Phases of Gauge Theories

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F.S. - 0804.0182

Why ?

Beyond Standard Model Physics

Cosmology

Unexpected

Phase Diagrams of 4-Dimensional Gauge Theories

All orders (non)supersymmetric beta function

Ladder approximation

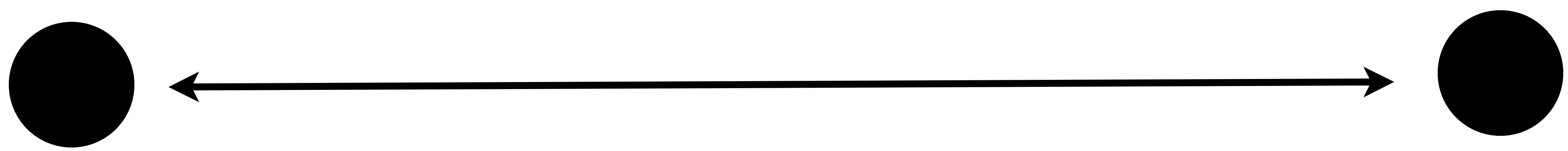
Lattice results

From Walking to Unparticle Physics

Alternative Large N limits

Phases of Gauge Theories

$$V(r)$$



Phases of Gauge Theories

Coulomb : $V(r) \propto \frac{1}{r}$

Free electric : $V(r) \propto \frac{1}{r \log(r)}$

Free magnetic : $V(r) \propto \frac{\log(r)}{r}$

$$\textbf{Higgs}: \qquad V(r) \propto \text{constant}$$

$$\textbf{Confining}: \qquad V(r) \propto \sigma r\ .$$

How to plot the Phase Diagram?

For SUSY Use:

The all orders beta function of NSVZ

Unitarity Bounds for Conformal Theories

Non-Renormalization of Superpotentials

't Hoofts Anomaly Matching Conditions

a-Maximization

Instanton Calculus

....

For Non SUSY Use:

Old Methods:

Schwinger - Dyson and any variation of it

Instanton Inspired Calculus (Ruled out!)

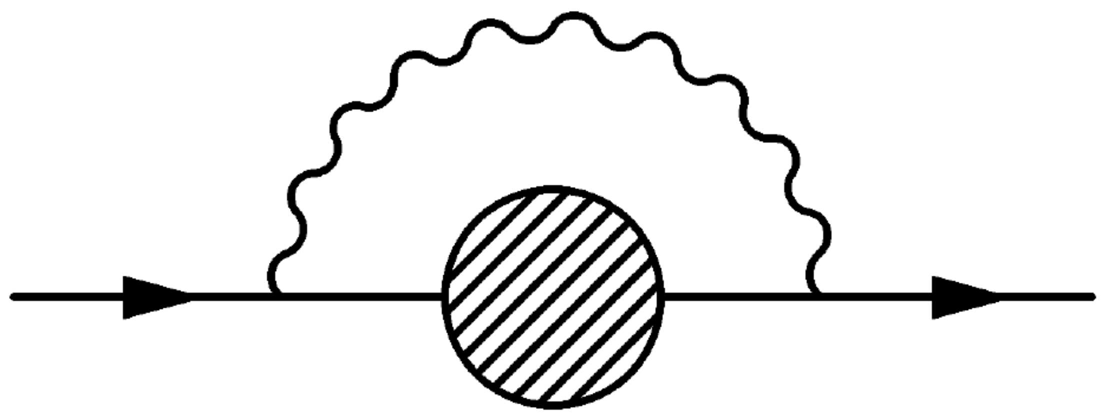
New Methods:

The all orders beta function conjecture

Unitarity of the Operators for Conformal Theories

First Principle Lattice Simulations

Rainbow - Schwinger-Dyson



The full nonperturbative fermion propagator reads:

$$iS^{-1}(p) = Z(p)\left(\not{p} - \Sigma(p)\right)$$

The Euclidianized gap equation in Landau gauge is:

$$\Sigma(p) = 3C_2(R) \int \frac{d^4k}{(2\pi)^4} \frac{\alpha\left((k-p)^2\right)}{(k-p)^2} \frac{\Sigma(k^2)}{Z(k^2)k^2 + \Sigma^2(k^2)}$$

$$Z(k^2) = 1$$

$$\beta(\alpha) \simeq 0 \qquad \alpha(\mu) \approx \alpha_c$$

The two solutions are:

$$\Sigma(p) \propto p^{-\gamma(\mu)} \, , \qquad \Sigma(p) \propto p^{\gamma(\mu)-2}$$

Running Hard Mass

Running Soft Mass

The critical coupling, i.e. when Chiral symmetry breaks is

$$\alpha_c \equiv \frac{\pi}{3C_2(R)}$$

$$\gamma = -d\ln m/d\ln\mu$$

$$\gamma(\mu) = 1 - \sqrt{1 - \frac{\alpha(\mu)}{\alpha_c}} \sim \frac{3C_2(R)\alpha(\mu)}{2\pi}$$

In practice, use the perturbative expressions and saturate:

$$\gamma(2-\gamma) = 1 \qquad \beta(\alpha) \simeq 0$$

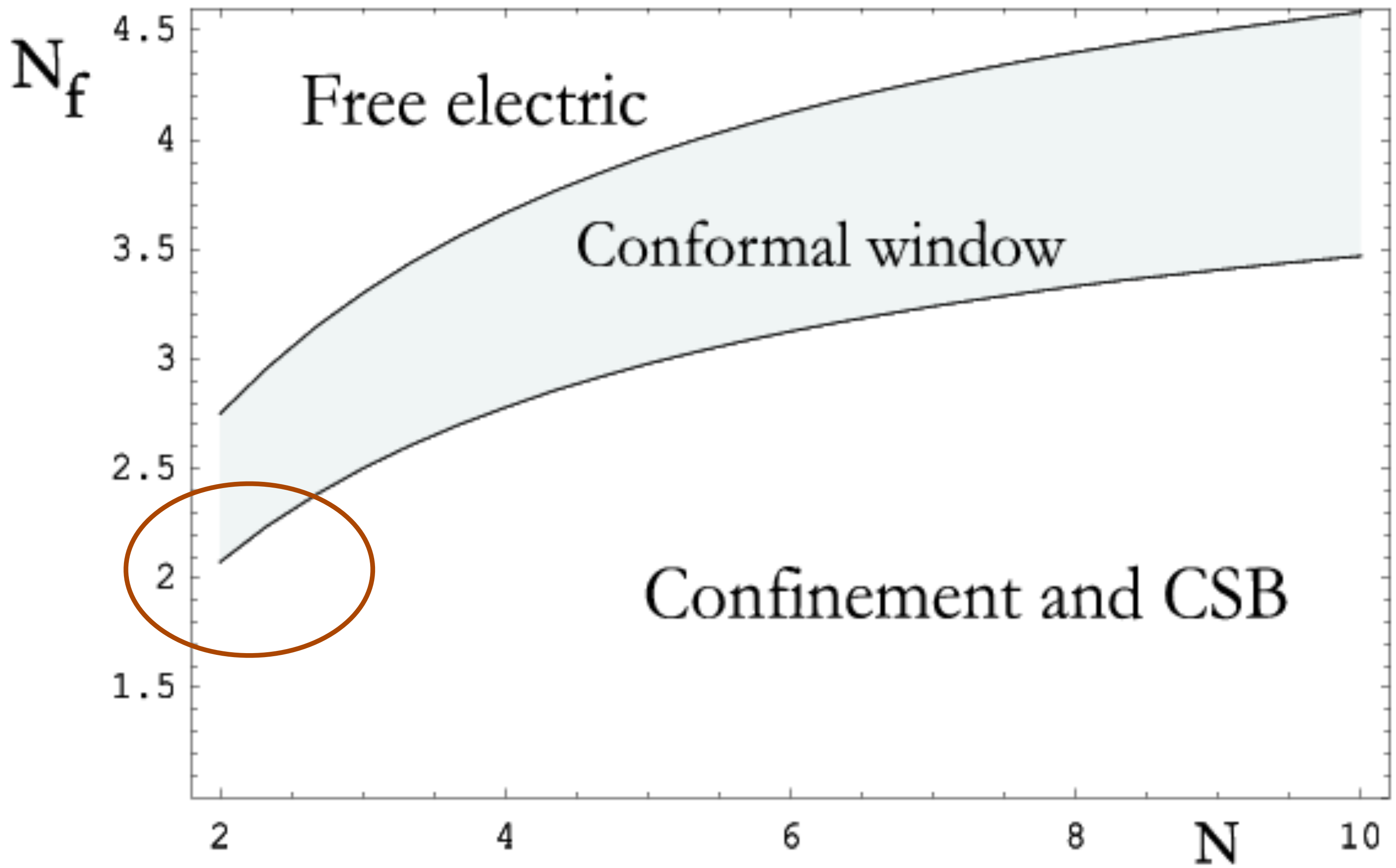
2-index Symmetric Theory

	$SU(N)$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	$U_A(1)$
$Q_{\{ij\}}$	$\Box\Box$	$\Box$	1	1	1
$\widetilde{Q}^{\{ij\}}$	$\overline{\Box\Box}$	1	$\overline{\Box}$	-1	1
G_μ	Adj	0	0	0	0

Here Q and $\widetilde{Q}$ are Weyl fermions.

The A-type is obtained by substituting $\Box\Box$ with $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$.

Phase Diagram for Symmetric Rep.

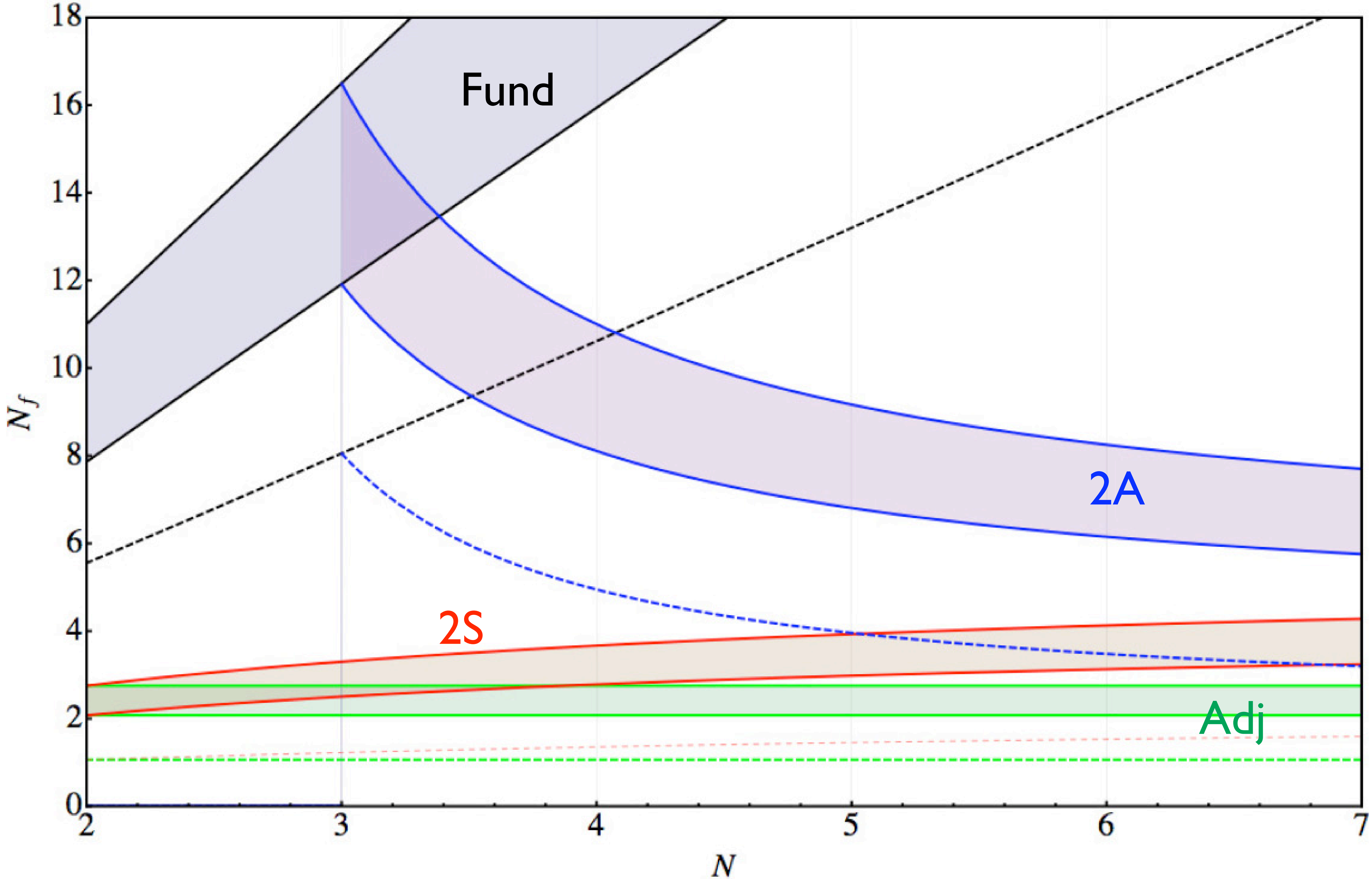


F.S. - Tuominen 04

Using Ladder Approximation

Is this the minimal walking theory?

Non-SUSY Phase Diagram for HDRs



Ladder approximation

Dietrich and F.S. 06

SUSY - Diagram

Exact Susy Beta Function

$$\beta(g) \;=\; -\frac{g^3}{16\pi^2}\frac{\beta_0 + 2T(r)N_f\gamma(g^2)}{1-\frac{g^2}{8\pi^2}C_2(G)}$$

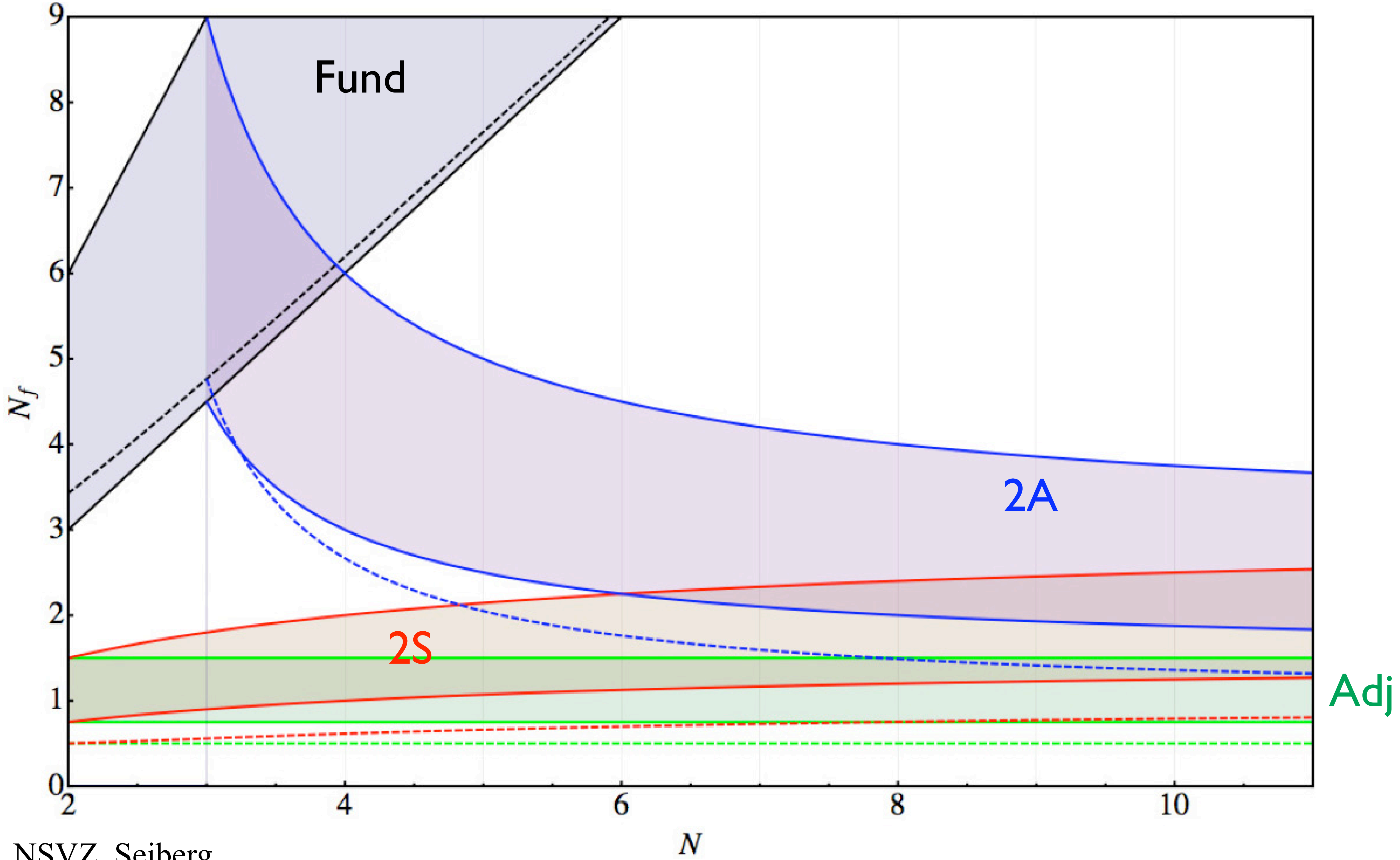
$$\gamma(g^2) \;=\; -\frac{g^2}{4\pi^2}C_2(r) + O(g^4)$$

$$\gamma(g^2) = -d\ln Z(\mu)/d\ln\mu$$

$$\beta_0 = 3C_2(G) - 2T(r)N_f$$

NSVZ

SUSY Phase Diagram for HDRs



NSVZ, Seiberg
Intriligator-Seiberg

Ryttov and F.S. 07

Beta Function

Ryttov and F.S. 07

All orders beta function conjecture

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3}\, T(r)\, N_f\, \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta_0'}{\beta_0}\right)}$$

$$\gamma = -d\ln m / d\ln \mu$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(r) N_f$$

$$\beta_0' = C_2(G) - T(r) N_f$$

Ryttov and F.S. 07

Recovering SYM

$$\beta(g) = -\frac{g^3}{(4\pi)^2} 3N \frac{1 - \frac{\gamma_{\rm Adj}}{9}}{1 - \frac{g^2}{8\pi^2}\frac{4N}{3}}$$

$$\beta_{SYM}(g) = -\frac{g^3}{(4\pi)^2} \frac{3N}{1 - \frac{g^2}{8\pi^2}N}$$

$$\gamma_{\rm Adj} = \frac{g^2}{8\pi^2} \frac{3N}{1 - \frac{g^2}{8\pi^2}N}$$

Ryttov and F.S. 07

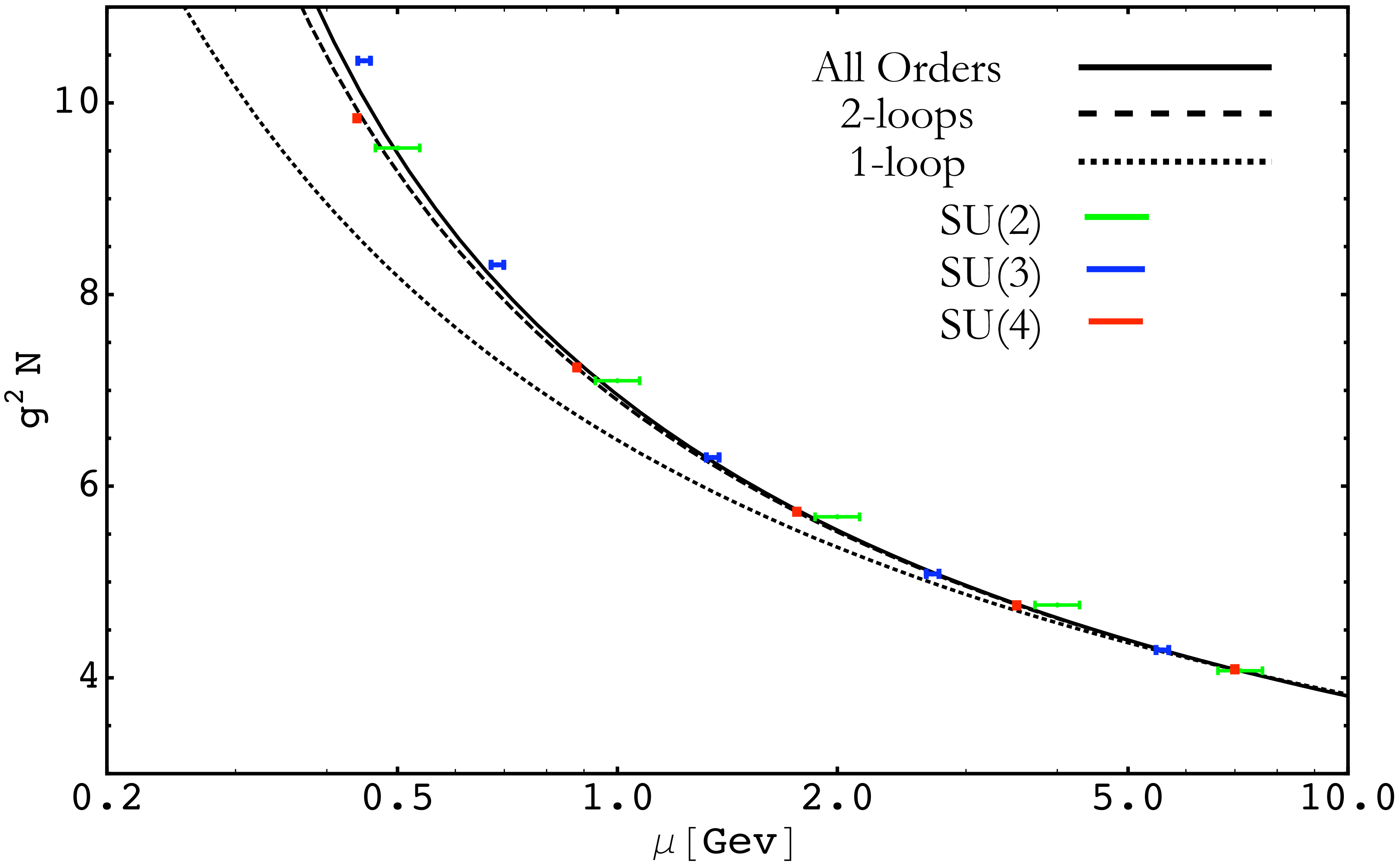
YM

$$\beta_{YM}(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0}{1 - \frac{g^2}{(4\pi)^2}\frac{\beta_1}{\beta_0}}$$

$$\beta_0 = \frac{11N}{3} \qquad \beta_1 = \frac{34N^2}{3}$$

Ryttov and F.S. 07

Running in Yang - Mills for different N



Luscher, Sommer, Wolff, Weisz, 92	SU(2)
Luscher, Sommer, Weisz, Wolff 94	SU(3)
Lucini and Moraitis 07	SU(4)

Bounds on the Conformal Window

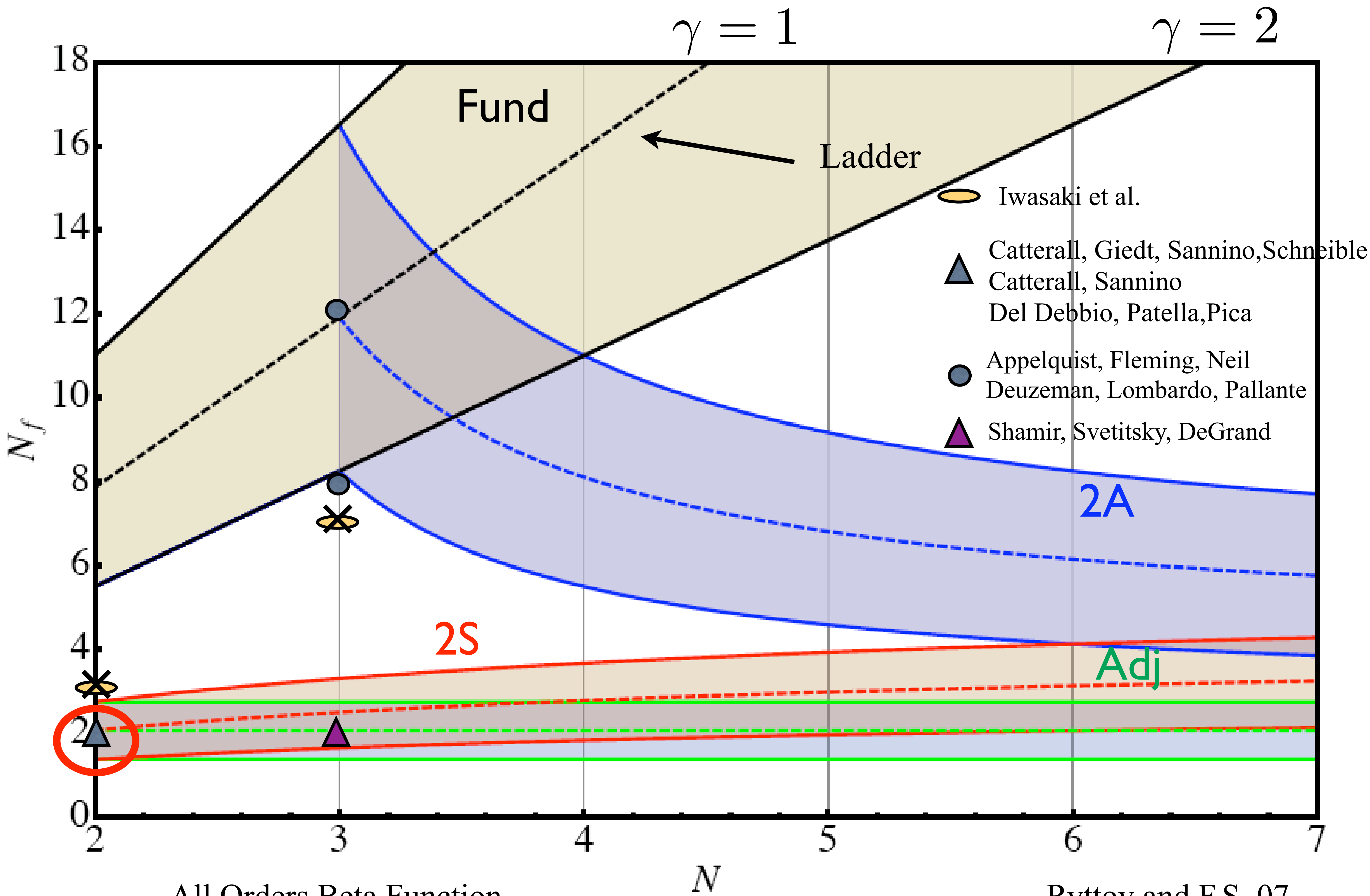
$$\beta = 0 \quad \longrightarrow \quad \gamma = \frac{11C_2(G) - 4T(r)N_f}{2T(r)N_f}$$

Unitarity of the Conformal Operators demands:

$$\gamma \leq 2$$

Ryttov and F.S. 07

Non-SUSY Phase Diagram Bound



All Orders Beta Function

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More Beta Functions

‘t Hooft 2-loops beta function (H)

2-index large N, Armoni, Shifman, Veneziano (ASV)

YM, Bochicchio, (B)

Comparison

	YM	2-index Large - N	Any-Rep.	Conforma Window
H	x	x	x	-
ASV	-	x	-	-
B	x	-	-	-
RS	x	x	x	x

A Measure in Theory Space

$$R_{FP} = \frac{A_{\text{Conformal}}}{A_{\text{AF}}}$$

Rep. independent in SUSY

$$R_{FP} = \frac{1}{2}$$

In non-SUSY is rep. independent within the ladder approximations

$$R_{FP}[F] = \frac{3}{11} \simeq 0.27 \ , \quad R_{FP}[G] = R_{FP}[A] = R_{FP}[S] = \frac{27}{110} \simeq 0.24$$

All order beta function for non-SUSY predicts $R_{FP} = \frac{1}{2}$

A New Universality ?

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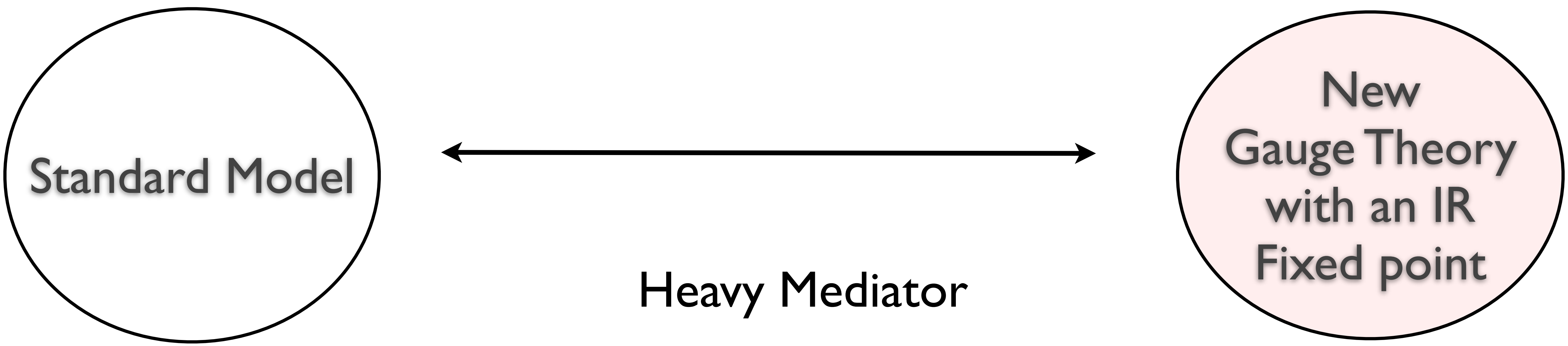
Unexpected

Landscape of the Unparticle World

Cartoon of the Unparticle World



Georgi 07

$$\frac{1}{M_{\mathcal{U}}^{d_{UV}+d_{SM}-4}} \mathcal{O}_{SM}\ \mathcal{O}_{UV} + \text{contact terms}$$

$$\frac{\Lambda_{\mathcal{U}}^{\gamma_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{UV}+d_{SM}-4}} \mathcal{O}_{SM}\ \mathcal{O}_{IR} \equiv \frac{\mathcal{O}_{SM}\ \mathcal{O}_{IR}}{{M'_{\mathcal{U}}}^{d_{UV}+d_{SM}-4-\gamma_{\mathcal{U}}}}$$

A simple example

$$\mathcal{O}_{UV} = \bar{\psi}\psi\,, \quad \text{with} \quad d_{UV} = 3$$

$$\mathcal{O}_{IR} \quad \text{with} \quad d_{IR} = 3 - \gamma_{\mathcal{U}}$$

$$\gamma_{\mathcal{U}} = \gamma$$

$$\frac{\mathcal{O}_{IR}}{M_Z^{3-\gamma}} \left[c_{S1} \bar{f} \gamma_\mu D^\mu f - \frac{c_{\gamma\gamma}}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

Use observations to constraint coefficients

The results depend on the value of the anomalous dimension

Use the Landscape:

"No nonsupersymmetric" spinorial type unparticle.

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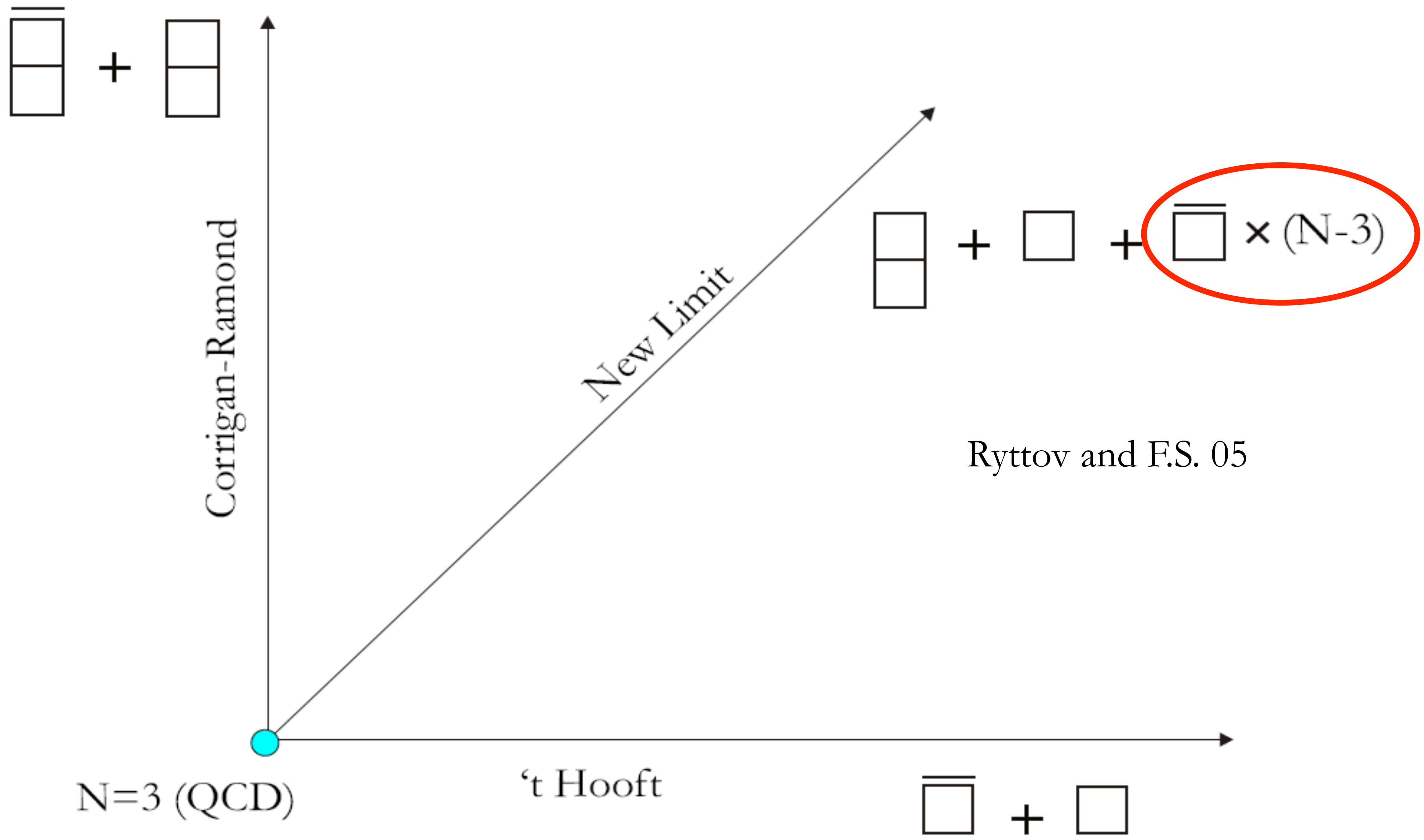
Information on nonperturbative aspects of HDRs

3 Large N Limits

Armoni-Shifman-Veneziano



F.S. 05, [Temperature]
Frandsen, Kouvaris, F.S 06, [Density]
F.S. & Schechter 07 [pion-pion scattering]
Kiritsis-Papavassiliou 90

Unsal, 07
Unsal, Yaffe, 06
Kovtun, Unsal, Yaffe 03

Higgsless versus Higgsful

Higgsless:

$$\frac{M_H}{M_V} > 1$$

Higgsful:

$$\frac{M_H}{M_V} \leq 1$$

Spectrum of Hadronic/Technihadronic States

Using 't Hooft Large N and Unitarity in Pion-Pion Scattering in QCD

Vector Meson is a quark-antiquark state: $\rho(770)$

Broad Sigma of multiquark nature $f_0(600)$

F.S. & Schechter, 95
Harada, F.S. and Schechter, 03
Caprini, Colangelo,Leutwyler 05
Maiani,Piccinini, Polosa, Riquer 04
F.S. and Schechter, 07

Higgsless: ‘t Hooft Extension

$$M_{T\rho} = \frac{\sqrt{2}v_0}{F_\pi}\frac{\sqrt{3}}{\sqrt{N_D N_{TC}}}m_\rho \qquad v_0 \sim 250\text{GeV}$$

$$M_{Tf_0} = \frac{\sqrt{2}v_0}{F_\pi\sqrt{N_D}}\left(\frac{N_{TC}}{\sqrt{3}}\right)^{\frac{p-1}{2}} m_{f_0} \qquad p \geq 1$$

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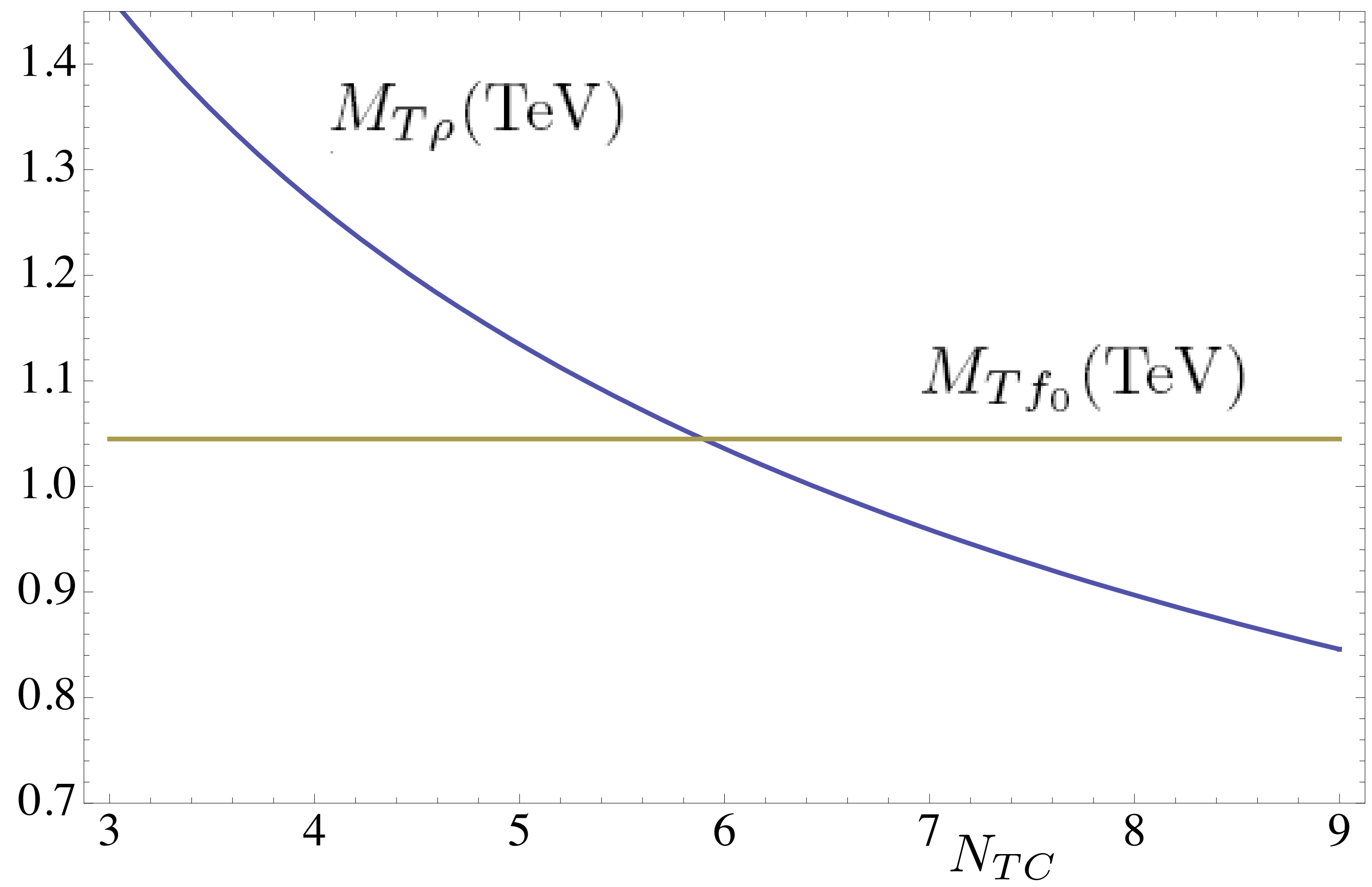
$$N_D = \frac{N_{TF}}{2}$$

$$N_D = 2$$

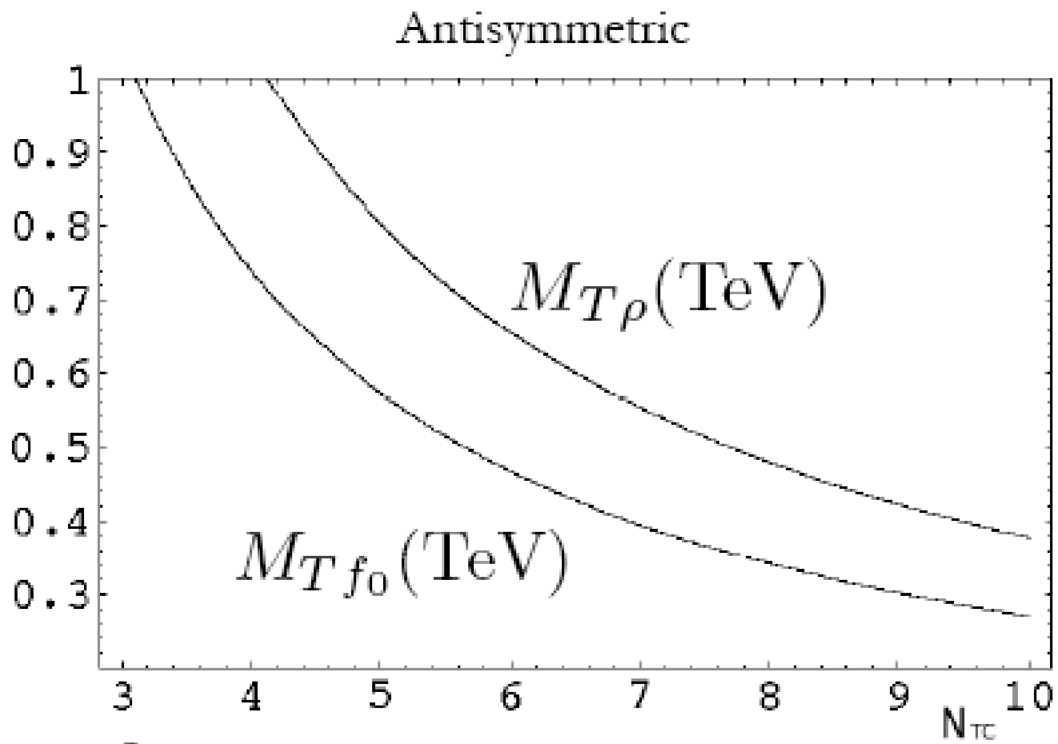


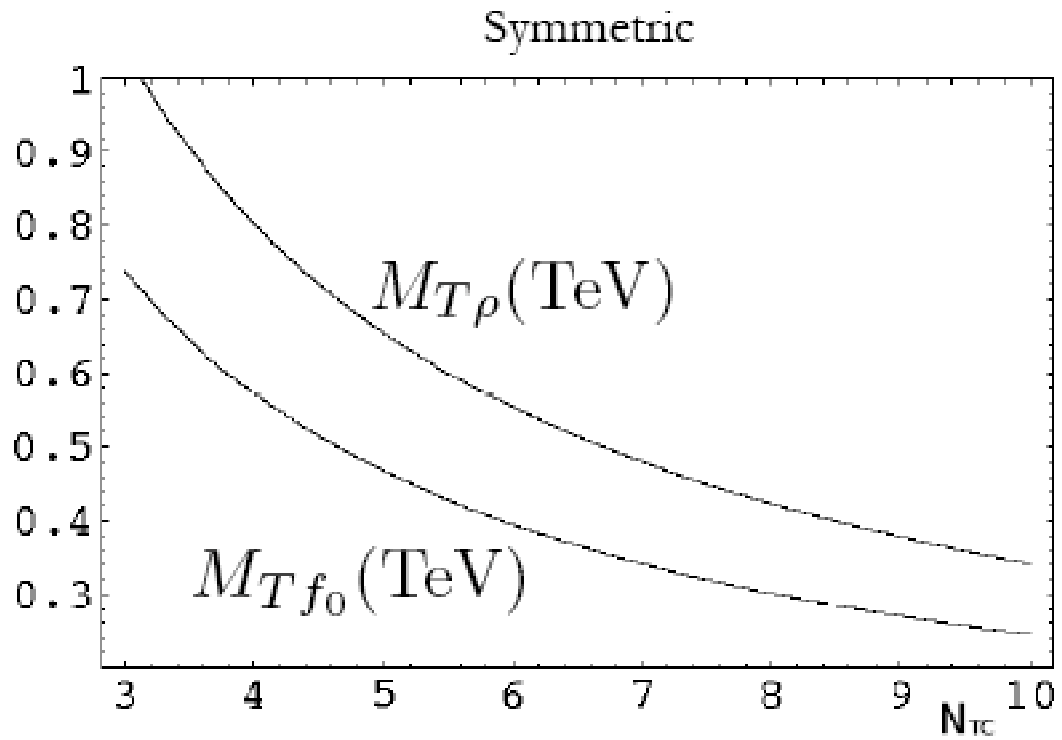
M_H/M_V < 1 in MWT theories

$$M_{T\rho} = \frac{\sqrt{2}v_0}{F_\pi}\frac{\sqrt{3}\sqrt{2}}{\sqrt{N_D\,N_{TC}(N_{TC}\mp 1)}}m_\rho$$

$$M_{Tf_0} = \frac{\sqrt{2}v_0}{F_\pi}\frac{\sqrt{3}\sqrt{2}}{\sqrt{N_D\,N_{TC}(N_{TC}\mp 1)}}m_{f_0}$$

F.S. 07





$N_D = 2$

Summary

- Phases of Gauge Theories
- Non Perturbative Methods to explore PGT
- All orders beta function
- Comparison with Lattice Data
- Unparticle Physics
- Alternative Large N limits
- Higgsless versus Higgsful