Effective theory for quantum gravity and inflation

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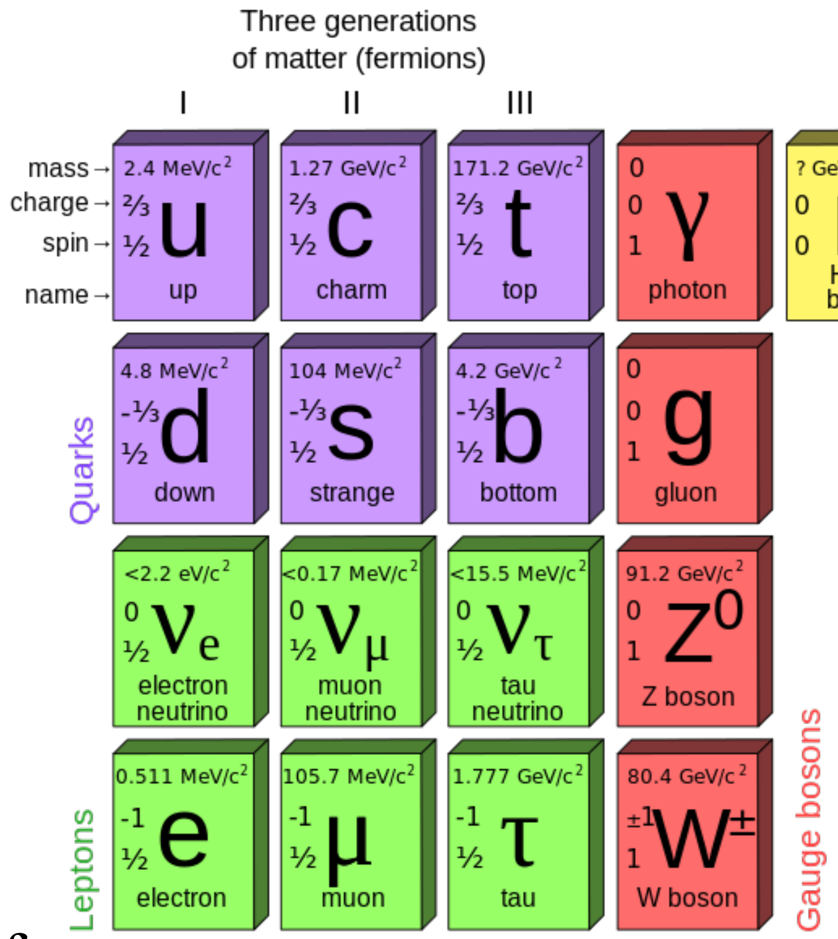
Introduction

- Quantum gravity is tough because of the lack of experimental guidance.
- It is crucial to find a way to make the link to data.
- Effective field theory techniques are a conservative tools to probe poorly known theories.
- In particular we can probe the symmetries of quantum gravity:
  - is there an approximate shift symmetry which prevents these higher dimensional operators?
  - Are Lorentz invariance and CPT invariance valid symmetries at the Planck scale?
- Inflation might helps us to probe whether space-time is quantized and also whether general relativity is a purely classical theory or whether it needs to be quantized.

Outline

- Definition of the effective theory for quantum gravity
- Application of this effective action to Higgs inflation
- Could Higgs inflation be a complete model of the world?
- Quantum gravity and chaotic inflation and ϕ^4 inflation after BICEP2
- Quantum gravity and Grand Unification.

The standard model is finally complete!



$m_H \approx 125$ GeV

However no sign of new physics

So let's try to be minimalist!

Effective theory approach

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques.
- Making simple assumptions about the symmetry and particle content we get

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Let's try to explain as much of our universe with this as possible.
  - Collider physics (e.g. LHC) ✔
  - Expansion of the universe ✔
  - Dark matter (right handed neutrinos or Planck size primordial BHs) ✔?
  - Inflation ?
    - Starobinsky inflation
    - Higgs inflation

Quick review of Higgs inflation

- Since we know of one scalar field in nature it is natural to try to describe inflation with it.

- The SM Higgs potential

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

  is not flat enough!
- But a nonminimal coupling will change the shape of the potential

$$S = -\int d^4x \sqrt{-g} \left(\frac{1}{2} M^2 + \xi H^\dagger H \right) R$$

Quick review of Higgs inflation

- In Einstein frame the action becomes

$$S_E = \int d^4x\sqrt{-\hat{g}}\Big\{-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi)\Big\}$$

- with

$$U(\chi) = \frac{1}{\Omega(\chi)^4}\frac{\lambda}{4}\left(h(\chi)^2 - v^2\right)^2$$

- For small Higgs values $h \simeq \chi$ and $\Omega^2 \simeq 1$

the potential is the same as for the initial Higgs one, however for large field values $h \gg M_P/\sqrt{\xi}$

$$h \simeq \frac{M_P}{\sqrt{\xi}}\exp\left(\frac{\chi}{\sqrt{6}M_P}\right)$$

i.e. the potential is exponentially flat

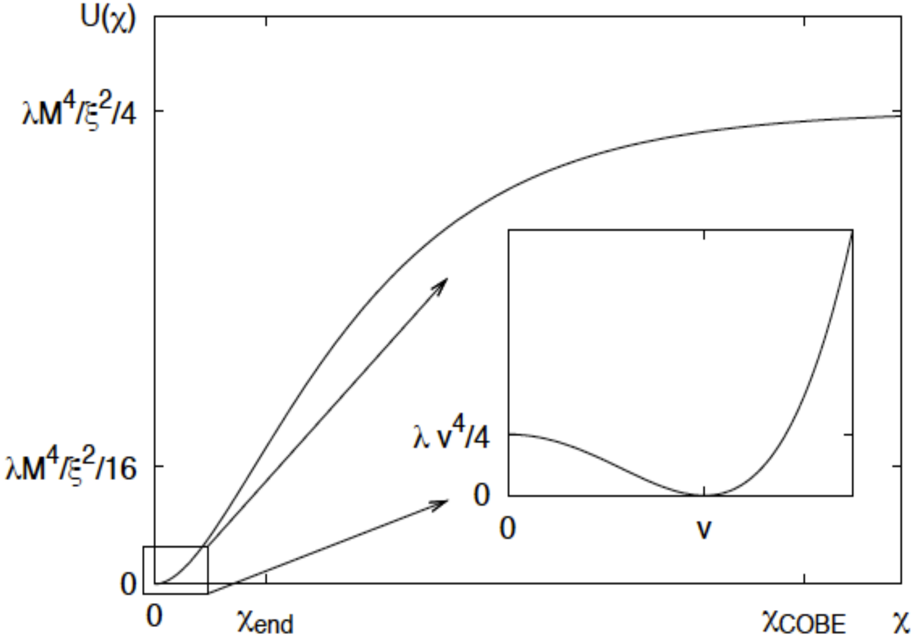


Fig. 1. Effective potential in the Einstein frame.

From 0710.3755
(Bezrukov&Shaposhnikov)

Standard analysis, slow role parameters:

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4M_P^4}{3\xi^2 h^4} ,$$

$$\eta = M_P^2 \frac{d^2U/d\chi^2}{U} \simeq -\frac{4M_P^2}{3\xi h^2} ,$$

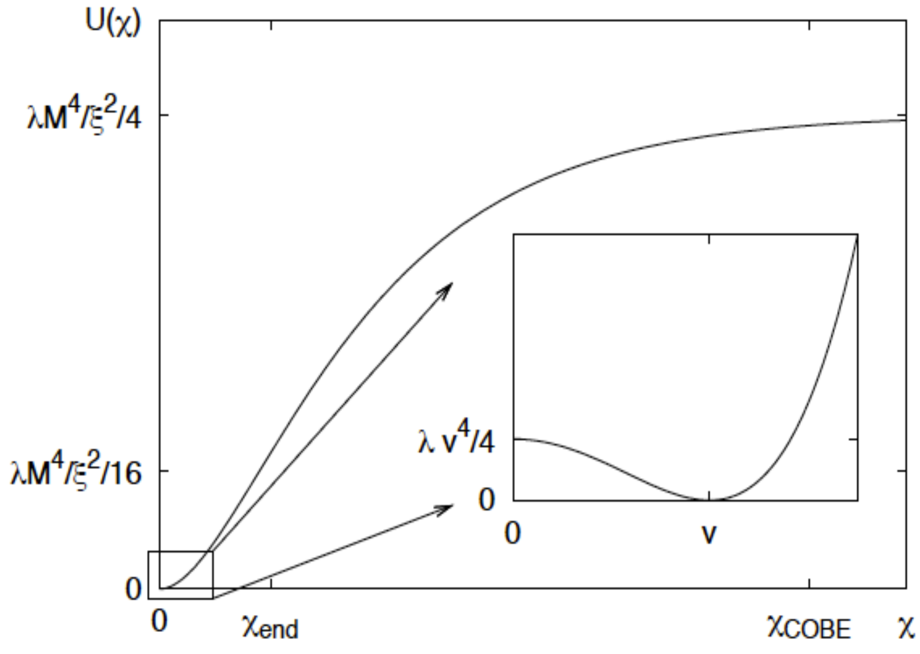
$$\zeta^2 = M_P^4 \frac{(d^3U/d\chi^3)dU/d\chi}{U^2} \simeq \frac{16M_P^4}{9\xi^2 h^4} .$$

Number of e-foldings:

$$N = \int_{h_{\rm end}}^{h_0} \frac{1}{M_P^2}\frac{U}{dU/dh}\left(\frac{d\chi}{dh}\right)^2 dh \simeq \frac{6}{8}\frac{h_0^2 - h_{\rm end}^2}{M_P^2/\xi}$$

$$\xi \simeq \sqrt{\frac{\lambda}{3}}\frac{N_{\rm COBE}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

$$\xi \sim 10^4$$



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$$\zeta^2 = M_P^4 \frac{(d^3U/d\chi^3)dU/d\chi}{U^2} \simeq \frac{16M_P^4}{9\xi^2 h^4}.$$

Absence of a second minimum for the potential beyond SM vacuum:

$$m_h > \left(129.5 + 1.8\frac{M_t - 173.2\,\mathrm{GeV}}{0.9\,\mathrm{GeV}} - 0.6\frac{\alpha_s - 0.1184}{0.0007} \pm 2\right)\mathrm{GeV}$$

What do we know about ξ?

- Let's consider the SM with a nonminimal coupling to R

$$S = -\int d^4x\,\sqrt{-g}\left[\left(\frac{1}{2}M^2 + \xi H^\dagger H\right) R - (D^\mu H)^\dagger (D_\mu H) + \mathcal{L}_{SM} + \mathcal{O}(M_P^{-2})\right]$$

- We can always go from the Jordan frame to the Einstein frame

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}\,, \quad \sqrt{-\tilde{g}} = \Omega^d \sqrt{-g}\,.$$

$$R = \Omega^2\left[\tilde{R} - 2(n-1)\tilde{\Box}\omega - (n-1)(n-2)\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right]$$

$$\omega \equiv \ln\Omega\,, \quad \tilde{\Box}\omega = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\omega)$$

$$\Omega^2 = (M^2 + 2\xi H^\dagger H)/M_P^2$$

What do we know about ξ?

- In the Einstein frame, the action reads

$$S = -\int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} M_P^2 \tilde{R} - \frac{3\xi^2}{M_P^2 \Omega^4} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) - \frac{1}{\Omega^2} (D^\mu H)^\dagger (D_\mu H) + \frac{\mathcal{L}_{SM}}{\Omega^4} \right]$$

- One notices that the Higgs boson kinetic term is not canonically normalized. We need to diagonalize this term.

- Let me now use the unitary gauge

$$H = \frac{1}{\sqrt{2}} (0, h + v)^\top$$

- The Planck mass is defined by

$$(M^2 + \xi v^2) = M_P^2$$

What do we know about ξ?

- To diagonalize the Higgs boson kinetic term:

$$\frac{d\chi}{dh} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2 v^2}{M_P^2 \Omega^4}}$$

- To leading order in Ω^{-1} $\qquad \Omega^2 = (M^2 + 2\xi H^\dagger H)/M_P^2$

$$h = \frac{1}{\sqrt{1+\beta}}\,\chi \qquad \beta = 6\xi^2 v^2 / M_P^2$$

What do we know about ξ?

- The couplings of the Higgs boson to particles of the SM are rescaled! E.g.

$$y h \bar{\psi}\psi \to \frac{y}{\sqrt{1+\beta}} \chi \bar{\psi}\psi$$

- For a large nonminimal coupling, the Higgs boson decouples from the Standard Model:

$$\xi^2 \gg M_P^2/v^2 \simeq 10^{32}$$

What do we know about ξ?

- The decoupling can also be seen in the Jordan frame:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{L}^{(2)} = -\frac{M^2 + \xi v^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h^\rho_\rho - h^\mu_\mu \Box h^\nu_\nu\right)$$
$$+ \frac{1}{2}(\partial_\mu h)^2 + \xi v(\Box h^\mu_\mu - \partial_\mu\partial_\nu h^{\mu\nu})h.$$

$$h = \frac{1}{\sqrt{1+\beta}}\chi,$$
$$h_{\mu\nu} = \frac{1}{M_P}\tilde{h}_{\mu\nu} - \frac{2\xi v}{M_P^2\sqrt{1+\beta}}\bar{g}_{\mu\nu}\chi.$$

same renormalization factor!

LHC Bound on ξ

- The LHC experiments produce fits to the data assuming that all Higgs boson couplings are modified by a single parameter (arXiv:1209.0040 [hep-ph]):

$$\kappa = 1/\sqrt{1+\beta}$$

- In the narrow width approximation, one finds:

$$\begin{aligned}\sigma(ii \to h \to ff) &= \sigma(ii \to h) \cdot \mathrm{BR}(h \to ff) \\ &= \kappa^2\, \sigma_{\mathrm{SM}}(ii \to h) \cdot \mathrm{BR}_{\mathrm{SM}}(h \to ff).\end{aligned}$$

LHC Bound on ξ

- Current LHC data allows to bound

$$\mu = \sigma/\sigma_{\rm SM} = \quad 1.4 \pm 0.3 \qquad \text{ATLAS}$$

$$0.87 \pm 0.23 \qquad \text{CMS}$$

- Combining these two bounds one gets:

$$\mu = 1.07 \pm 0.18$$

- which excludes

$$|\xi| > 2.6 \times 10^{15} \text{ at the 95\% C.L.}$$

Atkins & xc, PRL 110 (2013) 051301

LHC Bound on ξ

- At a 14 TeV LHC with an integrated luminosity of 300 fb^{-1}, could lead to an improved bound on the nonminimal coupling:

$$|\xi| < 1.6 \times 10^{15}$$

- while an ILC with a center of mass energy of 500 GeV and an integrated luminosity of 500 fb^{-1}, could give

$$|\xi| < 4 \times 10^{14}$$

- It seems tough to push the bound below this limit within the foreseeable future.

Up to what energy scale is Higgs inflation valid?
Effective theory approach

Cutoff for the model

- Using the linearized fields, one obtains the kinetic terms for the graviton, the Higgs boson and its pseudo-Goldstone bosons:

$$
\begin{aligned}
L^{(2)} \;=\; & -\frac{M_P(\bar{\phi})^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h - h\Box h\right) \\
& +\frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + \frac{1}{2}\left(\partial_\mu \pi^0\right)\left(\partial^\mu \pi^0\right) + \left(\partial_\mu \pi^+\right)\left(\partial^\mu \pi^-\right) \\
& +\xi\bar{\phi}\left(\Box h - \partial_\lambda \partial_\rho h^{\lambda\rho}\right)\phi \;,
\end{aligned}
$$

- We see that the pseudo-Goldstone bosons are canonically normalized, but there is a mixing between the kinetic terms of the graviton and that of the Higgs boson

Cutoff for the model

- Diagonalizing this term leads to a rescaling of the couplings of the Higgs boson to all particles of the standard model.

- Also to a rescaling of the non-minimal coupling to R

$$\frac{\xi}{M_P(\bar{\phi})\left(1+\frac{6\xi^2\bar{\phi}^2}{M_P^2(\bar{\phi})}\right)}\hat{\phi}^2\Box\hat{h}$$

- The coefficient of this operator can be identified as the cutoff of the effective theory. It behaves as
  - M/ξ for small Higgs background field values
  - $\xi\bar{\phi}^2/M$ for intermediate Higgs background field values
  - $\sqrt{\xi}\bar{\phi}$ for large Higgs background field values

Perturbative Unitarity

- Look at gravitational scattering of the Higgs doublet:

- Expand the amplitude into partial-waves

$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'} \quad |\mathrm{Re}\ a_J| \leq 1/2$$

- From the J=0 partial wave, one gets in today's background, i.e. small Higgs vev, flat spacetime:

$$\Lambda \simeq \bar{M}_P/\xi$$

Perturbative Unitarity

- From the J=0 partial wave, one gets in today's background, i.e. small Higgs vev, flat:

$$\Lambda \simeq \bar{M}_P/\xi$$

- For $\xi=10^4$, unitarity breaks down at 10^{14} GeV
- Do we need new physics? Strong dynamics? Is the potential still flat enough?
- First important observation, the bound is background dependent.
- In inflationary background, one finds

$$\bar{M}_P/\sqrt{\xi}$$

- The tightest bound on ξ is the one obtained in flat space-time and for a small Higgs vev

Self-healing of unitarity

- Perturbative unitarity of the S-matrix implies at one-loop:

$$|T^{\text{tree}}|^2 = \text{Im}\left(T^{1-\text{loop}}\right)$$

- This is called a cutting relation and is a test of unitarity.

- Using partial waves, for the J=0 wave:

$$|a_{0,\text{tree}}|^2 = \text{Im}\left(a_{0,1-\text{loop}}\right)$$

- Let’s verify this relation

Self-healing of unitarity

- We look at the gravitational scattering between the Higgs boson and its pseudo-Goldstone bosons.

- At tree level

$$A_{\text{tree}} = \frac{8\pi G_N(\bar{\phi})}{s}\left[s^2\left(6\xi_1\xi_2 + \xi_1 + \xi_2\right) + ut\right]$$

- where we defined the background dependent Newton's constant:

$$G_N(\bar{\phi}) = \frac{1}{8\pi(M^2 + \xi\bar{\phi}^2)}$$

Self-healing of unitarity

- At one-loop

$$A_{1-\text{loop}}(\xi_A, \xi_B, \xi) = -\frac{G_N^2(\bar{\phi})}{15}\left[s^2 F(\xi_A, \xi_B, \xi) - ut\right] \log(-s)$$

$$\begin{aligned} F(\xi_A, \xi_B, \xi) &= 1 + 10\xi + 5\xi_A + 5\xi_B + 30\xi^2 + 60\xi\xi_A + 60\xi\xi_B + 30\xi_A\xi_B \\ &\quad +180\xi^2\xi_A + 180\xi^2\xi_B + 360\xi\xi_A\xi_B + 1080\xi^2\xi_A\xi_B. \end{aligned}$$

- In the standard model

$$A_{1-\text{loop}}(\phi + \phi \to \pi^i + \pi^i) = A_{1-\text{loop}}(\xi_H, \xi_G, \xi_H) + 3A_{1-\text{loop}}(\xi_H, \xi_G, \xi_G)$$

Self-healing of unitarity

- We next look at the partial wave decomposition and find

$$a_{0,\mathrm{tree,max}} = \frac{G_N(\bar{\phi})\, s}{3}\left(9\xi_H^2 + 3\xi_H + 27\xi_G^2 + 9\xi_G + 1\right)$$

- The Im part of the one-loop diagram is

$$\mathrm{Im}\left(a_{0,1-\mathrm{loop,max}}\right) = \frac{G_N^2(\bar{\phi})\, s^2}{9}\left(9\xi_H^2 + 3\xi_H + 27\xi_G^2 + 9\xi_G + 1\right)^2$$

- And we thus verify the Cutkosky cutting relation implied by unitarity:

$$|a_{0,\mathrm{tree,max}}|^2 = \mathrm{Im}\left(a_{0,1-\mathrm{loop,max}}\right)$$

Self-healing of unitarity

- We have shown that at one-loop unitarity is restored.

- One can rewrite the J=0 partial wave as:

$$a_0 = a_0^{(1)} \left(1 + \frac{\text{Re } a_0^{(2)}}{a_0^{(1)}} + i a_0^{(1)} \right)$$

- It is the first term of a geometrical series. If one resums it, one finds:

$$a_0 = \frac{a_0^{(1)}}{1 - \text{Re } a_0^{(2)}/a_0^{(1)} - i a_0^{(1)}}$$

- Which fulfills

$$|a_0|^2 = \text{Im}(a_0)$$

Self-healing of unitarity

- One can also resum the infinite series of 1-loop polarization diagrams

- 1-loop corrected graviton propagator:

$$iD^{\alpha\beta\mu\nu}_{1-loop} = \frac{i}{2q^2}(1+2F_2(q^2))[L^{\alpha\mu}L^{\beta\nu}+L^{\alpha\nu}L^{\beta\mu}-L^{\alpha\beta}L^{\mu\nu}] - i\frac{F_1(q^2)}{4}L^{\alpha\beta}L^{\mu\nu}$$

- In the large ξ and N limits but keeping N ξG_N small, I get

$$A_{dressed} = \frac{48\pi G_N(\bar{\phi})s\xi^2}{1+\frac{2}{\pi}G_N(\bar{\phi})s\xi^2\log(-s/\mu^2)}$$

- The dressed amplitude fulfills exactly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

XC & Casadio 2014

Bounds on the effective action

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Electroweak symmetry breaking:

$$(M^2 + \xi v^2) = M_P^2 \qquad M_P = 2.4335 \times 10^{18} \text{ GeV}$$

- Several energy scales:
  - $\Lambda_C \sim 10^{-12}$ GeV cosmological constant
  - M_P or equivalently Newton's constant $G = 1/(8\pi M_P^2)$
  - $M_\star$ energy scale up to which one trusts the effective theory

- Dimensionless coupling constants ξ, c_1, c_2 etc

What values to expect for the coefficients?

- It all depends whether they are truly new fundamental constants or whether the operators are induced by quantum gravitational effects.
  - If fundamental constants, they are arbitrary
  - If induced by quantum gravity we can estimate their magnitude.
- Usually induced dimension four operators are expected to be small

$$\exp(-M_P/\lambda)$$

λ is some low energy scale

- However, $\xi H^\dagger H \mathcal{R}$ translates into $\xi H^\dagger H h \Box h / M_P^2$ in terms of the graviton h. $\mathcal{R}^2$-type operators lead to $h \Box h h \Box h / M_P^4$

- We thus expect the coefficients of these operators to be $O(1)$.

- Naturalness arguments would imply $M_\star \sim \Lambda_C$. However, there is not sign of new physics at this energy scale.

What do experiments tell us?

- In 1977, Stelle has shown that one obtains a modification of Newton's potential at short distances from R^2 terms

$$\Phi(r) = -\frac{Gm}{r}\left(1 + \frac{1}{3}e^{-m_0 r} - \frac{4}{3}e^{-m_2 r}\right)$$

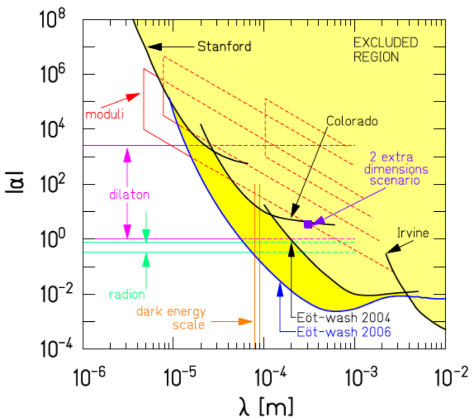
$$m_0^{-1} = \sqrt{32\pi G\left(3c_1 - c_2\right)}$$

$$m_2^{-1} = \sqrt{16\pi G c_2}$$

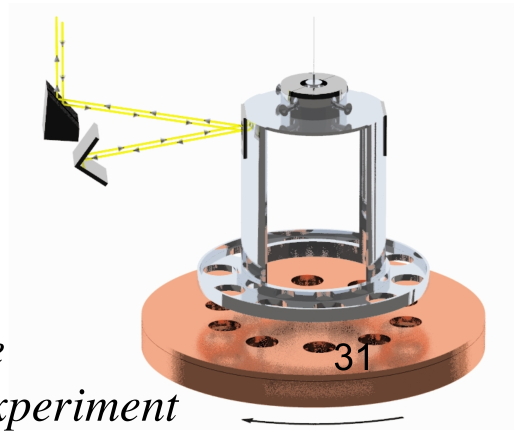
$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 + \alpha \exp\left(-r/\lambda\right)\right]$$

c_1 and c_2 $<10^{61}$

xc, Hsu and Reeb (2008)



NB: Bound has improved by 10 order of magnitude since Stelle's paper!



Schematic drawing of the Eöt-Wash Short-range Experiment

Can better bounds be obtained in astrophysics?

- Bounds on Earth are obtained in weak curvature, binary pulsar systems are probing high curvature regime.

- Approximation: Ricci scalar in the binary system of pulsars by G M/(r^3c^2) where M is the mass of the pulsar and r is the distance to the center of the pulsar.

- But: if the distance is larger than the radius of the pulsar, then the Ricci scalar vanishes. This is a rather crude estimate.

Can better bounds be obtained in astrophysics?

- Let me be optimistic and assume one can probe gravity at the surface of the pulsar. I take r=13.1km and M=2 solar masses.
- I now request that the R^2 term should become comparable to the leading order Einstein-Hilbert term ($1/2\ M_P^2\ R$)
- One could reach bounds of the order of 10^{78} only on c_1 or c_2
- Such limits are obviously much weaker that those obtained on Earth.

Summary of current status of GR coupled to SM

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Planck scale $(M^2 + \xi v^2) = M_P^2$ $M_P = 2.4335 \times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays; energy scale up to which one trusts the effective theory.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 5 \times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6 \times 10^{15}$ [xc & Atkins, 2013]

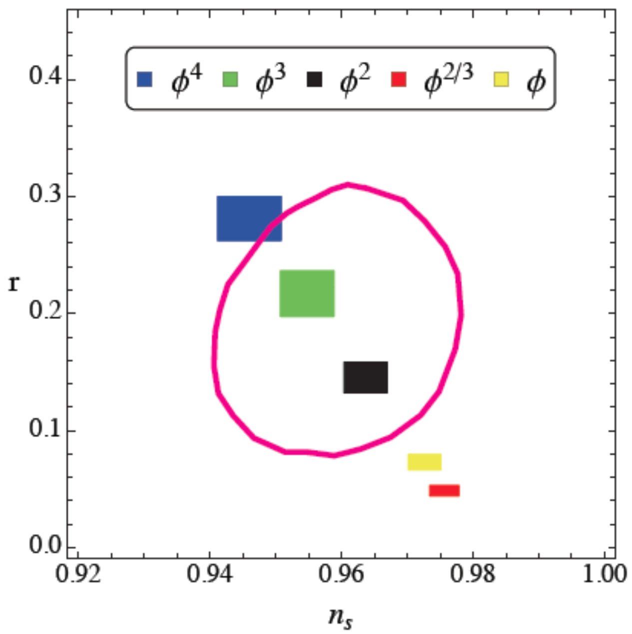
    Higgs inflation requires $\xi \sim 10^4$.

Quantum Gravity and models of inflation

- Effective action for the inflaton:

$$S = \int d^x \sqrt{-g} \left(\frac{\bar{M}_P^2}{2} R + f(\phi) F(R, R_{\mu\nu}) + g^{\mu\nu} \partial_\mu \phi \partial^\nu \phi + V_{ren}(\phi) + \sum_{n=5}^{\infty} c_n \frac{\phi^n}{\bar{M}_P^{n-4}} \right)$$

$$V_{ren} \supset v^3 \phi + m^2 \phi^2 + \lambda_3 \phi^3 + \lambda_4 \phi^4$$



Predictions for various polynomial forms of V_{ren} with $N \in [50, 60]$.
The pink circle corresponds to the 95% CL from BICEP2.

Quantum Gravity and models of inflation

- In inflationary models, one often focuses on one specific term and one sets the remaining Wilson coefficients to zero (or advocate a shift symmetry).

- However in quantum field theory, with the exception of dimension three and four operators higher dimensional operators will be generated by quantum corrections.

- The Wilson coefficients of dimension 3&4 operators can be tiny as seen before, however those of higher dimensional operators are expected to be order unity.

Quantum Gravity effects on chaotic inflation (CI)

- We consider the potential

$$V(\tilde{\phi}) = \bar{M}_P^4 \left(\tilde{m}^2 \tilde{\phi}^2 + c_n \tilde{\phi}^n \right) ,$$

- with

$$\tilde{\phi} = \phi / \bar{M}_P \ , \ \tilde{m} = m / \bar{M}_P \ .$$

- For illustration let's take the dimension 6 operator

$$c_6 = \alpha_m \tilde{m}^2 \rightarrow V(\tilde{\phi}) = \bar{M}_P^4 \tilde{m}^2 \tilde{\phi}^2 \left(1 + \alpha_m \tilde{\phi}^4 \right)$$

- Effective theory is valid if

$$|\alpha_m| \tilde{\phi}^4 < 1$$

Quantum Gravity effects on chaotic inflation (CI)

- The higher-dimensional operator term modifies the slow-role conditions:

$$\epsilon = \frac{1}{16\pi}\left(\frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)^2 = \frac{1}{4\pi}\frac{1}{\tilde{\phi}^2}\left(\frac{1+3\,\alpha_m\tilde{\phi}^4}{1+\alpha_m\tilde{\phi}^4}\right)^2 = \epsilon_{CI} + \frac{\alpha_m\tilde{\phi}^4}{\pi\tilde{\phi}^2} + \mathcal{O}(\alpha_m\tilde{\phi}^4)^3$$

with the usual CI parameter given by $\epsilon_{CI} = 1/(4\pi\tilde{\phi}^2)$

- The second slow-roll parameter, which is zero in usual CI, reads

$$\eta = \frac{1}{8\pi}\left(\frac{V''(\tilde{\phi})}{V(\tilde{\phi})} - \frac{1}{2}\left(\frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)^2\right) \simeq \frac{5}{2\pi\tilde{\phi}^2}\,(\alpha_m\tilde{\phi}^4).$$

Quantum Gravity effects on chaotic inflation (CI)

- The condition for the end of inflation is modified;

$$\tilde{\phi}_E^2 = \frac{1}{4\pi}\left(1 + \frac{\alpha_m}{4\pi}\right)$$

- The number of e-foldings

$$N = 2\sqrt{\pi}\int_{\tilde{\phi}_E}^{\tilde{\phi}_I} \frac{1}{\sqrt{\epsilon}} = 2\pi\tilde{\phi}_I^2\left(1 - \frac{2\,\alpha_m\tilde{\phi}_I^4}{3}\right) - \frac{1}{2} - \frac{5\,\alpha_m}{48\,\pi^2}$$

- value of the field at the beginning of inflation with with N e-foldings,

$$\tilde{\phi}_N^2 \simeq \tilde{\phi}_{N,CI}^2 + \frac{N^3}{12\pi^3}\alpha_m \simeq \frac{N}{2\pi}\left(1 + \frac{N^2\alpha_m}{6\pi^2}\right) \qquad \tilde{\phi}_{N,CI}^2 = \frac{1+2N}{4\pi}$$

Quantum Gravity effects on chaotic inflation (CI)

- The convergence of the effective theory implies

$$|\alpha_m|\tilde{\phi}_N^4 \simeq \frac{N^2|\alpha_m|}{4\pi^2} \lesssim 1 \to |\alpha_m|^{EFT} \lesssim 2 \times 10^{-2}$$

- NB: for values of α_m close to this bound, and negative, cancellations could lead to a value of the field below the Planck mass:

$$\phi < \bar{M}_P \text{ for } N \simeq 60$$

- while there is no simultaneous cancellation in the potential:

$$V_N \simeq \frac{\tilde{m}^2 N}{2\pi}\left(1 + \frac{5}{3}\frac{N^2\alpha_m}{4\pi^2}\right)$$

Quantum Gravity effects on chaotic inflation (CI)

- The scalar power spectrum is affected as well:

$$P_{\mathcal{R}}^{1/2} = \frac{4\sqrt{24\pi}}{3} \frac{V(\tilde{\phi}_N)^{3/2}}{V'(\tilde{\phi}_N)} \simeq P_{\mathcal{R},\mathcal{CI}}^{1/2} \left(1 - \frac{5}{6} \frac{N^2 \alpha_m}{4\pi^2}\right)$$

- where

$$P_{\mathcal{R},\mathcal{CI}}^{1/2} = 2\sqrt{\tfrac{2}{3\pi}} N \tilde{m}$$

- The usual limit on the inflaton mass:

$$\tilde{m} \simeq 4 \times 10^{-7} \rightarrow m \sim 10^{12} \text{ GeV}.$$

Quantum Gravity effects on chaotic inflation (CI)

- Finally one obtains the spectral index

$$n_s - 1 = (n_s - 1)_{CI} \left(1 - \frac{5}{3}\frac{N^2\alpha_m}{4\pi^2}\right)$$

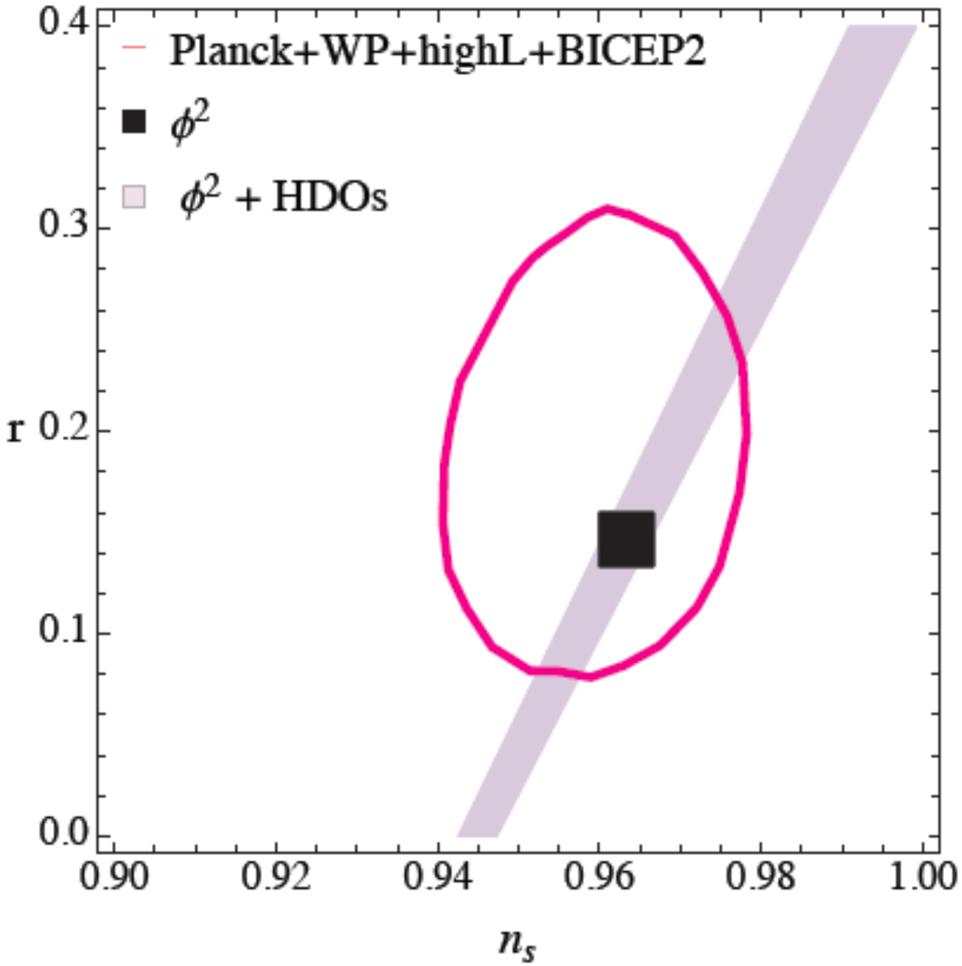
- And the tensor-to-scalar ratio

$$r = r_{CI} \left(1 + \frac{10}{3}\frac{N^2\alpha_m}{4\pi^2}\right)$$

- Which are constrained by BICEP2

$$\alpha_m^{BICEP2} \in [-2, 3] \times 10^{-3}$$

Quantum Gravity effects on chaotic inflation (CI)



Effect of quantum gravity is quite dramatic.

$$\alpha_m^{BICEP2} \in [-2, 3] \times 10^{-3}$$

$$c_6 \lesssim 10^{-9}$$

[xc, and Sanz (2014)]

Quantum Gravity effects on ϕ^4 inflation

Same calculation as before:

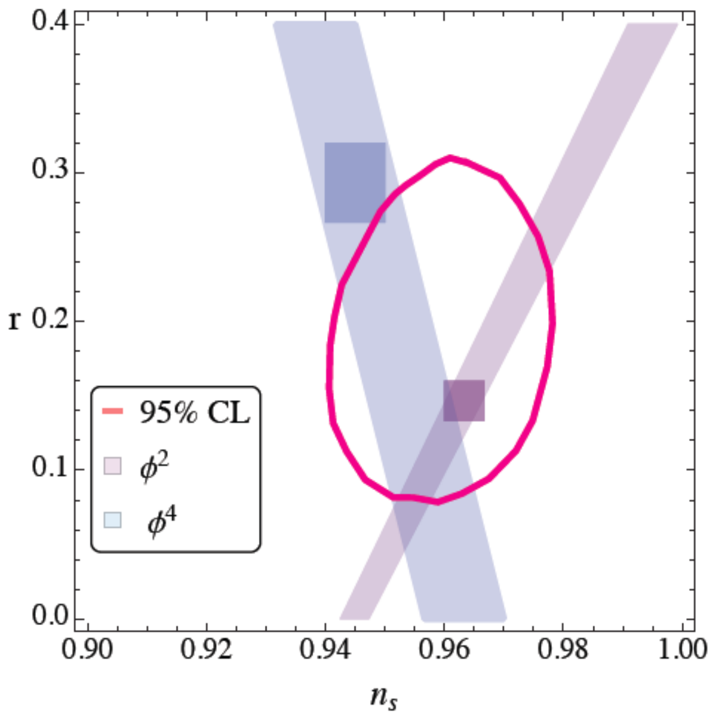
$$V(\tilde{\phi}) = \bar{M}_P^4 \left(\lambda \tilde{\phi}^4 + c_n \tilde{\phi}^n\right)$$

$$c_6 = \alpha_\lambda \lambda \to V(\tilde{\phi}) = \bar{M}_P^4 \lambda \tilde{\phi}^4 \left(1 + \alpha_\lambda \tilde{\phi}^2\right)$$

$$|\alpha_\lambda|^{EFT} \lesssim 0.06$$

$$\begin{aligned} \epsilon &= \epsilon_{\phi^4}\left(1 + \alpha_\lambda \tilde{\phi}^2\right) \\ \eta &= \eta_{\phi^4}\left(1 + 5\alpha_\lambda \tilde{\phi}^2/2\right) \end{aligned}$$

$$\alpha_\lambda^{BICEP} \in [-0.06, 0] \to c_6 < 10^{-15}$$



The effect of higher-dimensional operators on ϕ^4 and ϕ^2 potentials shown in blue and purple respectively.
The darker boxes corresponds to potentials without higher-dimensional operators, and the pink circle is the area of 95% CL from BICEP2.

[xc, and Sanz (2014)]

Grand unification through gravitational effects

[xc, Hsu and Reeb (2008,2010)]

- Generically speaking there are many dimension five operators:

$$\mathcal{L} = \frac{c_i}{4M_{Pl}} H_i^{ab} G_{\mu\nu}^a G^{b\mu\nu}$$

- Modified unification condition:

$$\alpha_G = (1+\epsilon_1)\,\alpha_1(M_X) = (1+\epsilon_2)\,\alpha_2(M_X) = (1+\epsilon_3)\,\alpha_3(M_X)\ .$$

$$\epsilon_1 = \frac{\epsilon_2}{3} = -\frac{\epsilon_3}{2} = \frac{\sqrt{2}}{5\sqrt{\pi}} \frac{c\eta}{\sqrt{\alpha_G}} \frac{M_X}{\hat{M}_{\rm Pl}}$$

- Unification without supersymmetry can easily be obtained.
- Unification scale is typically quite high and potentially close to the Planck mass.
- No problem with proton decay.
- Nice feature of non-SUSY unification: avoid Landau pole above the unification scale.

Yukawa couplings

- Dimension 5 terms in SU(5)

Ψ and f are fermion fields in **10** and **5** respectively scalar fields in the **24** and **5** representations

$$\mathcal{O}_5 = \frac{a_1}{\hat{\mu}_\star}\{\phi_{mn}\bar{f}^{mk}H^l_k\Psi^n_l\} + \frac{a_2}{\hat{\mu}_\star}\{\phi_{mn}H^{mk}\bar{f}^l{}_k\Psi^n_l\} + \frac{a_3}{\hat{\mu}_\star}\varepsilon^{mnpql}\{\Psi_{mn}\Psi_{pq}H_k\phi^k_l\},$$

- New unification condition:

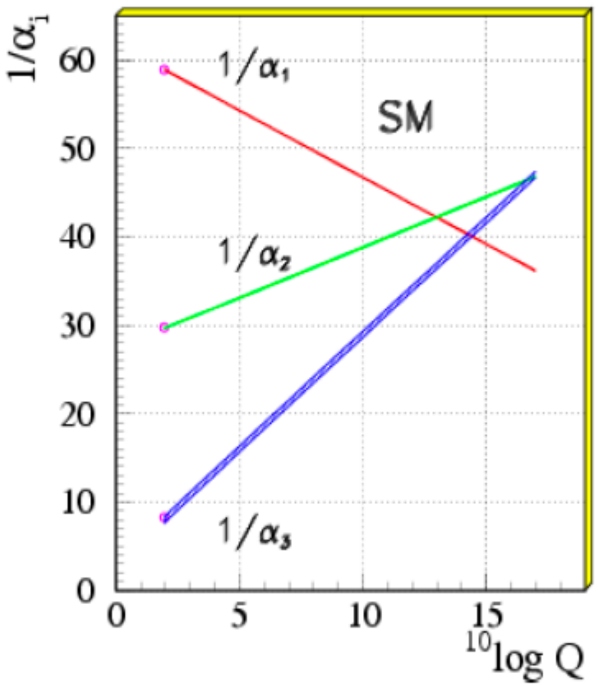
$$m_d(M_X)[1+\frac{3}{2}\zeta_1-\zeta_2] = m_e(M_X)[1+\frac{3}{2}\zeta_1+\frac{3}{2}\zeta_2]$$

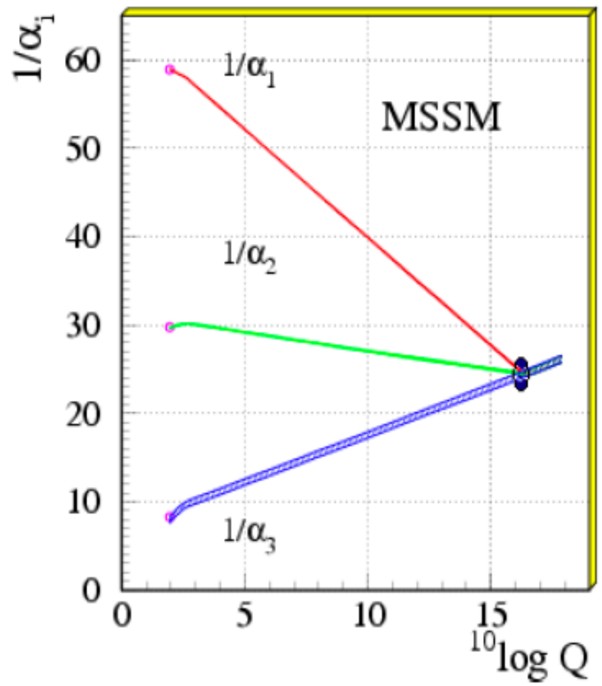
$$\zeta_i = \frac{-2\sqrt{2}}{5G_d g_u}\frac{M_X}{\bar{M}_{Pl}}a_i\eta$$

[xc and Yang (2011)]

Unification of the couplings of the Standard Model?

One of LEP's most impressive result





Standard Model does not work

But the minimal Supersymmetric (SUSY) Standard Model works beautifully

This is not quite correct because of quantum gravity!

Quantum Gravity and GUT

- Quantum gravity can help to unify the gauge couplings and Yukawa couplings.

- It spoils predictions done using low energy data.

- LEP does not favor SUSY unification: Extrapolation from low energy data is too naïve.

- If no BSM is discovered, gravity induced unification should be taken very seriously

- Impossible to make any prediction without knowing the full details of the unification group and symmetry breaking pattern.

Conclusions

- We have discussed a conservative effective action for quantum gravity within several frameworks
  - Standard model
  - Inflationary models
  - Grand Unified Models.
- We have seen that the effects of quantum gravity can be huge in inflationary models and in grand unified theories.
- They are relatively modest within the standard model (as expected).
- It's tough to probe QG using low energy experiments while if BICEP2 is correct, we have a good chance of testing the symmetries of quantum gravity.

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Thanks for your attention!