Universality Classes for Quantum Gravity

Frank Saueressig

Research Institute for Mathematics, Astrophysics and Particle Physics

Radboud University Nijmegen

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

M. Demmel, F.S. and O. Zanusso, arXiv:1401.5495

First FLAG Meeting “The Quantum and Gravity”

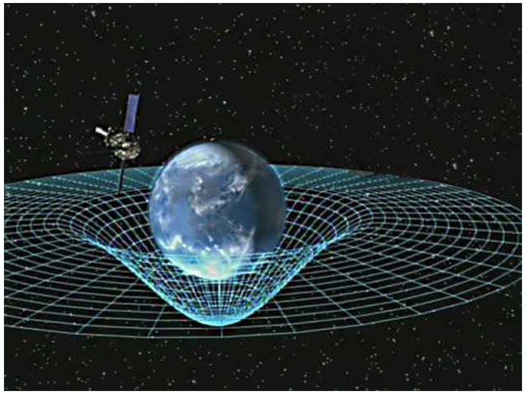
Bologna, May 28, 2014

Outline

- Motivations for Quantum Gravity
- Quantum Gravity from a Wilsonian perspective
- Asymptotic Safety program
  - fixed functionals of $f(R)$-gravity
- projectable Hořava-Lifshitz gravity
  - restoring Lorentz-symmetry in the IR
  - matter induced UV fixed point
- Conclusions

Motivations for Quantum Gravity

Classical General Relativity



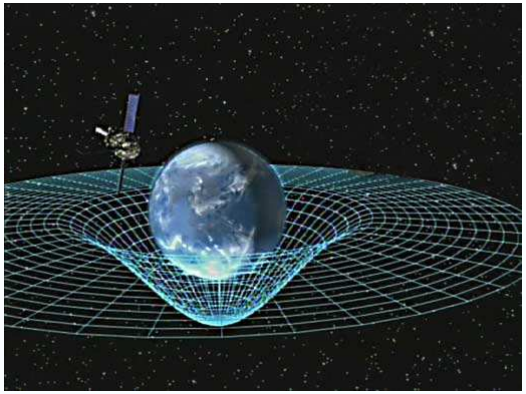
Based on Einsteins equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{space-time curvature}} = \underbrace{-\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}}_{\text{matter content}}$$

- Newton's constant: $G_N = 6.67 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$
- cosmological constant: $\Lambda \approx 10^{-35}\ \mathrm{s}^{-2}$

Classical General Relativity

Based on Einsteins equations

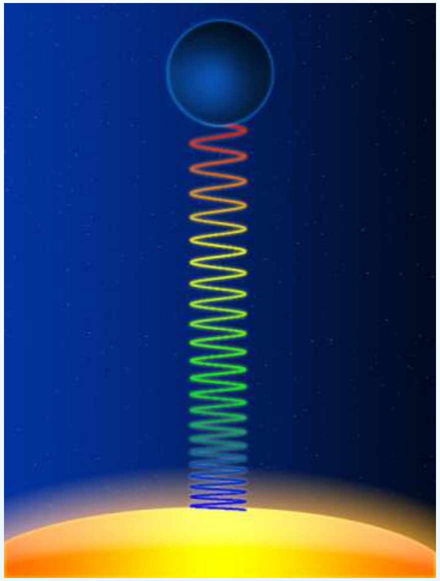


$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{space-time curvature}} = \underbrace{-\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}}_{\text{matter content}}$$

- Newton's constant: $G_N = 6.67 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$
- cosmological constant: $\Lambda \approx 10^{-35}\ \mathrm{s}^{-2}$

Passed highly non-trivial experimental tests:

- perihelion precession of Mercury
- deflection of light by sun
- gravitational redshift
- light travel time delay
- equivalence principle
- binary pulsars (strong gravitational fields)
- . . .



Motivations for Quantum Gravity

1. internal consistency

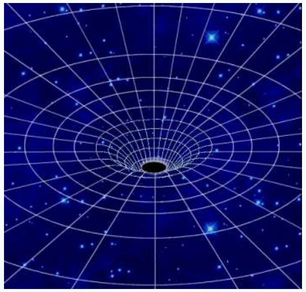
$$\underbrace{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + \Lambda\, g_{\mu\nu}}_{\text{classical}} \quad = \quad 8\,\pi\, G_N \quad \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

Motivations for Quantum Gravity

1. internal consistency

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + \Lambda\, g_{\mu\nu}}_{\text{classical}} \quad = \quad 8\,\pi\, G_N \quad \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

2. singularities in solutions of Einstein equations
   - black hole singularities
   - Big Bang singularity

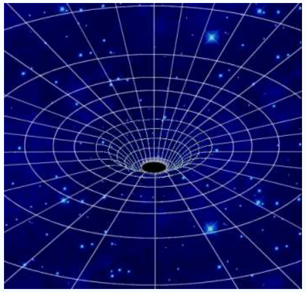


Motivations for Quantum Gravity

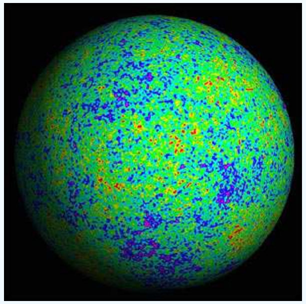
1. internal consistency

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + \Lambda\, g_{\mu\nu}}_{\text{classical}} \quad = \quad 8\,\pi\, G_N \quad \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

2. singularities in solutions of Einstein equations
    - black hole singularities
    - Big Bang singularity



3. cosmological observations:
    - initial conditions for structure formation
    - small positive cosmological constant

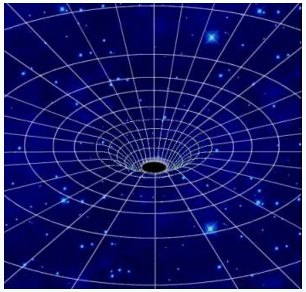


Motivations for Quantum Gravity

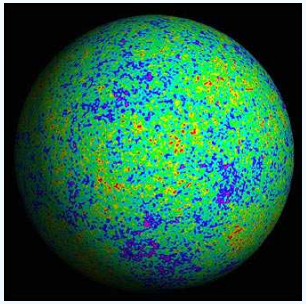
1. internal consistency

$$\underbrace{R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R + \Lambda\, g_{\mu\nu}}_{\text{classical}} \quad = \quad 8\,\pi\, G_N \quad \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

2. singularities in solutions of Einstein equations
   - black hole singularities
   - Big Bang singularity



3. cosmological observations:
   - initial conditions for structure formation
   - small positive cosmological constant



General Relativity is incomplete

Quantum Gravity may give better answers to these puzzles

Perturbative quantization of General Relativity

Dynamics of General Relativity governed by Einstein-Hilbert action

$$S^{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^dx \sqrt{g} \left[-R + 2\Lambda \right]$$

- Newton's constant G_N has negative mass-dimension

Perturbative quantization of General Relativity

Dynamics of General Relativity governed by Einstein-Hilbert action

$$S^{\mathrm{EH}} = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} \left[- R + 2\Lambda \right]$$

- Newton's constant G_N has negative mass-dimension

Wilsonian picture of perturbative renormalization:

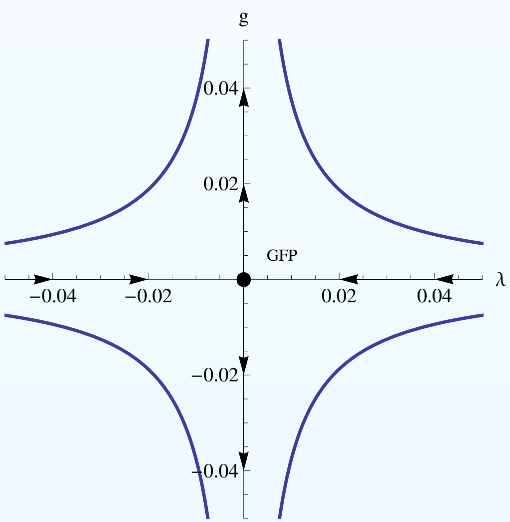
$\Rightarrow$ dimensionless coupling constant attracted to GFP (free theory) in UV

- introduce dimensionless coupling constants

$$g_k = k^{d-2} G_N \,, \quad \lambda_k \equiv \Lambda k^{-2}$$

- GFP: flow governed by mass-dimension:

$$k\partial_k g_k = (d-2)\, g + \mathcal{O}(g^2)$$
$$k\partial_k \lambda_k = -\, 2\lambda + \mathcal{O}(g)$$



Perturbative quantization of General Relativity

Dynamics of General Relativity governed by Einstein-Hilbert action

$$S^{\rm EH} = \frac{1}{16\pi G_N} \int d^dx \sqrt{g} \left[- R + 2\Lambda \right]$$

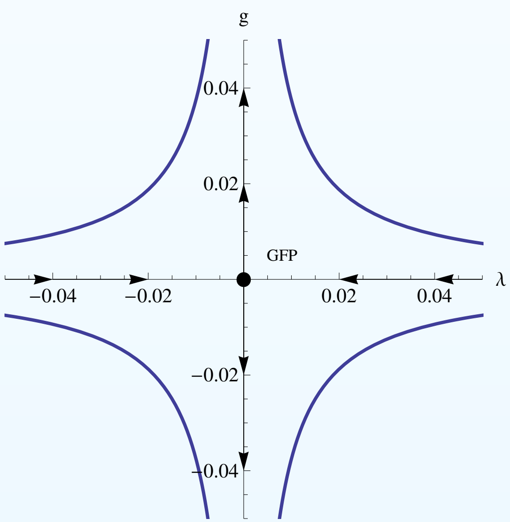
- Newton's constant G_N has negative mass-dimension

Wilsonian picture of perturbative renormalization:

$\Rightarrow$ dimensionless coupling constant attracted to GFP (free theory) in UV

- introduce dimensionless coupling constants

$$g_k = k^{d-2} G_N \, , \quad \lambda_k \equiv \Lambda k^{-2}$$



- GFP: flow governed by mass-dimension:

$$k\partial_k g_k = (d-2)\, g + \mathcal{O}(g^2)$$
$$k\partial_k \lambda_k = -2\lambda + \mathcal{O}(g)$$

General Relativity is perturbatively non-renormalizable

General Relativity is non-renormalizable: conclusions

a) Treat gravity as effective field theory: [J. Donoghue, gr-qc/9405057]

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{\rm Pl}^2$

General Relativity is non-renormalizable: conclusions

a) Treat gravity as effective field theory: [J. Donoghue, gr-qc/9405057]

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{\rm Pl}^2$

b) UV-completion requires new physics:

- string theory:
  - requires: supersymmetry, extra dimensions
- loop quantum gravity:
  - keeps Einstein-Hilbert action as “fundamental”
  - new quantization scheme
- causal sets

General Relativity is non-renormalizable: conclusions

a) Treat gravity as effective field theory: [J. Donoghue, gr-qc/9405057]

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{\rm Pl}^2$

b) UV-completion requires new physics:

- string theory:
  - requires: supersymmetry, extra dimensions
- loop quantum gravity:
  - keeps Einstein-Hilbert action as “fundamental”
  - new quantization scheme
- causal sets

c) Gravity makes sense as Quantum Field Theory:

- UV-completion beyond perturbation theory: Asymptotic Safety
- UV-completion by relaxing symmetries: Hořava-Lifshitz

General Relativity is non-renormalizable: conclusions

a) Treat gravity as effective field theory: [J. Donoghue, gr-qc/9405057]

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{\rm Pl}^2$

b) UV-completion requires new physics:

- string theory:
  - requires: supersymmetry, extra dimensions
- loop quantum gravity:
  - keeps Einstein-Hilbert action as “fundamental”
  - new quantization scheme
- causal sets

c) Gravity makes sense as Quantum Field Theory:

- UV-completion beyond perturbation theory: Asymptotic Safety
- UV-completion by relaxing symmetries: Hořava-Lifshitz

Quantum Gravity

from a Wilsonian perspective

Renormalization from the Wilsonian perspective

- theory $\Leftarrow$ specify
  - a) field content (e.g. graviton)
  - b) symmetries (e.g. coordinate transformations)

Renormalization from the Wilsonian perspective

- theory $\Leftarrow$ specify
  a) field content (e.g. graviton)
  b) symmetries (e.g. coordinate transformations)

- action = specific combination of interaction monomials $\mathcal{O}$
  - build from field content
  - compatible with symmetries (e.g. $\mathcal{O} = \int d^4x \sqrt{g} R$)

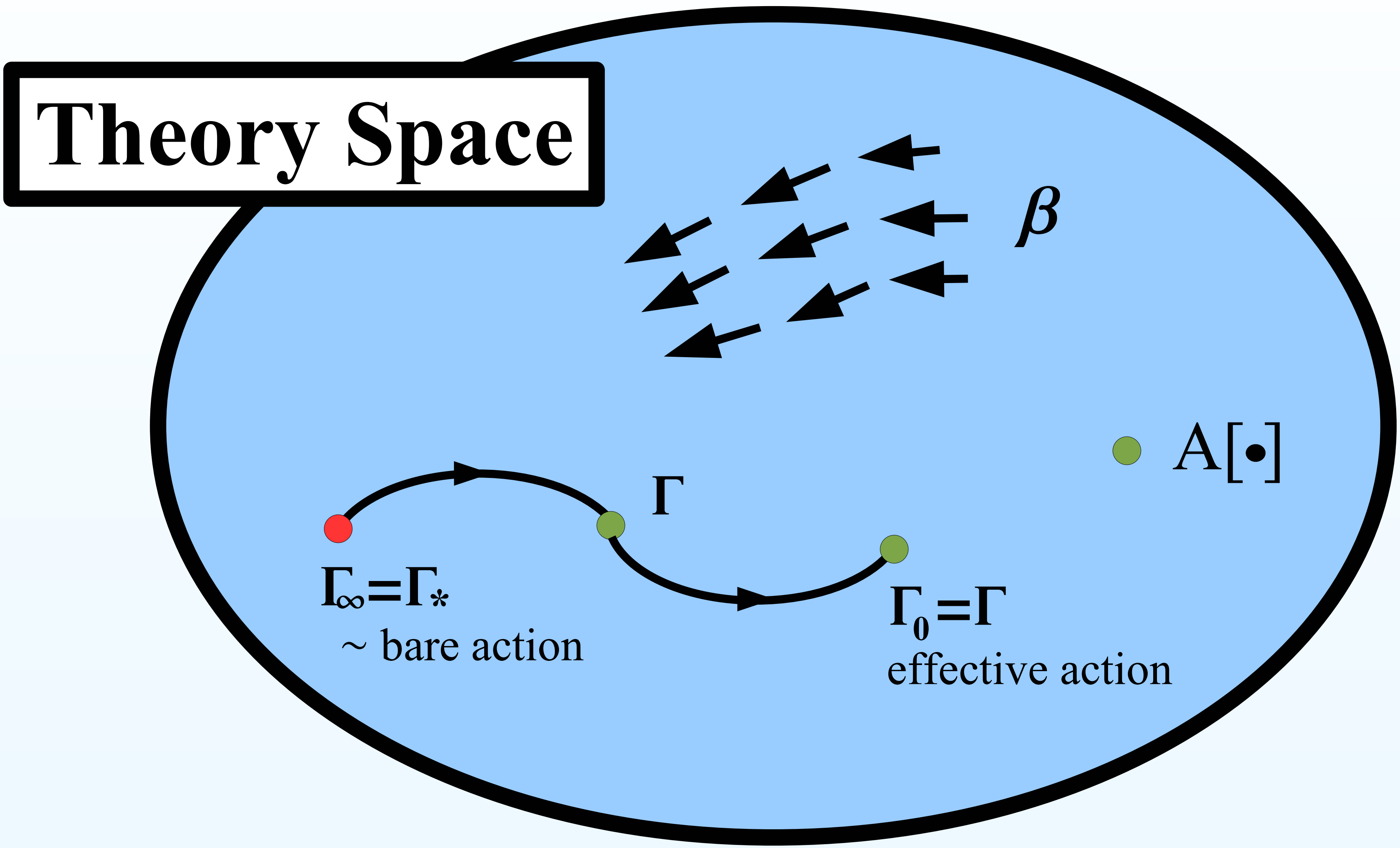
Renormalization from the Wilsonian perspective

- theory $\Leftarrow$ specify
  - a) field content (e.g. graviton)
  - b) symmetries (e.g. coordinate transformations)
- action = specific combination of interaction monomials $\mathcal{O}$
  - build from field content
  - compatible with symmetries (e.g. $\mathcal{O} = \int d^4x \sqrt{g} R$)
- theory space:
  - space containing all actions
  - coordinates: coupling constants $\{g_i\}$ (e.g. $G_N, \Lambda, \ldots$)

Renormalization from the Wilsonian perspective

- theory $\Leftarrow$ specify
  - a) field content (e.g. graviton)
  - b) symmetries (e.g. coordinate transformations)
- action = specific combination of interaction monomials $\mathcal{O}$
  - build from field content
  - compatible with symmetries (e.g. $\mathcal{O} = \int d^4x \sqrt{g} R$)
- theory space:
  - space containing all actions
  - coordinates: coupling constants $\{g_i\}$ (e.g. $G_N, \Lambda, \ldots$)
- renormalization group flow:
  - connects physics at different scales k
  - for coupling constants: (e.g. $k\partial_k g_i = \beta_i(g_i)$)

Theory space underlying the Functional Renormalization Group



Fixed points of the RG flow

Central ingredient in Wilsons picture of renormalization

Definition:

- fixed point $\{g_i^*\} \Longleftrightarrow \beta$-functions vanish $(\beta_{g_i}(\{g_i\})|_{g_i=g_i^*} \stackrel{!}{=} 0)$

Fixed points of the RG flow

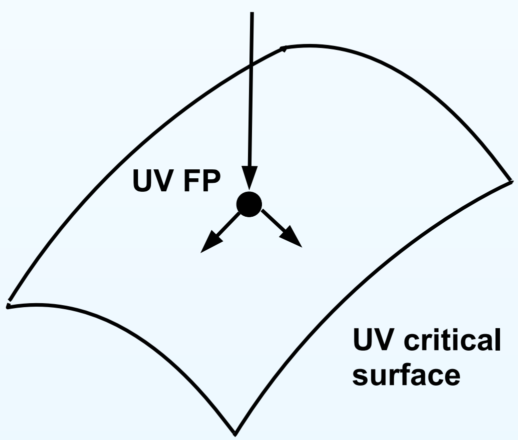
Central ingredient in Wilsons picture of renormalization

Definition:

- fixed point $\{g_i^*\} \Longleftrightarrow \beta$-functions vanish $\left(\beta_{g_i}(\{g_i\})\big|_{g_i=g_i^*} \stackrel{!}{=} 0 \right)$

Properties:

- well-defined continuum limit
  - trajectory captured by FP in UV has no unphysical UV divergences
- 2 classes of RG trajectories:
  - relevant $=$ attracted to FP in UV
  - irrelevant $=$ repelled from FP in UV
- predictivity:
  - number of relevant directions
    $=$ free parameters (determine experimentally)



Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

- non-Gaussian Fixed Point (NGFP)
  - fundamental theory: interacting
  - non-perturbatively renormalizable field theories

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

- non-Gaussian Fixed Point (NGFP)
  - fundamental theory: interacting
  - non-perturbatively renormalizable field theories

Gravity

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

- non-Gaussian Fixed Point (NGFP)
  - fundamental theory: interacting
  - non-perturbatively renormalizable field theories

Gravity

- anisotropic Gaussian Fixed Point (aGFP)
  - fundamental theory: Hořava-Lifshitz gravity
  - Lorentz-violating renormalizable field theory

Proposals for UV fixed points

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

- non-Gaussian Fixed Point (NGFP)
  - fundamental theory: interacting
  - non-perturbatively renormalizable field theories

Gravity

- anisotropic Gaussian Fixed Point (aGFP)
  - fundamental theory: Hořava-Lifshitz gravity
  - Lorentz-violating renormalizable field theory

Gravity

Asymptotic Safety Program

Asymptotic Safety as viable theory for Quantum Gravity

Requirements:

a) non-Gaussian fixed point (NGFP)

- controls the UV-behavior of the RG-trajectory
- ensures the absence of UV-divergences

Asymptotic Safety as viable theory for Quantum Gravity

Requirements:

a) non-Gaussian fixed point (NGFP)

- controls the UV-behavior of the RG-trajectory
- ensures the absence of UV-divergences

b) finite-dimensional UV-critical surface $\mathcal{S}_{\rm UV}$

- fixing the position of a RG-trajectory in $\mathcal{S}_{\rm UV}$
  $\Longleftrightarrow$ experimental determination of relevant parameters
- ensures predictivity

Asymptotic Safety as viable theory for Quantum Gravity

Requirements:

a) non-Gaussian fixed point (NGFP)

- controls the UV-behavior of the RG-trajectory
- ensures the absence of UV-divergences

b) finite-dimensional UV-critical surface $\mathcal{S}_{\mathrm{UV}}$

- fixing the position of a RG-trajectory in $\mathcal{S}_{\mathrm{UV}}$
  $\Longleftrightarrow$ experimental determination of relevant parameters
- ensures predictivity

c) classical limit:

- RG-trajectories have part where GR is good approximation
- recover gravitational physics captured by General Relativity:
  (perihelion shift, gravitational lensing, nucleo-synthesis, ...)

Asymptotic Safety as viable theory for Quantum Gravity

Requirements:

a) non-Gaussian fixed point (NGFP)

- controls the UV-behavior of the RG-trajectory
- ensures the absence of UV-divergences

b) finite-dimensional UV-critical surface $\mathcal{S}_{\mathrm{UV}}$

- fixing the position of a RG-trajectory in $\mathcal{S}_{\mathrm{UV}}$
  $\Longleftrightarrow$ experimental determination of relevant parameters
- ensures predictivity

c) classical limit:

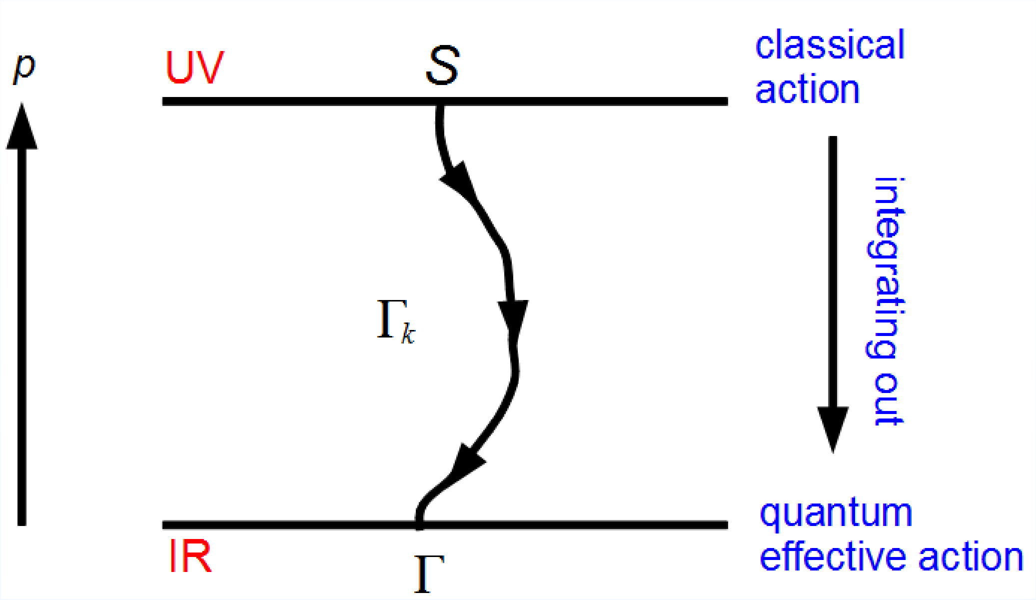
- RG-trajectories have part where GR is good approximation
- recover gravitational physics captured by General Relativity:
  (perihelion shift, gravitational lensing, nucleo-synthesis, ...)

Quantum Einstein Gravity (QEG)

Effective average action Γ_k for gravity

C. Wetterich, Phys. Lett. **B301** (1993) 90
M. Reuter, Phys. Rev. D **57** (1998) 971

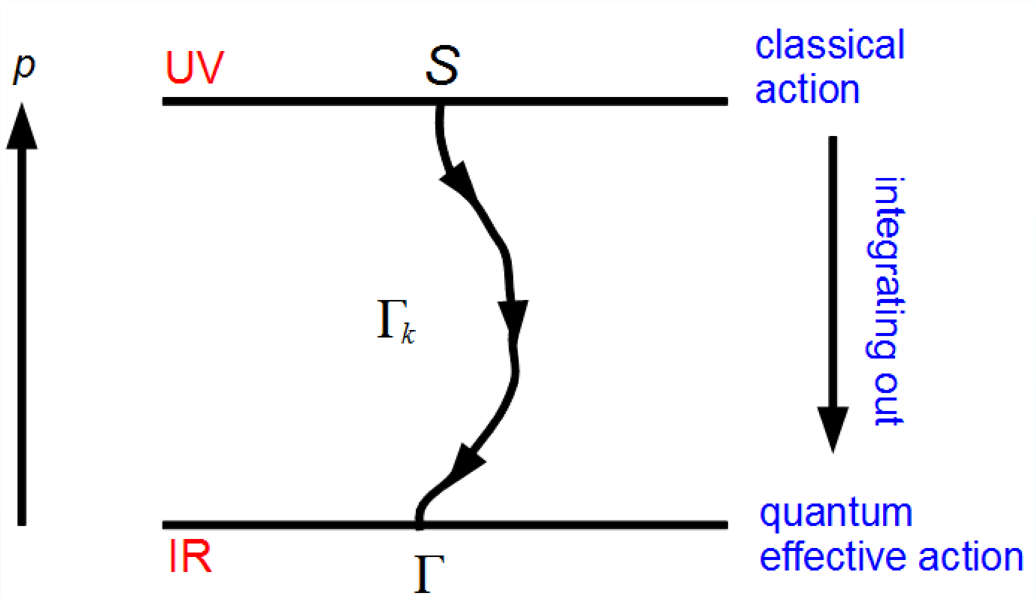
central idea: integrate out quantum fluctuations shell-by-shell in momentum-space



Effective average action Γ_k for gravity

C. Wetterich, Phys. Lett. **B301** (1993) 90
M. Reuter, Phys. Rev. D **57** (1998) 971

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space



- scale-dependence governed by functional renormalization group equation

$$k\partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

  - effective vertices in incorporate quantum-corrections with $p^2 > k^2$

The Einstein-Hilbert truncation

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g}\,[-R + 2\Lambda(k)] + S^{\text{gf}} + S^{\text{gh}}$$

microscopic theory $\Longleftrightarrow$ fixed points of the β-functions

$$\beta_g(g^*, \lambda^*) = 0\ , \qquad \beta_\lambda(g^*, \lambda^*) = 0$$

- Gaussian Fixed Point:
  - at $g^* = 0, \lambda^* = 0 \Longleftrightarrow$ free theory
  - saddle point in the g-λ-plane
- non-Gaussian Fixed Point ($\eta_N^* = -2$):
  - at $g^* > 0, \lambda^* > 0 \Longleftrightarrow$ “interacting” theory
  - UV attractive in g_k, λ_k

The Einstein-Hilbert truncation

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g} \, [-R + 2\Lambda(k)] + S^{\text{gf}} + S^{\text{gh}}$$

microscopic theory $\Longleftrightarrow$ fixed points of the β-functions

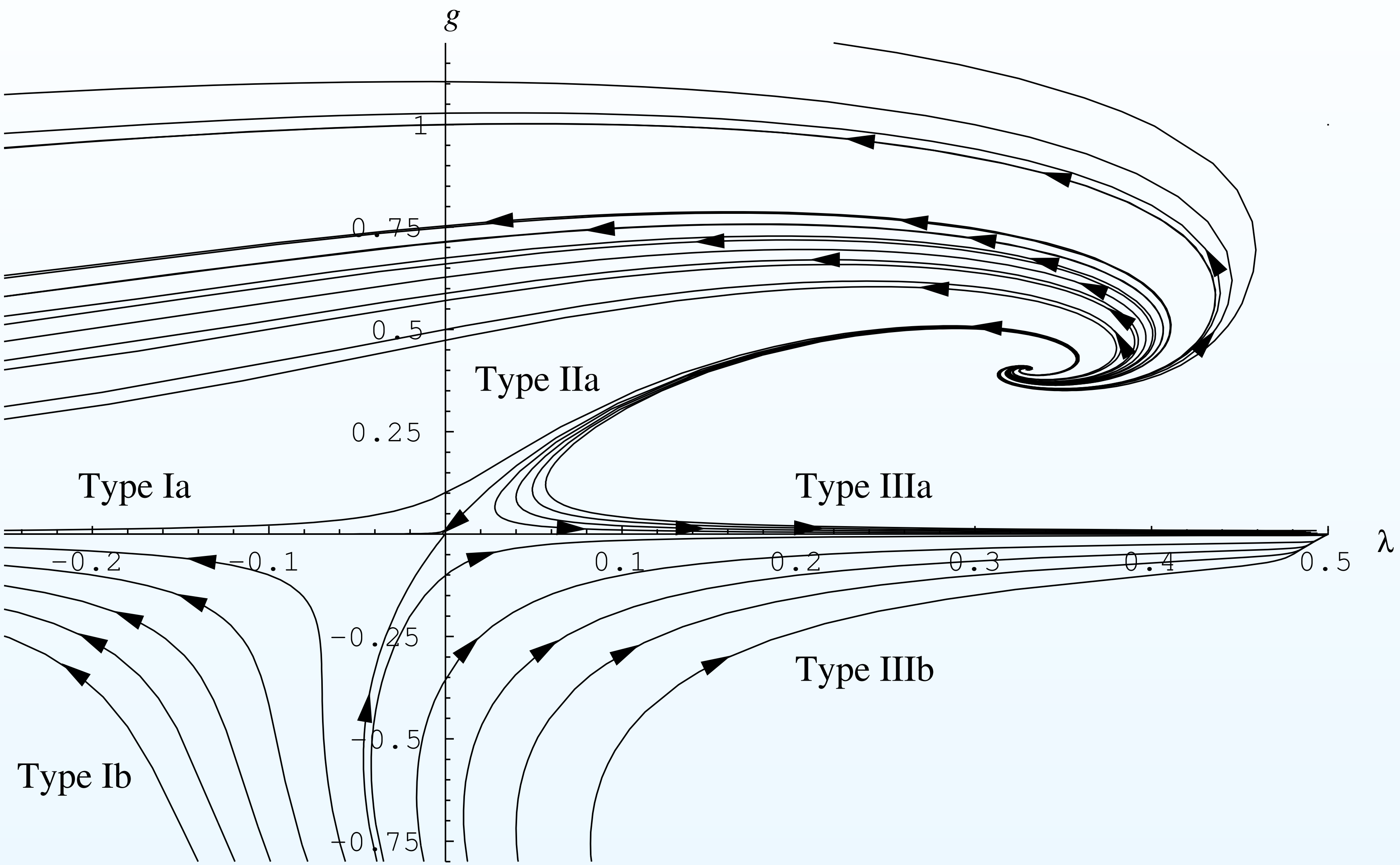
$$\beta_g(g^*, \lambda^*) = 0 \, , \qquad \beta_\lambda(g^*, \lambda^*) = 0$$

- Gaussian Fixed Point:
  - at $g^* = 0, \lambda^* = 0 \Longleftrightarrow$ free theory
  - saddle point in the g-λ-plane
- non-Gaussian Fixed Point ($\eta_N^* = -2$):
  - at $g^* > 0, \lambda^* > 0 \Longleftrightarrow$ “interacting” theory
  - UV attractive in g_k, λ_k

Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity

Einstein-Hilbert-truncation: the phase diagram

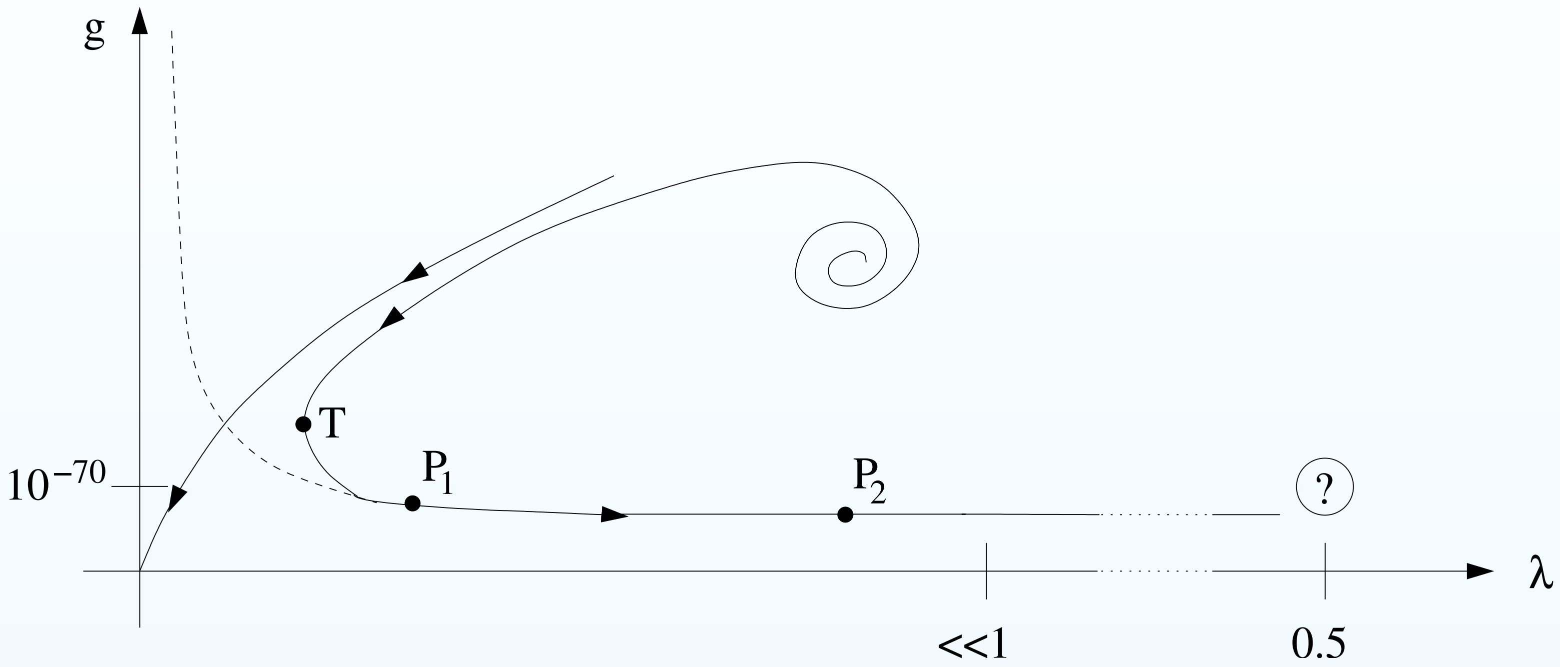
M. Reuter, F. S., Phys. Rev. D **65** (2002) 065016, hep-th/0110054



The RG trajectory realized in Nature

M. Reuter, H. Weyer, JCAP 0412 (2004) 001, hep-th/0410119

measurement of G_N, Λ in classical regime:



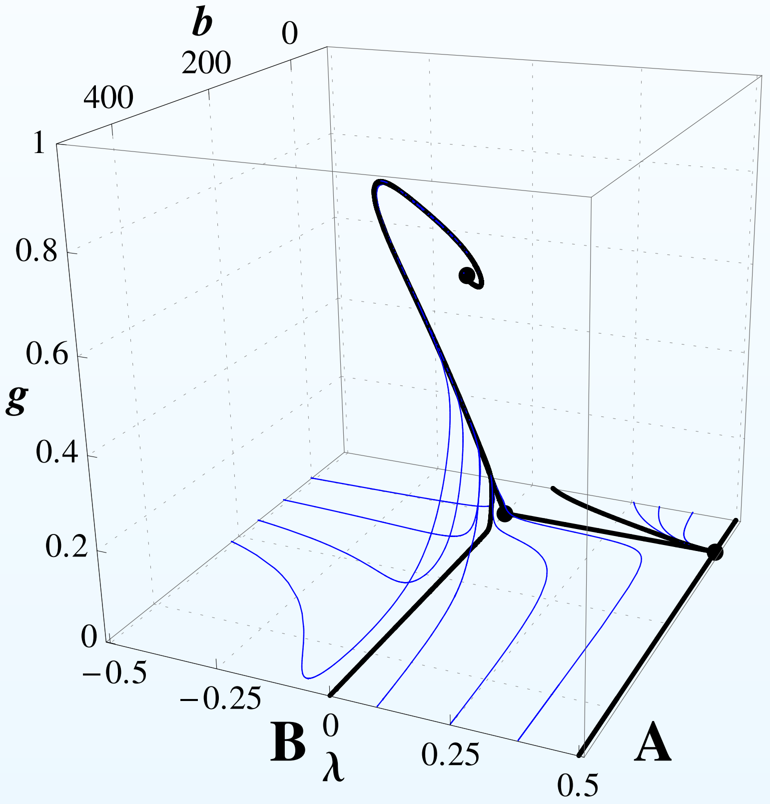
- originates at NGFP (quantum regime: $G(k) = k^{2-d} g_*, \Lambda(k) = k^2 \lambda_*$)
- passing *extremely* close to the GFP
- long classical GR regime (classical regime: $G(k) = \text{const}, \Lambda(k) = \text{const}$)
- $\lambda \lesssim 1/2$: IR fixed point?

Charting the RG-flow of the R^2-truncation

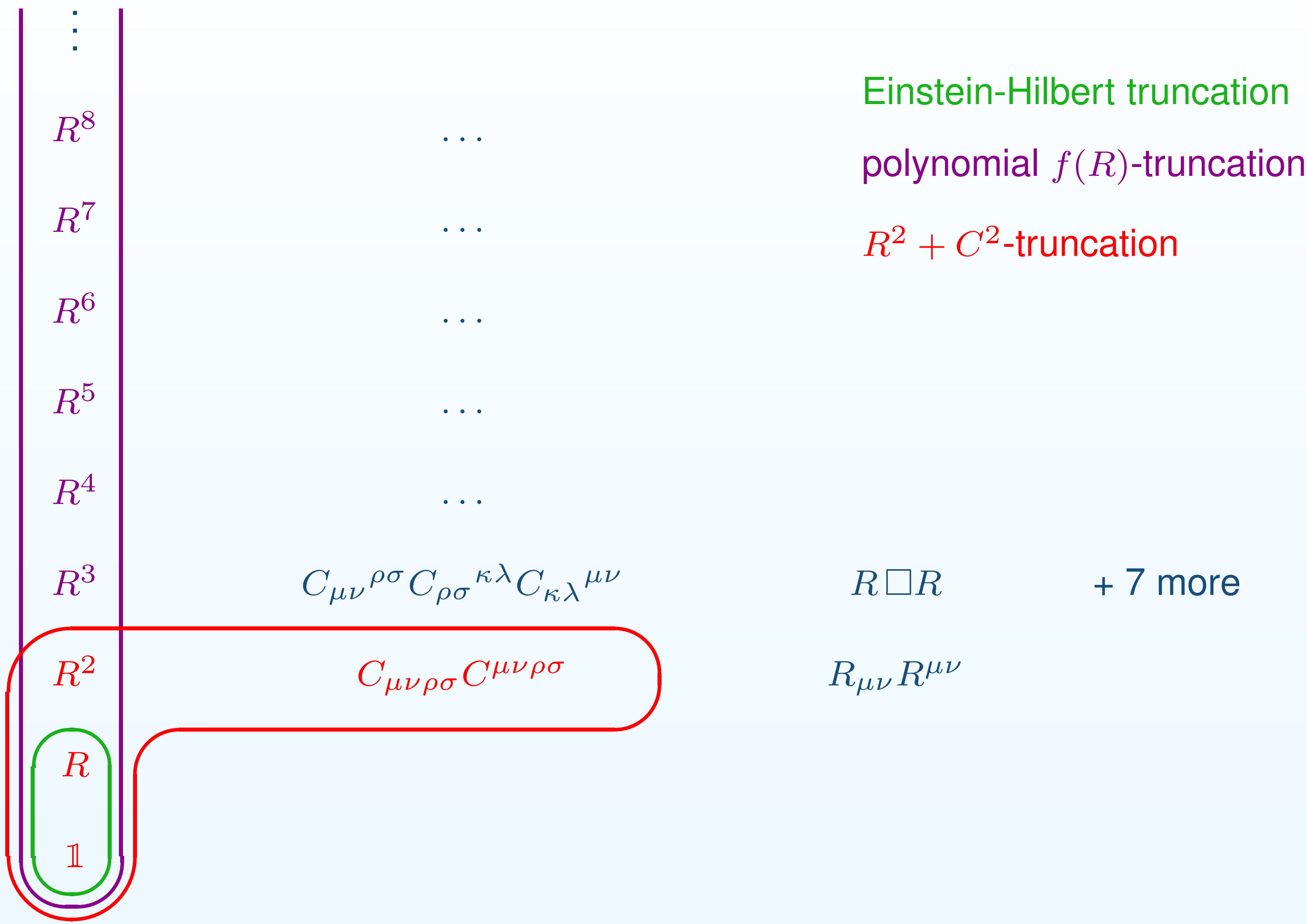
O. Lauscher, M. Reuter, Phys. Rev. D66 (2002) 025026, hep-th/0205062
S. Rechenberger, F.S., Phys. Rev. D86 (2012) 024018, arXiv:1206.0657

Extending Einstein-Hilbert truncation with higher-derivative couplings

$$\Gamma_k^{\rm grav}[g] = \int d^4x \sqrt{g} \left[\frac{1}{16\pi G_k} \left(-R + 2\Lambda_k\right) + \frac{1}{b_k} R^2 \right]$$



Charting the theory space spanned by $\Gamma_k^{\mathrm{grav}}[g]$



Charting the theory space spanned by $\Gamma_k^{\text{grav}}[g]$

Einstein-Hilbert truncation

polynomial $f(R)$-truncation

$\vdots$			
R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\kappa\lambda} R_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	+ 7 more
R^2	$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

finite-dimensional truncations

polynomial expansions of $f(R)$-gravity

[A. Codello, R. Percacci, C. Rahmede, '07]
[P. Machado, F. Saueressig, '07]
[A. Codello, R. Percacci, C. Rahmede, '09]
[A. Bonanno, A. Contillo, R. Percacci, '11]
[K. Falls, D. F. Litim, K. Nikolakopoulos, C. Rahmede, '13]

Polynomial expansion of $f(R)$-gravity

[A. Codello, R. Percacci, C. Rahmede, '07]

[P. Machado, F. Saueressig, '07]

Flow equation for $f(R)$-gravity:

$$\Gamma_k^{\text{grav}}[g] = \int d^4x \sqrt{g}\, f_k(R)$$

- complicated partial differential equation governing k-dependence of $f_k(R)$

UV properties of RG flow:

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} \bar{u}_n R^n + \dots$
- expand flow equation $\Longrightarrow$ β-functions for $g_n = \bar{u}_n k^{2n-4}$

$$k\partial_k g_n = \beta_{g_n}(g_0, g_1, \dots)\,, \quad n = 0, \dots, N$$

- reduces search for NGFP to algebraic problem

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \ldots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \ldots) \,, \quad i = 0, \ldots, N$$

- NGFP can be traced through extensions of truncation subspace

N	g_0^*	g_1^*	g_2^*	g_3^*	g_4^*	g_5^*	g_6^*
1	0.00523	-0.0202					
2	0.00333	-0.0125	0.00149				
3	0.00518	-0.0196	0.00070	-0.0104			
4	0.00505	-0.0206	0.00026	-0.0120	-0.0101		
5	0.00506	-0.0206	0.00023	-0.0105	-0.0096	-0.00455	
6	0.00504	-0.0208	0.00012	-0.0110	-0.0109	-0.00473	0.00238

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \ldots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \ldots)\,, \quad i = 0, \ldots, N$$

- NGFP can be traced through extensions of truncation subspace

N	g_0^*	g_1^*	g_2^*	g_3^*	g_4^*	g_5^*	g_6^*
1	0.00523	-0.0202					
2	0.00333	-0.0125	0.00149				
3	0.00518	-0.0196	0.00070	-0.0104			
4	0.00505	-0.0206	0.00026	-0.0120	-0.0101		
5	0.00506	-0.0206	0.00023	-0.0105	-0.0096	-0.00455	
6	0.00504	-0.0208	0.00012	-0.0110	-0.0109	-0.00473	0.00238

NGFP is stable under extension of truncation subspace

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \ldots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \ldots) \,, \quad i = 0, \ldots, N$$

- linearized RG flow at NGFP $\Longrightarrow$ three UV relevant directions

N	Re $\theta_{0,1}$	Im $\theta_{0,1}$	θ_2	θ_3	θ_4	θ_5	θ_6
1	2.38	2.17					
2	1.26	2.44	27.0				
3	2.67	2.26	2.07	-4.42			
4	2.83	2.42	1.54	-4.28	-5.09		
5	2.57	2.67	1.73	-4.40	-3.97 + $4.57i$	-3.97 - $4.57i$	
6	2.39	2.38	1.51	-4.16	-4.67 + $6.08i$	-4.67 - $6.08i$	-8.67

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \ldots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \ldots) \,, \quad i = 0, \ldots, N$$

- linearized RG flow at NGFP $\Longrightarrow$ three UV relevant directions

N	Re $\theta_{0,1}$	Im $\theta_{0,1}$	θ_2	θ_3	θ_4	θ_5	θ_6
1	2.38	2.17					
2	1.26	2.44	27.0				
3	2.67	2.26	2.07	-4.42			
4	2.83	2.42	1.54	-4.28	-5.09		
5	2.57	2.67	1.73	-4.40	-3.97 + $4.57i$	-3.97 - $4.57i$	
6	2.39	2.38	1.51	-4.16	-4.67 + $6.08i$	-4.67 - $6.08i$	-8.67

NGFP is stable under extension of truncation subspace

good evidence: fundamental theory has finite number of relevant parameters

infinite-dimensional truncations

RG flow of $f(R)$-gravity

[D. Benedetti and F. Caravelli, JHEP 1206 (2012) 017, arXiv:1204.3541]
[M. Demmel, F. Saueressig and O. Zanusso, JHEP 1211 (2012) 131, arXiv:1208.2038]
[J. A. Dietz and T. R. Morris, JHEP **1301** (2013) 108, arXiv:1211.0955]
[D. Benedetti, Europhys. Lett. 102 (2013) 20007, arXiv:1301.4422]
[M. Demmel, F. Saueressig and O. Zanusso, arXiv:1302.1312]
[J. A. Dietz and T. R. Morris, JHEP **1307** (2013) 064, arXiv:1306.1223]
[D. Benedetti and F. Guarnieri, arXiv:1311.1081]
[I. H. Bridle, J. A. Dietz and T. R. Morris, arXiv:1312.2846]
[M. Demmel, F. Saueressig and O. Zanusso, arXiv:1401.5495]

RG flows of $f(R)$-gravity

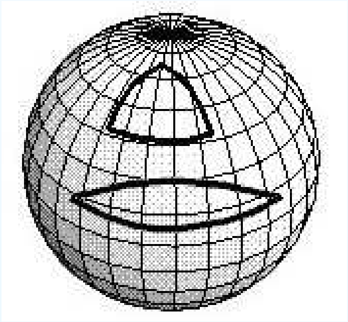
toy model: 3-dimensional, conformally reduced gravity

$$\Gamma_k[g] = \int d^3x \sqrt{g}\, f_k(R)$$

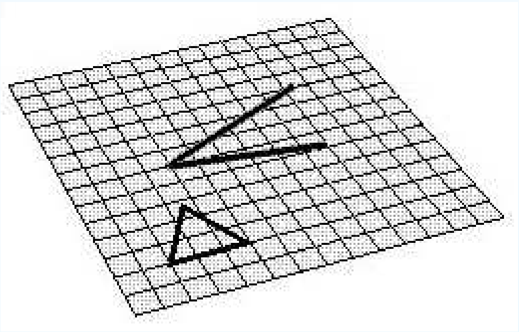
flow equation:

$$\int d^3x \sqrt{g}\, [k\partial_k\, f_k(R)] = \tfrac{1}{2}\mathrm{Tr}\, W[\Box]$$
$$= \tfrac{1}{2}\int_0^\infty ds\, \tilde{W}(s)\, \mathrm{Tr}\, \mathrm{e}^{-s\Box}$$

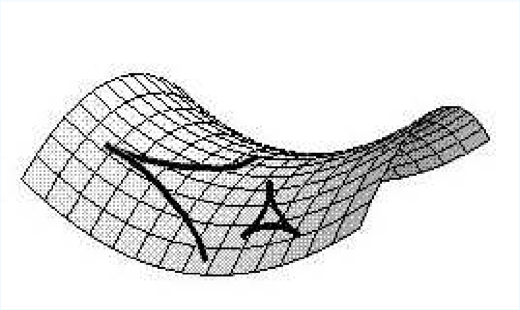
$f(R)$-ansatz: evaluate trace on maximally symmetric spaces



S^3



R^3

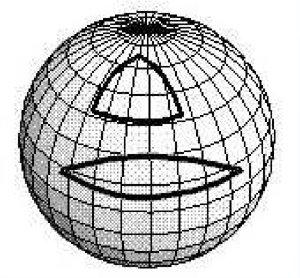


H^3

Laplacian Δ on S^3

Laplacian has discrete spectrum

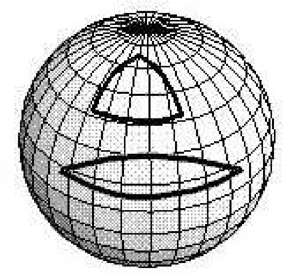
$$\lambda_l = l(l+2)\frac{R}{6}\,, \qquad D_l = (l+1)^2\,, \qquad l = 0, 1, \ldots$$



Laplacian Δ on S^3

Laplacian has discrete spectrum

$$\lambda_l = l(l+2)\frac{R}{6}\,, \qquad D_l = (l+1)^2\,, \qquad l = 0, 1, \ldots$$



Operator trace can be expressed through the heat-kernel

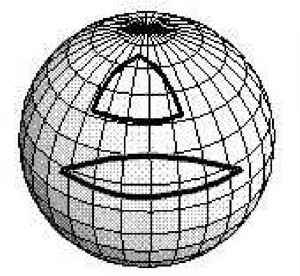
$$\mathrm{Tr}\, \mathrm{e}^{-s\Delta} = \int d^3x \sqrt{g}\, K(x,s)\,.$$

local heat-kernel

$$\begin{aligned} K(x,s) =& (4\pi s)^{-3/2}\, \mathrm{e}^{sR/6} \\ =& (4\pi s)^{-3/2}\, \left(1 + \tfrac{1}{6} sR + \ldots\right) \end{aligned}$$

Laplacian Δ on S^3

Laplacian has discrete spectrum



$$\lambda_l = l(l+2)\frac{R}{6}\,, \qquad D_l = (l+1)^2\,, \qquad l = 0, 1, \ldots$$

Operator trace can be expressed through the heat-kernel

$$\mathrm{Tr}\, \mathrm{e}^{-s\Delta} = \int d^3x \sqrt{g}\, K(x,s)\,.$$

local heat-kernel

$$\begin{aligned} K(x,s) =& (4\pi s)^{-3/2}\, \mathrm{e}^{sR/6} \\ =& (4\pi s)^{-3/2}\, \left(1 + \tfrac{1}{6} sR + \ldots\right) \end{aligned}$$

non-local contributions (diffusing particle returning multiple times)

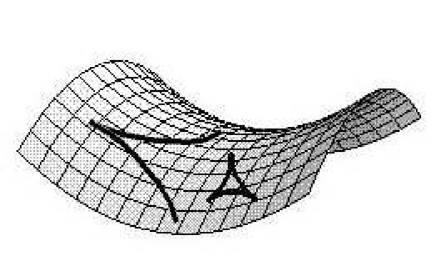
$$K(x,s) = (4\pi s)^{-3/2}\, \mathrm{e}^{sR/6} \sum_{n=-\infty}^{\infty} \left(1 - \frac{12\pi^2 n^2}{sR}\right) \mathrm{e}^{\frac{-6n^2\pi^2}{sR}}$$

- crucial for correct asymptotic for $s \to \infty$

Laplacian Δ on H^3

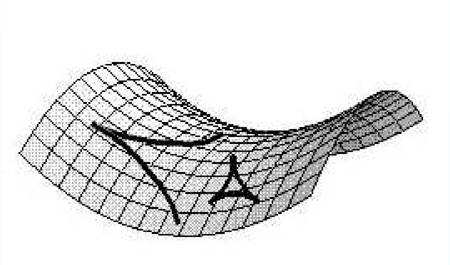
Laplacian has continuous spectrum

$$\rho \in [\lambda_c, \infty]\,, \qquad \lambda_c = -\frac{R}{6} > 0$$



Laplacian Δ on H^3

Laplacian has continuous spectrum



$$\rho \in [\lambda_c, \infty]\,, \qquad \lambda_c = -\frac{R}{6} > 0$$

Operator trace can be expressed through the heat-kernel

$$\mathrm{Tr}\, \mathrm{e}^{-s\Delta} = \int d^3x \sqrt{g}\, K(x,s)\,.$$

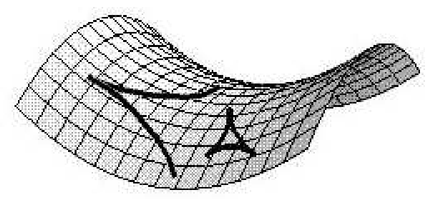
exact heat-kernel:

$$K(x,s) = (4\pi s)^{-3/2}\, \mathrm{e}^{sR/6}$$
$$= (4\pi s)^{-3/2}\, \left(1 + \tfrac{1}{6} sR + \ldots\right)$$

- H^3 is non-compact: no winding modes!

Laplacian Δ on H^3

Laplacian has continuous spectrum



$$\rho \in [\lambda_c, \infty]\,, \qquad \lambda_c = -\frac{R}{6} > 0$$

Operator trace can be expressed through the heat-kernel

$$\mathrm{Tr}\,\mathrm{e}^{-s\Delta} = \int d^3x \sqrt{g}\, K(x,s)\,.$$

exact heat-kernel:

$$K(x,s) = (4\pi s)^{-3/2}\, \mathrm{e}^{sR/6}$$
$$= (4\pi s)^{-3/2}\, \left(1 + \tfrac{1}{6} sR + \ldots\right)$$

- H^3 is non-compact: no winding modes!

"local heat-kernel" is same for any $\bar{g}$

background covariance: β-functions independent of $\bar{g}$

Flow equation on S^3

choose coarse-graining operator

$$\Box = \Delta + \mathbb{E}\,, \qquad \mathbb{E} = 0, R/6$$

write flow in dimensionless quantities

$$R = k^2\, r\,, \qquad f_k(R) = k^3\, \varphi_k(r)$$

obtain partial-differential equation for φ ($\mathbb{E} = 0$):

$$\dot{\varphi}_k + 3\varphi - 2r\varphi' = \sum_{l=0}^{\infty} (l+1)^2\, \theta\left(1 - \tfrac{1}{6} l(l+2) r\right)\, \mathcal{N}\left(l, r, \varphi', \varphi'', \varphi''', \dot{\varphi}', \dot{\varphi}''\right)$$

- first order in $\partial_t \equiv k\partial_k$
- third order in r
- integrates out fluctuations of sphere “mode by mode”

Constructing fixed functionals

fixed functionals are k-stationary, global solutions of the PDE

truncation	flow	fixed points
finite-dimensional	ODE	algebraic
infinite-dimensional	PDE	ODE

Constructing fixed functionals

fixed functionals are k-stationary, global solutions of the PDE

truncation	flow	fixed points
finite-dimensional	ODE	algebraic
infinite-dimensional	PDE	ODE

non-linear third-order ODE determining φ_* ($\mathbb{E} = R/6$):

$$3\varphi - 2r\varphi' = \begin{cases} \frac{3\, r^{3/2}}{4\sqrt{6}\pi^2} \sum_{n\geq 1} \theta\left(1 - \frac{r}{6} n^2\right) \frac{\hat{b}_1\, n^2 + \hat{b}_2\, n^4 + \hat{b}_3\, n^6}{27\varphi + 6(6-7r)\varphi' + 16(3-2r)^2\varphi''}\,, & r \in [0,6] \\ \frac{1}{35\,\pi^2} \frac{252\varphi' + 20(72-49r)\varphi'' - 32r(15-14r)\varphi'''}{27\varphi + 6(6-7r)\varphi' + 16(3-2r)^2\varphi''}\,, & r \in [-\infty, 0]\,. \end{cases}$$

Constructing fixed functionals

fixed functionals are k-stationary, global solutions of the PDE

truncation	flow	fixed points
finite-dimensional	ODE	algebraic
infinite-dimensional	PDE	ODE

non-linear third-order ODE determining φ_* ($\mathbb{E} = R/6$):

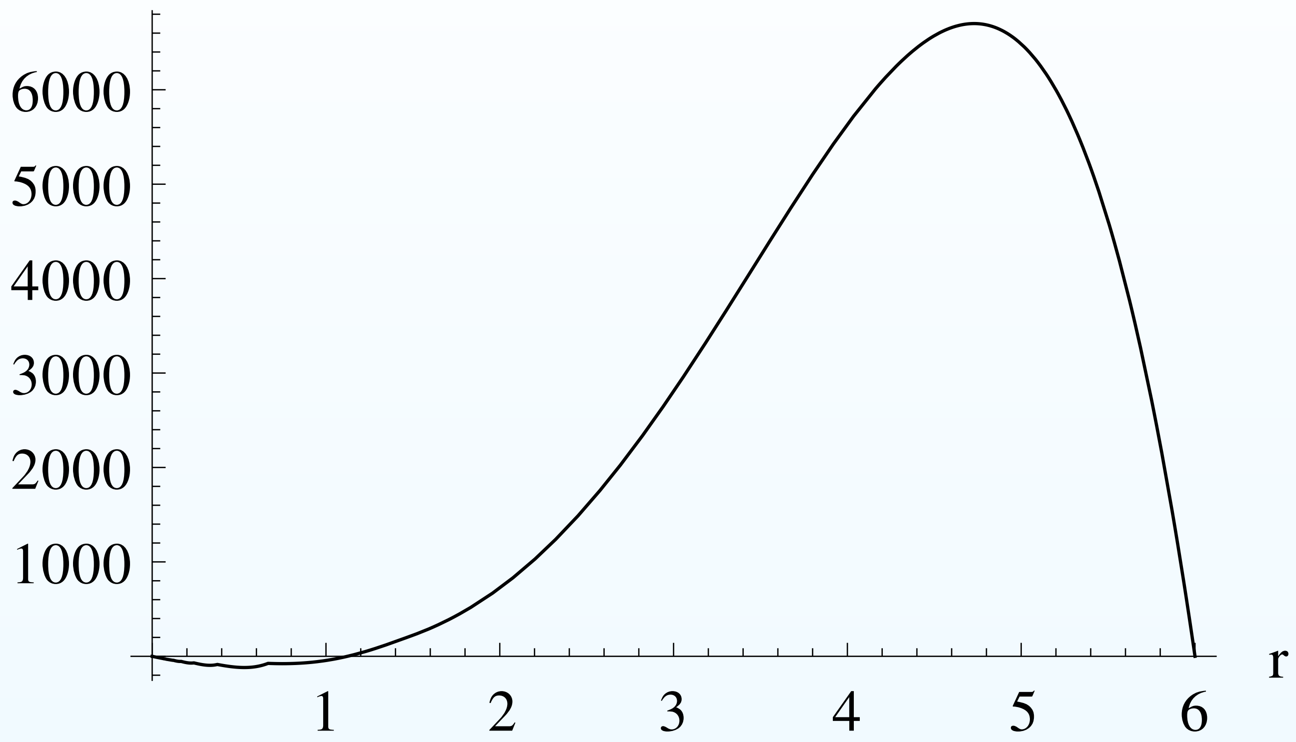
$$3\varphi - 2r\varphi' = \begin{cases} \frac{3\,r^{3/2}}{4\sqrt{6}\pi^2} \sum_{n\geq 1} \theta\left(1 - \frac{r}{6}n^2\right) \frac{\hat{b}_1\, n^2 + \hat{b}_2\, n^4 + \hat{b}_3\, n^6}{27\varphi + 6(6-7r)\varphi' + 16(3-2r)^2\varphi''}\,, & r \in [0,6] \\ \frac{1}{35\,\pi^2} \frac{252\varphi' + 20(72-49r)\varphi'' - 32r(15-14r)\varphi'''}{27\varphi + 6(6-7r)\varphi' + 16(3-2r)^2\varphi''}\,, & r \in [-\infty, 0]\,. \end{cases}$$

expect set of discrete solutions, iff

order of ODE - number of singularities = 0

Checking the singularity index

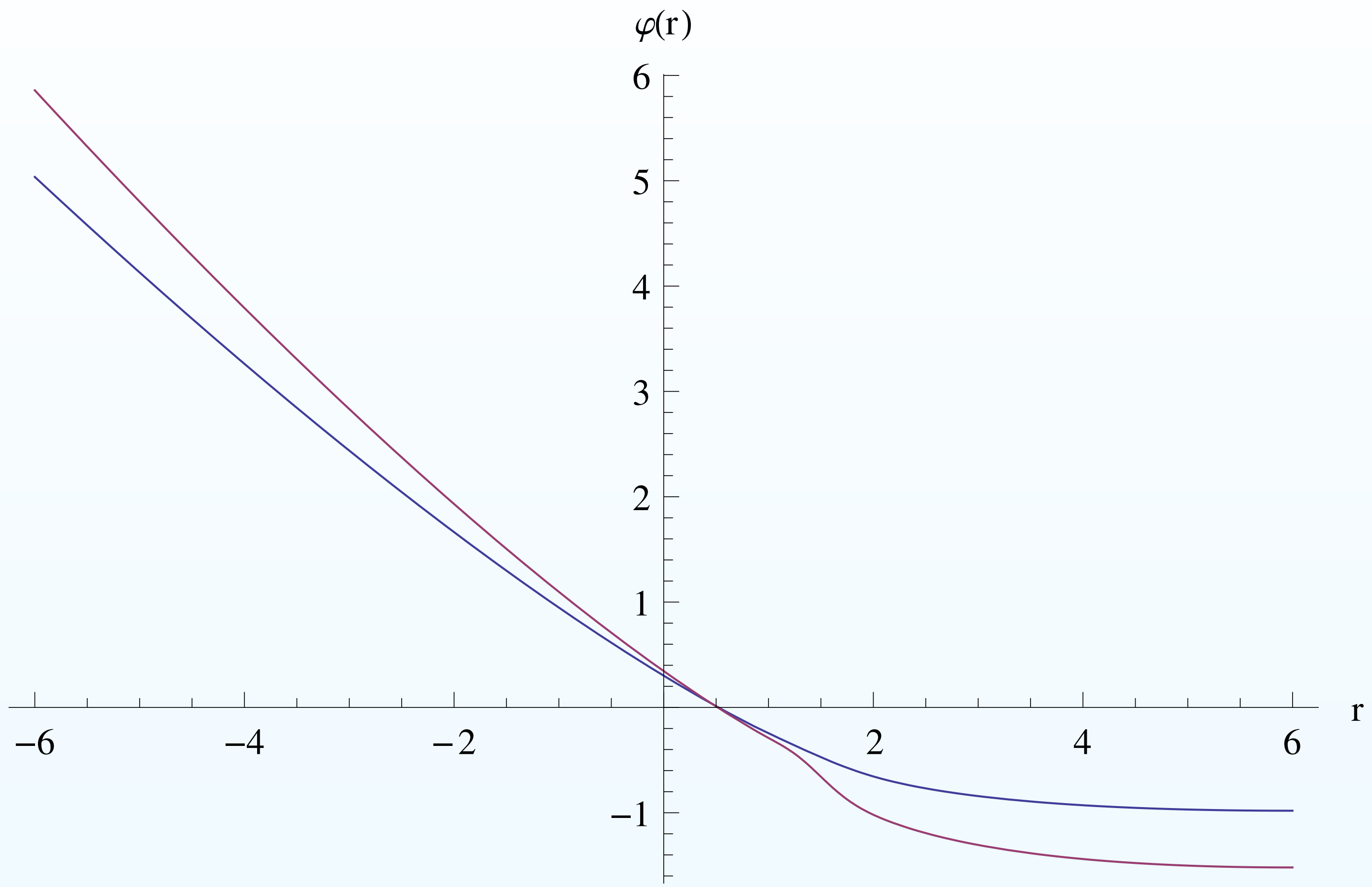
- $r > 0$ has three fixed singularities



- $r < 0$ has no fixed singularities

order of ODE - number of singularities $\stackrel{!}{=}$ 0

Fixed functionals obtained from shooting method



two global solutions $\varphi_{*,1}$, $\varphi_{*,2}$ with positive λ_*, g_*

key results: Asymptotic Safety

pure gravity:

- evidence for Asymptotic Safety
  - $\Rightarrow$ non-Gaussian fixed point provides UV completion of gravity
- low number of relevant parameter:
  - $\Rightarrow$ dimensionality of UV-critical surface $\simeq 3$
- perturbative counterterms:
  - gravity + matter: asymptotic safety survives 1-loop counterterm

key results: Asymptotic Safety

pure gravity:

- evidence for Asymptotic Safety
  - $\Rightarrow$ non-Gaussian fixed point provides UV completion of gravity
- low number of relevant parameter:
  - $\Rightarrow$ dimensionality of UV-critical surface $\simeq$ 3
- perturbative counterterms:
  - ○ gravity + matter: asymptotic safety survives 1-loop counterterm

gravity coupled to matter:

- non-Gaussian fixed point compatible with standard-model matter

[R. Percacci and D. Perini, hep-th/0207033]

[P. Dona, A. Eichhorn and R. Percacci, arXiv:1311.2898]

- prediction of the Higgs mass $m_H \simeq 126$ GeV

[M. Shaposhnikov and C. Wetterich, arXiv:0912.0208]

Is

Asymptotic Safety

the only possibility for a

quantum field theory of gravity?

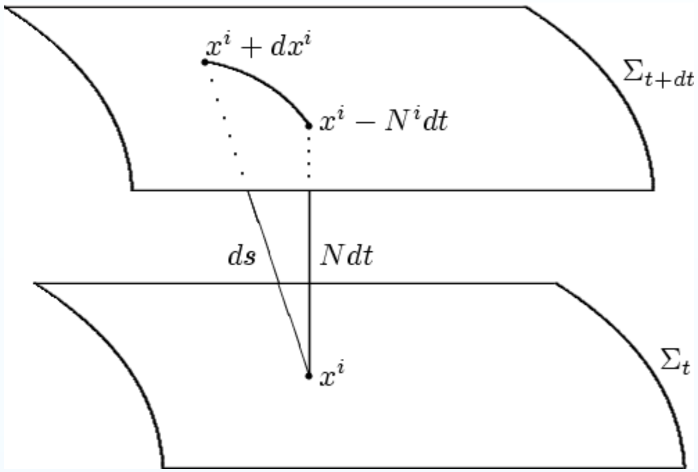
Hořava-Lifshitz Gravity

Asymptotic Safety

and its connections

Foliation structure via ADM-decomposition

Preferred “time”-direction via foliation of space-time



- foliation structure $\mathcal{M}^{d+1} = S^1 \times \mathcal{M}^d$ with $y^\mu \mapsto (\tau, x^a)$:

$$ds^2 = N^2 dt^2 + \sigma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

- fundamental fields: $g_{\mu\nu} \mapsto (N, N_i, \sigma_{ij})$

$$g_{\mu\nu} = \begin{pmatrix} N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix}$$

projectable Hořava-Lifshitz gravity in a nutshell

P. Hořava, Phys. Rev. D79 (2009) 084008, arXiv:0901.3775

central idea: find a perturbatively renormalizable quantum theory of gravity

fundamental fields: $\{N(t), N_i(t,x), \sigma_{ij}(t,x)\}$

symmetry: Diff$(\mathcal{M}, \Sigma) \subset$ Diff$(\mathcal{M})$

- breaks Lorentz-invariance at high energies

projectable Hořava-Lifshitz gravity in a nutshell

P. Hořava, Phys. Rev. D79 (2009) 084008, arXiv:0901.3775

central idea: find a perturbatively renormalizable quantum theory of gravity

fundamental fields: $\{N(t), N_i(t,x), \sigma_{ij}(t,x)\}$

symmetry: Diff($\mathcal{M}, \Sigma$) $\subset$ Diff($\mathcal{M}$)

- breaks Lorentz-invariance at high energies

Can construct the effective average action for projective HL-gravity

S. Rechenberger and F.S., JHEP 03 (2013) 010, arXiv:1212.5114

- scale-dependence governed by functional renormalization group equation

$$k\partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

Hořava-Lifshitz gravity as viable theory for Quantum Gravity

Requirements:

a) anisotropic Gaussian Fixed Point (aGFP)

- controls the UV-behavior of the RG-trajectory
- regulates UV-divergences without introducing ghosts

Hořava-Lifshitz gravity as viable theory for Quantum Gravity

Requirements:

a) anisotropic Gaussian Fixed Point (aGFP)

- controls the UV-behavior of the RG-trajectory
- regulates UV-divergences without introducing ghosts

b) finite-dimensional UV-critical surface $\mathcal{S}_{\rm UV}$

- guaranteed by perturbation theory

Hořava-Lifshitz gravity as viable theory for Quantum Gravity

Requirements:

a) anisotropic Gaussian Fixed Point (aGFP)

- controls the UV-behavior of the RG-trajectory
- regulates UV-divergences without introducing ghosts

b) finite-dimensional UV-critical surface $\mathcal{S}_{\rm UV}$

- guaranteed by perturbation theory

c) classical limit

- Lorentz-symmetry is restored in the IR
- decoupling of extra degree of freedom

Hořava-Lifshitz gravity as viable theory for Quantum Gravity

Requirements:

a) anisotropic Gaussian Fixed Point (aGFP)

- controls the UV-behavior of the RG-trajectory
- regulates UV-divergences without introducing ghosts

b) finite-dimensional UV-critical surface $\mathcal{S}_{\rm UV}$

- guaranteed by perturbation theory

c) classical limit

- Lorentz-symmetry is restored in the IR
- decoupling of extra degree of freedom

Hořava-Lifshitz (HL) gravity

RG flows for Hořava-Lifshitz gravity

finite temperature type computations

ADM-decomposed Einstein-Hilbert truncation

ADM-decomposed Einstein-Hilbert action:

$$\Gamma_k^{\mathrm{ADM}} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int dt d^3x N \sqrt{\sigma} \Big[\epsilon^{-1} \underbrace{K_{ij}}_{\text{extrinsic curvature}} \left[\sigma^{ik} \sigma^{jl} - \sigma^{ij} \sigma^{kl} \right] K_{kl} - \underbrace{R}_{\text{intrinsic curvature}} + 2\Lambda_k \Big]$$

- lives on foliation $S^1_T \times \mathcal{M}^{(3)}$
- running couplings: G_k, Λ_k
- signature parameter ϵ

ADM-decomposed Einstein-Hilbert truncation

ADM-decomposed Einstein-Hilbert action:

$$\Gamma_k^{\mathrm{ADM}} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int dt d^3x N \sqrt{\sigma} \Big[\epsilon^{-1} \underbrace{K_{ij}}_{\text{extrinsic curvature}} \left[\sigma^{ik}\sigma^{jl} - \sigma^{ij}\sigma^{kl}\right] K_{kl} - \underbrace{R}_{\text{intrinsic curvature}} + 2\Lambda_k \Big]$$

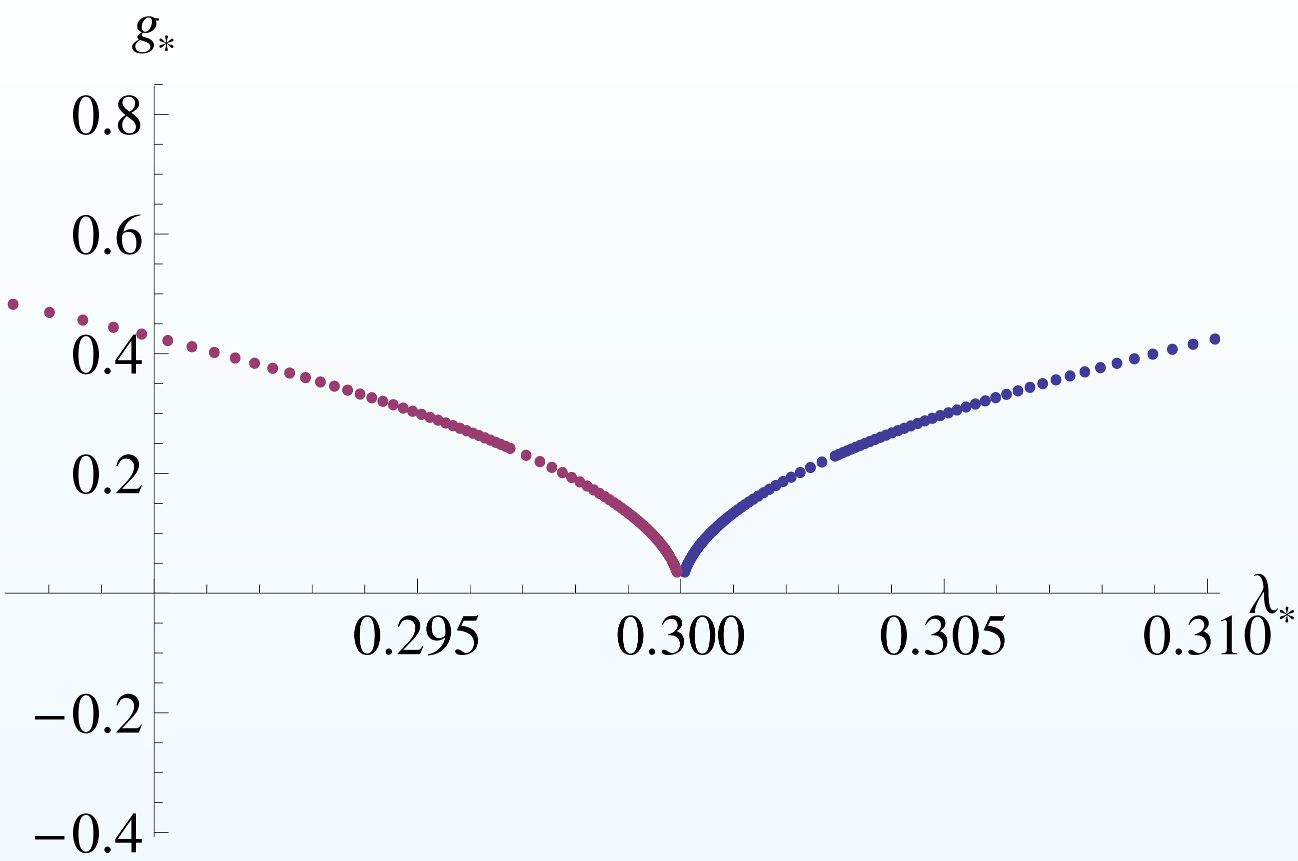
- lives on foliation $S^1_T \times \mathcal{M}^{(3)}$
- running couplings: G_k, Λ_k
- signature parameter ϵ

β-functions depend parametrically on $m = \frac{2\pi}{Tk}$:

$$k\partial_k g_k = \beta_g(g, \lambda; m), \qquad k\partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

- m: anisotropy between cutoff in spatial/time direction

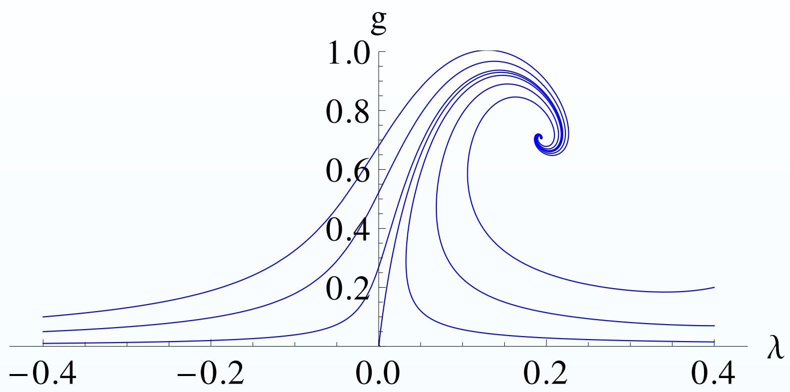
result: signature dependence of NGFP



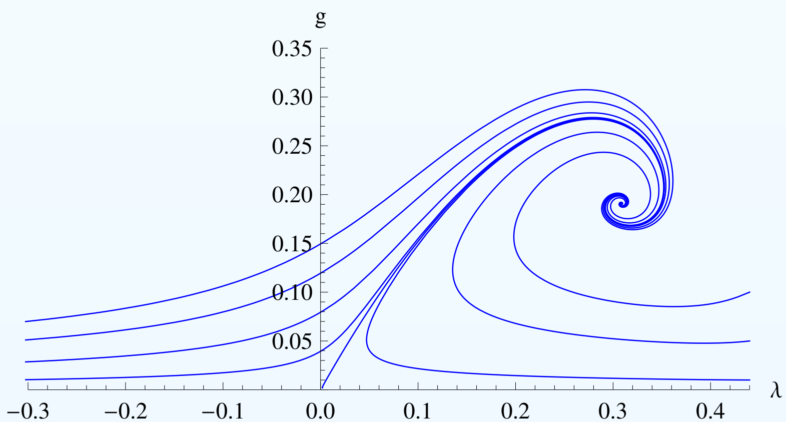
for m finite NGFPs separate:

- $\epsilon = +1$: Euclidean signature (blue)
- $\epsilon = -1$: Lorentzian signature (magenta)

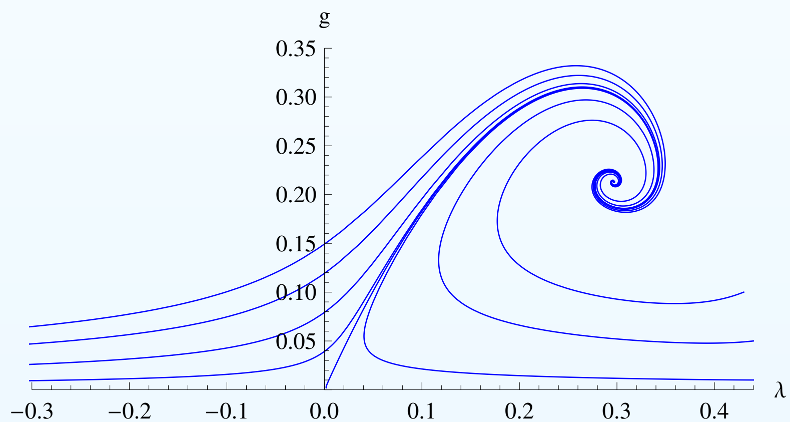
phase diagrams



covariant computation



Euclidean



Lorentzian

RG flows for Hořava-Lifshitz gravity

including anisotropy

Hořava-Lifshitz gravity: recovering general relativity in the IR

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

RG flow of anisotropic Einstein-Hilbert truncation

$$\Gamma_k^{\rm grav}[N, N_i, \sigma_{ij}] = \frac{1}{16\pi G_k} \int dt d^3x N \sqrt{g} \left[K_{ij} K^{ij} - \lambda_k K^2 - R + 2\Lambda_k \right]$$

Fixed points of the beta functions:

- line of GFPs

$$\tilde{G}_* = 0 \,, \qquad \tilde{\Lambda}_* = 0 \,, \qquad \lambda = \lambda_*$$

  - one IR attractive, one IR repulsive, one marginal direction

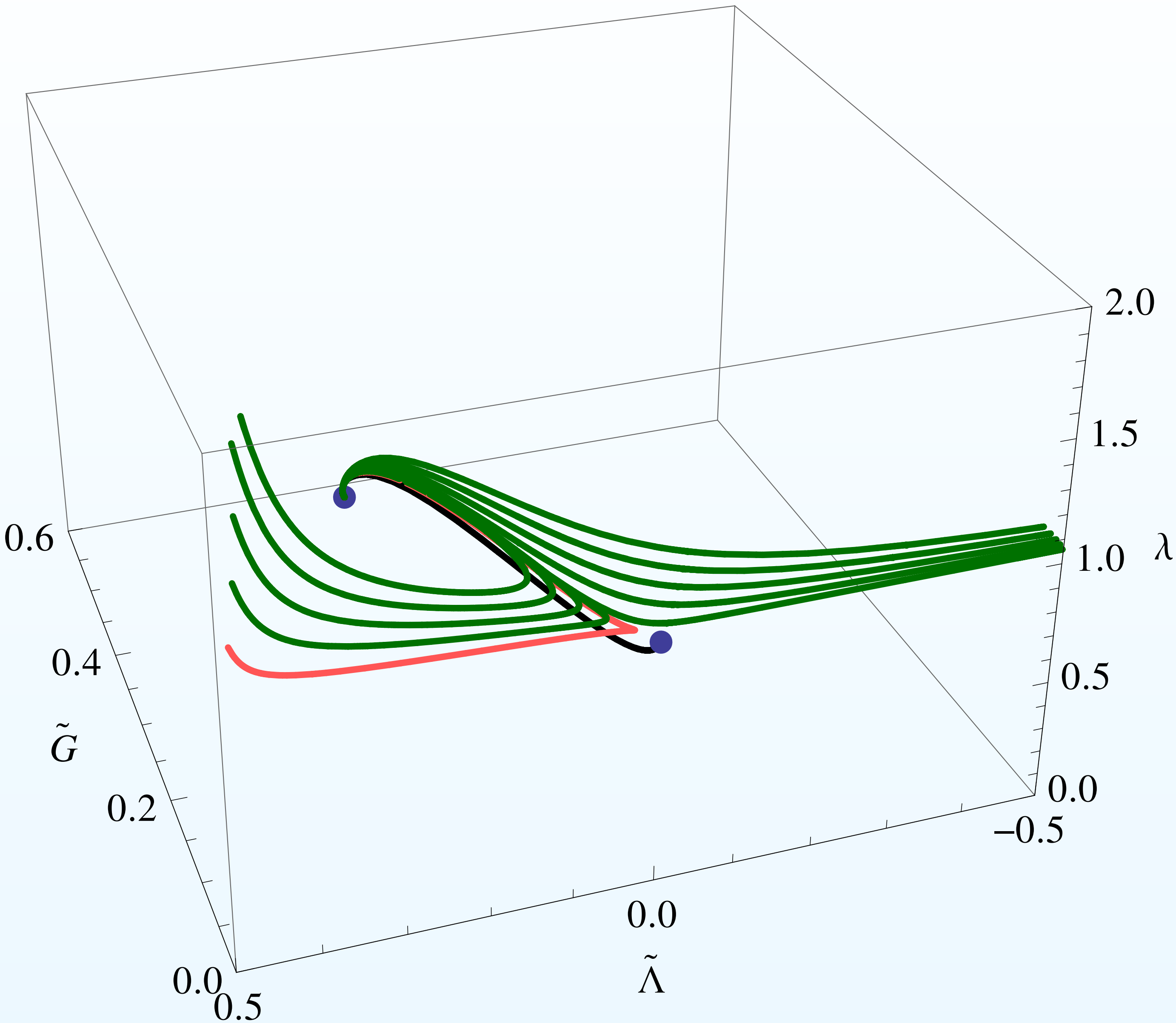
- NGFP underlying Asymptotic Safety

$$\tilde{G}_* = 0.49 \,, \qquad \tilde{\Lambda}_* = 0.17 \,, \qquad \lambda_* = 0.44$$

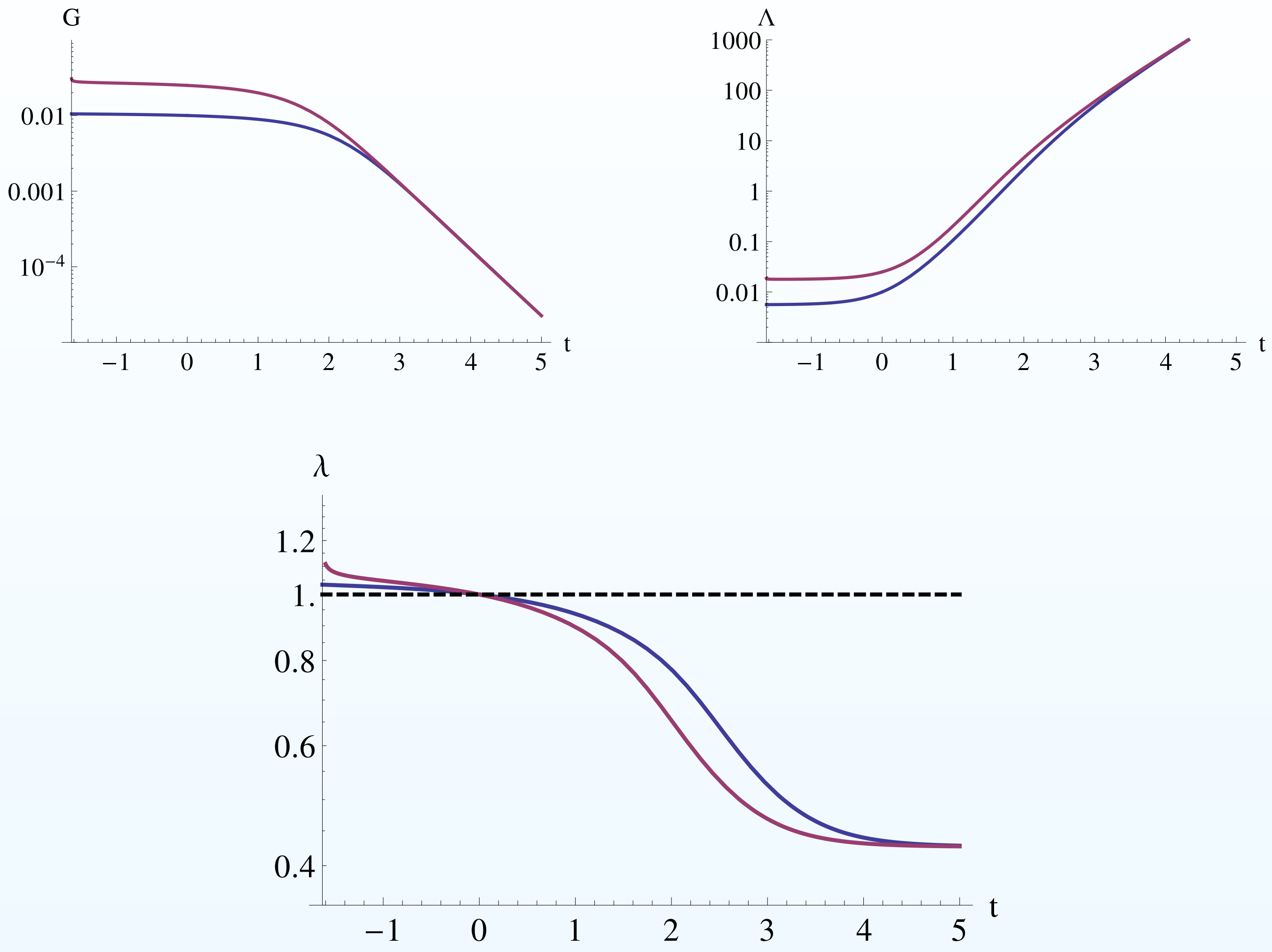
  - three UV-attractive eigendirections

anisotropic GFP providing UV-limit of HL-gravity not in truncation

Hořava-Lifshitz gravity: recovering general relativity in the IR



Scale-dependence of dimensionful couplings



GFP governs IR-behavior of HL-gravity

small value of cosmological constant makes λ compatible with experiments

RG flows for projectable HL gravity

anisotropic heat-kernels

Zooming into the aGFP in $D = 3 + 1$

Compute matter-induced gravitational β-functions

$$\Gamma_k = \Gamma_k^{\rm HL} + S^{\rm LM}$$

for two wave-function renormalizations and 8 potential couplings

$$\Gamma_k^{\rm HL} = \frac{1}{16\pi G_k} \int dt d^3x \sqrt{\sigma} \, \left[\left(K_{ij} K^{ij} - \lambda_k \, K^2 \right) - g_7 R \, \Delta_x \, R - g_8 R_{ij} \, \Delta_x \, R^{ij} + \ldots \right]$$

$$S^{\rm LM} = \frac{1}{2} \int dt d^3x \sqrt{\sigma} \, \left[\phi \left(\Delta_t + (\Delta_x)^z \right) \phi \right]$$

key ingredient: anisotropic Laplace operator

$$D = \Delta_t + (\Delta_x)^z$$

$$\Delta_t = -\sqrt{\sigma}^{-1} \, \partial_t \sqrt{\sigma} \partial_t \, , \qquad \Delta_x = -\sigma^{ij}(t,x) D_i D_j$$

Zooming into the aGFP in $d = 4$

Compute matter-induced gravitational β-functions

$$\Gamma_k = \Gamma_k^{\mathrm{HL}} + S^{\mathrm{LM}}$$

for two wave-function renormalizations and 8 potential couplings

$$\Gamma_k^{\mathrm{HL}} = \frac{1}{16\pi G_k} \int dt d^3x \sqrt{\sigma} \left[\left(K_{ij} K^{ij} - \lambda_k K^2 \right) - g_7 R \Delta_x R - g_8 R_{ij} \Delta_x R^{ij} + \dots \right]$$

$$S^{\mathrm{LM}} = \frac{1}{2} \int dt d^3x \sqrt{\sigma} \left[\phi \left(\Delta_t + (\Delta_x)^z \right) \phi \right]$$

Gravitational propagators in flat space: $\sigma_{ij} = \delta_{ij} + \sqrt{16\pi G_k}\, h_{ij}$

$$[\mathcal{G}_{s=2}(\omega, \vec{p})] \propto \omega^2 - g_8\, \vec{p}^{\,6}$$

$$[\mathcal{G}_{s=0}(\omega, \vec{p})] \propto \left(\tfrac{1}{3} - \lambda_k \right) \left(\omega^2 - \left(\tfrac{1}{3} - \lambda_k \right)^{-1} \left(\tfrac{8}{9}\, g_7 + \tfrac{1}{3} g_8 \right) \vec{p}^{\,6} \right)$$

Heat kernel expansion of anisotropic operators

FRGE computations use heat-kernel expansion of Laplacian $\Delta \equiv -g^{\mu\nu} D_\mu D_\nu$

$$\mathrm{Tr} e^{-s\Delta} \simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \sum_{n \geq 0} s^n \, a_{2n}$$

$$\simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \, \left[1 + \tfrac{s}{6} R + \ldots\right]$$

Heat kernel expansion of anisotropic operators

FRGE computations use heat-kernel expansion of Laplacian $\Delta \equiv -g^{\mu\nu} D_\mu D_\nu$

$$\mathrm{Tr} e^{-s\Delta} \simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \sum_{n\geq 0} s^n \, a_{2n}$$
$$\simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \left[1 + \tfrac{s}{6} R + \ldots\right]$$

Heat kernel expansion of anisotropic operators

$$D \equiv \Delta_t + (\Delta_x)^z$$

- compute from off-diagonal heat-kernel techniques

D. Anselmi, A. Benini, JHEP 07 (2007) 099

D. Benedetti, K. Groh, P. F. Machado, F. Saueressig, JHEP 06 (2011) 079

$$\mathrm{Tr} e^{-sD} \simeq \frac{s^{-\frac{1}{2}(1+d/z)}}{(4\pi)^{(d+1)/2}} \int dt d^d x \sqrt{\sigma} \left[\tfrac{s}{6} \left(e_1 K^2 + e_2 K_{ij} K^{ij} \right) + \sum_{n\geq 0} s^{n/z} \, b_n \, a_{2n} \right]$$

$$e_1 = \frac{d - z + 3}{d+2} \frac{\Gamma(\frac{d}{2z})}{z \, \Gamma(\frac{d}{2})} \,, \qquad e_2 = -\frac{d + 2z}{d+2} \frac{\Gamma(\frac{d}{2z})}{z \, \Gamma(\frac{d}{2})}$$

Heat kernel coefficients for anisotropic operators

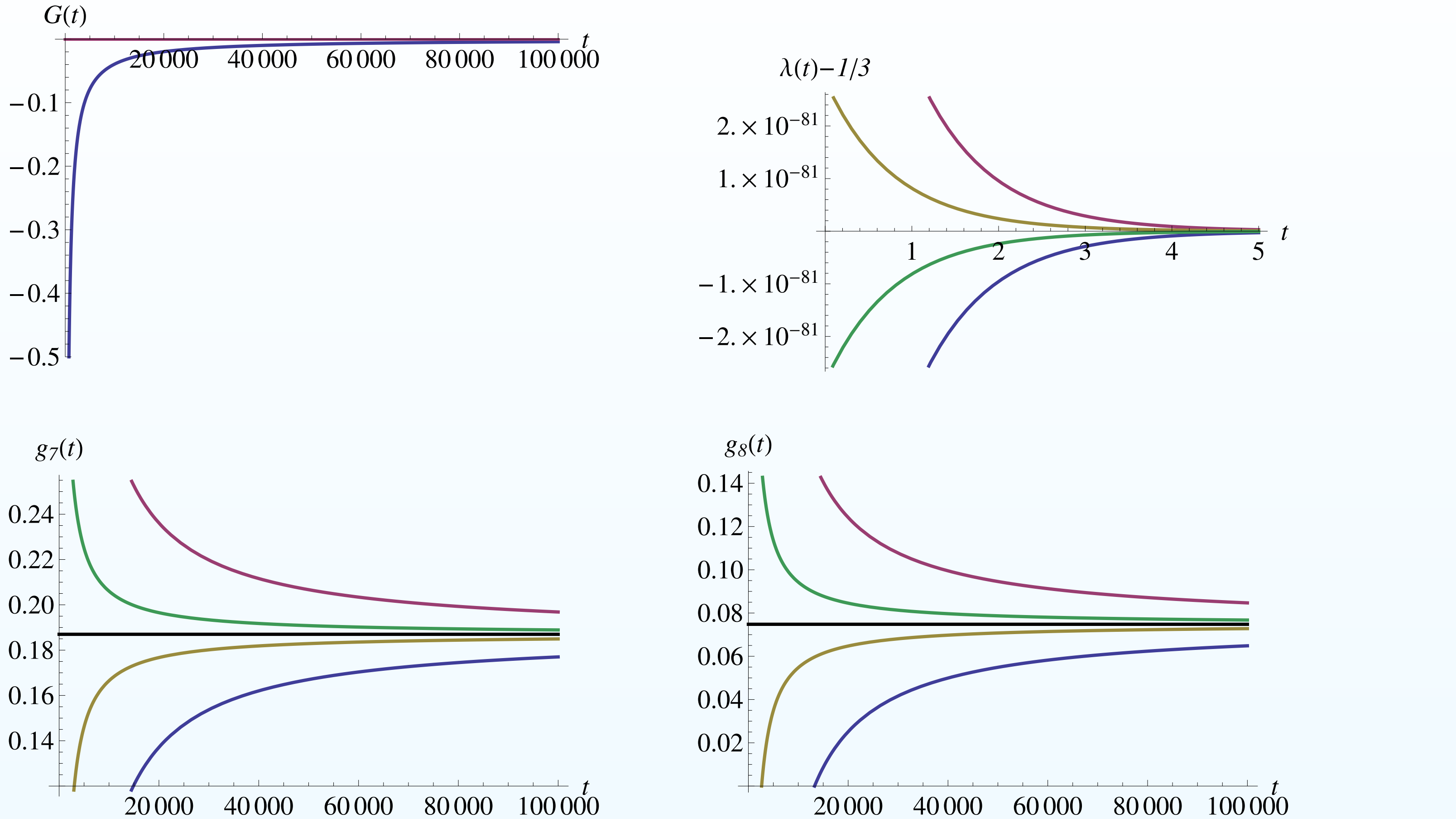
	$d=2$			$d=3$			
	$z=1$	$z=2$	$z=3$	$z=1$	$z=2$	$z=3$	$z=4$
b_0	1	$\frac{\sqrt{\pi}}{2}$	$\Gamma(\frac{4}{3})$	1	$\frac{4}{3\sqrt{\pi}}\Gamma(\frac{7}{4})$	$\frac{2}{3}$	$\frac{4}{3\sqrt{\pi}}\Gamma(\frac{11}{8})$
b_1	1	1	1	1	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{5}{4})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{7}{6})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{9}{8})$
b_2	1	0	0	1	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{3}{4})$	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{5}{6})$	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{7}{8})$
b_3	1	-2	0	1	$-\frac{2}{\sqrt{\pi}}\Gamma(\frac{5}{4})$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{\pi}}\Gamma(\frac{5}{8})$
b_4	1	0	6	1	$-\frac{4}{\sqrt{\pi}}\Gamma(\frac{7}{4})$	$\frac{9}{2\sqrt{\pi}}\Gamma(\frac{7}{6})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{11}{8})$

- $z=1$: reproduces standard heat-kernel
- $z=2, d=2$: reproduces

[M. Baggio, J. de Boer and K. Holsheimer, arXiv:1112.6416]

- d even: zero coefficients in heat kernel expansion

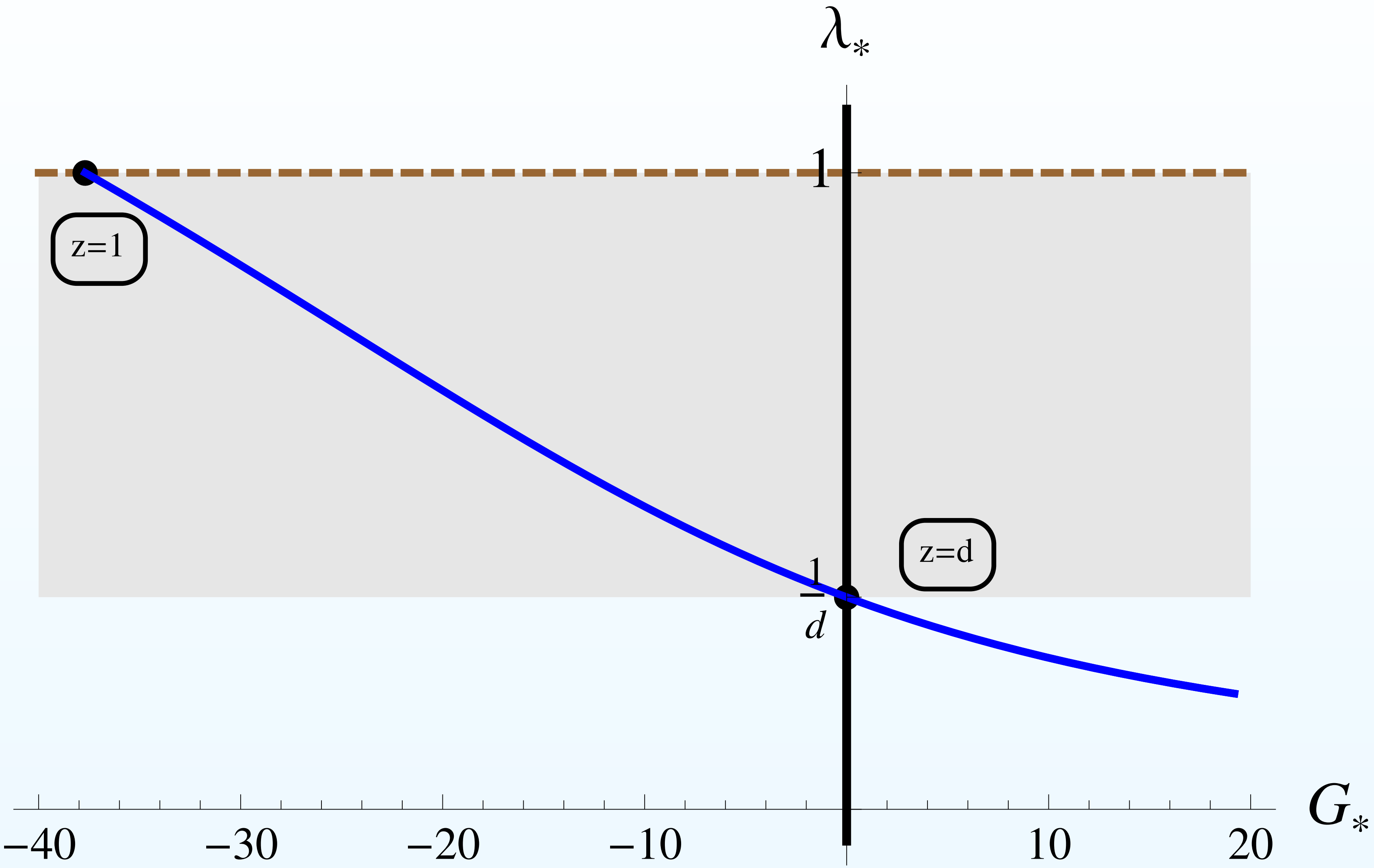
matter-induced RG flows in $d = z = 3$



UV attractive anisotropic GFP

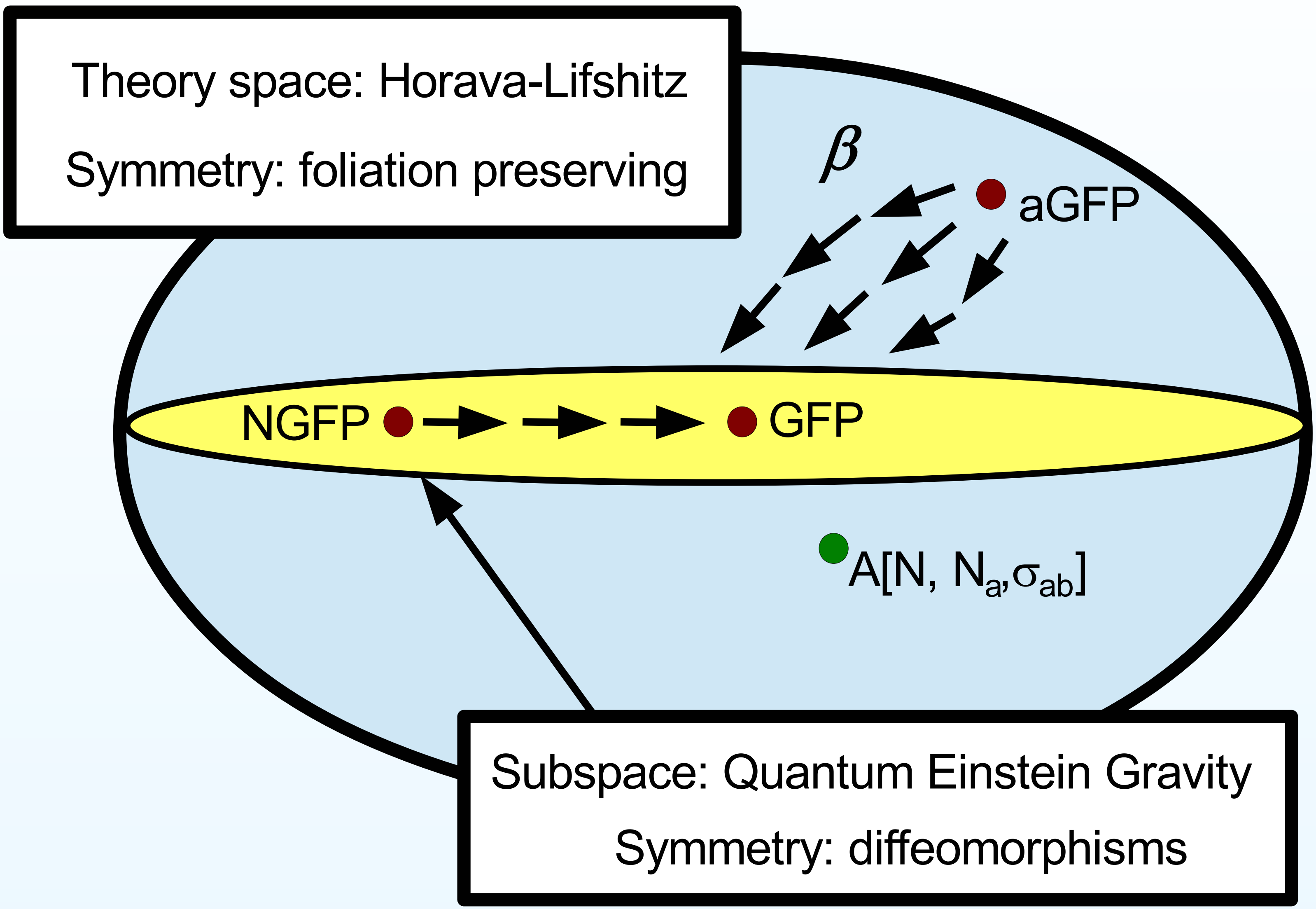
$$G^* = 0, \quad \lambda^* = 1/3, \quad g_7^* = \frac{5\pi}{84}, \quad g_8^* = \frac{\pi}{42}$$

Tracing the anisotropic Gaussian fixed point in z



Conclusions

Asymptotic Safety and Hořava-Lifshitz gravity live in same space



Conclusions

many proposals for quantum gravity within QFT:

- Asymptotic Safety
- Causal Dynamical Triangulations
- Hořava-Lifshitz gravity
- Shape Dynamics

differences:

- field content (metric, vielbein, ADM-variables, . . .)
- symmetry group (diffeomorphisms, foliation preserving diff.)

??? Which formulations describe the same Universality Class ???

Conclusions

many proposals for quantum gravity within QFT:

- Asymptotic Safety
- Causal Dynamical Triangulations
- Hořava-Lifshitz gravity
- Shape Dynamics

differences:

- field content (metric, vielbein, ADM-variables, . . .)
- symmetry group (diffeomorphisms, foliation preserving diff.)

??? Which formulations describe the same Universality Class ???

RG techniques crucial for solving this question

Conclusions

Asymptotic safety

- strong evidence for NGFP from finite-dimensional truncations
- progress towards controlling infinite-dimensional RG flows

Conclusions

Asymptotic safety

- strong evidence for NGFP from finite-dimensional truncations
- progress towards controlling infinite-dimensional RG flows

Hořava-Lifshitz (HL) gravity

- Quantum Einstein Gravity spans subspace of HL gravity
- GFP capable of providing IR-completion
- aGFP: $\lambda_* = d^{-1}$ acts as UV-attractor

Conclusions

Asymptotic safety

- strong evidence for NGFP from finite-dimensional truncations
- progress towards controlling infinite-dimensional RG flows

Hořava-Lifshitz (HL) gravity

- Quantum Einstein Gravity spans subspace of HL gravity
- GFP capable of providing IR-completion
- aGFP: $\lambda_* = d^{-1}$ acts as UV-attractor

!!! WORK AHEAD !!!