Precision physics with jet observables

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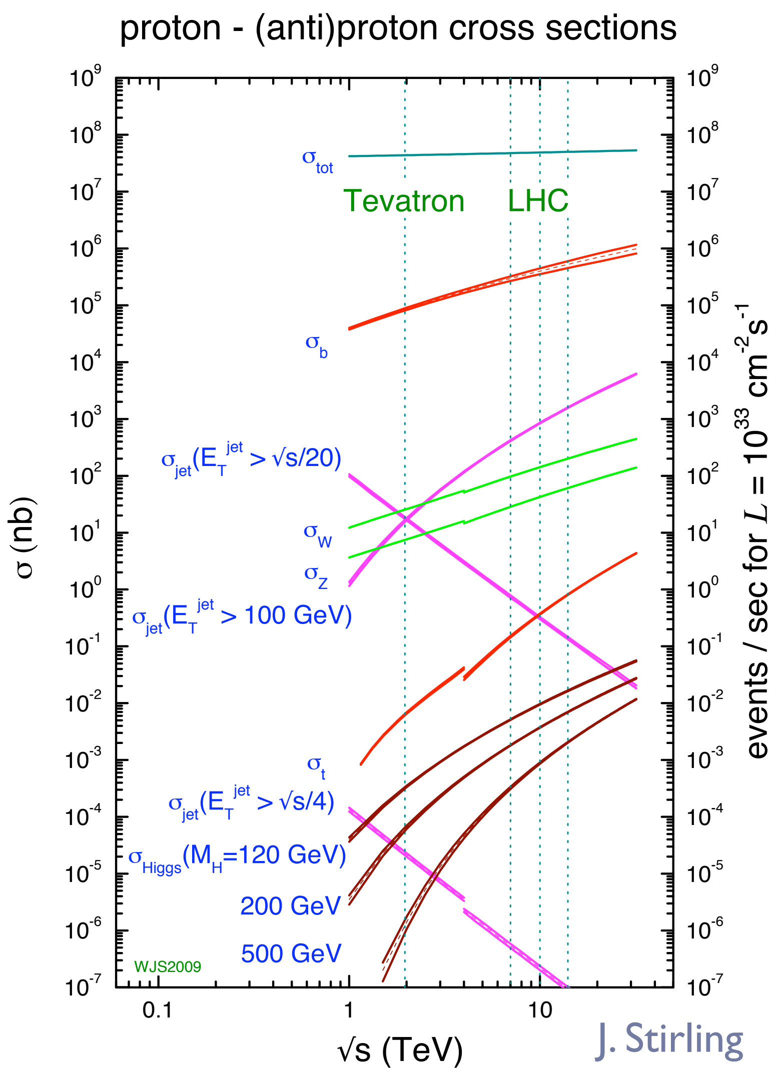
Benchmark processes at colliders

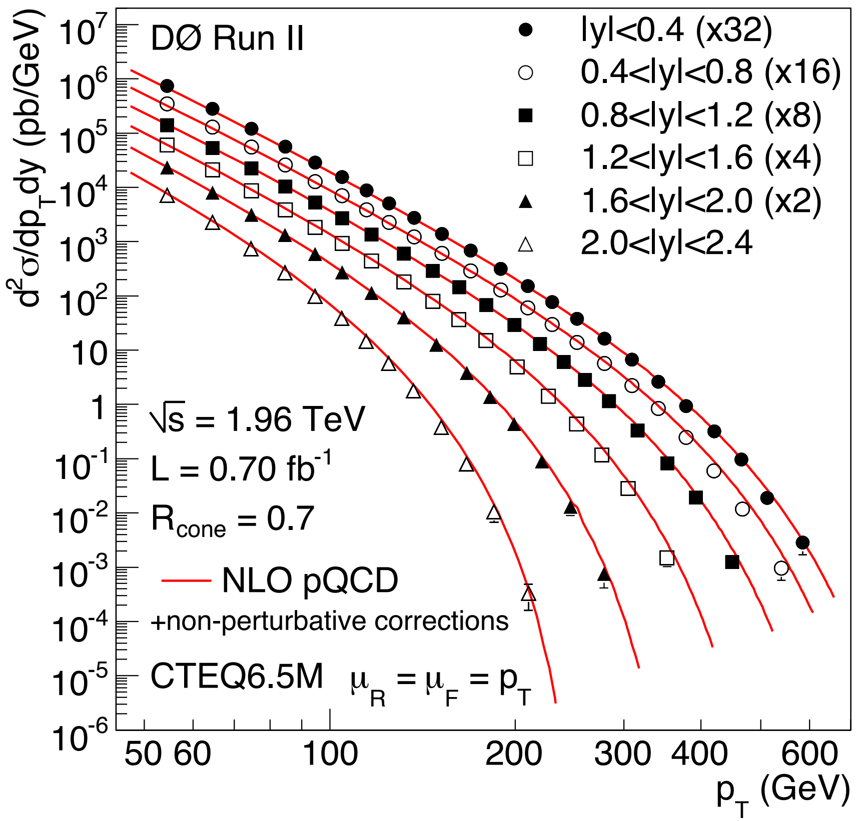
- Large production rates for Standard Model processes
  - jets
  - top quark pairs
  - vector bosons
- Allow precision measurements
  - masses
  - couplings
  - parton distributions
- Require precise theory: NNLO

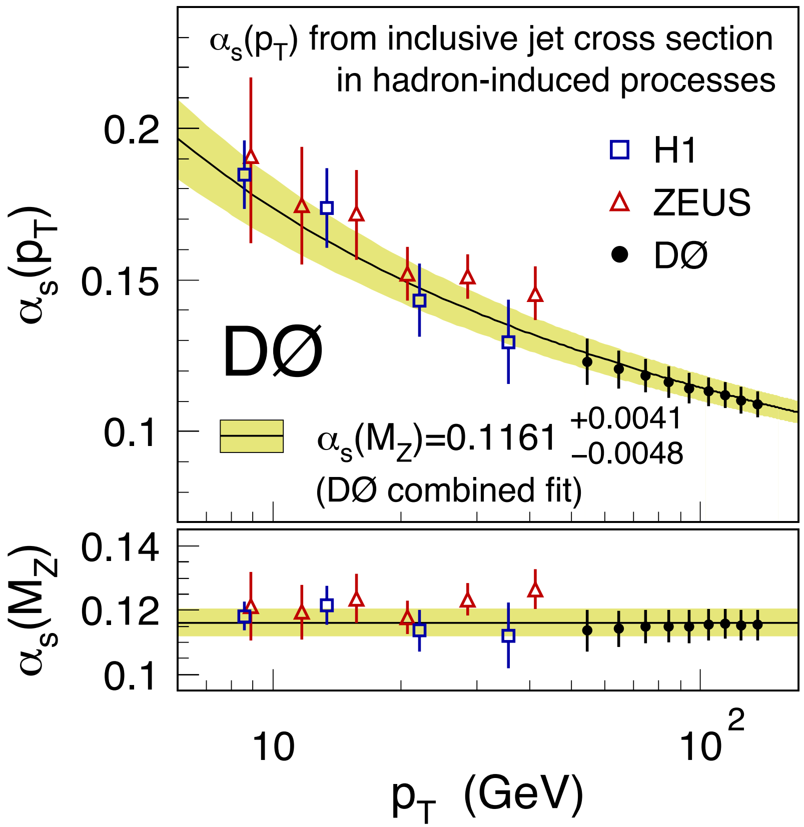


J. Stirling

Jet cross sections at hadron colliders

Precision observable at the Tevatron

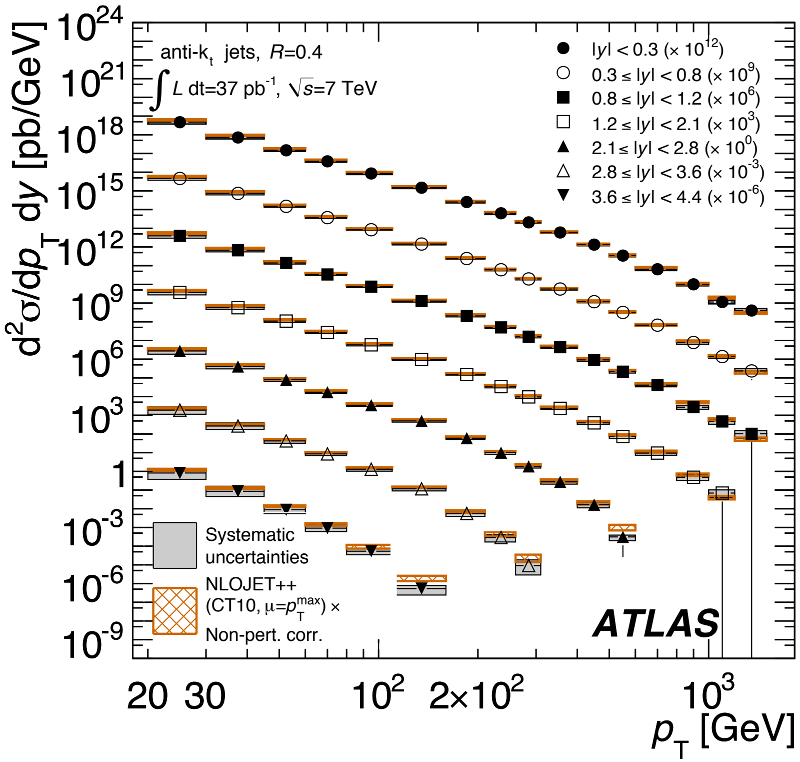


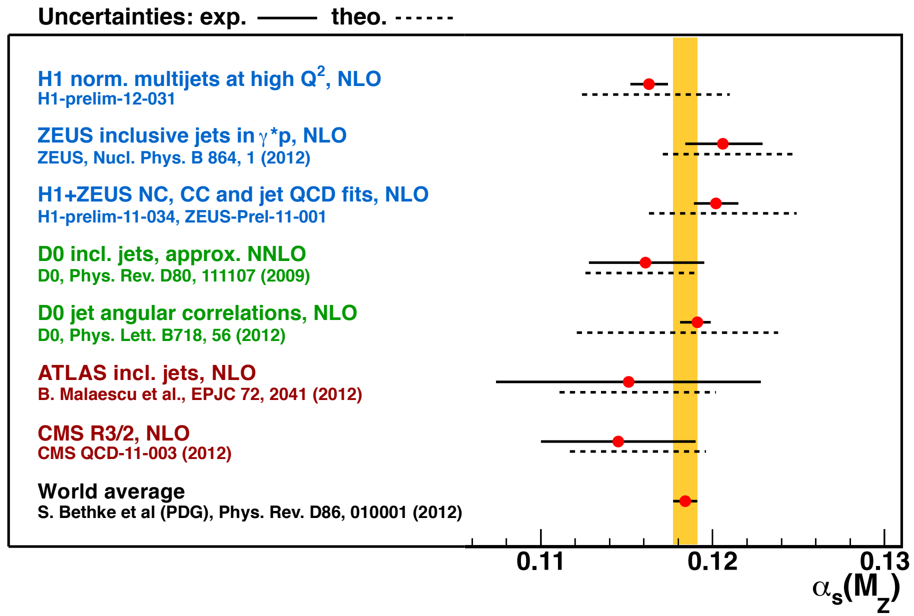


- Constrain parton distributions
- Measure strong coupling constant

Jet cross sections at hadron colliders

ATLAS 2010 results: single jet inclusive

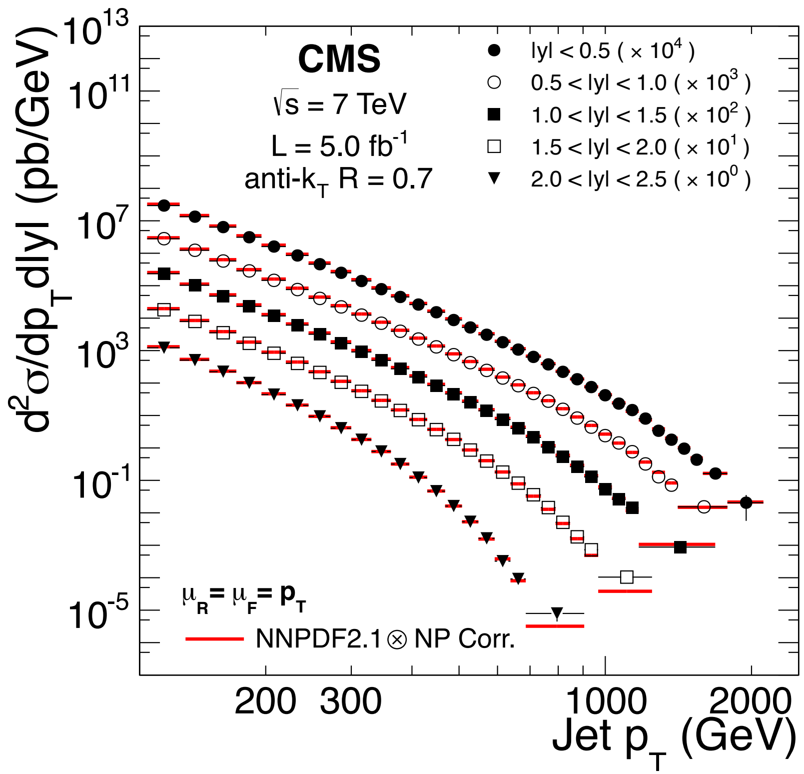


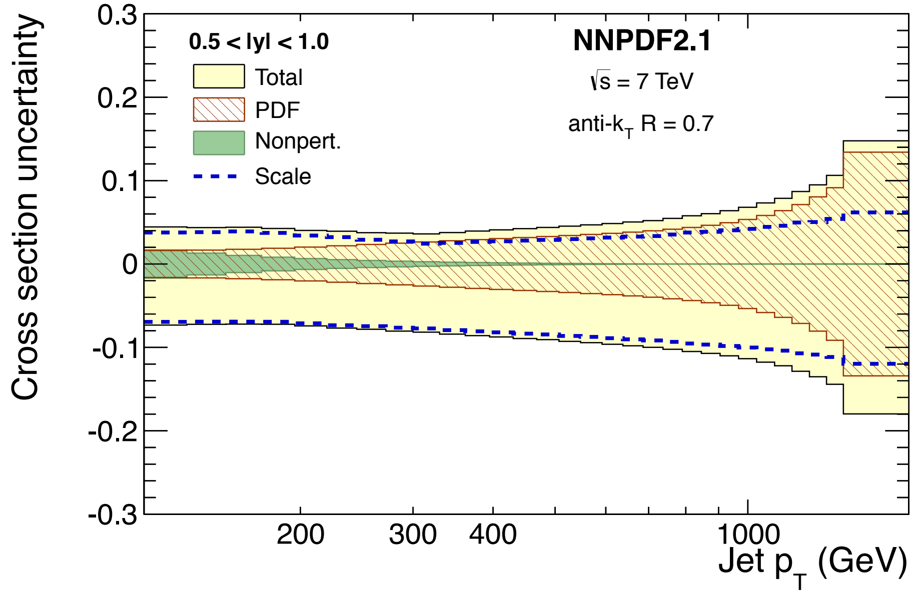


- On track to a multiple-differential high-precision measurement
- Limiting factor in interpretation will be theory accuracy

Jet cross sections at hadron colliders

CMS 2011 results: single jet inclusive

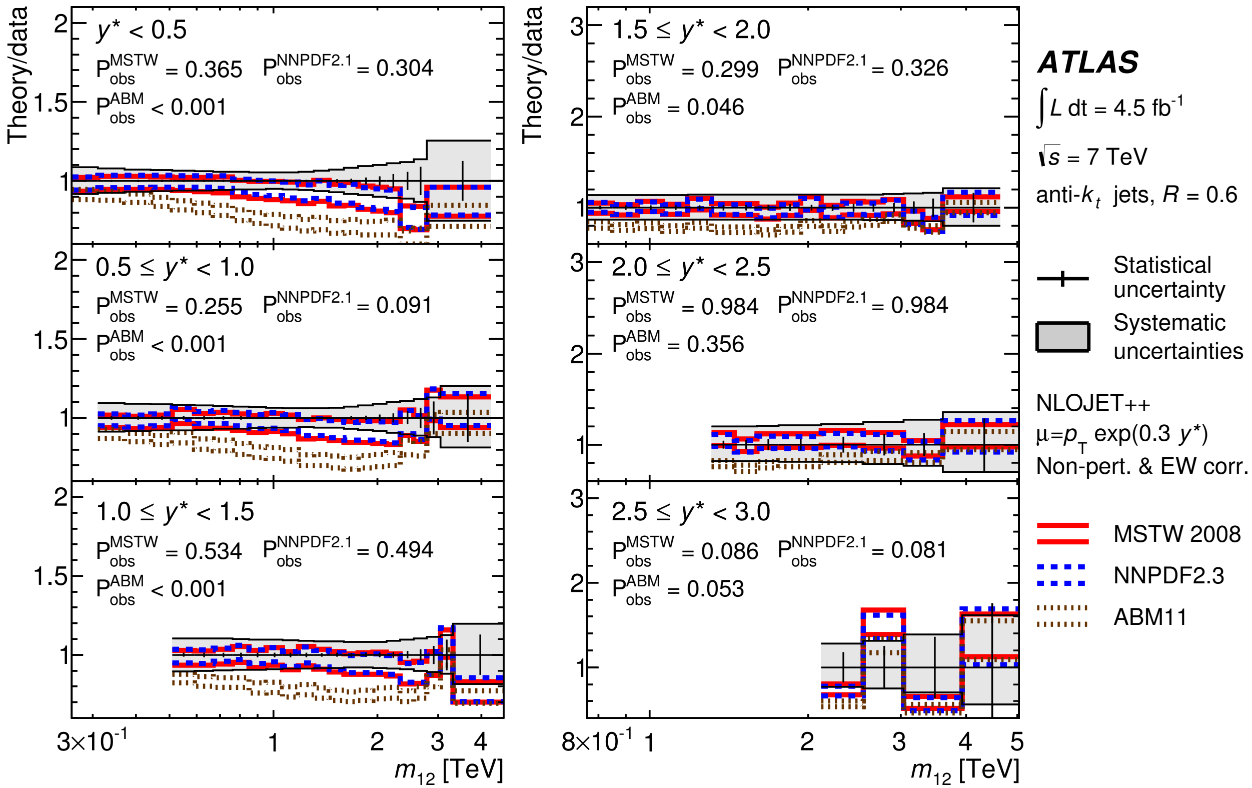




- uncertainty on NLO prediction comparable to spread from partons
- need improved theory for precise extraction of parton distributions from jets

Jet cross sections at hadron colliders

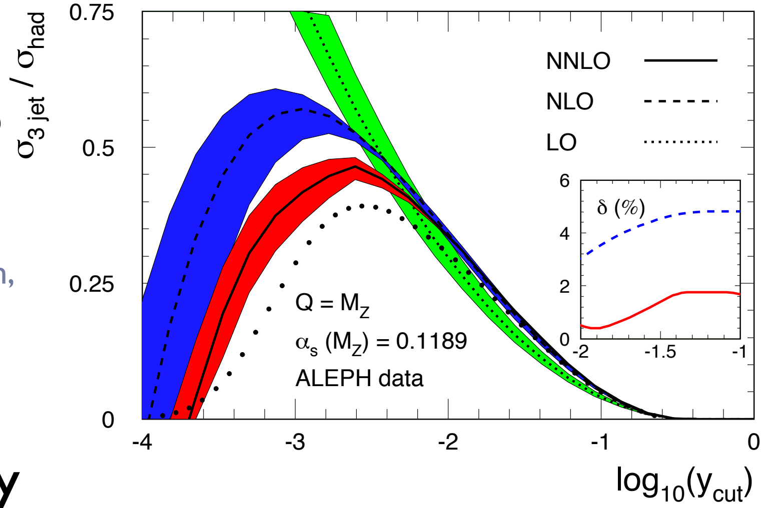
ATLAS 2011 results: dijet mass distribution



- Dijet production: differential probe of parton distributions

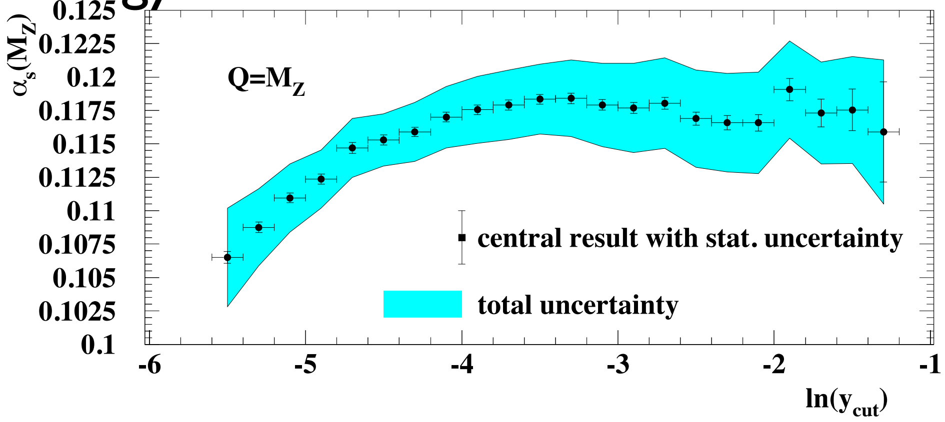
NNLO corrections to $e^+e^- \rightarrow$ 3 jets

- Calculation of NNLO corrections to $e^+e^- \rightarrow$ 3 jets and related event shapes (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)



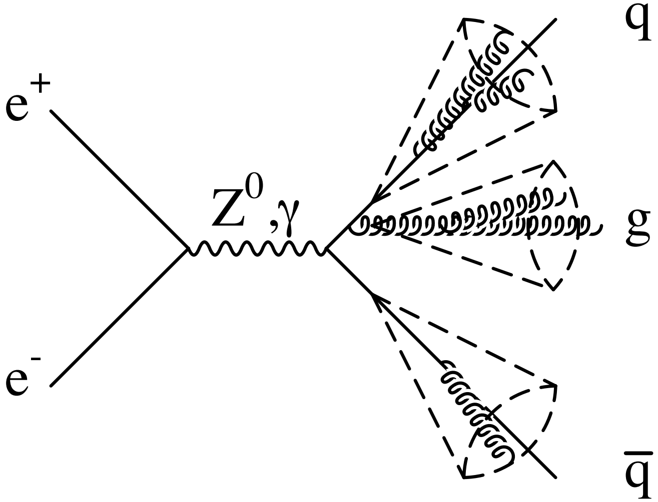
- Revisit LEP precision phenomenology (with G. Dissertori, G. Luisoni, H. Stenzel)

- Strong coupling from R_{3jet}

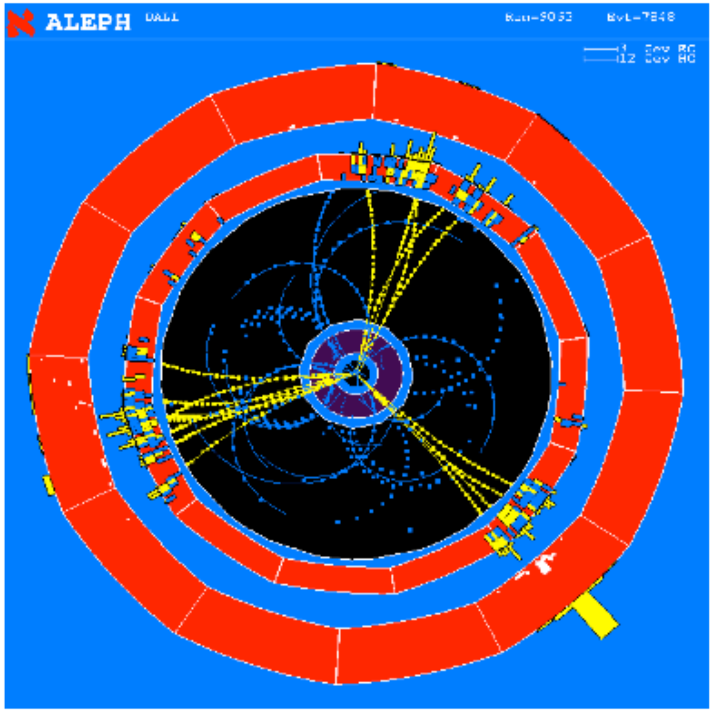


$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\mathrm{exp}) \pm 0.0015\,(\mathrm{theo})$$

Jets: $e^+e^- \rightarrow$ 3 jets



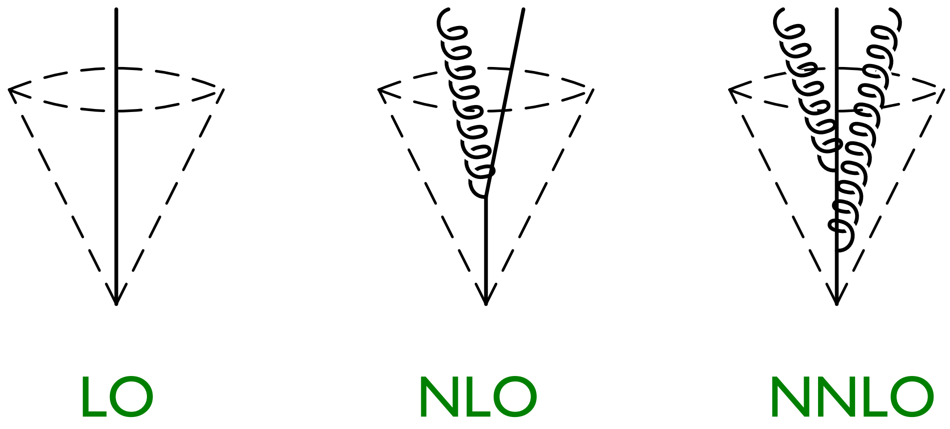
Theory



Experiment

Jets in perturbative QCD

- Partons are combined into jets with the same jet algorithm as in experiment



  - No algorithm dependence at leading order
  - Theoretical description more accurate with increasing order
  - Current status: at most three partons in one jet

Infrared safety

- Formal requirement on QCD observables (jets, event shapes) defined through final state momenta
  - Value of observable should not change if a soft particle is emitted or a particle splits into two collinear particles (Sterman-Weinberg criteria)
  - Ensures cancellation of infrared divergences at each order in perturbative QCD (KLN theorem)

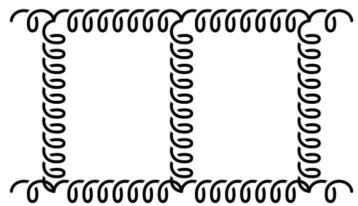
- Observables that fulfil these criteria are called infrared-safe

Soft $(E_{n+1} \rightarrow 0)$: $O_{n+1}(p_1,\dots,p_n,p_{n+1}) \rightarrow O_n(p_1,\dots,p_n)$

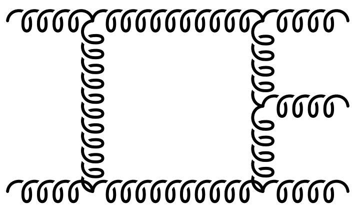
Collinear $(p_n \,//\, p_{n+1})$: $O_{n+1}(p_1,\dots,p_n,p_{n+1}) \rightarrow O_n(p_1,\dots,p_n+p_{n+1})$

Ingredients to jet production at NNLO

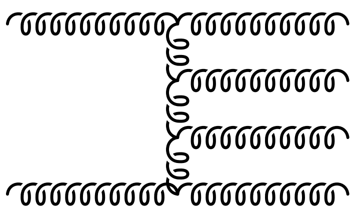
- Two-loop matrix elements
  (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
  - Explicit infrared poles from loop integrals



- One-loop matrix elements
  (Z. Kunszt, A. Signer, Z. Trocsanyi)
  - Explicit infrared poles from loop integral
  - Implicit infrared poles from real radiation



- Tree-level matrix elements
  - Implicit infrared poles from real radiation



Two-loop matrix elements

- Generation of diagrams (QGRAF: P. Noguiera, FORM: J. Vermaseren)
  - Expressed in terms of two-loop Feynman integrals
- Reduction to master integrals
  - Integration-by-parts identities

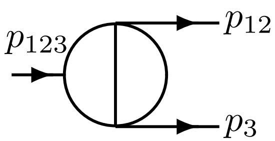
$$\int \frac{d^d k}{(2\pi)^d} \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0 \qquad \text{with } a^\mu = k^\mu, l^\mu; b^\mu = k^\mu, l^\mu, p_i^\mu$$

  - Complemented by Lorentz invariance and symmetry
  - Solution based on lexicographic ordering (S. Laporta)
    - AIR (C. Anastasiou, A. Lazopoulos)
    - FIRE (A. Smirnov)
    - Reduze (A. von Manteuffel, C. Studerus)

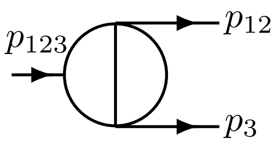
Two-loop matrix elements

- Master integrals from differential equations
  - Differentiate integrand with respect to masses and momenta
  - Apply integration-by-parts identities

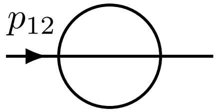
$$s_{123}\frac{\partial}{\partial s_{123}}$$



$$= \frac{d-4}{2}\,\frac{2s_{123}-s_{12}}{s_{123}-s_{12}}$$



$$-\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



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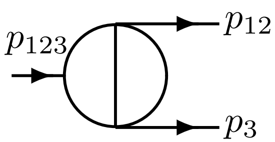
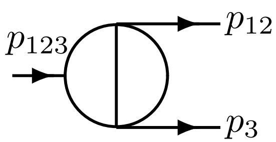
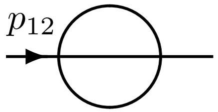
$$s_{12}\frac{\partial}{\partial s_{12}}$$



$$= -\frac{d-4}{2}\,\frac{s_{12}}{s_{123}-s_{12}}$$



$$+\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



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  - Integrate differential equations and match boundary

Two-loop matrix elements for jet production

- Analytic 2 →2 results for processes with jets
  - Di-jet production (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
  - Vector-boson-plus-jet production (L. Garland, E.W.N. Glover, A. Koukoutsakis, E. Remiddi, L. Tancredi, E. Weihs, TG)
  - Higgs-boson-plus-jet production (E.W.N. Glover, M. Jaquier, A. Koukoutsakis, TG)
- Top quark pair production
  - Numerical representation (P. Bärnreuther, M. Czakon)
  - Analytical work ongoing (R. Bonciani, A. Ferroglia, C. Studerus, A. von Manteuffel, TG)
- Next frontier: automation and 2 → 3
  - Unitarity-based methods (P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)
  - Classification of integral basis (H. Johansson, D. Kosower, K. Larsen)

Real radiation at NNLO: factorization

- Single unresolved radiation at one loop
  - One-loop correction to collinear splitting factors (Z. Bern, V. Del Duca, W. Kilgore, C. Schmidt)
  - One-loop correction to soft eikonal factor (S. Catani, M. Grazzini)
- Double unresolved radiation factors at tree level (J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini)
  - Double soft
  - Soft/Collinear
  - Triple collinear
  - Double single collinear
- Require method to extract singular contributions

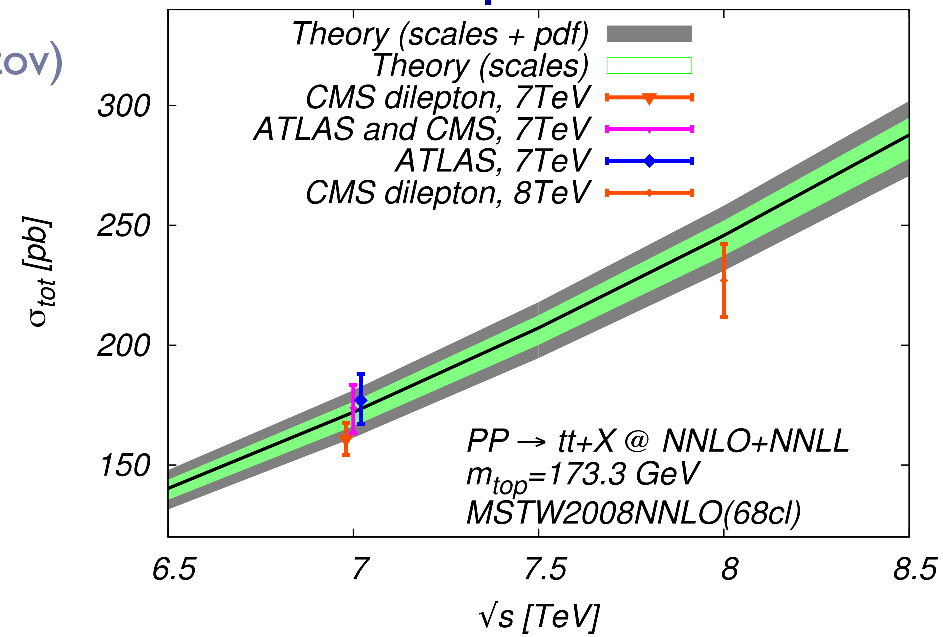
Real radiation at NNLO: methods

- Sector decompostion
  (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)
  - $pp \to H$, $pp \to V$, including decays
    (C. Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)
- q_T-subtraction (S. Catani, M. Grazzini)
  - $pp \to H$, $pp \to V$, $pp \to \gamma\gamma$, $pp \to VH$
    (S. Catani, L. Cieri, D. de Florian, G. Ferrera M. Grazzini, F. Tramontano)
- Sector-improved subtraction schemes
  (M. Czakon; R. Boughezal, K. Melinkov, F. Petriello)
  - $pp \to t\bar{t}$ (M. Czakon, P. Fiedler, A. Mitov)
  - $pp \to H+j$ (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
- Antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - $e^+e^- \to 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)
  - $pp \to 2j$ (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)

NNLO results on top quark pair production

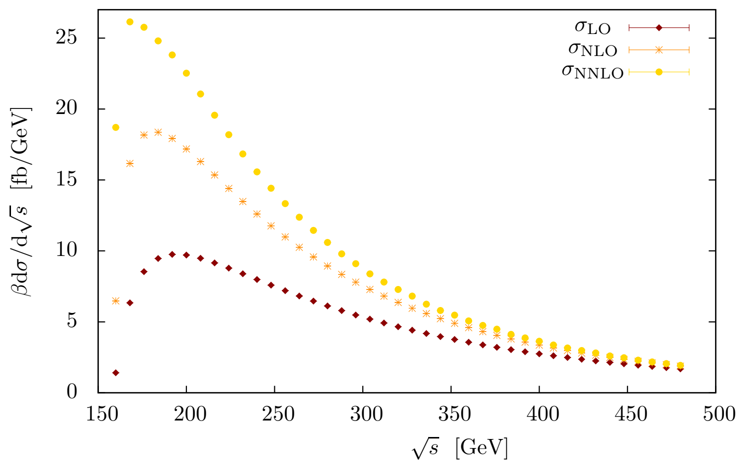
- Large production cross section at the LHC
  - Expected experimental error of ~5% on total cross section
  - NLO+NLL predictions yield uncertainty of ~10%

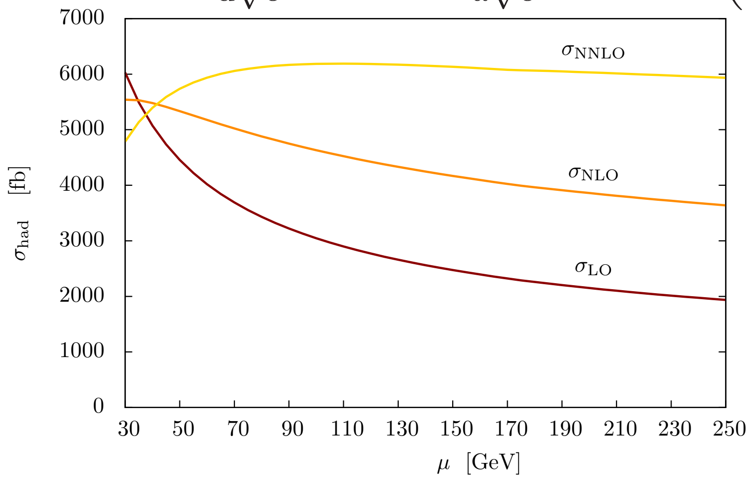
- NNLO calculation of total cross section completed recently (M. Czakon, P. Fiedler, A. Mitov)
  - Based on sector-improved subtraction
  - Numerical cancellation of infrared poles
  - Differential distributions in progress



NNLO results on Higgs+jet production

- First results of H+jet total cross section (gluons only)
  (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
  - Based on sector-improved subtraction
  - Numerical cancellation of infrared singularities
- Cross section multiplied by gluon luminosity
  - Large corrections at partonic threshold $\beta \frac{\mathrm{d}\sigma_{\mathrm{had}}}{\mathrm{d}\sqrt{s}} = \beta \frac{\mathrm{d}\sigma(s, \alpha_s, \mu_R, \mu_F)}{d\sqrt{s}} \times \mathcal{L}\left(\frac{s}{s_{\mathrm{had}}}, \mu_F\right)$





- Stabilization of scale dependence for total cross section

NNLO Infrared Subtraction

Structure of NNLO cross section

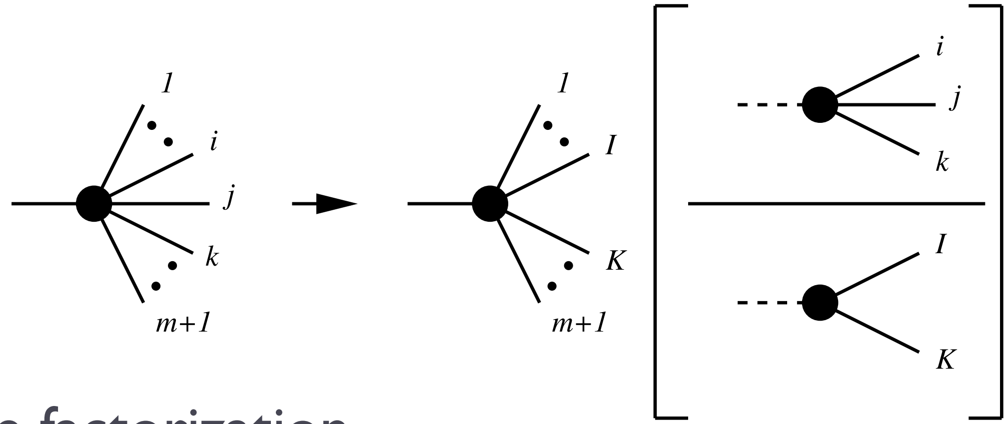
$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{MF,1}_{NNLO} \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{MF,2}_{NNLO}
\end{aligned}
$$

- Real and virtual contributions: $\mathrm{d}\sigma^{R}_{NNLO}, \mathrm{d}\sigma^{V,1}_{NNLO}, \mathrm{d}\sigma^{V,2}_{NNLO}$
- Subtraction term for double real radiation: $\mathrm{d}\sigma^{S}_{NNLO}$
- Subtraction term for one-loop single real radiation: $\mathrm{d}\sigma^{VS,1}_{NNLO}$
- Mass factorization terms: $\mathrm{d}\sigma^{MF,1}_{NNLO}, \mathrm{d}\sigma^{MF,2}_{NNLO}$

- Each line finite and free of poles
  $\longrightarrow$ numerical implementation

Antenna subtraction

- Subtraction terms constructed from antenna functions
  - Antenna function contains all emission between two partons



  - Phase space factorization

$$d\Phi_{m+1}(p_1,\ldots,p_{m+1};q) = d\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_K,\ldots,p_{m+1};q)\cdot d\Phi_{X_{ijk}}(p_i,p_j,p_k;\tilde{p}_I+\tilde{p}_K)$$

  - Integrated subtraction term

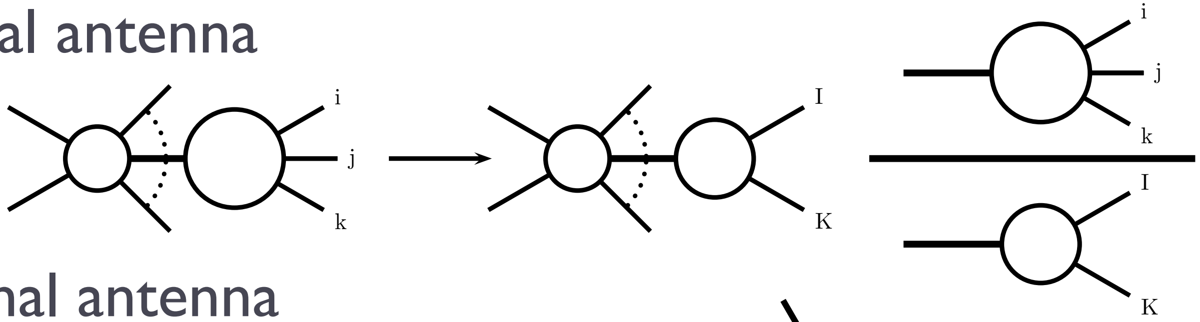
$$\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$$

Antenna functions

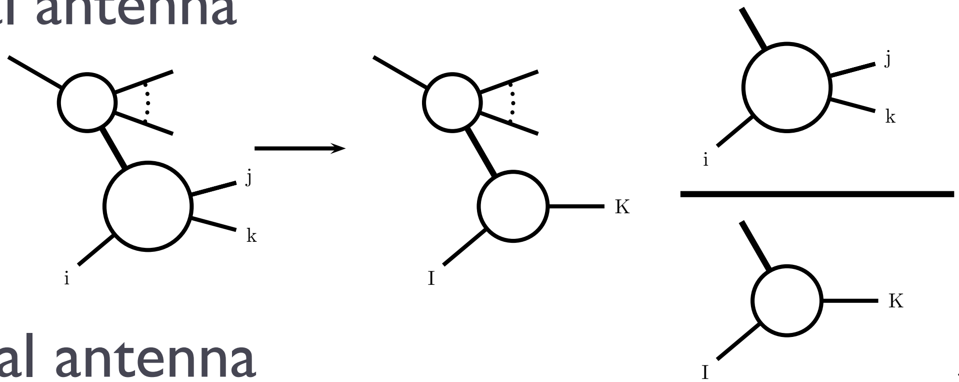
- Colour-ordered pair of hard partons (radiators)
  - Hard quark-antiquark pair
  - Hard quark-gluon pair
  - Hard gluon-gluon pair
- NLO (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover)
  - Three-parton antenna: one unresolved parton
- NNLO (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - Four-parton antenna: two unresolved partons
  - Three-parton antenna at one loop
  - Products of NLO antenna functions
  - Soft antenna function

Antenna subtraction: incoming hadrons

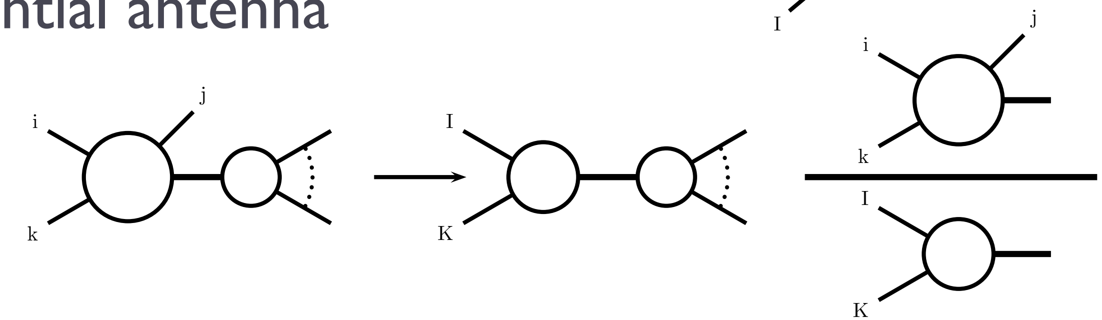
- Three antenna types (A. Daleo, D. Maitre, TG)
  - Final-final antenna



  - Initial-final antenna



  - Initial-intial antenna

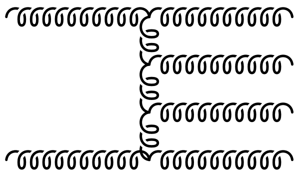


Integrated NNLO antenna functions

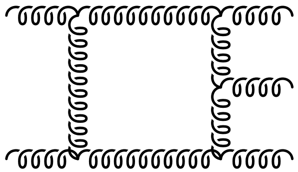
- Analytical integration over unresolved part of phase space only
  - phase space integrals reduced to masters (C. Anastasiou, K. Melnikov)
  - Final-final: $q \to k_1 + k_2 + k_3 (+k_4)$, one scale: q^2
    - $1 \to 4$ tree level
    - $1 \to 3$ one loop
  - Initial-final: $q + p_1 \to k_1 + k_2 (+k_3)$, two scales: q^2, x
    (A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG)
    - $2 \to 3$ tree level
    - $2 \to 2$ one loop
  - Initial-initial: $p_1 + p_2 \to q + k_1 (+k_2)$, three scales: q^2, x_1, x_2
    (R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann, TG)
    - $2 \to 3$ tree level
    - $2 \to 2$ one loop

NNLO corrections to pp → 2j

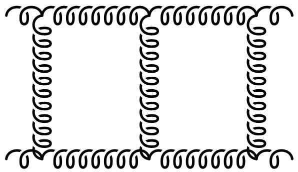
- Leading colour gluons-only as proof of concept
  (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - Double real radiation
    - Subtraction terms constructed and implemented
    - Azimuthal correlations from gluon splitting



  - Single real radiation at one loop
    - Subtraction terms constructed and implemented
    - Interplay of antenna functions and mass factorization



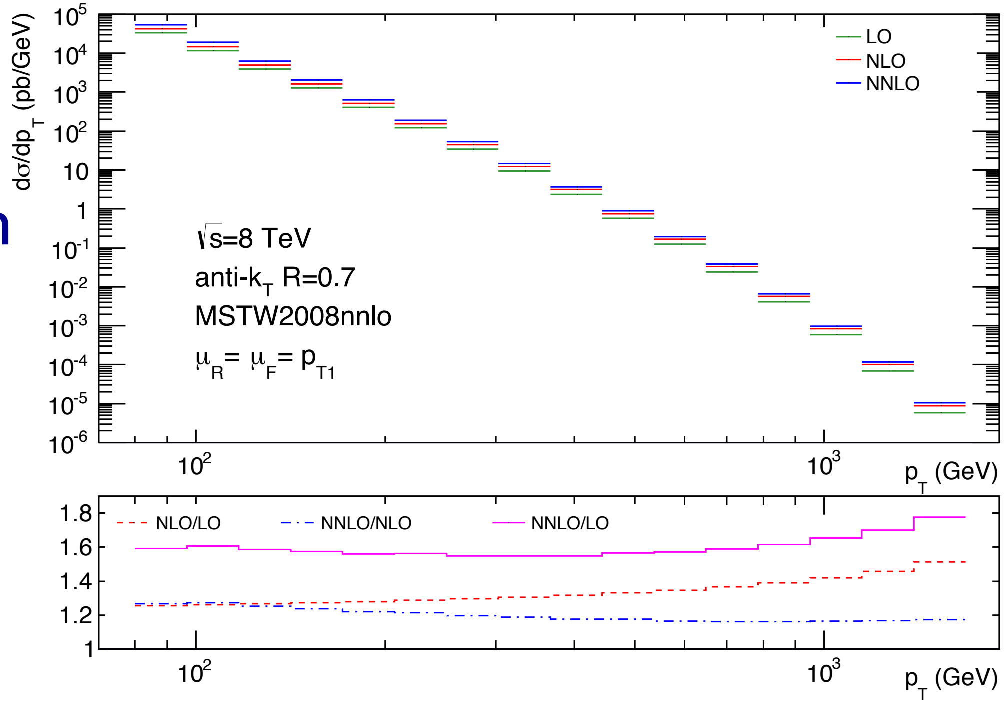
  - Two-loop contributions
    - Added integrated subtraction terms from above
    - Observe analytic cancellation of all infrared poles



- All implemented in parton-level event generator

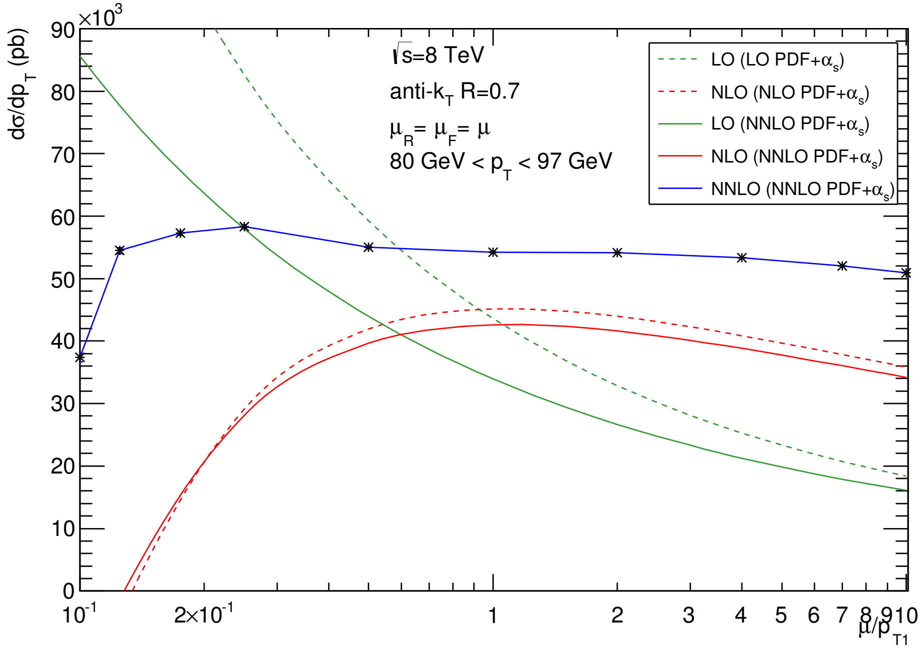
pp → 2jets at NNLO

- First results at NNLO available
  - gg → gg subprocess
    (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - Developed a new parton-level event generator NNLOJET
  - using antenna subtraction
    - analytic cancellation of infrared poles
- Inclusive jet p_T distribution
  - NNLO/NLO differential K-factor flat over the whole p_T range



pp → 2jets at NNLO

- **Inclusive jet p_T distribution: scale dependence** (gluons only)
  (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - Dynamical scale choice: leading jet p_T

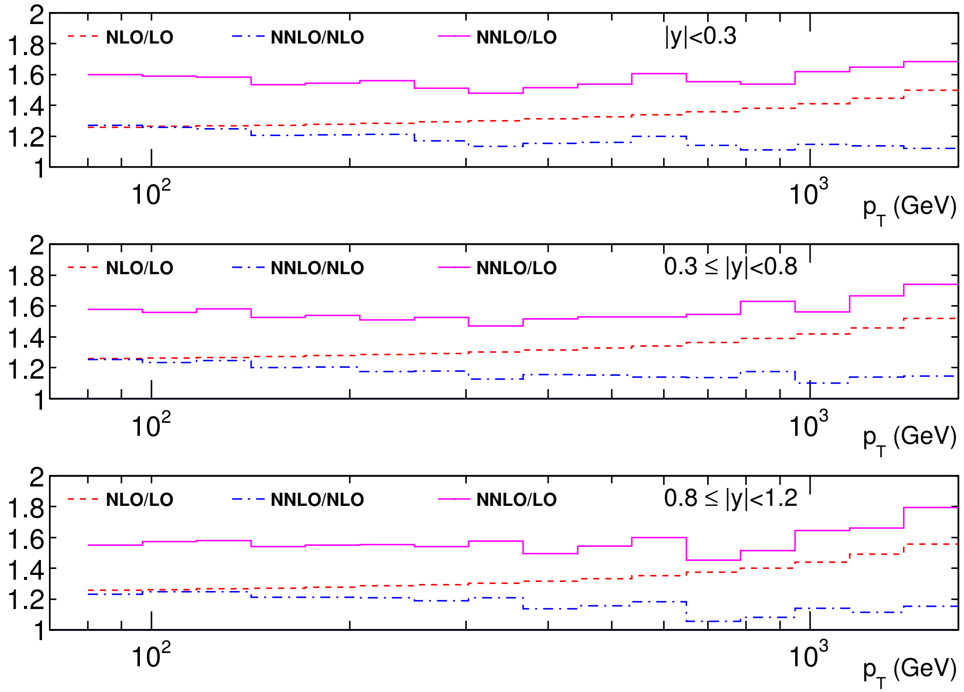


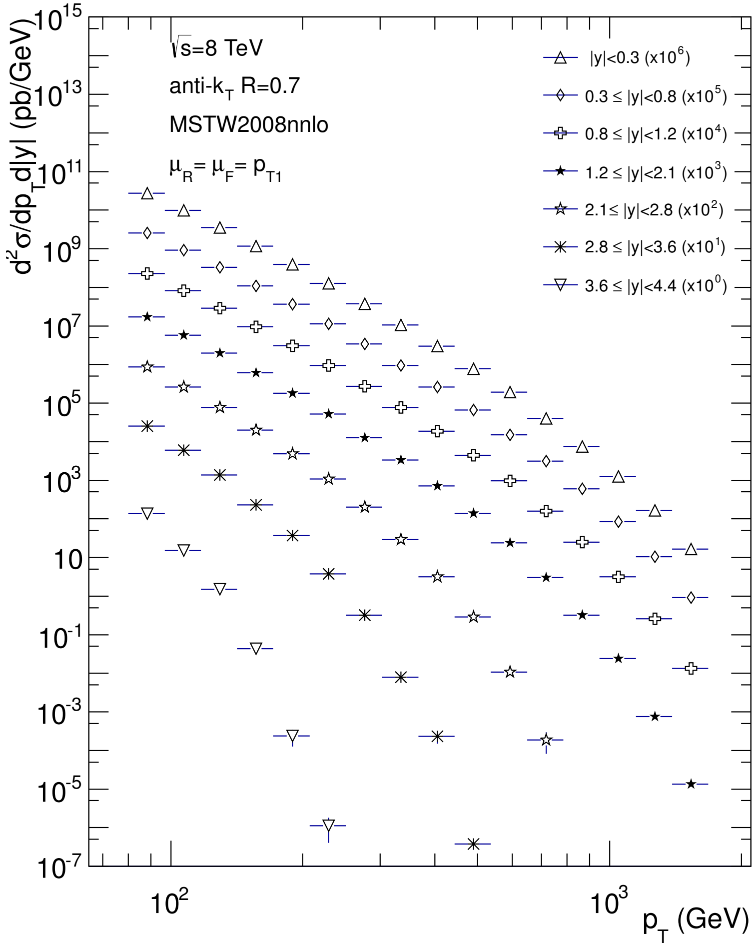
- Stabilization at NNLO

NNLO corrections to pp → 2j

Inclusive jet production:
double differential distributions

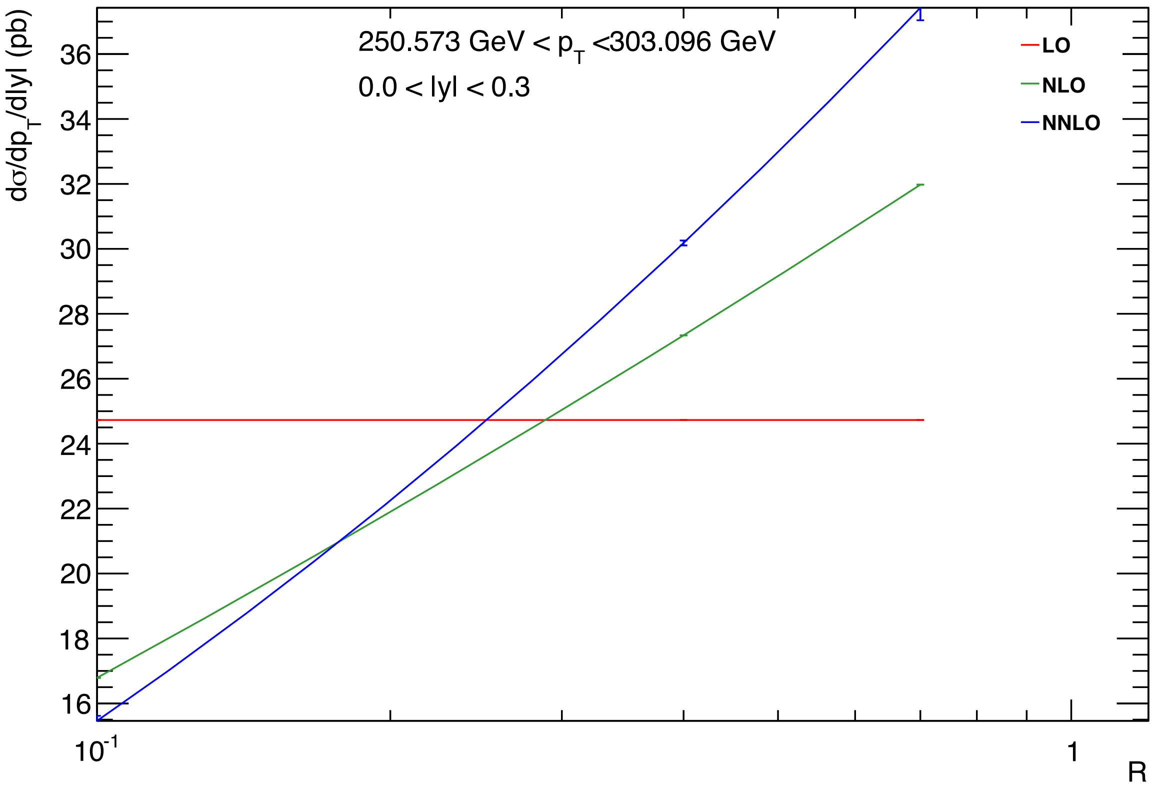
R = 0.7





NNLO corrections to pp → 2j

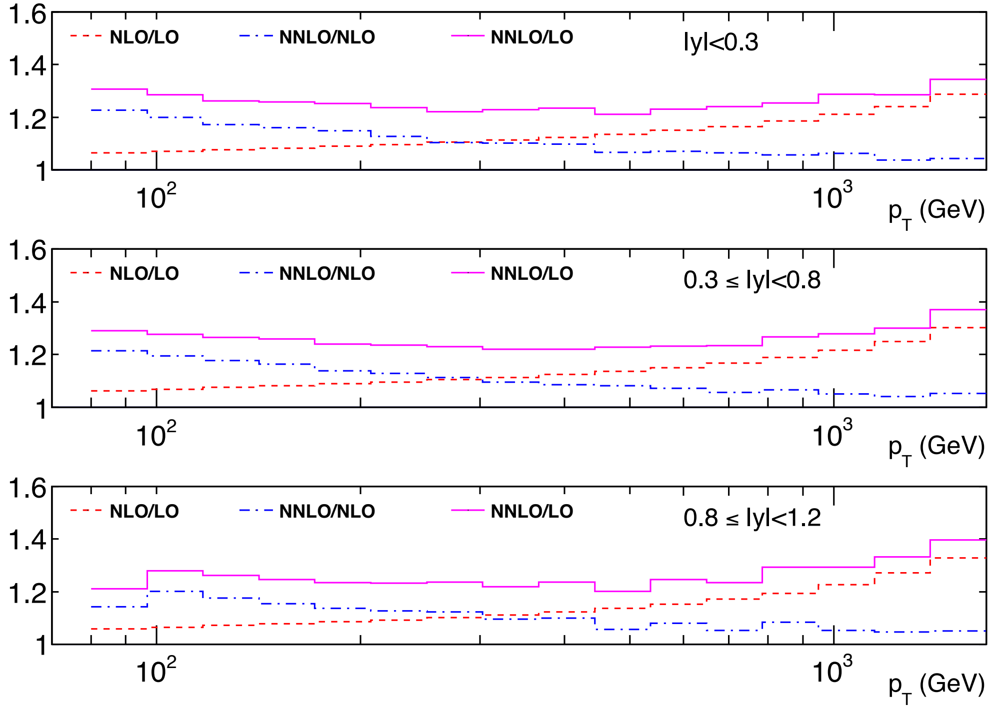
Single-jet inclusive: jet size dependence in anti-k_T algorithm

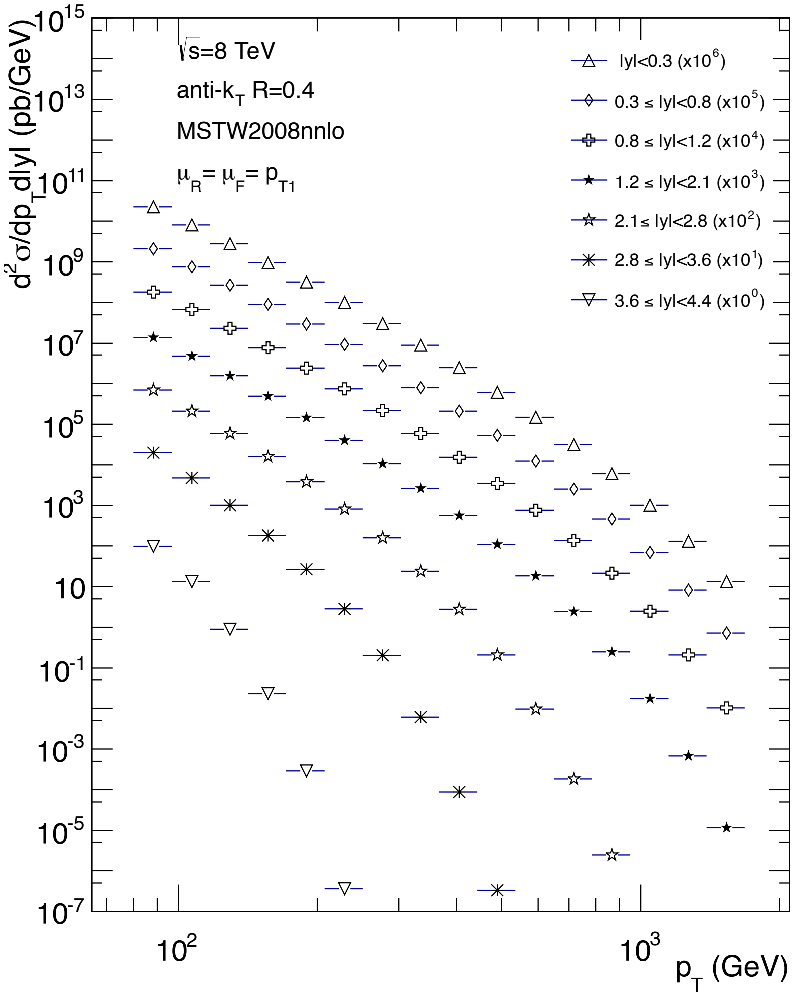


NNLO corrections to pp → 2j

Inclusive jet production:
double differential distributions

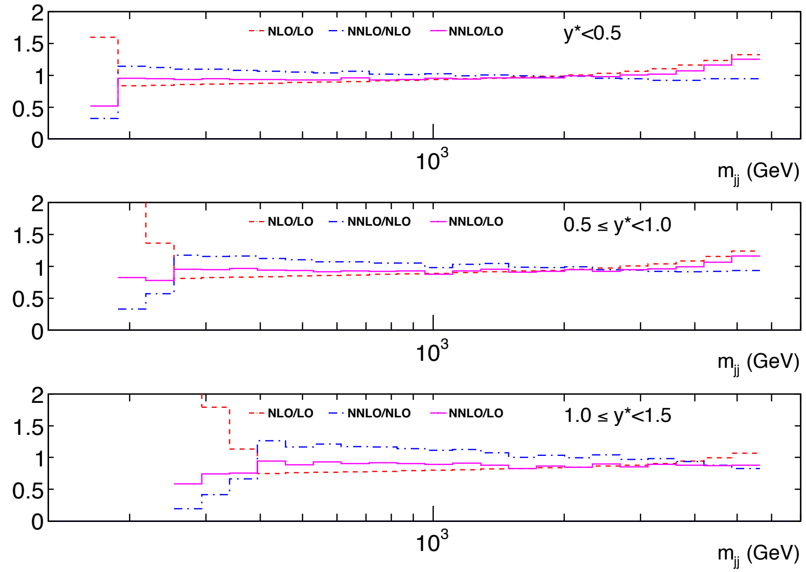
R = 0.4

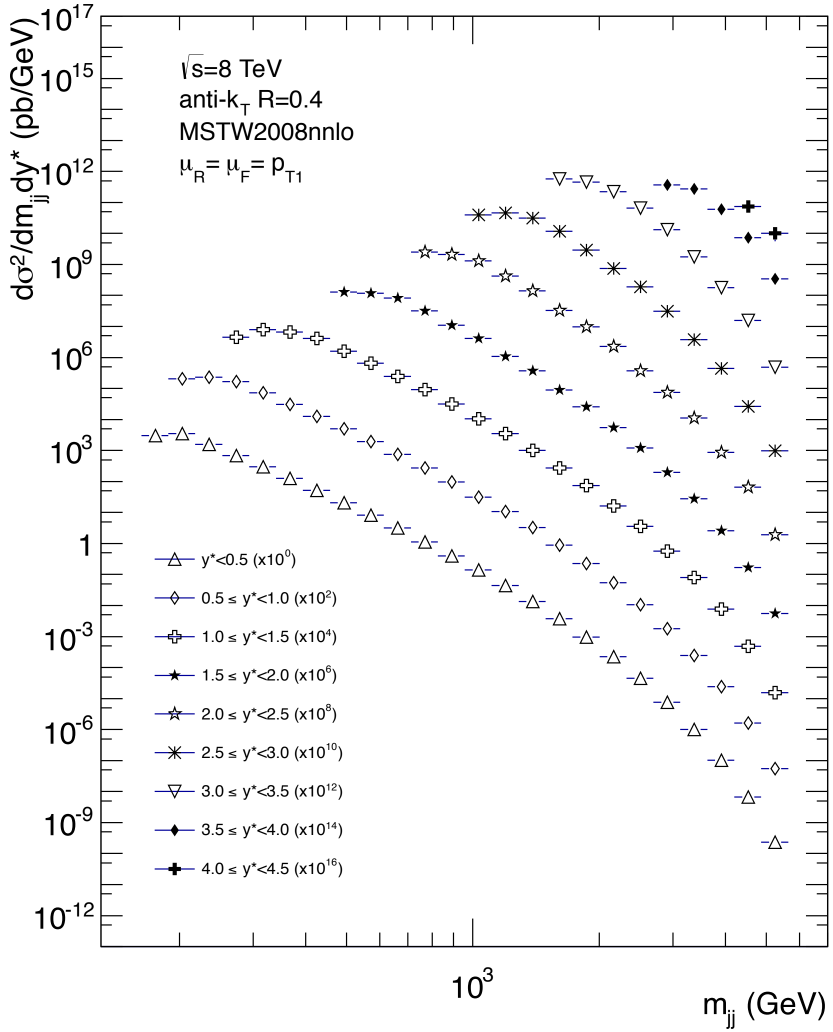




NNLO corrections to pp → 2j

Exclusive di-jet production





Outlook: next steps

- Current status of pp → 2j: gluons only
  - Serves as proof-of-principle
  - Implementation of all parton-level processes at NNLO
    (J. Currie, E.W.N. Glover, S. Wells)
  - Towards automated generation of subtraction terms
    - Systematic understanding of infrared cancellations
      (J. Currie, E.W.N. Glover)
- Other processes of similar complexity: 2 → 2
  - pp → H+j
  - pp → V+j
- Higher-multiplicity processes: two-loop virtuals needed

Conclusions

- NNLO corrections to precision observables at LHC
  - Various methods have been applied successfully
  - Healthy competition between groups

- Current frontier: 2 $\rightarrow$ 2 QCD processes
  - Top quark pairs
  - Higgs-plus-jet cross section (gluons only)
  - Single-jet inclusive and di-jet cross sections (gluons only)

- Precision phenomenology with jet observables starting
  - Measurements of coupling constants
  - Determination of parton distributions