Quantum quenches from excited states in the Ising chain

Leda Bucciantini

Università di Pisa

— New Frontiers in Theoretical Physics, Cortona, 28-31 May 2014 —

with Pasquale Calabrese and Marton Kormos
arXiv: 1401.7250 J. Phys. A.: Math. Theor. **47** (2014) 175002

Thermalization in quantum many body systems

$$Thermalization: \qquad \langle \hat{A}(t \to \infty) \rangle = \mathrm{Tr}(\hat{A}\rho_{\mu-\mathrm{can}})$$

Many numerical and experimental evidences supporting thermalization in some quantum systems [Rigol et al (2009), Trotzky et al. (2012)...] ...

but why should a quantum system thermalize?

Thermalization in quantum many body systems

$$\textit{Thermalization}: \qquad \langle \hat{A}(t \to \infty) \rangle = \mathrm{Tr}(\hat{A}\rho_{\mu-\mathrm{can}})$$

Many numerical and experimental evidences supporting thermalization in some quantum systems [Rigol et al (2009), Trotzky et al. (2012)...] ...

but why should a quantum system thermalize?

Long-Standing Questions

[Von Neumann '29; Birkhoff '30]

- Does an isolated quantum system equilibrate to a statistical ensemble for large times, starting from an arbitrary initial state?
- How do correlation functions depend on time?

Thermalization in quantum many body systems

$$\textit{Thermalization}: \qquad \langle \hat{A}(t \to \infty) \rangle = \mathrm{Tr}(\hat{A}\rho_{\mu-\mathrm{can}})$$

Many numerical and experimental evidences supporting thermalization in some quantum systems [Rigol et al (2009), Trotzky et al. (2012)...] ...

but why should a quantum system thermalize?

Long-Standing Questions

[Von Neumann '29; Birkhoff '30]

- Does an isolated quantum system equilibrate to a statistical ensemble for large times, starting from an arbitrary initial state?
- How do correlation functions depend on time?

Out of equilibrium quantum physics

Quantum quenches

- prepare a many-body quantum system in an eigenstate $|\psi_0\rangle$ of a pre-quenched hamiltonian H
- from $t = 0$ let it evolve unitarily with a different post-quenched hamiltonian H'

$$|\psi(t)\rangle = e^{-iH't}|\psi_0\rangle, \qquad [H, H'] \neq 0$$

Evolution from an out of equilibrium state $|\psi_0\rangle$!

Quantum quenches

- prepare a many-body quantum system in an eigenstate $|\psi_0\rangle$ of a pre-quenched hamiltonian H
- from $t = 0$ let it evolve unitarily with a different post-quenched hamiltonian H'

$$|\psi(t)\rangle = e^{-iH't}|\psi_0\rangle, \qquad [H, H'] \neq 0$$

Evolution from an out of equilibrium state $|\psi_0\rangle$!

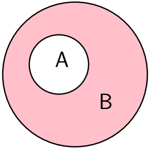
Main results of quantum quenches literature

1. Relaxation
2. Light-cone spread

1. Relaxation

Can the whole system attain stationary behaviour?

A $\cup$ B: initial pure state + unitary evolution



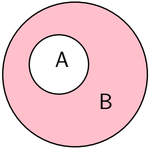
- It can never relax as a whole (pure state $\forall t$)
- First taking B infinite, then $t \to \infty$
  a finite subsystem A can relax!

Only local observables relax!

Physical picture: B acts like a "thermal" bath on A
No time averaging involved!

$A \cup B$: initial pure state + unitary evolution

- It can never relax as a whole (pure state $\forall t$)
- First taking B infinite, then $t \to \infty$
  a finite subsystem A can relax!



Only local observables relax!

Physical picture: B acts like a "thermal" bath on A
No time averaging involved!

Reduced Density Matrix of A

$$\rho_A(t) \equiv \mathrm{Tr}_B\left[\rho_{A\cup B}(t)\right]$$

- stationary and allows for an ensemble description (mixed state)
- determines all local correlation functions

...To which ensemble?

Common Belief

Non Integrable Systems

$$\rho_A = \rho_{\mathrm{can}} = \frac{e^{-\beta H}}{Z}$$

Thermal ensemble
only one integral of motion E
few info on the whole Initial state

[Deutsch '91; Srednicki '95]

...To which ensemble?

Common Belief

Non Integrable Systems

$$\rho_A = \rho_{\text{can}} = \frac{e^{-\beta H}}{Z}$$

Thermal ensemble
only one integral of motion E
few info on the whole Initial state

[Deutsch '91; Srednicki '95]

Integrable Systems

$$\rho_A = \rho_{\text{GGE}} = \frac{e^{-\sum_m \beta_m I_m}}{Z}$$

Non thermal ensemble
all local integrals of motions I_m
full info on the whole Initial state

[Rigol et al '07; Eisert; Cramer...]

...To which ensemble?

Common Belief

Non Integrable Systems

$$\rho_A = \rho_{\text{can}} = \frac{e^{-\beta H}}{Z}$$

Thermal ensemble
only one integral of motion E
few info on the whole Initial state

[Deutsch '91; Srednicki '95]

Integrable Systems

$$\rho_A = \rho_{\text{GGE}} = \frac{e^{-\sum_m \beta_m I_m}}{Z}$$

Non thermal ensemble
all local integrals of motions I_m
full info on the whole Initial state

[Rigol et al '07; Eisert; Cramer...]

... based on

- theoretical, experimental and numerical outcomes

  [Rigol, Muramatsu, Olshanii; Cazalilla; Calabrese, Cardy; Fioretto, Mussardo; Caux, Mossel...]

...To which ensemble?

Common Belief

Non Integrable Systems

$$\rho_A = \rho_{\text{can}} = \frac{e^{-\beta H}}{Z}$$

Thermal ensemble
only one integral of motion E
few info on the whole Initial state

[Deutsch '91; Srednicki '95]

Integrable Systems

$$\rho_A = \rho_{\text{GGE}} = \frac{e^{-\sum_m \beta_m I_m}}{Z}$$

Non thermal ensemble
all local integrals of motions I_m
full info on the whole Initial state

[Rigol et al '07; Eisert; Cramer...]

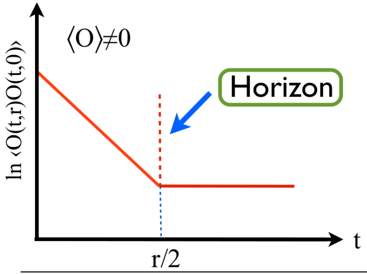
... based on

- theoretical, experimental and numerical outcomes

  [Rigol, Muramatsu, Olshanii; Cazalilla; Calabrese, Cardy; Fioretto, Mussardo; Caux, Mossel...]

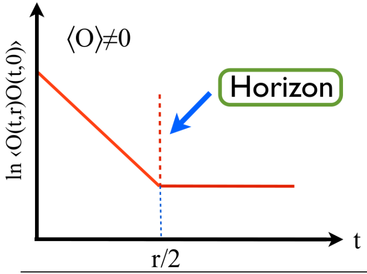
Main test: exact solution of the full dynamics (free theories, TFIC, XY...)

2. Light-cone spread



Equal time two point function for fixed separation r

- exponential decay in time for $t \lesssim r/2$
- saturation to t-independent values for $t \gtrsim r/2$



Equal time two point function for fixed separation r

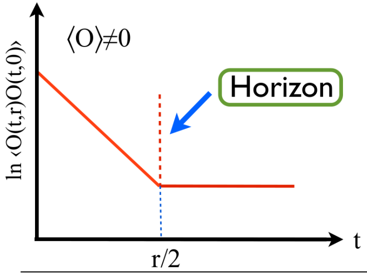
- exponential decay in time for $t \lesssim r/2$
- saturation to t-independent values for $t \gtrsim r/2$

Physical Interpretation

[Calabrese, Cardy '07]

$E_{\psi_0} \gg E_{\text{GS}}$, $|\psi_0\rangle$ acts as a source of excitations

- quasi-particle emitted on scales $E_{\psi_0}^{-1}$ are entangled
- they move classically with light-cone trajectories and spread
- for $t \lesssim r/2$ causally disconnected regions
- after a transient $t \gtrsim r/2$ observables freeze-out



Equal time two point function for fixed separation r

- exponential decay in time for $t \lesssim r/2$
- saturation to t-independent values for $t \gtrsim r/2$

Physical Interpretation

[Calabrese, Cardy '07]

$E_{\psi_0} \gg E_{\mathrm{GS}}$, $|\psi_0\rangle$ acts as a source of excitations

- quasi-particle emitted on scales $E_{\psi_0}^{-1}$ are entangled
- they move classically with light-cone trajectories and spread
- for $t \lesssim r/2$ causally disconnected regions
- after a transient $t \gtrsim r/2$ observables freeze-out

Horizon effect predicts freeze-out of n (>2)-point functions

Objective

Study the time evolution of local observables after a quench
[1 & 2-point functions, entanglement entropy ...]

Objective

Study the time evolution of local observables after a quench

[1 & 2-point functions, entanglement entropy ...]

Starting from an initial excited state

Objective

Study the time evolution of local observables after a quench
[1 & 2-point functions, entanglement entropy ...]

Starting from an initial excited state

In the Transverse field Ising chain
[solvable but non-trivial as free theories]

Objective

Study the time evolution of local observables after a quench

[1 & 2-point functions, entanglement entropy ...]

Starting from an initial excited state

Let's discuss first this point

In the Transverse field Ising chain

[solvable but non-trivial as free theories]

Initial state

In the literature, only focus on ground states of some local hamiltonian as I.S.

Initial state

In the literature, only focus on ground states of some local hamiltonian as I.S.

Why should we focus on excited ones?

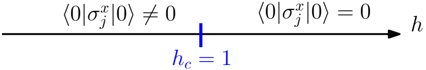
- They are much more common than ground states
- Different behaviour of entanglement entropy in equilibrium:

$S_{\rm GS} \simeq$ Area law versus $S_{\rm exc} \simeq$ Volume law

- Look for universal behaviour
- Room for new effects

Quenched Transverse field Ising chain

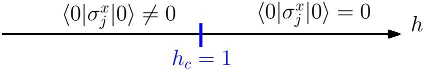
$$H(h) = -\frac{1}{2}\sum_{j=1}^{N}\left[\sigma_j^x\sigma_{j+1}^x + h\sigma_j^z\right] \quad + \quad \text{PBC}$$



$|0\rangle$: ground state of $H(h)$

Quenched Transverse field Ising chain

$$H(h) = -\frac{1}{2}\sum_{j=1}^{N}[\sigma_j^x \sigma_{j+1}^x + h\sigma_j^z] \quad + \quad \text{PBC}$$



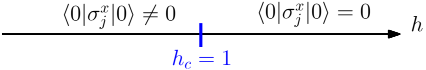
$|0\rangle$: ground state of $H(h)$

From interacting spins σ_i to free spinless fermions b_k

$$H(h) = \sum_k \epsilon_h(k)\big(b_k^\dagger b_k - \frac{1}{2}\big) \qquad \epsilon_h^2(k) = 1 + h^2 - 2h\cos\frac{2\pi k}{N}$$

Quenched Transverse field Ising chain

$$H(h) = -\frac{1}{2}\sum_{j=1}^{N}[\sigma_j^x\sigma_{j+1}^x + h\sigma_j^z] \quad + \quad \mathrm{PBC}$$



$|0\rangle$: ground state of $H(h)$

From interacting spins σ_i to free spinless fermions b_k

$$H(h) = \sum_k \epsilon_h(k)\big(b_k^\dagger b_k - \frac{1}{2}\big) \qquad \epsilon_h^2(k) = 1 + h^2 - 2h\cos\frac{2\pi k}{N}$$

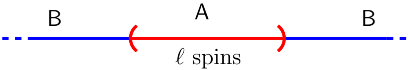
Interaction quench $h \to h'$

Initial state: $\quad |\psi_0\rangle = |m_k\rangle \equiv \prod_k (b_k^\dagger)^{m_k}|0\rangle$

- excited state of pre-quenched hamiltonian $H(h)$
- Z_2-invariant: $\langle\psi_0|\sigma_j^x|\psi_0\rangle = 0$
- m_k: fermionic initial occupation number of k-mode

Our results

Local relaxation in the TFIC from excited states

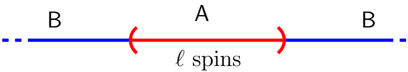


"A" is a block of ℓ contiguous spins

$$\rho_A(t) = \mathrm{Tr}_B\big(|\psi_0(t)\rangle\langle\psi_0(t)|\big)$$

$$|\psi_0(t)\rangle = e^{-iH(h')t}|\psi_0\rangle$$

Local relaxation in the TFIC from excited states



"A" is a block of ℓ contiguous spins

$$\rho_A(t) = \mathrm{Tr}_B \left(|\psi_0(t)\rangle\langle\psi_0(t)| \right) \qquad |\psi_0(t)\rangle = e^{-iH(h')t}|\psi_0\rangle$$

Result: GGE works even for excited states!

$$\rho_{\mathrm{GGE},A} = \rho_A(\infty)$$

Idea:

Free systems $\to$ Wick's thm $\to$ just need to prove it for propagators!

- exactly solvable dynamics
- ensemble averages $\qquad \rho_{\mathrm{GGE,A}} = \frac{e^{-\sum_k \lambda_k n_k}}{Z}$

n_k: post-quench conserved fermionic occupation number operators

Local conserved charges from excited states

$$\langle I_n^+ \rangle = \int_{-\pi}^{+\pi} \frac{dk}{4\pi} \cos(nk)\epsilon_k \left[1 + m_k^S \cos\Delta_k\right] \qquad m_k^S \equiv m_{-k} + m_k - 1$$

$$\langle I_n^- \rangle = -\int_{-\pi}^{+\pi} \frac{dk}{4\pi} \sin[(n+1)k] m_k^A \qquad m_k^A \equiv m_{-k} - m_k$$

Two classes of IS

- $m_k^A = 0$: Only $\langle I_n^+ \rangle \neq 0$ (GS belongs to this class!)
- $m_k^A \neq 0$: Both $\langle I_n^+ \rangle$ and $\langle I_n^- \rangle \neq 0$

Result: Doubling of non zero local conservation laws wrt ground state

Local conserved charges from excited states

$$\langle I_n^+ \rangle = \int_{-\pi}^{+\pi} \frac{dk}{4\pi} \cos(nk)\epsilon_k \left[1 + m_k^S \cos\Delta_k\right] \qquad m_k^S \equiv m_{-k} + m_k - 1$$

$$\langle I_n^- \rangle = -\int_{-\pi}^{+\pi} \frac{dk}{4\pi} \sin[(n+1)k] m_k^A \qquad m_k^A \equiv m_{-k} - m_k$$

Two classes of IS

- $m_k^A = 0$: Only $\langle I_n^+ \rangle \neq 0$ (GS belongs to this class!)
- $m_k^A \neq 0$: Both $\langle I_n^+ \rangle$ and $\langle I_n^- \rangle \neq 0$

Result: Doubling of non zero local conservation laws wrt ground state

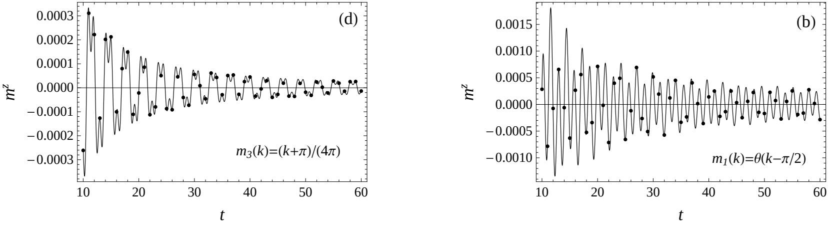
Does the increased number of conservation laws in m_k^A alter the asymptotic time dependence of correlations?

Transverse magnetization

$$m^z(t) = \underbrace{\int_{-\pi}^{\pi} \frac{dk}{4\pi} e^{i\theta_k} m_k^S \cos\Delta_k}_{\text{stationary part}} - i \underbrace{\int_{-\pi}^{\pi} \frac{dk}{4\pi} e^{i\theta_k} m_k^S \sin\Delta_k \cos(2\epsilon_k t)}_{\text{time-dependent}}$$

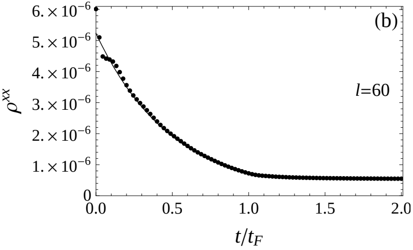
Asymptotic behaviour: stationary phase approximation

$m(k)$ analytic	$m(k)$ non-analytic
$m^z(t) \simeq t^{-\frac{3}{2}} + \mathcal{O}(t^{-\frac{2n+1}{2}})$	$m^z(t) \simeq t^{-1} + \mathcal{O}(t^{-\frac{2n+1}{2}})$
AS GROUND STATE	NOVELTY!



Longitudinal spin-spin function

$$\rho^{xx}(\ell, t) \equiv \langle \Psi_0(t) | \sigma_n^x \sigma_{\ell+n}^x | \Psi_0(t) \rangle$$



$$m(k) = \frac{k^2}{(2\pi)^2}$$

$$\ell = 60$$

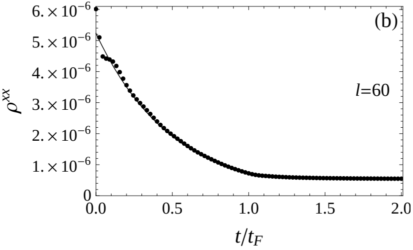
$$h = 1/3, \quad h' = 2/3$$

$$t_F = \ell/(2v_{\max})$$

$$v_{\max} = \min[h, 1]$$

Longitudinal spin-spin function

$$\rho^{xx}(\ell, t) \equiv \langle \Psi_0(t) | \sigma_n^x \sigma_{\ell+n}^x | \Psi_0(t) \rangle$$



$$m(k) = \frac{k^2}{(2\pi)^2}$$

$$\ell = 60$$

$$h = 1/3, \quad h' = 2/3$$

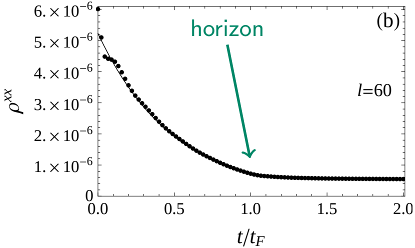
$$t_F = \ell/(2v_{\max})$$

$$v_{\max} = \min[h, 1]$$

Results

Longitudinal spin-spin function

$$\rho^{xx}(\ell, t) \equiv \langle \Psi_0(t) | \sigma_n^x \sigma_{\ell+n}^x | \Psi_0(t) \rangle$$



$$m(k) = \frac{k^2}{(2\pi)^2}$$

$$\ell = 60$$

$$h = 1/3, \quad h' = 2/3$$

$$t_F = \ell/(2v_{\max})$$

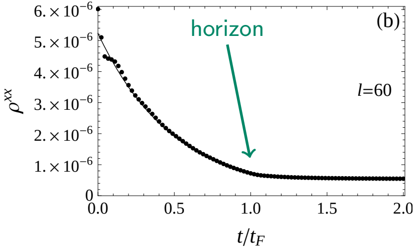
$$v_{\max} = \min[h, 1]$$

Results

- Emergent light-cone spreading of correlations (as for GS)

Longitudinal spin-spin function

$$\rho^{xx}(\ell, t) \equiv \langle \Psi_0(t) | \sigma_n^x \sigma_{\ell+n}^x | \Psi_0(t) \rangle$$



$$m(k) = \frac{k^2}{(2\pi)^2}$$
$$\ell = 60$$
$$h = 1/3, \quad h' = 2/3$$
$$t_F = \ell/(2v_{\max})$$
$$v_{\max} = \min[h, 1]$$

Results

- Emergent light-cone spreading of correlations (as for GS)
- Common behaviour $\forall m_k$ analyzed (stepfunction, linear, quadratic)...

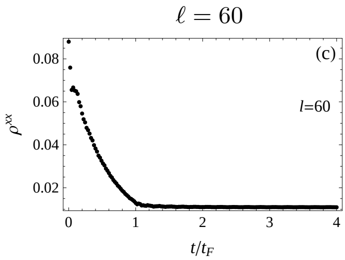
...EXCEPT ONE!

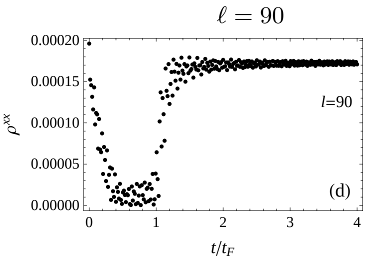
The anomalous state: $m_k = \theta(k - \frac{\pi}{2})$

Different behaviour for different ℓ!

The anomalous state: $m_k = \theta(k - \frac{\pi}{2})$

Different behaviour for different ℓ!



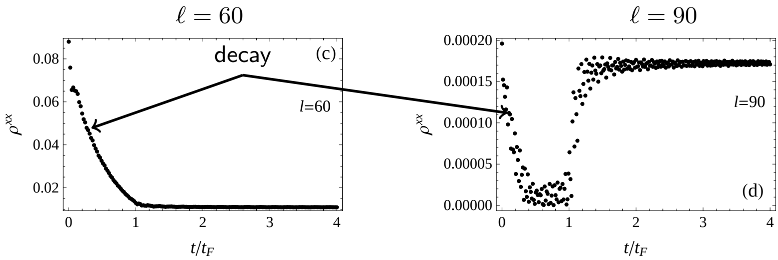


Still open problems

- Is it related to $\boxed{\langle I_n^- \rangle \neq 0}$?
- But other $m_k^A \neq 0$ display usual light-cone effect...

The anomalous state: $m_k = \theta(k - \frac{\pi}{2})$

Different behaviour for different ℓ!

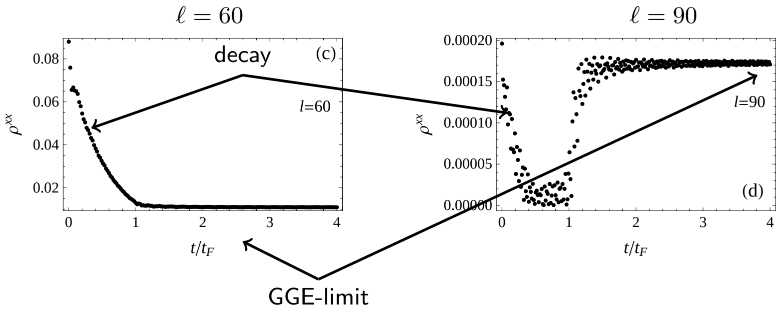


Still open problems

- Is it related to $\boxed{\langle I_n^- \rangle \neq 0}$?
- But other $m_k^A \neq 0$ display usual light-cone effect...

The anomalous state: $m_k = \theta(k - \frac{\pi}{2})$

Different behaviour for different ℓ!

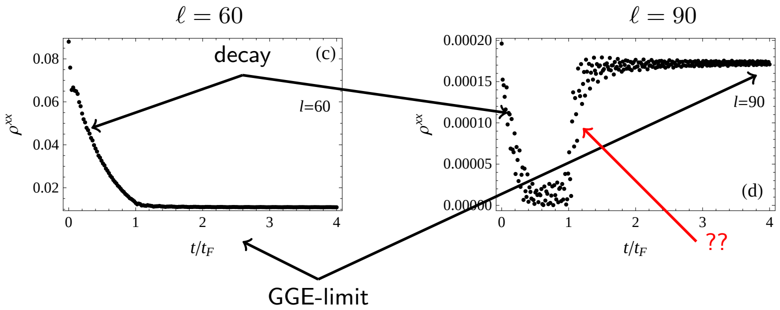


Still open problems

- Is it related to $\boxed{\langle I_n^- \rangle \neq 0}$?
- But other $m_k^A \neq 0$ display usual light-cone effect...

The anomalous state: $m_k = \theta(k - \frac{\pi}{2})$

Different behaviour for different ℓ!



Still open problems

- Is it related to $\boxed{\langle I_n^- \rangle \neq 0}$?
- But other $m_k^A \neq 0$ display usual light-cone effect...

Analytical full time evolution of $\rho^{xx}(\ell, t)$

- Focus on quenches within the ferromagnetic phase $h, h_0 < 1$
- Method: multi-dimensional stationary phase [Fagotti, Essler, Calabrese '08]
- Extension only to $m_k^A = 0$

Analytical full time evolution of $\rho^{xx}(\ell, t)$

- Focus on quenches within the ferromagnetic phase $h, h_0 < 1$
- Method: multi-dimensional stationary phase [Fagotti, Essler, Calabrese '08]
- Extension only to $m_k^A = 0$

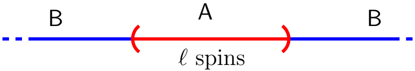
$$\rho^{xx}_{m_k}(\ell,t) \simeq C_{m_k} \overbrace{\exp\left[\ell \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left(1 - 2|\epsilon'_k|\frac{t}{\ell}\right) \ln(|m_k^S|)\theta(\ell - 2|\epsilon'_k|t)\right]}^{\text{typical of excited states}}$$
$$\times \exp\left[2t \int_{-\pi}^{\pi} \frac{dk}{2\pi} |\epsilon'_k| \ln[|\cos\Delta_k m_k^S|]\theta(\ell - 2|\epsilon'_k|t)\right]$$
$$\times \exp\left[\ell \int_{-\pi}^{\pi} \frac{dk}{2\pi} \ln[|\cos\Delta_k m_k^S|]\theta(2|\epsilon'_k|t - \ell)\right]$$

Universal properties:

- $t \ll t_F$, evolution in t does not depend on m_k^S (first two lines)
- $t \gg t_F$, constant in time (third line)
- At fixed time, exponential decreasing with ℓ

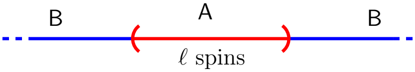
Entanglement Entropy

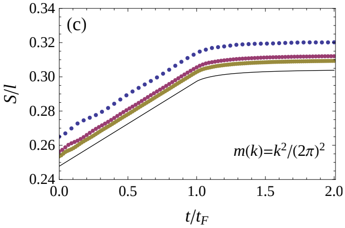
$S_{\mathsf{A}} = -\mathrm{Tr}[\rho_{\mathsf{A}} \ln \rho_{\mathsf{A}}]$



Entanglement Entropy

$$S_{\mathsf{A}} = -\mathrm{Tr}[\rho_{\mathsf{A}} \ln \rho_{\mathsf{A}}]$$





- Light-cone behaviour
- Dependence on m_k^S
- $S_\ell/\ell \neq 0$ at $t = 0$ due to excitations

Conclusions & Outlooks

We have considered quenches from excited states

Validity of GGE

Horizon effect for S_ℓ and ρ_ℓ^{xx}

Still open problems

Non-trivial dependence for m_k^A?

Excitations in truly interacting models?