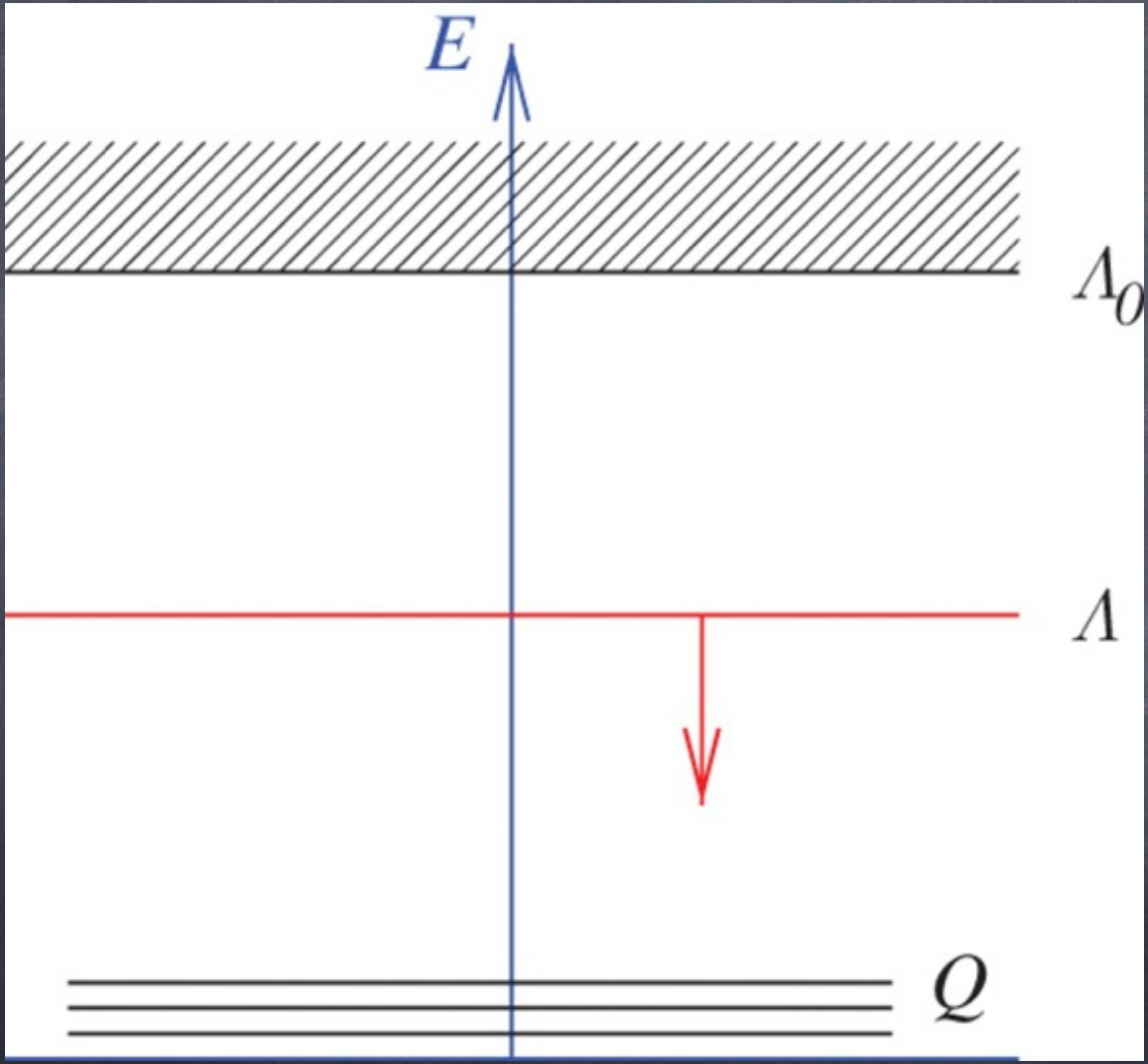
Models of Asymptotically Safe Inflation

Alfio Bonanno - INAF - Catania

Wilsonian renormalization



All theories are effective!

Kadanoff blocking generates a flow for infinitely many couplings $\lambda_i(k)$

$$\beta_i = \sum_j M_{ij} \, (\lambda_j - \lambda_j^*) + \text{subleading}$$

$$\lambda_i(k) = \lambda_i^* + \sum_n c_n \, V_i^n \, k^{\vartheta_n} + \text{subleading}$$

Predictability means finite dimensional UV-critical surface!

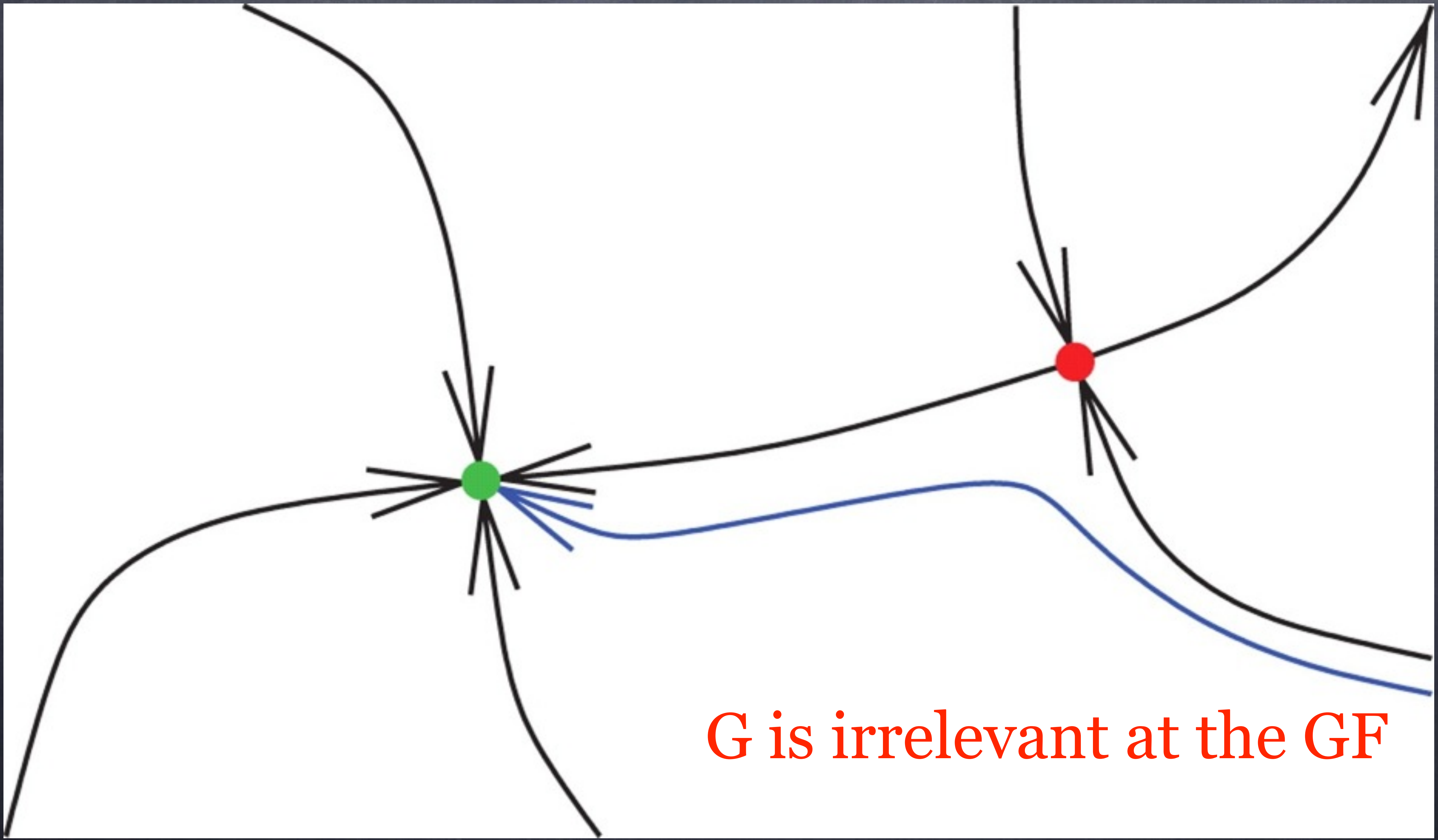
NONPERTUBATIVE UV FP

- SCALAR THEORY K.G. Wilson & Kogut, Phys.Rep. 12 C, 1974
- LARGE N Gross-Neveu model in d<4 K.G.Wilson, PRD10, '73

Gravity: Asymptotic Safety

- Gravity in d=2+epsilon S.Weinberg, 1972
- LARGE N L.Smolin Nuc.Phys.B208 (1982) 439; R.Percacci PRD73 (2006)

Multiple FPs



Quantum Einstein Gravity

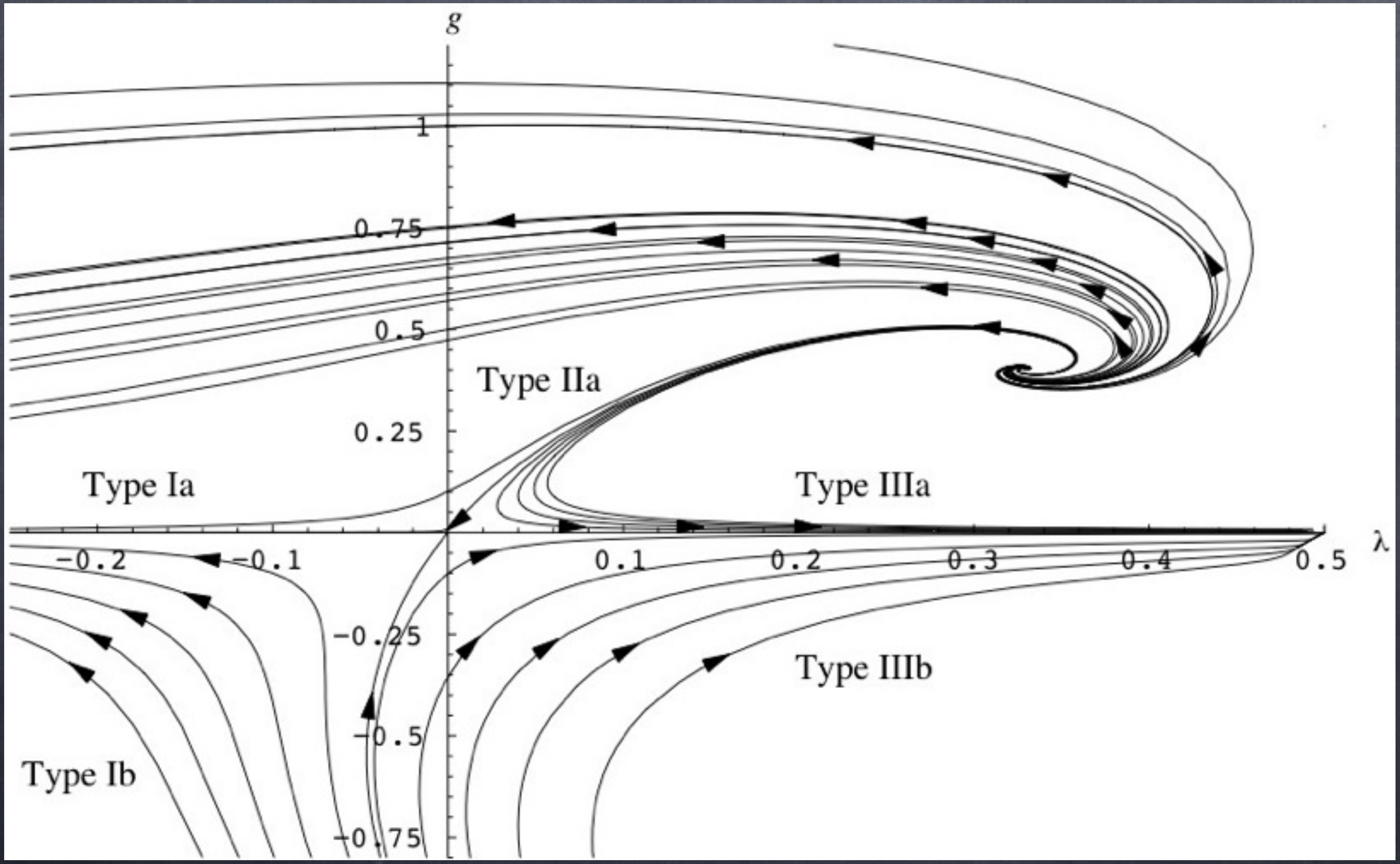
$$g_k := k^2 G_k \,, \quad \lambda_k := \Lambda_k k^{-2}$$

$$k\,\partial_k g_k = (\eta_N + 2) g_k \,,$$

$$k\,\partial_k \lambda_k = -(2 - \eta_N)\lambda_k - \frac{g_k}{\pi}\left[5\ln(1 - 2\lambda_k) - 2\zeta(3) + \frac{5}{2}\eta_N\right]$$

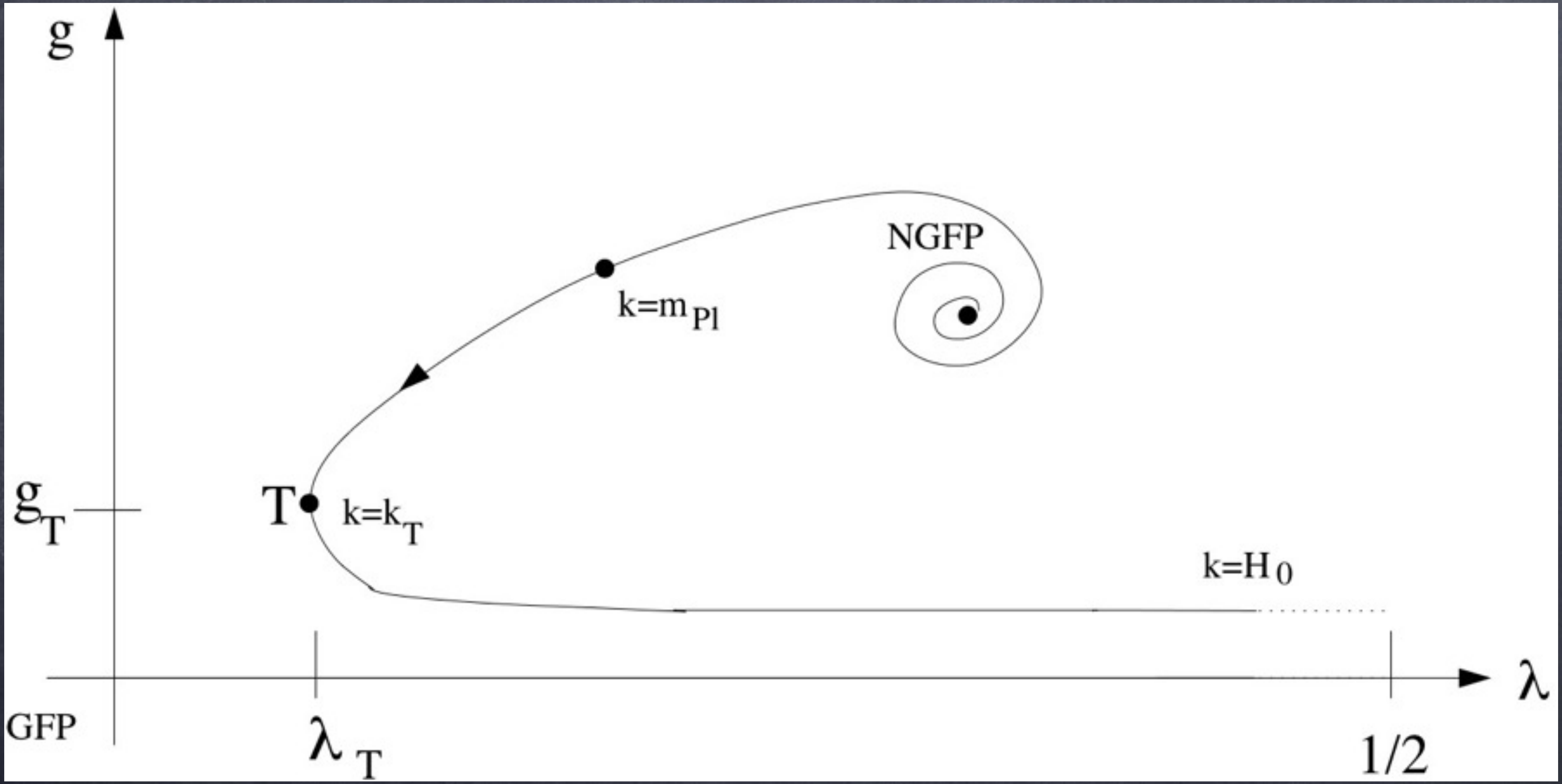
$$\eta_N = -\frac{2g_k}{6\pi + 5g_k}\left[\frac{18}{1-2\lambda_k} + 5\ln(1 - 2\lambda_k) - \zeta(2) + 6\right.$$

two attractive relevant directions with complex critical exponents



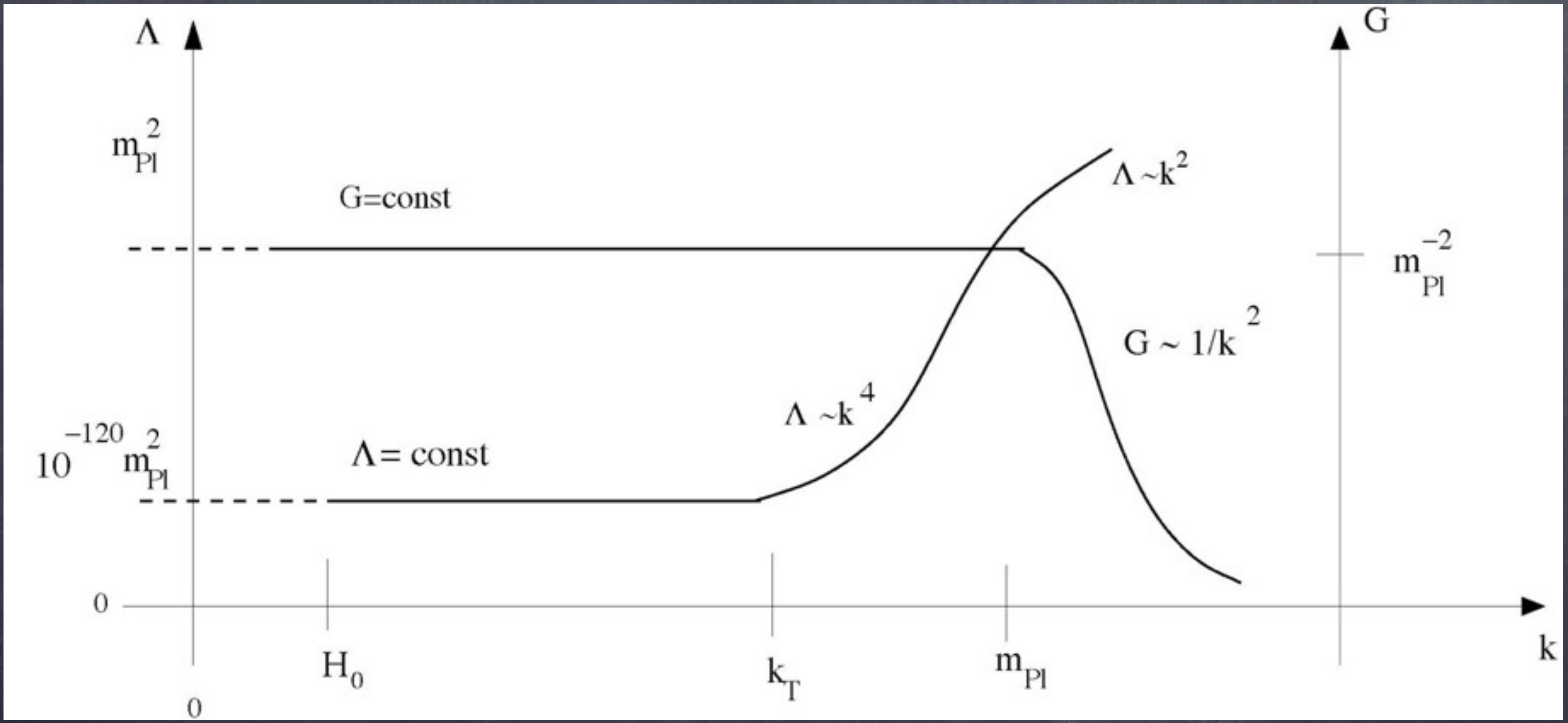
Saueressig & Reuter, PRD, 2002

Cosmological consequences



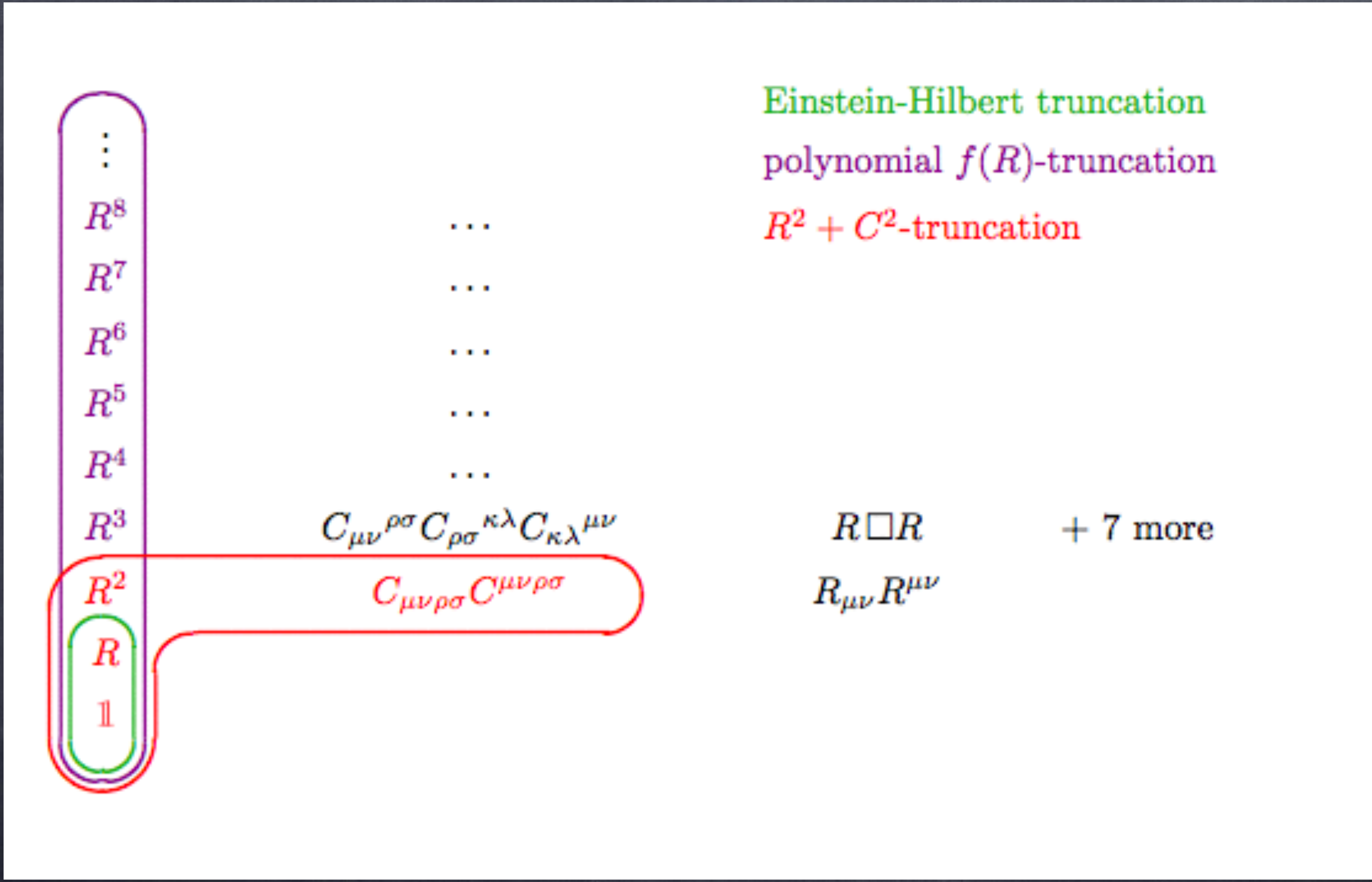
assume k = 1 / t

Complete cosmic history



AB & M Reuter, 2003, 2007, 2008. AB, Esposito, Rubano, 2003,2004,2005; AB, Contillo, Percacci 2010

More general truncations



Asymptotically Safe Inflation

$$I_\Lambda[g] = -\int d^4x \sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R + g_{2a}(\Lambda) R^2$$
$$+ g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu} + \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \ldots\Big]$$

Optimal cutoff: radiative corrections just beginning to be important and higher order terms just beginning to be less important

Objective: to obtain a dS solution which is unstable but lasts N>60 e-folds

S.Weinberg PRD 2010, Thie & Xu, PRD 2010

Beyond polynomial truncations

- Project on maximally symmetric spaces
- Effective action
- Solve a non-linear PDE flow equation
- SEE NEXT TALK!

How to extract physical information ?

RG-improvement of the standard QCD Lagrangian

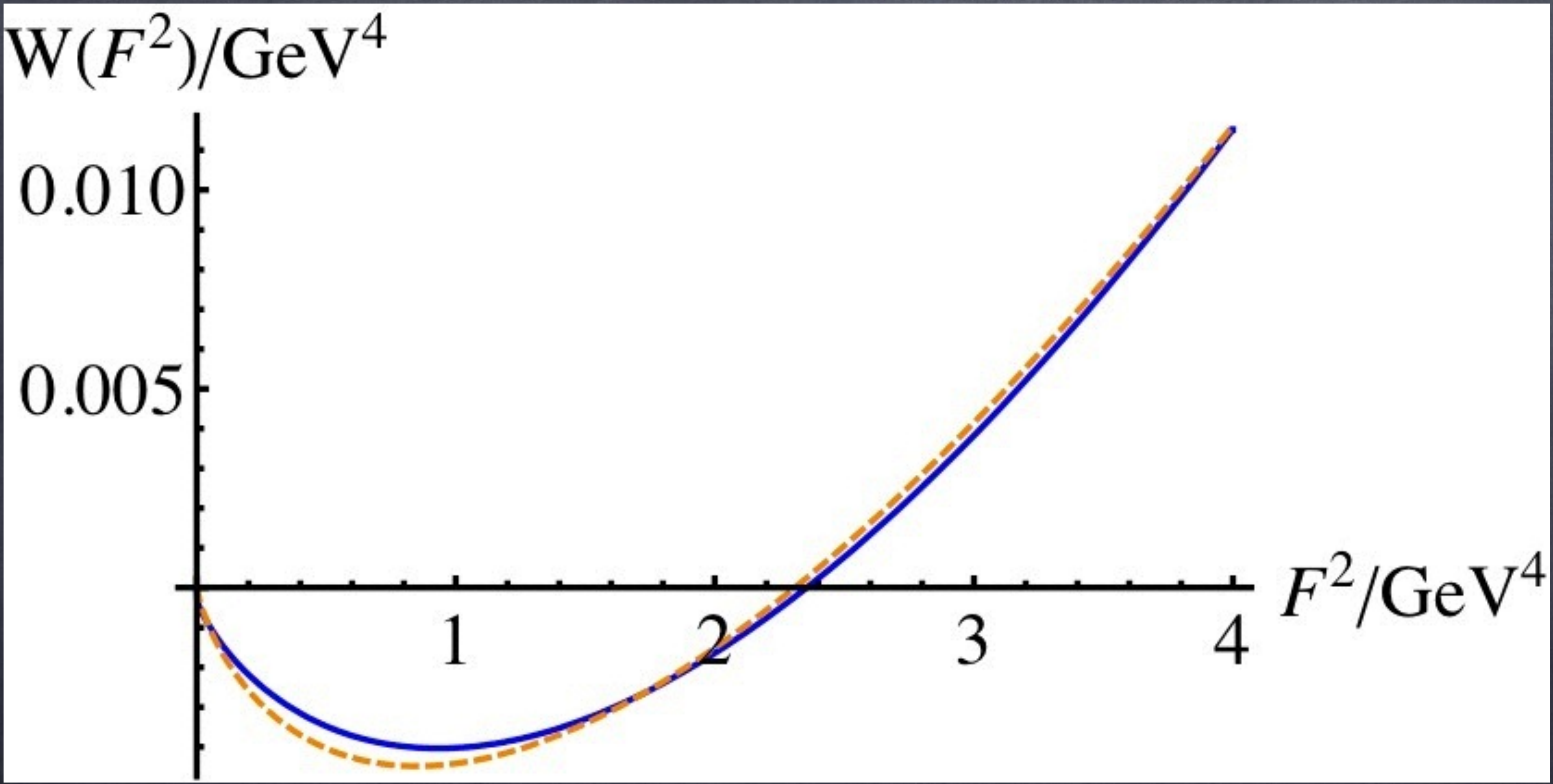
$$\mathcal{L}_{\rm eff}^{\rm QCD} = \frac{\mathcal{F}}{2g^2_{\rm running}} \qquad g^2_{\rm running} = \frac{2g^2(\mu^2)}{1+\frac{1}{4}\,b\,g^2(\mu^2)\log\left(\frac{\mathcal{F}}{\mu^4}\right)}$$

$$k^2 \propto \mathcal{F}^{1/2}$$

Use the field-strength as a cutoff as in the "leading-log" model

$$\mathcal{L}_{\rm eff}^{\rm QCD}(\mathcal{F}) = \frac{\mathcal{F}}{g^2}\left[1 + \frac{1}{4} b\, g^2 \log\left(\frac{\mathcal{F}}{\mu^4}\right)\right] = \frac{1}{8}\, b\, \mathcal{F} \log\left(\frac{\mathcal{F}}{{\rm e}\kappa^2}\right)$$

$$\mathcal{F} = -\frac{1}{2}(D_\mu A_\nu^a - D_\nu A_\mu^a)^2, \qquad \kappa^2 = \frac{\mu^4}{\rm e} \exp\left(-\frac{4}{bg^2}\right)$$



Eichhorn, Gies and Pawlowski, 2011

Apply the same approach in QEG

Einstein-Hilbert truncation: $$\mathcal{L}^{\rm EH} = \frac{1}{16\pi G}(R - 2\Lambda)$$

Linearized flow around NGFP:

$$(\lambda, g)^{\mathbf{T}} = (\lambda_*, g_*)^{\mathbf{T}} + 2\{[\mathrm{Re}C\cos(\theta''t) + \mathrm{Im}C\sin(\theta''t)]\mathrm{Re}\,V$$
$$+[\mathrm{Re}C\cos(\theta''t) - \mathrm{Im}C\sin(\theta''t)]\mathrm{Im}\,V\}\,e^{-\theta' t}$$

$$t = \ln(k/k_0) \qquad \theta = \theta' \pm i\theta''$$

(AB, PRD 2012)

Substitute this solution in the EH Lagrangian after identifying k with the field strength

$$\mathcal{L}_{\rm eff}^{\rm QEG}(R) = R^2 + bR^2 \cos\left[\alpha \log\left(\frac{R}{\mu}\right)\right] \left(\frac{R}{\mu}\right)^{\beta}$$

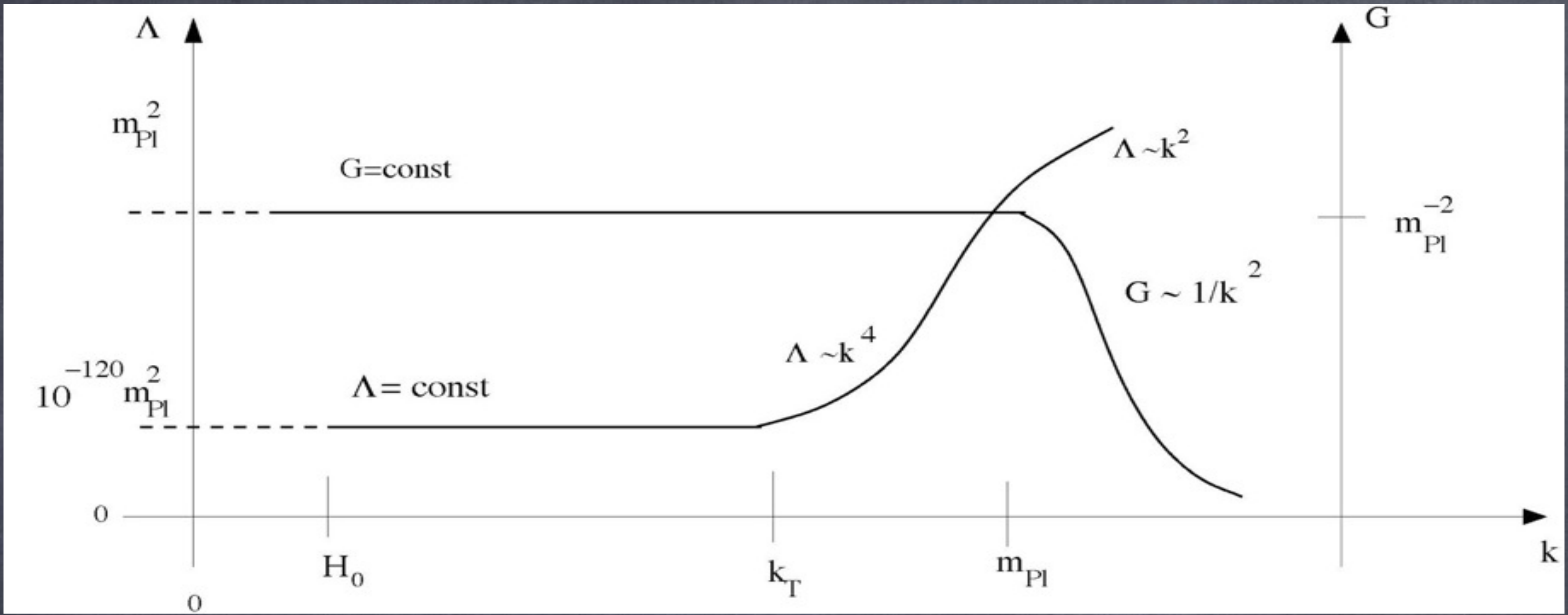
$$\alpha = \theta''/2, \ \beta = -\theta' < 0$$

μ is a renormalization scale

$$2.1 < \theta' < 3.4, \qquad 3.1 < \theta'' < 4.3$$

Dietz and Morris, 2013

$$f(R) = R^2 + R + A\ R \cos\log R^2 + B\ R \sin\log R^2$$



look for de Sitter solutions:

$$\bar{H} = \sqrt{\frac{\mu}{12}}\ \exp\left[\frac{1}{2\alpha}\left(\tan^{-1}\frac{\beta}{\alpha} + n\pi\right)\right], \quad n \in \mathbb{Z}$$

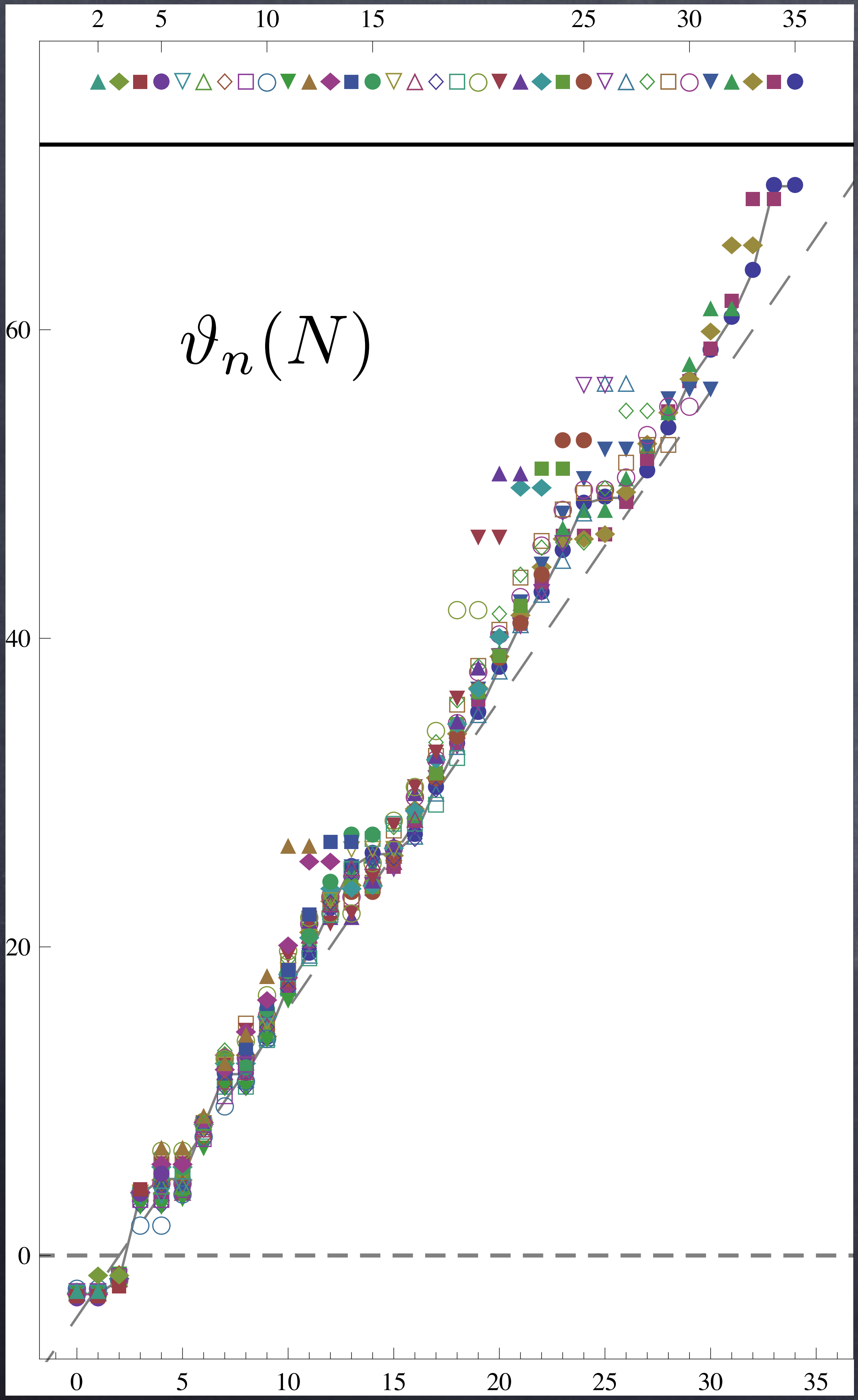
Look for unstable solutions with growth time >>1/H so that inflation comes to an end after enough e-folds

$$H(t) = \bar{H} + \delta \exp(\xi \bar{H} t) \qquad \xi^2 + \xi\, 3\, e^{\frac{n\pi}{2\alpha}} + A = 0$$

$$1/\xi \approx e^{-n\pi/\theta''}$$

The stability of the solutions does not depend on μ

For negative values of n, A is always negative !



$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n k^{d_n} \int d^4x \sqrt{g} R^n$$

$$\vartheta_n(N), 0 \leq n \leq N-1$$

Only three operators are relevant at NGFP

$$\vartheta_0, \quad \vartheta_1, \quad \vartheta_2$$

Falls, Litim, Nikolakopoulos, Rahmede, 2012

Approximate Effective Action for the N=34 truncation

$$\mathcal{L}_{\text{eff}} = R + \frac{1}{6M^2}R^2 + \frac{\lambda}{3M\sqrt{3}}R^{2-\frac{\vartheta_2}{2}}$$

$$\vartheta_2 \approx 1.2$$

Starobinsky model + Planck scale modification

conformal frame representation

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) \qquad f(R) = R + F(R)$$

The above action is equivalent to

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\varphi R - U(\varphi)\right]$$

where

$$\varphi \equiv 1 + F_{,\chi}(\chi)$$

Study the theory in the conformal frame

$$g^{E}_{\mu\nu} = \varphi g_{\mu\nu}$$

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{1}{2\kappa^2}R_E - \frac{1}{2}g^{\mu\nu}_E\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$V(\phi) = \frac{1}{2\kappa^2\varphi^2}\left[(\varphi - 1)\chi(\varphi) - F(\chi(\varphi))\right]$$

$$\varphi = e^{\sqrt{2/3}\kappa\phi}$$

Inflationary potential

$$V(\phi) =$$

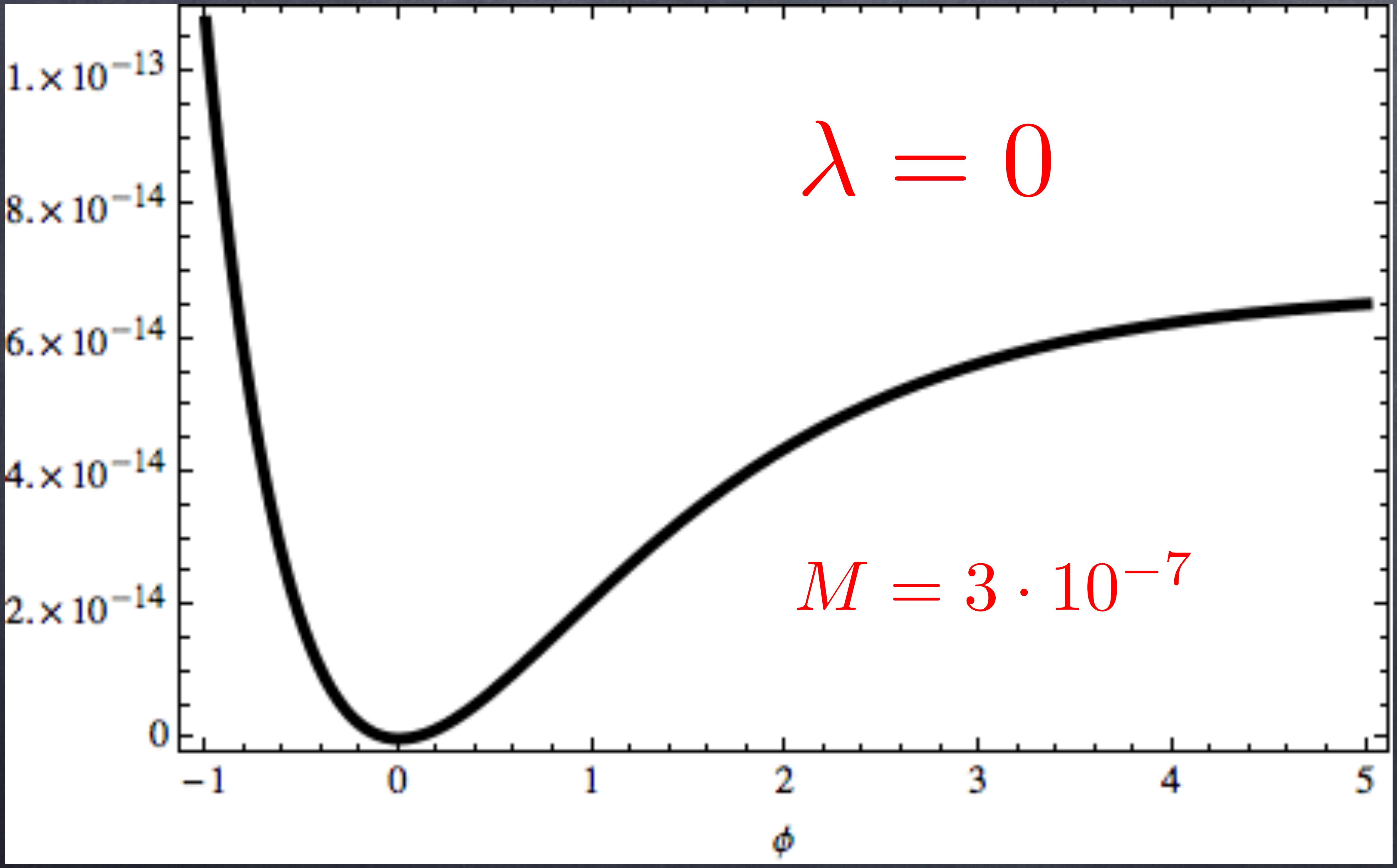
$$-\frac{e^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{256\kappa^2 M}\left[6\lambda^3 M\sqrt{M^4\left(16e^{\sqrt{\frac{2}{3}}\kappa\phi}+\lambda^2-16\right)}\right.$$

$$+3\lambda^2 M^3\left(16e^{\sqrt{\frac{2}{3}}\kappa\phi}+\lambda^2-16\right)+$$

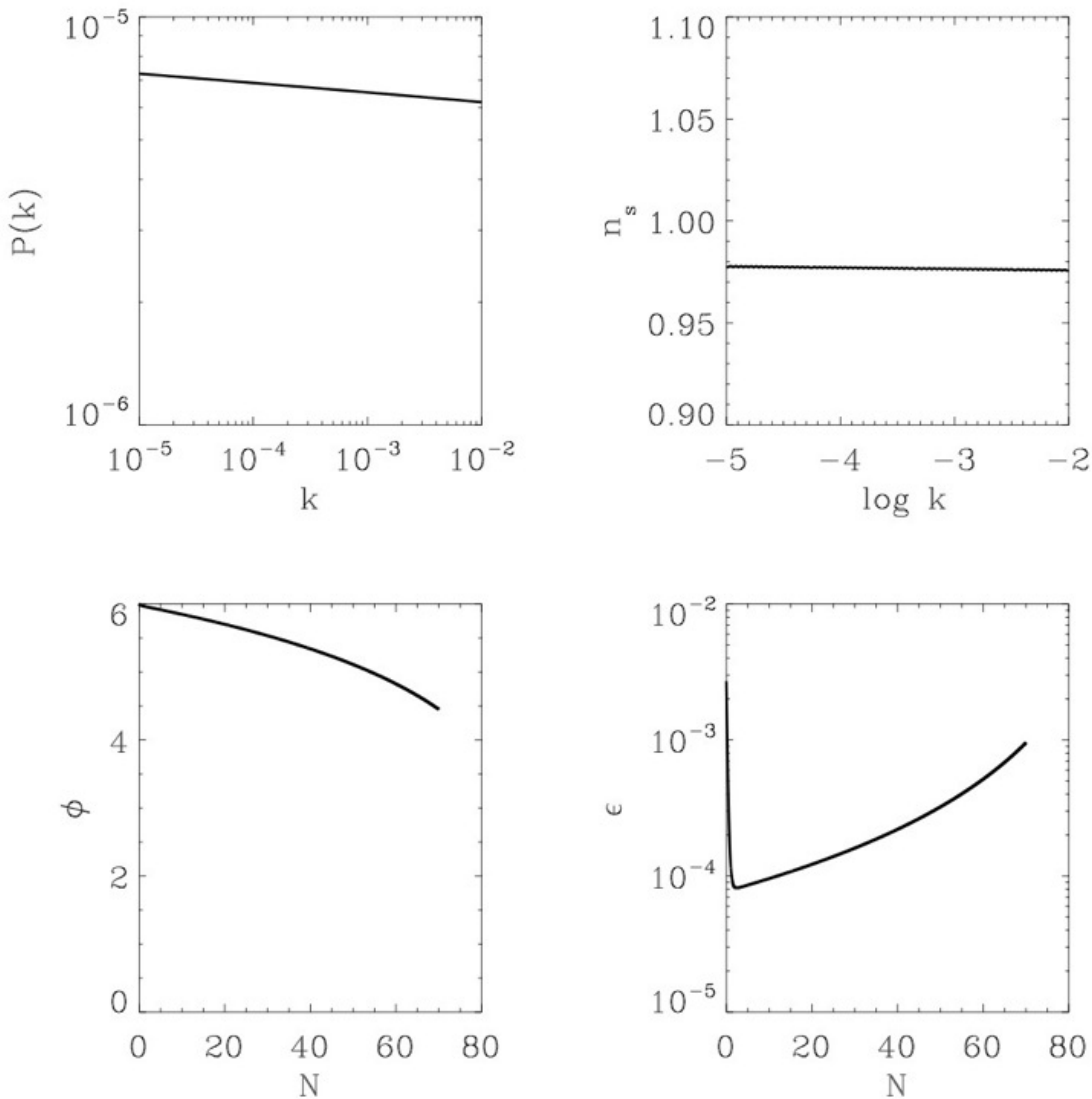
$$384M^3e^{\sqrt{\frac{2}{3}}\kappa\phi}-192M^3e^{2\sqrt{\frac{2}{3}}\kappa\phi}+3\lambda^4M^3-192M^3$$

$$+4\sqrt{2}\lambda\left(\lambda\sqrt{M^4\left(16e^{\sqrt{\frac{2}{3}}\kappa\phi}+\lambda^2-16\right)}+\right.$$

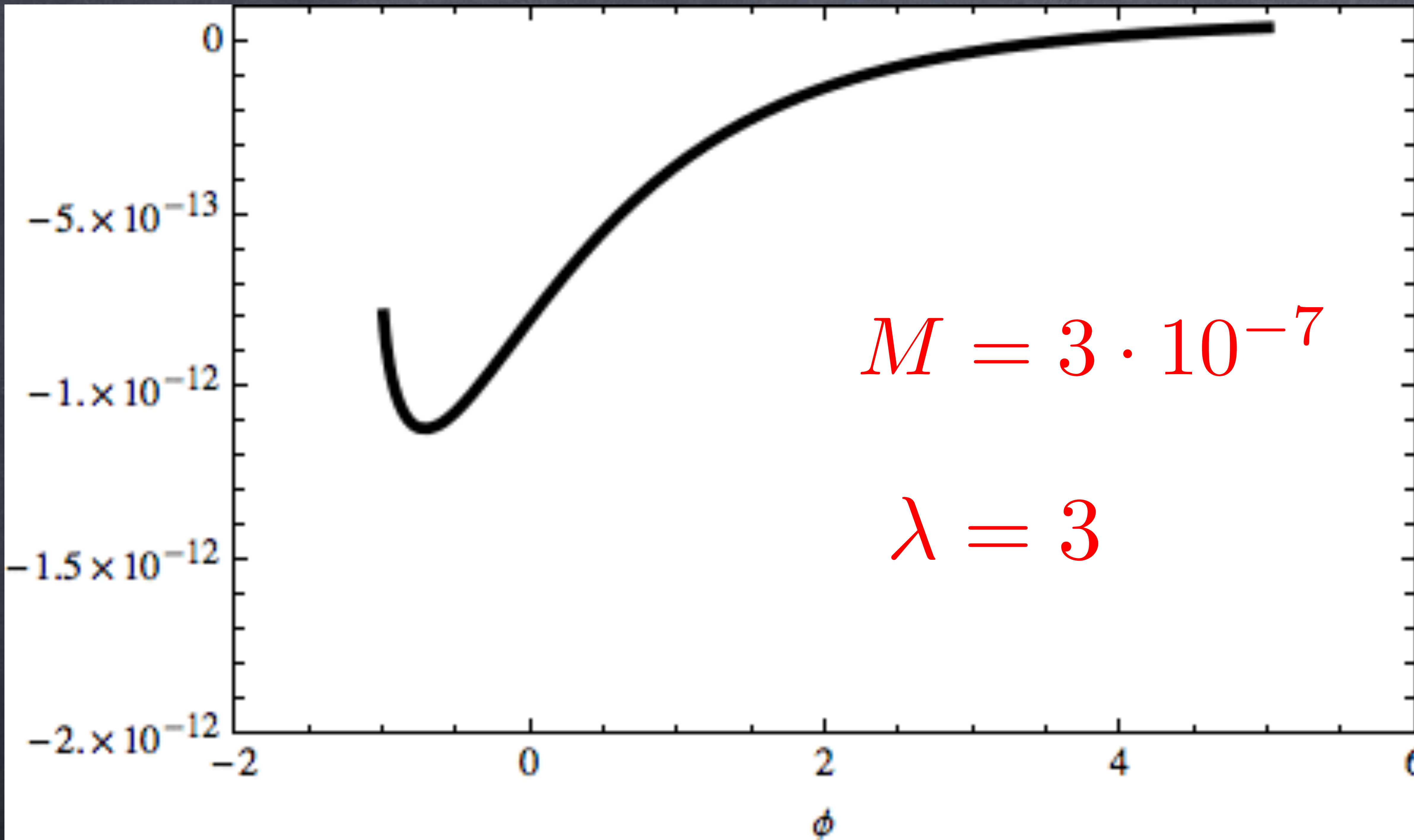
$$\left.\left.M^2\left(8e^{\sqrt{\frac{2}{3}}\kappa\phi}+\lambda^2-8\right)\right)^{3/2}\right]$$



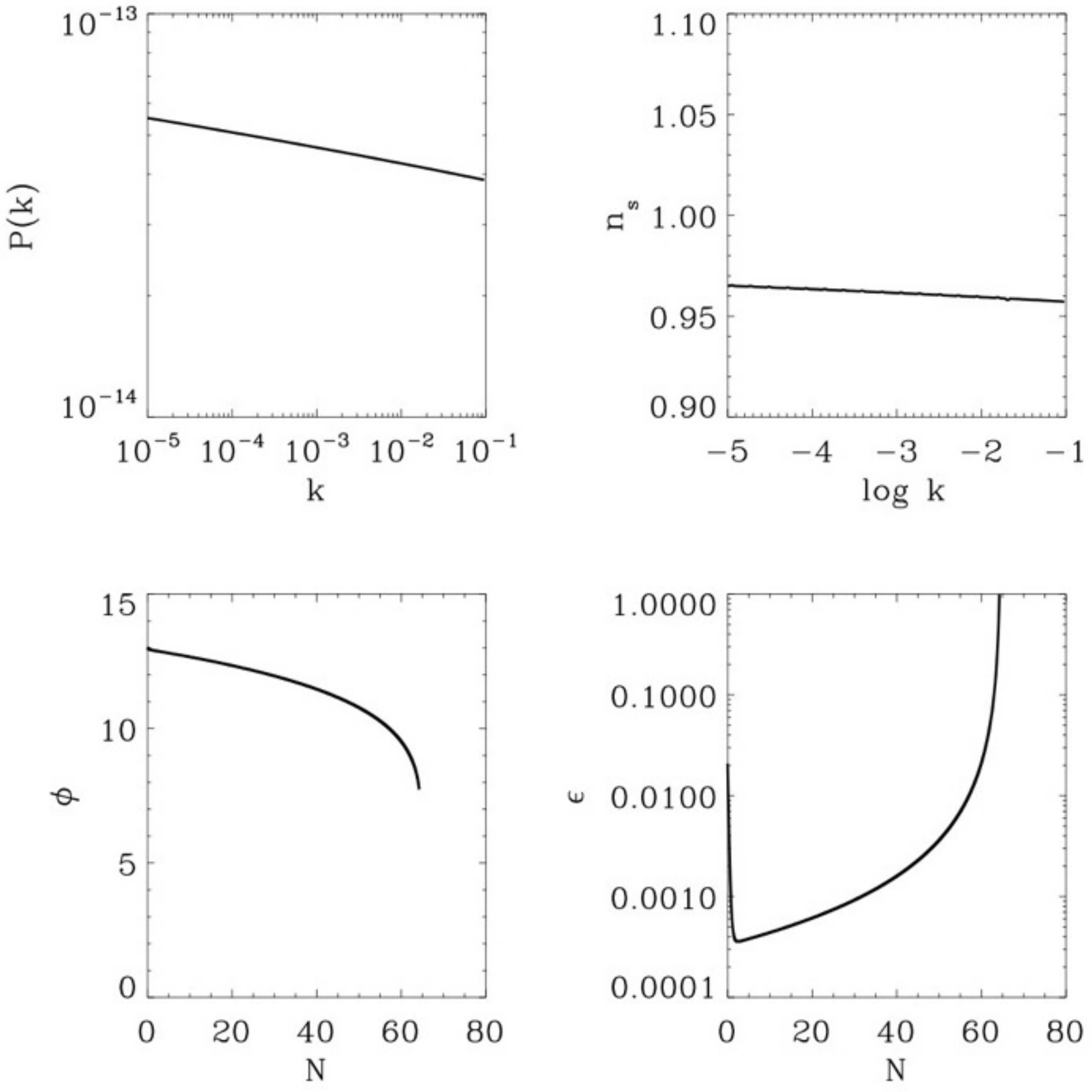
1.×10^−13
8.×10^−14
6.×10^−14
4.×10^−14
2.×10^−14
0
−1
0
1
2
3
4
5
ϕ
λ = 0
M = 3 · 10^−7



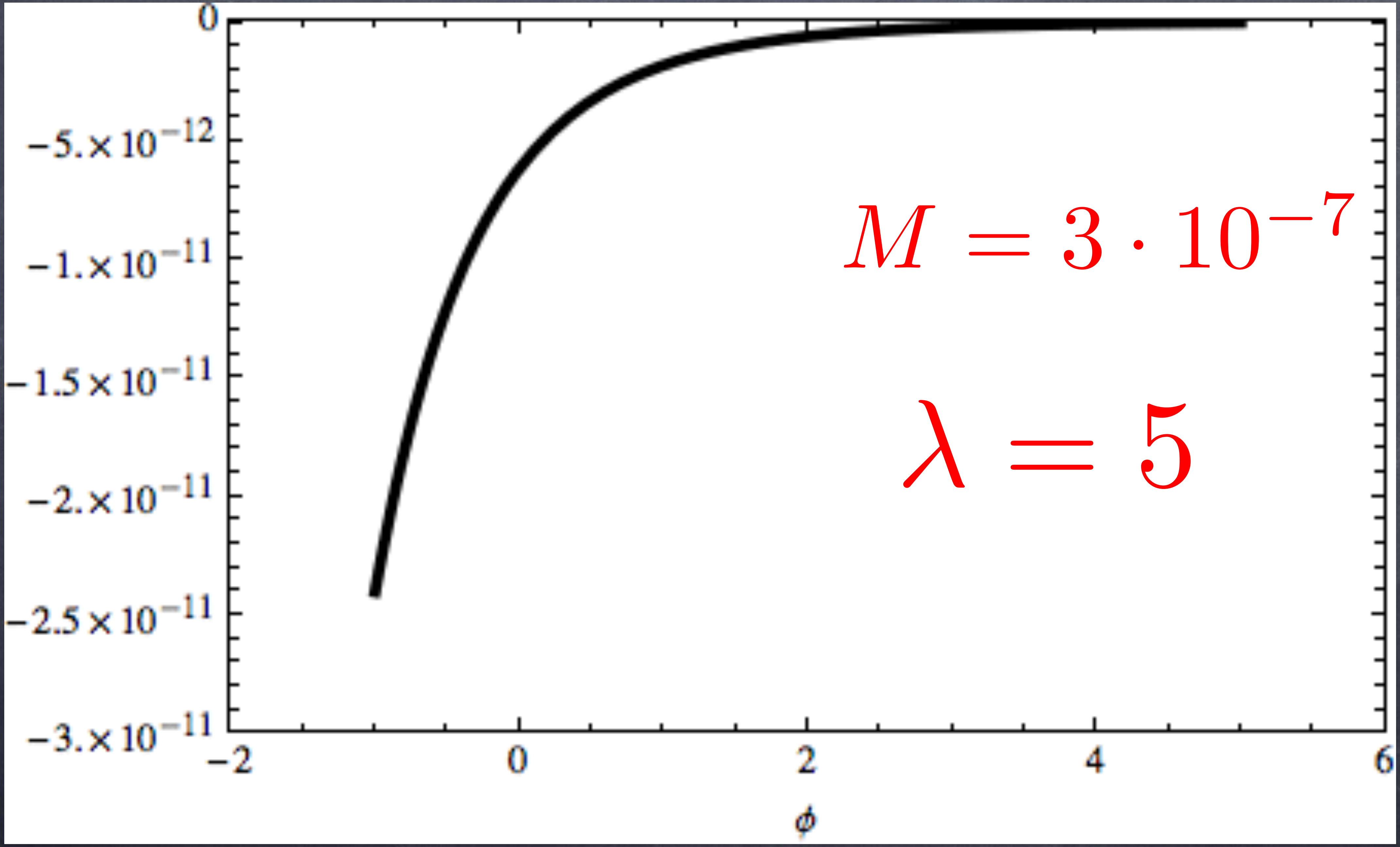
10^{-5}
10^{-6}
P(k)
10^{-5} 10^{-4} 10^{-3} 10^{-2}
k
1.10
1.05
1.00
0.95
0.90
n_s
−5 −4 −3 −2
log k
6
4
2
0
ϕ
0 20 40 60 80
N
10^{-2}
10^{-3}
10^{-4}
10^{-5}
ϵ
0 20 40 60 80
N



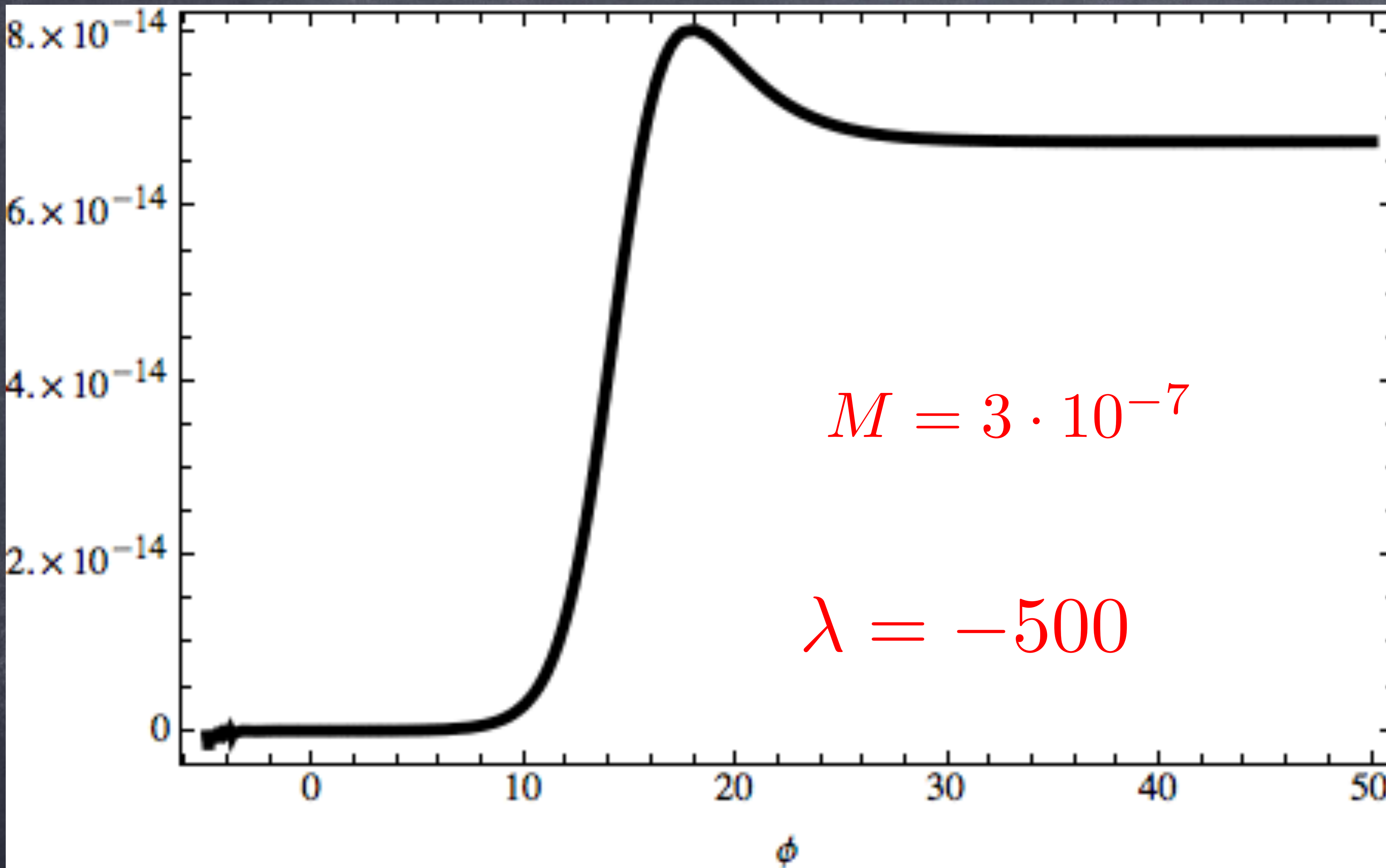
0
−5.×10⁻¹³
−1.×10⁻¹²
−1.5×10⁻¹²
−2.×10⁻¹²
−2
0
2
4
6
φ
$M = 3 \cdot 10^{-7}$
$\lambda = 3$



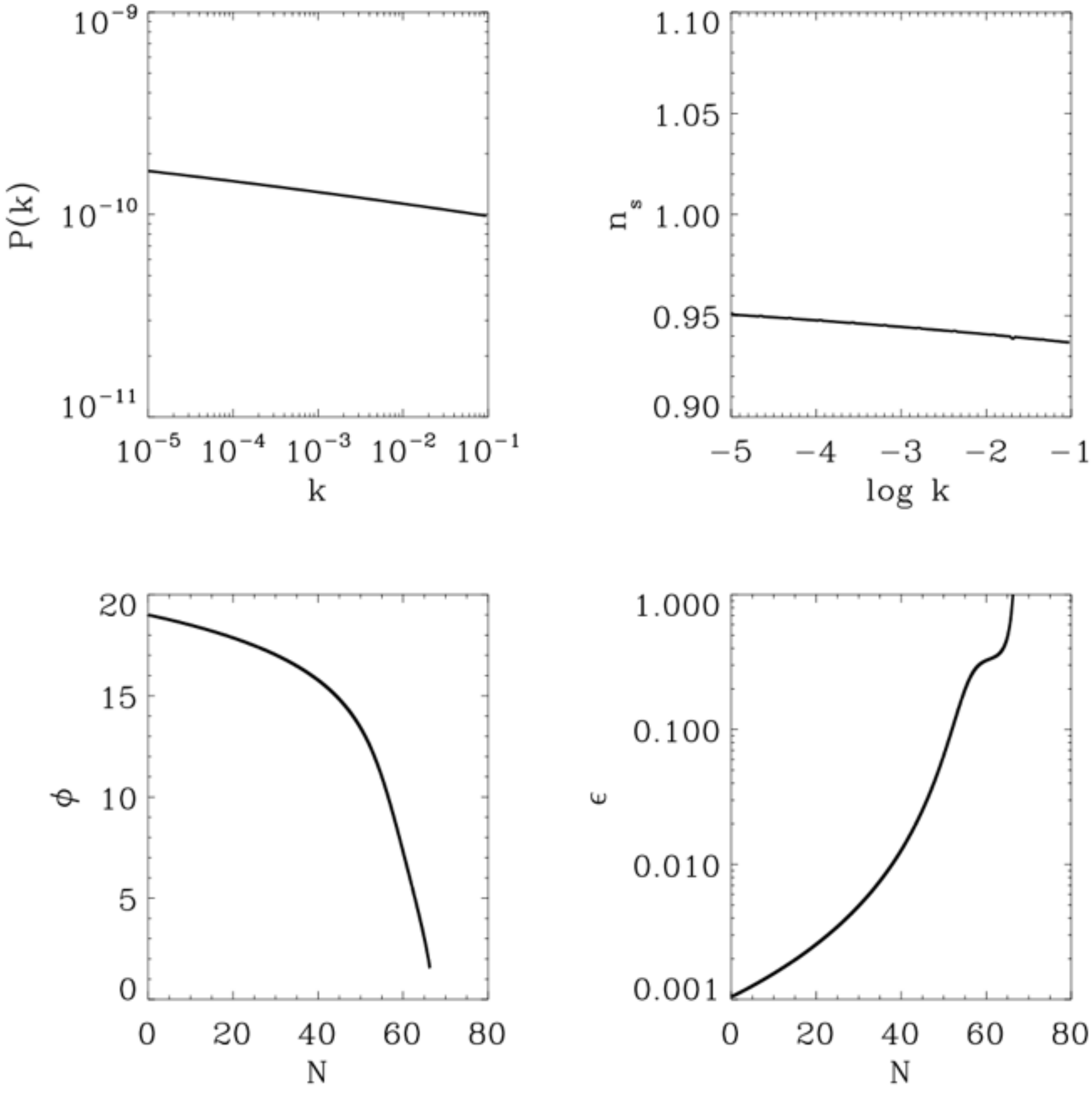
P(k)
k
n_s
log k
φ
N
ε
N



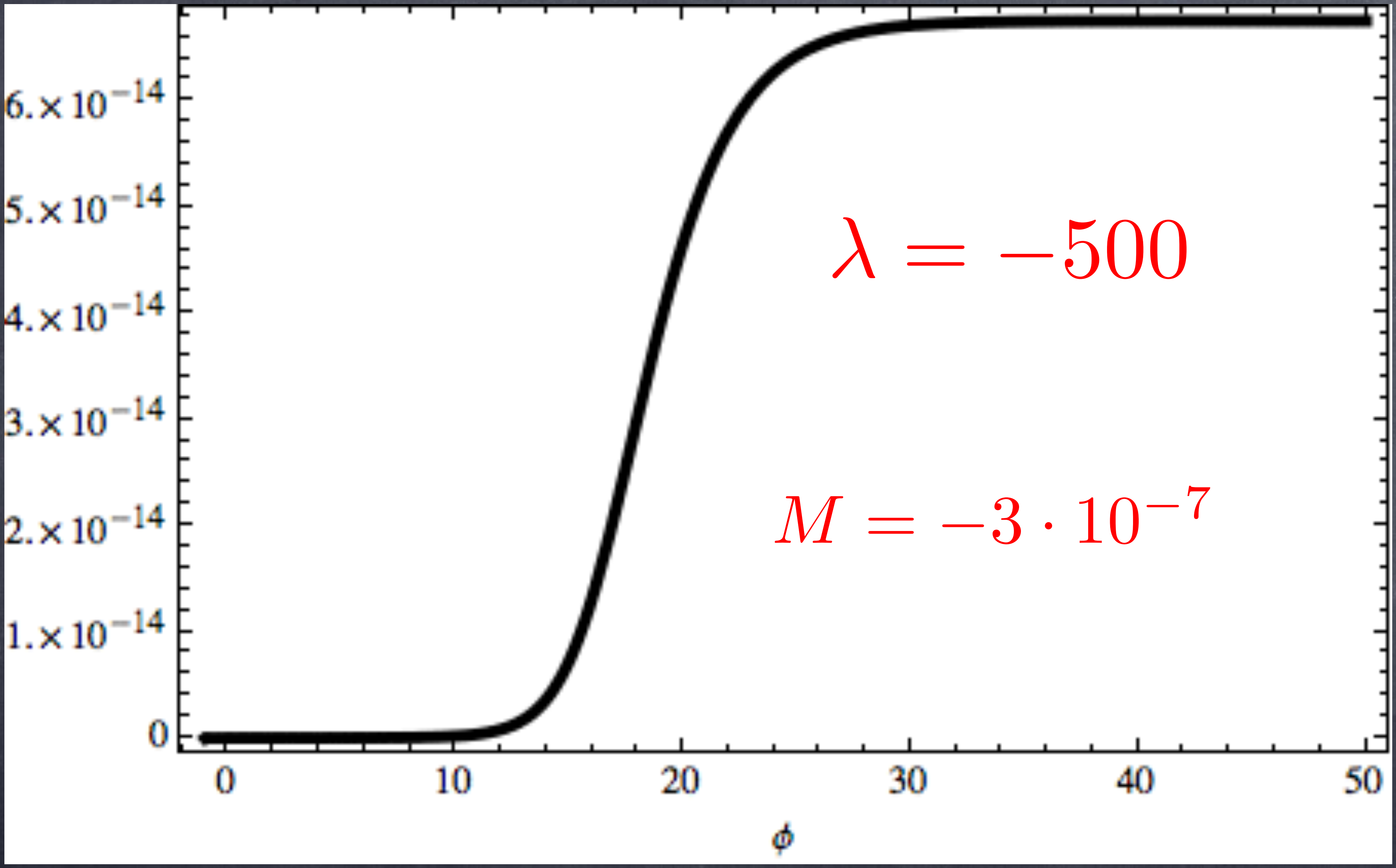
0
$-5.\times 10^{-12}$
$-1.\times 10^{-11}$
-1.5×10^{-11}
$-2.\times 10^{-11}$
-2.5×10^{-11}
$-3.\times 10^{-11}$
-2
0
2
4
6
ϕ
$M = 3 \cdot 10^{-7}$
$\lambda = 5$



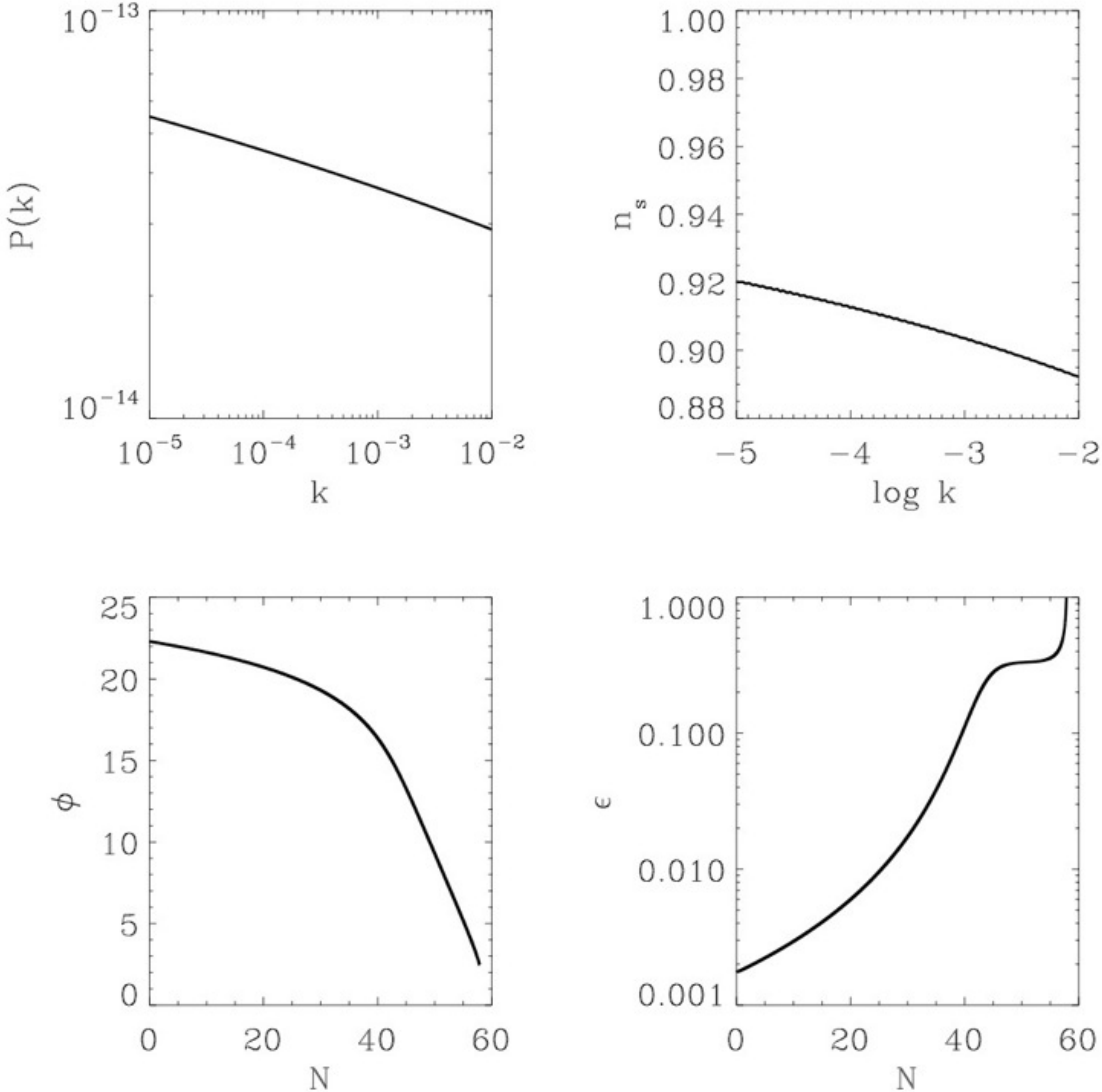
$8.\times 10^{-14}$
$6.\times 10^{-14}$
$4.\times 10^{-14}$
$2.\times 10^{-14}$
0
0
10
20
30
40
50
ϕ
$M = 3 \cdot 10^{-7}$
$\lambda = -500$



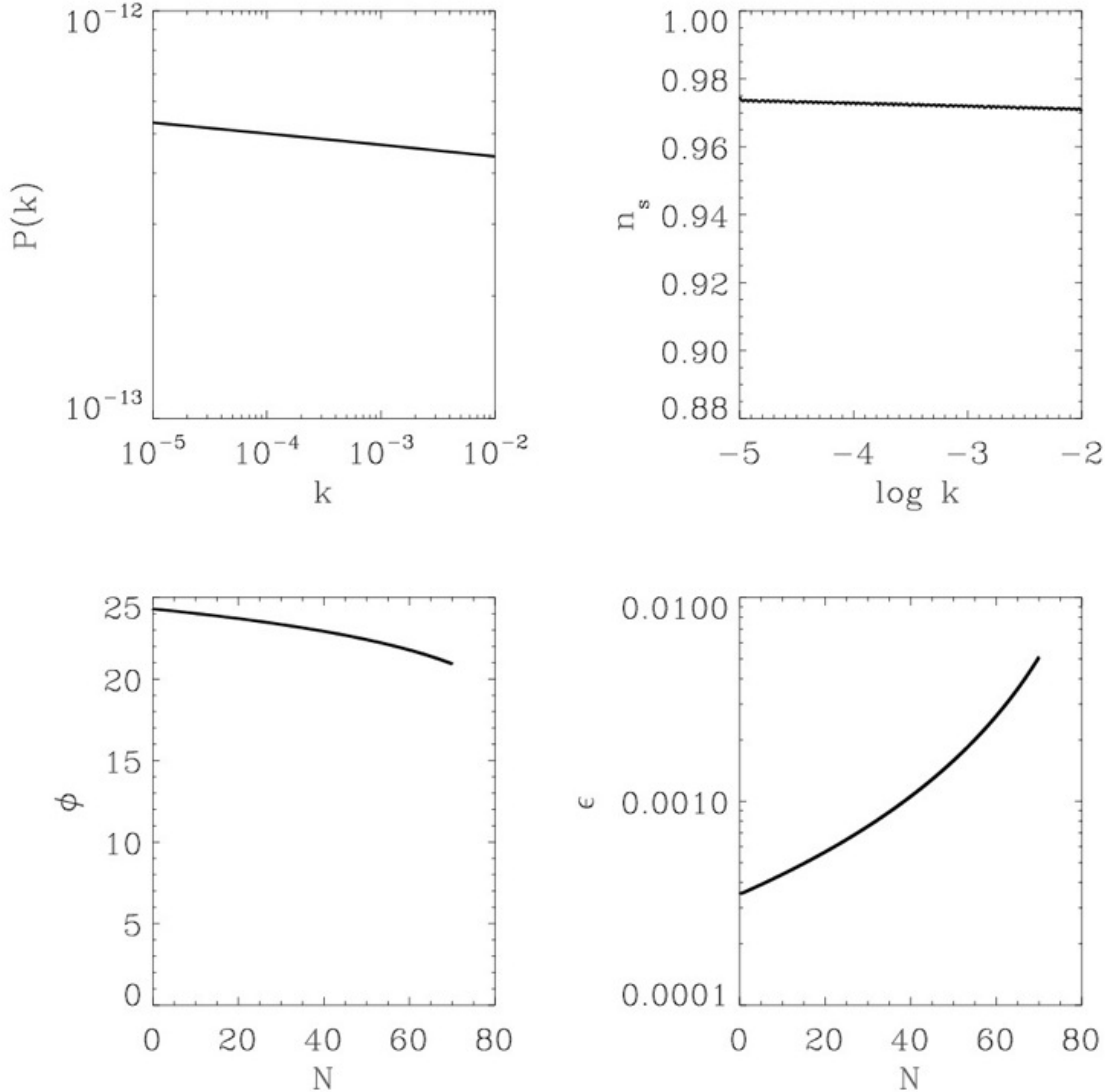
P(k)
10⁻⁹
10⁻¹⁰
10⁻¹¹
10⁻⁵ 10⁻⁴ 10⁻³ 10⁻² 10⁻¹
k
n_s
1.10
1.05
1.00
0.95
0.90
−5 −4 −3 −2 −1
log k
φ
20
15
10
5
0
0 20 40 60 80
N
ε
1.000
0.100
0.010
0.001
0 20 40 60 80
N



6.×10^−14
5.×10^−14
4.×10^−14
3.×10^−14
2.×10^−14
1.×10^−14
0
0
10
20
30
40
50
ϕ
λ = −500
M = −3 · 10^−7

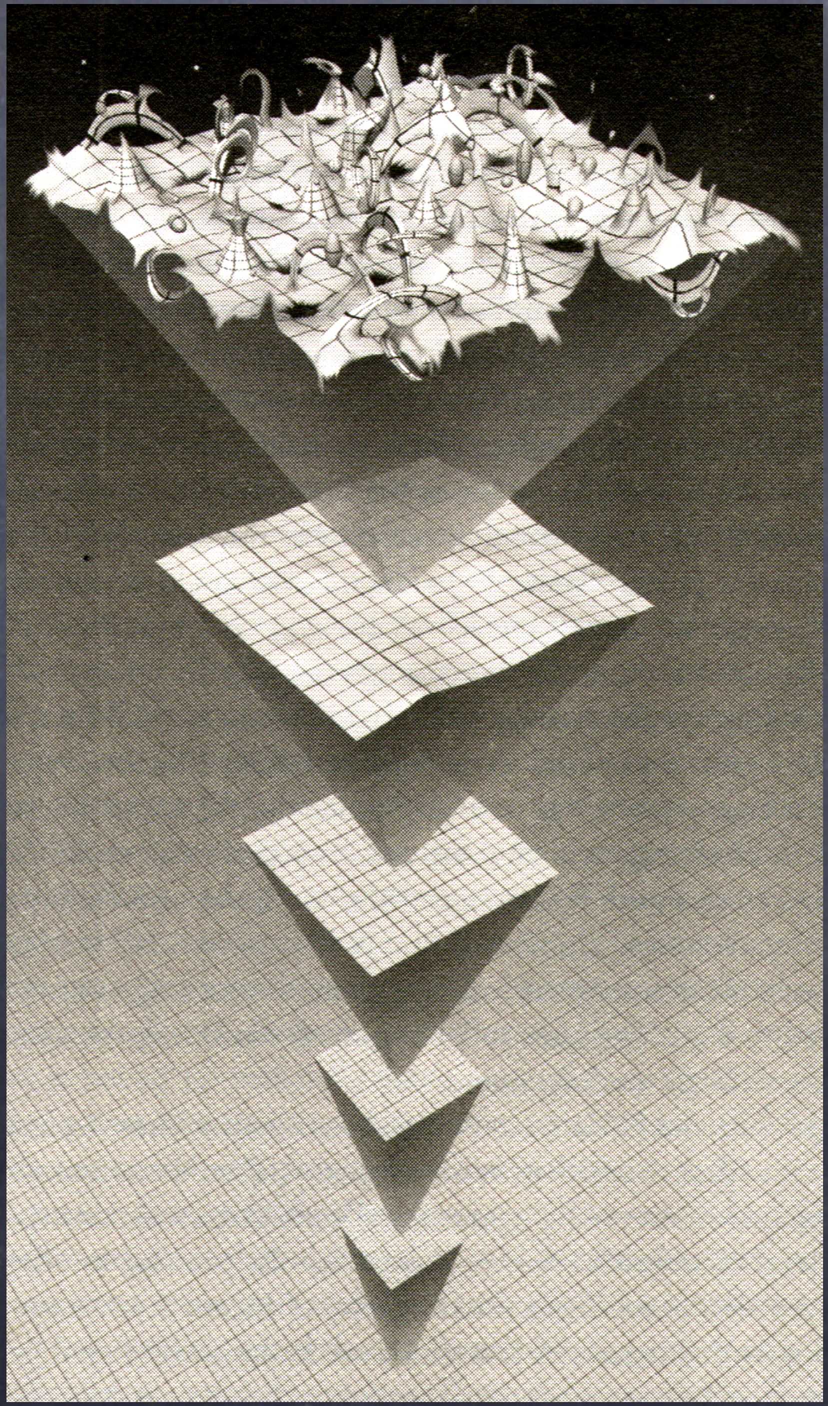


10^{-13}
10^{-14}
P(k)
10^{-5} 10^{-4} 10^{-3} 10^{-2}
k
1.00
0.98
0.96
0.94
0.92
0.90
0.88
n_s
−5 −4 −3 −2
log k
25
20
15
10
5
0
ϕ
0 20 40 60
N
1.000
0.100
0.010
0.001
ϵ
0 20 40 60
N



P(k)
10^{-12}
10^{-13}
10^{-5} 10^{-4} 10^{-3} 10^{-2}
k
n_s
1.00 0.98 0.96 0.94 0.92 0.90 0.88
−5 −4 −3 −2
log k
ϕ
25 20 15 10 5 0
0 20 40 60 80
N
ϵ
0.0100 0.0010 0.0001
0 20 40 60 80
N

Is the AS - vacuum "flat" ?



Asymptotic Safety predicts the presence of at least another relevant direction in the UV critical manifold

$$L = \frac{1}{16\pi G}(-R + 2\Lambda) + \beta R^2 \qquad \beta > 0$$

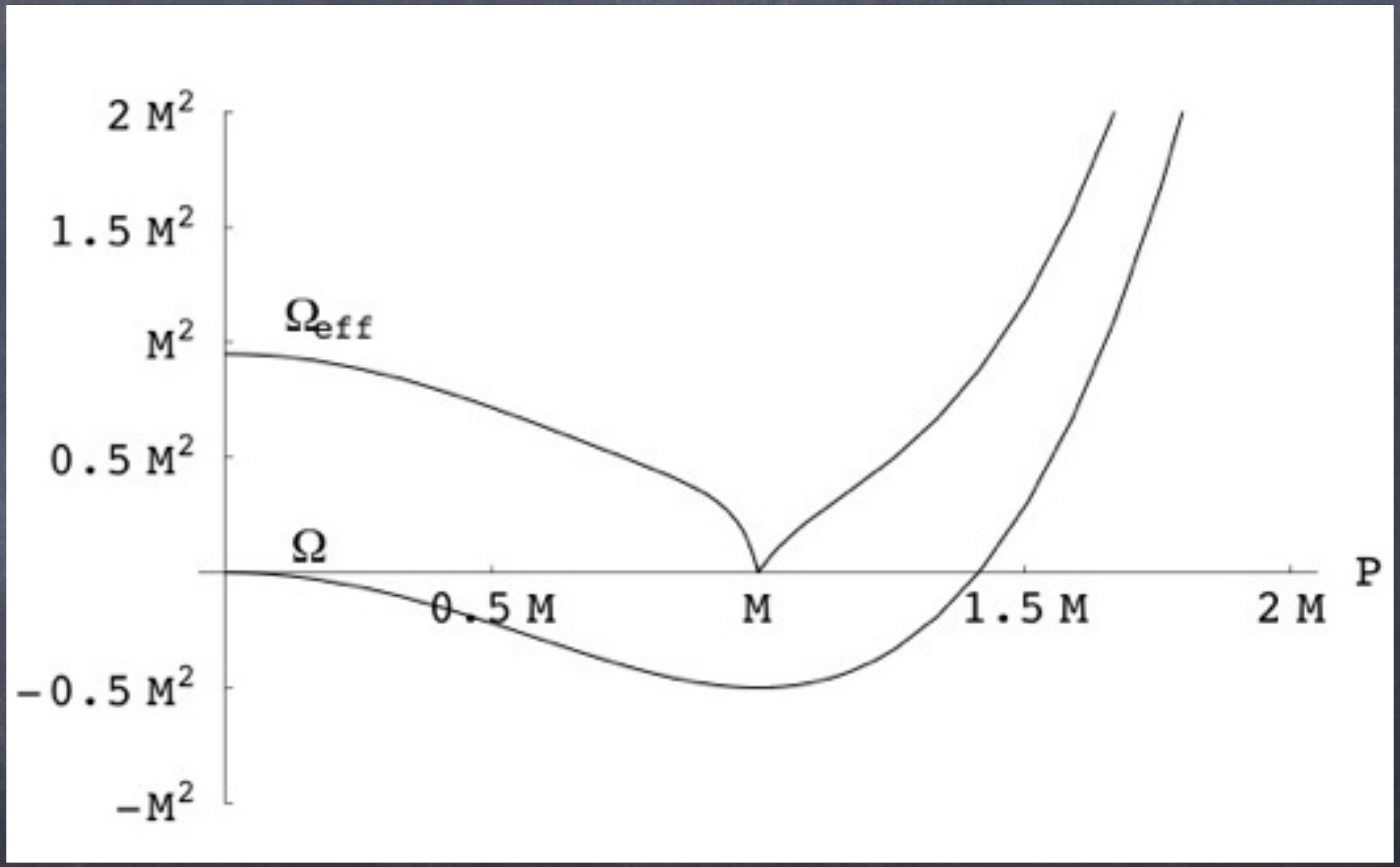
- Ref: Codello, Percacci, Rahmede 2007

Kinetic “condensate” model

$\langle 0|\partial_\mu\phi\partial^\mu\phi|0\rangle \neq 0$

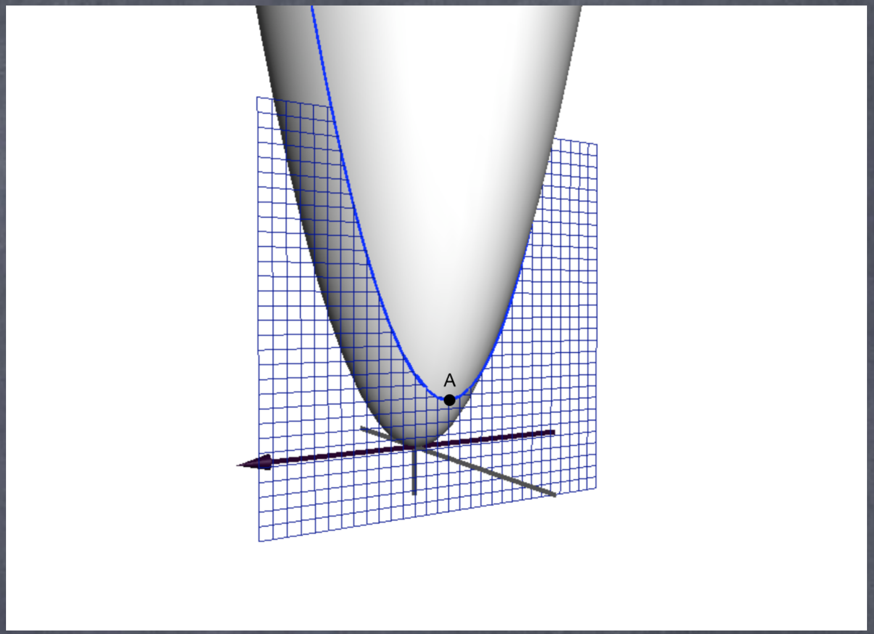
$$S[g] = S_{\rm EH} + \beta\int d^4x\sqrt{\beta}R^2 \;\Rightarrow\; \Omega(-\Box) = \Box + \Box\Box/2M^2$$

$$\Omega(p^2) = -p^2 + \frac{(p^2)^2}{2M^2}$$



Lauscher, Reuter and Wetterich, 2000

Conformally reduced R+R2 gravity



$$g_{\mu\nu} = \tfrac{1}{3}(4\pi G)\, e^{2\sigma(x)}\, \delta_{\mu\nu}$$

$$S[\sigma] = \int d^4x \left\{ \tfrac{1}{2} e^{2\sigma} \left(\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma \right) + 36\beta \left(\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma \right)^2 + \tfrac{u}{4!}\, e^{4\sigma} \right.$$

$$e^{2\sigma}\left[\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma\right] + \frac{u}{6}\, e^{4\sigma} + 72\beta\left[\Box\Box\sigma + 2(\partial_\mu\partial_\nu\sigma)(\partial^\mu\partial^\nu\sigma) - 2(\Box\sigma)(\Box\sigma)\right.$$

$$\left. -2(\partial_\mu\sigma)(\partial^\mu\sigma)\Box\sigma - 4(\partial_\mu\sigma)(\partial_\nu\sigma)(\partial^\mu\partial^\nu\sigma)\right] = 0$$

Lattice-regulated model in 2-dim

$$S[\sigma(x)] = \sum_x \Big\{ \frac{u}{4!}\, e^{4\sigma(x)}$$

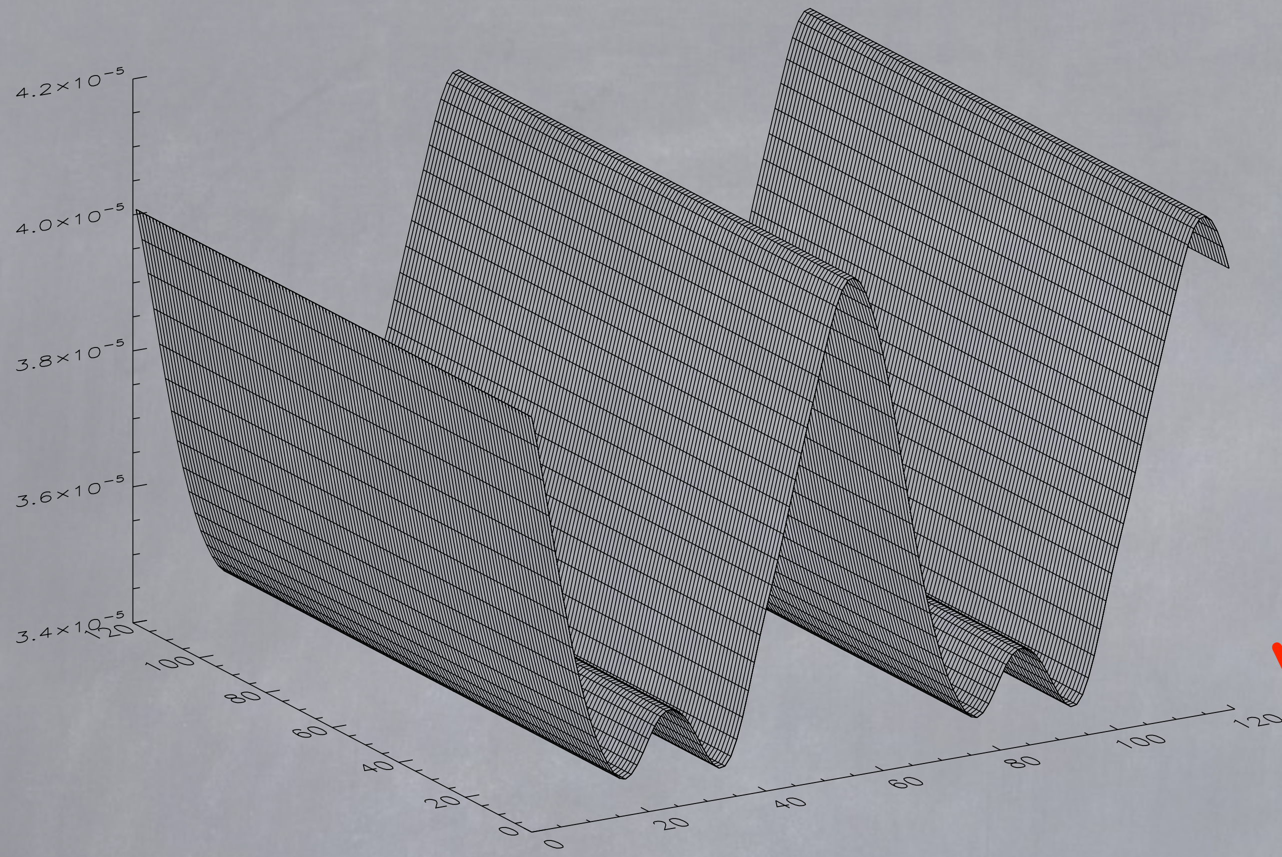
$$+\sum_\mu \frac{1}{2}\Big[e^{2\sigma(x)}(\sigma(x+e_\mu)+\sigma(x-e_\mu)-2\sigma(x)+(\sigma(x+e_\mu)-\sigma(x))^2)$$

$$+\sum_\nu 36\beta(\sigma(x+e_\mu)+\sigma(x-e_\mu)-2\sigma(x))(\sigma(x+e_\nu)+\sigma(x-e_\nu)-2\sigma(x))$$

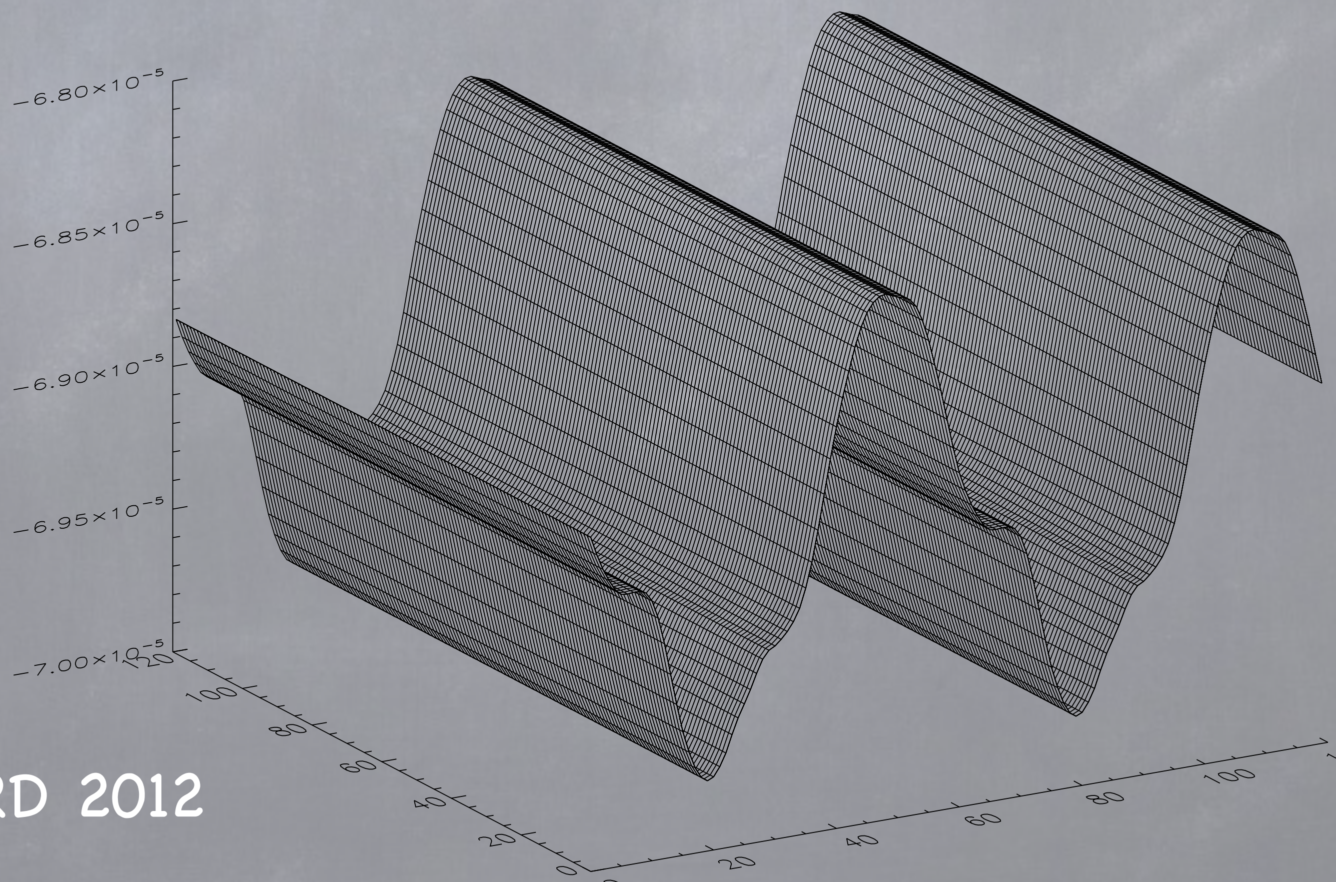
$$+(\sigma(x+e_\mu)+\sigma(x-e_\mu)-2\sigma(x))(\sigma(x+e_\nu)-\sigma(x))$$

$$+(\sigma(x+e_\nu)+\sigma(x-e_\nu)-2\sigma(x))(\sigma(x+e_\mu)-\sigma(x))$$

$$+(\sigma(x+e_\mu)-\sigma(x))(\sigma(x+e_\nu)-\sigma(x))^2\Big]\Big\}$$



"Lasagne"-type of vacuum!



2-D simulations

Bonanno & Reuter, PRD 2012

Conclusions

- Starobinsky inflation is a prediction of AS
- No rehating is needed but a transition to radiation dominated universe is obtained
- Spectral index is generally < 1 with some "running"
- tensor to scalar ratio < 0.1
- Vacuum structure at transplanckian scale might still change things even in standard inflation !