Horizon wave-function the way to the quantum hoop conjecture

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Abstract

We address the issue of **black hole formation by collision of quantum particles**. We introduce the *horizon wave-function* for quantum mechanical states representing a single localised particle, from which we derive a GUP. For two highly-boosted non-interacting particles that collide in (1+1)-dimensions, this wave-function determines a probability that the system becomes a black hole depending on the initial momenta and spatial separation between the particles, thus *extending the hoop conjecture to quantum mechanics* and yielding corrections to its classical counterpart.

ArXiv:1305.3195, 1306.5298 [EPJ C], 1311.5698 [PLB], 1405.4192,…
Collaboration: R. Casadio, A. Giugno ,A. Orlandi, F. Scardigli, …

Plan of the talk

1. Physical system: gravitational collapse of quantum matter

2. Hawking radiation: lessons from the semiclassical picture

3. Hoop conjecture: Schwarzschild radius of a classical particle

4. Problem: Schwarzschild radius of a quantum particle?

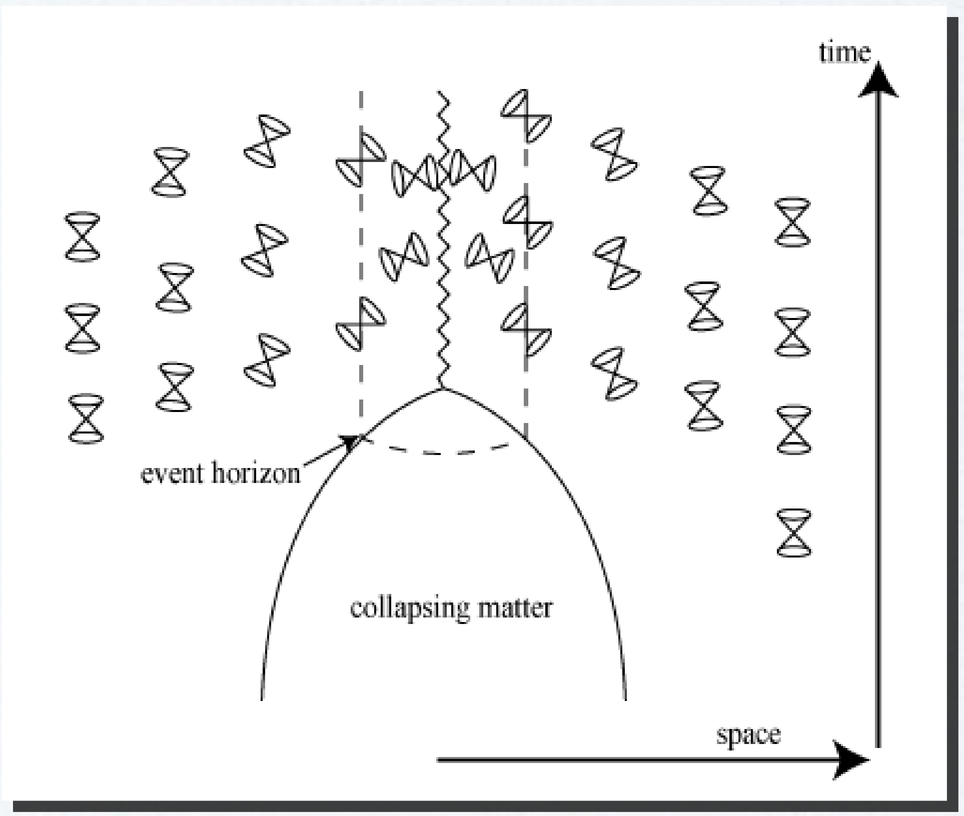
5. Single particle: horizon wave-function and the GUP

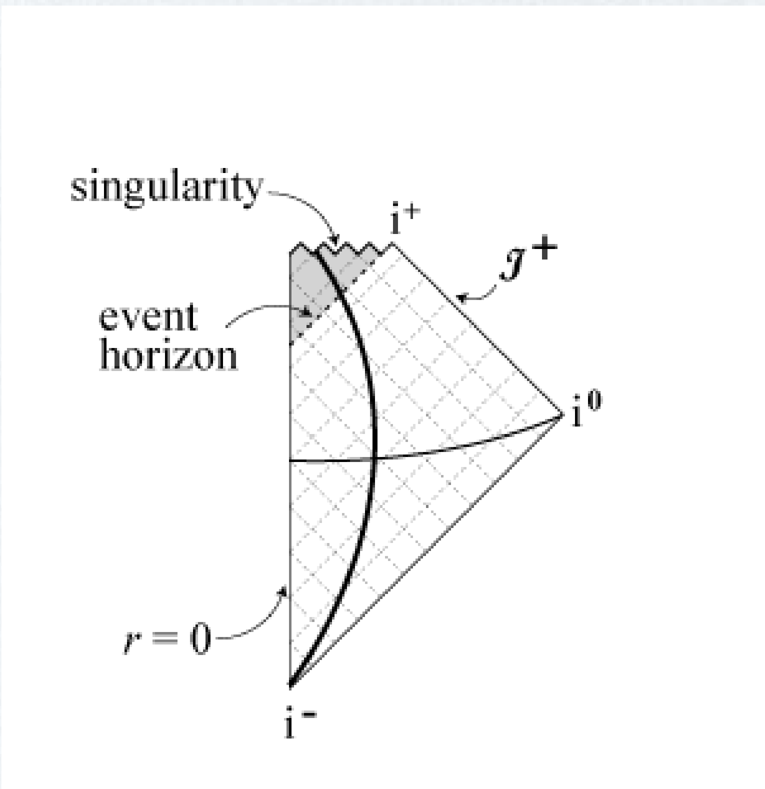
6. 2-particle collision: horizon wave-function and the quantum hoop

7. Summary and outlook

1) Gravitational collapse

Standard classical picture: classical matter and "geometrical" space-time*



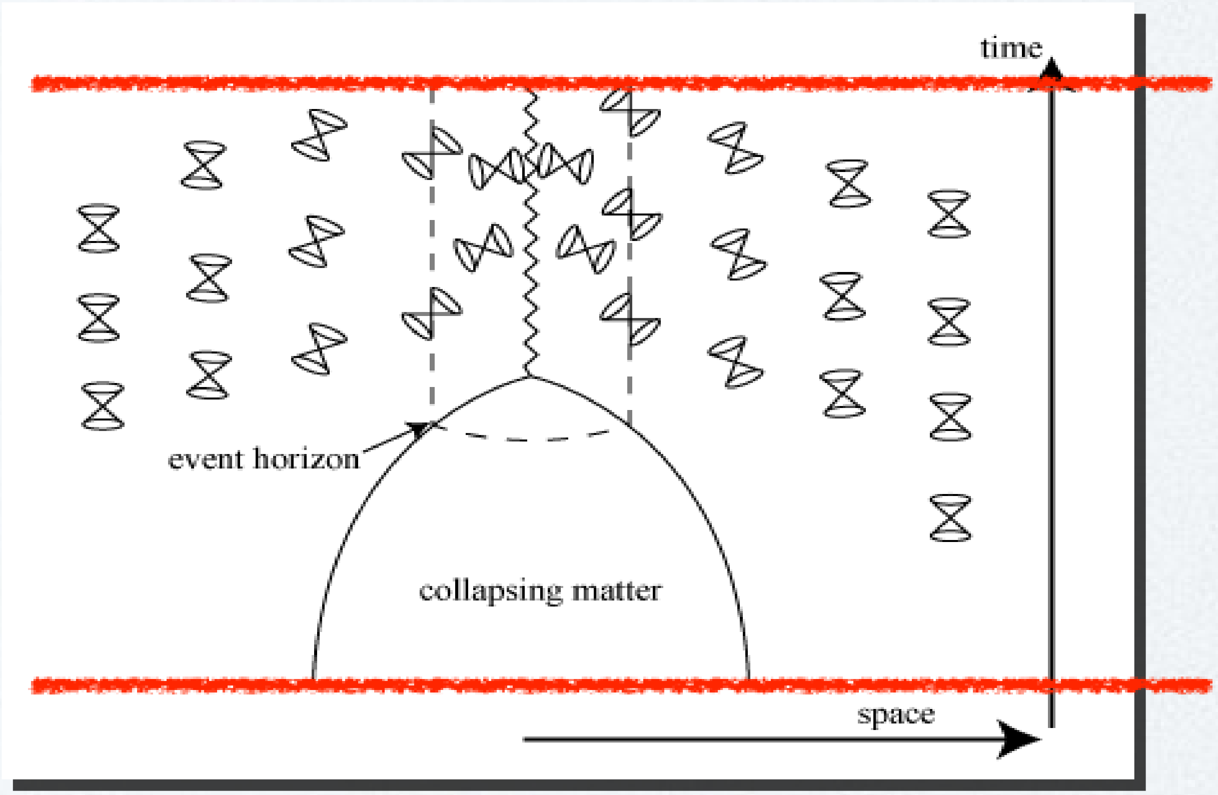


*Prototype background: $ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \, d\Omega^2$

But matter is **quantum**...!

1) Gravitational collapse

Standard semiclassical picture: classical matter and "geometrical" space-time + foreground quantum particles



$$|0; t = +\infty\rangle$$

$$e^{-\frac{i}{\hbar} \int \hat{H}\, dt}$$

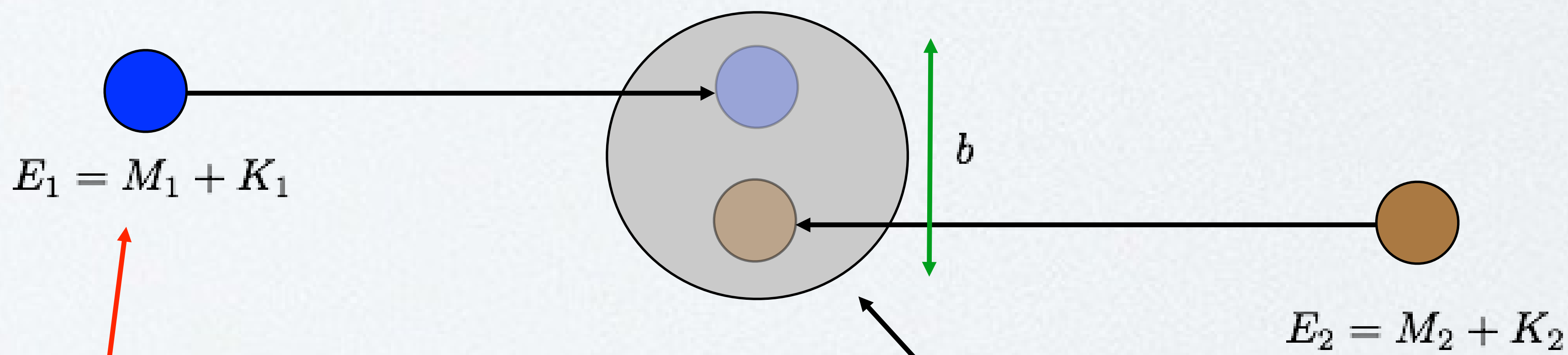
$$|0; t = -\infty\rangle$$

$$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$$

2) Hoop conjecture

Thorne's hoop conjecture (1972):

A black hole forms when the impact parameter *b* of two colliding objects (of negligible spatial extension) is shorter than the radius of the would-be-horizon (Schwarzschild radius, for negligible angular momentum) corresponding to the total energy *E*

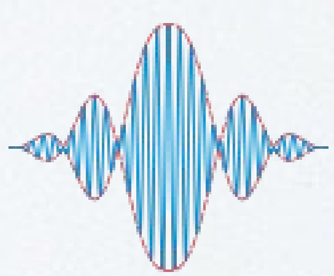


Quantum mechanical particle

?

$$b \lesssim 2\,\ell_p\,\frac{E}{m_p} \equiv R_H$$

Classical geometry



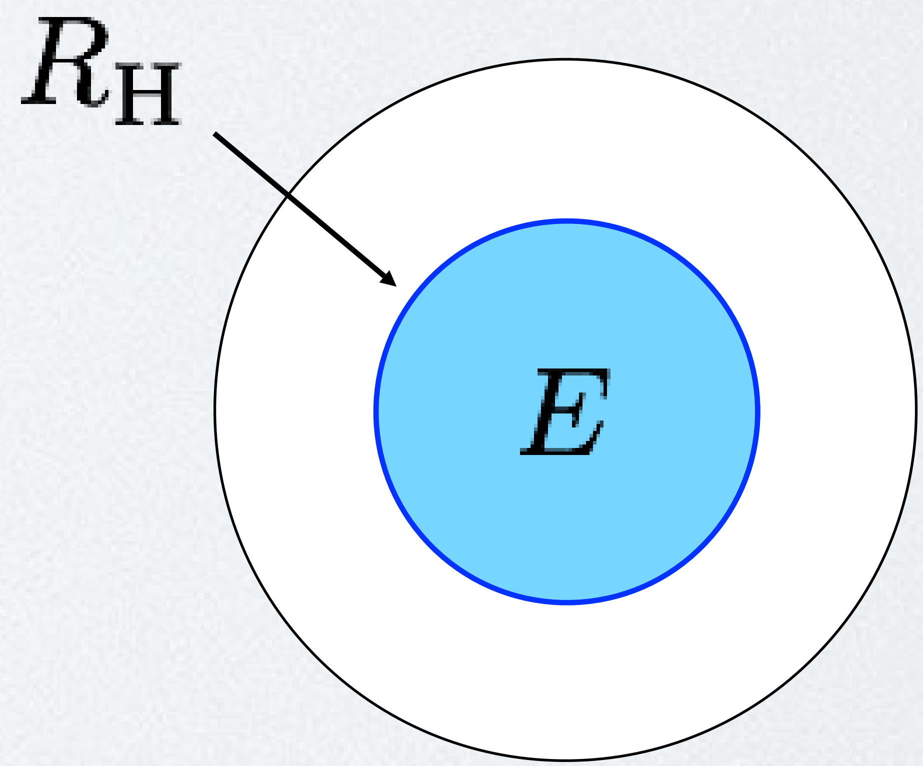
2) Hoop conjecture

Classical spherically symmetric system:

$$E(r) = \frac{4}{3}\pi \int_0^r \rho(t, r')\, r'^2\, dr'$$

Misner-Sharp mass

$$R_{\mathrm{H}} = 2\,\ell_{\mathrm{p}}\,\frac{E}{m_{\mathrm{p}}}$$



Surface is a "Horizon" if: $4\pi R_{\mathrm{H}}^2 = 4\pi r^2$

Schwarzschild radius   Areal radius

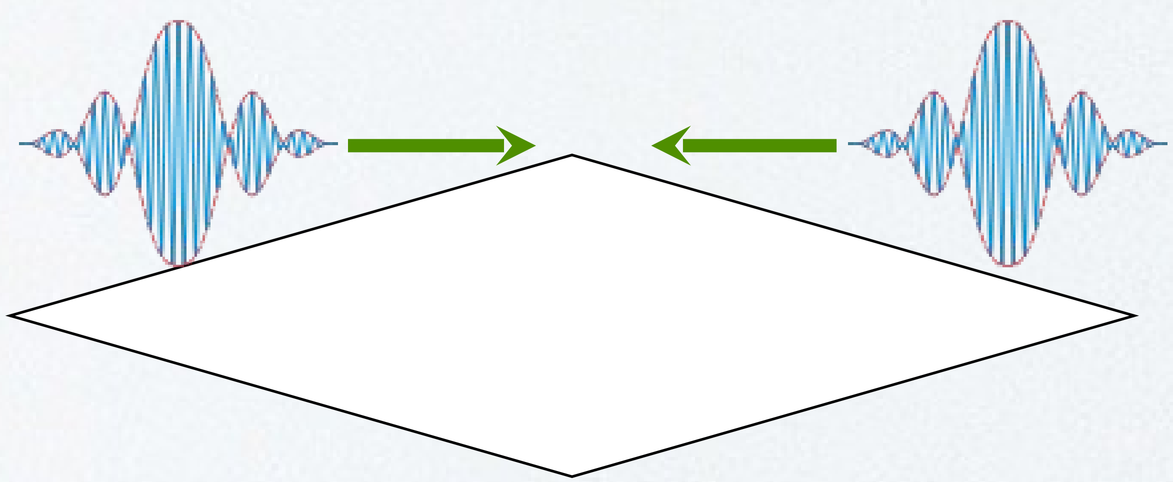
3) Horizon of QM particle

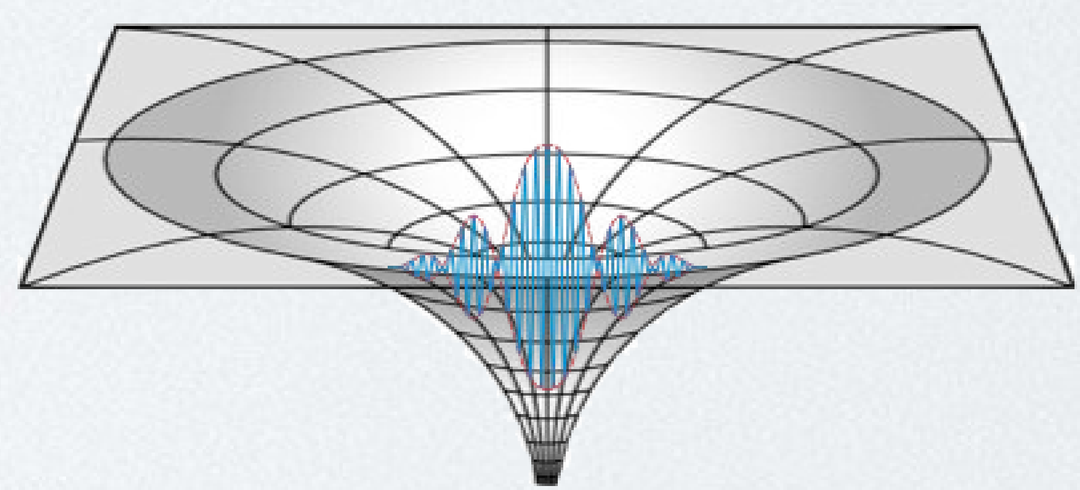
What is the Schwarzschild radius of QM particles?

From *Generalized uncertainty principles* (GUPs): $\Delta x \gtrsim \ell_{\rm p} \frac{m_{\rm p}}{\Delta p} + \alpha\, \ell_{\rm p} \frac{\Delta p}{m_{\rm p}}$

To Dvali's *classicalization* (2010):

At high (~Planckian) energy, quantum particle scatterings lead to formation of "classicalons" and quantum degrees of freedom disappear (*no UV divergences*).
For gravity, "classicalons" = black holes = BEC of gravitons





4) Horizon wave-function

[ArXiv:1305.3195]

1) Localised particle at rest: $\langle x|\psi_{\rm S}\rangle$ ~ packet

Energy (modes) of choice!

2) Spectral decomposition: $|\psi_{\rm S}\rangle = \sum_E C(E)\,|E\rangle$

Schwarzschild-link

$$R_{\rm H} = 2\,\ell_{\rm p}\,\frac{E}{m_{\rm p}}$$

3) Horizon wave-function: $\langle R_{\rm H}|\psi_{\rm H}\rangle \sim C(R_{\rm H})$

4) Horizon wave-function

[ArXiv:1305.3195]

Localised particle at rest:

Gaussian wave-function:

$$\psi_{\rm S}(r) = \frac{e^{-\frac{r^2}{2\,\ell^2}}}{\ell^{3/2}\,\pi^{3/4}}$$

Energy spectrum:

$$|\psi_{\rm S}\rangle = \sum_E C(E)\,|E\rangle$$

Fourier transform:

$$\psi_{\rm S}(p) = \frac{e^{-\frac{p^2}{2\,\Delta^2}}}{\Delta^{3/2}\,\pi^{3/4}} \qquad \Delta = \frac{\hbar}{\ell} \sim m$$

$E^2 = p^2 + m^2$ (flat space)

Horizon wave-function:

$$R_{\rm H} = 2\,\ell_{\rm p}\,\frac{E}{m_{\rm p}}$$

$$\psi_H(R_{\rm H}) = \frac{\ell^{3/2}\,e^{-\frac{\ell^2\,R_{\rm H}^2}{8\,\ell_{\rm p}^4}}}{2^{3/2}\,\pi^{3/4}\,\ell_{\rm p}^3}$$

4) Horizon wave-function

[ArXiv:1305.3195]

Probability particle is inside its own horizon:

$$P_{<}(r < R_{\rm H}) = P_{\rm S}(r < R_{\rm H})\, P_{\rm H}(R_{\rm H})$$

$$P_{\rm S}(r < R_{\rm H}) = 4\,\pi \int_0^{R_{\rm H}} |\psi_{\rm S}(r)|^2\, r^2\, dr$$

$$P_{\rm H}(R_{\rm H}) = 4\,\pi\, R_{\rm H}^2\, |\psi_{\rm H}(R_{\rm H})|^2$$

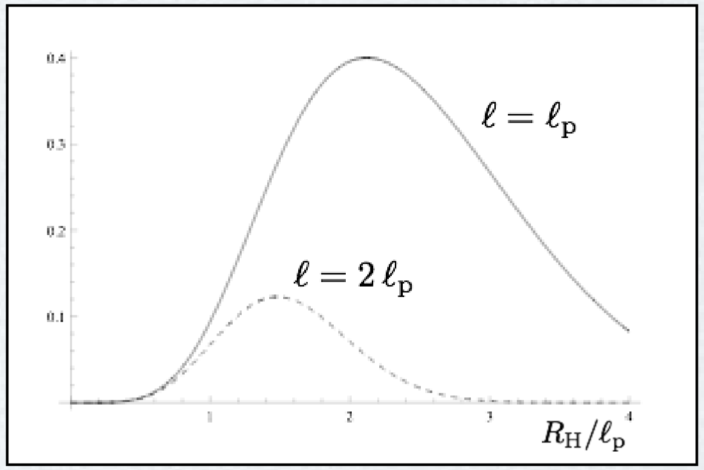
Probability particle is a Black Hole:

$$P_{\rm BH} = \int_0^{\infty} P_{<}(r < R_{\rm H})\, dR_{\rm H}$$

4) Horizon wave-function

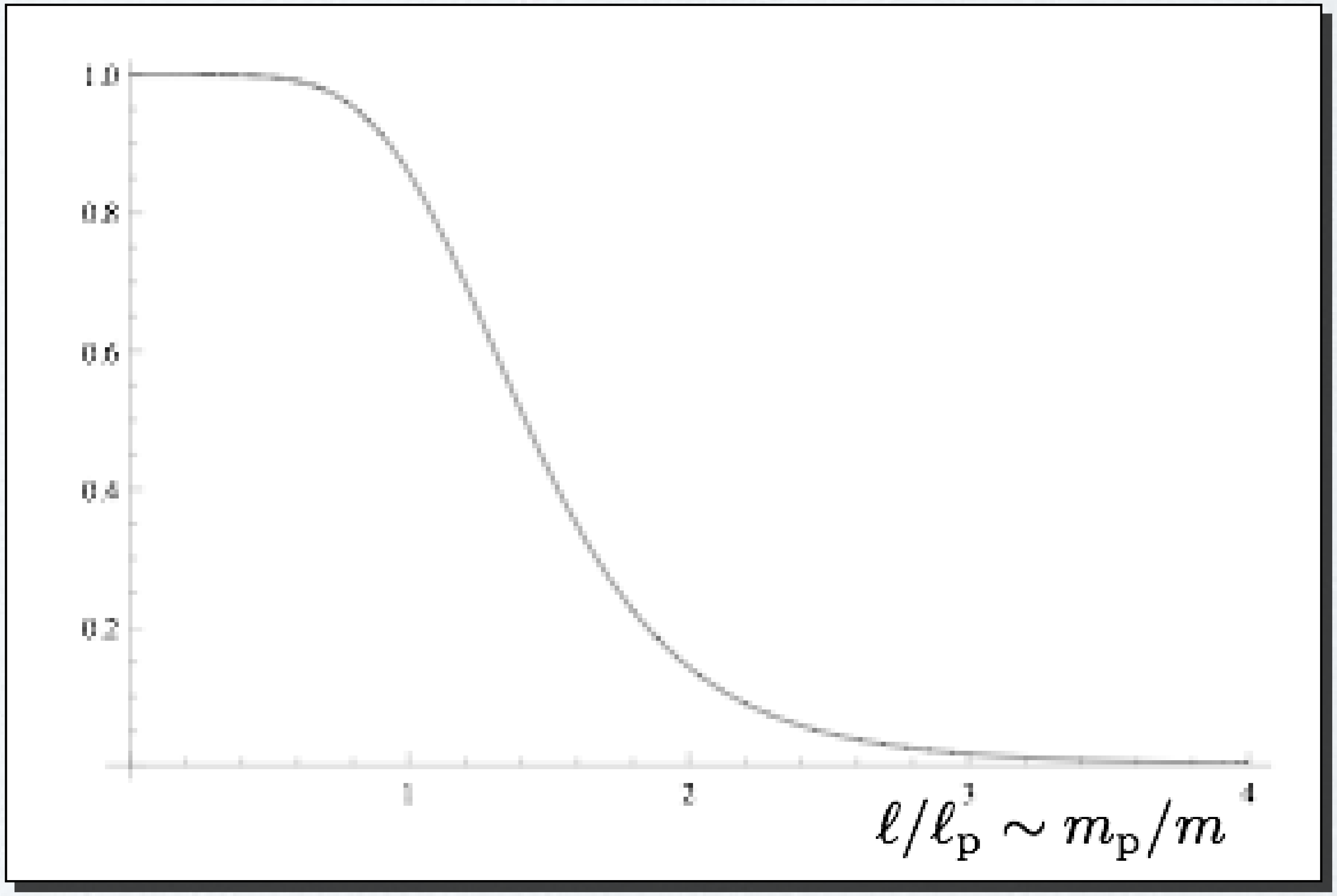
[ArXiv:1305.3195]

$$\psi_H(R_{\rm H}) = \frac{\ell^{3/2}\, e^{-\frac{\ell^2 R_{\rm H}^2}{8\,\ell_{\rm p}^4}}}{2^{3/2}\,\pi^{3/4}\,\ell_{\rm p}^3}$$



$$P_<(r < R_{\rm H}) = \frac{\ell^3 R_{\rm H}^2}{2\sqrt{\pi}\,\ell_{\rm p}^6}\, e^{-\frac{\ell^2 R_{\rm H}^2}{4\,\ell_{\rm p}^4}} \left[{\rm Erf}\left(\frac{R_{\rm H}}{\ell}\right) - \frac{2\,R_{\rm H}}{\sqrt{\pi}\,\ell}\, e^{-\frac{R_{\rm H}^2}{\ell^2}} \right]$$

$$P_{\rm BH}(\ell) = \frac{2}{\pi}\left[\arctan\left(2\,\frac{\ell_{\rm p}^2}{\ell^2}\right) + 2\,\frac{\ell^2\,(4 - \ell^4/\ell_{\rm p}^4)}{\ell_{\rm p}^2\,(4 + \ell^4/\ell_{\rm p}^4)^2}\right]$$



"Fuzzy" minimum mass:

$$M_{\rm BH} \gtrsim m_{\rm p}$$

4) GUP

[ArXiv:1306.5298, EPJ C]

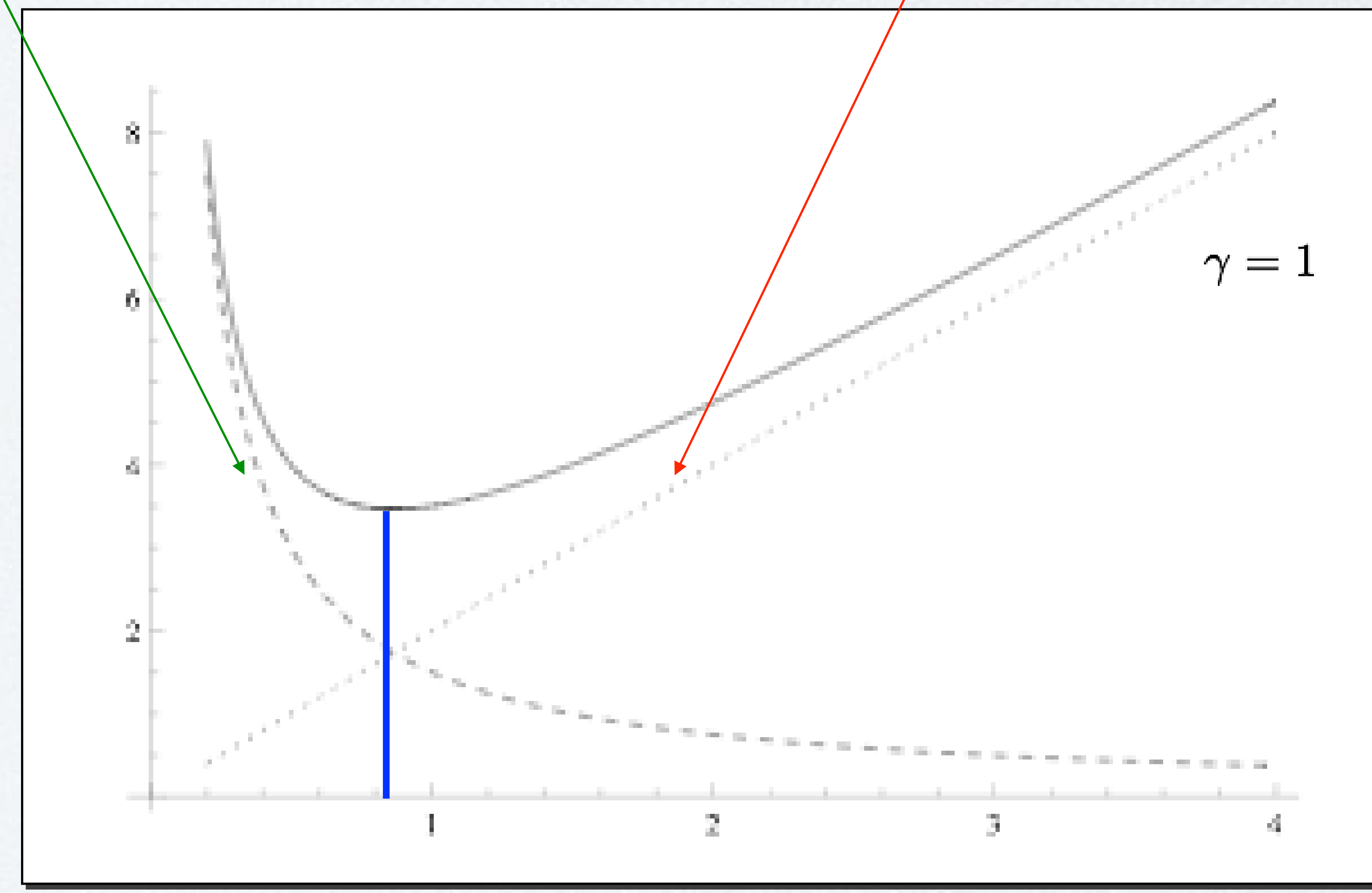
Two uncertainties:

$$\langle \Delta r^2 \rangle = \langle \psi_S | r^2 | \psi_S \rangle - \langle \psi_S | r | \psi_S \rangle^2 = \left(\frac{3\,\pi - 8}{2\,\pi} \right) \ell^2$$

$$\langle \Delta R_H^2 \rangle = 4 \left(\frac{3\,\pi - 8}{2\,\pi} \right) \frac{\ell_p^4}{\ell^2}$$

$$\Delta r \equiv \sqrt{\langle \Delta r^2 \rangle} + \gamma \sqrt{\langle \Delta R_H^2 \rangle} = \left(\frac{3\,\pi - 8}{2\,\pi} \right) \ell_p \frac{m_p}{\Delta p} + 2\,\gamma\,\ell_p \frac{\Delta p}{m_p}$$

$$\langle \Delta p^2 \rangle = \left(\frac{3\,\pi - 8}{2\,\pi} \right) m_p^2 \frac{\ell_p^2}{\ell^2}$$



Minimum length

N.B. Uncertainty **derived** with standard canonical commutators: $[q, p] = i\,\hbar$
(gravity is more than kinematics...?)

5) Collisions

[ArXiv:1311.5698]

1) Two localised particles: $\psi_{\rm S}(x_1,x_2)=\psi_{\rm S}(x_1)\,\psi_{\rm S}(x_2)$

$$\psi_{\rm S}(x_i)=e^{-i\,\frac{P_i\,x_i}{\hbar}}\,\frac{e^{-\frac{(x_i-X_i)^2}{2\,\ell_i}}}{\sqrt{\pi^{1/2}\,\ell_i}}$$

$$\Delta_i=\hbar/\ell_i$$

2) Two particles in momentum space: $\psi_{\rm S}(p_i)=e^{-i\,\frac{p_i\,X_i}{\hbar}}\,\frac{e^{-\frac{(p_i-P_i)^2}{2\,\Delta_i}}}{\sqrt{\pi^{1/2}\,\Delta_i}}$

$$|\psi_{\rm S}^{(1,2)}\rangle=\prod_{i=1}^{2}\left[\int_{-\infty}^{+\infty}dp_i\,\psi_{\rm S}(p_i,t)\,|p_i\rangle\right]$$

X_i

P_i

5) Collisions

[ArXiv:1311.5698]

3) Unnormalised horizon wave-function:

$$|\psi_{\rm S}\rangle = \sum_E C(E)\,|E\rangle$$

$$C(E) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \psi_{\rm S}(p_1)\,\psi_{\rm S}(p_2)\,\delta(E - E_1 - E_2)\,dp_1\,dp_2$$

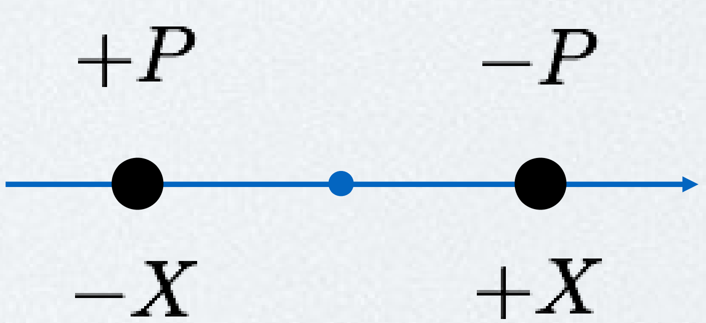
4) Centre-mass and relativistic limit:

$$\ell_i = \frac{\hbar}{\sqrt{P_i^2 + m_i^2}} \simeq \frac{\ell_{\rm p}\,m_{\rm p}}{|P_i|}$$

$$\Delta_i \simeq |P_i|$$

$$P_1 = -P_2 \equiv P > 0$$

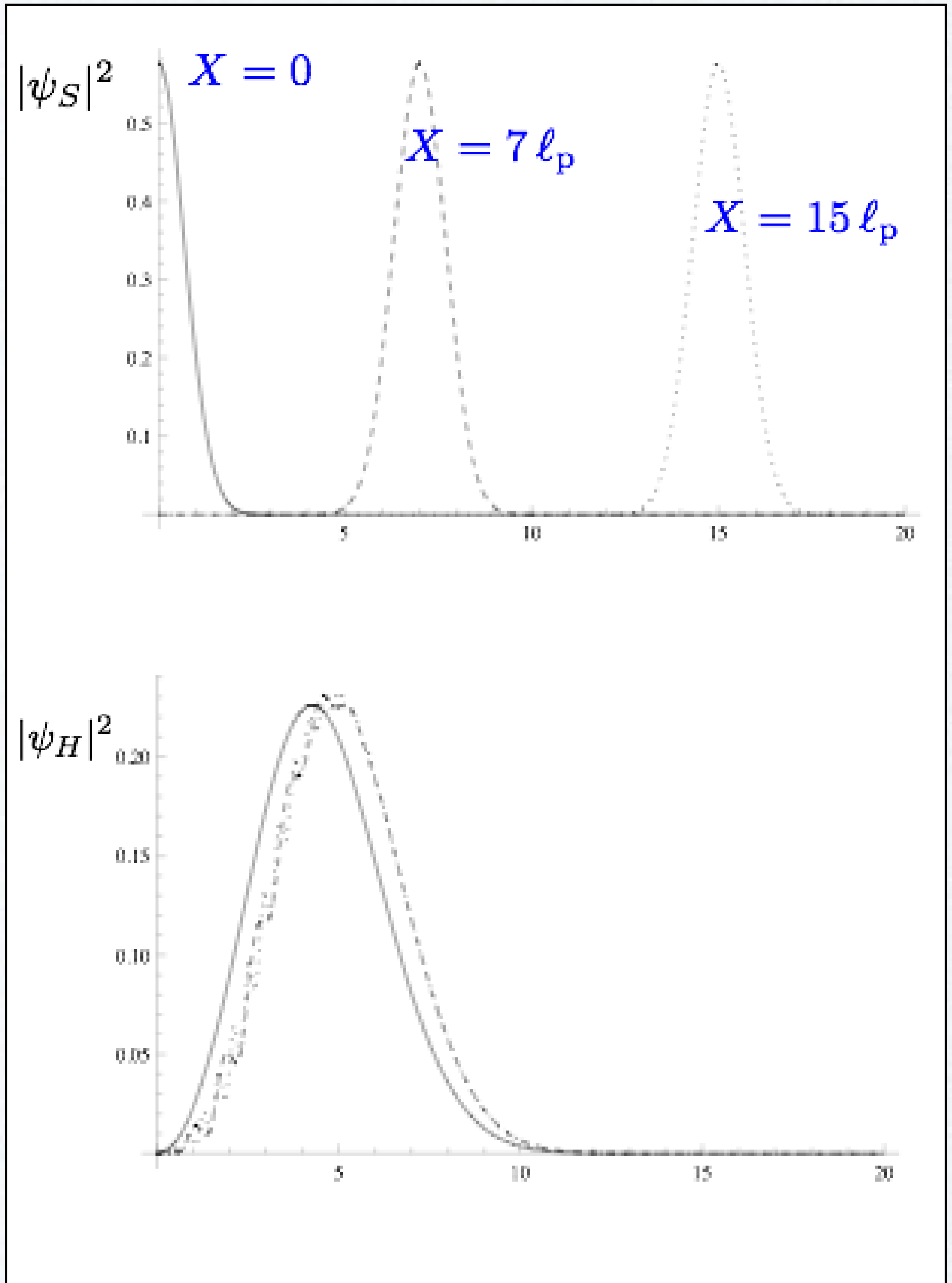
$$X_1 \simeq -X_2 \equiv X > 0$$



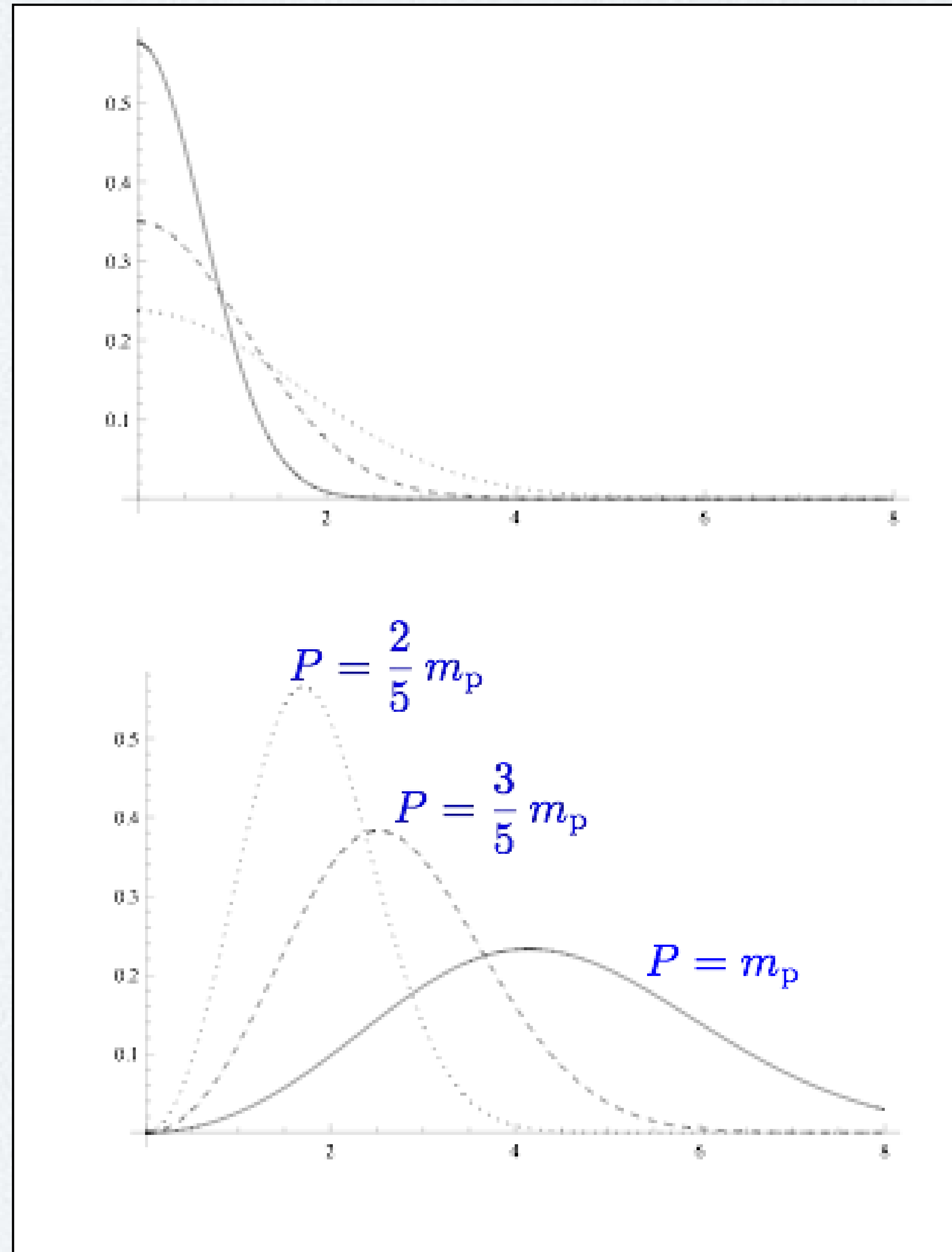
5) Collisions

[ArXiv:1311.5698]

Horizon wave-function:



$P = m_p$

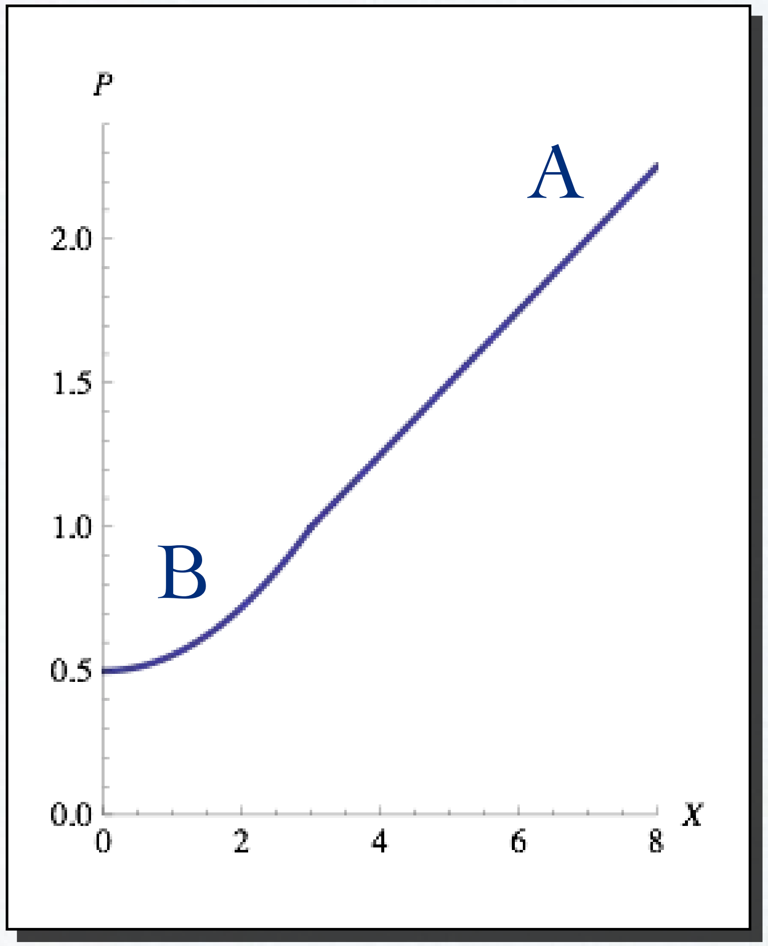


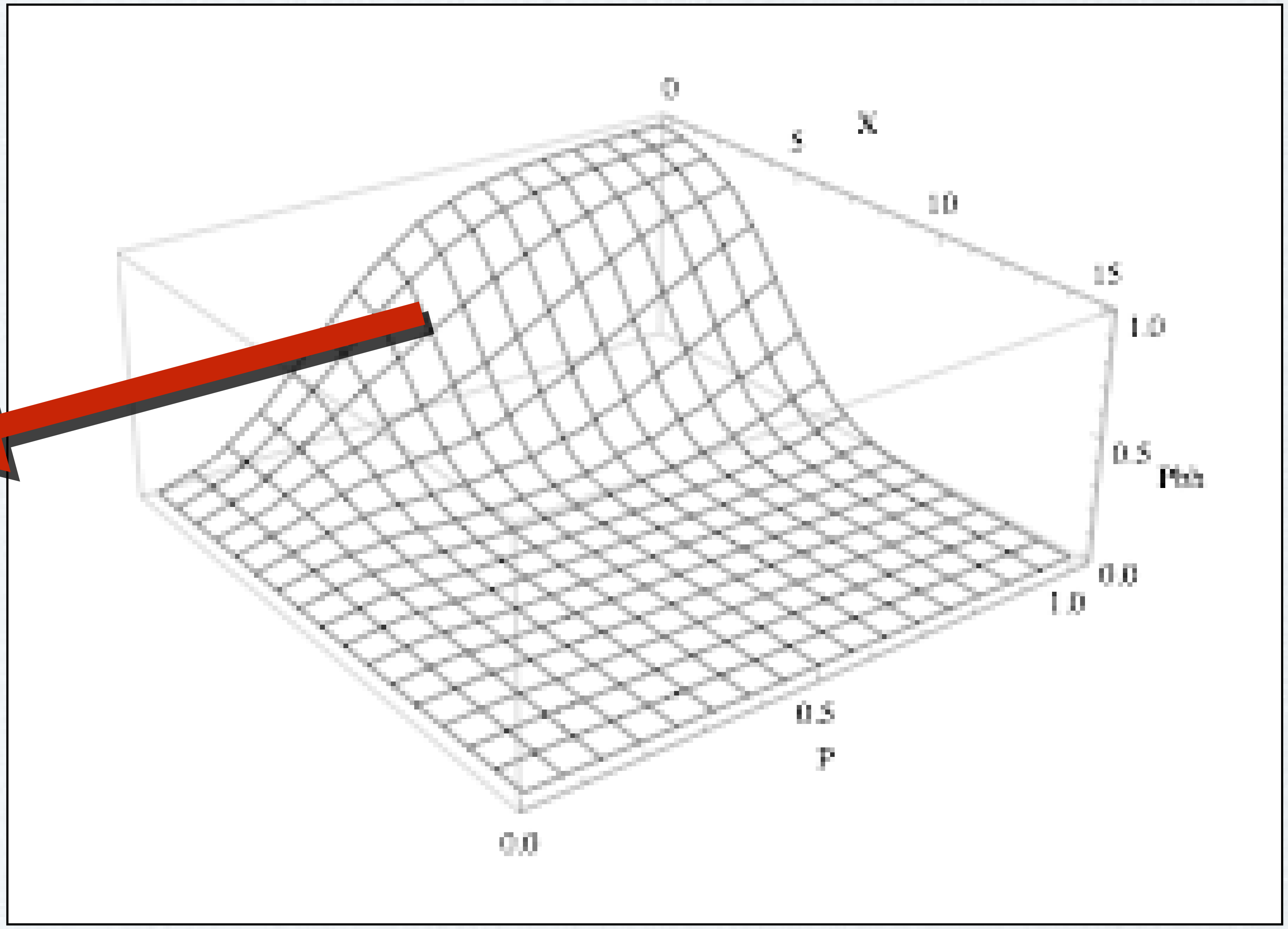
$X = 0$

5) Collisions

[ArXiv:1311.5698]

6) Hoop conjecture:





A) classical

$$P_{\rm BH}(X, 2P \gtrsim 2\, m_{\rm p}) \gtrsim 80\%$$

$$X \lesssim 2\,\ell_{\rm p}\,(2P/m_{\rm p}) - \ell_{\rm p} \simeq R_{\rm H}(2P)$$

B) quantum

$$P_{\rm BH}(X, 2P \lesssim 2\, m_{\rm p}) \gtrsim 80\%$$

$$2P - m_{\rm p} \gtrsim \frac{m_{\rm p}\, X^2}{9\,\ell_{\rm p}}$$

Summary and outlook

1. Horizon wave-function describes spherical particle/BH + **GUP**
2. Horizon wave-function yields **quantum hoop conjecture** for 2-particle collisions (in flat 1+1 dimensions)
3. Account for particle(s) self-gravity (BEC BHs - arXiv:1405.4192)
4. Generalise to non-spherical systems (and spin)
5. Analyse (2-)particle collisions with angular momentum+spin
6. (Hope for?) quantum description of gravitational collapse