Temperature of the CMB temperature in the CFT driven cosmology and the a-theorem

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Plan

Cosmological initial conditions – density matrix of the Universe:

- microcanonical ensemble in cosmology
- initial conditions for the Universe via EQG statistical sum

CFT driven cosmology:

- constraining landscape of Λ
- inflation and generation of thermally corrected CMB spectrum – temperature of the CMB temperature
- a-theorem and CFT cosmology – role of higher-spin conformal fields and renormalization group flow

Two known prescriptions for a *pure* initial state:

no-boundary wavefunction (Hartle-Hawking);

$$\Psi[\,g_{ij},\varphi\,] = \int D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

"tunneling" wavefunction (Linde, Vilenkin, Rubakov, Zeldovich-Starobinsky, …)

semiclassical solution of the minisuperspace
Wheeler-DeWitt equation,
outgoing wave prescription,
etc.

Both no-boundary (EQG path integral) and tunneling (WKB approximation) do not have a clear operator interpretation

The ***path integral*** **formulation** ***of the microcanonical ensemble*** **in quantum cosmology**

A.B. & A.Kamenshchik, JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006);
A.B., Phys. Rev. Lett. 99, 071301 (2007)

We apply it to the Universe dominated by:

massless matter conformally coupled to gravity →

- **bounded** range of the **primordial** Λ with band structure;
- dynamical elimination of **vacuum** no-boundary and tunneling states;
- inflation and **thermal corrections** to CMB power spectrum
- application of ***a*-theorem** – RG flow and enhancement of thermal effect

A.B, J JCAP 1310 (2013) 059, arXiv:1308.4451

Microcanonical ensemble in cosmology and EQG path integral

$H_\mu = 0$ constraints on initial value data – corner stone of any diffeomorphism invariant theory.

Physical states: $\hat{H}_\mu|\Psi\rangle = 0 \quad \hat{H}_\mu \equiv \hat{H}_\perp(\mathbf{x}),\ \hat{H}_i(\mathbf{x})$

operators of the Wheeler-DeWitt equations

$\mu = (\perp\mathbf{x},\ i\mathbf{x})$
$\mathbf{x}$ – spatial coordinates

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

$$|\Psi\rangle \to \hat{\rho}, \qquad \hat{H}_\mu\,\hat{\rho} = 0$$

$$\hat{\rho} = e^{\Gamma}\prod_\mu \delta(\hat{H}_\mu)$$

Statistical sum

$$e^{-\Gamma} = \mathrm{Tr}\prod_\mu \delta(\hat{H}_\mu)$$

A.O.B., Phys.Rev.Lett. 99, 071301 (2007)

Motivation: aesthetical (minimum of assumptions – Occamistic razor)

A simple analogy — an unconstrained system with a conserved Hamiltonian $\widehat{H}$ in the microcanonical state with a fixed energy E

$$\hat{\rho} \sim \delta(\widehat{H} - E)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value --- zero

↓

The microcanonical ensemble with

$$\hat{\rho} \sim \prod_\mu \delta(\widehat{H}_\mu)$$

is a natural candidate for the quantum state of the closed Universe – ultimate equipartition in the physical phase space of the theory --- *Sum over Everything*.

Path integral representation of the statistical sum

$$-i\infty < N < i\infty, \quad g^{44} = +N^2$$

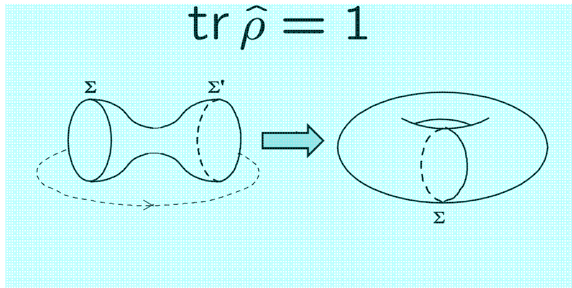
Euclidean metric

$$\mathrm{Tr} \prod_{\mu} \delta(\widehat{H}_{\mu}) \equiv e^{-\Gamma} = \int_{\text{periodic}} D[\, g_{\mu\nu}, \phi\,]\, e^{-S_E[\, g_{\mu\nu}, \phi\,]}$$

EQG density matrix
D.Page (1986)

Spacetime topology in the statistical sum:

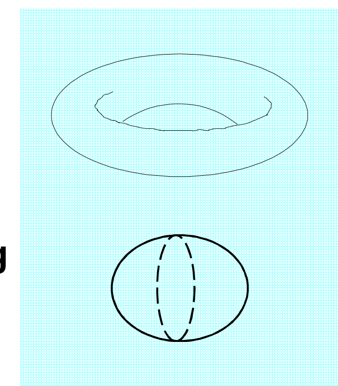
S^3 topology of a spatially closed cosmology



$$e^{-\Gamma} = \int_{\text{periodic}} D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

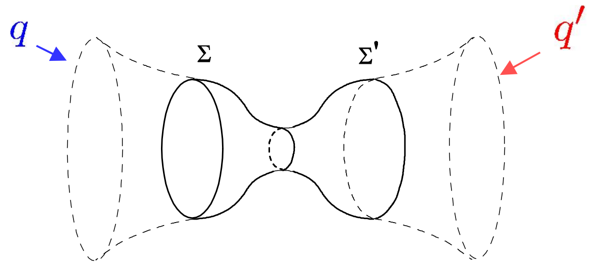
on $S^3 \times S^1$ (thermal)

including as a limiting (vacuum) case S^4



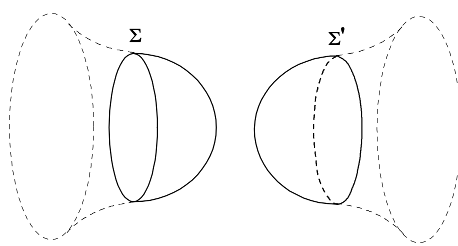
Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

$\hat{\rho}_{\mathrm{mixed}}$
$= \rho_{\mathrm{mixed}}(q, q')$

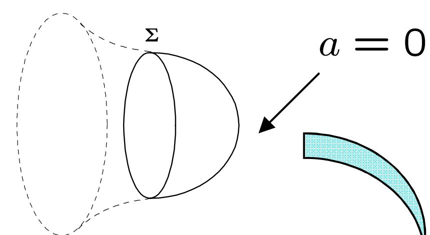


pinching a tubular spacetime

$|\Psi_{HH}\rangle\langle\Psi_{HH}|$
$= \rho_{HH}(q, q')$



$\Longleftrightarrow$ $|\Psi_{HH}\rangle$



density matrix representation of a pure Hartle-Hawking state – vacuum state of zero temperature $T \sim 1/\eta = 0$

$$\eta = \int_{a=0} d\tau \, \frac{N}{a} \to \infty$$

(see below)

Path integral calculation:

Disentangling the minisuperspace sector

Euclidean FRW metric

3-sphere of a unit size

$$ds^2 = N^2\,d\tau^2 + a^2\,d^2\Omega^{(3)}$$

lapse **scale factor**

$$[\,g,\phi\,] = [\,a(\tau), N(\tau); \Phi(x)\,]$$

minisuperspace background

quantum “matter” – cosmological perturbations:

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$$

Decomposition of the statistical sum path integral:

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,a, N\,]\; e^{-S_{\text{eff}}[\,a,\,N\,]}$$

$$e^{-S_{\text{eff}}[\,a,\,N]} = \int_{\text{periodic}} D\Phi(x)\; e^{-S_E[\,a,\,N;\,\Phi(x)\,]}$$

quantum effective action of Φ on minisuperspace background spin-dependent coefficients

$$g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2}g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss-Bonnet term

Weyl term

$$\beta = \frac{1}{360}(2N_0 + 11N_{1/2} + 124N_1)$$

N*$_s$ # of fields of spin *s

$$S_{\text{eff}} = \text{classical part} + \Gamma_A + \Gamma_{EU}$$

anomaly contribution

Einstein universe contribution

Application to the CFT driven cosmology

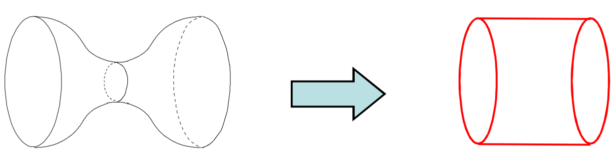
$$S_E[g_{\mu\nu},\phi] = -\frac{1}{16\pi G}\int d^4x\, g^{1/2}\left(R-2\Lambda\right) + S_{CFT}[g_{\mu\nu},\phi]$$

$\Lambda=3H^2$ -- primordial cosmological constant

$N_s \gg 1$ conformal fields of spin s=0,1,1/2

Conformal invariance → exact calculation of S_{CFT}

Assumption of N_{cdf} conformally invariant, $N_{cdf} \gg 1$, quantum fields and recovery of the action from the conformal anomaly and the action on a static Einstein Universe



A.A.Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur & Mottola;
……

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \Longrightarrow d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$$

conformal time

Full quantum effective action on FRW background

nonlocal (thermal) part

$$S_{\rm eff}[\,a,N\,] = \int d\tau\, N\mathcal{L}(a,a') + F(\eta)$$

$$\mathcal{L}(a,a') = -aa'^2 - a + \frac{\Lambda}{3}a^3 + B\left(\frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a}\right)$$

classical part

conformal anomaly part

vacuum (Casimir) energy – from static EU

$$B = \frac{3\beta}{4m_P^2}$$

-- coefficient of the Gauss-Bonnet term in the conformal anomaly

$$F(\eta) = \pm\sum_{\omega} \ln\left(1 \mp e^{-\omega\eta}\right),$$

energies of field oscillators on S^3

Inverse (comoving) temperature

$$\eta = \oint d\tau\, \frac{N}{a}$$

$$a' \equiv \frac{1}{N}\frac{da}{d\tau}$$

-- time reparameterization invariance (1D diffeomorphism)

Effective Friedmann equation for saddle points of the path integral:

$$\frac{\delta S_{\rm eff}[\,a, N\,]}{\delta N(\tau)} = 0$$

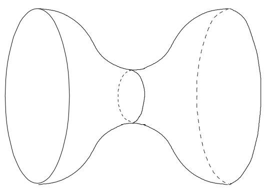
amount of radiation constant

$$\frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{B}{2}\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4}, \quad \mathcal{C} = \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

"bootstrap" equation:
$\eta=\eta[a(\tau)]$

$$B = \frac{3\beta}{4m_P^2}$$ -- coefficient of the Gauss-Bonnet term in the conformal anomaly

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and vacuum Hartle-Hawking instantons (S^4)



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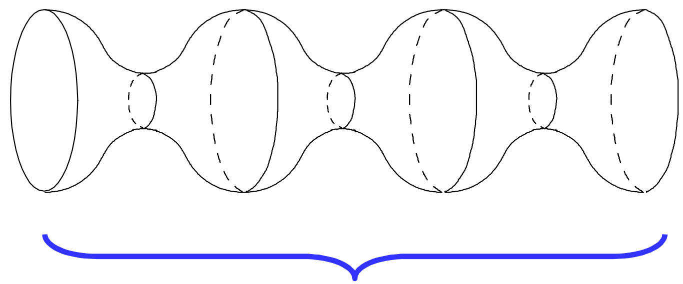
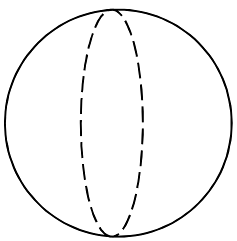




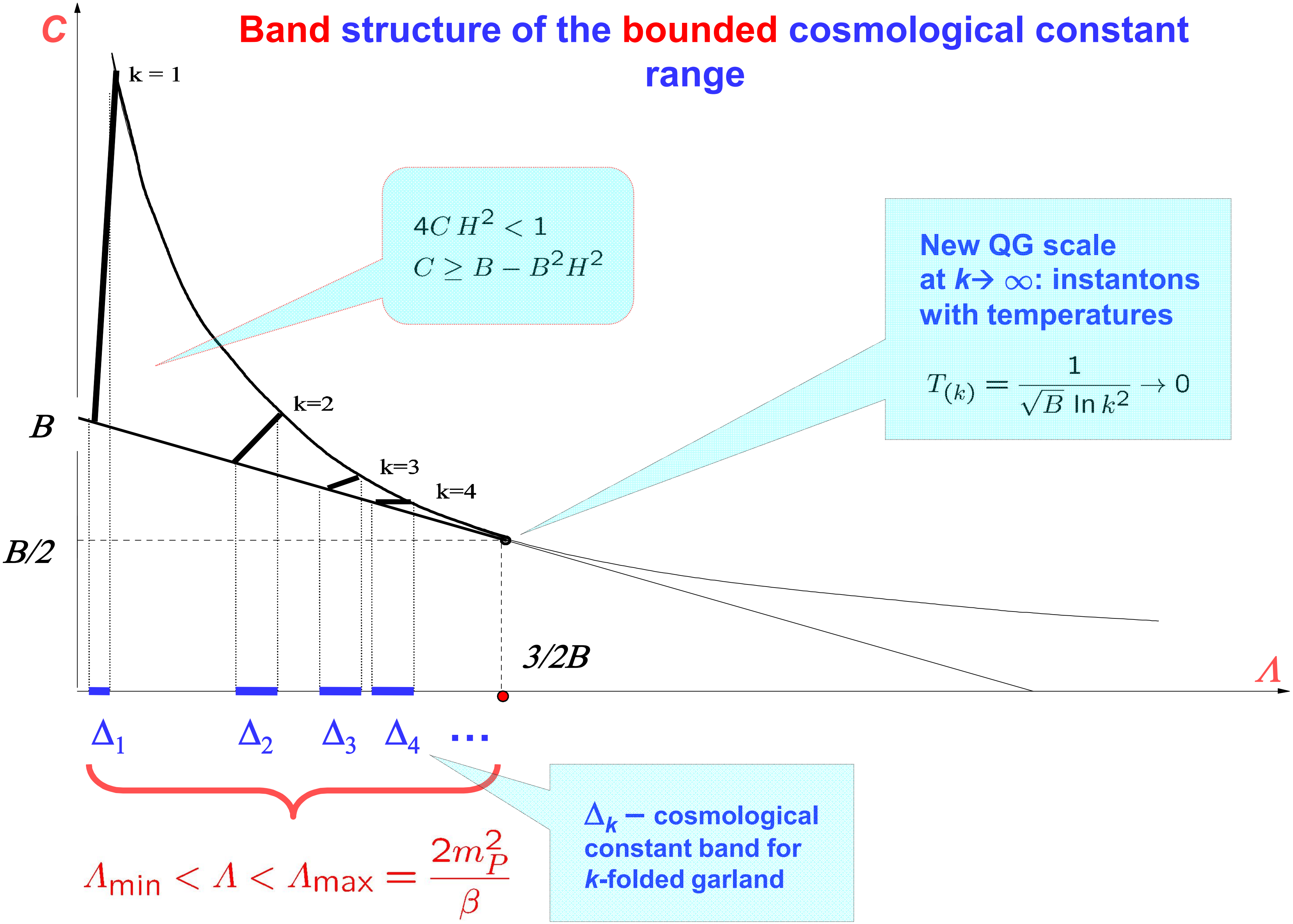
1- fold, k=1

k- folded garland, k=1,2,3,…

S^4

does not contribute: weight e^{-i}

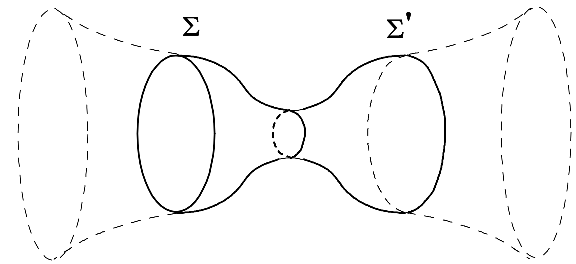
Band structure of the bounded cosmological constant range



Inflationary evolution and "*hot*" CMB

Lorentzian Universe with initial conditions set by the saddle-point instanton. Analytic continuation of the instanton solutions:

$$\tau = it, \;\; a(t) = a_E(it)$$



Expansion and quick dilution of primordial radiation

Inflation via Λ as a composite operator – inflaton potential and a slow roll

$$\Lambda \simeq \frac{V(\varphi)}{M_P^2}$$

Decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter:

$$\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4} \Rightarrow \frac{8\pi G}{3}\,\varepsilon$$

matter energy density

Primordial CMB spectrum with thermal corrections:

standard red CMB spectrum

thermal contribution

$$\delta_\phi^2(k) = \langle\, \hat{\phi}_k(t)\hat{\phi}_k(t)\,\rangle_{\text{thermal}}$$

$$\sim \langle\, \hat{a}_k^\dagger \hat{a}_k + \hat{a}_k \hat{a}_k^\dagger \,\rangle_{\text{thermal}} |u_k(t)|^2 = |u_k(t)|^2 \big(1 + 2N_k(\eta)\big)$$

$$N_k(\eta) = \frac{1}{e^{k\eta} - 1}$$

$$k_l \leftrightarrow l \;\Rightarrow\; C_l^2 \to C_l^2 \left(1 + \frac{2}{e^{k_l \eta} - 1}\right)$$

additional reddening of the CMB spectrum

Spectral index:

$$n_s(k) = 1 + \frac{d}{d\ln k}\ln\delta_\phi^2(k) = n_s^{\text{vac}}(k) + \Delta n_s^{\text{thermal}}(k),$$

$$\Delta n_s^{\text{thermal}}(k) = \frac{d}{d\ln k}\ln\left(1 + 2N_k(\eta)\right)$$

$$N_{k_l} \simeq \exp\left[-\frac{2l}{(\Omega_0 - 1)^{1/2}}\left(\frac{1}{180\,\tilde{\beta}}\right)^{1/6}\right] \simeq \exp\left[-\frac{10\,l}{(3\,\tilde{\beta})^{1/6}}\right]$$

<<1

$$\Delta n_s^{\text{thermal}}(k_l) \simeq -\frac{20\,l}{(3\,\tilde{\beta})^{1/6}}e^{-10\,l/(3\,\tilde{\beta})^{1/6}} \ll 1$$

$$\tilde{\beta} = \frac{\sum_s \beta_s N_s}{\mathbb{N}_{\text{dof}}}$$

β per one degree of freedom

$$\frac{1}{60} \leq 3\,\tilde{\beta}_{\text{low-spin}} \leq \frac{31}{30} \simeq 1$$

$$\beta_s = \begin{cases} \frac{1}{180} & s=0 \\ \frac{11}{360} & s=\frac{1}{2} \\ \frac{31}{90} & s=1 \\ -\frac{274}{90} & s=\frac{3}{2} \\ \frac{87}{10} & s=2 \\ \dots & s>1 \end{cases}$$

For $s=0, \frac{1}{2}, 1$:

$3\tilde{\beta} \lesssim 1$

thermal part is negligible:

$$n_l < e^{-\frac{l}{\sqrt{\Omega_0 - 1}}} \sim e^{-10\,l}$$

$s=\frac{3}{2}$: **conformal gravitino**

$s=2$: **conformal graviton**

contribution of higher conformal spins might have thermal imprint on CMB ?

$s^6 \to \infty$

$$\tilde{\beta}_s = \frac{\beta_s}{\nu_s} \sim s^4 \to \infty$$

$$\beta_s = \frac{1}{360}\nu_s^2(3+14\nu_s), \quad \nu_s = s(s+1), \quad s = 1,2,3,\dots,$$

$$\beta_s = \frac{1}{720}\nu_s(12+45\nu_s+14\nu_s^2), \quad \nu_s = -2\left(s+\frac{1}{2}\right)^2, \quad s = \frac{1}{2},\frac{3}{2},\frac{5}{2},\dots$$

A.A.Tseytlin, arXiv:1309.0785

CFT cosmology and the a-theorem

Free CFT ⟶ interacting CFT

$$e^{iW[g_{\mu\nu}]} = \int D\phi \, e^{iS_{CFT}[g_{\mu\nu},\phi]}$$

$$\langle T^{\mu}_{\mu} \rangle \equiv \frac{2}{g^{1/2}} g^{\mu\nu} \frac{\delta W}{\delta g^{\mu\nu}} = aE - c\, C^2_{\mu\nu\alpha\beta} + b\Box R.$$

Renormlization group flow: $a,\, b,\, c \quad \Rightarrow \quad a(\mu^2),\, b(\mu^2),\, c(\mu^2)$

UV ↓ IR ↓

$$a(\infty) - a(0) = \frac{1}{4\pi} \int_{s>0} ds \, \frac{\sigma(s)}{s^2} > 0$$

$$\sigma(s) = s\,\mathrm{Im}\mathcal{A}(s,t)_{t=0} > 0$$

Total cross section of the forward 2 ⟶ 2 dilaton scattering

Z.Komargodski and A.Schwimmer, arXiv:1107.3987

Z.Komargodski, arXiv:1112.4538

Cf. c-coefficient:

$$iW = \frac{i}{2(4\pi)^2}\int d^4x_L\, g^{1/2}\Big(c\,C^2_{\mu\nu\alpha\beta} - aE + \ldots\Big)$$

$$-\frac{i}{2(4\pi)^2}\int d^4x_L\, g^{1/2}\left(c\,C_{\mu\nu\alpha\beta}\ln\left(-\frac{\Box + i\varepsilon}{\mu^2}\right)C^{\mu\nu\alpha\beta} + \ldots\right)$$

$$\ln\left(-\frac{\Box + i\varepsilon}{\mu^2}\right) = \ln\left(\frac{|p^2|}{\mu^2}\right) - i\pi\theta(-p^2)$$

$$\mathrm{Im}\,W = \frac{1}{32\pi}\int d^4p\left(c\,|\,\widehat{C}_{\mu\nu\alpha\beta}\,|^2(p)\,\theta(-p^2) + \ldots\right) > 0 \quad \Rightarrow \quad c > 0$$

unitarity

dilaton = physical variable in cosmology

$$g_{\mu\nu} = e^{\sigma}\bar{g}_{\mu\nu},$$

dilaton action

$$\Gamma_R[g] - \Gamma_R[\bar{g}] = \frac{\gamma}{4(4\pi)^2}\int d^4x \bar{g}^{1/2}\,\sigma\,\bar{C}^2_{\mu\nu\alpha\beta}$$
$$+\frac{\beta}{2(4\pi)^2}\int d^4x\bar{g}^{1/2}\left\{\frac{1}{2}\sigma\bar{E} - \left(\bar{R}^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{R}\right)\partial_\mu\sigma\,\partial_\nu\sigma - \frac{1}{2}\bar{\Box}\sigma\,(\bar{\nabla}^\mu\sigma\,\bar{\nabla}_\mu\sigma) - \frac{1}{8}(\bar{\nabla}^\mu\sigma\,\bar{\nabla}_\mu\sigma)^2\right\}$$

Galileon type mode -- no higher derivative ghosts!

Cosmological expansion: UV ⟶ IR

$$a(\mu^2) = \frac{\beta(\mu^2)}{32\pi^2}, \qquad \beta_{\text{early}} \gg 1 \quad \to \quad \beta_{\text{now}} \sim O(1)$$

⇓

Large observable thermal imprint on CMB of invisible now higher spin fields ?

Conclusions

Microcanonical density matrix of the Universe

Application to the CFT driven cosmology with a large # of quantum species – thermal version of the no-boundary state

Initial conditions for inflation with a limited range of Λ -- cosmological landscape -- and generation of the thermal CMB spectrum, but too cold

SOME LIKE IT COOL