Open probems
in
gravitational physics

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Summary

- Foundation: gravity and space-time
- Shortcomings in General Relativity
- Alternatives, way out and extensions
- Metric or connections?
- The role of Equivalence Principle
- Testing EP at classical and quantum level
- Conclusions

Foundation: gravity and space-time

Einstein achieved a theory of gravity based on the following requirements:

- principle of equivalence → Gravity and Inertia are indistinguishable; there exist observers in free fall (inertial motion)
- principle of relativity → Special Relativity holds; the structure of the spacetime is pointwise Minkowskian
- principle of general covariance → “democracy” in Physics
- principle of causality → all physical phenomena propagate respecting the light cones
- Riemann’s teachings about the link between matter and curvature →

Foundation: gravity and space-time

Mathematical consequences:

- principle of equivalence → Inertial motion = geodesic motion
- principle of relativity → the spacetime M is endowed with a Lorentzian metric g
- principle of general covariance → tensoriality
- principle of causality → light cones structure generated by the metric g
- Riemann's teachings about the link between matter and curvature → the gravitational field is described by g → 10 equations
  Riem(g) has 20 (independent) components: too many!
  Ric(g) has 10 (independent) components: OK!

Foundation: gravity and space-time

The distribution of matter influences Gravity through 10 second order equations, the Einstein equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\,\pi\, G\, T_{\mu\nu}$$

A linear concomitant of the Riemann tensor, the Einstein tensor, equals the stress-energy tensor that reflects the properties of matter.

They have a structure that suitably reduces to Newtonian equations in the "weak field limit."

Foundation: gravity and space-time

Einstein was not happy with the fact that the gravitational field is not the fundamental object, but just a by-product of the metric. Using a method invented few years before by Attilio Palatini, he realizes that one can obtain field equations by working on a theory that depends on *two* variables, varied independently:

a metric g and a linear connection *Γ* assumed to be symmetric.

$$R \equiv R(g, \Gamma) = g^{\mu\nu} R_{\mu\nu}(\Gamma) \qquad \mathcal{L}_{PE} = g^{\mu\nu} R_{\mu\nu}(\Gamma, \partial\Gamma)$$

There are 10 + 40 independent variables and the equations are:

$$R_{(\mu\nu)} - \frac{1}{2} R\, g_{\mu\nu} = \kappa\, T_{\mu\nu}$$

$$\nabla_\alpha^\Gamma \left(\sqrt{g} g^{\mu\nu}\right) = 0$$

Shortcomings in General Relativity

Is still *g* the fundamental object of Gravity?

Einstein tries to consider directly the connection as the fundamental object of Gravity, but he never completed the process of "dethronizing" *g*. Similar attempts by Weyl….

Shortcomings in General Relativity

But after all, what are the problems with GR?

GR is *simple*, beautiful.. but seems to be not self-consistent at all scales:

- cosmological constant Λ
- Inflation
- Dark Matter + Dark Energy
- Quantum Gravity
- Consistency of EP at classical and quantum level

Today observations say that there is too few matter in the Universe! Thence the need, in order to save GR, for dark energy and dark matter:

$$G_{\mu\nu} = T_{\mu\nu} + T_{\mu\nu}^{dark}$$

Is there any way out to these shortcomings?

Alternatives, way out and extensions

A straightforward extension is considering f(R)-gravity where the Hilbert Lagrangian is replaced by any non-linear function of Ricci scalar (Starobinsky 1980, Capozziello 2002).

In these theories there is a second order part that resembles Einstein tensor (and reduces to it if and only if f(R) = R) and a fourth order "curvature part" (that reduces to zero if and only if f (R) = R):

$$f'(R(g))\,R_{\mu\nu}(g) - \frac{1}{2}f(R(g))g_{\mu\nu} - \nabla_\mu\nabla_\nu f'(R(g)) + g_{\mu\nu}\Box f'(R(g)) = \kappa\, T_{\mu\nu}$$

Higher order Gravity (4^{th})!

Pushing the 4th order part to the r.h.s. we get an "extra curvature stress-energy tensor" $T^{\text{curv}}_{\mu\nu}$

f(R)-gravity can be recast as scalar-tensor theories so the paradigm is that higher order terms can be dealt as scalar fields.

Alternatives, way out and extensions

From f(R) theories, GR is retrieved in (and only in) the particular case f(R)=R.

$f'(R) = 1$

$f''(R) = 0$

Second Order

Field Equations

degenerate theory

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 4\pi T_{\mu\nu}$$

Gravitational contribution

Matter contribution

Two approaches to the shortcomings: From Matter Side or from Gravitational Side!!!

$$G_{\mu\nu} = (8\pi G) T_{\mu\nu}$$

$G_{\mu\nu} \Rightarrow \tilde{G}_{\mu\nu}$

$T_{\mu\nu} \Rightarrow \tilde{T}_{\mu\nu}$

Extended theories of Gravity	Dark Energy, Dark Matter and Λ

WHY?

- QFT on curved spacetimes
- String/M-theory corrections
- Brane-world models, gravitational Higgs sector

$R,\ R^{\mu\nu}R_{\mu\nu},\ R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta},\ R\Box^{l}R,$

Curvature invariants or scalar fields have to be taken into account

- Cosmological constant (Λ)
- Time varing Λ
- Scalar field theories
- Phantom fields
- Phenomenological Theories
- Exotic matter

Metric or connections?

Let's return to our Questions:

- Was Einstein right in assuming the metric *g* of the space-time as the fundamental object to describe Gravity?

  What is the role of the connection Γ?

Metric or connections?

When Einstein formulated GR, the only geometrical field he could use was a (Lorentzian) metric *g*, the structure that Gauss (1830) and Riemann (1856) introduced in surfaces and higher-dimensional manifolds to define curvature.

At that time he had no other choice!

In GR $\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}_g$ are not equations. They express a founding issue.

Assumption on space-time structure: there is a connection Γ ; this connection has no dynamics; it is *a priori* the Levi-Civita connection of the metric *g*. Only *g* has dynamics. So the single object *g* determines at the same time the causal structure (light cones), the measurements (rods and clocks) and the free fall of test particles (geodesics). Spacetime is a couple (M,g).

Even if it was clear to Einstein that Gravity induces “freely falling observers” and that the principle of equivalence selects an object that cannot be a tensor, i.e. Γ , he was obliged to choose it as determined by the metric structure.

Metric or connections?

When in 1919 Levi-Civita introduced connections, Einstein had another choice. But he did not really take it. Why?

In Palatini's formalism, connection Γ and metric g are independent. Spacetime is a triple (M,g,Γ) where the metric determines rods and clocks (i.e., it sets the fundamental measurements of spacetime) while Γ determines the free fall

The second equation tells us *a posteriori* that Γ is the Levi-Civita connection of g. The first equation is then turned into the standard Einstein equation. That is why Einstein considered metric the fundamental object of Gravity

.. But this coincidence (between Γ and the Levi-Civita's connection of g) is due to the particular Lagrangian considered by Einstein, which is the *simplest*… but not the only one possible! In f(R)-gravity and other extended gravities g and Γ *can be independent!!*

Metric or connections?

Palatini method privileges the affine structure towards the metric structure

The Palatini Lagrangian contains only derivatives of Γ, that is the real dynamical field. The metric g has no dynamics since it enters the Lagrangian as a "Lagrange multiplier"

The metric g gains DYNAMICS from Γ !!

Dynamics of Γ tells us that a sort of Einstein's equation holds for the Ricci tensor of Γ

Dynamics is obtained by varying the Lagrangian with respect to the metric. These are 10 equations. Other 40 equations come out varying the Lagrangian with respect to the connection Γ. These additional equations govern the form of Γ and impose it to be the Levi-Civita connection of the metric. The first equation then transforms into the Einstein Equations.

In Palatini's formalism $\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}_g$ are now field equations.

The fact that Γ is the Levi-Civita connection of g is no longer an assumption but becomes the outcome of field equations!

Metric or connections?

Among the different Theories of Gravitation, we really should prefer the simplest (in the sense of the one with the simplest Lagrangian)?

Metric or connections?

The universality properties discovered for non-linear theories of Gravitation, written under the Palatini form, tell us that the true dynamical field is Γ and not the metric *g*.

The metric *g* is no longer a Lagrange multiplier, but still has no dynamics since it enters algebraically the Lagrangian. However *g* gains dynamics from the dynamics of the connection Γ.

The connection is the Gravitational Field and it is the fundamental field in the Lagrangian. The metric *g* enters the Lagrangian with an "ancillary role."

It reflects the fundamental need we have to define lenghts and distances, as well as areas and volumes. It defines rods & clocks, that we use to make experiments. It defines also the causal structure of spacetime.

But it has no dynamical role.

There is no reason whatsoever to assume *g* to be the potential for Γ.

Nor that it has to be a true field just because it appears in the action!

Metric or connections?

… but the Equivalence Principle selects the family of geodesics of Γ, that become "more fundamental" than the metric structure of g!

Equivalence Principle selects the true dynamical field. Rods & clocks follow up.

DISCRIMINATION AMONG COMPETING THEORIES OF GRAVITY CAN BE ACHIEVED BY THE KEY ROLE OF EQUIVALENCE PRINCIPLE!

The role of Equivalence Principle

We need to investigate the EP:

- discriminating among theories of gravity
- its validity at classical and quantum level
- investigating geodesic and causal structures

The role of Equivalence Principle

Einstein Equivalence Principle states:

- Weak Equivalence Principle is valid;
- the outcome of any local non-gravitational test experiment is independent of velocity of free-falling apparatus;
- the outcome of any local non-gravitational test experiment is independent of where and when in the Universe it is performed.

The role of Equivalence Principle

One defines as “local non-gravitational experiment” an experiment performed in a small-size freely falling laboratory

One gets that the gravitational interaction depends on the curvature of space-time, i.e. the postulates of any metric theory of gravity have to be satisfied

- space-time is endowed with a metric $g_{\mu\nu}$;
- the world lines of test bodies are geodesics of the metric;
- in local freely falling frames, called local Lorentz frames, the non-gravitational laws of physics are those of Special Relativity.

The role of Equivalence Principle

One of the predictions of this principle is the gravitational red-shift, experimentally verified by Pound and Rebka in 1960

Gravitational interactions are excluded from WEP and Einstein EP

In order to classify alternative theories of gravity, the Gravitational WEP and the Strong Equivalence Principle (SEP) has to be introduced

The role of Equivalence Principle

The SEP extends the Einstein EP by including all the laws of physics in its terms:

- WEP is valid for self-gravitating bodies as well as for test bodies (Gravitational Weak Equivalence Principle);
- the outcome of any local test experiment is independent of the velocity of the free-falling apparatus;
- the outcome of any local test experiment is independent of where and when in the Universe it is performed.

The SEP contains the Einstein Equivalence Principle, when gravitational forces are neglected.

The role of Equivalence Principle

Many authors claim that the only theory coherent with the SEP is GR

An extremely important issue is related to the consistency of EP with respect to the Quantum Mechanics.

Some phenomena, like neutrino oscillations could violate it at quantum level, if induced by the gravitational field.

GR is not the only theory of gravitation and, several alternative theories of gravity have been investigated from the 60's, considering the space-time to be "special relativistic" at a background level and treating gravitation as a Lorentz-invariant field on the background

The role of Equivalence Principle

Two different classes of experiments can be considered:

- the first ones test the foundations of gravitational theories (among them the EP)

- the second ones test the metric theories of gravity where space-time is endowed with a metric tensor and where the Einstein EP is valid.

For several fundamental reasons extra fields might be necessary to describe the gravitation, e.g. scalar fields or higher-order corrections in curvature invariants.

The role of Equivalence Principle

Two sets of equations can be distinguished

- The first ones couple the gravitational fields to the non-gravitational contents of the Universe, i.e. the matter distribution, the electromagnetic fields, etc...

- The second set of equations gives the evolution of non-gravitational fields.

Within the framework of metric theories, these laws depend only on the metric: this is a consequence of the EEP and the so-called ”minimal coupling”.

Several theories are characterized by the fact that a scalar field (or more than one scalar field) is coupled or not to gravity and ordinary matter

There are several reasons to introduce scalar fields:

* Scalar fields are unavoidable for theories aimed to unify gravity with the other fundamental forces: e.g. Superstring, Supergravity (SUGRA), M-theories.

* Scalar fields appear both in particle physics and cosmology:
  - the Higgs boson in the Standard Model
  - the dilaton entering the supermultiplet of higher dimensional gravity
  - the super-partner of spin ½ in SUGRA.

- The introduction of a scalar field gives rise typically to a possible “violation” of the Einstein Equivalence Principle (EEP).

In order to distinguish competing theories, a possibility is related to the so-called “fifth force” approach.
For example, the case of f(R)-gravity:

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{X}\mathcal{L}_m\right],$$

$$\mathcal{X} = \frac{16\pi G}{c^4}$$

The variation with respect to the metric tensor gives

$$f'R_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - f'_{;\mu\nu} + g_{\mu\nu}\Box f' = \frac{\mathcal{X}}{2}T_{\mu\nu}$$

$$3\Box f' + f'R - 2f = \frac{\mathcal{X}}{2}T$$

Trace equation

$$f(R) = \sum_n \frac{f^n(R_0)}{n!}(R - R_0)^n \simeq f_0 + f_1 R + f_2 R^2 + f_3 R^3 + \ldots .$$

In the Newtonian limit,
let us consider the perturbation of the metric with respect to the Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

The metric entries can be developed as

$$\begin{cases} g_{tt}(t,r) \simeq 1 + g_{tt}^{(2)}(t,r) + g_{tt}^{(4)}(t,r) \\ g_{rr}(t,r) \simeq -1 + g_{rr}^{(2)}(t,r) \\ g_{\theta\theta}(t,r) = -r^2 \\ g_{\phi\phi}(t,r) = -r^2 \sin^2\theta \end{cases},$$

As general solution:

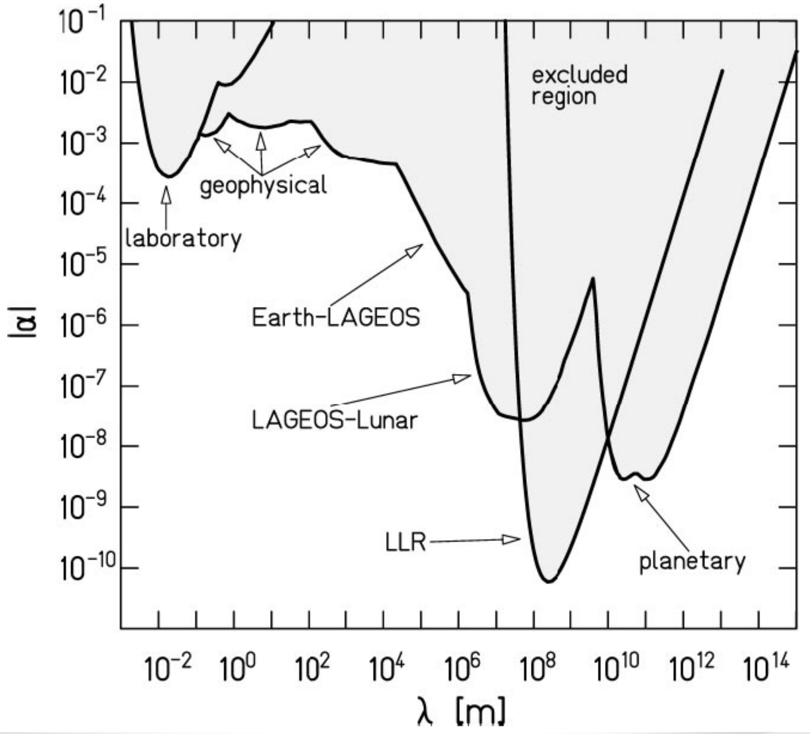
$$\Phi_{grav} = -\left(\frac{GM}{f_1 r} + \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{6\xi r}\right)$$

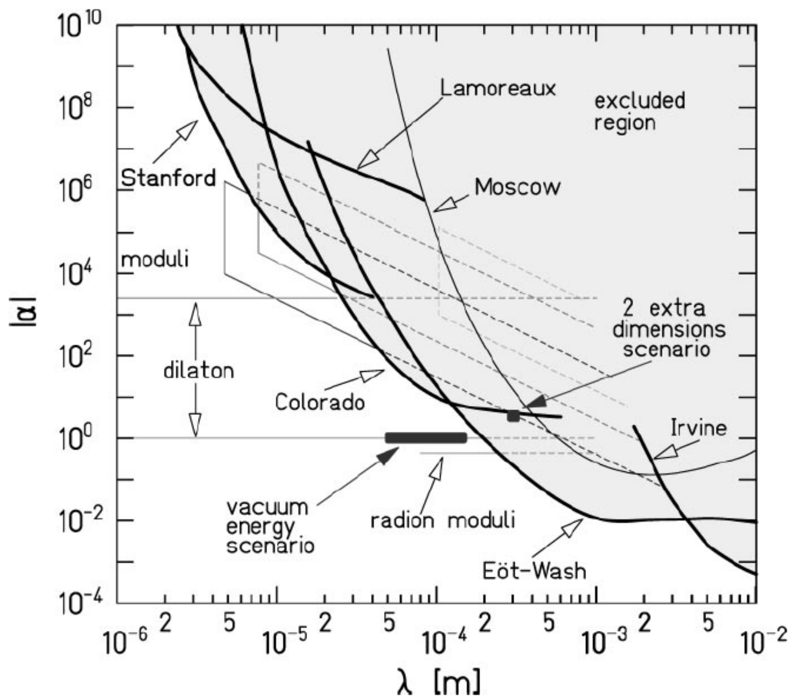
Fifth force $V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha e^{-r/\lambda}]$

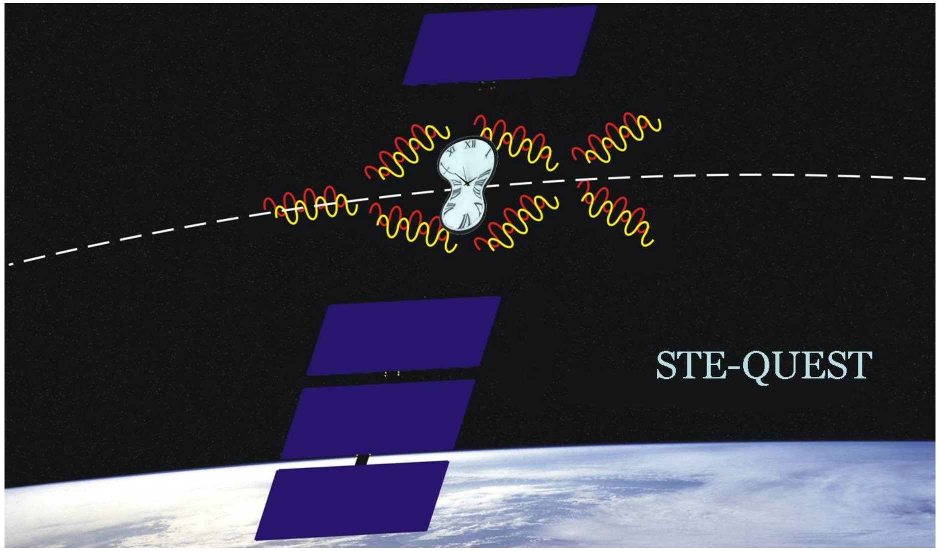
α is a dimensionless strength parameter

λ is a length scale or range

Experimental bounds







In conclusion, space and ground-based experiments should allow:

- to set and refine the bounds on space parameters
- to discriminate among competing theories
- dark side vs alternative gravities
- The role of ***g*** and ***Γ***
- fifth force
- to test EP and SEP at quantum level
- STE-QUEST experiment (next talk!)

……Wait and see…