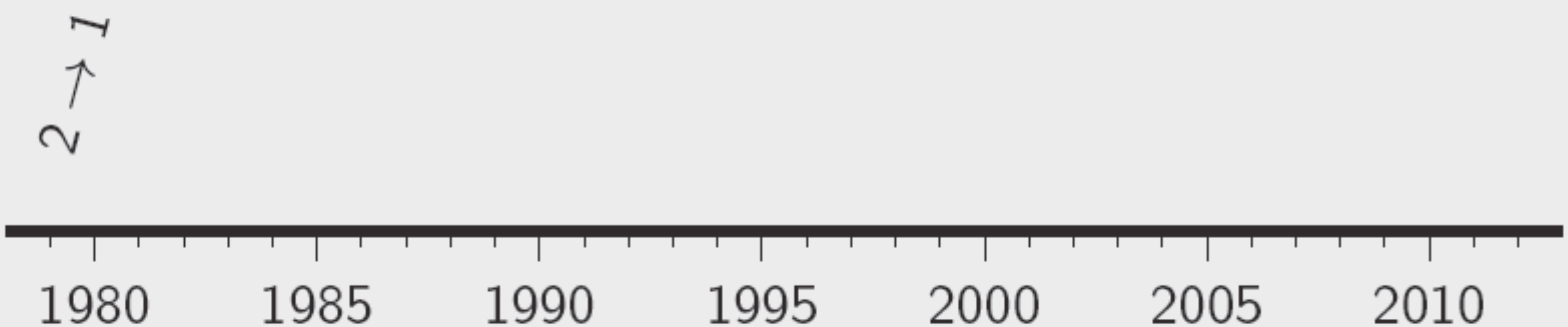
NLO REVOLUTION



NLO REVOLUTION

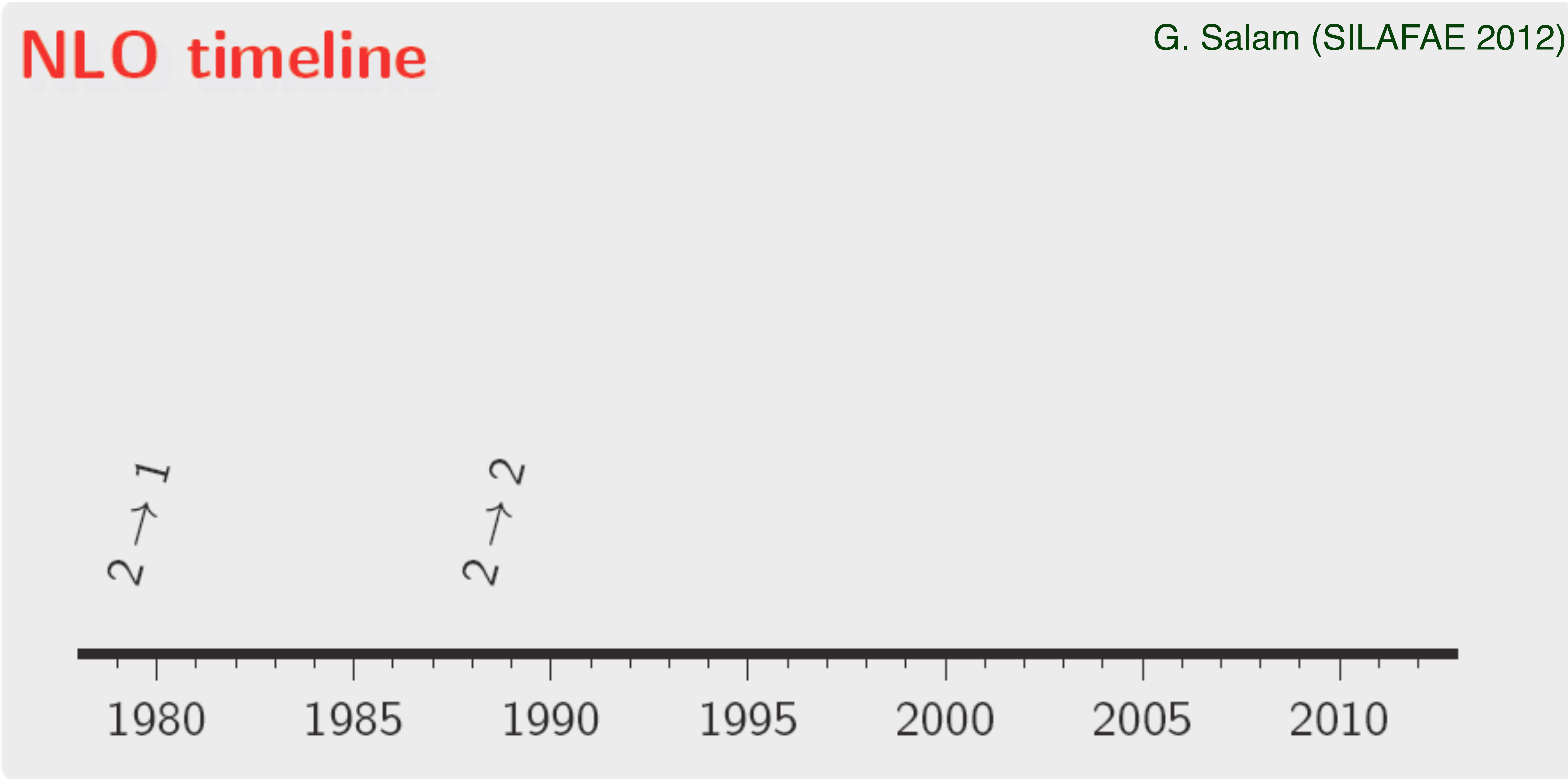
NLO timeline

G. Salam (SILAFAE 2012)



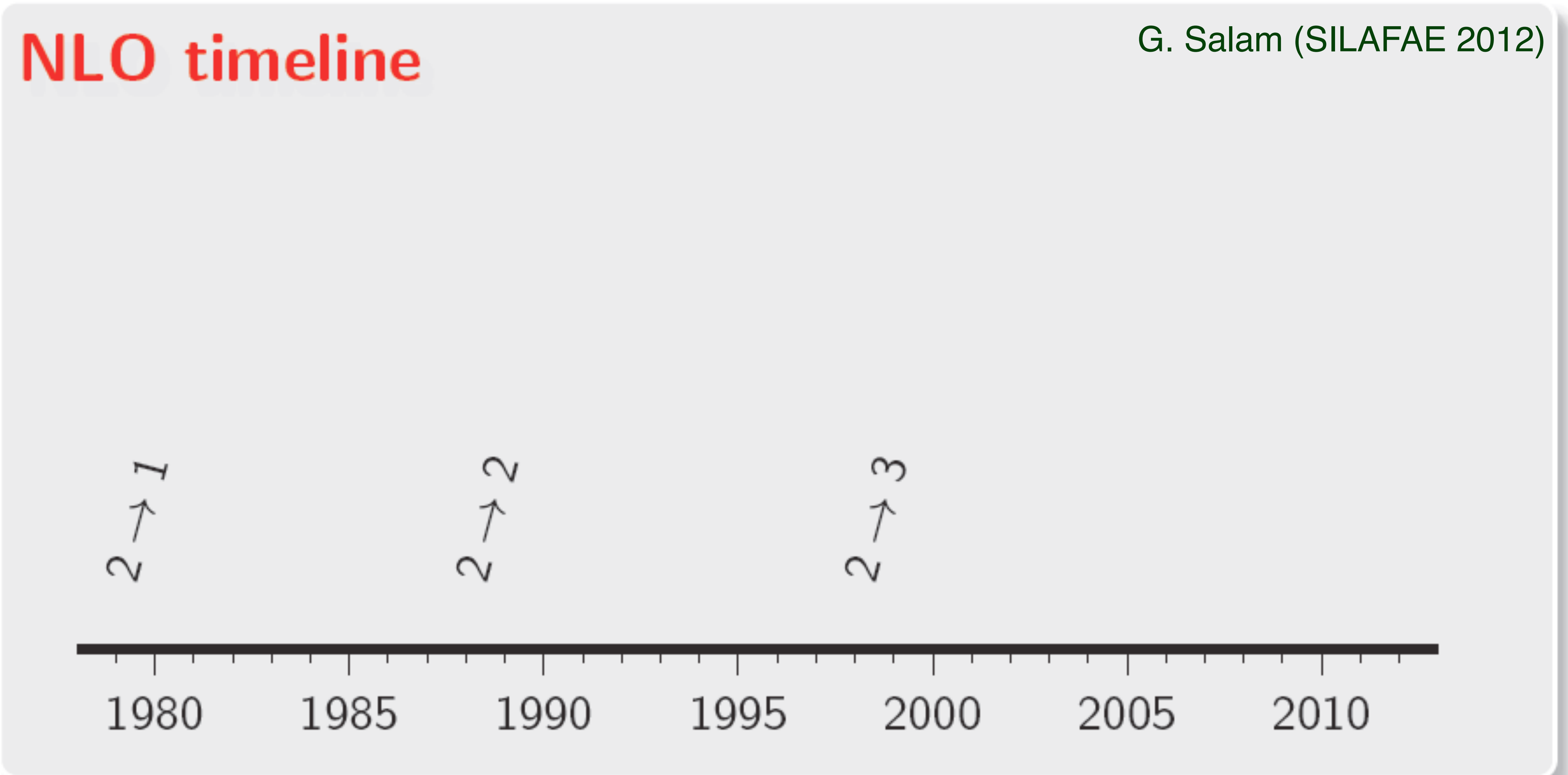
1979: NLO Drell-Yan [Altarelli, Ellis & Martinelli]
1991: NLO $gg \to$ Higgs [Dawson; Djouadi, Spira & Zerwas]

NLO REVOLUTION



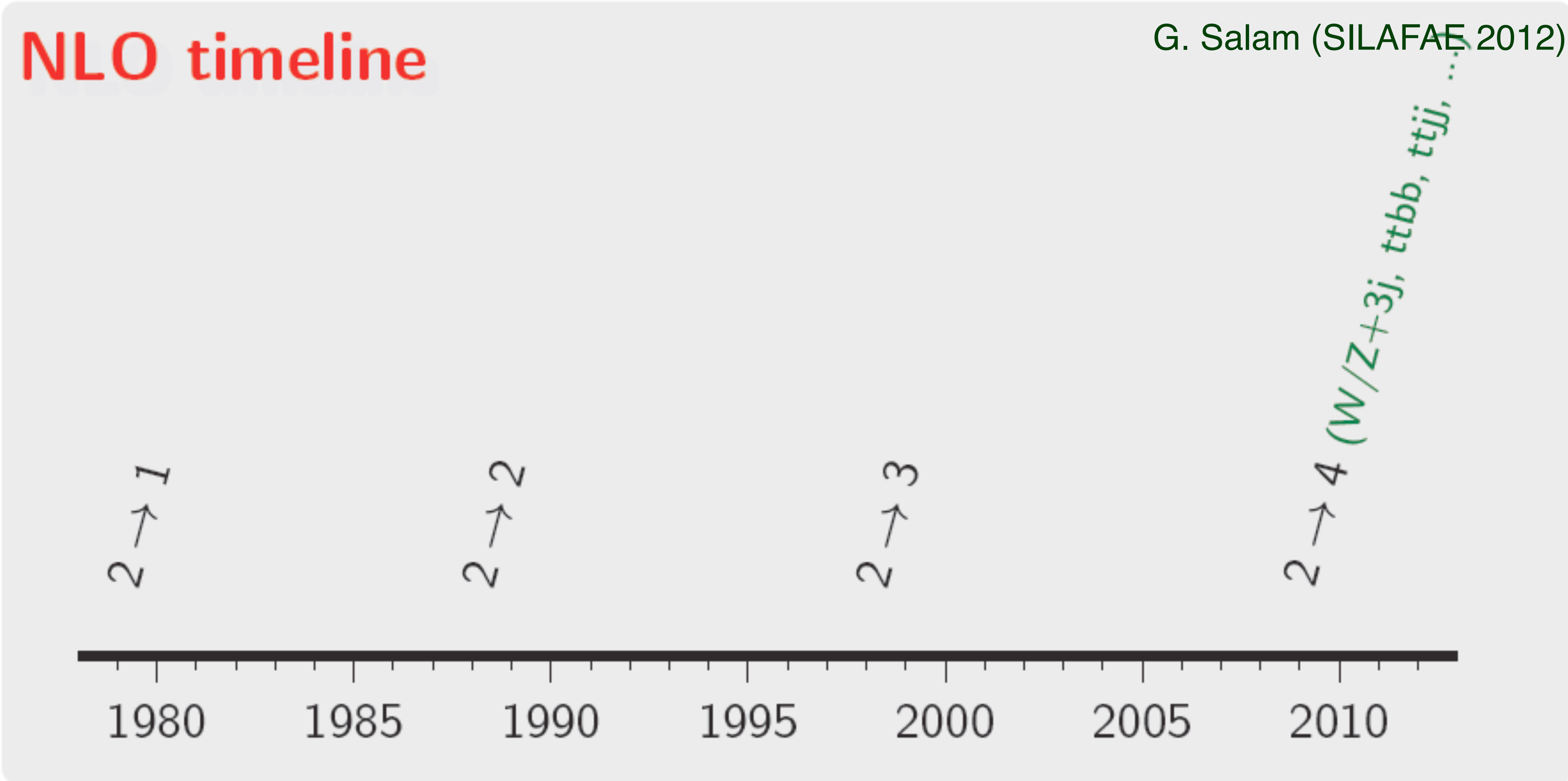
1987: NLO high-p_t photoproduction [Aurenche et al]
1988: NLO $b\bar{b}$, $t\bar{t}$ [Nason et al]
1988: NLO dijets [Aversa et al]
1993: Vj [JETRAD, Giele, Glover & Kosower]

NLO REVOLUTION



1998: NLO $Wb\bar{b}$ [MCFM: Ellis & Veseli]
2000: NLO $Zb\bar{b}$ [MCFM: Campbell & Ellis]
2001: NLO $3j$ [NLOJet++: Nagy]
...
2007: NLO $t\bar{t}j$ [Dittmaier, Uwer & Weinzierl '07]
...

NLO REVOLUTION



2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]

2009: NLO $W+3j$ [BlackHat+Sherpa: Berger et al] [unitarity]

2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]

2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]

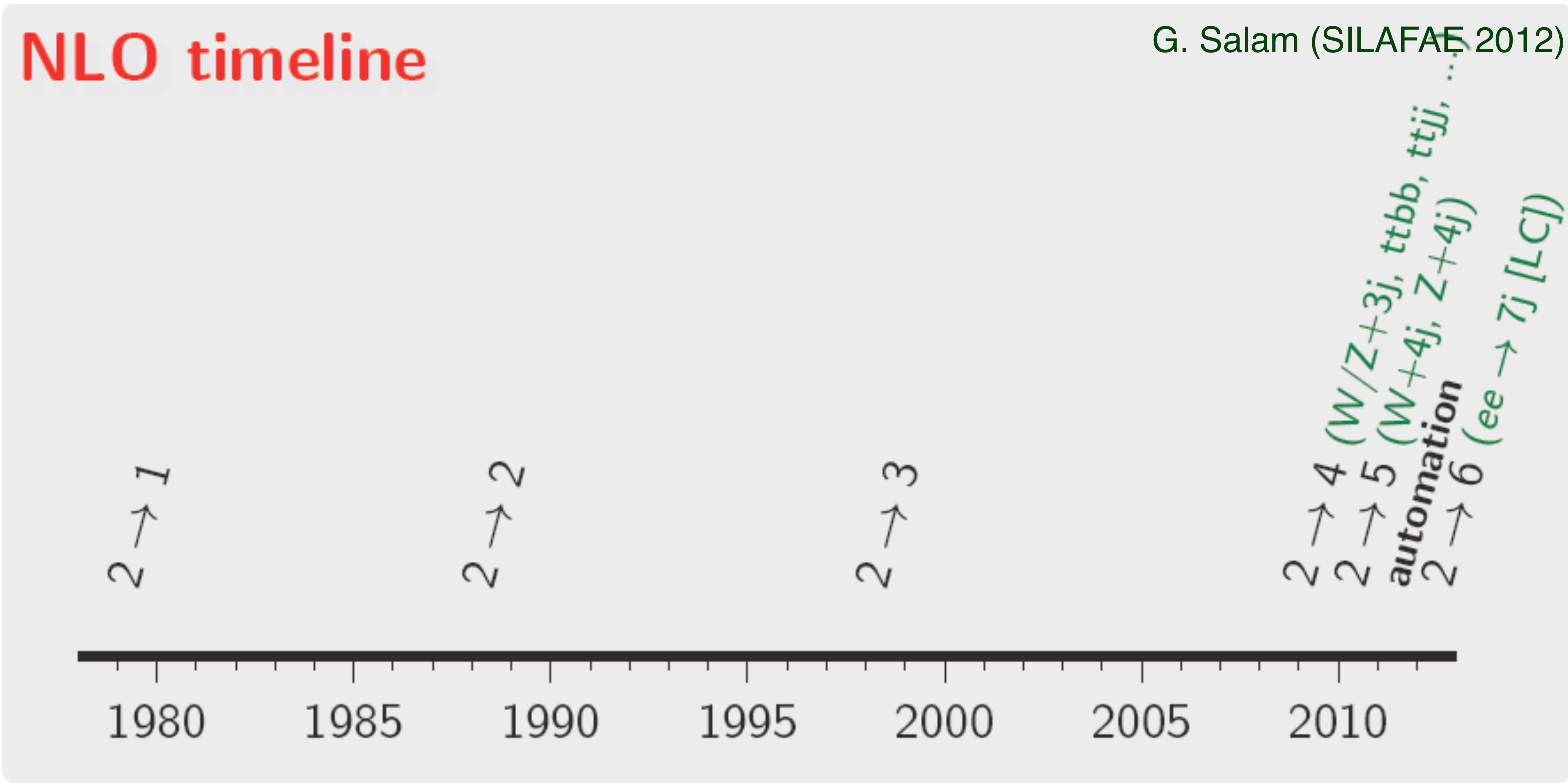
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]

2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]

2010: NLO $Z+3j$ [BlackHat+Sherpa: Berger et al] [unitarity]

...

NLO REVOLUTION



2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al] [unitarity]
2011/12: NLO $WWjj$ [Rocket: Melia et al; GoSaM+MadX Greiner et al] [unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al] [unitarity]
2011/12: NLO $4j$ [BlackHat/NGluons+Sherpa: Bern et al; Badger et al] [unitarity]
2011–: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011–: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011–: first automation [GoSam: Cullen et al] **(See Peraro's talk)** [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]

Color Decomposition & Spinor-Helicity Formalism

Color Decomposition

At tree-level

For the **n**-gluon tree-level amplitude, the color decomposition is

$$\boxed{\mathcal{A}_{\rm n}^{\rm tree}\left(\{k_i,a_i,h_i\}\right)=g^{n-2}\,{\rm Tr}\left(T^{a_1}T^{a_2}\cdots T^{a_n}\right)A_{\rm n}^{\rm tree}\left(1^{h_1},2^{h_2},\ldots,n^{h_n}\right)\quad + \text{ all non-cyclic permutations}}$$

Momenta

Color

Helicities

Color-ordered primitive amplitudes
only depends on Lorentz variables

Similarly, the **(n-2)**-gluon with 2 external quarks tree-level amplitude can be reduced to single strings of generators T^a in fundamental representation.

$$\boxed{\mathcal{A}_{\rm n}^{\rm tree}\left(\{k_i,a_i,h_i\}\right)=g^{n-2}\left(T^{a_1}T^{a_2}\cdots T^{a_n}\right)_{i_1}^{\bar{j}_n}A_{\rm n}^{\rm tree}\left(1_q^{h_1},2^{h_2},\ldots,n_{\bar{q}}^{h_n}\right)\quad + \text{ all non-cyclic permutations}}$$

Color Decomposition

At one-loop

For the **n**-gluon one-loop amplitude, the color decomposition is

Momenta

Helicities

Color

$$\mathcal{A}_n^{1-loop}(\{k_{i,}, h_i, a_i\}) =$$

$$= g^n \left[\sum_{\sigma \in S_n/Z_n} N_c \mathrm{Tr}\left(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}\right) A_{n;1}\left(\sigma\left(1^{h_1}\right), \ldots, \sigma\left(n^{h_n}\right)\right) \right.$$

$$+ \sum_{c=2}^{\lfloor n/2 \rfloor + 1} \sum_{\sigma \in S_n/S_{n;c}} \mathrm{Tr}\left(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(c-1)}}\right) \mathrm{Tr}\left(T^{a_{\sigma(c)}} \cdots T^{a_{\sigma(n)}}\right) A_{n;c}\left(\sigma\left(1^{h_1}\right), \ldots, \sigma\left(n^{h_n}\right)\right)$$

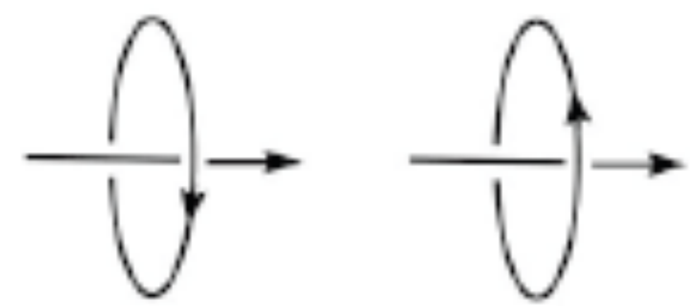
Color-ordered one-loop primitive amplitudes
only depends on Lorentz variables

Spinor-Helicity Formalism

Powerful formalism in 4D to write compact amplitudes in terms of 4D spinor products.

For a massless fermion of momentum p there are two solutions to the Dirac equation, spinors for right- and left- handed fermions

$$\boxed{\begin{aligned} U_R(p) &= \begin{pmatrix} 0 \\ u_R(p) \end{pmatrix} \\ U_L(p) &= \begin{pmatrix} u_L(p) \\ 0 \end{pmatrix} \end{aligned}}$$



Massless quarks, gluons, photons in $D = 4$ have two helicity states,

[Kleiss and Stirling (1985)]
[Xu, Zhang, Chang (1987)]
[Gastmans, Wu (1990)]

Spinor Representation

$$\bar{u}_L(p_i) = \langle i \qquad u_L(p_i) = i]$$
$$\bar{u}_R(p_i) = [i \qquad u_R(p_i) = i\rangle$$

Identities

$\langle ij\rangle = \bar{u}_L(p_i)u_R(p_j)$ $[ij] = \bar{u}_R(p_i)u_L(p_j)$ $\langle ij\rangle[ji] = s_{ij} = (p_i + p_j)^2$ $\langle ij\rangle\,[jk] = \langle i\,\lvert j\rvert\,k] = [k\,\lvert j\rvert\,i\rangle$	$[ii] = \langle ii\rangle = 0$ $\langle i\,\lvert j\rvert\,k] = [k\,\lvert j\rvert\,i\rangle$ $\langle ik] = [ki\rangle = 0$ $\langle ij\rangle = -\langle ji\rangle$ $[ij] = -[ji]$

Spinor-Helicity Formalism

[Berends, Kleiss, De Causmaecker, Gastmans, Wu (1981)]
[De Causmaecker, Gastmans, Troost, Wu (1982)]
[Xu, Zhang, Chang (1984)]

Polarisation Vectors

$$\varepsilon_+^{\mu}(k;q) = \frac{\langle q|\gamma^\mu|k]}{\sqrt{2}\langle qk\rangle}$$

$$\varepsilon_-^{\mu}(k;q) = -\frac{[q|\gamma^\mu|k\rangle}{\sqrt{2}[qk]}$$

$$\varepsilon_i^{\pm}\cdot k = 0$$ (required transversality)

$$\varepsilon_i^{\pm}\cdot q = 0$$ (Bonus)

$$\varepsilon_+^{\mu}\varepsilon_+^{*\nu} + \varepsilon_-^{\mu}\varepsilon_-^{*\nu} = -g^{\mu\nu} + \frac{k^\mu q^\nu + q^\mu k^\nu}{q\cdot k}$$ (Polarisation sum)

Are defined in terms of both the momentum vector ***k*** and an arbitrary reference vector ***q***.

Polarisation vectors for states of helicity +1 or -1

Under azimuthal rotation about k_i axis, helicity +1/2 $|i\rangle \to |i'\rangle = e^{i\phi/2}|i\rangle$

helicity -1/2 $|i] \to |i'] = e^{-i\phi/2}|i]$

and the polarisation vectors with helicity $\pm$

$$\varepsilon_+^{\mu}(i) \to \frac{\langle i'|\gamma^\mu|q]}{\sqrt{2}[qi']} = e^{i\phi}\varepsilon_+^{\mu}(i)$$

$$\varepsilon_-^{\mu}(i) \to \frac{[i'|\gamma^\mu|q\rangle}{\sqrt{2}[qi']} = e^{-i\phi}\varepsilon_-^{\mu}(i)$$

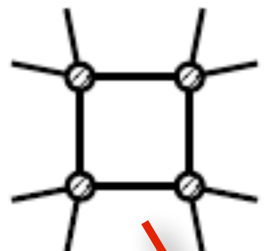
One-loop Amplitudes

Generalised Unitarity: isolate the leading discontinuity

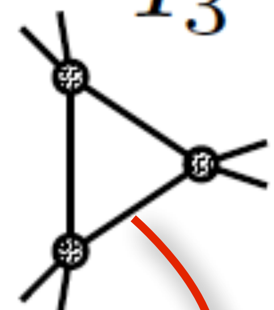
From Passarino-Veltman reduction theorem any one-loop amplitude in D=4 of massless degrees of freedom can be decomposed as:

[Passarino - Veltman (1979)]

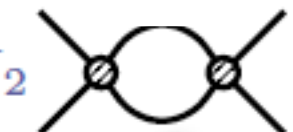
$$A_n^{(1)}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K_4} \; I_4$$



$$+ \sum_{K_3} C_{3;K_3} \; I_3$$



$$+ \sum_{K_2} C_{2;K_2} \; I_2$$



Scalar Master Integrals

Made of polylogarithmic functions

- In dimensional regularisation, the tadpole contributions arise only with internal masses.
- If an amplitude is determined by its branch cuts, it is said to be *cut-constructible*.
- All one-loop amplitudes are cut-constructible in dimensional regularisation.

Cutting $\quad n \times \dfrac{i}{p^2 + i\varepsilon} \to 2\pi\delta^{(+)}(p^2)$

n propagators are put on-shell

D-dimensional cut and rational terms

In $D = 4 - 2\epsilon$ we can do the decomposition

$$\ell^\nu = \bar{\ell}^\nu + \tilde{\ell}^\nu_{[-2\epsilon]}$$

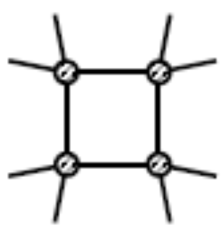
$D = 4$ $\qquad$ $D = -2\epsilon$

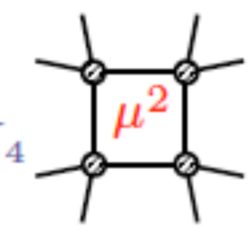
The on-shell condition

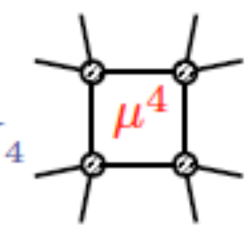
$$\ell^2 = \bar{\ell}^2 - \mu^2 = 0 \longrightarrow \bar{\ell}^2 = \mu^2$$

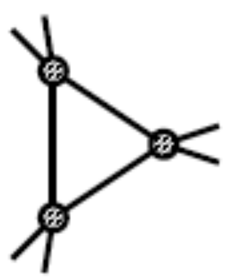
Mass term

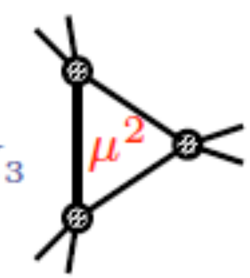
Any massless one-loop becomes

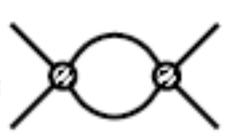
$$A_n^{(1),4-2\epsilon}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K_4}$$


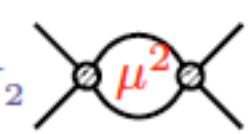
$$+ \sum_{K_4} C^{[2]}_{4;K_4}$$


$$+ \sum_{K_4} C^{[4]}_{4;K_4}$$


$$+ \sum_{K_3} C_{3;K_3}$$


$$+ \sum_{K_3} C^{[2]}_{3;K_3}$$


$$+ \sum_{K_2} C_{2;K_2}$$


$$+ \sum_{K_2} C^{[2]}_{2;K_2}$$


[Bern,Dixon,Dunbar,Kosower (1997)]
[Ossola,Papadopoulos,Pittau (2006)]
[Anastasiou, Britto, Feng, Kunszt, Mastrolia (2006)]
[Ellis, Giele,Kunszt,Melnikov (2008)]

Rational terms are extracted by using massive propagators

D-dimensional cut and rational terms

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$D = 4$ $\qquad$ $D = -2\epsilon$

The on-shell condition

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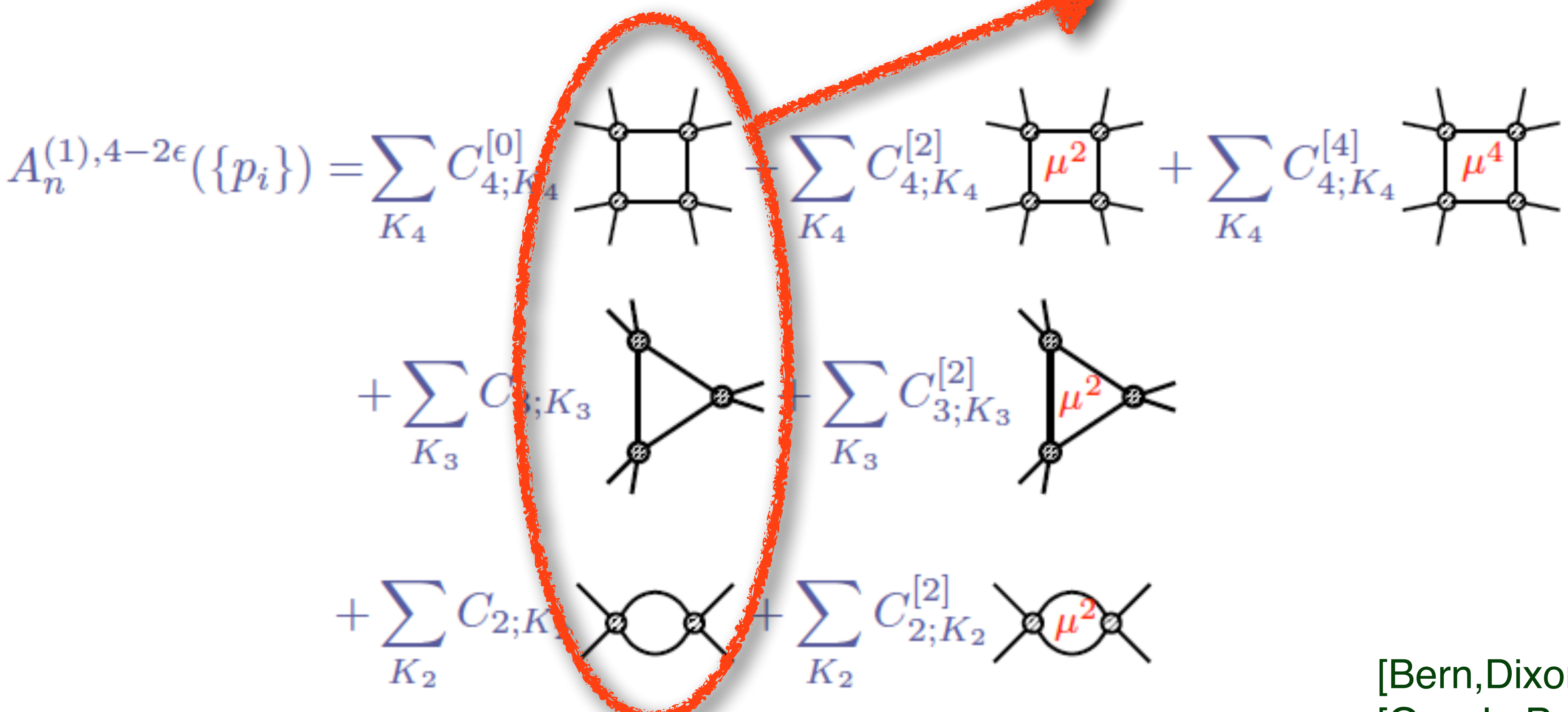
Mass term

Any massless one-loop becomes

Cut-constructible part



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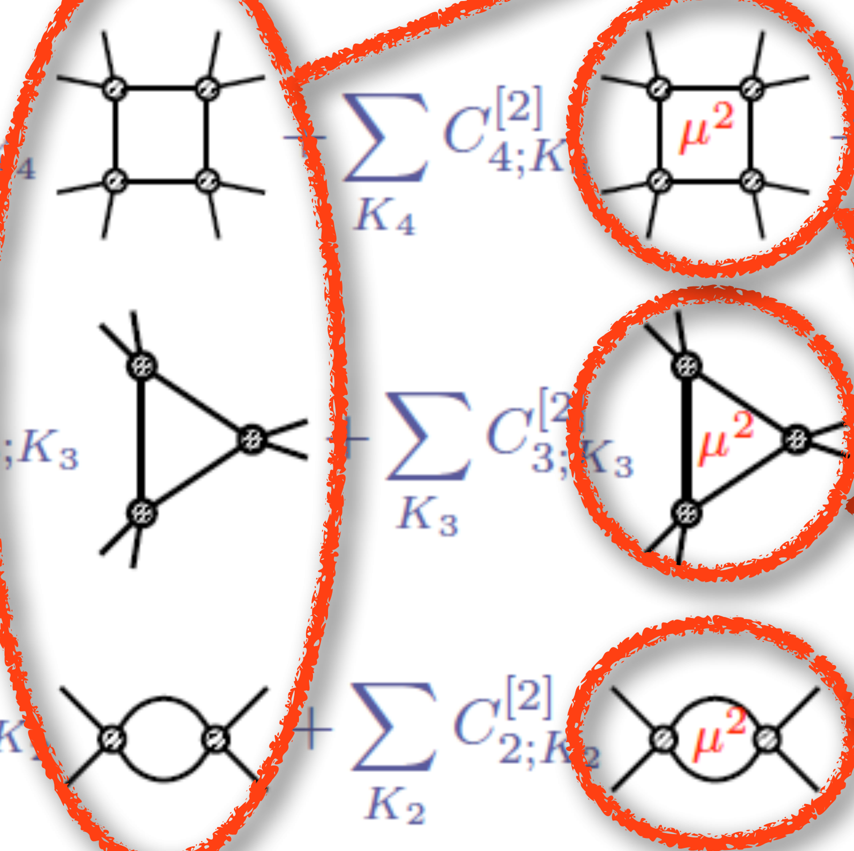

$$A_n^{(1),4-2\epsilon}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K_4} \; + \sum_{K_4} C^{[2]}_{4;K_4} \; + \sum_{K_4} C^{[4]}_{4;K_4} \; + \sum_{K_3} C_{3;K_3} \; + \sum_{K_3} C^{[2]}_{3;K_3} \; + \sum_{K_2} C_{2;K_2} \; + \sum_{K_2} C^{[2]}_{2;K_2}$$

In renormalisable theories:

Powers of loop momentum
<= # of propagators

[Bern,Dixon,Dunbar,Kosower (1997)]
[Ossola,Papadopoulos,Pittau (2006)]
[Anastasiou, Britto, Feng, Kunszt, Mastrolia (2006)]
[Ellis, Giele,Kunszt,Melnikov (2008)]

Rational terms are extracted by using massive propagators

Four Dimensional Formulation of Dimensional Regularisation (FDF)

Why should we consider a new formulation?

To compute amplitudes at1-loop and understand how to treat cuts in D-dimensions there are existing approaches

A: Separated computation of cut-constructible and rational terms

A1: Computing the rational term separately (using non gauge invariant terms)

— **R1 and R2 separation** [Ossola, Papadopoulos, Pittau(2008); Pittau, Draggiotis, Garzelli (2009)]
— **Supersymmetric decomposition** [Bern, Dixon, Kosower]

B: D-dimensional unitarity offers the determination of all pieces together

B1: 6-dimensional spinor-helicity formalism [Cheung and O'Connell(2009); Davies (2012)]

— **New rules for spinor products**
— **No automatic generator exists**

B2: Gamma algebra in extended dimension [Ellis,Giele,Kunszt,Melnikov (2008)]

— **The explicit representation of the polarisation states is avoid**
— **Gamma algebra has to be extended everywhere.**
— **Automatic generator has to be modified**

B3: Don't leave 4 dimensions! [Fazio, Mastrolia, Mirabella, WT (2014)]

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B: D-dimensional unitarity offers the determination of all pieces together

Four Dimensional Formulation of Dimensional Regularisation (FDF)

B3: Don't leave 4 dimensions! [Fazio, Mastrolia, Mirabella, WT (2014)]

— Explicit 4D representation of generalised polarisation and spinors
— 4D representation of D-reg loop propagators
— 4D Feynman rules + multiplicative Selection Rules
— Easy to implement in existing generators

The d-dimensional metric tensor can be split as

$$\bar{g}^{\mu\nu} = g^{\mu\nu} + \tilde{g}^{\mu\nu}$$

d-dimensional

4-dimensional

-2ε-dimensional

Where

$$\tilde{g}^{\mu\nu} g_{\mu\nu} = 0, \qquad \tilde{g}^{\mu}_{\mu} = -2\epsilon \xrightarrow[d\to 4]{} 0, \qquad g^{\mu}_{\mu} = 4 \qquad \tilde{q}^2 = \tilde{g}^{\mu\nu} \bar{q}_\mu \bar{q}_\nu = -\mu^2$$

Projections of the vectors q and $\tilde{q}$.

$$\tilde{q}^{\mu} g_{\mu\nu} = \tilde{g}^{\mu\sigma} \bar{q}_\sigma g_{\mu\nu} = 0$$

As well for the gamma matrices

$$[\tilde{\gamma}^{\alpha}, \gamma^5] = 0, \qquad \{\tilde{\gamma}^{\alpha}, \tilde{\gamma}^{\beta}\} = 2\,\tilde{g}^{\alpha\beta}, \qquad \{\tilde{\gamma}^{\alpha}, \gamma^{\mu}\} = 0.$$

In 4-dimension, one can infer:

$$\tilde{\gamma} \sim \gamma^5$$

And the Clifford algebra

$$\tilde{\gamma}^\mu \tilde{\gamma}_\mu \xrightarrow[d\to 4]{} 0 \qquad \text{while} \qquad \gamma^5 \gamma^5 = 1$$

Excludes any four-dimensional representation of the —2ε-subspace

—2ε-subspace → —2ε-Selection Rules (—2ε)-SRs

[Fazio, Mastrolia, Mirabella, WT (2014)]

—2ε-Selection Rules (—2ε)-SRs

[Fazio, Mastrolia, Mirabella, WT (2014)]

The d-dimensional gluon onto

- — A four-dimensional one
- — A colored scalar Sg

The Clifford algebra conditions are satisfied by imposing

$$\tilde{g}^{\alpha\beta} \to G^{AB}, \qquad \tilde{\ell}^{\alpha} \to i\,\mu\,Q^{A}\,, \qquad \tilde{\gamma}^{\alpha} \to \gamma^{5}\,\Gamma^{A}$$

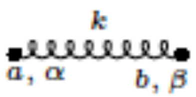
A,B := —2ε-dimensional vectorial indices traded for (—2ε)-SRs

$$\begin{aligned} G^{AB}G^{BC} &= G^{AC}, & G^{AA} &= 0, & G^{AB} &= G^{BA}, \\ \Gamma^{A}G^{AB} &= \Gamma^{B}, & \Gamma^{A}\Gamma^{A} &= 0, & Q^{A}\Gamma^{A} &= 1, \\ Q^{A}G^{AB} &= Q^{B}, & Q^{A}Q^{A} &= 1. \end{aligned}$$

—2ε-Selection Rules (—2ε)-SRs

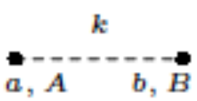
[Fazio, Mastrolia, Mirabella, WT (2014)]

Feynman Rules

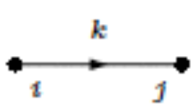


$$= -i\,\delta^{ab}\,\frac{g^{\alpha\beta}}{k^2-\mu^2+i0} \quad \text{(gluon)},$$

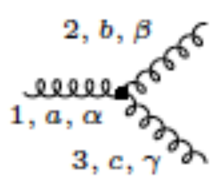
$$= i\,\delta^{ab}\,\frac{1}{k^2-\mu^2+i0} \quad \text{(ghost)},$$



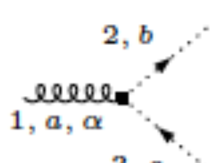
$$= -i\,\delta^{ab}\,\frac{G^{AB}}{k^2-\mu^2+i0}\,, \quad \text{(scalar)},$$



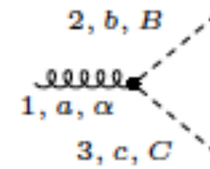
$$= i\,\delta^{ij}\,\frac{\not{k}+i\mu\gamma^5+m}{k^2-m^2-\mu^2+i0}\,,$$
(fermion),



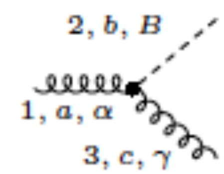
$$= -g\,f^{abc}\,\big[(k_1-k_2)^\gamma g^{\alpha\beta} + (k_2-k_3)^\alpha g^{\beta\gamma} + (k_3-k_1)^\beta g^{\gamma\alpha}\big]\,,$$



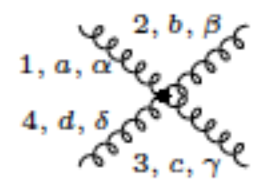
$$= -g\,f^{abc}\,k_2^\alpha\,,$$



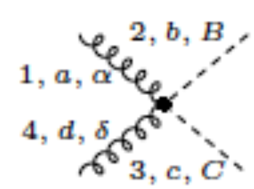
$$= -g\,f^{abc}\,(k_2-k_3)^\alpha\,G^{BC}\,,$$



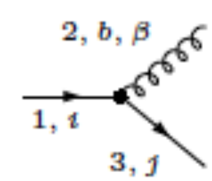
$$= \mp g\,f^{abc}\,(i\mu)\,g^{\gamma\alpha}\,Q^B\,,$$
$$(\tilde{k}_1 = 0, \quad \tilde{k}_3 = \pm\tilde{\ell})$$



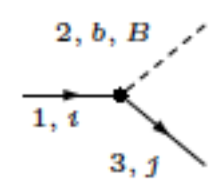
$$\begin{aligned} = -ig^2\big[& + f^{xad}\,f^{xbc}\,\left(g^{\alpha\beta}g^{\delta\gamma} - g^{\alpha\gamma}g^{\beta\delta}\right) \\ & + f^{xac}\,f^{xbd}\,\left(g^{\alpha\beta}g^{\delta\gamma} - g^{\alpha\delta}g^{\beta\gamma}\right) \\ & + f^{xab}\,f^{xdc}\,\left(g^{\alpha\delta}g^{\beta\gamma} - g^{\alpha\gamma}g^{\beta\delta}\right)\big]\,, \end{aligned}$$



$$= 2ig^2\,g^{\alpha\delta}\,\left(f^{xab}\,f^{xcd} + f^{xac}\,f^{xbd}\right)\,G^{BC}\,,$$



$$= -ig\,\left(t^b\right)_{ji}\,\gamma^\beta\,,$$



$$= -ig\,\left(t^b\right)_{ji}\,\gamma^5\,\Gamma^B\,.$$

Spinors

[Fazio, Mastrolia, Mirabella, WT (2014)]

The spinors of a d-dimensional fermion have to fulfill the completeness relation

$$\sum_{\lambda=1}^{2^{(d-2)/2}} u_{\lambda,(d)}\left(\bar{l}\right)\bar{u}_{\lambda,(d)}\left(\bar{l}\right) = \bar{l}\!\!\!/ + m$$

$$\sum_{\lambda=1}^{2^{(d-2)/2}} v_{\lambda,(d)}\left(\bar{l}\right)\bar{v}_{\lambda,(d)}\left(\bar{l}\right) = \bar{l}\!\!\!/ - m$$

The FDF allows us to express these relations as

$$\sum_{\lambda=\pm} u_{\lambda}\left(l\right)\bar{u}_{\lambda}\left(l\right) = l\!\!\!/ + i\mu\gamma^5 + m$$

$$\sum_{\lambda=\pm} v_{\lambda}\left(l\right)\bar{v}_{\lambda}\left(l\right) = l\!\!\!/ + i\mu\gamma^5 - m$$

Spinors

[Fazio, Mastrolia, Mirabella, WT (2014)]

Therefore, we can generalise the Dirac Equation

$$(\not{l} - m - i\mu\gamma^5)u_\lambda(l) = 0$$

$$l^2 = m^2 + \mu^2$$

$$\not{l} = \not{l}^\flat + \frac{l^2}{2l\cdot\bar{l}}\bar{\not{l}}, \qquad (l^\flat)^2 = (\bar{l})^2 = 0$$

Via generalised helicity spinors

$$u_-(\ell) = \left|\ell^\flat\right] + \frac{(m+i\mu)}{\langle \ell^\flat\, q_\ell\rangle}|q_\ell\rangle\,, \qquad u_+(\ell) = \left|\ell^\flat\right\rangle + \frac{(m-i\mu)}{[\ell^\flat\, q_\ell]}|q_\ell]\,,$$

$$\bar{u}_-(\ell) = \left\langle\ell^\flat\right| + \frac{(m-i\mu)}{[q_\ell\, \ell^\flat]}[q_\ell|\,, \qquad \bar{u}_+(\ell) = \left[\ell^\flat\right| + \frac{(m+i\mu)}{\langle q_\ell\, \ell^\flat\rangle}\langle q_\ell|\,,$$

Polarisation Vectors

[Fazio, Mastrolia, Mirabella, WT (2014)]

$$\sum_{i=1}^{d-1} \varepsilon^{\mu}_{i(d)}\left(\bar{l},\bar{\eta}\right)\varepsilon^{*\nu}_{i(d)}\left(\bar{l},\bar{\eta}\right) = -\bar{g}^{\mu\nu} + \frac{\bar{l}^{\mu}\bar{\eta}^{\nu}+\bar{l}^{\nu}\bar{\eta}^{\mu}}{\bar{l}\cdot\bar{\eta}}$$

We choose $\bar{\eta}^{\mu} = \bar{l}^{\mu} - \tilde{l}^{\mu}$ (gauge invariance in d-dimensions)

$$\sum_{i=1}^{d-1} \varepsilon^{\mu}_{i(d)}\left(\bar{l},\bar{\eta}\right)\varepsilon^{*\nu}_{i(d)}\left(\bar{l},\bar{\eta}\right) = \left(-g^{\mu\nu} + \frac{l^{\mu}l^{\nu}}{\mu^2}\right) - \left(\tilde{g}^{\mu\nu} + \frac{\tilde{l}^{\mu}\tilde{l}^{\nu}}{\mu^2}\right)$$

Propagator of a massive gluon

$$\left(-g^{\mu\nu} + \frac{l^{\mu}l^{\nu}}{\mu^2}\right) = \sum_{\lambda=\pm,0} \varepsilon^{\mu}_{\lambda}\left(l\right)\varepsilon^{*\nu}_{\lambda}\left(l\right)$$

Numerator of the cut propagator of the scalar Sg

$$\left(\tilde{g}^{\mu\nu} + \frac{\tilde{l}^{\mu}\tilde{l}^{\nu}}{\mu^2}\right) \longrightarrow \hat{G}^{AB} = G^{AB} - Q^{A}Q^{B}$$

Polarisation Vectors

[Fazio, Mastrolia, Mirabella, WT (2014)]

Analogous to the generalised spinors we can build
Generalised polarisation vectors for the internal lines

$$\varepsilon_+^\mu(l) = -\frac{[l^\flat|\gamma^\mu|\bar{l}\rangle}{\sqrt{2}[l^\flat \bar{l}]}, \qquad \varepsilon_-^\mu(l) = \frac{\langle l^\flat|\gamma^\mu|\bar{l}]}{\sqrt{2}\langle l^\flat \bar{l}\rangle}, \qquad \varepsilon_0^\mu(l) = \frac{l^{\flat\mu} - \bar{l}^\mu}{\mu}$$

which fulfil the well-known relations

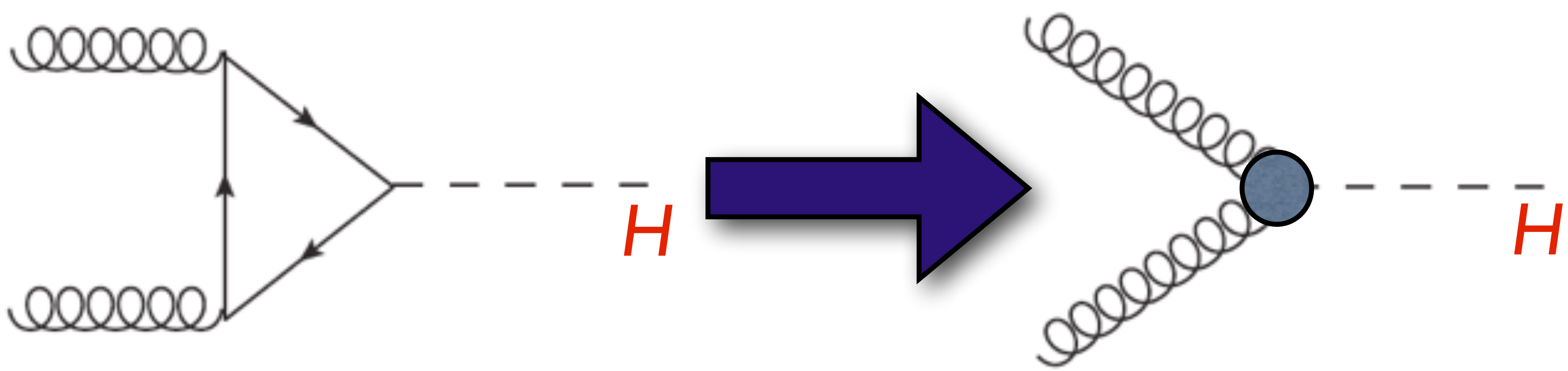
$$\begin{aligned} \varepsilon_\pm^2(\ell) &= \ \ 0\,, \\ \varepsilon_0^2(\ell) &= -1\,, \\ \varepsilon_\lambda(\ell)\cdot\ell &= \ \ 0\,. \end{aligned} \qquad \begin{aligned} \varepsilon_\pm(\ell)\cdot\varepsilon_\mp(\ell) &= -1\,, \\ \varepsilon_\pm(\ell)\cdot\varepsilon_0(\ell) &= \ \ 0\,, \end{aligned}$$

NLO QCD Corrections to Higgs to partons

- For 2 gluons $\longrightarrow$ Higgs, we use an effective operator with $m_{\text{top}} \to \infty$

[Wilczek (1977)]

$$gg \to H$$



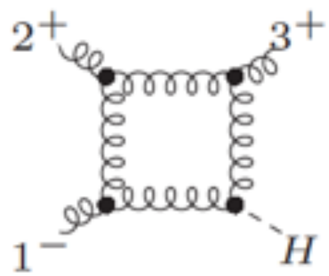
$$\mathcal{L}\left(Hgg\right) = \frac{\left(\sqrt{2}G_F\right)^{1/2}\alpha_s}{12\pi}\left(1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right) H F^a_{\mu\nu} F_a^{\mu\nu}$$

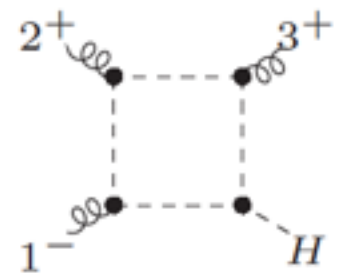
[Adler, Collins and Duncan (1977)]

$$A_4^{1-loop}\left(1^-, 2^+, 3^+, H\right)$$

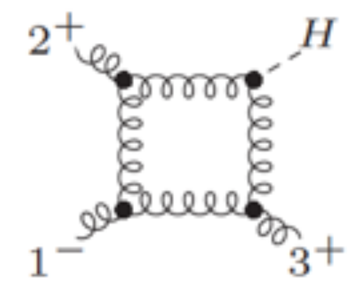
[Fazio, Mastrolia, Mirabella, WT (2014)]

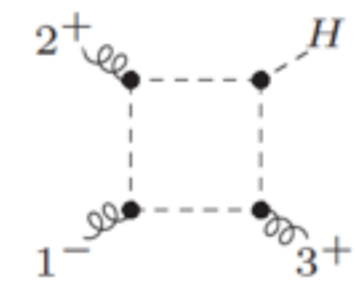
Box Contributions

$C_{1|2|3|H} =$ 

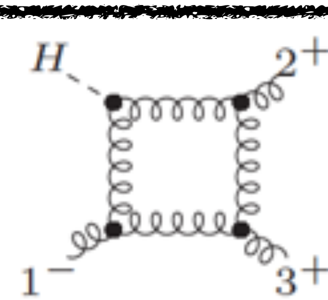
$+$ 

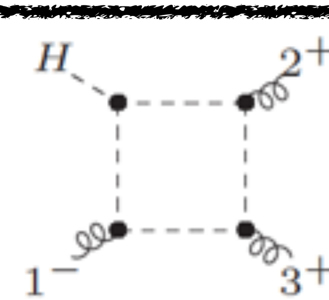
$$c_{1|2|3|H;0} = -\frac{1}{2} A_{4,H}^{\text{tree}} s_{12} s_{23}\,,$$
$$c_{1|2|3|H;4} = 0\,;$$

$C_{1|2|H|3} =$ 

$+$ 

$$c_{1|2|H|3;0} = -\frac{1}{2} A_{4,H}^{\text{tree}} s_{13} s_{12}\,,$$
$$c_{1|2|H|3;4} = 0\,;$$

$C_{1|H|2|3} =$ 

$+$ 

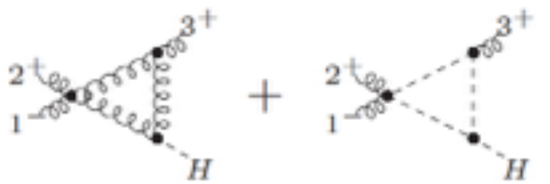
$,$

$$c_{1|H|2|3;0} = -\frac{1}{2} A_{4,H}^{\text{tree}} s_{23} s_{13}\,,$$
$$c_{1|H|2|3;4} = 0\,.$$

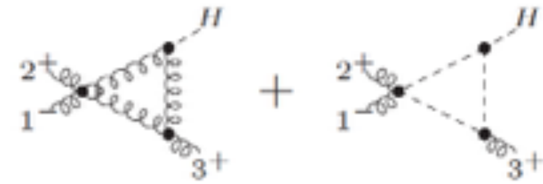
$$A_4^{1-loop}\left(1^-, 2^+, 3^+, H\right)$$

[Fazio, Mastrolia, Mirabella, WT (2014)]

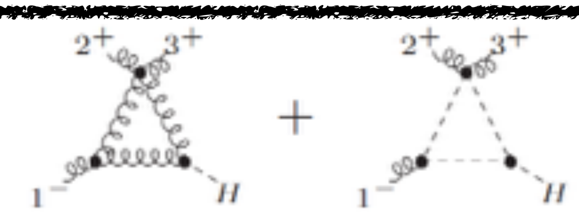
Triangle Contributions

$C_{12|3|H} =$


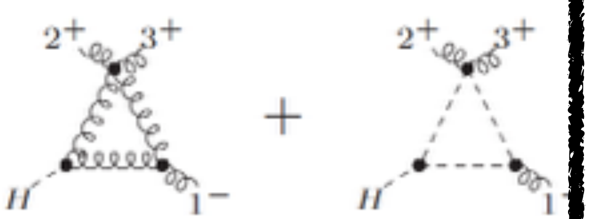
$$c_{12|3|H;0} = \frac{1}{2} A_{4,H}^{\text{tree}} \left(s_{13} + s_{23}\right),$$
$$c_{12|3|H;2} = 0;$$

$C_{12|H|3} =$


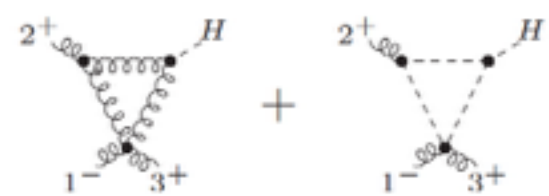
$$c_{12|H|3;0} = \frac{1}{2} A_{4,H}^{\text{tree}} \left(s_{13} + s_{23}\right),$$
$$c_{12|H|3;2} = 0;$$

$C_{1|23|H} =$


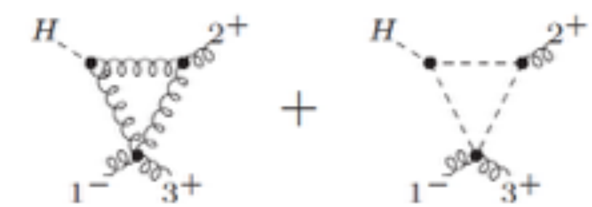
$$c_{1|23|H;0} = \frac{1}{2} A_{4,H}^{\text{tree}} \left(s_{12} + s_{13}\right)$$
$$c_{1|23|H;2} = 0;$$

$C_{1|H|23} =$


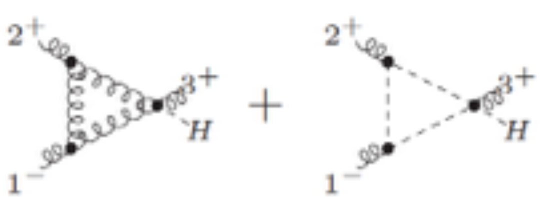
$$c_{1|H|23;0} = \frac{1}{2} A_{4,H}^{\text{tree}} \left(s_{12} + s_{13}\right),$$
$$c_{1|H|23;2} = 0;$$

$C_{2|H|31} =$


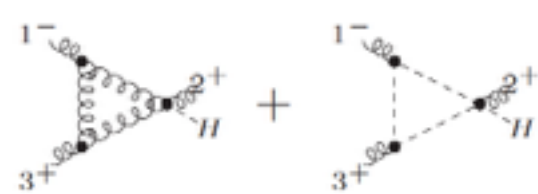
$$c_{2|H|31\,0} = \frac{1}{2} A_{4,H}^{\text{tree}} \left(s_{12} + s_{23}\right),$$
$$c_{2|H|31\,2} = 0;$$

$C_{H|2|31} =$


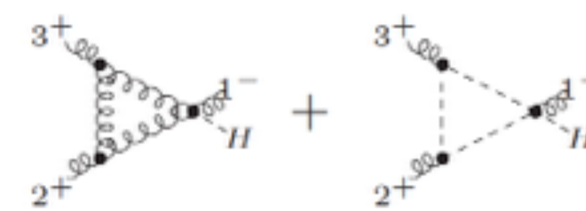
$$c_{H|2|31;0} = \frac{1}{2} A_{4,H}^{\text{tree}} \left(s_{12} + s_{23}\right),$$
$$c_{H|2|31;2} = 0;$$

$C_{1|2|3H} =$


$$c_{1|2|3H;0} = 0,$$
$$c_{1|2|3H;0} = 0;$$

$C_{1|2H|3} =$


$$c_{1|2H|3;0} = 0,$$
$$c_{1|2H|3;0} = 0;$$

$C_{1H|2|3} =$


$$c_{1H|2|3;0} = 0,$$
$$c_{1H|2|3;2} = -2A_{4,H}^{\text{tree}} \frac{s_{12}s_{13}}{s_{23}^2}.$$

$$A_4^{1-loop}(1^-, 2^+, 3^+, H)$$

[Fazio, Mastrolia, Mirabella, WT (2014)]

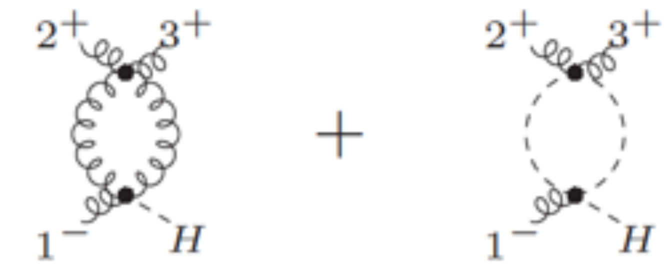
Bubble Contributions

$C_{12|3H} =$

$$c_{12|3H;\,0} = 0\,,$$

$$c_{12|3H;\,2} = 0\,;$$

$C_{23|H1} =$



$$c_{23|H1;\,0} = 0\,,$$

$$c_{23|H1;\,2} = 4A_{4,H}^{\text{tree}}\frac{s_{12}s_{13}}{s_{23}^3}\,;$$

$C_{H2|31} =$

$$c_{H2|31;\,0} = 0\,,$$

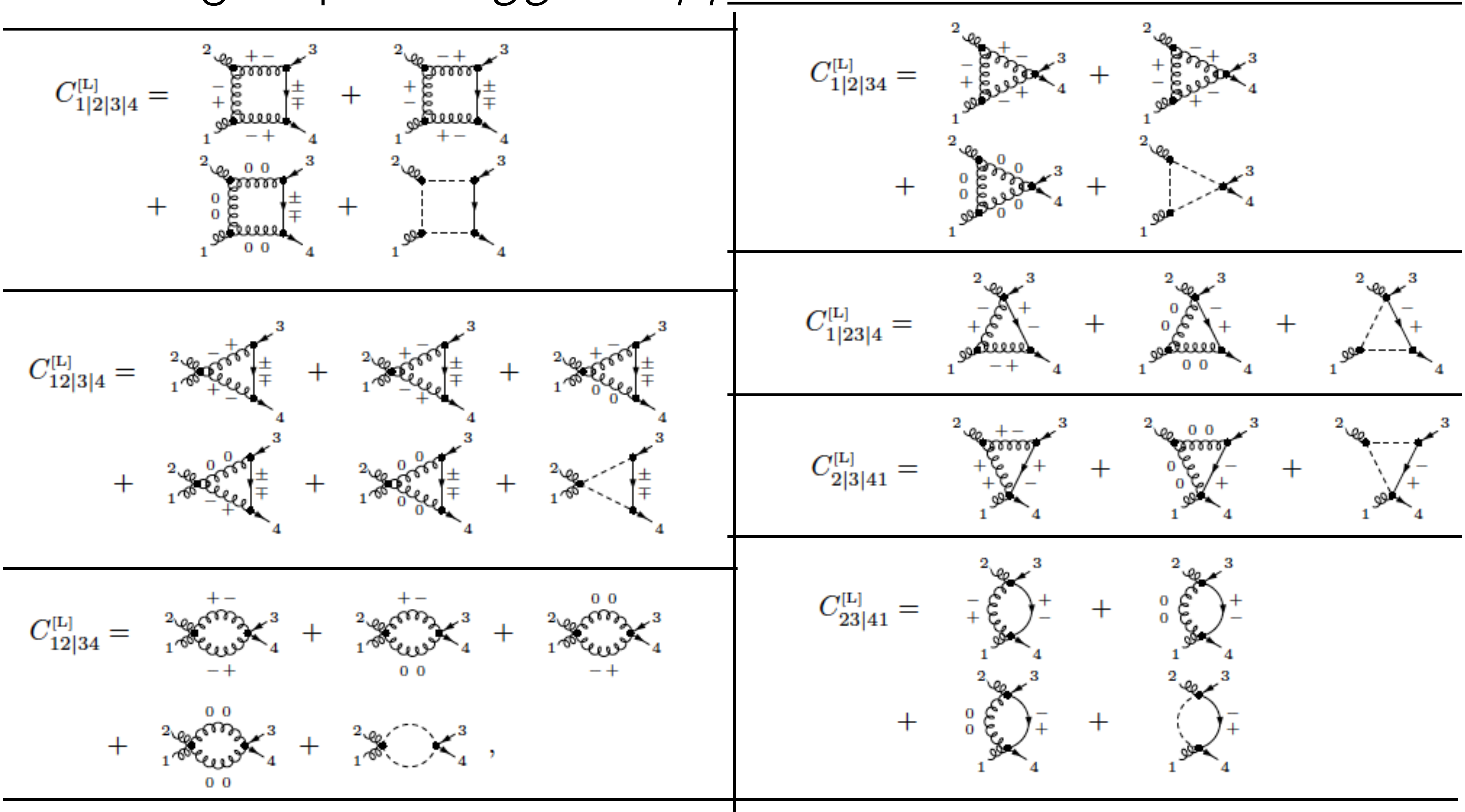
$$c_{H2|31;\,2} = 0\,.$$

The cut $C_{123|H}$ does not give any contribution

In agreement with [Schmidt (1997)]

The FDF has also been tested for the 2 —> 2 processes

Consider for instance the non-zero contributions to the left-turning amplitude $g\bar{g}$ —> qq



Conclusions and Perspectives

- A four-dimensional formulation (FDF) of dimensional regularisation has been introduced, particles that propagates inside the loop are represented by massive particles regularising the divergencies. Their interactions are described by generalised four dimensional Feynman Rules.

- Since we are studying a formulation in 4-dimensions we can use the existing automatic generators for amplitudes in 4-dimensions, where Feynman rules have to be modified.

- At one-loop level, we have implemented the FDF to reconstruct at once cut-constructible and the rational part of any dimensional regularised scattering amplitudes. FDF can be helpful in building more efficient generators for one-loop integrands (for instance within GoSam —> see Peraro's talk).

- The inclusion of the fermion mass for a one-loop amplitude like $0 \to ggt\bar{t}$ at one-loop in FDF will be analysed

- More loops and more jets in FDF is another goal to achieve

Thank you for your attention!