Hadronic effects in the exclusive $b \to s$ transitions

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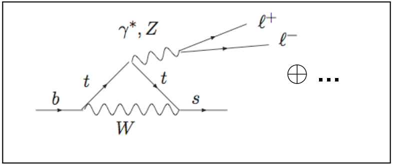
Outline

- “anatomy” of the exclusive $b \to s$ transitions
- $B \to K^{(*)}$ form factors, current accuracy
- nonlocal /nonfactorizable hadronic effects
  $\Rightarrow$ process-dependent function $\Delta C_9^{(B\to K^{(*)})}(q^2)$
- $B \to K\ell^+\ell^-$ at large hadronic recoil
- comments on $B \to K^*\ell^+\ell^-$ at large recoil
- low hadronic recoil region

$b \to s$ transitions at the quark level

- loops with virtual W, t, Z at scales $\geq m_b$

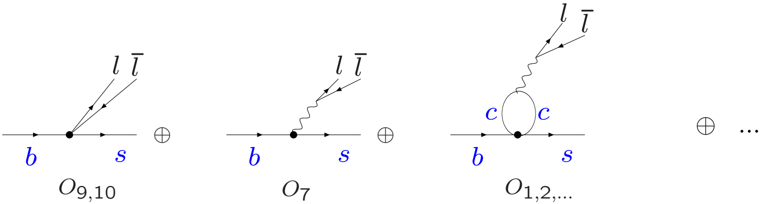


- set of effective pointlike operators

$$\Downarrow$$

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \Big|_{\mu \sim m_b}$$

- $b \to s\ell^+\ell^-$ at the quark level



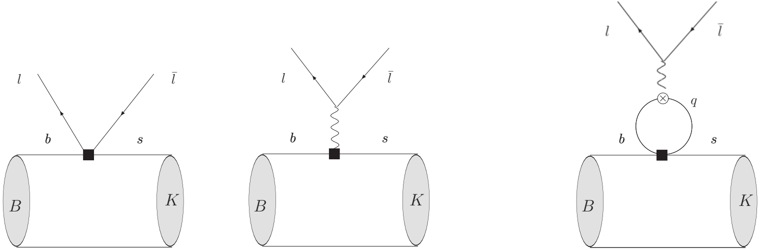
- "background" effects due to $O_{1,2}$ (charm loops),...

  interference of "ordinary" weak transition and e.m. interactions

Exclusive $b \to s$ transitions

- measured at LHCb with continuously improved precision

  - semileptonic modes: $B \to K\ell^+\ell^-$, $B \to K^*\ell^+\ell^-$,...
    theory: effective operators sandwiched between hadronic states



  - exclusive radiative mode: $B \to K^*\gamma$, "byproduct" of $B \to K^*\ell^+\ell^-$

  - leptonic decay mode: $B_s \to \mu^+\mu^-$
    single hadronic parameter: $f_{B_s} \sim \langle 0|\bar{s}\gamma_5 b|B_s\rangle$

Anatomy of the $B \to K^{(*)} \ell^+\ell^-$ decay amplitude

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \langle K^{(*)}\ell^+\ell^- \mid O_i \mid B\rangle$$

- isolating hadronic matrix elements:

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} \Bigg[(\bar{\ell}\gamma^\mu\gamma_5\ell)\, C_{10} \langle K^{(*)}|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle$$
$$+(\bar{\ell}\gamma^\mu\ell)\Big(C_9 \langle K^{(*)}|\bar{s}\gamma_\mu b|B\rangle + C_7 \frac{2m_b}{q^2} q^\nu \langle K^{(*)}|\bar{s} i\sigma_{\nu\mu}(1+\gamma_5)b|B\rangle$$
$$+\frac{8\pi^2}{q^2} \sum_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^\rho \Big)\Bigg]$$

- nonlocal matrix elements

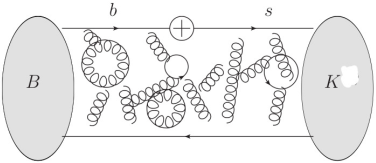
$$\boxed{\mathcal{H}_i^\rho(q,p) = \langle K^{(*)}(p)| i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle},$$

not all of them can be reduced to form factors and/or factorized!

$B \to K$ form factors

lepton-pair invariant mass q^2

$\Uparrow$



- hadronic matrix elements:

$\langle K(p)|\bar{s}\gamma_\mu b|B(p+q)\rangle \Rightarrow f^+_{BK}(q^2)$

$\langle K(p)|\bar{s}\sigma_{\mu\nu} b|B(p+q)\rangle \Rightarrow f^T_{BK}(q^2)$

- the kinematical region in $B \to K \ell^+\ell^-$,

$0 < q^2 < (m_B - m_K)^2 \simeq 23.0\,\text{GeV}^2$

- hadronic form factors described by nonperturbative QCD

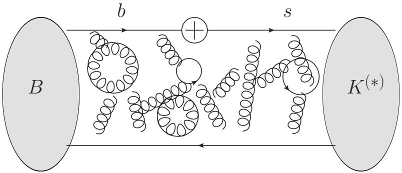
$B \to K^*$ form factors

- definitions: (alternatively: helicity form factors)

$$\langle K^*(p)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle \Rightarrow V_{BK^*}(q^2),\;\; A^{1,2,3}_{BK^*}(q^2)$$

$$\langle K^*(p)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|B(p+q)\rangle \Rightarrow T^{1,2,3}_{BK^*}(q^2)$$

- more complicated objects due to unstable final state K^*



- $K\pi \Leftrightarrow K^*$
  effectively resums in $\Gamma^{K^*}_{tot}$

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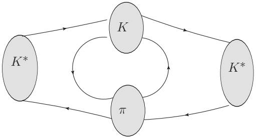


- resonance extraction
  in $B \to K\pi\ell^+\ell^-$,
  a model-dependent procedure:
  BW ansatz $\oplus$ nonresonant background,

QCD calculation of the form factors

- lattice QCD: small hadronic recoil (large q^2) region accessible

  most recent results:
  - $B \to K$ *HPQCD, P.Bouchard et al. (2013)*
  - $B \to K^*$ ("quenched"), *R. Horgan, Z. Liu, S. Meinel, M. Wingate (2013)*

- QCD light-cone sum rules (LCSR):
  form factors at large hadronic recoil ($q^2 \ll m_b^2$)

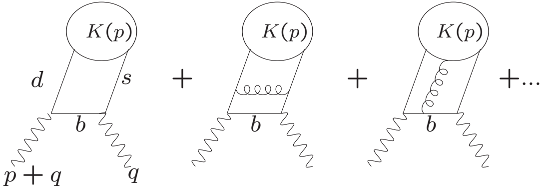
- universal tools:
  analyticity & unitarity, hadronic dispersion relations
  used for extrapolations in q^2

Light-Cone Sum Rules for $B \to K$ form factors

- introducing the correlation function

$$F_\mu(q,p) = i \int d^4x\, e^{iqx} \langle K(p) \mid T\{\bar{s}(x)\gamma_\mu b(x), \bar{b}(0) i\gamma_5 d(0)\} \mid 0 \rangle$$

$q^2, (p+q)^2 \ll m_b^2$



- *b-quark highly virtual*

  $\Rightarrow$ operator-product expansion near light-cone ($x^2 \sim 0$)

LCSR method: [I.Balitsky,V.Braun et al (1989); V.Chernyak, I.Zhitnitsky (1989)]

$B \to K$ form factor [V.Belyaev, A.K., R.Rückl (1993)]

Calculation of the correlation function

- the result of OPE

$$F_\mu(q,p) = \sum_{t=2,3,4,..} \int du\, T_\mu^{(t)}(q^2,(p+q)^2,m_b^2,\alpha_s,u)\, \varphi_K^{(t)}(u,\mu)$$

hard scattering ampl. $\otimes$ kaon light-cone DA

- nonperturbative objects: distribution amplitudes (DA's): vacuum-kaon hadronic matrix elements, e.g.,

$$\langle K(q)|\bar{s}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -iq_\mu f_K \int_0^1 du\, e^{iuqx}\varphi_K(u)\,.$$

- DA's, defined originally for the pion, $SU(3)_{fl}$ breaking in f_K/f_π and in DA's (Gegenbauer expansion)

- $t = 3,4$ contributions (soft gluon) power suppressed,

Calculation of the correlation function

- currently achieved accuracy

$$F(q^2,(p+q)^2) = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)}\right) \otimes \varphi_K^{(2)}$$

$$+\frac{\mu_\pi}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_K^{(3)} + \frac{\delta_\pi^2}{m_b\chi}T^{(4)} \otimes \varphi_K^{(4)} + \ldots$$

$\mu_K = m_K^2/(m_s+m_u), \quad m_b \gg \chi \gg \Lambda_{QCD}$

- LO twist 2,3,4 $q\bar{q}$ and $q\bar{q}G$ terms

  [V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]

- NLO $O(\alpha_s)$ twist 2, (collinear factorization)

  [A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]

- NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)

  [P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]

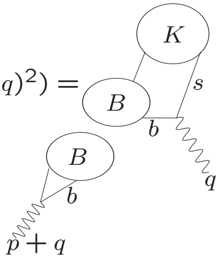
- part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 *[A. Bharucha (2012)]*

Derivation of LCSR

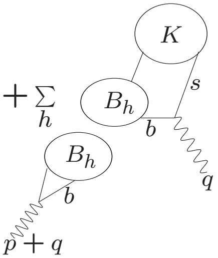
- Hadronic dispersion relation in the variable $(p+q)^2$, (fixed $q^2 \ll m_b^2$)

inserting the total set of B -states, isolating B-meson pole:

$F(q^2,(p+q)^2) =$



$+\sum_h$



$f_B \; f^+_{BK}(q^2)$ $\qquad\qquad$ $\sum_{B_h} \to duality \; (s_0^B)$

- quark-hadron duality approximation
- f_B taken from two-point QCD sum rule

How accurate are LCSR's

- the "raw" sum rule: {OPE = dispersion relation}

$$[F(q^2,(p+q)^2)]_{OPE} = \frac{m_B^2 f_B f_{BK}^+(q^2)}{m_B^2-(p+q)^2} + \int_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s,q^2)]_{OPE}}{s-(p+q)^2}$$

$\Uparrow$ $\overline{m}_b$, α_s, $\varphi_K^{(t)}(u)$, t=2,3,4;

$\Uparrow$ QCD SR for f_B

$\Uparrow$ quark-hadron duality

- the input for kaon DA's (Gegenbauer moments, normalization coeffs.):
  - two-point QCD sum rules
  - LCSR's for the kaon e.m. form factors

- "systematic error" of quark-hadron duality approximation
  (suppressed with Borel transformation, controlled by the m_B calculation)

- at $0 < q^2 \leq 12 - 14$ GeV^2 estimated uncertainties for $B \to \pi, K$ form factors amount to $\pm(12-15)\%$

$B_{(s)}$ and $D_{(s)}$ decay constants

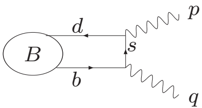
[P.Gelhausen, AK, A.A.Pivovarov, D.Rosenthal, 1305.5432 hep/ph]

Decay constant	Lattice QCD [ref.]	this work
f_B[MeV]	196.9 ± 9.1 [1] 186 ± 4 [2]	207^{+17}_{-9}
f_{B_s}[MeV]	242.0 ± 10.0 [1] 224 ± 5 [2]	242^{+17}_{-12}
f_{B_s}/f_B	1.229 ± 0.026 [1] 1.205 ± 0.007 [2]	$1.17^{+0.04}_{-0.03}$
f_D[MeV]	218.9 ± 11.3 [1] 213 ± 4 [2]	201^{+12}_{-13}
f_{D_s}[MeV]	260.1 ± 10.8 [1] 248.0 ± 2.5 [2]	238^{+13}_{-23}
f_{D_s}/f_D	1.188 ± 0.025 [1] 1.164 ± 0.018 [2]	$1.15^{+0.04}_{-0.05}$

[1]-Fermilab/MILC, [2]-HPQCD

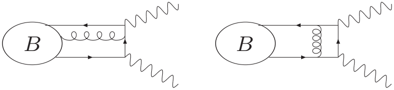
LCSR with B-meson distribution amplitudes

- In the correlation function:
  B-meson ⇒ on-shell state,
  light meson ⇒ current,



(a)

- *B*-meson DA's,
  defined in HQET,
  light mesons
  from quark-hadron duality



(b) (c)

- so far only tree-level
  calculations, 2,3-particle DA's

- all $B \to \pi, K^{(*)}, \rho$ form factors calculated at $q^2 \leq 10$ GeV^2
  [A.K., Th.Mannel, N.Offen (2007)]

- LCSR in SCET [F. De Fazio, Th. Feldmann T.Hurth (2006)]

Extrapolation to large q^2

- Series parameterization of form factors based on conformal mapping:

  ...., [Boyd,Grinstein,Lebed(1995)],... [Bourrely,Caprini, Lellouch(2008)]

$$z(q^2,\tau_0) = \frac{\sqrt{\tau_+ - q^2} - \sqrt{\tau_+ - \tau_0}}{\sqrt{\tau_+ - q^2} + \sqrt{\tau_+ - \tau_0}}$$

$$\tau_+ = (m_B + m_{K^{(*)}})^2\,, \qquad \tau_- = (m_B - m_{K^{(*)}})^2 \;\; \tau_0 = \tau_+ - \sqrt{\tau_+ - \tau_-}\sqrt{\tau_+}\,.$$

- we use BCL parameterization

$$F(q^2) = \frac{F(0)}{1 - q^2/m^2_{B_s(J^P)}}\Big\{1 + b_1\Big(z(q^2,t_0) - z(0,t_0) + \frac{1}{2}\big[z(q^2,t_0)^2 - z(0,t_0)^2\big]\Big)\Big\}\,,$$

- allows to extra(inter)polate the LCSR and lattice QCD FF's

$B \to K, K^{(*)}$ form factors from LCSR's

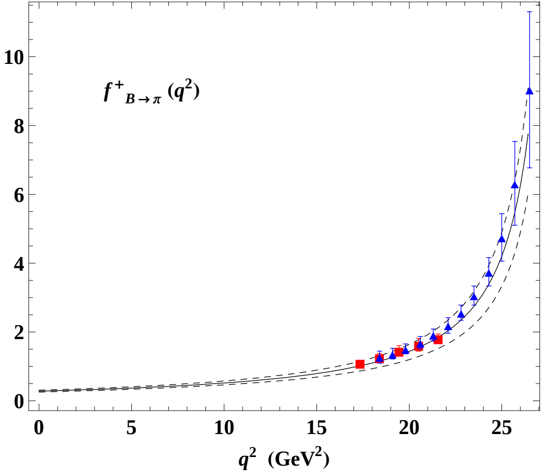
[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

form factor	$F^i_{BK^{(*)}}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12$ GeV^2
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B^*_s(1^-)$	LCSR with K DA's
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B^*_s(1^-)$	
V^{BK^*}	$0.36^{+0.23}_{-0.12}$	$-4.8^{+0.8}_{-0.4}$	$B^*_s(1^-)$	LCSR with B DA's
$A^{BK^*}_1$	$0.25^{+0.16}_{-0.10}$	$0.34^{+0.86}_{-0.80}$	$B_s(1^+)$	
$A^{BK^*}_2$	$0.23^{+0.19}_{-0.10}$	$-0.85^{+2.88}_{-1.35}$	$B_s(1^+)$	
$A^{BK^*}_0$	$0.29^{+0.10}_{-0.07}$	$-18.2^{+1.3}_{-3.0}$	$B_s(0^-)$	
$T^{BK^*}_1$	$0.31^{+0.18}_{-0.10}$	$-4.6^{+0.81}_{-0.41}$	$B^*_s(1^-)$	
$T^{BK^*}_2$	$0.31^{+0.18}_{-0.10}$	$-3.2^{+2.1}_{-2.2}$	$B_s(1^+)$	
$T^{BK^*}_3$	$0.22^{+0.17}_{-0.10}$	$-10.3^{+2.5}_{-3.1}$	$B_s(1^+)$	

correlations between normalization & slope out of the scope

$B \to \pi$ form factor: LCSR vs lattice QCD

[A.K, Th.Mannel, N.Offen, Y-M. Wang (2011)]



$q^2 \leq 12$ GeV^2 -LCSR, $q^2 > 12$ GeV^2 - [HPQCD, FNAL/MILC]

$B \to K$ form factor: LCSR vs lattice QCD

- dashed: LCSR, central input

[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

- solid: unitarity bounds for the z-transformed form factor,

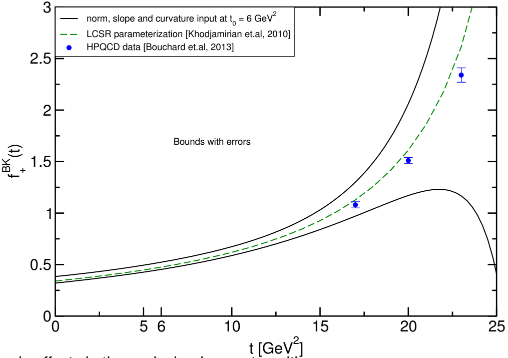
[L.Lellouch (1996); Th.Mannel,B.Postler(1998)]

(PRELIMINARY),
S.Imsong, AK, Th.Mannel,
work in progress

input for the bounds :

$f^+_{BK}(q^2 = 6.0~\text{GeV}^2)$

$\oplus$ slope $\oplus$ curvature



Status of the $B \to K^*$ form factors

- LCSR's for $B \to K^*$ form factors
  [P. Ball, V.Braun (1998), P.Ball, R. Zwicky (2004,...)]
  DA's of vector mesons worked out up to twist 4
  P. Ball, V. M. Braun, Y. Koike and K. Tanaka, (1998),...
- input $\oplus$ duality uncertainties
  in the same ballpark as for $B \to K$
- $\Gamma_V = 0$ approximation ("quenched") $\Rightarrow$ additional uncertainty
- the same in LCSRs with B-meson DA's (see above table)
  BW ansatz (a model !) with $\Gamma_V \neq 0$
  for the K^* pole can be included in the sum rule...

Comments on $B \to V$ form factors

- the problem more general, concerns also experiment:
  ρ (K^*) are strongly coupled to 2π ($K\pi$) in P-wave:

  - a simple constraint on the invariant mass of 2π ($K\pi$) state
  may not be accurate enough

  - scalar, tensor resonances in 2π ($K\pi$):
  a more detailed angular analysis is needed

- on the theory side:

  - $B \to \pi\pi\ell\nu_\ell$ ($B \to K\pi\ell\ell$) in terms of generic
    $B \to \pi\pi$ ($B \to K\pi$) form factors,
  including the partial expansion and resonance contributions;

  S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, 1310.6660

  - estimating $B \to$ light scalar, tensor meson form factors

- more work in this direction is on the way:
  LCSR's with 2-pion DAs

Hadronic input in $B \to K\ell^+\ell^-$

- after inserting the $B \to K$ form factors:

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{\pi} V_{tb}V_{ts}^* \Bigg[\bar{\ell}\gamma_\mu \ell\, p^\mu \Bigg(C_9 f^+_{BK}(q^2)$$

$$+\frac{2(m_b+m_s)}{m_B+m_K} C_7^{eff} f^T_{BK}(q^2) + \sum_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^{(BK)}(q^2)\Bigg) + \bar{\ell}\gamma_\mu\gamma_5 \ell\, p^\mu C_{10} f^+_{BK}(q^2)\Bigg]$$

- new nonlocal $B \to K$ hadronic matrix elements:

$$\langle K(p)|i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle = [(p\cdot q)q^\rho - q^2 p^\rho]\mathcal{H}_i^{(BK)}(q^2)$$

$j^\rho_{em} = \sum\limits_{q=u,d,s,c,b} e_q \bar{q}\gamma^\rho q$, O_i -quark-gluon effective operators

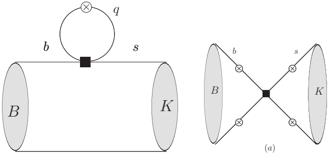
- a (process- and q^2-dependent) correction to C_9:

$$\boxed{\Delta C_9^{(BK)}(q^2) = \frac{\sum\limits_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^{(BK)}(q^2)}{f^+_{BK}(q^2)}}$$

The (topological) diagrams

- LO: factorizable loop, weak annihilation

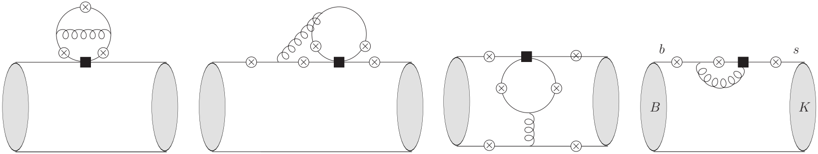
$\otimes$ -virtual photon



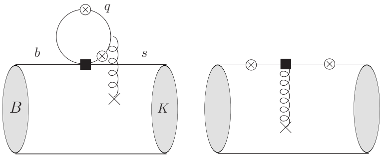
the hierarchy of operators

(Wilson coeffs): $O_i = O_{1,2}^{(c)}$, O_{8g}, $O_{3,4,5,6}^{(q)}$, $O_{1,2}^{(u)}$

- NLO: partially factorizable ...



- soft (low virtuality) gluons, nonfactorizable



...

Theory of nonlocal effects

- earlier estimates: simple loop, neglecting nonfactorizable effects, adding resonances to the loop,

  $\Rightarrow$ kink in the branching fraction at $q^2 = 4m_c^2$, double counting,...

- QCD factorization approach at large recoil region $q^2 \ll M^2_{J/\psi}$

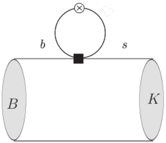
  [M.Beneke, Th.Feldmann, D.Seidel (2001)]

- estimating soft-gluon nonfactorizable effects
- OPE at spacelike $q^2 \Rightarrow$ hadronic disperison relations at timelike q^2

  [A.K., Th. Mannel, A. Pivovarov ,Yu-M. Wang, (2010)];

  [A.K., Th. Mannel and Yu-M. Wang, (2013)]

Charm loops in $B \to K^{(*)}\ell^+\ell^-$



- the largest nonlocal effect:

  $O^c_{1,2} = (\bar{s}\Gamma^a_\mu c)(\bar{c}\Gamma^{\mu a} b) \quad j^\rho_{em,c} = e_c \bar{c}\gamma^\rho c$

- virtual c-quarks $\Rightarrow$ quark-loop diagrams, applicable at $q^2 \ll 4m_c^2$ (large recoil of $K^{(*)}$)

- operator-product expansion (OPE) near the light-cone

  $$\boxed{i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}}$$

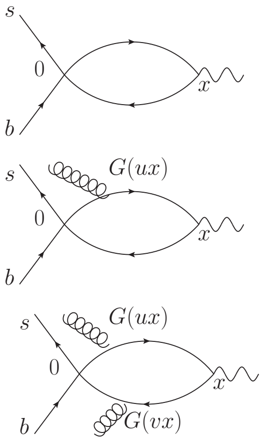
- $q^2 \ll 4m_c^2, \quad \langle x^2 \rangle \sim 1/(2m_c - \sqrt{q^2})^2$

- $q^2 \to 4m_c^2$ OPE diverges,
  i.e. the quark-loop approximation invalid

- at $q^2 \to m^2_{J/\psi}$, $\bar{c}c$ loop becomes a hadronic state:
  $B \to K^{(*)}\ell^+\ell^-$ = $\{ B \to J/\psi K \otimes J/\psi \to \ell^+\ell^- \}$

Scheme of the operator-product expansion

- T- product of $\bar{c}c$-operators expanded near $x^2 \sim 0$:



the simple loop

loop-function $\otimes\ \bar{s}\Gamma b$

one-gluon emission:

nonlocal operator $\sim G_{\mu\nu}(ux)$, $0 < u < 1$

$$\widetilde{\mathcal{O}}(q) \sim \bar{s}\left(\frac{1}{4m_c^2 - q^2 - q\cdot(iD)}\right) G_{\mu\nu}\, b$$

two-gluon emission

$$\sim \frac{\Lambda_{QCD}^2}{4m_c^2 - q^2} \times \{\text{one-gluon term}\}$$

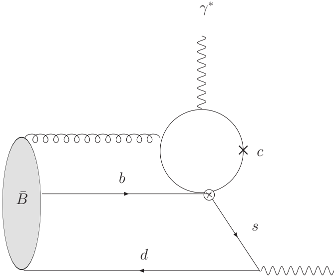
$\oplus\ O(\alpha_s)$ perturbative corrections

LCSR for the soft-gluon hadronic matrix element

- soft-gluon emission from charm loop reduced to an effective nonlocal operator $\widetilde{\mathcal{O}}(q)$

- the correlation function: (in terms of B meson DAs)

$$\mathcal{F}^{(B\to K)}(p,q) = i\int d^4y e^{ip\cdot y}\,\langle 0|T\{j^K(y)\widetilde{\mathcal{O}}(q)\}|B(p+q)\rangle\,,$$



- hadronic dispersion relation in the kaon channel $\oplus$ duality

$$\mathcal{F}^{(B\to K)}(p,q) = \frac{f_K}{m_K^2-p^2}\tilde{A}(q^2) + \int_{s_h}^{\infty} ds\,\frac{\tilde{\rho}(s,q^2)}{s-p^2}$$

Charm-loop effect in $B \to K\ell^+\ell^-$ in terms of ΔC_9

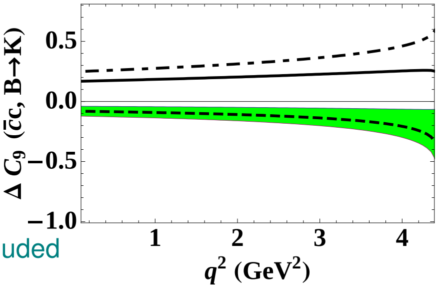
- the LO loop $\oplus$ soft-gluon contribution:

$$\Delta C_9^{(\bar{c}c,B\to K)}(q^2) = \frac{\mathcal{H}^{(B\to K)}(q^2)}{f_{BK}^+(q^2)} = (C_1 + 3C_2)\, g(m_c^2, q^2) + C_1 \frac{\tilde{A}(q^2)}{f_{BK}^+(q^2)}$$

$g(m_c^2, q^2)$ the standard c-loop function

$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}$,
$(\mu = m_b)$

loop (dash-dotted),
soft-gluon (dotted),
total (solid)



- $O(\alpha_s)$ contribution not yet included
- cf. $C_9(\mu = m_b) \simeq 4.4$

Accessing the large q^2 region

- analyticity of the hadronic matrix element in q^2,
  $\oplus$ unitarity $\Rightarrow$ hadronic dispersion relation:

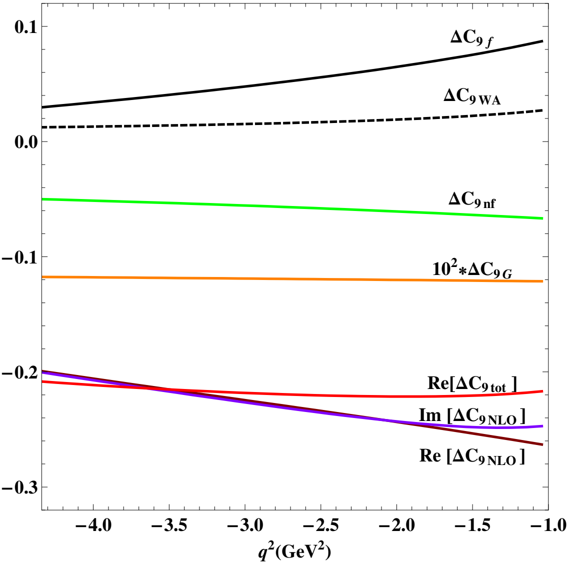
$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2 \Big[\sum_{\psi=J/\psi,\psi(2S),..} \frac{f_\psi A_{B\psi K}}{m_\psi^2 (m_\psi^2 - q^2 - i m_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)} \Big]$$

- the residues $|A_{B\psi K}|$ and $|f_\psi|$ determined by $BR(B\to\psi K)$, $BR(\psi\to\ell^+\ell^-)$, eff.pole ansatz for the rest
- complex FSI phase in each $A(B\to\psi K)$, (Im part in $(p+q)^2$)
  destructive interferences between different ψ terms possible !
- we fit disp. relation for $\mathcal{H}^{(B\to K)}$ at $q^2 \ll 4m_c^2$
  to the calculated OPE result $\Rightarrow$ model-dependence inevitable
  results stable at $q^2 < 6-8$ GeV^2

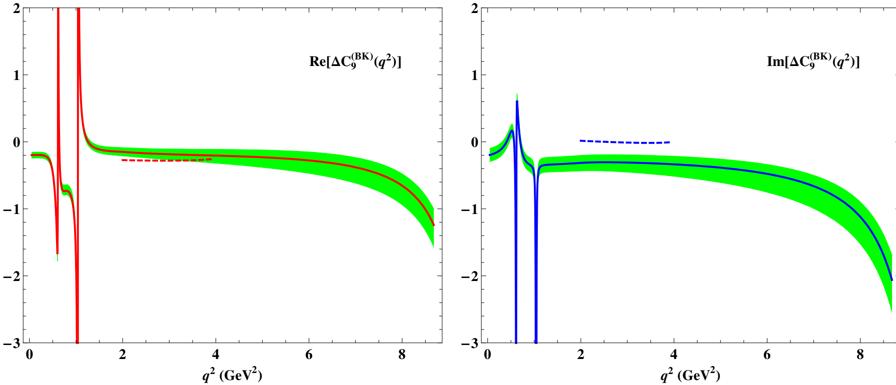
A complete hadronic input for $B \to K\ell^+\ell^-$

- all operators $O_{1,2}$, O_{8g}, O_{3-6} included
- quark-loop soft-gluon effects at $q^2 < 0$ calculated
- soft-gluon emission from gluonic penguin operator (new LCSR calculation)
- hard-gluon effects estimated at $q^2 < 0$ employing QCDF (partly cross-checked with LCSRs)
- LO weak annihilation (small effect)
- dispersion relation includes $V = \rho, \omega, \phi$ in addition to $V = J/\psi, \psi'$; the parameters fitted with $q^2 < 0$ calculation and with measured $BR(B \to VK)$.

$\Delta C_9^{(BK)}(q^2 < 0)$



$\Delta C_9(q^2)$ below J/ψ region



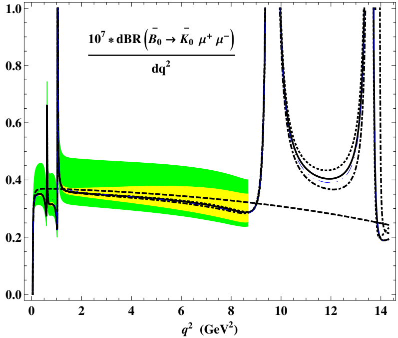
the red (blue) solid curve corresponds to the Re (Im) part obtained from the hadronic dispersion relation, fitted to the QCD calculation at q2 < 0 (central input, default parametrization). The shaded areas indicate the uncertainties. The dashed curves correspond to the prediction of QCDF.

$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

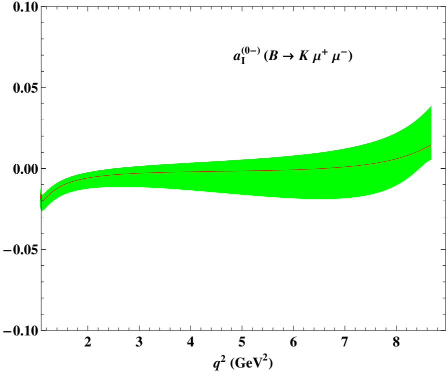
long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



$[q^2_{min}, q^2_{max}]$	Belle	CDF	LHCb	LHCb	this work
[0.05, 2.0]	$0.81^{+0.18}_{-0.16} \pm 0.05$	$0.33 \pm 0.10 \pm 0.02$	$0.21^{+0.27}_{-0.23}$	$0.56 \pm 0.05 \pm 0.03$	$0.71^{+0.22}_{-0.08}$
[2.0, 4.3]	$0.46^{+0.14}_{-0.12} \pm 0.03$	$0.77 \pm 0.14 \pm 0.05$	$0.07^{+0.25}_{-0.21}$	$0.57 \pm 0.05 \pm 0.02$	$0.80^{+0.27}_{-0.11}$
[4.3, 8.68]	$1.00^{+0.19}_{-0.08} \pm 0.06$	$1.05 \pm 0.17 \pm 0.07$	1.2 ± 0.3	$1.00 \pm 0.07 \pm 0.04$	$1.39^{+0.53}_{-0.22}$
[1.0, 6.0]	$1.36^{+0.23}_{-0.21} \pm 0.08$	$1.29 \pm 0.18 \pm 0.08$	$0.65^{+0.45}_{-0.35}$	$1.21 \pm 0.09 \pm 0.07$	$1.76^{+0.60}_{-0.23}$

Isospin asymmetry: $\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ vs $B^- \to K^- \ell^+ \ell^-$



- integrated over $1.0 < q^2 < 6.0$ GeV^2.

Belle (2009)	BaBar (2012)	LHCb (2012)	this work
$-0.41^{+0.25}_{-0.20} \pm 0.07$	$-0.41 \pm 0.25 \pm 0.01$	$-0.35^{+0.23}_{-0.27}$	$(-0.4)\% \div (-0.3)\%$

Nonlocal effects in $B \to K^* \ell^+ \ell^-$

- $B \to K^* \ell^+ \ell^-$, a rich set of observables, sensitive to new physics
- calculated: $\Delta C_9^{(\bar{c}c, B\to K^*)}(q^2)$ (simple c-loop $\oplus$ soft-gluon effect)
- three kinematical structures for the nonfactorizable part

(cf. three $B \to K^*$ form factors),

$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) - C_1 \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)} ,$$

- nonfactorizable part enhanced by $1/q^2$ factor

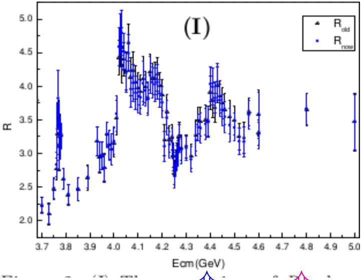
$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\ \mathrm{GeV}^2) = 0.7^{+0.6}_{-0.4} \qquad \Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\ \mathrm{GeV}^2) = 0.8^{+0.6}_{-0.4}$$

$$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\ \mathrm{GeV}^2) = 1.1^{+1.1}_{-0.7}$$

- to be completed with $O(\alpha_s)$ (QCDF), other operators, topologies
- timelike $q^2 \Rightarrow$ dispersion relation with $B \to K^*\psi, K^*\phi, \ldots$

Can we use the low-recoil, $q^2 > m^2_{\psi(2S)}$, region ?

- charmonium states above $\psi(2S)$: $\psi(3770), \psi(4040), \psi(4160), \psi(4415)$



measurement of $R(e^+e^- \to hadrons)$,
from: BESS Collab. 0705.4500 [hep-ex]

$\sqrt{q^2} = m_B - m_{K*} = 4.389\text{GeV}$

$\sqrt{q^2} = m_B - m_K = 4.784\text{GeV}$

- nonlocal effects: local OPE valid at $|q^2| \sim m_b^2 \gg 4m_c^2$,

B.Grinstein, D. Pirjol (2004); Beilich,Buchalla, Feldmann (2011)

- a duality ansatz *Beilich,Buchalla, Feldmann (2011)*

nonfactorizable effects (FSI phases in $B \to \psi K$) not included...

Summary

- $B \to K$ form factors accessible on the lattice and with LCSR's,
  $B \to K^*$ form factors demand additional efforts
- nonlocal effects accessible combining QCDF (perturbative factorizable effects) and LCSRs (soft-gluon effects)
  full analysis for $B \to K^* \ell\ell$ to be done
- hadronic dispersion relation fitted to OPE $\oplus$ data on $B \to \psi K$ allows to access large recoil region
- $B \to K\ell\ell$ at large recoil (small q^2): nonlocal effects small,
  a hint of a tension with LHCb (BR bins, isospin asymmetry)
- $B \to K^*\ell\ell$ at large recoil (small q^2):
  nonlocal effects are larger and have larger errors,
  influencing "form-factor independent" and "clean" observables ?

BACKUP FILES

B-meson DA's

- defined in HQET:

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

- key input parameter: the inverse moment of ϕ_+^B

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV

  [V.Braun, D.Ivanov, G.Korchemsky,2004]

- QCD sum rule based model for 3-particle DA's

[A.K., T.Mannel, N.Offen (2007)]

The local OPE limit

- $\omega \to 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\widetilde{\mathcal{O}}_\mu^{(0)}(q) = I_{\mu\rho\alpha\beta}(q)\bar{s}_L\gamma^\rho \widetilde{G}_{\alpha\beta} b_L \ ,$$

$$I_{\mu\rho\alpha\beta}(q, m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha} g_{\rho\beta}) \times \frac{1}{16\pi^2}\int_0^1 dt \, \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained
in $B \to X_s\gamma$ in *[M.Voloshin (1997)]*
in $B \to K^*\gamma$ *[A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]*

- the neccesity of resummation was discussed before
[Z. Ligeti, L. Randall and M.B. Wise,(1997);
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
J. W. Chen, G. Rupak and M. J. Savage,(1997);
G. Buchalla, G. Isidori and S.J. Rey (1997)]

Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

$$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_{1,2}\,,$$

$$\left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2}\,,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2}\,,$$
$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2}\,. \qquad (2)$$

[P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]