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K_{l3} systematics (2004 data)

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Outline

- Last change in events selection
- Last (final?) central fit results for 3 parameterizations
- Systematic uncertainty of experimental nature

Last (hopefully) change in selection

Remainder:

from Manuel talk at NA48 weekly, 08/03/2012):

• A cut (for $K_{\mu 3}$ only) to define a new central value inside the range of systematical dependence of the fit results on DCH radius cut:
R_{track}(DCH) < 27 cm. (slide 20)

For K_{e3} there is no so large R(DCH) dependence, so for K_{e3} MM analysis stay at R_{track}(DCH) < 13 cm (we always use 15 cm).

In our April 2014 talk it has been shown, that if one apply the increased radius cut for R(DCH) < 27 cm for $K_{\mu 3}$, the problems with $K_{\mu 3}$ fit quality is solved, and $K_{\mu 3}$ — K_{e3} compatibility is reached.

But why?

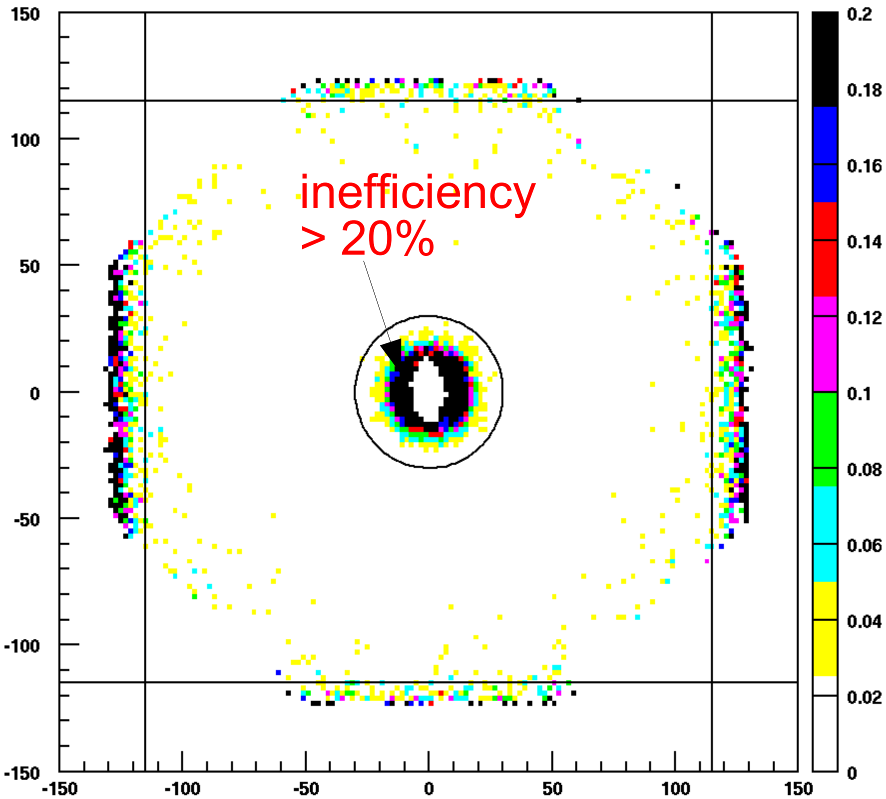
The important difference between $K_{\mu 3}$ and K_{e3} is MUV rather than DCH.

Let us look on MUV.

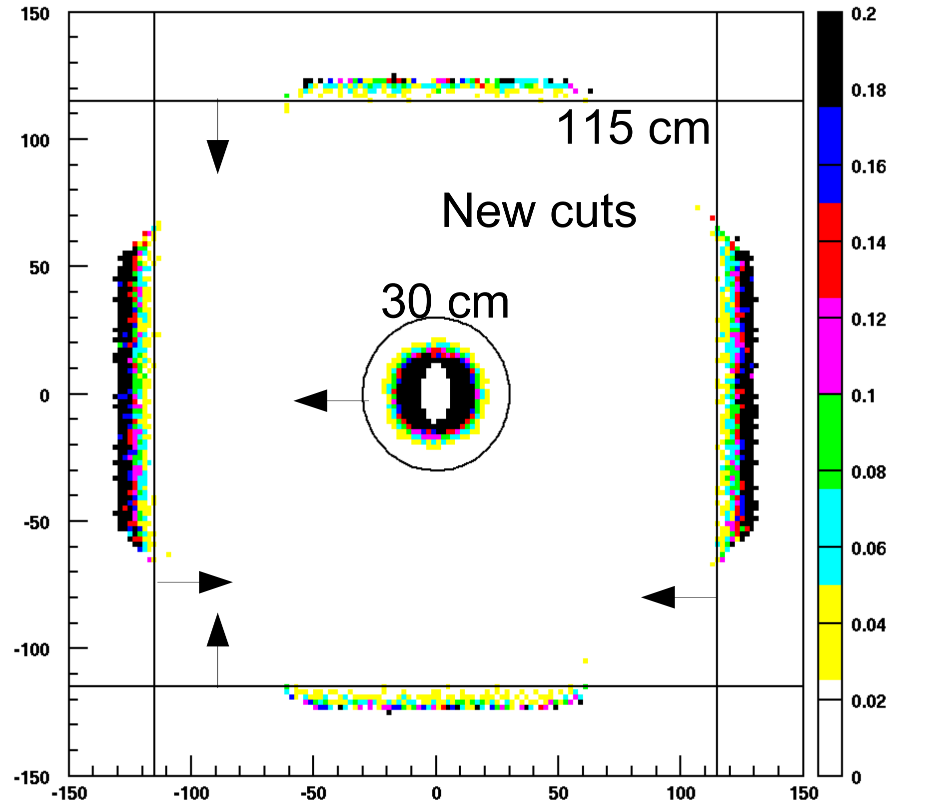
We have a multiple scattering of muon on the path from last DCH to MUV (~ 25 cm of transversal deviation). So for the track, that is measured from DCH, there are large MUV-inefficient zones near the MUV geometrical borders. And large inefficiency is difficult for MC simulation (beam geometry dependence, border efficiency details etc.)

How to check it?

- Remove MUV cut in $K_{\mu3}$ selection.
- Plot the DCH-measured track impact points on MUV plane, both for all selected $K_{\mu3}$ events and for events without MUV muon.
- Divide these two plots to see the MUV transversal behavior of resulting inefficiency



Data MUV inefficiency (plus background)

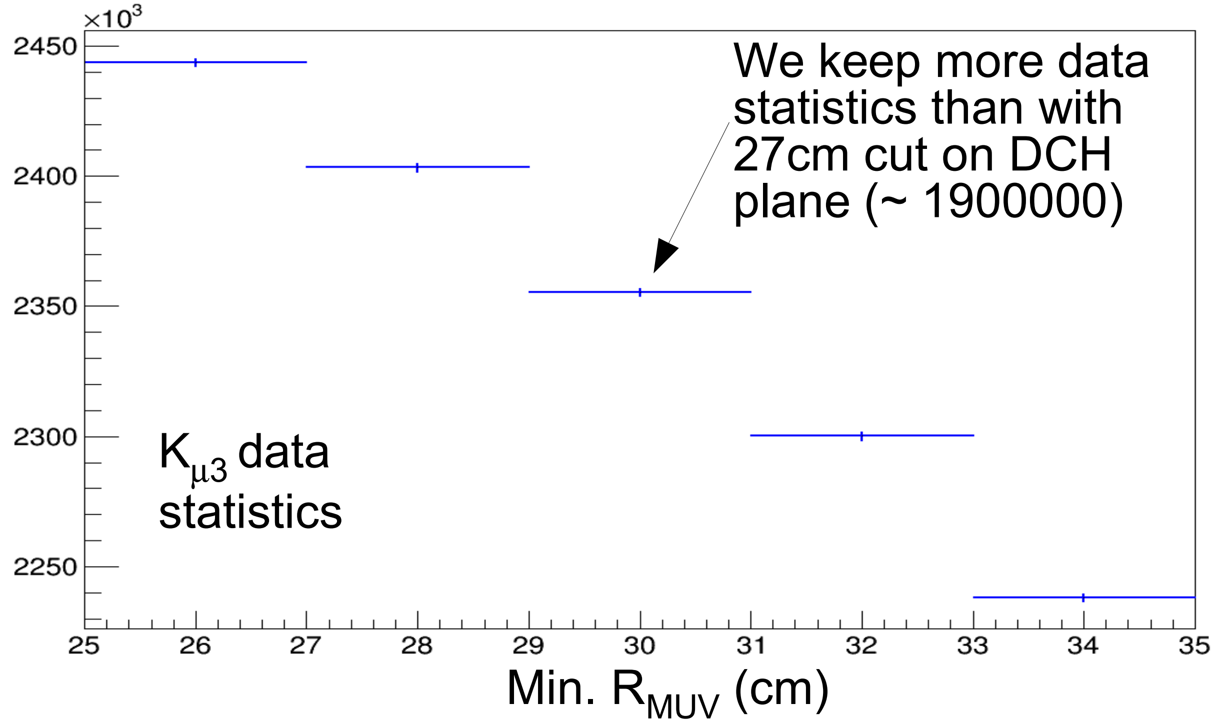


MC MUV inefficiency

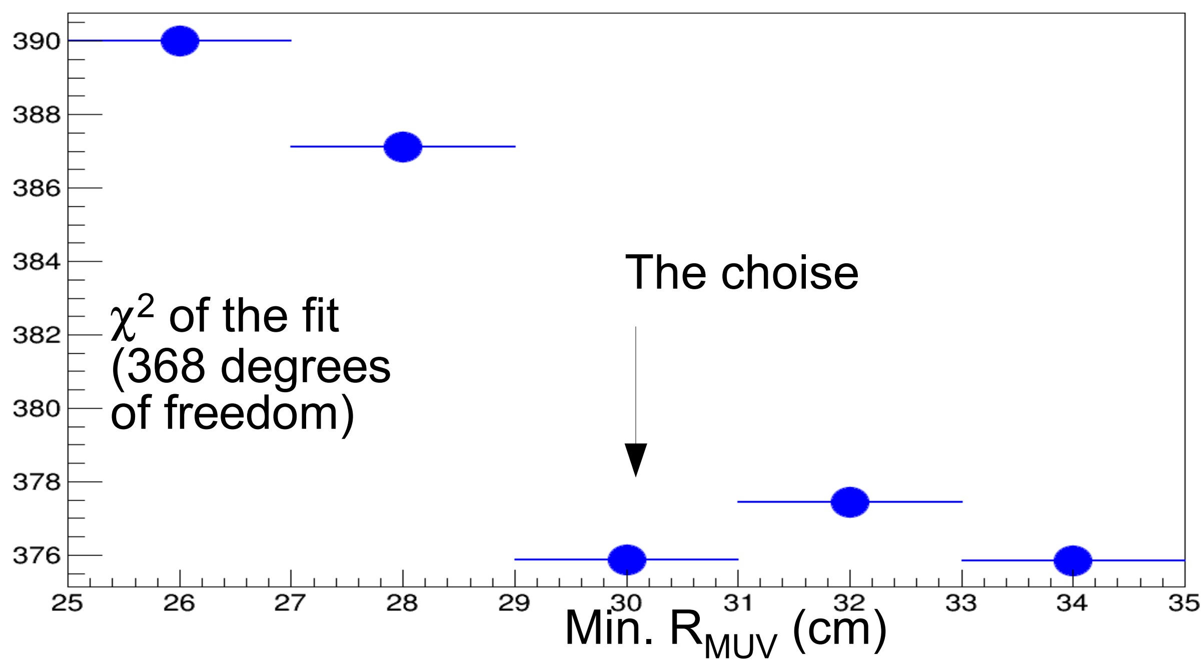
So we return back the DCH radius cut for $K_{\mu 3}$ to standard 15 cm value.

And apply the new cuts on MUV plane:

- |x,y| < 115 cm
- R_{MUV} > 30 cm



MUV minumum radius R_{MUV} cut choise (only for $K_{\mu 3}$):

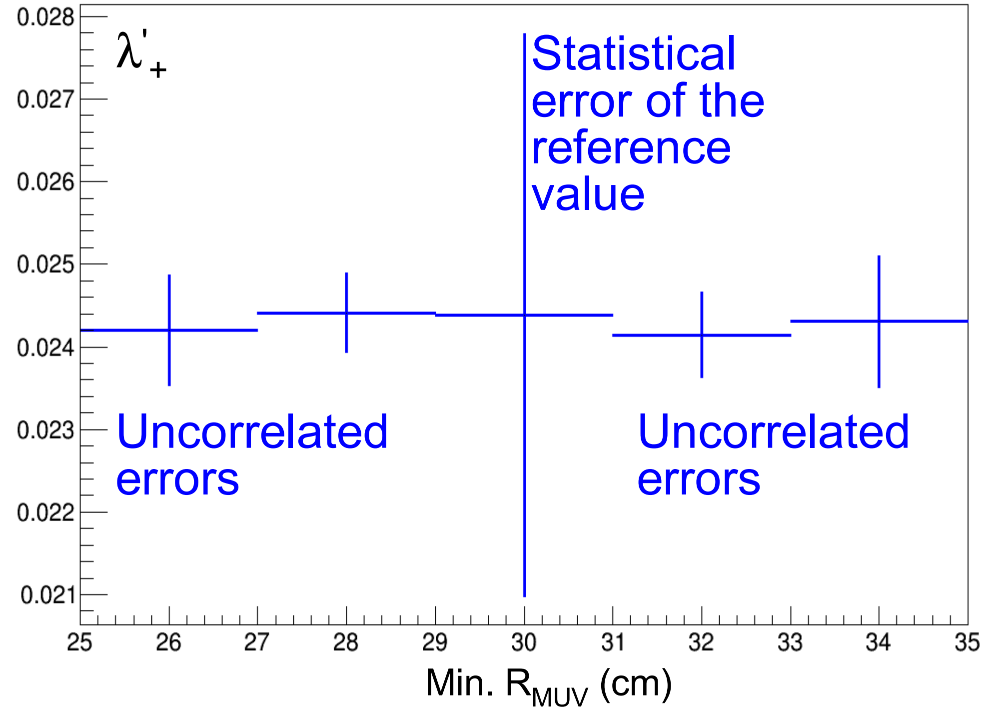


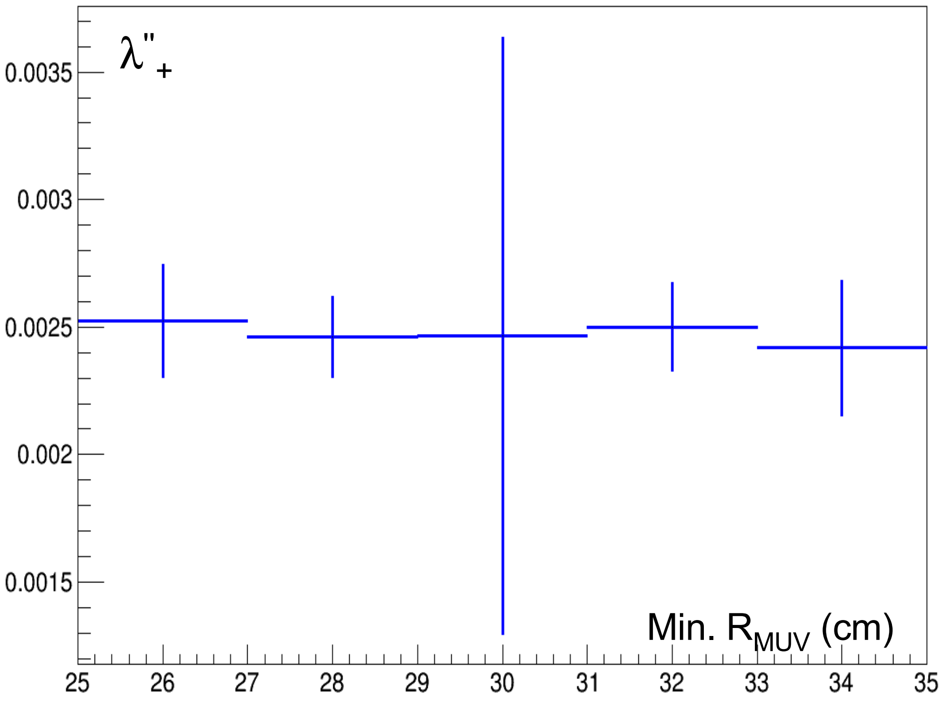
Results stability *vs* the cut value

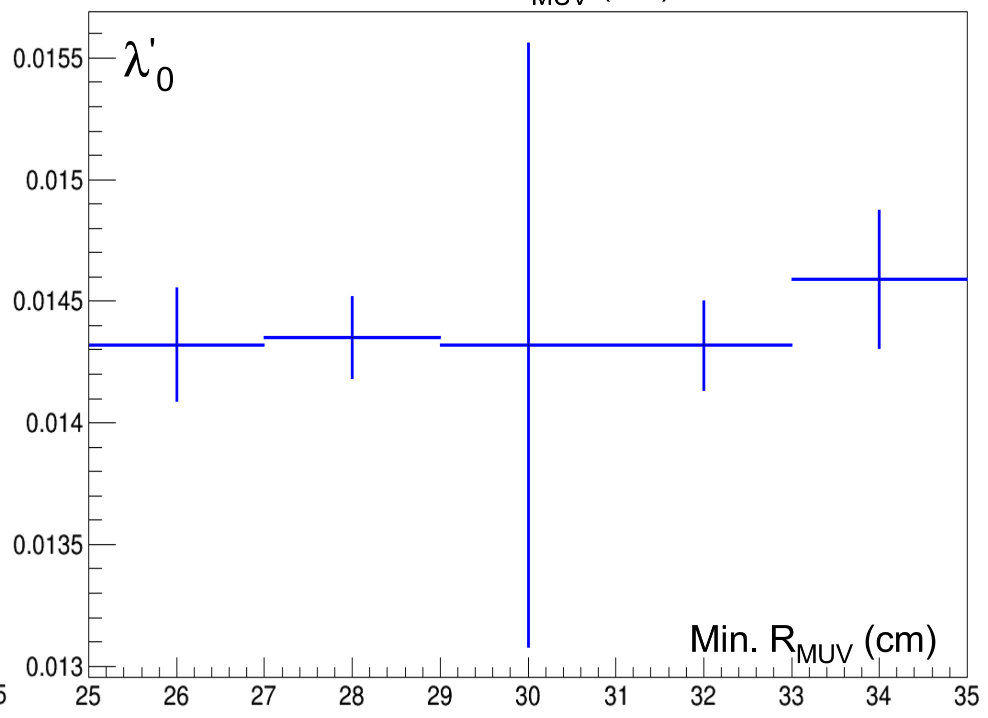
Uncorrelated error:
$\sigma_{un}^2 = |\sigma_{stat}^2 - \sigma_{ref}^2|$

Fit results are quite stable with the R_{MUV} cut variation. Shifts are within uncorrelated errors => compatible with purely statistical effect.

So we don't add the corresponding component to systematic uncertainty.







Selection (MC standard)

Min bias **trigger**: 1 track and E_{LKr} > 10 GeV ((sevt->trigWord >> 11) & 1)

N of good clusters > 1 :

- LKr standard nonlinearity correction for Data clusters (user_lkrcalcor_SC)
- LKr small final nonlinearity correction for MC clusters, from $\pi^+\pi^0\pi^0$
  (see April 2007 talk of Di Lella and Madigozhin)
- LKr scale corrections from K_{e3} E/P (different for Data and MC, subpermill precision)
- Cluster status <= 4
- Cluster energy >= 3 GeV
- Distance to dead cell >= 2 cm
- Radius at LKr >= 15 cm
- In LKr acceptance (defining outer cut for the full acceptance)
- Distance to any in-time (within 10 ns) track impact point at LKr >= 15 cm
- Distance to any another in-time (within 5 ns) cluster >= 10 cm

In Monte Carlo everithing is in-time

N of good tracks > 0 :

- Pe >= 5 GeV, Pμ >= 10 GeV (muon case cut applied after identification)
- Track momenta α,β corrections both for data and MC
- If there is the associated LKr cluster, its cluster status <=4
- Track quality >= 0.6
- Relative error of momentum measurement <= 6%
- Distance to dead cell >= 2 cm
- Radius at every DCH(1,2,3,4) >= 15 cm
- **Reject DCH tracks with 0 cm < X(DCH4) < 6 cm && Y(DCH4)>0** (inefficient band)
- **$K_{\mu3}$ DCH track: for all 3 MUV planes R_{MUV} > 30 cm, $|X_{MUV},Y_{MUV}|$<115 cm.**
- LKr impact point is in LKr acceptance

π^0 selection

- Check all the pairs of good in-time (within 5 ns) clusters

- Calculate π^0 time t_π (average of two γ ones) and reject the combination, if there is a good extra cluster in 5 nanoseconds around t_π (to suppress $\pi^+\pi^0\pi^0$ and showers).

- Make the projectivity correction for the experimental data and MC.

- Reject the pair, if the distance between the clusters is < 20 cm

- $E_{\pi 0}$ > 15 GeV for trigger efficiensy (trigger E LKr > 10 GeV).

- Calculate Z from two γ, assuming π^0 mass

- $-1600 \text{ cm} < Z < 9000 \text{ cm}$

- DCH flunge gamma cut for both γ

Track selection and identification

For each good π^0 check all the good tracks:

- In-time with π^0 (within 10 ns)
- There is no extra good track within 8 ns around the track time (against showers).
- If 0.9 < E/P < 1.1 ~~and no muon associated~~, it is electron (K_{e3})
- If E/P < 0.2 and there is a muon associated, it is muon ($K_{\mu 3}$)

First iteration decay vertex position:

- $Z_{decay} = Z\ (\pi^0)$
- X_{decay}, Y_{decay} = impact point of reconstructed charged track on the transversal plane, defined by Z_{decay}

Blue field correction:

With the «first iteration vertex», we implement the Blue field correction, obtain corrected track slopes and recalculate vertex X,Y again.

Beam position correction:

We know the position of beam axis in space, it is always displaced slightly from the nominal Z axis. For the CMC tuning, these positions were measured for each run from $3\pi^{\pm}$ data many years ago.

We use these data to calculate all the relevant values with respect to the current run beam axis rather than with respect to nominal Z arrow. First of all, we calculate the vertex (x,y) with respect to the beam center X_b,Y_b at this Z_n.

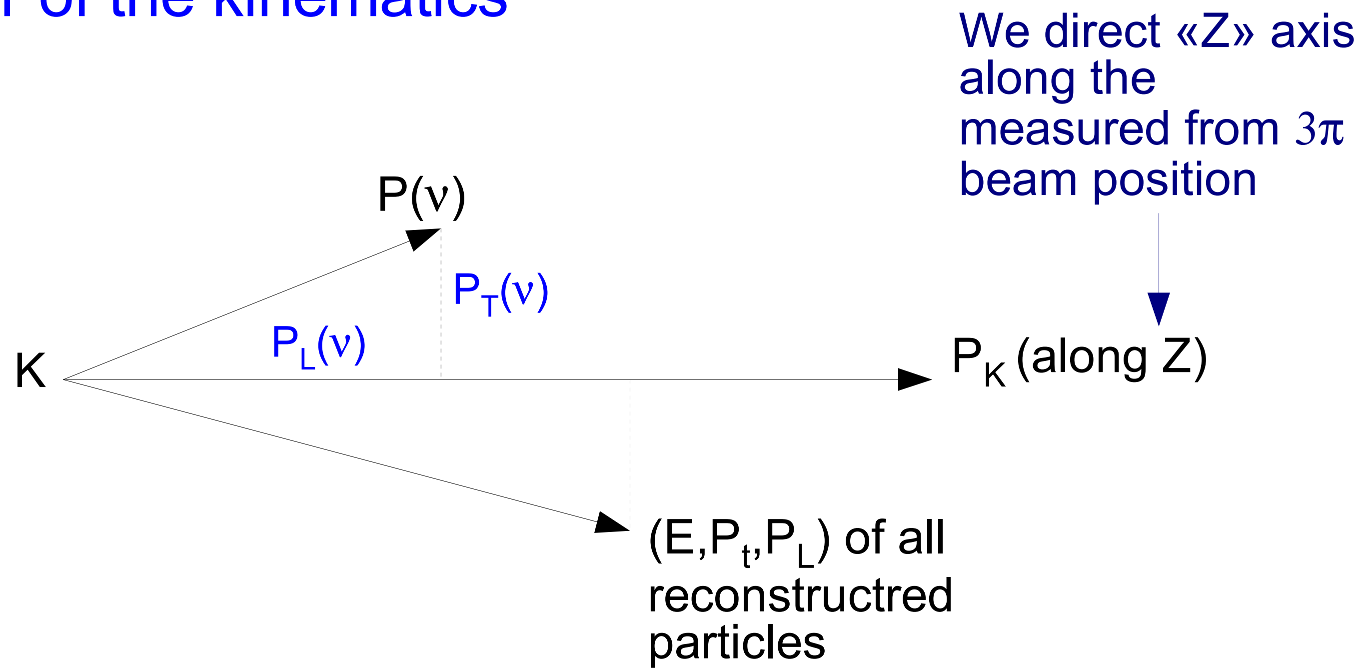
Vertex position cut:

$$\mathrm{SQRT}(\ ((X-a_X(Z))/\sigma_x(Z))^2 + ((Y-a_Y(Z))/\sigma_Y(Z))^2\ ) < 3.0$$

Here a_X, a_Y, σ_X and σ_Y are the functions of Z and represent the average position and width of the beam with respect to standard ($3\pi^{+-}$) beam position.

They are obtained by Gaussian fit (-1.1,+1.1 cm) of Z slices, separately for MC and Data, for X and Y and for positive and negative beams. Then these points are parametrised as functions of Z by polinomes of 5-th degree of Z.

Reminder of the kinematics



Assumptions:

- $P_T(\nu) = - P_t$
- $M(\nu) = 0$

May be two solutions of quadratic equation for P_K:

$P_{1,2} = (\phi P_z \pm \mathrm{SQRT}(D)) / (E^2 - P_z^2)$, and we choose the closest one to P_{beam}

Here:
$\phi = 0.5\ (M_K^2 + E^2 — P_t^2 — P_Z^2)$,
$D = (\phi^2 P_Z^2 — (E^2 — P_z^2)(M_K^2 E^2 — \phi^2))$

But sometimes may be no real solutions — if D<0.

It happens mainly when $P_L(\nu)^2 < 0$ (April 2014 talk), that depends on correctness of the assumed kaon direction of flight

Final stage of selection is not changed

- $P_L(\nu)^2 > 0$

- Quadratic equation for P_K is solved, if no solutions, the combination is taken with zero discriminant. But with the above $P_L(\nu)^2$ requirement, such a cases are rare.

- Average beam momentum P_b measured from $3\pi^\pm$ decays for each run is used to choose the best P_K solution (closest to P_b from two ones).

- -7.5 GeV/c $< (P_K — P_b) <$ 7.5 GeV/c

- For $K_{\mu 3}$, the cut against $K^\pm \to \pi^\pm\pi^0$ with $\pi^\pm \to \mu^\pm\bar{\nu}$:
$m(\pi^+\pi^0) < 0.47$ GeV and $m(\pi^+\pi^0) < (0.6 — P_t(\pi^0))$ GeV;

- For $K_{\mu 3}$, one more cut against $K^\pm \to \pi^\pm\pi^0$ with $\pi^\pm \to \mu^\pm\bar{\nu}$:
$m(\mu^\pm\bar{\nu}) > 0.18$ GeV;

- For both $K_{\mu 3}$ and K_{e3}: a cut against $\pi^\pm\pi^0\pi^0$: $(P_2\text{-}P_1) < 60$ GeV
<=> in terms of P_K equation discriminant squared $\mathbf{d = ((P_2\text{-}P_1)/2)^2}$: $d < 900\ \text{GeV}^2$;

- For K_{e3}, the $\boldsymbol{\nu}$ transversal momentum with respect to beam axis must be
$P_t >= 0.02$ GeV : a cut against $K^\pm \to \pi^\pm\pi^0$ with $\pi^\pm$ misidentified as e (when $E/P > 0.9$).

In every event, separately for K_{e3} and $K_{\mu 3}$, the combination with the minimum $\Delta P = |P_K — P_b|$ is choosen as the best candidate.

Statistics of selected events

$R_{DCH} > 27$ cm for $K_{\mu3}$

Statistics
In units of 10^6

	MM	Fit 1	Fit 2	Fit 3	Fit 4	**Fit 5**
K_{e3}	4.0	5.6	5.8	4.1	4.0	**4.0**
$K_{\mu3}$	2.5	3.8	3.8	2.7	1.9	**2.3**

We have almost the same $K_{\mu3}$ statistics as MM analysis, but a subpermill background in $K_{\mu3}$ (Bg fraction ~ 0.0003), is reached.

Reminder: Matrix element depends on two form factors $f_+(t)$ and $f_-(t)$:

$$M = \tfrac{1}{2} G_F V_{us} \left(f_+(t)(P_K+P_\pi)^\mu \bar{u}_l\gamma_\mu(1+\gamma_5)u_\nu + f_-(t)m_l\bar{u}_l(1+\gamma_5)u_\nu\right)$$

$t = q^2$ — *square of momentum transfer to the lepton system*

in K_{e3} decays one can measure only $f_+(t)$ (small m_e)

Usually form factors are re-formulated in terms of the vector and scalar exchange contributions:

1) $f_+(t)$: vector form factor - change only E^*_π — dependence of Dalitz plot
2) $f_0(t) = f_+(t) + f_-(t)\, t/(m_K^2 - m_\pi^2)$: scalar form factor, measured from $K_{\mu 3}$

All these formfactors are usually normalized to $f_+(0)$

Reminder: Probability unperturbed (without rad. corr.)

$$d^2N/(dE_l\, dE_\pi) \sim A\, f_+^{\,2}(t) + B\, f_+(t)\, f_-(t) + C\, f_-^{\,2}(t)\ ,$$

Where $f_-(t) = (f_+(t) - f_0(t))(m_K^2 - m_\pi^2)/t$,
E_l is charged lepton energy and E_π is π^0 energy in kaon rest frame,
$t = (P_K - P_\pi)^2 = M_K^{\,2} + M_\pi^{\,2} - 2\, M_K\, E_\pi$

$A = M_K(2\, E_l E_\nu - M_K(E_\pi^{max} - E_\pi)) + M_l^{\,2}\, ((E_\pi^{max} - E_\pi)/4 - E_\nu)$
$B = M_l^{\,2}(E_\nu - (E_\pi^{max} - E_\pi)/2)$
$C = M_l^{\,2}(E_\pi^{max} - E_\pi)/4$
$E_\pi^{max} = (M_K^{\,2} + M_\pi^{\,2} - M_l^{\,2})/(2\, M_K)$
$E_\nu = M_K - E_l - E_\pi$

For K_{e3} the terms B and C are negligible (small lepton mass).

Parameterizations of Form Factors:

FF Parameterisation (PDG)	$f_+(t,$parameters$)$	$f_0(t,$parameters$)$
Quadratic (linear for $\bar{f}_0(t)$)	$1+\lambda'_+\, t/m^2_\pi + ½\, \lambda''_+(t/m^2_\pi)^2$	$1 + \lambda'_0\, t/m^2_\pi$
Pole	$M_v^2 / (M_v^2 - t)$	$M_s^2 / (M_s^2 - t)$
Dispersive H(t), G(t): functions, fixed from theory and another experiments (with some uncertainty)	$\exp(\, (\Lambda_+ + H(t))\, t/m^2_\pi\,)$	$\exp(\, (\ln[C] - G(t))\, t/(m_K^2 - m^2_\pi)\,)$

Quadratic Parameterization

Fit results (x 10^{-3}), errors are statistical only.

Fits with background correction (negligible effect in our fits)

Fit	$K_{e3}\ \lambda'_+$	$K_{e3}\ \lambda''_+$	K_{e3} χ^2/ndf	$K_{\mu 3}\ \lambda'_+$	$K_{\mu 3}\ \lambda''_+$	$K_{\mu 3}\ \lambda'_0$	$K_{\mu 3}$ χ^2/ndf	$K_{\mu 3}$ fit probability
MM	27.2±0.7	0.7±0.3		26.3 ± 3.0	1.2 ± 1.1	15.7 ± 1.4		
3	23.3±0.8	1.91±0.31	728.2/721	26.3±3.2	1.9±1.1	14.8±1.2	411.6/368	0.058
4	23.4±0.8	1.89±0.32	722.5/720	26.4±3.8	1.61±1.3	15.7±1.4	369.6/364	0.409
5	23.4±0.8	1.90±0.31	713.9/720	24.3±3.4	2.46±1.2	14.3±1.2	375.9/368	0.377

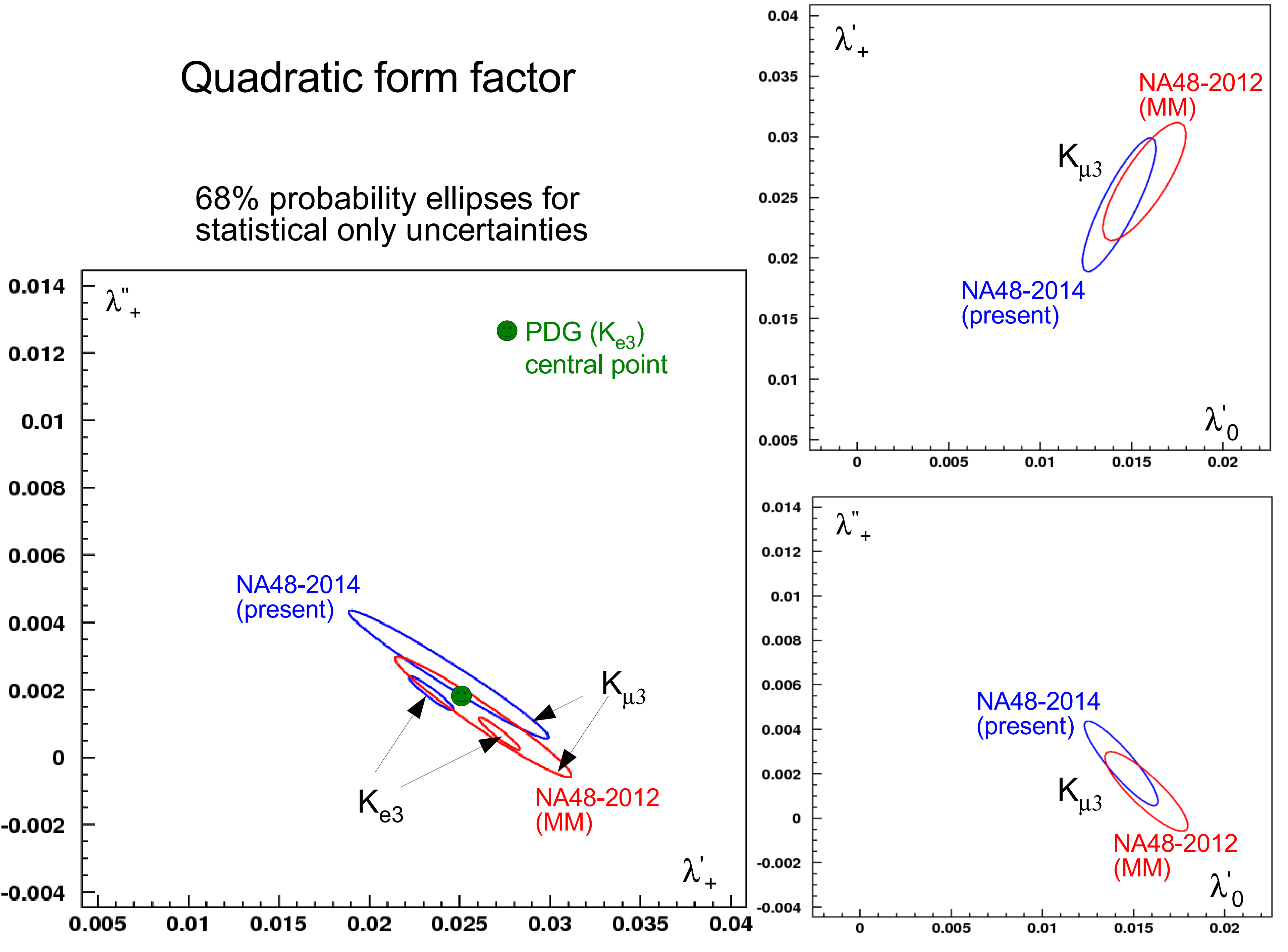
Compatibility between K_{e3} and $K_{\mu 3}$ now looks good, but actually it is not so easy to compare multidimensional points with their errors and correlations.

If one fit $K_{\mu 3}$ data with λ'_+ and λ''_+ values fixed from K_{e3}, one obtain:
λ'_0 = (15.3±0.5) x 10^{-3} and χ^2/ndf = 389.6/370, that corresponds to probability 0.232.
=> compatibility between $K_{\mu 3}$ and K_{e3} is satisfactory even with the $K_{\mu 3}$ statistical error only.

The increased R_{MUV} cut for $K_{\mu 3}$ (instead of proposed by MM R_{DCH} cut) solves the last problems apart from discrepance with MM in K_{e3}.

Quadratic form factor

68% probability ellipses for statistical only uncertainties



The experiment-related systematics

Contribution	*Approach to uncertainty calculation*
Background	Effect of the first order background correction.
Lkr nonlinearity	½ of difference between the cases with the final extra nonlinearity correction and without it (see below).
Lkr scale	Effect of scale shift by 0.001 (well detectable from K_{e3} E/P).
Beam width	Difference in results between the 3σ (standard) and 3.5σ cuts around the beam axis.
Dalitz plot resolution	Difference between the event-weighting fit technique and Dalitz plot acceptance correction approach (possible difference in tails effect).
Kaon energy spectra	Effect of the first order kaon spectrum correction, based on K_{e3} (see below).
P_K average	Effect of $\langle P_K \rangle$(beam) possible mismeasurement (see below)
Beam direction	Effect of inclusion of the events with $P_L(\nu)^2<0$ (~ events without P_K solutions)
Acceptance	Effect of all radius cuts increasing for MC by the factor of 1.003, that corrects Z distribution (see below)
Trigger efficiency	Effect of quadratically smoothed trigger efficiency (Sergey last talk).
Accidentals	Effect of doubling the time widows for data tracks and clusters

LKr Nonlinearity

Use 2004 $\pi^0\pi^0\pi^{+-}$ data
(done for cusp analysis):

$22 < E(\pi^0_1) < 26$ GeV
$E(\pi^0_2) < E(\pi^0_1)$
$E(\gamma)^{max} < 0.55\, E(\pi^0)$ for both π^0

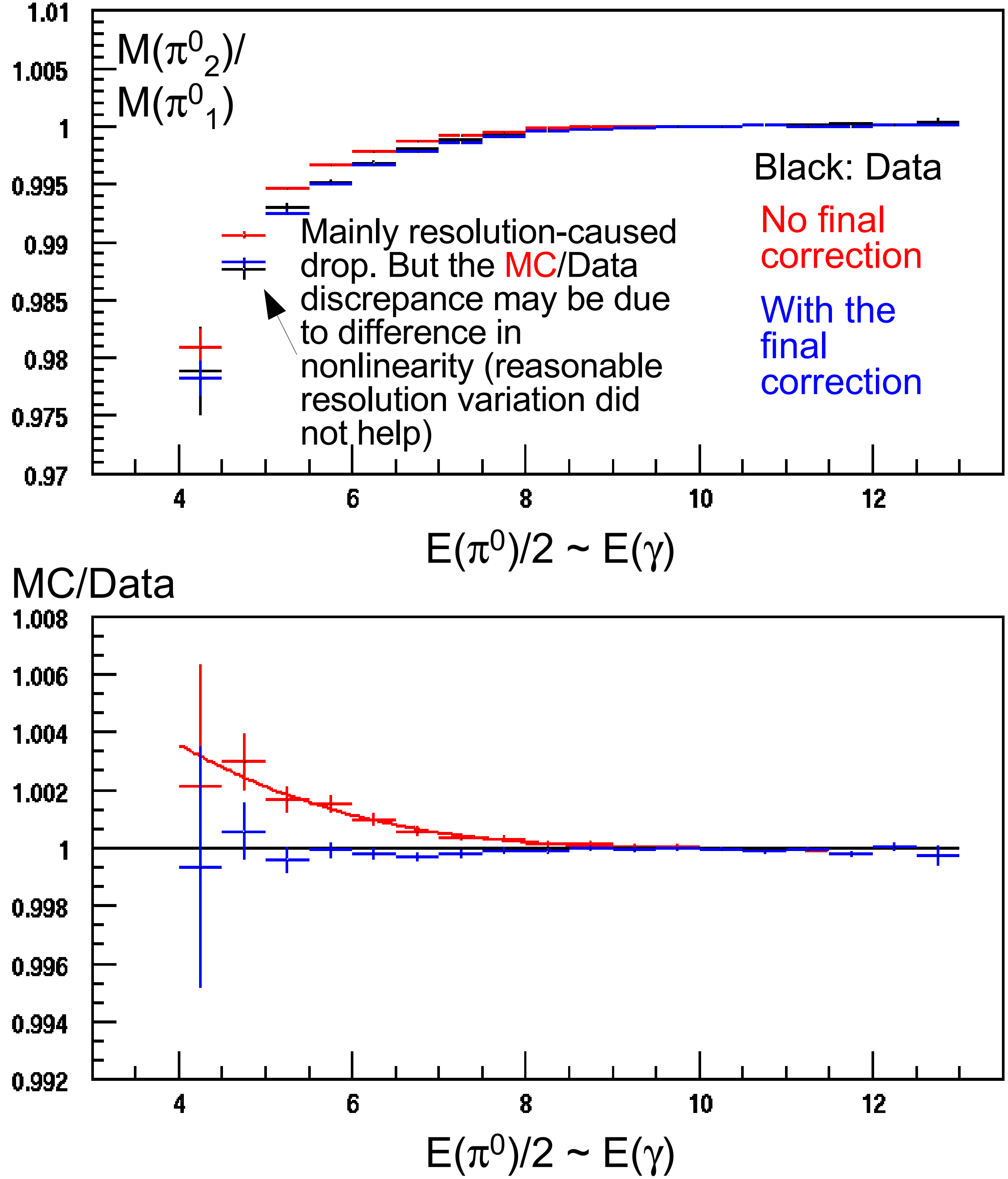
Final correction for MC:

P_0 1.0170
P_1 -0.48025E-02
P_2 0.45538E-03
P_3 -0.14474E-04

E: cluster energy in GeV

$f=P_0+P_1E+P_2E^2+P_3E^3$
if(f > 1) E= E/f

½ of the final correction effect is taken as the nonlinearity-related uncertainty.

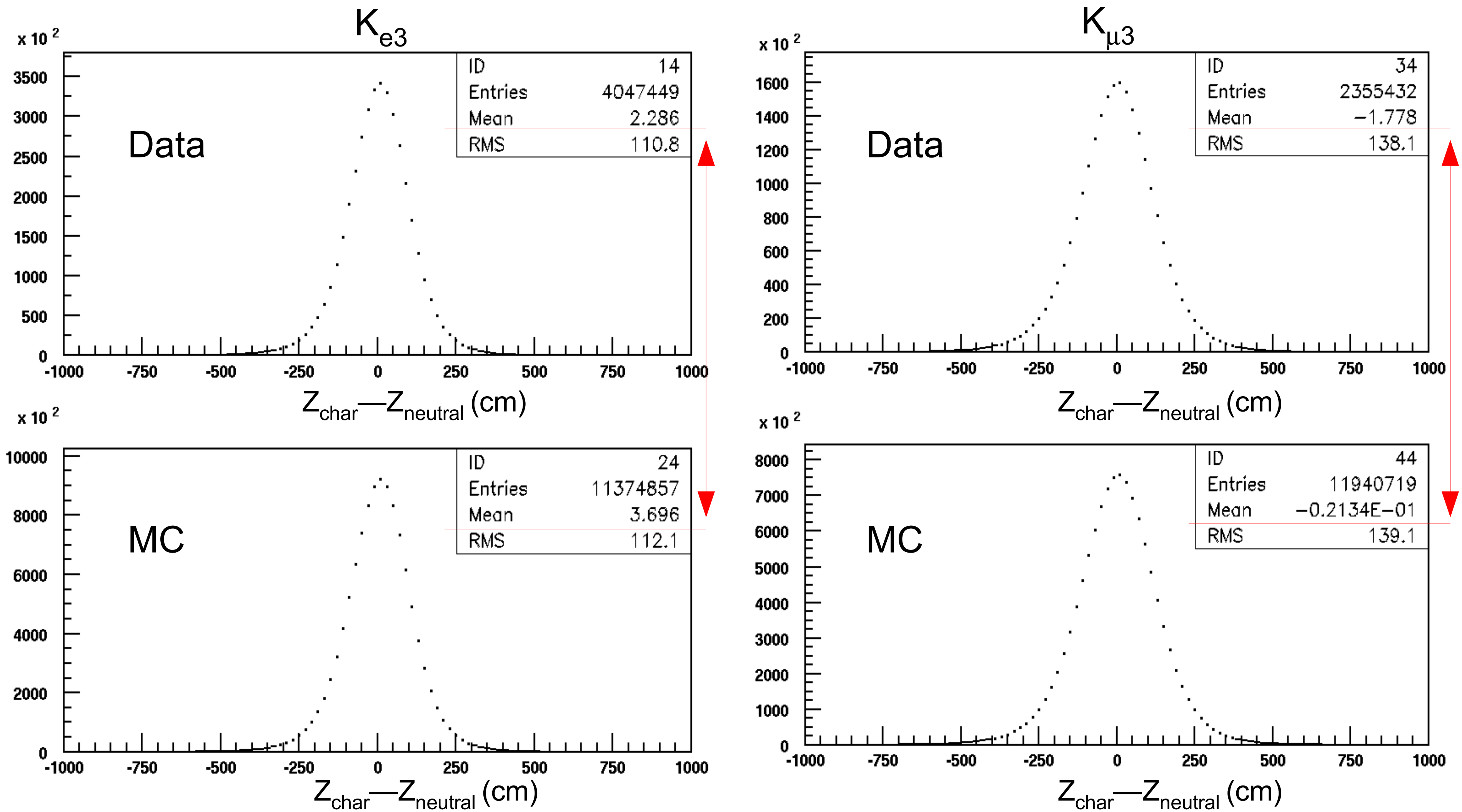


Z difference Data/MC comparisons

Z_{char} (from track CDA to beam axis) — $Z_{neutral}$ (from π^0 to LKr distance)

Sensitive both to LKr (scale and nonlinearity) and beam geometry simulation.

MC-Data discrepance is about +1.5 cm for both modes.



Acceptance

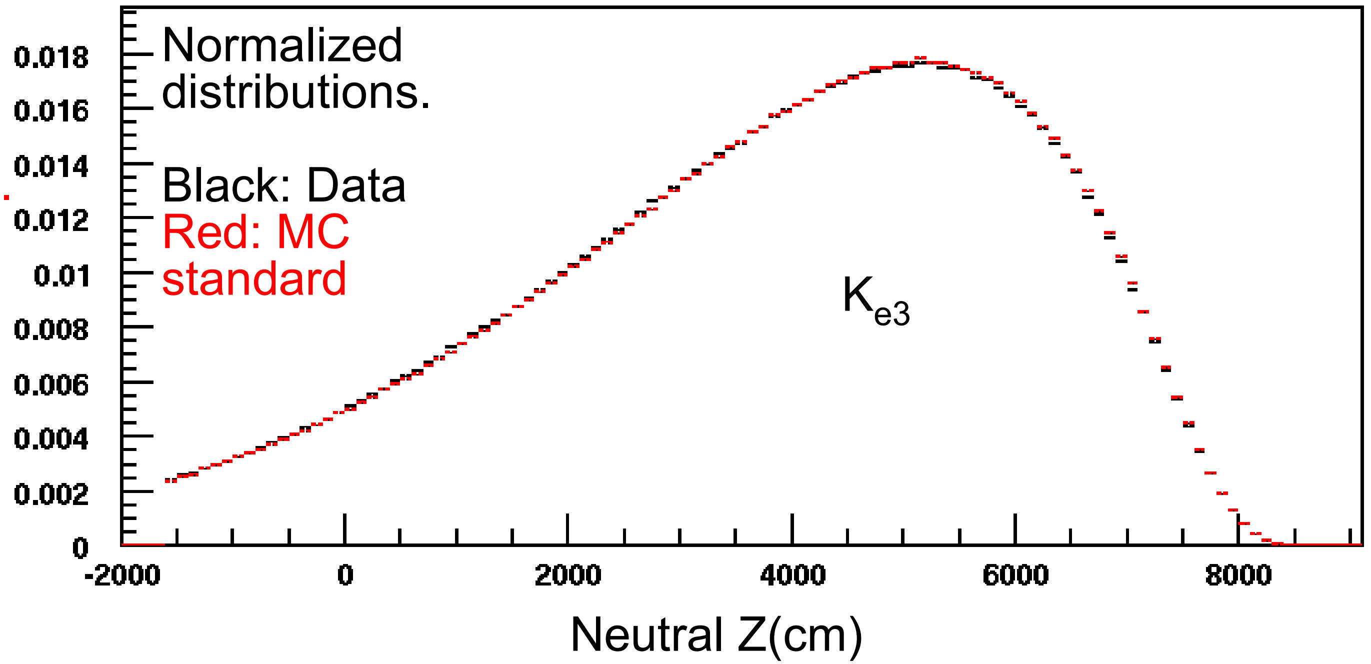
Yes, MC looks shifted forward by 15-20 cm (from fits).

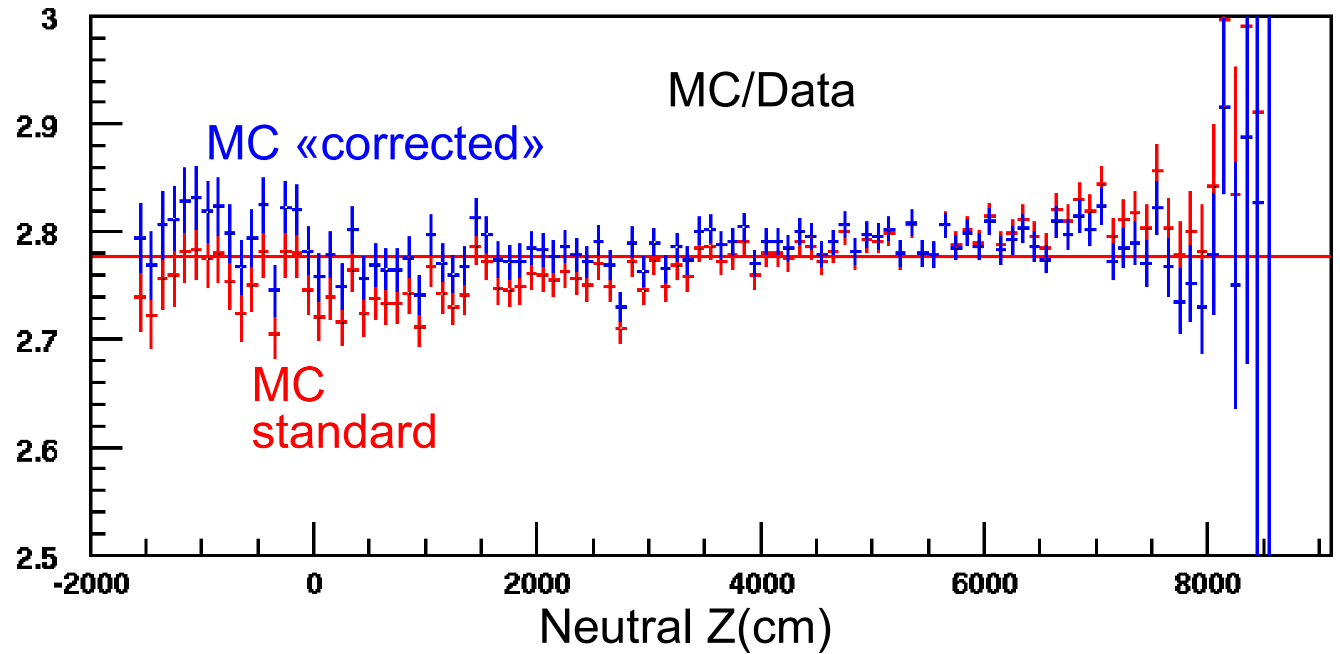
It is not an LKr scale effect for neutral Z, as ($Z_{charged}$-$Z_{neutral}$) is reproduced in MC much better (1.5 cm).

Generally, it may be some acceptance simulation problem(s) (including possible detector misplacing in MC).

The «correction» is reached increasing all the radius cuts for MC by the factor of 1.003.

Shifts of fit results = acceptance simulation uncertainty.

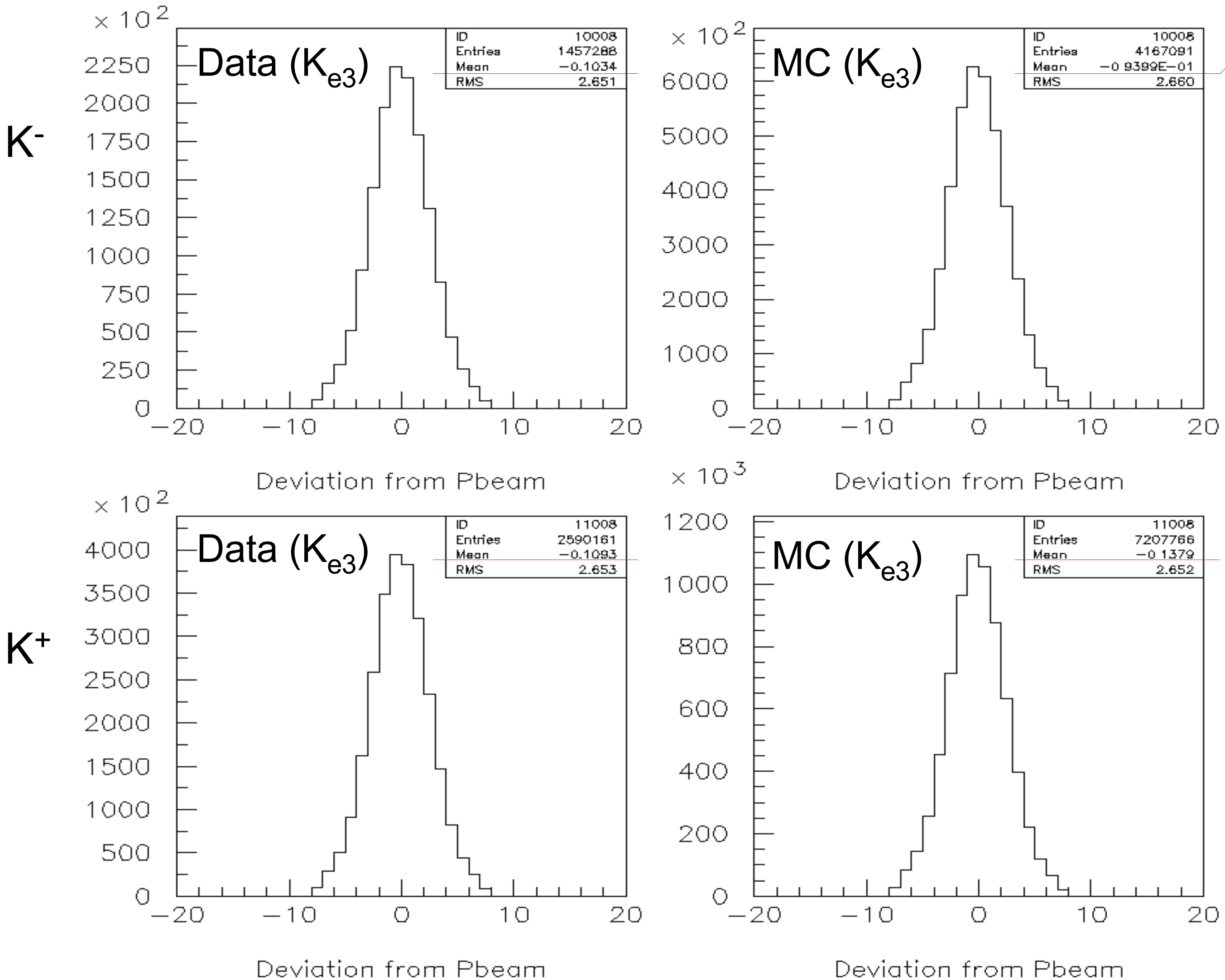




Kaon beams average P (measured from charged $K_{3\pi}$)

We apply a cut and choose the best P_K using the earlier measured average beam momenta (P_{beam}) taken from $K_{3\pi+-}$

Well, mainly it is reproduced, but not ideally

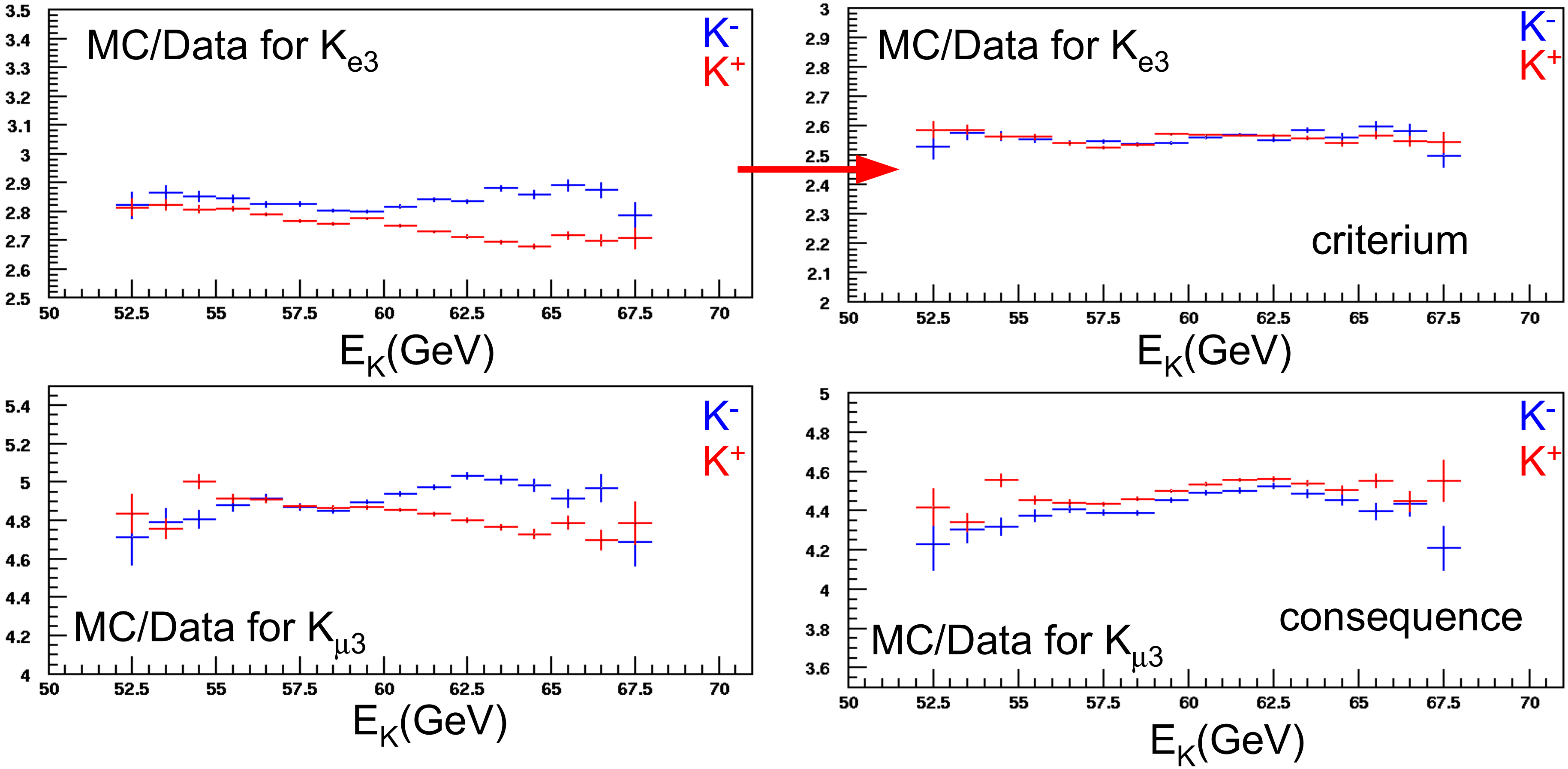


To check sensitivity, we shift P_{beam} by -0.1 GeV.

The shifts of fit results are regarded as the contributions to systematics.

Kaon beams P spectra uncertainty

Correction of MC E_K spectra by means of events rejection based on true kaon energy



The shift of final results due to correction by MC events rejection

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Kaon momenta related uncertainty

Table 1: Fit results for the Quadratic Parametrization ($\times 10^3$)

	$\lambda'_+(K_{e3})$	$\lambda''_+(K_{e3})$	$\lambda'_+(K_{\mu 3})$	$\lambda''_+(K_{\mu 3})$	$\lambda_0(K_{\mu 3})$
central values	23.42	1.90	24.38	2.46	14.32
stat.error	0.77	0.31	3.41	1.17	1.24
resolution	0.07	0.05	0.85	0.46	0.81
beam direction	0.72	0.25	2.81	0.76	0.91
background	0.11	0.02	0.29	0.05	0.02
LKr nonlinearity	0.39	0.12	0.35	0.12	0.04
LKr scale	0.06	0.19	0.40	0.01	1.05
acceptance	0.11	0.02	1.22	0.34	0.37
beam width	0.60	0.16	1.83	0.38	0.51
trigger	0.15	0.07	1.07	0.34	0.16
kaon spectra	0.05	0.02	0.26	0.08	0.09
Pk average	0.05	0.02	0.01	0.01	0.06
Accidental track	0.00	0.00	0.00	0.00	0.00
Accidental cluster	0.02	0.01	0.04	0.01	0.01
syst. error	1.04	0.39	3.88	1.09	1.74
total error	1.30	0.50	5.16	1.60	2.14

Fit results and contributions to systematic uncertainty for Quadratic Parameterisation are shown in the Table 1. Central values fit quality for Quadratic Parameterisation: $\chi^2/NDF(K_{e3}) = 713.9/720$; $\chi^2/NDF(K_{\mu 3}) = 375.9/368$.

If one fit $K_{\mu 3}$ data with λ'_+ and λ''_+ values fixed from K_{e3}, one obtain (with stat. error): $\lambda'_0 = (15.3 \pm 0.5) \times 10^{-3}$ and $\chi^2/ndf = 389.6/370$, that corresponds to probability 23%.

We ignore conservatively the correlations between systematic effects for different fit parameters. It leads to dilution (weakening) of the final correlations.

Another possible approaches (may be to decide):
1. Use the statistics uncertainty correlation coefficients for the full errors.
2. Sum the systematic effects «covariance matrices» $\Delta P_i * \Delta P_j$., where P_i are the fit parameters

Statistics uncertainty correlation for $(\lambda'_+(K_{e3}),\lambda''_+(K_{e3}))$ is found to be -0.946.

Table 2: Statistics uncertainty correlation coefficients for $K_{\mu 3}$ Quadratic Parametrization

	$\lambda''_+(K_{\mu 3})$	$\lambda_0(K_{\mu 3})$
$\lambda'_+(K_{\mu 3})$	-0.979	0.860
$\lambda''_+(K_{\mu 3})$		-0.900

Full uncertainty correlation for $(\lambda'_+(K_{e3}),\lambda''_+(K_{e3}))$ is found to be -0.355.

Table 3: Full uncertainty correlation coefficients for $K_{\mu 3}$ Quadratic Parametrization

	$\lambda''_+(K_{\mu 3})$	$\lambda_0(K_{\mu 3})$
$\lambda'_+(K_{\mu 3})$	-0.473	0.330
$\lambda''_+(K_{\mu 3})$		-0.382

Table 4: Fit results for the Pole Parametrization ($\times 10^3$)

	$m_V(K_{e3})$	$m_V(K_{\mu3})$	$m_S(K_{\mu3})$
central values	888.3	854.7	1192.1
stat.error	3.5	8.2	20.5
resolution	0.8	6.0	15.0
beam direction	1.7	7.0	5.4
background	1.0	1.8	1.4
LKr nonlinearity	1.4	0.0	2.7
LKr scale	5.3	4.3	32.5
acceptance	0.9	2.7	1.2
beam width	3.0	8.6	4.0
trigger	0.4	1.3	18.4
kaon spectra	1.3	0.4	0.5
Pk average	0.0	0.4	2.3
Accidental track	0.0	0.0	0.0
Accidental cluster	0.1	0.1	0.1
syst. error	6.8	13.7	41.0
total error	7.6	16.0	45.9

Fit results and contributions to systematic uncertainty for Pole Parameterisation are shown in the Table 4. Central values fit quality for Pole Parameterisation: $\chi^2/NDF(K_{e3}) = 716.0/721$; $\chi^2/NDF(K_{\mu3}) = 372.3/369$.

Statistics uncertainty correlation for $(m_V(K_{\mu3}),m_S(K_{\mu3}))$ is found to be -0.352.

Full uncertainty correlation for $(m_V(K_{\mu3}),m_S(K_{\mu3}))$ is found to be -0.080.

If one fit $K_{\mu3}$ data with m_V value fixed from K_{e3}, one obtain (with stat. error):
$m_S = (1165.9 \pm 17.6) \times 10^{-3}$ and $\chi^2/ndf = 387.1/370$, that corresponds to probability 26%.

Table 5: Fit results for the Dispersive Parametrization ($\times 10^3$)

	$\Lambda_+(K_{e3})$	$\Lambda_+(K_{\mu3})$	$ln[C](K_{\mu3})$
central values	23.03	25.14	188.88
stat.error	0.21	0.56	5.84
resolution	0.04	0.41	4.31
beam direction	0.11	0.49	1.75
background	0.06	0.13	0.36
LKr nonlinearity	0.08	0.00	0.80
LKr scale	0.31	0.30	10.66
acceptance	0.05	0.19	0.41
beam width	0.18	0.59	1.39
trigger	0.02	0.09	5.18
kaon spectra	0.07	0.03	0.17
Pk average	0.00	0.03	0.63
Accidental track	0.00	0.00	0.00
Accidental cluster	0.01	0.00	0.04
syst. error	0.40	0.95	12.86
total error	0.45	1.10	14.13

Fit results and contributions to systematic uncertainty for Dispersive Parameterisation are shown in the Table 5. Central values fit quality for Dispersive Parameterisation: $\chi^2/NDF(K_{e3}) = 715.9/721$; $\chi^2/NDF(K_{\mu3}) = 370.0/369$.

Statistics uncertainty correlation for ($\Lambda_+(K_{\mu3})$,$ln[C](K_{\mu3})$) is found to be -0.291.

Full uncertainty correlation for ($\Lambda_+(K_{\mu3})$,$ln[C](K_{\mu3})$) is found to be -0.061.

If one fit $K_{\mu3}$ data with Λ_+ value fixed from K_{e3}, one obtain (with stat. error): *ln[C]*= $(195.19 \pm 5.5) \times 10^{-3}$ and χ^2/ndf = 384.2/370, that corresponds to probability 29%.

Conclusion

- With the present version of selection, there are no big problems with all the usual fits and with e-μ compatibility.
- It may be a “draft final” central result.
- Preliminary check of experimental systematics is done.

To do

- Theory-related systematic uncertainty
- Try beam focusing correction
- Understand the difference with MM analysis (in progress).

Intentions

• To propose a «draft final» result on the next Collaboration meeting (official «second preliminary» result after the shown 2012 one would look strange).

So please think about internal referee for the final paper (not right now, but rather soon).