“When Higgs met Einstein”

Massimiliano Rinaldi
Dipartimento di Fisica & TIFPA-INFN
Università di Trento

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Outline

- Higgs-Hilbert-Einstein Lagrangian
- Spherically Symmetric Solutions: the "Higgs Monopole"
- Dark Energy and Dark Matter
- Conclusions

SU(2) Higgs-Hilbert-Einstein Lagrangian

$$\frac{\mathcal{L}_J}{\sqrt{g}} = \frac{1}{2}(m^2 + 2\xi \mathcal{H}^\dagger \mathcal{H})R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{F^2}{4} - V + \ldots$$

$$V = \lambda \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2} \right)^2 \qquad \mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \phi + i\chi_3 \end{pmatrix}$$

We assume that Higgs and gauge fields share the **symmetries of spacetime** (i.e. they are background fields).

Our goal is to study:

- Spherically symmetric and static solutions

- Cosmological solutions

The "Higgs Monopole"

- A. Füzfa, M. Rinaldi, and S. Schlögel, PRL 111, 121103 (2013).
- S. Schlogel, M. Rinaldi, F. Staelens and A. Fuzfa, arXiv:1405.5476 [gr-qc].

$$\mathcal{L} = \sqrt{g}\left[\frac{F(H)}{2\kappa}R - \frac{1}{2}(\partial H)^2 - V(H)\right] + \mathcal{L}_M\left[g_{\mu\nu}, \Psi_m\right]$$

perfect fluid

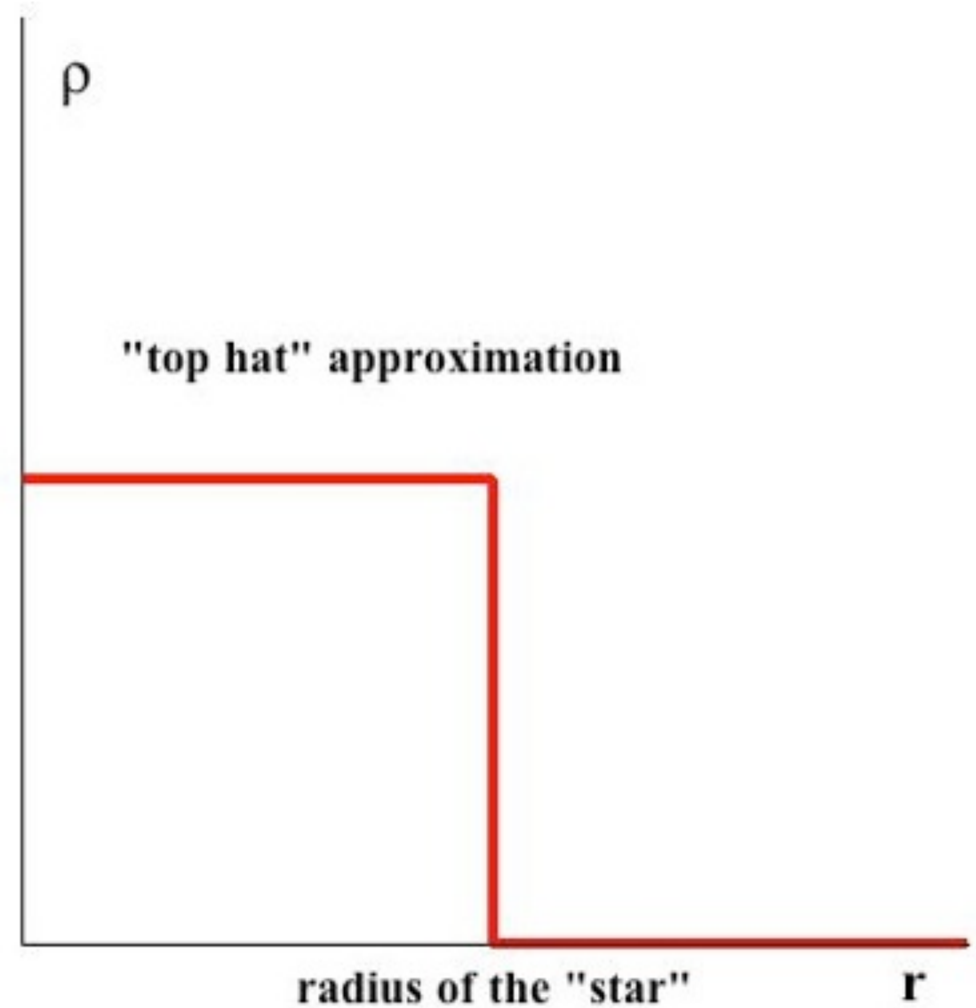
U-gauge

$$V(H) = \frac{\lambda_{sm}}{4}(H^2 - v^2)^2$$

$$F(H) = 1 + \frac{\xi}{m_p^2}H^2$$

we look for solutions that are

- unitary gauge
- spherically symmetric
- asymptotically flat
- nonsingular



Equations of motion

$$\mathcal{L} = \sqrt{g}\left[(m^2 + \xi H^2)R - \frac{1}{2}(\partial H)^2 - \frac{\lambda}{4}\left(H^2 - v^2\right)^2\right] + \mathcal{L}_m$$

$$\Box H + \frac{\xi H R}{8\pi} = \frac{dV}{dH}$$

$$\left(1 + \frac{\xi}{m_p^2} H^2\right) G_{\mu\nu} = \kappa \left[T_{\mu\nu}^{(H)} + T_{\mu\nu}^{(\xi)} + T_{\mu\nu}^{(mat)}\right]$$

✶Without matter : no-hair theorem, Schwarzschild

✶With matter :

$H(r) = 0 \ \forall \ r \ \Rightarrow$ GR - de Sitter

$H(r) = v \ \forall \ r, \ \xi = 0 \ \Rightarrow$ Asympt. Flat

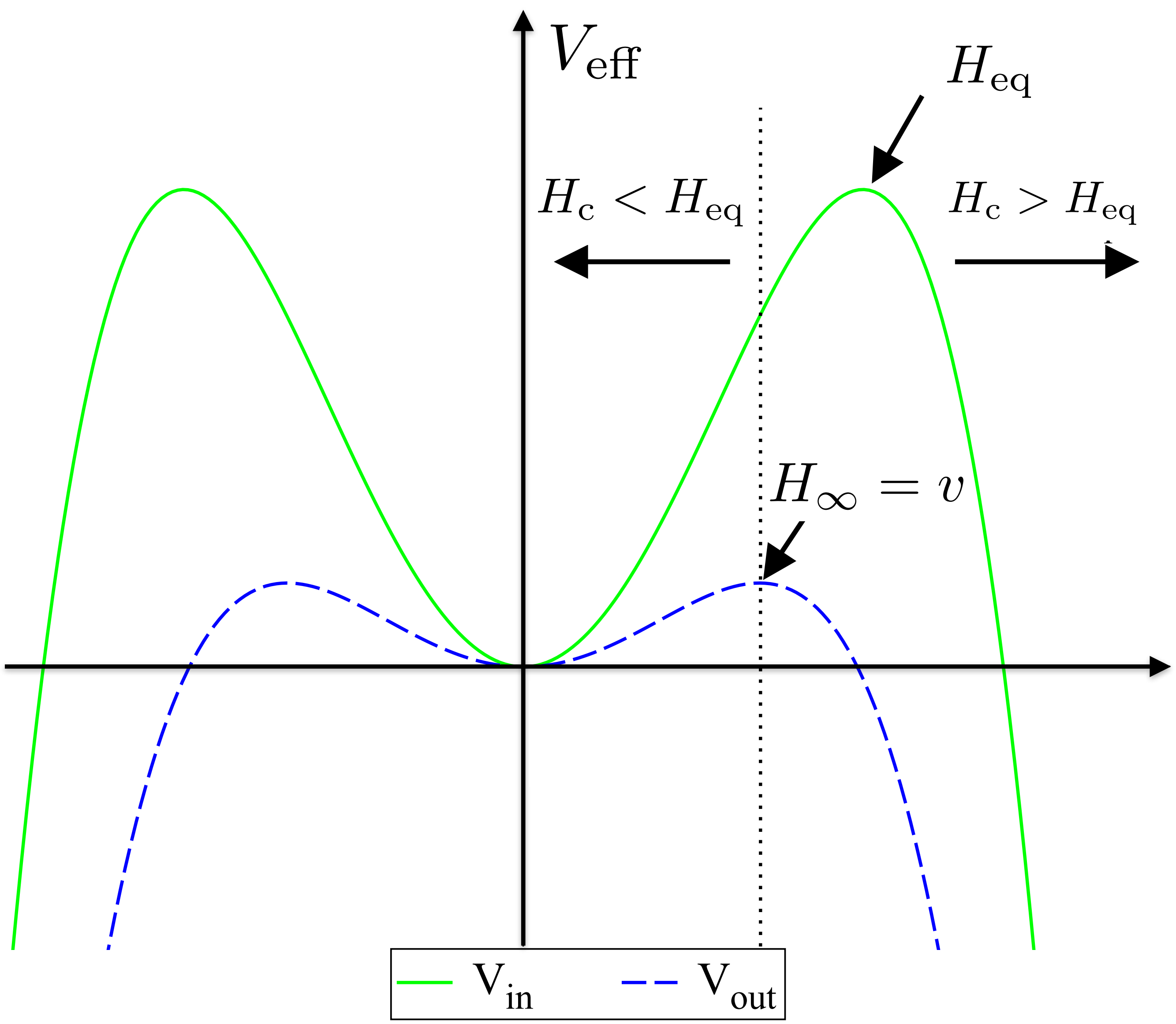
$H(r) \neq 0, v, \ \xi \neq 0 \ \Rightarrow$ GR - Flat ??

Effective potential $(-,+,+,+)$

$$\frac{d^2H}{dr^2} \simeq -\frac{dV_{\text{eff}}}{dH}$$

$$V_{\text{eff}} \simeq -\frac{\lambda}{4}(H^2 - v^2)^2 + \frac{\xi R H^2}{16\pi}$$

Vanishes (almost) outside or for minimal coupling



$H_c =$ central value

$H_{\text{eq}} =$ equilibrium points

The monopole smoothly interpolates between H_{eq} and H_∞ with binding energy:

$$E_{\text{bin}} = E_{\text{bar}} - E_{\text{ADM}} > 0$$

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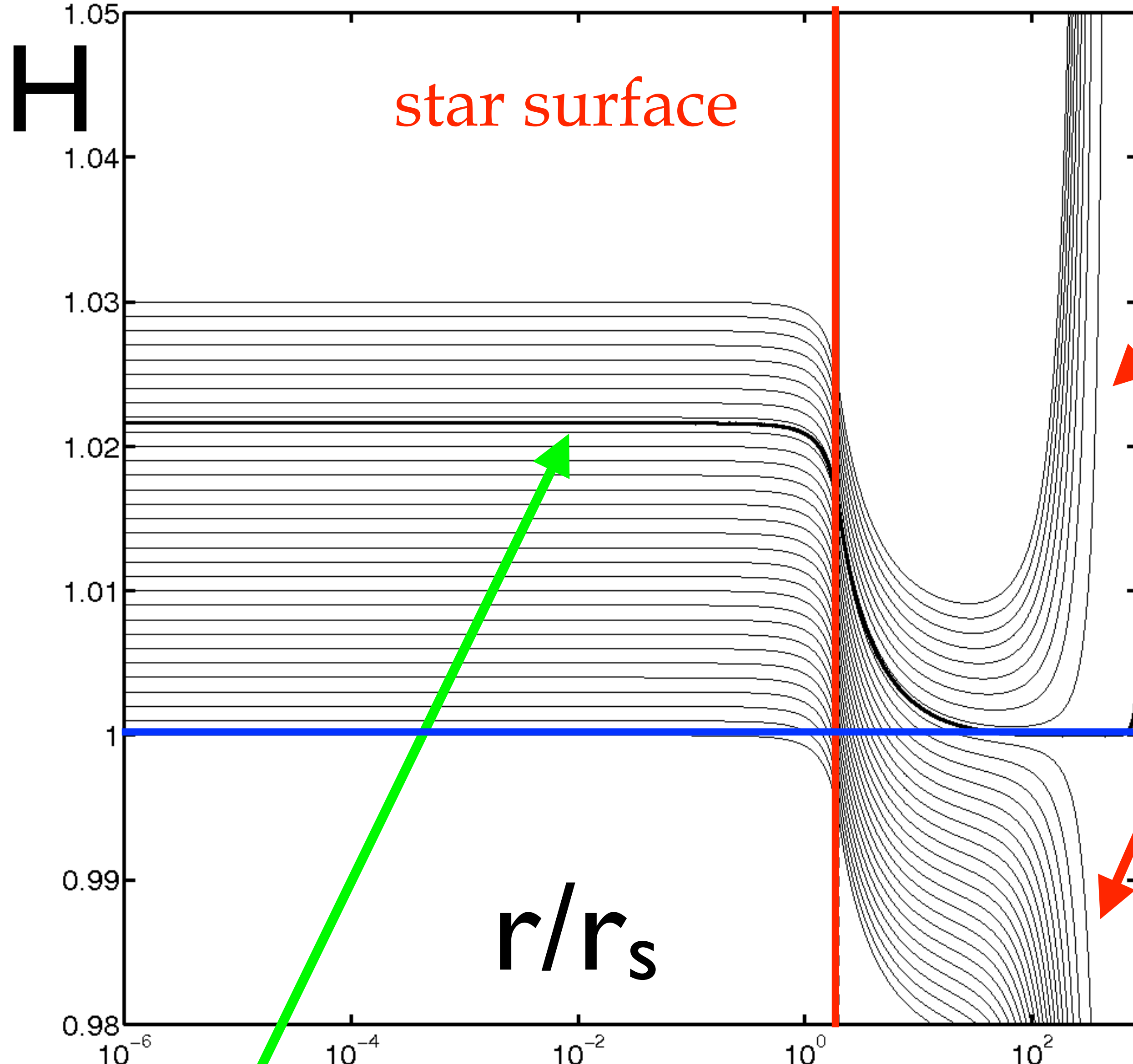
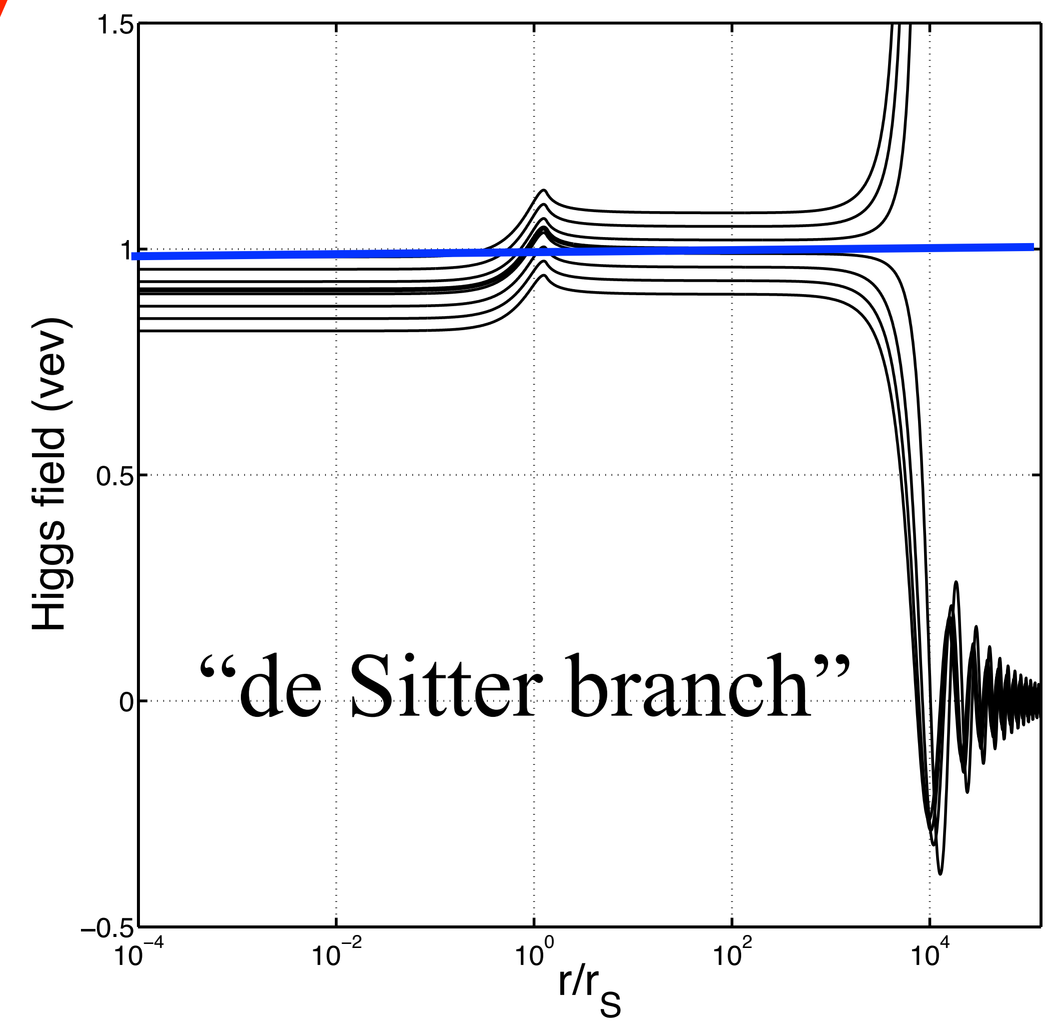
Numerical analysis: shooting method



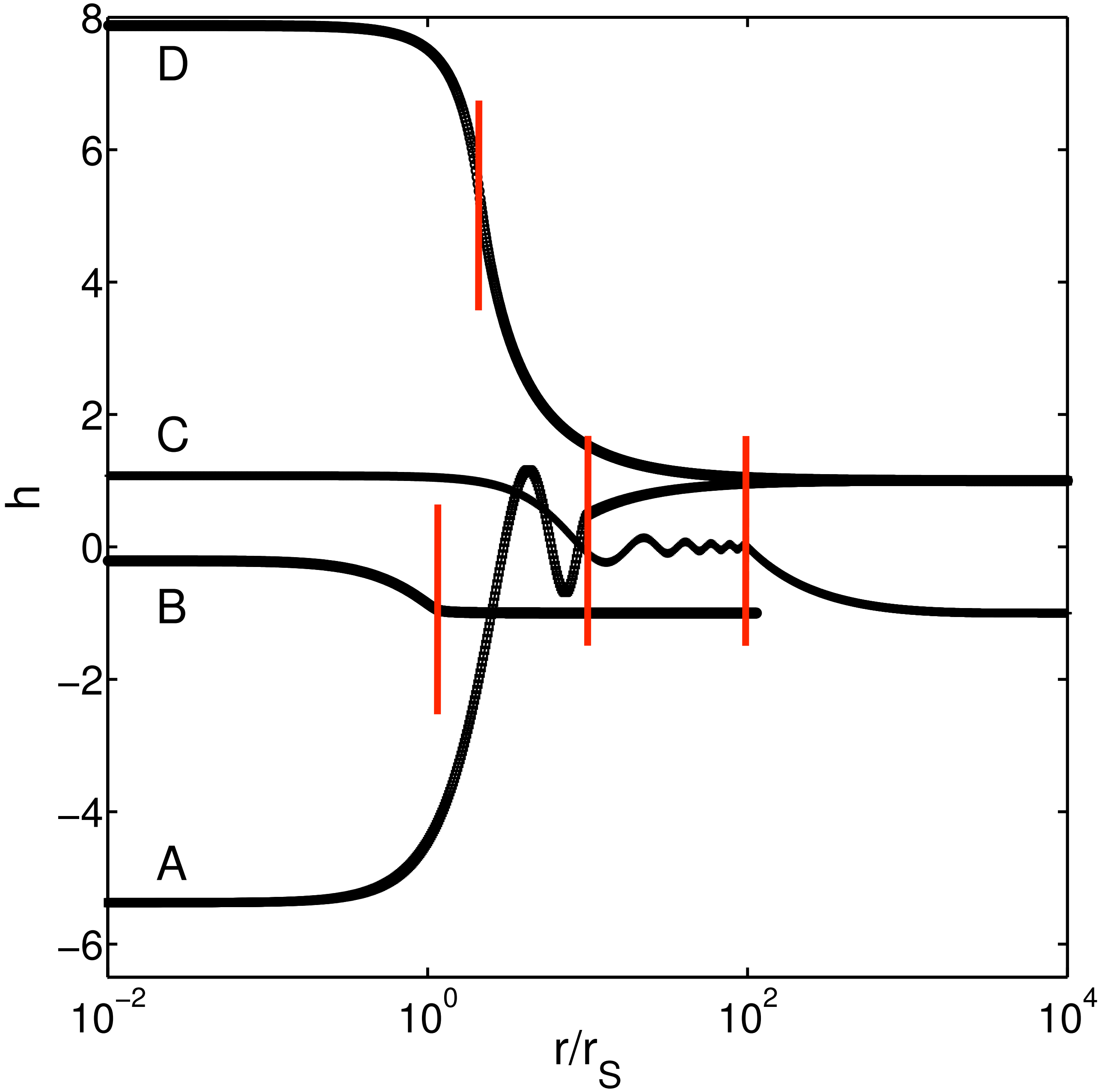
Unstable solutions

Monopole

(fixed mass and radius)



Monopole family



<u>Physical parameters:</u>

m = baryonic mass
s = compactness
ξ = non-minimal coupling
hc = central Higgs field value

	h_c	ξ	m	s
F	0.91	10	10^6 kg	0.75
A	- 5.37	10^4	10^3 kg	0.1
B	- 0.21	10	10^6 kg	0.88
C	1.077	10^6	10^6 kg	0.01
D	7.88	60	10^4 kg	0.47

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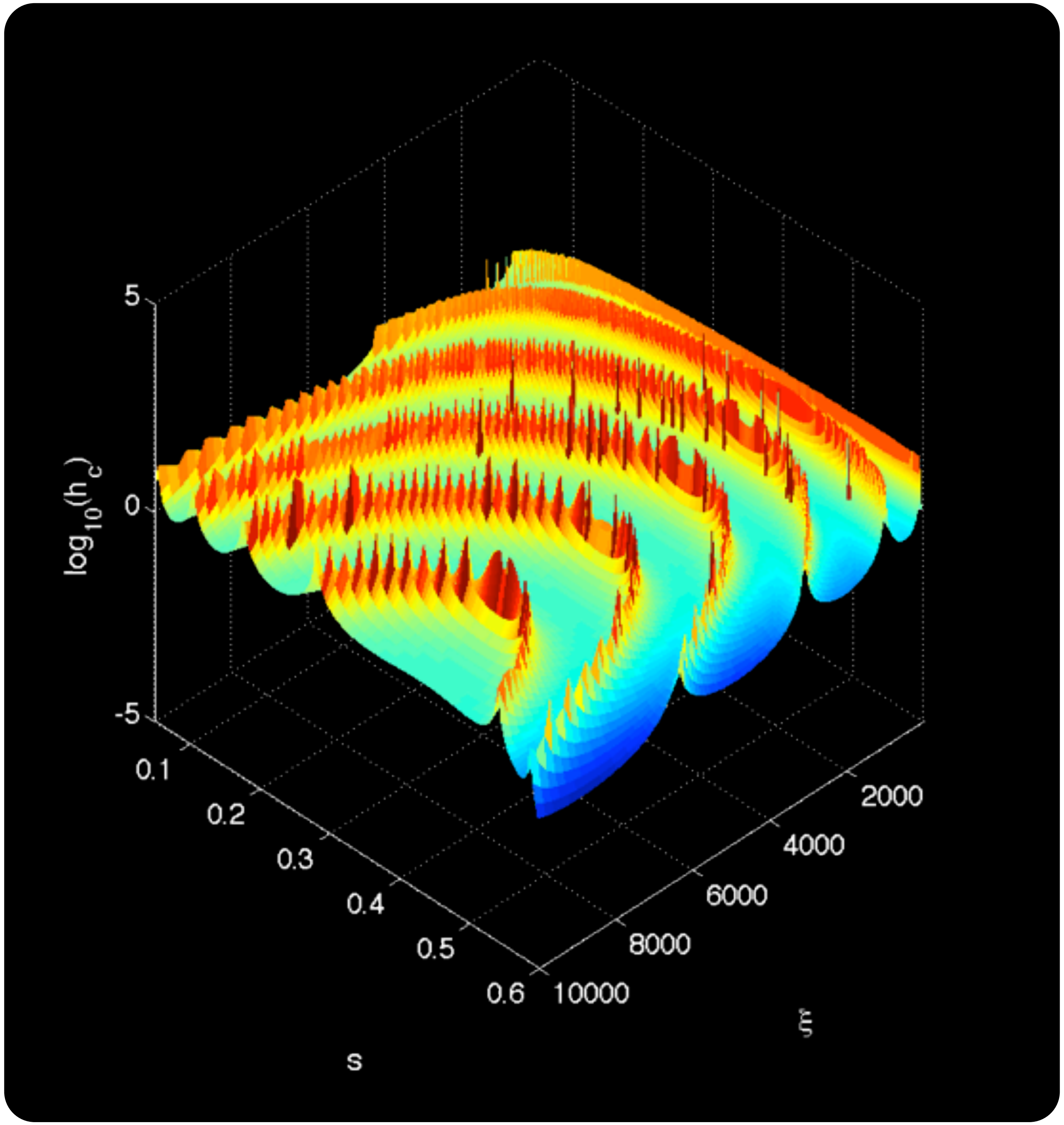
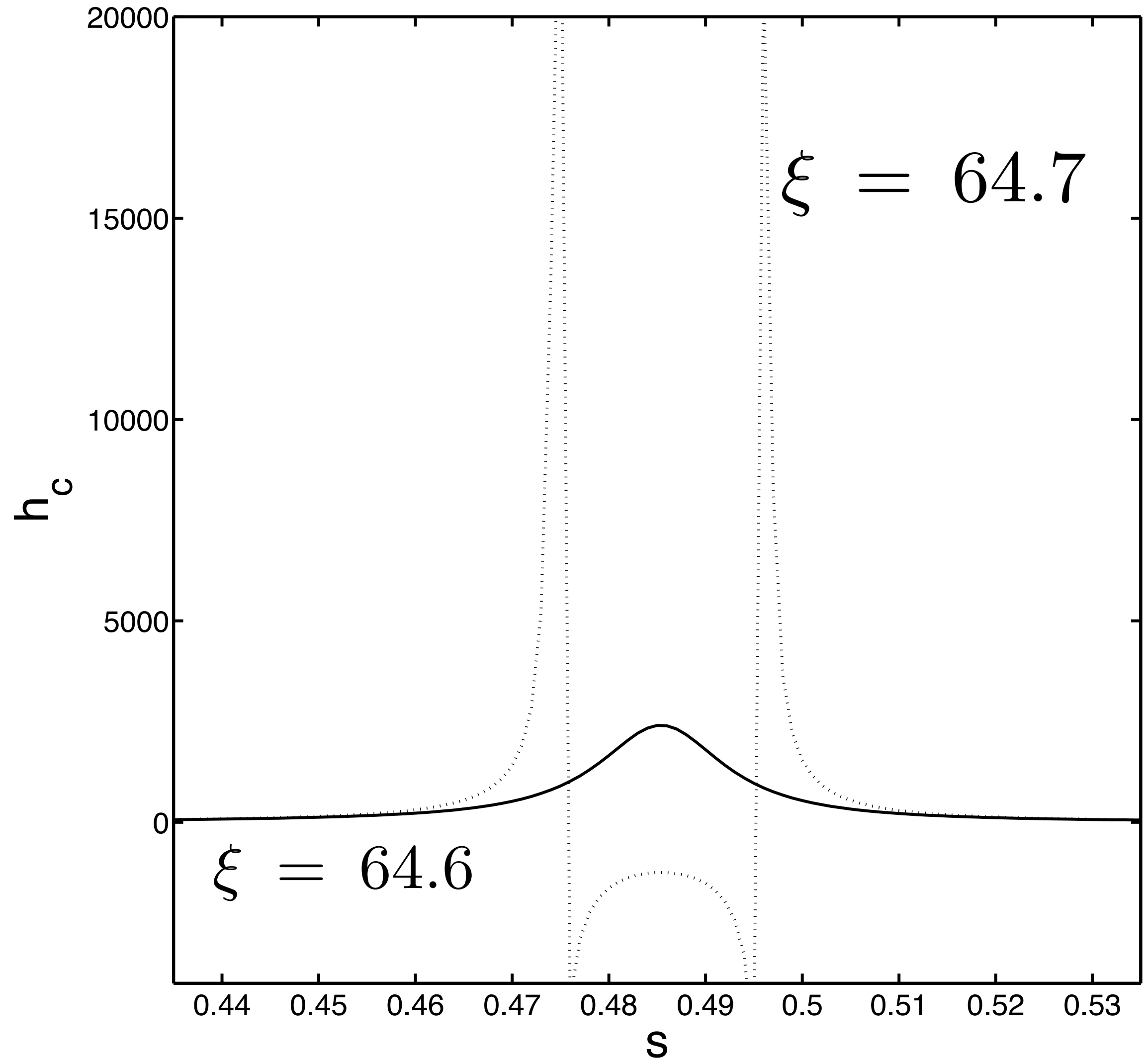
Higgs amplification mechanism



For large nonminimal coupling and for some combination of mass and radius **the Higgs field diverges**!



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Higgs amplification mechanism

Analytic approximation and modeling of the amplification mechanism:

- Inner solution given by pure GR (excellent approx for small compactness *R*/*rs*)
- Exterior solution governed by Higgs field only $$H_{\rm ext} \sim \frac{Q}{r} e^{-r/L}$$ Yukawa form
- Continuity of *H* and *H'* at the boundary

Resonances determined by the implicit equation:

$$\left(H_c + \frac{B}{A}\right)\left[\sqrt{\frac{\alpha}{|A|}} \sin\left(\frac{\sqrt{|A|}}{s}\right) + \cos\left(\frac{\sqrt{|A|}}{s}\right)\right] = \left(1 + \frac{B}{A}\right)\left(1 + \frac{\sqrt{\alpha}}{s}\right)$$

$$A = \frac{\alpha}{2}\left(3H_c^2 - 1\right) - \frac{\langle R\rangle \xi}{8\pi}$$

$$B(H_c) = -\alpha H_c^3$$

$$\alpha = 2\lambda_{\rm sm} r_s^2 m_{pl}^2 \tilde{v}^2$$

$$H_c = \left|\cos\sqrt{\frac{\xi\langle R\rangle}{8\pi s^2}}\right|^{-1}$$

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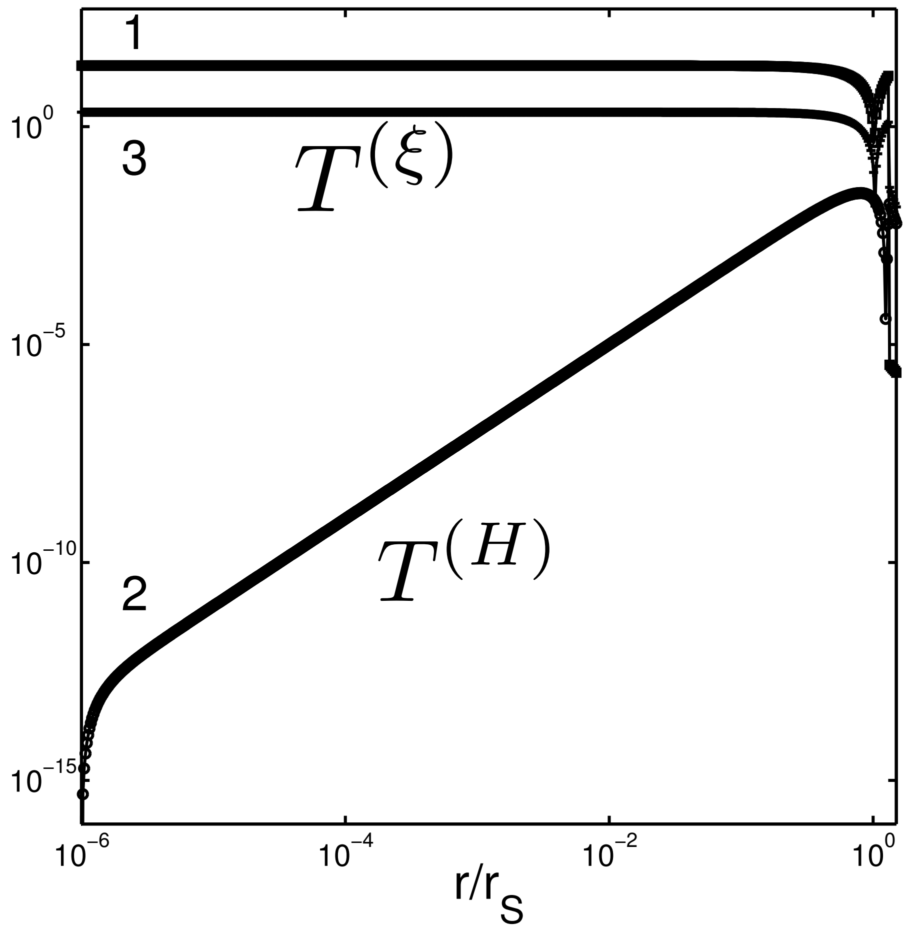
Higgs amplification mechanism

- Note that the amplification mechanism is different from **spontaneous scalarization:** it depends only on the scalar potential (absent in previous works)
- **Spontaneous scalarization** is greatly reduced because of the potential
- The **analytic approximation** is very accurate for small compactness

$$\left(1+\frac{\xi}{m_p^2}H^2\right)G_{\mu\nu}=\kappa\left[T_{\mu\nu}^{(H)}+T_{\mu\nu}^{(\xi)}+T_{\mu\nu}^{(mat)}\right]$$

Single contributions inside the body

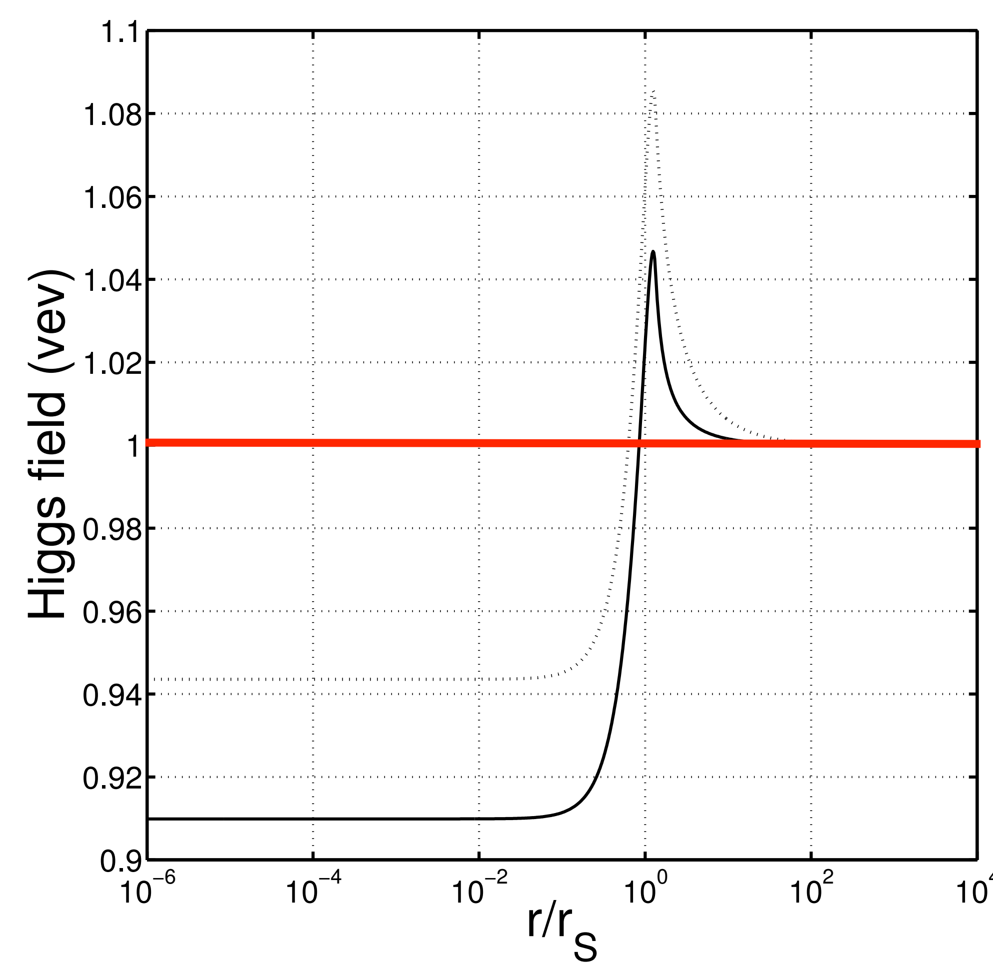


$$s=0.75$$

$$\xi=10$$

$$m=10^6\,kg$$

Full numerical vs approximation



Other effects

- **PPN analysis** reveal negligible deviations
- **TOV equation**: solve the equations near the center with the approximation

$$H(r) \simeq H_0 + H_2 r^2$$

- We find that the baryonic energy density must satisfy:

$$V(H_c) < \rho_{\rm bar} < \rho_{\rm max}$$

- Recall that in GR:

$$0 < \rho_{\rm bar} < \rho_{\rm max}$$

- Is this preventing initial collapse?

Higgs Monopoles: summary

• Higgs gravity yields new particle-like, non singular, spherically symmetric, and stable solutions

• Spontaneous scalarization greatly reduced (sort of screening mechanism)

• New **general amplification mechanism** of the central value of the Higgs field

Open issues

• Effects of SU(2) structure of the theory

• Effects on the equation of state of a varying Higgs effective vacuum

• Formation mechanism and stability of these objects: dark matter?

• Interaction with other particles and stability wrt to small fluctuations

The dark aftermath of Higgs inflation

It does not matter how slowly you go
as long as you do not stop
(Confucius)

MR, arXiv: 1309.7332 - EPJ Plus 129 56
MR, arXiv: 1404.0532

Higgs-Einstein-Hilbert SU(2) Lagrangian (J-frame)

$$\frac{\mathcal{L}_J}{\sqrt{g}} = \frac{1}{2}(m^2 + 2\xi \mathcal{H}^\dagger \mathcal{H})R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{F^2}{4} - \lambda \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2} \right)^2$$

Low energy SU(2) Lagrangian (J-frame=E-frame)

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_p^2}{2} R - (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \frac{F^2}{4} - \lambda \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v^2}{2} \right)^2$$

FLRW metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

Can we impose the unitary gauge?

The **unitary gauge** is NOT compatible with **FLRW**:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T^{(H)}_{\mu\nu} + T^{(A)}_{\mu\nu} + \dots$$

The stress tensor of **massive gauge fields is not diagonal**.
For example U(1) gauge field we must impose

$$T^{(A)}_{12} = T^{(A)}_{13} = T^{(A)}_{23} = 0$$

Unbroken symmetry: 1+4 = 5 DOF
Broken + U-gauge: 1+3+1 - 3 = 2 DOF

All background fields are physical, including the Goldstone bosons!
E.g. **multifield Higgs inflation**, see Kaiser et. al.

We can choose a "diagonal" gauge:

$$A^b{}_0 = 0, \qquad A^b{}_i = \delta^b{}_i\, f(t)$$

Galtsov and Volkov
PLB 256,17 1991

Equations of motion:

$$f^4 + 2a^2 \dot{f}^2 = K_f$$

gauge contribution, radiation like

$$\dot{H} = -\frac{1}{2M_p^2}\left[\dot{\mathcal{H}}^\dagger \dot{\mathcal{H}} + \frac{K_f}{a^4} + \rho(1+\omega)\right]$$

$$H^2 = \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\mathcal{H}}^\dagger \dot{\mathcal{H}} + V + \frac{3K_f}{4a^4} + \rho\right]$$

Friedmann equations

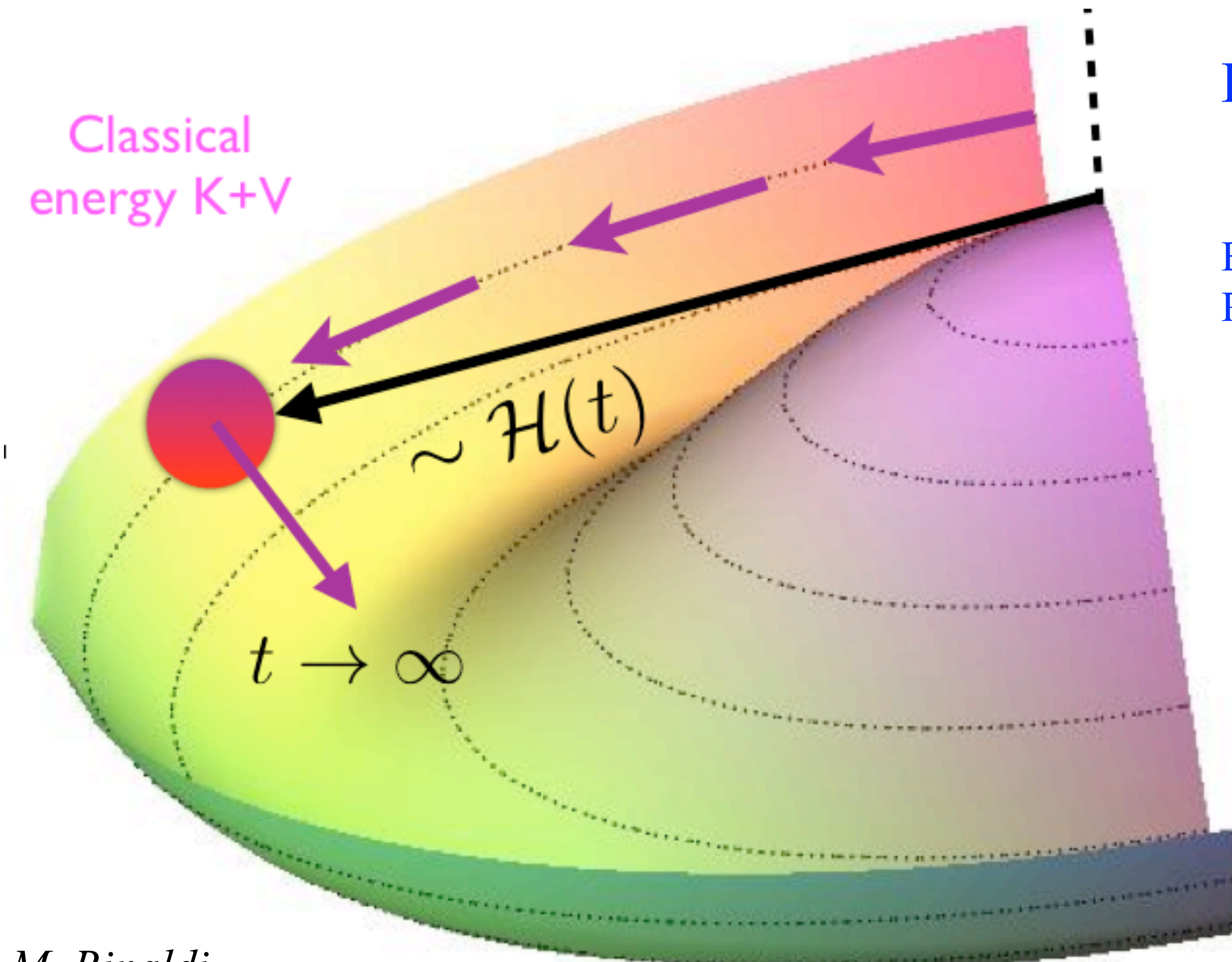
$$\boxed{\mathcal{H}^\dagger \dot{\mathcal{H}} - \dot{\mathcal{H}}^\dagger \mathcal{H} = \frac{Q}{a^3}}$$

SU(2) current conservation; it gives cosmic acceleration.

Dark energy - deceleration parameter:

$$q = -1 - \frac{\dot{H}}{H^2} \simeq \frac{2\dot{\mathcal{H}}^\dagger \dot{\mathcal{H}} - 2V}{\dot{\mathcal{H}}^\dagger \dot{\mathcal{H}} + 2V}$$

$$\boxed{\mathcal{H}^\dagger \dot{\mathcal{H}} - \dot{\mathcal{H}}^\dagger \mathcal{H} = \frac{Q}{a^3}}$$



Dynamics very similar to "Spintessence"

Boyle, Caldwell, Kamionkowski, Phys. Lett. B 545 (2002) 17.

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Abelian U(1) case

$$H(t) = \chi(t)\, e^{i\theta(t)}$$

Cosmological equations

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\chi}^2}{2} + \frac{\mathcal{Q}^2}{2\chi^2 a^6} + V + \rho_m\right),$$

$$\dot{H} = -\frac{\kappa^2}{2}\left[\dot{\chi}^2 + \frac{\mathcal{Q}^2}{\chi^2 a^6} + \rho_m(1+\omega_m)\right],$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\mathcal{Q}^2}{\chi^3 a^6} + \frac{dV}{d\chi} = 0,$$

$$\dot{\rho}_m + 3H\rho_m(\omega_m + 1) = 0,$$

"charge" $\dot{\theta} = \frac{\mathcal{Q}}{\chi^2 a^3}.$

Dynamical system

$$\frac{dx}{dN} = -3x + \frac{z^2}{w} + \sqrt{\frac{3}{2}}Ly^2 + \frac{3}{2}xP,$$

$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}}Lxy + \frac{3}{2}yP,$$

$$\frac{dz}{dN} = -3z - \frac{zx}{w} + \frac{3}{2}zP,$$

$$\frac{dw}{dN} = x,$$

$$\frac{\kappa^2\rho_m}{3H^2} = 1 - x^2 - y^2 - z^2,$$ Hamiltonian constraint

where: $N = \ln a,\quad x = \kappa\dot{\chi}/(\sqrt{6}H),\quad y = \kappa\sqrt{V}/(\sqrt{3}H),$

$z = \kappa\mathcal{Q}/(\sqrt{6}\chi a^3 H),\quad w = \kappa\chi/\sqrt{6},\quad L = -(1/\kappa)(d\ln V/d\chi)$

$P = (1-\omega_m)(x^2+z^2) + (1+\omega_m)(1-y^2)$

$$\omega_\chi = \frac{x^2+z^2-y^2}{x^2+z^2+y^2}$$

DE equation of state

$$q = -1 + \frac{3}{2}(x^2+y^2)(1-\omega_m) - \frac{3}{2}(1+\omega_m)y^2$$

deceleration parameter

Cosmological equations

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\chi}^2}{2} + \frac{\mathcal{Q}^2}{2\chi^2 a^6} + V + \rho_m\right),$$

$$\dot{H} = -\frac{\kappa^2}{2}\left[\dot{\chi}^2 + \frac{\mathcal{Q}^2}{\chi^2 a^6} + \rho_m(1+\omega_m)\right],$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\mathcal{Q}^2}{\chi^3 a^6} + \frac{dV}{d\chi} = 0,$$

$$\dot{\rho}_m + 3H\rho_m(\omega_m + 1) = 0,$$

"charge" $\dot{\theta} = \dfrac{\mathcal{Q}}{\chi^2 a^3}.$

Dynamical system

$$\frac{dx}{dN} = -3x + \frac{z^2}{w} + \sqrt{\frac{3}{2}}Ly^2 + \frac{3}{2}xP,$$

$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}}Lxy + \frac{3}{2}yP,$$

$$\frac{dz}{dN} = -3z - \frac{zx}{w} + \frac{3}{2}zP,$$

$$\frac{dw}{dN} = x,$$

Only one (saddle) fixed point!

$$\frac{\kappa^2\rho_m}{3H^2} = 1 - x^2 - y^2 - z^2,$$

Hamiltonian constraint

where: $N = \ln a$, $x = \kappa\dot{\chi}/(\sqrt{6}H)$, $y = \kappa\sqrt{V}/(\sqrt{3}H)$,

$z = \kappa\mathcal{Q}/(\sqrt{6}\chi a^3 H)$, $w = \kappa\chi/\sqrt{6}$, $L = -(1/\kappa)(d\ln V/d\chi)$

$P = (1-\omega_m)(x^2+z^2) + (1+\omega_m)(1-y^2)$

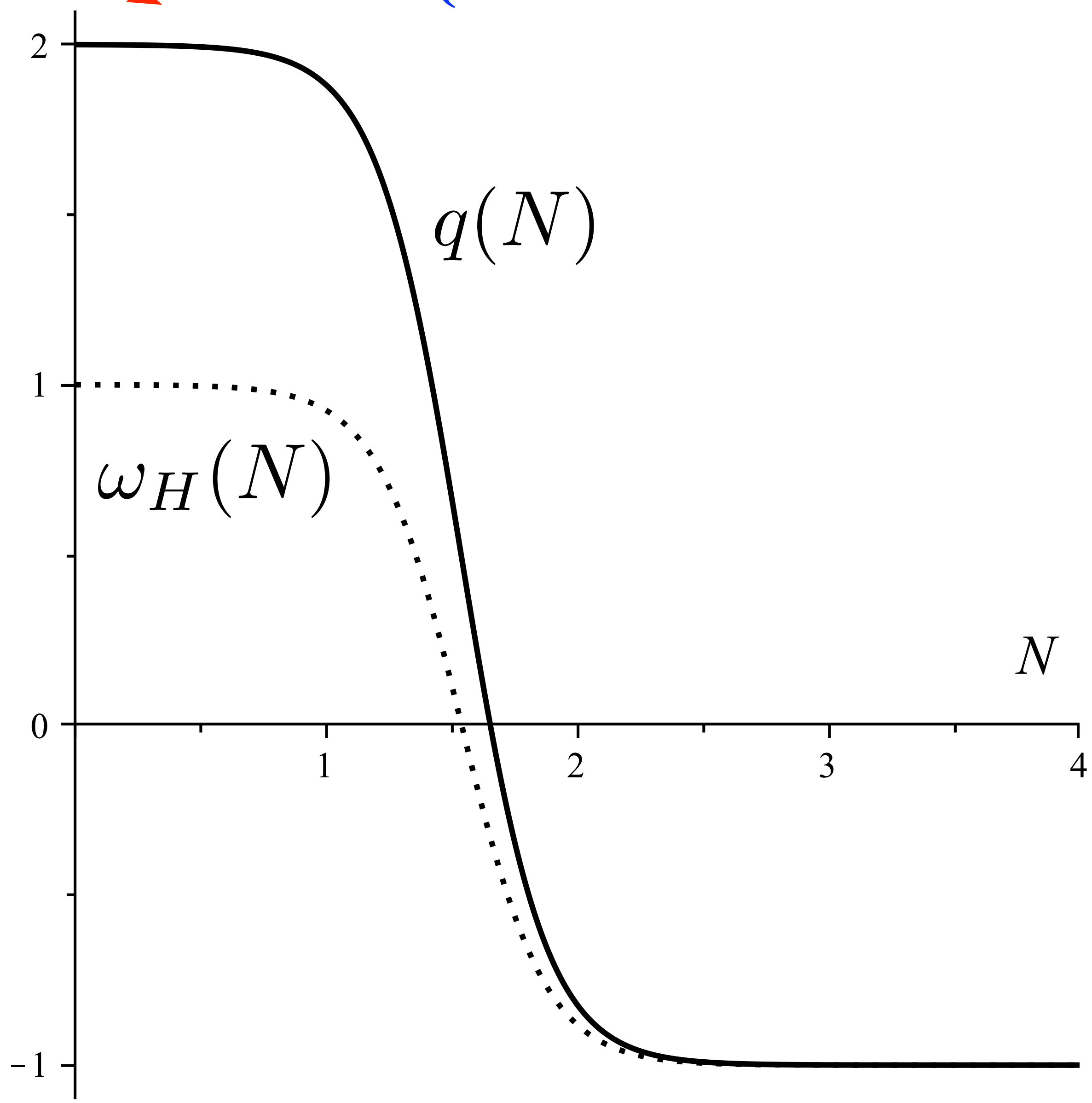
$$\omega_\chi = \frac{x^2+z^2-y^2}{x^2+z^2+y^2}$$

DE equation of state

$$q = -1 + \frac{3}{2}(x^2+y^2)(1-\omega_m) - \frac{3}{2}(1+\omega_m)y^2$$

deceleration parameter

Unique, unstable fixed point (radiation/matter dominated Universe)



By chasing its vacuum the Higgs makes the Universe accelerate

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SU(2) case
Work in progress, stay tuned.

Higgs dark energy: summary

- The Goldstone components of the **background Higgs** play a dynamical role
- During inflation they are negligible (see Kaiser et al)
- At late times the effects become important
- In the simplified U(1) case there is a **dominant dark energy** era in the future

Open issues

- As in quintessence, there are instabilities that maybe lead to Q-balls nucleation: **dark matter**?
- We expect SU(2) to behave as U(1). Dynamical analysis in progress.
- Fitting the data should constrain the model.

Thank you.

Extra material

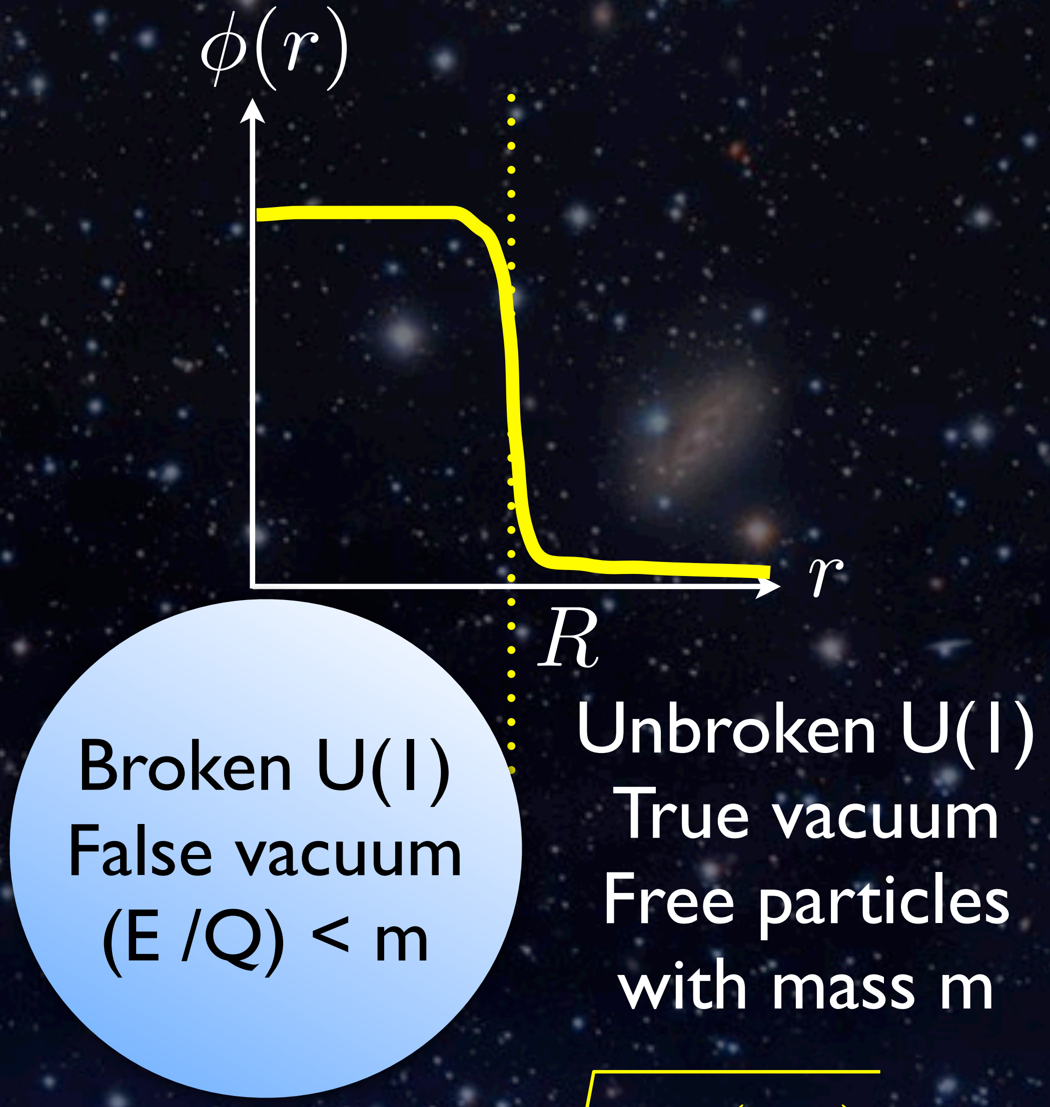
Gravitational collapse into Q-balls

A theory of two scalar field with global SO(2) symmetry can develop stable, non-topological solitonic solution, provided $\min[V(\phi)/\phi^2]$ is at point $\phi_0 \neq 0$.

S. R. Coleman, Nucl. Phys. B 262 (1985) 263

The same happens in gauged U(1) theories, provided the charge and the coupling are not too large (superconducting Q-balls).

K. -M. Lee, J. A. Stein-Schabes, R. Watkins
L. M. Widrow, Phys. Rev. D 39 (1989) 1665



$$E = Q\sqrt{\frac{2V(\phi_0)}{\phi_0^2}} < m$$

$$Q = \omega\phi_0^2 V(\phi_0)$$

These solutions are valid in flat space but can form also in curved space!

Metric and field perturbations

$$\begin{cases} ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)a^2\delta_{ij}dx^i dx^j \\ \chi(t) \to \chi(t) + \delta\chi(t,\vec{x}) \\ \theta(t) \to \theta(t) + \delta\theta(t,\vec{x}) \end{cases}$$

Perturbed system

$$\ddot{\delta\chi} + 3H\dot{\delta\chi} + \left(V'' - \dot{\theta}^2 - a^{-2}\nabla^2\right)\delta\chi =$$

$$4\dot{\chi}\dot{\Phi} - 2\Phi V' + 2\chi\dot{\theta}\dot{\delta\theta},$$

$$\ddot{\delta\theta} + 3H\dot{\delta\theta} - a^{-2}\nabla^2\delta\theta =$$

$$4\dot{\theta}\dot{\Phi} - 2\frac{\dot{\delta\chi}}{\chi}\dot{\theta} + 2\frac{\dot{\chi}}{\chi}\left(\frac{\delta\chi}{\chi}\dot{\theta} - \dot{\delta\theta}\right),$$

$$a^{-2}\nabla^2\Phi - 3H\dot{\Phi} - 3H^2\Phi =$$

$$\frac{\kappa^2}{2}\left[\dot{\chi}\dot{\delta\chi} + V'\delta\chi + \chi^2\dot{\theta}\dot{\delta\theta} + \chi\dot{\theta}^2\delta\chi - \Phi(\dot{\chi}^2 + \chi^2\dot{\theta}^2)\right]$$

$$k < k_J \;\Rightarrow\; \omega^2 > 0$$

Instability and collapse

Ansatz

$$\begin{cases} \delta\chi = \delta\chi_0 \exp(\omega t + i\vec{k}\cdot\vec{x}) \\ \delta\theta = \delta\theta_0 \exp(\omega t + i\vec{k}\cdot\vec{x}) \end{cases}$$

Jeans critical value

$$k_J^2 \simeq 8\lambda v^2\kappa^2(\chi^2 - v^2) + \mathcal{O}((\chi^2 - v^2)^2)$$

$\dfrac{V(\chi)}{\chi^2}$ has a minimum at $\chi \neq 0$ $\Rightarrow$ necessary condition for Q-balls