Journey to the un-world

Piero Nicolini

Frankfurt Institute for Advanced Studies (FIAS)
&
Institut für Theoretische Physik, Goethe Universität

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Talk based on

A.M. Frassino, P.N. and O. Panella, arXiv:1311.7173 [hep-ph].
P.N. and J. Mureika, in preparation
P.N. and E. Spallucci, Phys. Lett. B 695, 290 (2011)
P.N., Phys. Rev. D 82, 044030 (2010)

FIAS Frankfurt Institute for Advanced Studies

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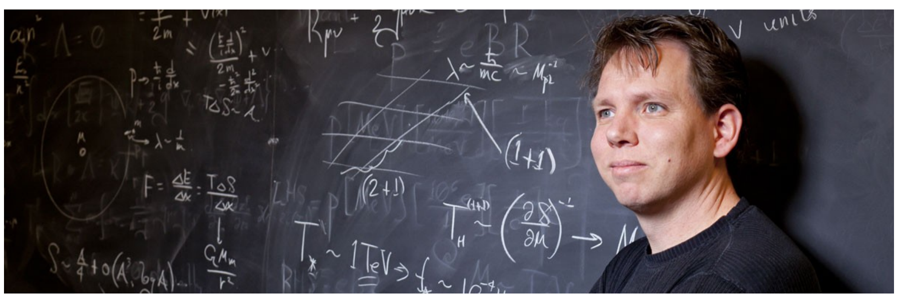

Jonas R. Mureika
(LMU, LA)

Un-world

scale invariance is less common in physics

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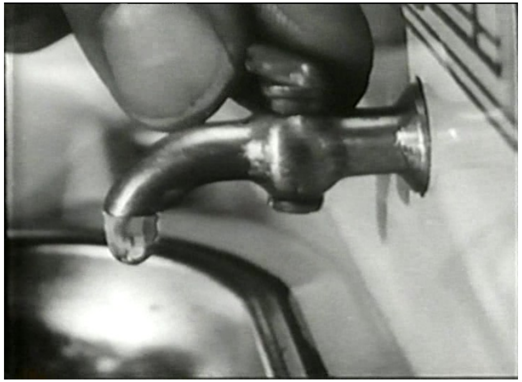
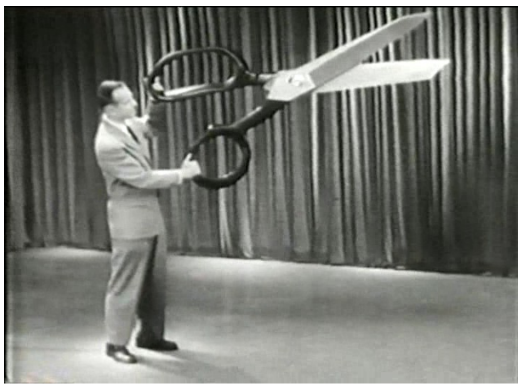
Scale invariance?



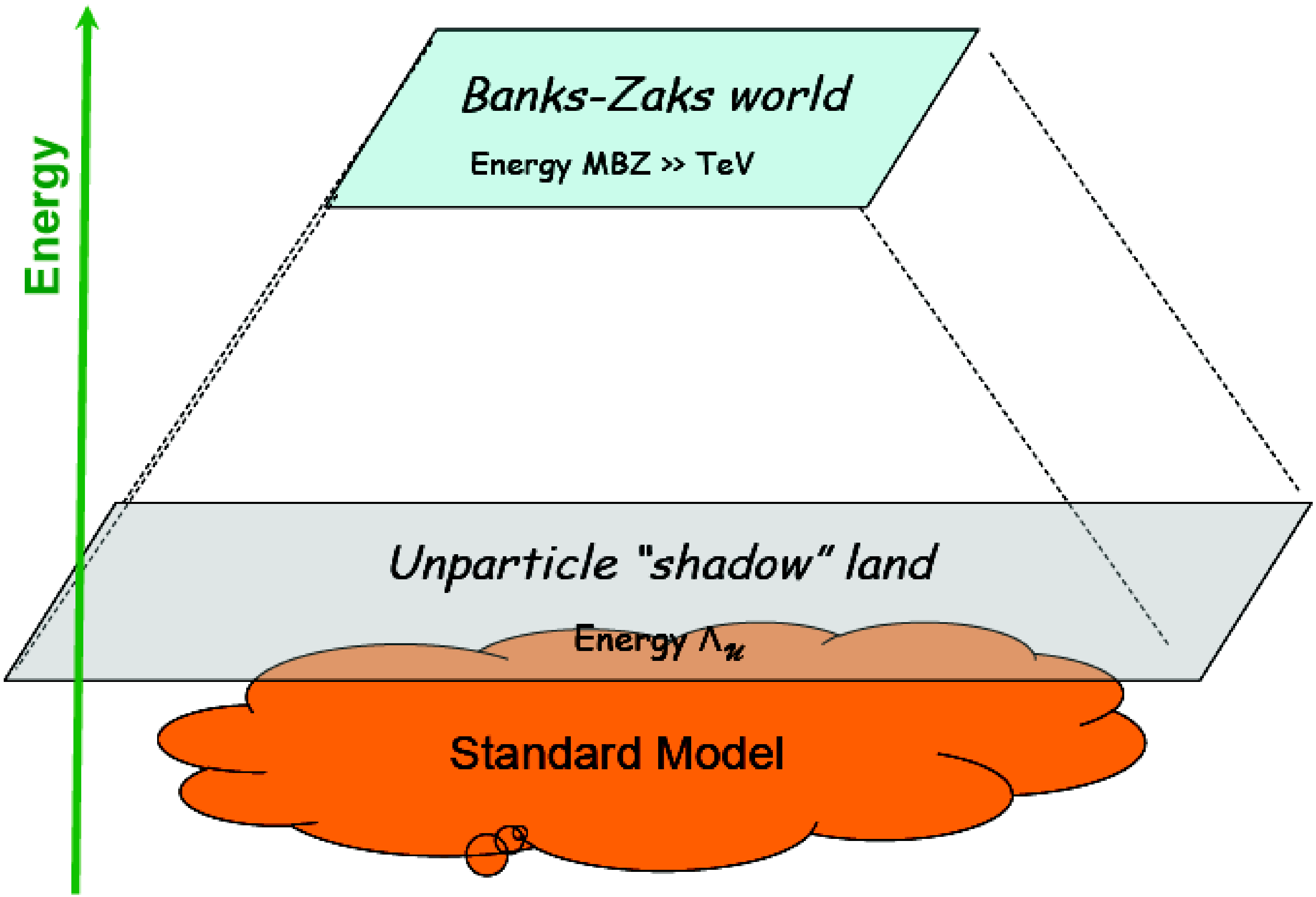


What will the LHC find?

- The Higgs Boson: 95%.
- Supersymmetry: 60%.
- Something that has never been predicted: 50%.
- Dark Matter: 15%.
- Warped Extra Dimensions: 10%.
- Absolutely Nothing: 3%.

[S. Carroll, Discover, Aug 4th, 2008]

- Large Extra Dimensions: 1%.
- **Unparticles: 0.5%.**
- Evidence for or against String Theory: 0.5%.
- Black Holes: 0.1%.
- Dark Energy: 0.1%.
- Stable Black Holes that eat up the Earth, destroying all living organisms in the process: 10^{-25} %.



Energy
Banks-Zaks world
Energy MBZ >> TeV
Unparticle "shadow" land
Energy $\Lambda_{\mathcal{U}}$
Standard Model

Unparticle physics primer

[H. Georgi, PRL 98 (2007) 221601]

- Scale-invariant high energy particle sector, weakly interacting with standard model:

$$\mathcal{L} = \frac{1}{(M_{\mathcal{U}})^k} \mathcal{O}_{\rm SM} \mathcal{O}_{\rm BZ} \qquad k = d_{SM} + d_{BZ} - 4$$

- Below some scale, undergoes dimensional transmutation $\mathcal{O}_{BZ} \rightarrow C_{\mathcal{U}} \lambda^{d_{BZ} - d_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}}$ to become an “unparticle” field $\mathcal{O}_{\mathcal{U}}$ (scalar, vector, tensor, spinor) at energy scale $\Lambda_{\mathcal{U}}$:

$$\mathcal{L} = \frac{\kappa}{M_{\mathcal{U}}^{k_{\mathcal{U}}}} \mathcal{O}_{SM} \mathcal{O}_{\mathcal{U}} \quad , \quad k_{\mathcal{U}} = d_{SM} + d_{\mathcal{U}} - 4$$

$$\lambda = (\Lambda_{\mathcal{U}} / M_{\mathcal{U}})^k$$

Unparticle physics primer

- Consider the vacuum matrix element

$$\langle 0|\mathcal{O}_\mathcal{U}(x)\mathcal{O}_\mathcal{U}^\dagger(0)|0\rangle = \int \frac{d^4p}{(4\pi)^4} e^{iPx} |\langle 0|\mathcal{O}_\mathcal{U}(0)|P\rangle|^2 \rho(P^2)$$

- Unparticle Phase Space: $x \to \lambda x$ $\quad O_U(\lambda x) \to \lambda^{-d_U} O_U(x)$

$$A_{d_\mathcal{U}}\, \theta(P^0)\, \theta(P^2)\, (P^2)^{d_\mathcal{U}-2}$$

$d_\mathcal{U} > 1$

$d_U = 1 + \varepsilon \qquad \to 2\pi\theta(P^0)\delta(P^2)$

- SM Particle Phase Space:

$$A_n\, \theta(P^0)\, \theta(P^2)\, (P^2)^{n-2}$$

$n = 1, 2, 3, \ldots$

The unparticle interpretation

$$\mathsf{D}_{\mathcal{U}}(x,x') = \frac{A_{d_{\mathcal{U}}}}{2\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm^2 (m^2)^{d_{\mathcal{U}}-2}\, \mathsf{D}(x,x';m^2)$$

$$A_{d_{\mathcal{U}}} = \frac{16\,\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\,\Gamma(2d_{\mathcal{U}})}$$

- It is a linear continuous superposition of Feynman propagators of fixed mass m
- When the conformal dimension $d_{\mathcal{U}} \to 1$ then the un-particle propagator reduces to that of an ordinary massless field.

Unparticle stuff is a composite particle with a *continuous mass spectrum*

Netwon's law corrections

- Static limit potential (from QFT amplitudes):

$$\Phi(r) = \frac{GM}{r}\left[1 \pm \Gamma_{\mathcal{U}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right]$$

"Ungravity" interaction scale

$$R_* = \Lambda_{\mathcal{U}}^{-1}\left(\frac{M_{Pl}}{\Lambda_{\mathcal{U}}}\right)^{\frac{1}{d_{\mathcal{U}}-1}}\left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{\frac{d_{BZ}}{d_{\mathcal{U}}-1}}$$

- Interactions will depend on mass scales $M_{\mathcal{U}} > \Lambda_{\mathcal{U}}$ and mass dimensions d_{BZ}, $d_{\mathcal{U}}$.

Goldberg and Nath, PRL ***100****, 031803 (2008); Das* et al., PRD **076001** (2008); *Deshpande et al.*, PLB **659**, 888 (2008) *Nicolini, PRD* ***82*** *(2010) 044030*

Exact solutions in GR

Given a non-local action $S \sim \int d^4x\sqrt{-g}\, G^{\mu\nu}\, \frac{\mathcal{A}^{-1}(\Box)}{\Box}\, R_{\mu\nu}$

[Biswas et al. PRL **108**, 031101 (2012); Barvinski, PLB**710**, 12 (2012); Modesto,PRD**86**:044005 (2012)]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\mathcal{T}_{\mu\nu} \qquad \mathcal{T}_{\mu\nu} \equiv \mathcal{A}^{-2}(\Box)T_{\mu\nu}$$

[Modesto, Moffat, PN, PLB**695**, 397 (2011)]

$$R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R = \kappa^2\left[1 + \frac{A_{d_U}\Lambda_U^{2-2d_U}}{(2d_U - 1)\sin(\pi d_U)}\frac{\kappa_*^2}{\kappa^2}(-\Box)^{d_U-1}\right]T^\mu{}_\nu$$

$$\equiv \kappa^2 T^\mu{}_\nu + \kappa_*^2\frac{A_{d_U}}{\sin(\pi d_U)}T_U{}^\mu_\nu$$

*Gaete, Helayel-Neto, Spallucci, PLB**693** (2010) 155*

Unparticle enhanced Schwarzschild metric

- **General solutions**
  - Scalar and tensor unparticle:

$$ds^2 = \left[1 - \frac{2GM}{r}\left(1 + \Gamma_{d_{\mathcal{U}}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right)\right] dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}\left(1 + \Gamma_{d_{\mathcal{U}}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right)} + r^2 d\Omega^2$$

  - Vector unparticle:

$$g_{00} = 1 - \frac{2GM_{\rm BH}}{r}\left(1 - \left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right) \quad , \quad g_{rr} = -g_{00}^{-1}$$

- ***Black hole solutions!***

*Mureika, PLB **660** (2008) 561-566*

*Gaete, Helayel-Neto, Spallucci, PLB**693** (2010) 155*

*Mureika and Spallucci, PLB**693** (2010) 129*

Continuous extradimensions

Mureika, PLB **660** (2008) 561-566

Gaete, Helayel-Neto, Spallucci, PLB**693** (2010) 155

- Metric:

$$ds^2 = \left[1 - \frac{2GM}{r}\left(1 + \Gamma_{d_\mathcal{U}}\left(\frac{R_*}{r}\right)^{2d_\mathcal{U}-2}\right)\right] dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}\left(1 + \Gamma_{d_\mathcal{U}}\left(\frac{R_*}{r}\right)^{2d_\mathcal{U}-2}\right)} + r^2 d\Omega^2$$

- When $r << R_U$:

$$ds^2 \sim -\left(1 - \left(\frac{R_\mathcal{U}}{r}\right)^{2d_\mathcal{U}-1}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{R_\mathcal{U}}{r}\right)^{2d_\mathcal{U}-1}} + d\Omega^2$$

- Looks like spacetime of ($2d_\mathcal{U}$-2) extra dimensions!

$$ds^2 = -\left(1 - \left(\frac{R_S}{r}\right)^{n+1}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{R_S}{r}\right)^{n+1}} + d\Omega^2$$

If $\Lambda_\mathcal{U} \sim$ 1 TeV, new EW physics without extra dimensions!

- Horizon (M_{Pl} independent):

$$r_H \approx \left(\frac{2M_{BH}\Gamma_{d_\mathcal{U}}}{M_\mathcal{U}^2 \Lambda_\mathcal{U}^{-1}}\right)^{\frac{1}{2d_\mathcal{U}-1}} \Lambda_\mathcal{U}^{-1}$$

Unparticle BH thermodynamics

*Gaete, Helayel-Neto, Spallucci, PLB**693** (2010) 155*
*Mureika and Spallucci, PLB**693** (2010) 129*

- **Hawking temperature:**

$$T_{d_\mathcal{U}} = \frac{1}{4\pi\left[1 \pm \left(\frac{R_\mathcal{U}}{r}\right)^{2d_\mathcal{U}-2}\right]} \cdot \left[1 \pm (2d_\mathcal{U} - 1)\left(\frac{R_\mathcal{U}}{r}\right)^{2d_\mathcal{U}-2}\right]$$

- **Thermal limits:**

$$T_{d_U} \simeq T_H = \frac{1}{4\pi r_H}$$

Weak coupling
($r >> R_\mathcal{U}$)

Area dimension

$$T_{d_U} \simeq \frac{2d_U - 1}{4\pi r_H}$$

Strong coupling
($r << R_\mathcal{U}$)

$$T_H = \frac{D-3}{4\pi r_H} = \frac{1+1+[d]_{\text{area}}-3}{4\pi r_H} = \frac{[d]_{\text{area}}-1}{4\pi r_H}$$

BH entropy

- **First law:**

$$dM = T_{d_{\mathcal{U}}}\, dS \quad \Longrightarrow \quad \frac{\partial M}{\partial r} dr = T_{d_{\mathcal{U}}}\, dS$$

- **Thermal limits:**

Weak coupling (r >> R)

$$dS \sim 2\pi r dr \quad \Longrightarrow \quad S = \pi r^2 = \frac{A}{4}$$

Strong coupling (r << R)

$$dS \simeq \frac{2\pi}{R_{\mathcal{U}}^{2d_U - 2}} r^{2d_U - 1} dr$$

$$S \sim r_H^{2d_{\mathcal{U}}}$$

"Fractal Area"

Unparticle black hole horizon is a *fractal surface* of area $2d_U$

Unparticle black hole horizon is a *fractal surface* of area $2d_U$

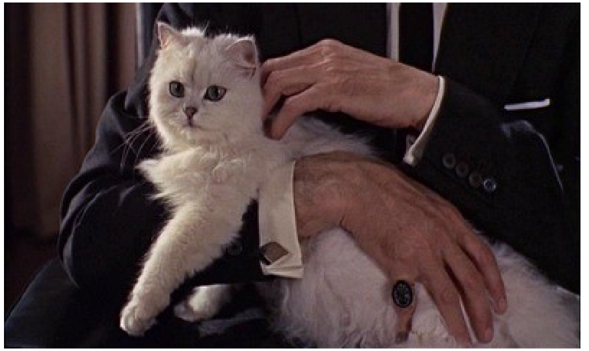
$$d_{\mathcal{U}} = 1 \quad \longrightarrow \quad A_H = \pi r^2$$

$$d_{\mathcal{U}} > 1 \quad \longrightarrow \quad A_H = \pi r^{2 d_{\mathcal{U}}}$$

Fractal dimensions

- Hausdorff

- Spectral



- QM, string theory, NC geometry

  Abbott and Wise, Am.J.Phys. 49 (1981) 37-39

  Ansoldi, Aurilia and Spallucci, PRD 56, 2352 (1997)

  Nicolini and Niedner, PRD 83, 024017 (2011)

- CDT, LQG, ASG, NC geometry

  Ambjorn et al. PRL 95 (2005) 171301

  Lauscher and Reuter, JHEP 0510 (2005) 050

  Modesto, CQG 26 (2009) 242002

  Benedetti PRL 102 (2009) 111303

  Modesto and Nicolini, PRD (2010)

Spectral vs Unspectral

[Nicolini & Spallucci, PLB (2011)]

$$\Delta K(x, y; s) = \frac{\partial}{\partial s} K(x, y; s)$$

$$K(x, y; 0) = \frac{\delta^d(x-y)}{\sqrt{\det g_{ab}}}$$

$$P(s) = \frac{\int d^d x \sqrt{\det g_{ab}}\, K(x, x; s)}{\int d^d x \sqrt{\det g_{ab}}}$$

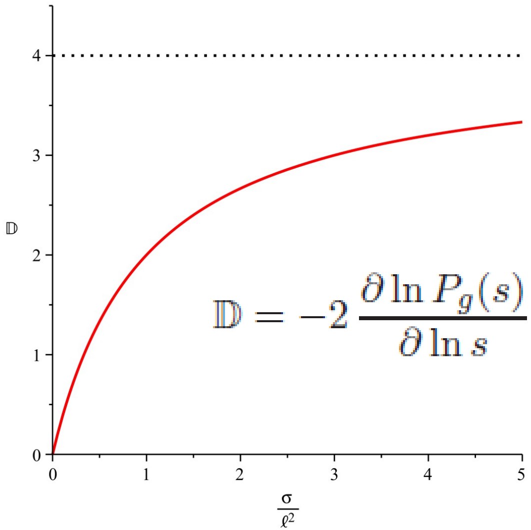
$$\Delta_U K_U(x, y; s) = \frac{\partial}{\partial s} K_U(x, y; s)$$

$$\Delta_U = \Delta - (d_U - 1)/s$$

time dependent "heat source" $(d_U - 1)/s$.

$$\mathbb{D}_U = -2s \frac{\int d^d x \sqrt{\det g_{ab}}\, \Delta K_U(x, x; s)}{\int d^d x \sqrt{\det g_{ab}}\, K_U(x, x; s)} + \frac{2\Gamma(d_U)}{\Gamma(d_U - 1)}$$

$$= \mathbb{D} + 2d_U - 2$$



$$\mathbb{D} = -2 \frac{\partial \ln P_g(s)}{\partial \ln s}$$

$s >> \ell \implies \mathbb{D}_{\mathcal{U}} = d - 2 + 2d_{\mathcal{U}}$ — Classical geometry

$s \sim \ell^2 \implies \mathbb{D}_{\mathcal{U}} = 2d_{\mathcal{U}} - 2 + \frac{d}{2}$ — Planck-scale geometry

$s << \ell^2 \implies \mathbb{D}_{\mathcal{U}} = 2d_{\mathcal{U}} - 2$ — Post-Planck regime

Entropic ungravity

Nicolini, PRD **82** (2010) 044030

- Entropy gravity: *[Verlinde JHEP (2011) 1104:029; Padmanabhan, MPLA 25 (2010) 1129]*

$$E = F\Delta x = T\Delta S \quad \Rightarrow \quad F_{entropic} = T\frac{\Delta S}{\Delta x} \quad \Longrightarrow \quad F = T\cdot\frac{\Delta S}{\Delta x} = \frac{mc}{\hbar}\left(\frac{2\pi MG\hbar}{2\pi R^2 c}\right) = \frac{GmM}{R^2}$$

- Holographic corrections

$$\Delta S_\Omega = k_B \Delta A\left(\frac{c^3}{4\hbar G} + \frac{\partial s}{\partial A}\right) \quad \Longrightarrow \quad F = \frac{GMm}{r^2}\left[1 + 4\ell_P^2 \frac{\partial s}{\partial A}\right]$$

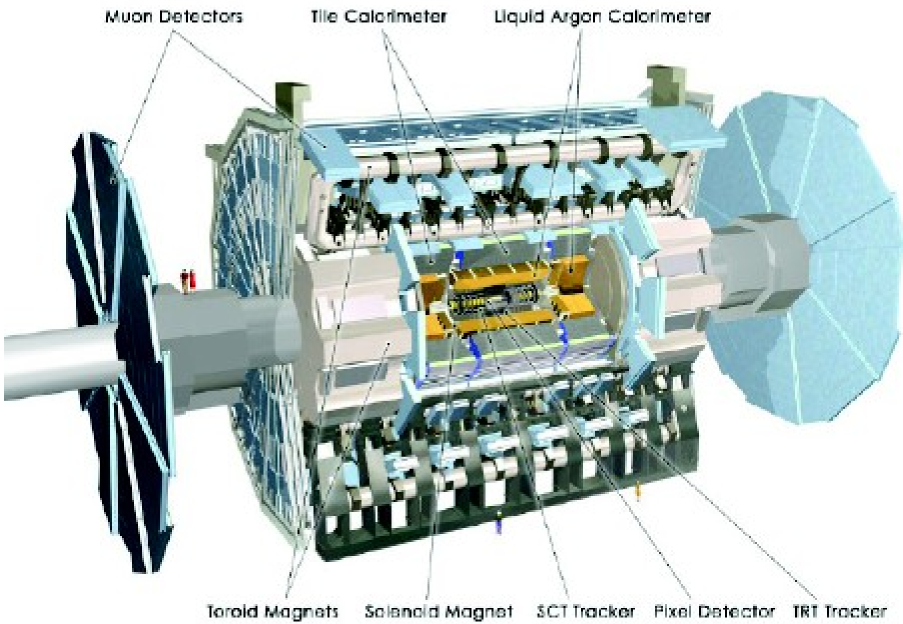
- Ungravity corrections:

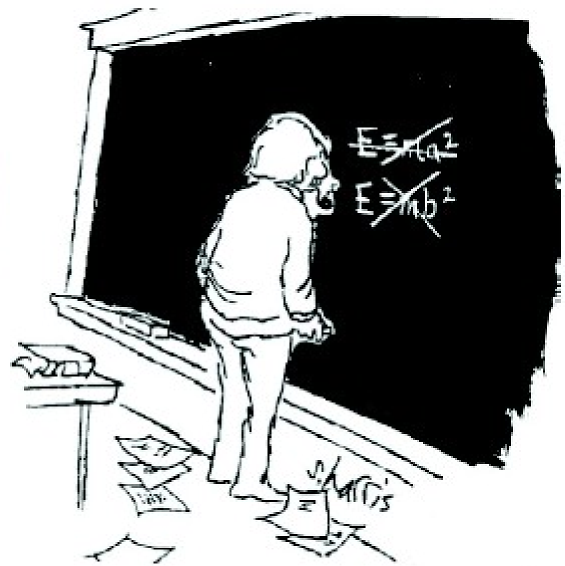
$$S = \frac{k_B c^3}{\hbar G}\frac{\pi R^{2-2d_U}}{d_U \Gamma_U} r_H^{2d_U}$$

$$F = \frac{GMm}{r^2}\left[1 - \frac{\Gamma_U\left(\frac{R}{r_H}\right)^{2d_U-2}}{1+\Gamma_U\left(\frac{R}{r_H}\right)^{2d_U-2}}\right]$$

Experimentally testable!

Un-particle physics tests

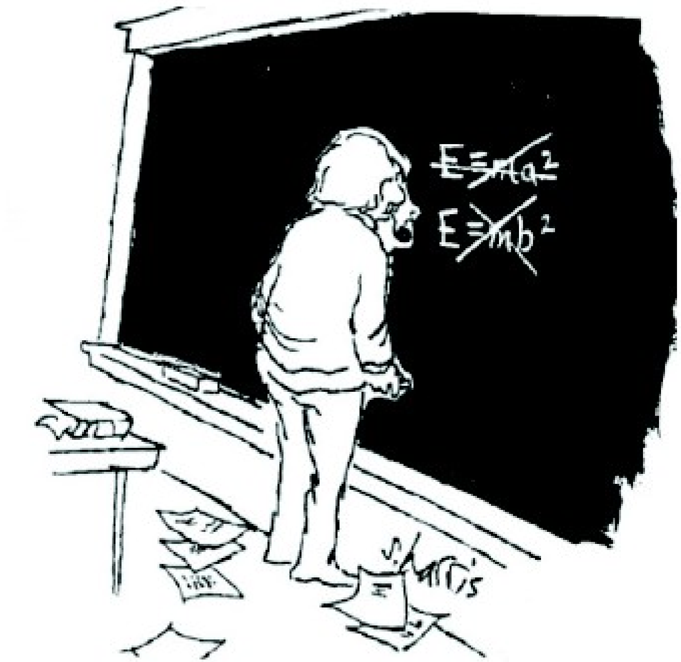


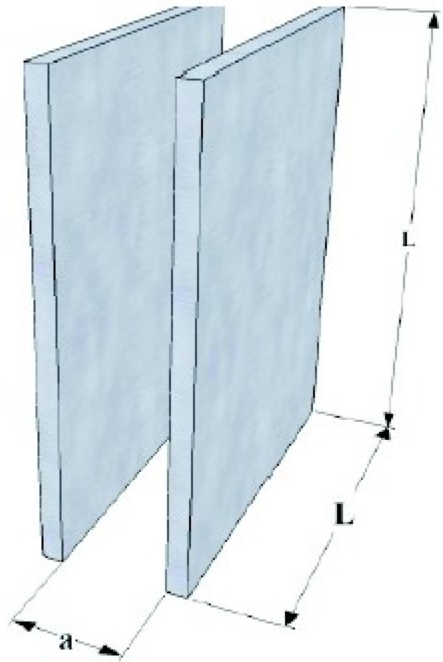


- The interaction term depends on a dimensionless coupling constant $\lambda = (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^k$

Un-particle Casimir effect

We will discuss the Casimir effect assuming the existence of a scalar unparticle sector weakly coupled to the standard model fields





Scalar case scenario

Working in Euclidean space one can use a Schwinger representation for $\mathbf{D}(p; m^2)$ to get:

$$\mathbf{D}_{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm^2 (m^2)^{d_{\mathcal{U}}-2} \int_0^\infty ds\, e^{-s(p^2+m^2)}$$

$$= \frac{16\pi^{5/2}\Gamma(d_{\mathcal{U}}+1/2)\Gamma(2-d_{\mathcal{U}})}{(2\pi)^{2d_{\mathcal{U}}+1}(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}\Gamma(2d_{\mathcal{U}})} (p^2)^{d_{\mathcal{U}}-2}$$

we can relate the unparticle Casimir energy $\mathcal{E}_{\mathcal{U}}^{C}$ with the standard Casimir energy of a particle field of fixed mass $\mathcal{E}^{C}(m^2)$:

$$\boxed{\mathcal{E}_{\mathcal{U}}^{C} = \frac{A_{d_{\mathcal{U}}}}{2\pi(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} \int_0^\infty dm^2 (m^2)^{d_{\mathcal{U}}-2}\, \mathcal{E}^{C}\,(m^2)}$$

Un-Casimir effect

[Frassino, Nicolini & Panella (2013)]

$$\mathcal{E}^{C}_{\mathcal{U}} = -\frac{1}{8\pi^2}\frac{1}{a}\frac{A_{d_\mathcal{U}}}{\pi(\Lambda^2_\mathcal{U})^{d_\mathcal{U}-1}}\sum_{n=1}^{\infty}\frac{1}{n^2}\int_0^\infty dm\, m^{2d_\mathcal{U}-1}K_2(2amn)$$

$$\mathcal{E}^{C}_{\mathcal{U}}(a) = -\frac{1}{a^3}\frac{d_\mathcal{U}\,\zeta(2+2d_\mathcal{U})}{(4\pi)^{2d_\mathcal{U}}}\frac{1}{(a\Lambda_\mathcal{U})^{2d_\mathcal{U}-2}}$$

The total attractive energy reads

$$\mathcal{E}^{C}(a) = -\frac{\pi^2}{720a^3}\left[1+\frac{720\,d_\mathcal{U}\,\zeta(2+2d_\mathcal{U})}{\pi^2(4\pi)^{2d_\mathcal{U}}}\frac{1}{(a\Lambda_\mathcal{U})^{2d_\mathcal{U}-2}}\right]$$

Plate fractalization

$$\mathbb{D} = -\frac{\partial \log\left(\mathcal{E}^C(a)\right)}{\partial \log a} - 1$$

$$\mathbb{D} = \frac{2 + (2d_{\mathcal{U}})L}{1 + L}; \qquad L = \frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2 (4\pi)^{2d_{\mathcal{U}}}} \frac{1}{(a\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}}$$

$\mathbb{D} \to 2$: For large plate separation $a \gg 1/\Lambda_{\mathcal{U}}$ or $d_{\mathcal{U}} \to 1$

$\mathbb{D} \to 2d_{\mathcal{U}}$ in the unparticle dominated case $a \ll 1/\Lambda_{\mathcal{U}}$

Estimate of un-particle scale

If Δ_{Cas} is the relative error of the experimental measurement we obtain

$$\frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2(4\pi)^{2d_{\mathcal{U}}}} \frac{1}{(a\Lambda_{\mathcal{U}})^{2d_{\mathcal{U}}-2}} \leq \Delta_{\mathrm{Cas}}.$$

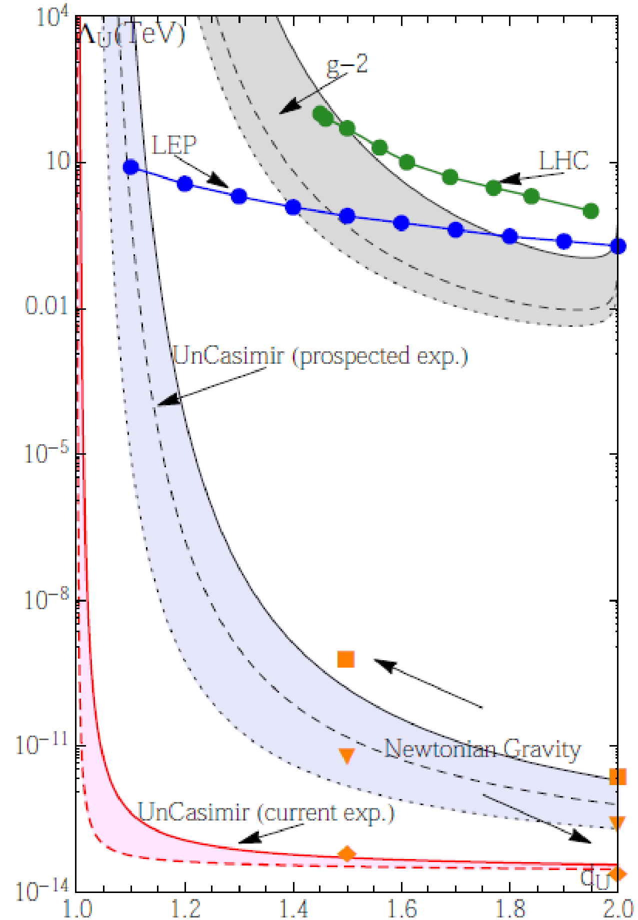
Therefore we get (for $d_{\mathcal{U}} \neq 1$)

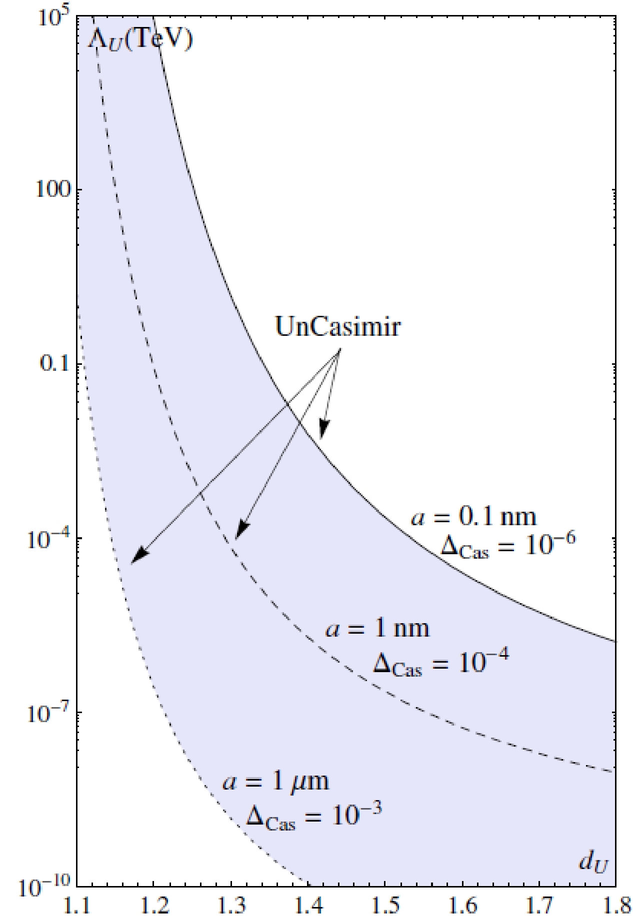
$$\Lambda_{\mathcal{U}} \geq \frac{1}{a} \left[\frac{720\, d_{\mathcal{U}}\, \zeta(2+2d_{\mathcal{U}})}{\pi^2(4\pi)^{2d_{\mathcal{U}}}} \frac{1}{\Delta_{\mathrm{Cas}}} \right]^{\frac{1}{2d_{\mathcal{U}}-2}}$$

strong dependence on the parameter $d_{\mathcal{U}}$:

- $d_{\mathcal{U}}$ slightly above 1 $\Rightarrow$ the bound on $\Lambda_{\mathcal{U}}$ is very strong
- $d_{\mathcal{U}}$ increases $\Rightarrow$ the bound exponentially decreases

Exclusion curves





The coupling is hidden!

- It disappears only in the perfect conductor limit

$$\gamma \equiv \frac{1}{\omega_{\rm pl}^2}\frac{e^2}{\hbar}\frac{c}{a^2} \quad \sim 10^{-5}$$

$$\alpha_{\rm EM} \approx 1/137 \gg \gamma = 10^{-5}$$

- Three scenarios

$$\lambda \gg 10^{-5}$$

: $\lambda \sim \gamma$ one has the critical case.

$$\lambda \ll \gamma$$

Conclusions

- Un particles offer new intriguing scenarios
- The Casimir effect offers a reliable testbed
- Vector case Casimir calculation (in preparation)

Molte grazie!

nicolini@fias.uni-frankfurt.de