Magnetic Properties of Strongly Interacting Matter

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Based on work done in collaboration with C. Bonati, M. Mariti, F. Negro and F. Sanfilippo

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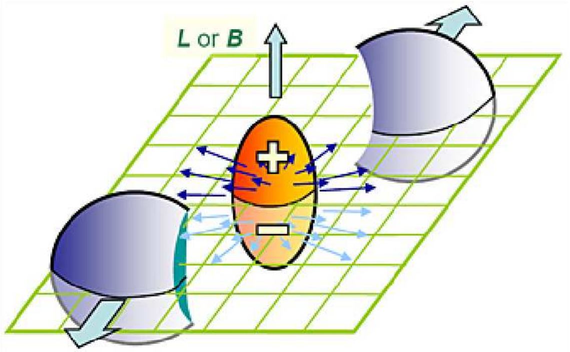
OUTLINE

1. Strong interactions in e.m. backgrounds and overview of lattice results

2. LQCD in a magnetic field and technical issues with the free energy determination

3. Magnetic susceptibility and the equation of state of strongly interacting matter: results and discussion

1 – Introduction

- Strong interactions are described by QCD, the theory of quarks and gluons.
- Quarks are also subject to electroweak interactions, which in general induce small corrections to strong interaction dynamics, but exceptions are expected in presence of strong e.m. backgrounds, a situations which is relevant to many contexts:
  - Large magnetic fields ($B \sim 10^{10}$ Tesla) are expected in a class of neutron stars known as magnetars (Duncan-Thompson, 1992).
  - Large magnetic fields ($B \sim 10^{16}$ Tesla, $\sqrt{|e|B} \sim 1.5$ GeV), may have been produced at the cosmological electroweak phase transition (Vachaspati, 1991).
  - 

in non-central heavy ion collisions, largest magnetic fields ever created in a laboratory (B up to 10^{15} Tesla at LHC) with a possible rich associated phenomenology: chiral magnetic effect (Vilenkin, 1980; Kharzeev, Fukushima, McLerran and Warringa, 2008).

E.m. fields affect quarks directly and gluons only at the 1-loop level.

However non-perturbative effects can be non-trivial in the gluon sector as well.

Various model computations predict a rich phenomenology:

- Effects on the QCD vacuum structure (e.g., on chiral symmetry breaking)
- Effects on the QCD phase diagram (location and nature of the deconfinement transition, possible emergence of new phases)
- Effects on the QCD equation of state: is strongly interacting matter paramagnetic or diamagnetic?

LQCD is the ideal tool for a non-perturbative investigation of such issues. QCD+QED studies of the e.m. properties of hadrons go back to the early days of LQCD

- G. Martinelli, G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. B 116, 434 (1982).

- C. Bernard, T. Draper, K. Olynyk and M. Rushton, Phys. Rev. Lett. 49, 1076 (1982).

Recent years have seen an increasing activity on the subject.

Overview of lattice results

I focus here on thermodynamical and vacuum properties:

- **QCD vacuum response:**
  - B**-induced increase of chiral symmetry breaking (magnetic catalysis):**

    - P. V. Buividovich *et al*, **Phys. Lett. B 682, 484 (2010), Nucl. Phys. B 826, 313 (2010)**

    - M. D. and F. Negro, Phys. Rev. D 83, 114028 (2011)

    - G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D 86, 071502 (2012)
  - B**-induced anisotropies in gluon action:**

    - E. -M. Ilgenfritz *et al*, **Phys. Rev. D 85, 114504 (2012)**

    - G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP 1304, 130 (2013)

    - E. -M. Ilgenfritz, M. Muller-Preussker, B. Petersson and A. Schreiber, arXiv:1310.7876 [hep-lat]
  - $\vec{E} \cdot \vec{B} \neq 0$ **induced effective** θ **term in the QCD vacuum:**

    - M. D., M. Mariti and F. Negro, Phys. Rev. Lett. 110, 082002 (2013)

- **QCD phase diagram and B-dependence of T_c:**

  - simulations on coarse lattices and unphysical m_π show an increase of T_c.

  - Improved studies at the physical point show a decrease of T_c and the likely related appearance of a new phenomenon around T_c: inverse magnetic catalyis

  - The origin of the discrepancy is still not completely clarified.

  - An increase of the strength of the transition is observed in all studies

  - No B-induced splitting of deconfinement / χSB is observed by any study

  - M. D., S. Mukherjee, F. Sanfilippo, Phys. Rev. D 82, 051501 (2010)

  - G. S. Bali *et al*, JHEP 1202, 044 (2012)

  - E. -M. Ilgenfritz *et al*, Phys. Rev. D 85, 114504 (2012)

  - G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP 1304, 130 (2013)

  - F. Bruckmann, G. Endrodi and T. G. Kovacs, JHEP 1304, 112 (2013)

  - E. -M. Ilgenfritz, M. Muller-Preussker, B. Petersson and A. Schreiber, arXiv:1310.7876 [hep-lat].

- **QCD equation of state:**

  Is strongly interacting matter paramagnetic or diamagnetic?

  this is the main topic of this talk.

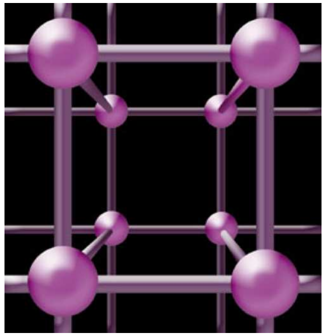
2 – Lattice QCD in a few words

The starting point is the path-integral approach to Quantum Mechanics and Quantum Field Theory, opened by R. Feynman in 1948

$$\langle 0|O|0\rangle \Rightarrow \int \mathcal{D}\varphi e^{-S[\varphi]} O[\varphi]$$

The QCD path integral is discretized on a finite space-time lattice

$\Longrightarrow$ finite number of integration variables



For QCD, integration variables are 3×3 unitary matrices, $U_\mu(n)$, living on lattice links (elementary parallel transporters)

(K.G. Wilson, 1974)

The path-integral is then computed by Monte-Carlo algorithms which sample field configurations proportionally to $e^{-S[U]}$

$$\langle O\rangle = \frac{1}{Z}\int \mathcal{D}U e^{-S[U]} O[U] \quad \simeq \quad \bar{O} = \frac{1}{M}\sum_{i=1}^{M} O[U^{\{i\}}]$$

The thermal QCD partition function is naturally rewritten in terms of an Euclidean path integral with a compactified temporal extension

$$S_{QCD} = \int d^4x \left(\sum_f \bar{\psi}_i^f \left(D_{ij}^{\mu} \gamma_{\mu}^E + m_f \delta_{ij} \right) \psi_j^f + \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \right) \to \bar{\psi} M[U] \psi + S_G[U]$$

$$Z(V,T) = \mathrm{Tr} \left(e^{-\frac{H_{\mathrm{QCD}}}{T}} \right) \Rightarrow \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-(S_G[U] + \bar{\psi} M[U] \psi)} = \int \mathcal{D}U e^{-S_G[U]} \det M[U]$$

As long as $\mathcal{D}U e^{-S_G} \det M[U]$ is positive, it can be interpreted as a probability distribution $\mathcal{D}U \mathcal{P}[U]$ over gauge link configurations, which can be sampled by proper algorithms

LQCD in electromagnetic background fields

An e.m. background field a_μ modifies the continuum covariant derivative as follows:

$$D_\mu = \partial_\mu + i\,g A_\mu^a T^a \quad \longrightarrow \quad \partial_\mu + i\,g A_\mu^a T^a + i\,q a_\mu$$

in the lattice formulation, the simplest symmetric discretization is

$$D_\mu \psi \rightarrow \frac{1}{2a}\Big(U_\mu(n) u_\mu(n) \psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu}) u_\mu^*(n-\hat{\mu}) \psi(n-\hat{\mu})\Big)$$

$U_\mu \in SU(3)$

$\mathbf{u}_\mu \simeq \exp(\mathbf{i\,q\,a}_\mu(\mathbf{n})) \in \mathbf{U(1)}$ **depends on the quark charge q.**

The thermal partition function of QCD is written as usual in terms of an euclidean path integral, with

$$T = \frac{1}{\tau} = \frac{1}{N_t a(\beta, m)}$$

where τ is the extension of the compactified time

$$Z = \mathrm{Tr}\left(e^{-\frac{H}{T}}\right) \Rightarrow \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-(S_G[U]+\bar{\psi}M[U,u]\psi)} = \int \mathcal{D}U e^{-S_G[U]} \det M[U,u]$$

where M is the fermion matrix

- **u fields affect gluon fields through the quark determinant and are not dynamical in the following (no integration): quenched QED approach.**

- **By loop expansion of the determinant (loop $\in U(3)$) or by $\not{D}^\dagger\gamma_5 = \gamma_5\not{D}$:**
  $\det M[U,u] > 0 \quad \Longrightarrow$ **MC simulations are feasible (with a caveat for electric fields)**

Some limitations and constraints

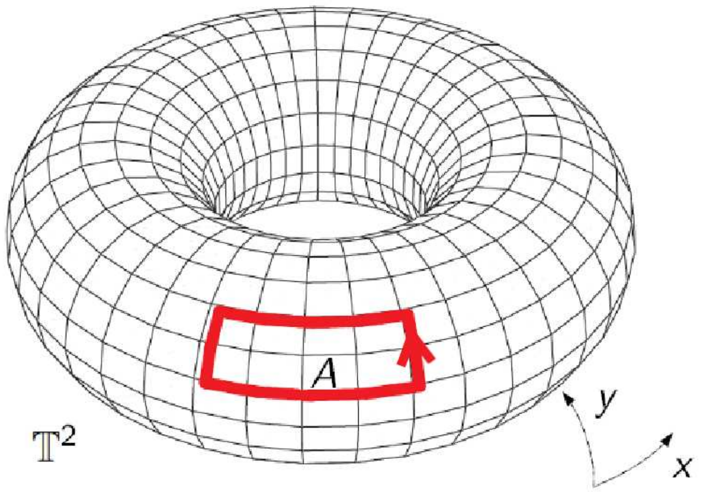
- **Field quantization on compact manifolds:**

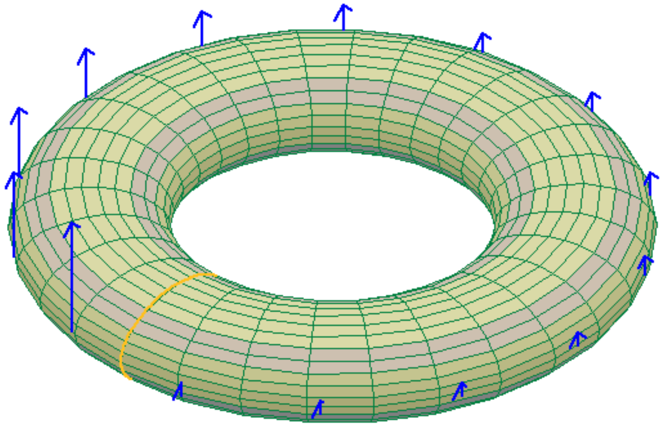
  – **To minimize finite size effects, one usually works on a compact manifold, like a torus (periodic b.c.).**
  Like for magnetic monopoles, consistency conditions for the gauge phases picked up by charged particles impose a quantized field flux through each closed surface ('t Hooft, 1979).

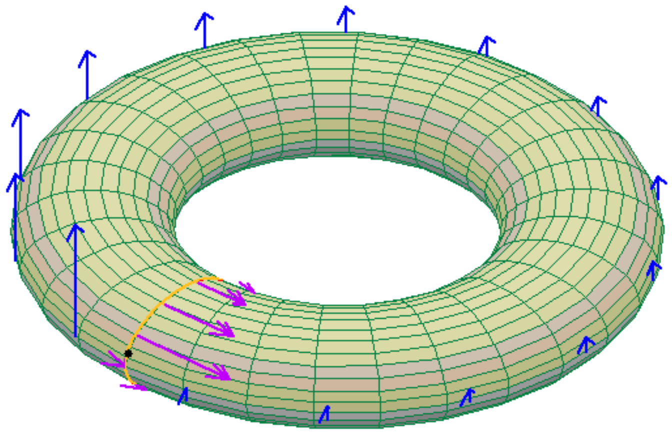
  – **e.g. for $\vec{B} = B\hat{z}$ on a torus populated by particles of charge q:**

$$qB = \frac{2\pi b}{L_x L_y a^2}$$

where b is an integer





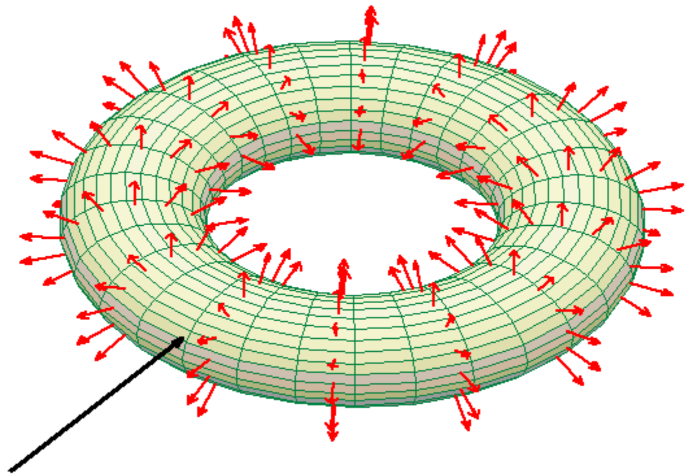


Consider an $l_x \times l_y$ torus and a realization of $\vec{B} = B\hat{z}$: $A_x = 0$, $A_y = Bx$

- this is discontinous at $x = 0$: that can be cured by adding $A(x) = -\delta(x)Bl_xy$
- but then $A(x)$ is discontinous in $y = 0$, and that cannot be cured any more

Particles looping around the origin will take a wrong $-qBl_xl_y$ additional phase

we are left with a uniform field plus a Dirac string, which is invisible only for quantized fields



– **The lattice $U(1)$ links corresponding to the choice above are the following:**

$u_y(B,q)(n) = e^{i\,a^2 qB\,n_x}$; $u_\mu(B,q)(n) = 1$ for $\mu = x, z, t$; $u_x(B,q)(n)|_{n_x = L_x} = e^{-i\,a^2 q L_x B\,n_y}$

they corresponds to a uniform field plus a Dirac string in the origin of each xy surface, which is invisible for integer b.

The Dirac string can actually be moved anywhere on the torus, for integer b this is done by a simple gauge transformation.

- **UV limitations from discretization:**

the plaquette sets the minimum explorable flux on the lattice, which is defined up to a 2π phase, thus fixing a sort of first Brillouin zone:

$$-\frac{\pi}{a^2} < qB < \frac{\pi}{a^2}$$

3 – The magnetic susceptibility of strongly interacting matter

Which kind of material is ”strongly interacting matter”?

A question strictly related to the equation of state of the system as a function of B

- **DIAMAGNETIC? free energy density f increases with B, pressure decreases**
- **PARAMAGNETIC? free energy density decreases with B, pressure increases**

The question is, in principle, simple and well posed:

We need the magnetization $M = -\partial f / \partial B$ and the magnetic susceptibility $\chi = -\partial^2 f / \partial B^2$ which are in principle perfectly computable equilibrium quantities.

$\chi > 0 \implies$ **PARAMAGNETIC** $\quad \chi < 0 \implies$ **DIAMAGNETIC**

PROBLEM: in the usual lattice setup (compact manifold with periodic b.c.), B is quantized and the derivative is not well defined.

Previous studies

- P. V. Buividovich *et al*, Nucl. Phys. B 826, 313 (2010)

  G. S. Bali *et al*, Phys. Rev. D 86, 094512 (2012)

  Only the spin component of the magnetization is computed

$$M^{spin} = \frac{1}{2} \sum_f \frac{q_f}{m_f} \left\langle \bar{\psi}_f \sigma_{xy} \psi_f \right\rangle$$

  diamagnetic behavior at $T = 0$ and $T \neq 0$, but "orbital" contribution unknown.

- G. S. Bali *et al*, JHEP 1304, 130 (2013)

  total vacuum magnetization computed from pressure differences in directions orthogonal or parallel to B

  Perturbative anisotropic lattice coefficients needed

  outcome: the magnetic susceptibility of the QCD vacuum is zero, but higher order terms in the free energy are paramagnetic

  (recently extended also to finite T, see later)

OUR APPROACH

C. Bonati, M. D., M. Mariti, F. Negro and F. Sanfilippo, Phys. Rev. Lett. 111, 182001 (2013) [arXiv:1307.8063]

- **The idea is to reconstruct directly the B-dependent part of the free energy density in place of its derivatives** $\Delta f(B,T) = -\frac{T}{V} \log\left(\frac{Z(B,T,V)}{Z(0,T,V)}\right)$

- **However, a direct determination of the ratio of partition functions is hardly feasible**

$$\frac{Z(B,T,V)}{Z(0,T,V)} = \frac{\int \mathcal{D}U e^{-S_G[U]} \det M[U,B]}{\int \mathcal{D}U e^{-S_G[U]} \det M[U,0]} = \left\langle \frac{\det M[U,B]}{\det M[U,0]} \right\rangle_{B=0}$$

difficulties emerge both in computing the observable and in correctly sampling it

- **A standard trick is to rewrite the ratio as the product of intermediate, easily computable ratios of interpolating partition functions (like the 't Hooft loop), possibly also a continuous interpolation $\longrightarrow$ derivative method (like for the pressure)**

$$\log\left(\frac{Z'}{Z}\right) = \log\left(\frac{Z'}{Z_N}\frac{Z_N}{Z_{N-1}}\dots\frac{Z_2}{Z_1}\frac{Z_1}{Z}\right) = \log\frac{Z}{Z_N} + \dots + \log\frac{Z_1}{Z} \to \int_Z^{Z'} dx \frac{d\log Z(x)}{dx}$$

NOTICE: Any interpolation is good! Provided the reconstruction is unambiguous

- Our idea is to extend the definition of $f(b)$ also to non-integer, unphysical values of b, and to obtain physical differences as follows:

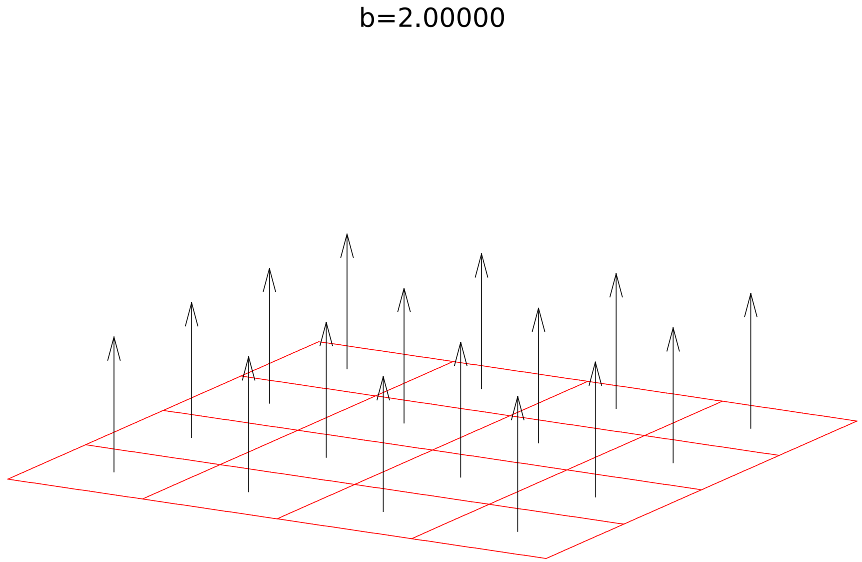
$$f(b_2) - f(b_1) = \int_{b_1}^{b_2} \frac{\partial f(b)}{\partial b} \mathrm{d}b \,,$$

with b_1 and b_2 integers, computing the integrand on a grid of points.

- $\partial f / \partial b$ is not the ”magnetization”, but just a derivative of the interpolating free energy. As long as the $f(b)$ is differentiable, the procedure is unambiguous.

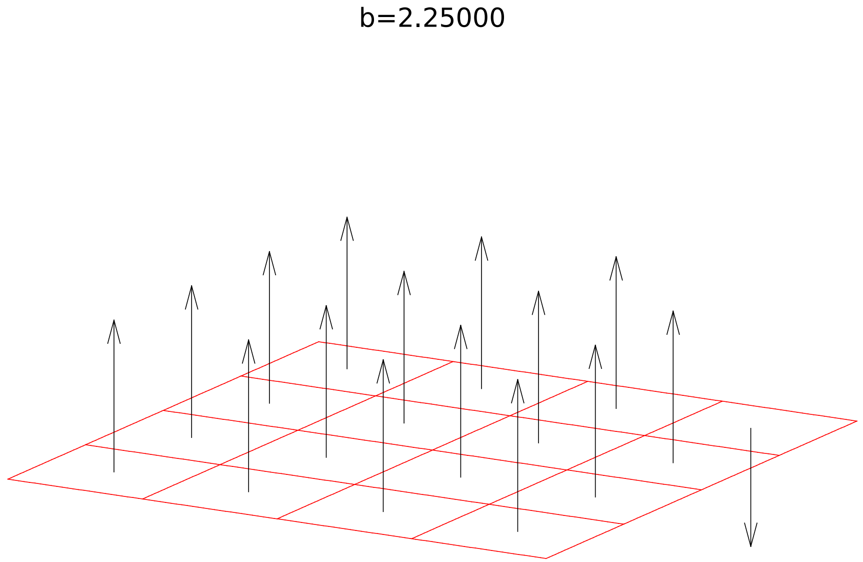
- In practice, our choice for the interpolating f corresponds to the same $U(1)$ field defined above, which for non-integer b describes a uniform field plus a (visible) Dirac string.

  On a finite lattice, analyticity is always guaranteed.



example of interpolating magnetic field on a 4×4 lattice torus

the plaquette in the up-right angle is pierced by the Dirac string



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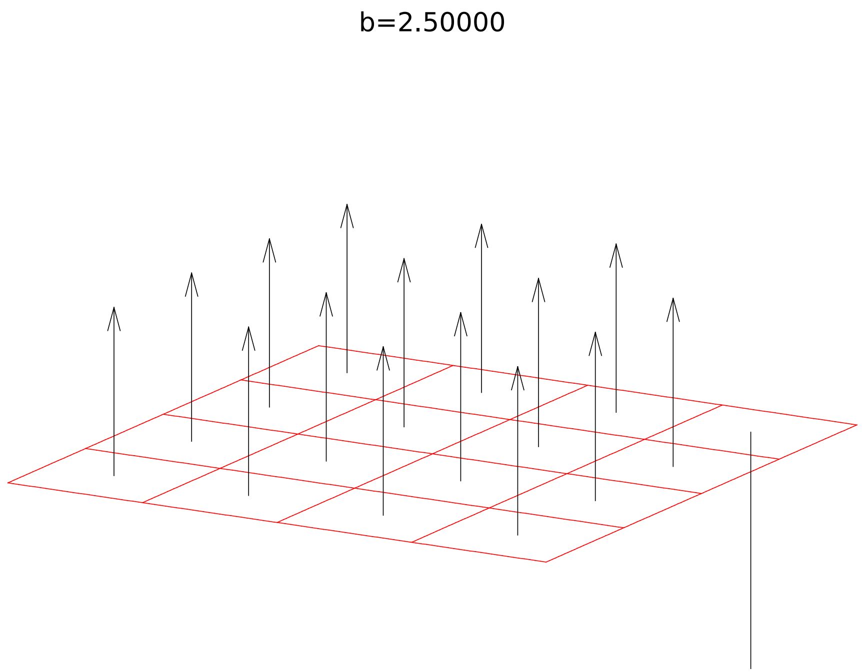

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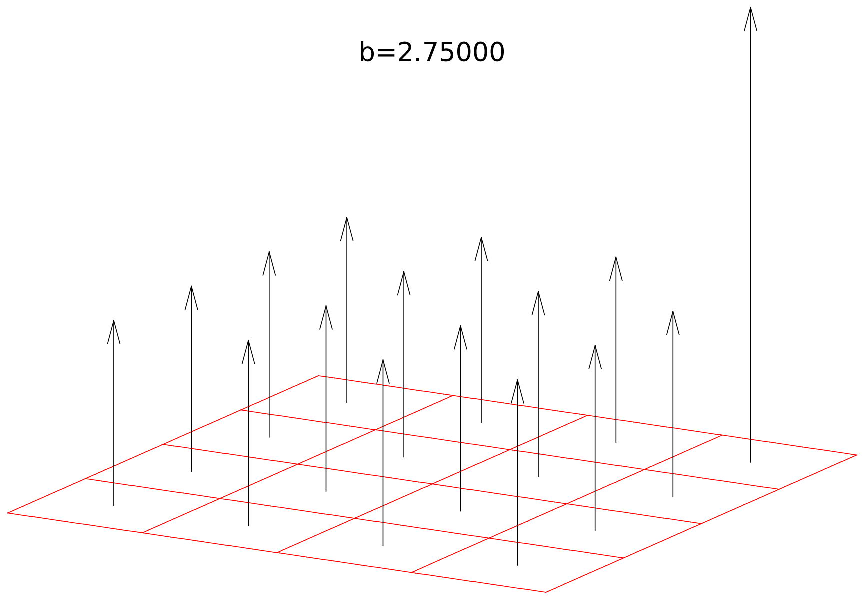

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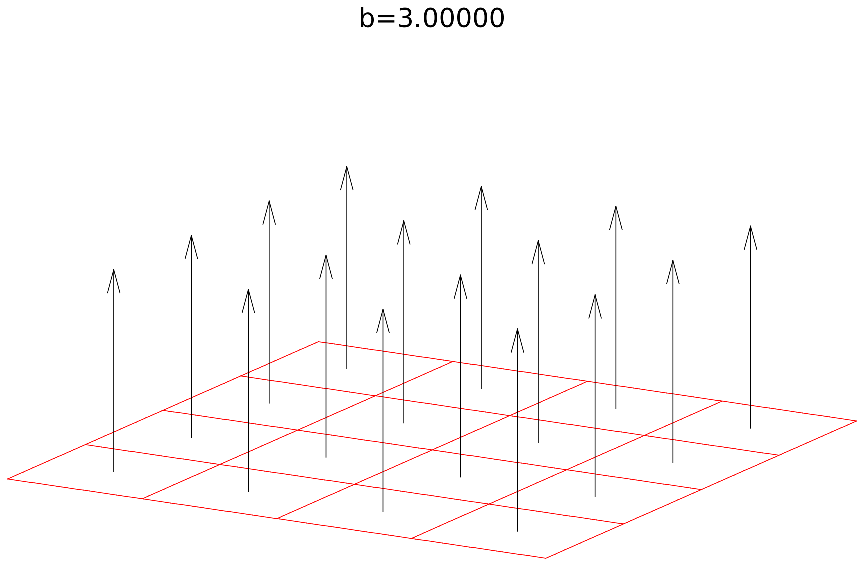


example of interpolating magnetic field on a 4×4 lattice torus

the plaquette in the up-right angle is pierced by the Dirac string

In practice:

- **We have considered QCD with fermions in the rooted staggered formulation**

$$Z \equiv \int \mathcal{D}U e^{-S_G} \prod_f \det D^{\frac{1}{4}}[U, m_f, q_f]$$

where the product runs over the different flavors

- **The Dirac operator is**

$$\begin{aligned} D^{(q)}_{i,j} \equiv am\delta_{i,j} \quad &+ \quad \frac{1}{2}\sum_{\nu=1}^{4} \eta_\nu(i) \left(u^{(q)}_\nu(i)\, U_\nu(i)\delta_{i,j-\hat{\nu}}\right. \\ &- \quad \left. u^{*(q)}_\nu(i-\hat{\nu})\, U^\dagger_\nu(i-\hat{\nu})\delta_{i,j+\hat{\nu}}\right) \end{aligned}$$

$q_u = 2|e|/3$ **and** $q_{d/s} = -|e|/3$**.**

- **The derivative of the interpolation can be expressed as**

$$M \equiv -a^4 \frac{\partial f}{\partial b} = \frac{1}{4L_t L_s^3} \sum_f \left\langle \left\{ \frac{\partial D^{(m_f,q_f)}}{\partial b} D^{(m_f,q_f)^{-1}} \right\} \right\rangle$$

Renormalization

- B-dependent divergences do not cancel when taking the difference $\Delta f \equiv f(B) - f(0)$, and must be properly subtracted.

- We are interested in the magnetic properties of the strongly interacting thermal medium, which may be probed experimentally. Therefore, our prescription is to subtract the vacuum ($T = 0$) contribution

$$\Delta f_R(B,T) = \Delta f(B,T) - \Delta f(B,0)$$

no further divergences, depending both on B and on T, appear

- Divergences are really removed only if the contributions to f_R are evaluated at a fixed value of the lattice spacing.

Effects of QED quenching

- for a linear homogeneous, isotropic medium, the magnetization is proportional to the field (SI units)

$$\boldsymbol{\mathcal{M}} = \tilde{\chi}\boldsymbol{B}/\mu_0\,; \quad \boldsymbol{\mathcal{M}} = \chi\boldsymbol{H}\,; \quad \boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{\mathcal{M}}\,; \quad \chi = \tilde{\chi}/(1-\tilde{\chi})$$

- After subtraction of the magnetic field energy in vacuum, one has

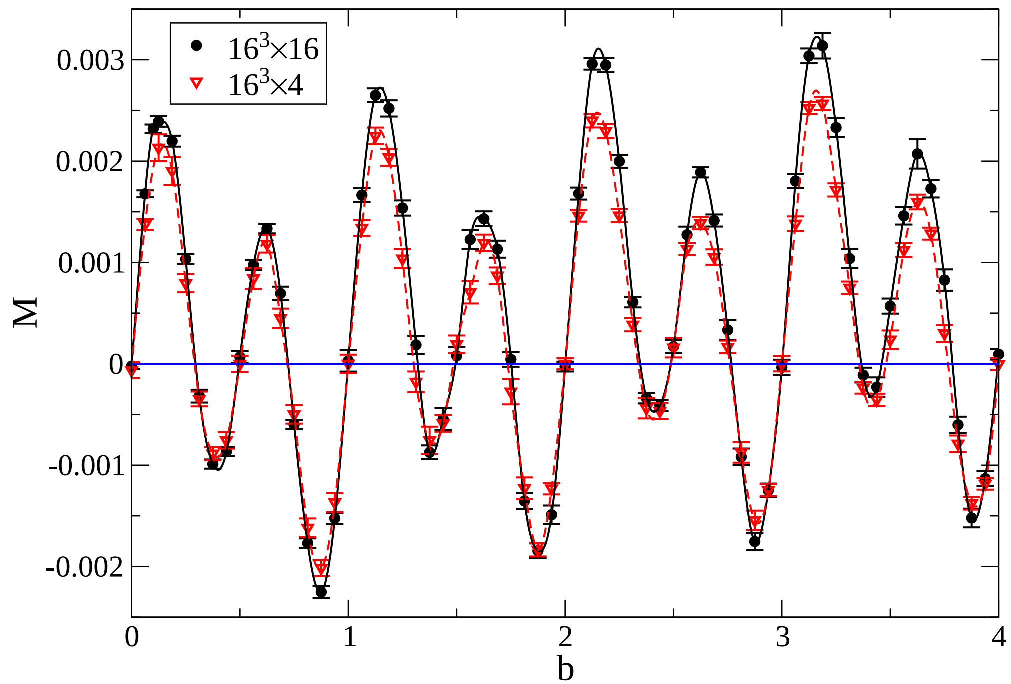
$$\Delta f_R = -\int \boldsymbol{\mathcal{M}}\cdot \mathrm{d}\boldsymbol{B} = -\frac{\tilde{\chi}}{\mu_0}\int \boldsymbol{B}\cdot\mathrm{d}\boldsymbol{B} \simeq -\frac{\tilde{\chi}}{2\mu_0}\boldsymbol{B}^2 = -\frac{\hat{\chi}}{2}(e\boldsymbol{B})^2$$

in the small field limit. Last expression defines the susceptibility in natural units.

- B is the total field felt by the medium. No backreaction from the medium (QED quenching) $\Longrightarrow$ it coincides with the external field added to the Dirac operator

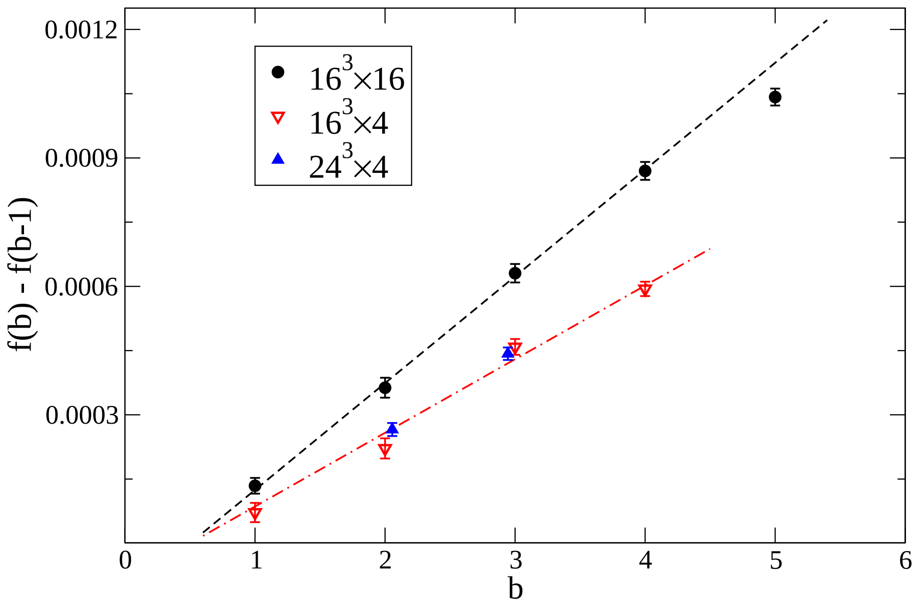
- The determination of $\tilde{\chi}$ is not affected by quenching effects, however, in a real medium, the backreaction would lead to an increase of Δf_R by a factor $1/(1-\tilde{\chi})^2$

RESULTS: we have first explored our method for $N_f = 2$ unimproved staggered fermions



$M \equiv a^4 \partial f / \partial b$ **on a** $T = 0$ **and a** $T \neq 0$ **lattice** $a \approx 0.188$ **fm**
$m_\pi \approx 480$ **MeV,** $T \simeq 262$ **MeV The lines are third order spline interpolations.**

- **Oscillating behavior caused by Dirac string becoming more or less visible, two harmonics due to different u and d quark charges**
- **The area spanned between integer values gives the free energy difference Δf**



- **restricting to regions where $a^4\Delta f(b) \simeq c_2\, b^2$ for both $T = 0$ and $T \neq 0$ (linear response region)**

$$a^4\,(f(b) - f(b-1)) \equiv \int_{b-1}^{b} M(\tilde{b})\mathrm{d}\tilde{b} \simeq c_2\,(2b-1)$$

- **Finally:** $c_{2R} = c_2(T) - c_2(T=0)$ **and**

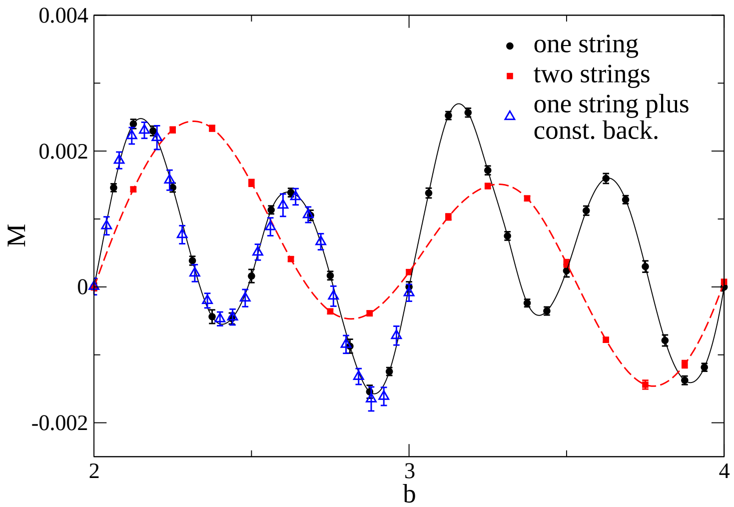
$$\tilde{\chi} = -\frac{|e|^2 \mu_0 c}{18\hbar\pi^2}\, L_s^4\, c_{2R}\ ; \qquad \hat{\chi} = -L_s^4\, c_{2R}\,/(18\pi^2)$$

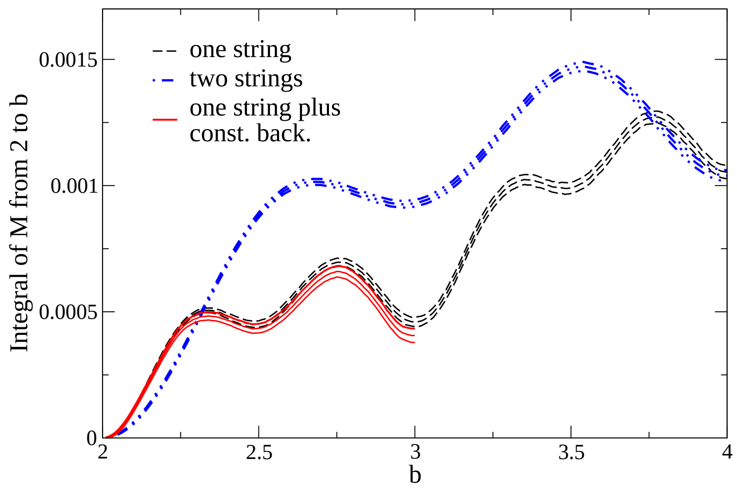
Stability checks

- **Right:** Results change within errors if we refine the grid of points or change the order of the spline integrator
- **Below** (M and $\int M$): Stability within errors if we change the interpolating free energy: comparison with a "two Dirac strings" interpolation and with adding a constant A_μ background

s	16 points	32 points
1	0.000596(16)	0.000594(12)
2	0.000594(17)	0.000593(12)
3	0.000592(17)	0.000594(12)
4	0.000592(17)	0.000594(13)

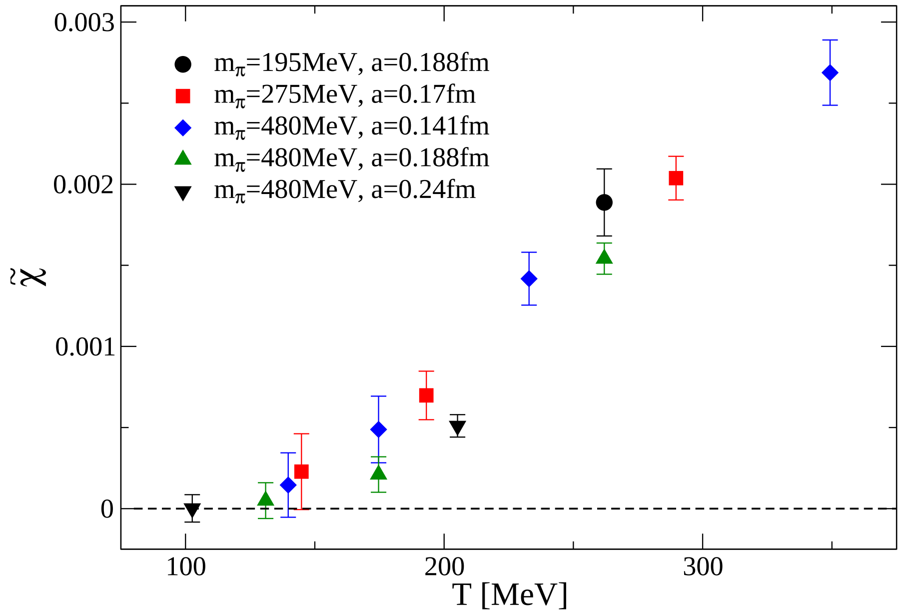
↓





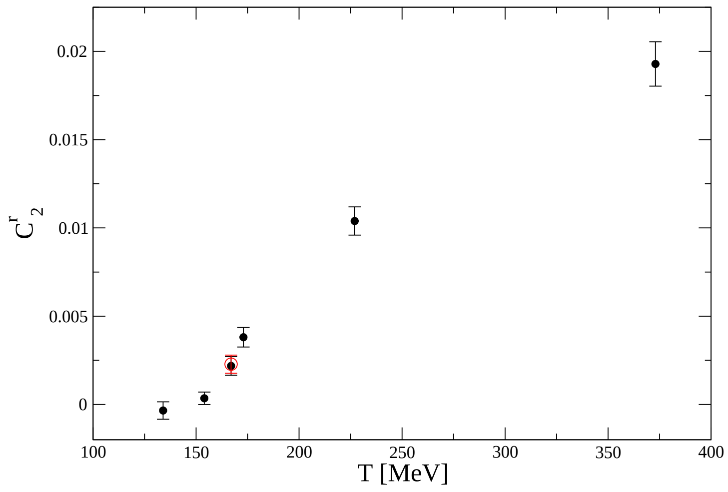
Final results for $N_f = 2$ QCD, standard unimproved staggered fermions

Simulations performed on GPU farms in Genoa, Pisa and Rome (QUONG)



- $\tilde{\chi}$ **is small or vanishing below T_c, while it steeply rises above deconfinement**
- **numbers indicate strong paramagnetism, one can compare, e.g.,** $\tilde{\chi} \simeq 2.8 \times 10^{-4}$ **for Platinum and** $\tilde{\chi} \simeq 3.9 \times 10^{-3}$ **for Liquid Oxygen.**
- **Data show only a mild dependence on the lattice spacing and on the pion mass**
- **The fact that the free energy of the deconfined phase decreases with B can account for the fact that T_c decreases with B.**

Results confirmed by different approaches by other groups



L. Levkova and C. DeTar, arXiv:1309.1142:

- A constant B for half of the lattice, and a constant $-B$ for the other half.
- Zero magnetic flux for every B: no quantization is required. But interface effects at the boundary must be kept under control.

Consistent results, based on $\langle\bar{\psi}\psi\rangle$ integration, presented in G. S. Bali, F. Bruckmann, G. Endrodi and A. Schafer, arXiv:1310.8145 [hep-lat].

Consistent results, based on the pressure anisotropy, presented in G. S. Bali, F. Bruckmann, G. Endrodi and A. Schafer, arXiv:1311.2559 [hep-lat].

Extension to $N_f = 2 + 1$ QCD with physical quark masses

C. Bonati, M. D., M. Mariti, F. Negro and F. Sanfilippo, arXiv:1310.8656 [hep-lat].

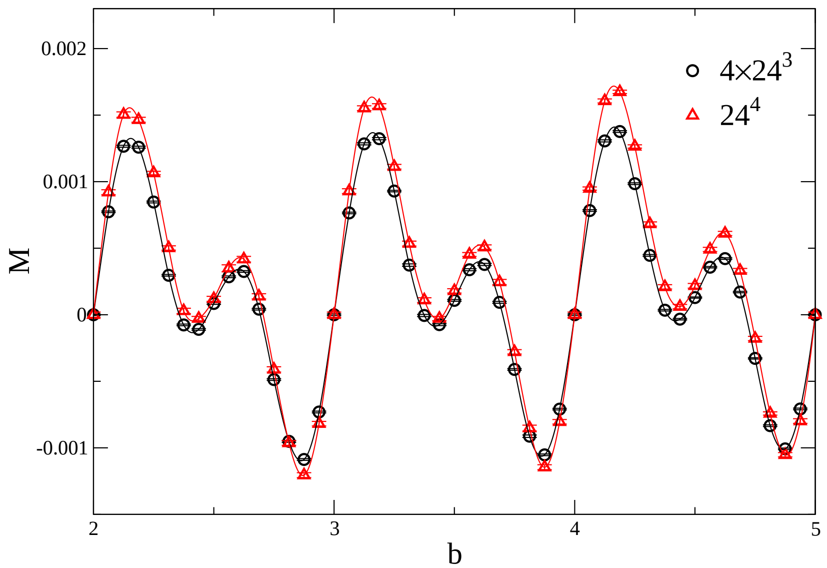
In order to refine our study and check for effects related to the quark mass spectrum and the UV cutoff, we have repeated our analysis with an improved discretization, adopting the same action used by the Budapest-Regensburg-Wuppertal collaboration (see, e.g., Aoki et al., JHEP 0906 (2009) 088)

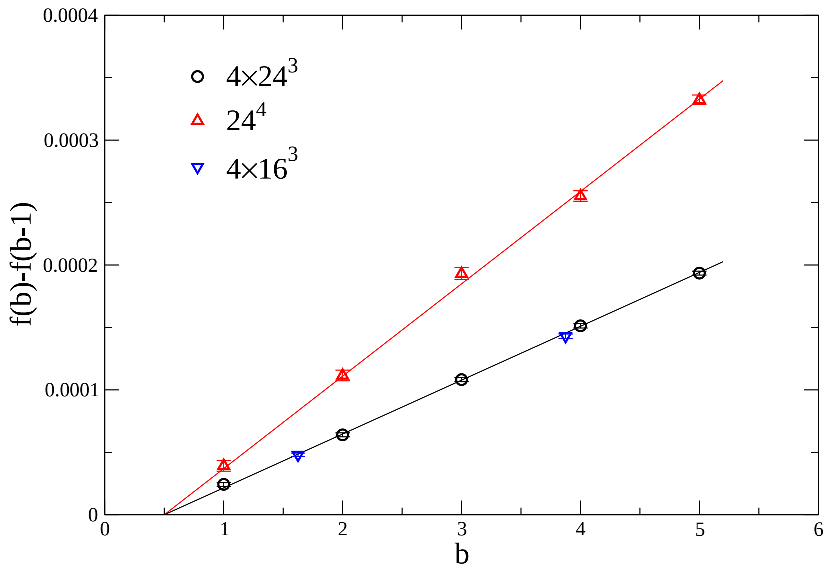
- Tree level Symanzik improved gauge action
- $N_f = 2 + 1$ stout rooted staggered quarks (2 stouting levels), with physical light and strange quark masses

We have used three different spacings, $aL_s \sim 5$ fm and temperatures in the range $90 - 400$ MeV (by varying L_t):

L_s	a(fm)	β	$am_{u/d}$	am_s
24	0.2173(4)	3.55	0.003636	0.1020
32	0.1535(3)	3.67	0.002270	0.0639
40	0.1249(3)	3.75	0.001787	0.0503

Simulations performed on the Fermi BlueGene/Q machine at CINECA

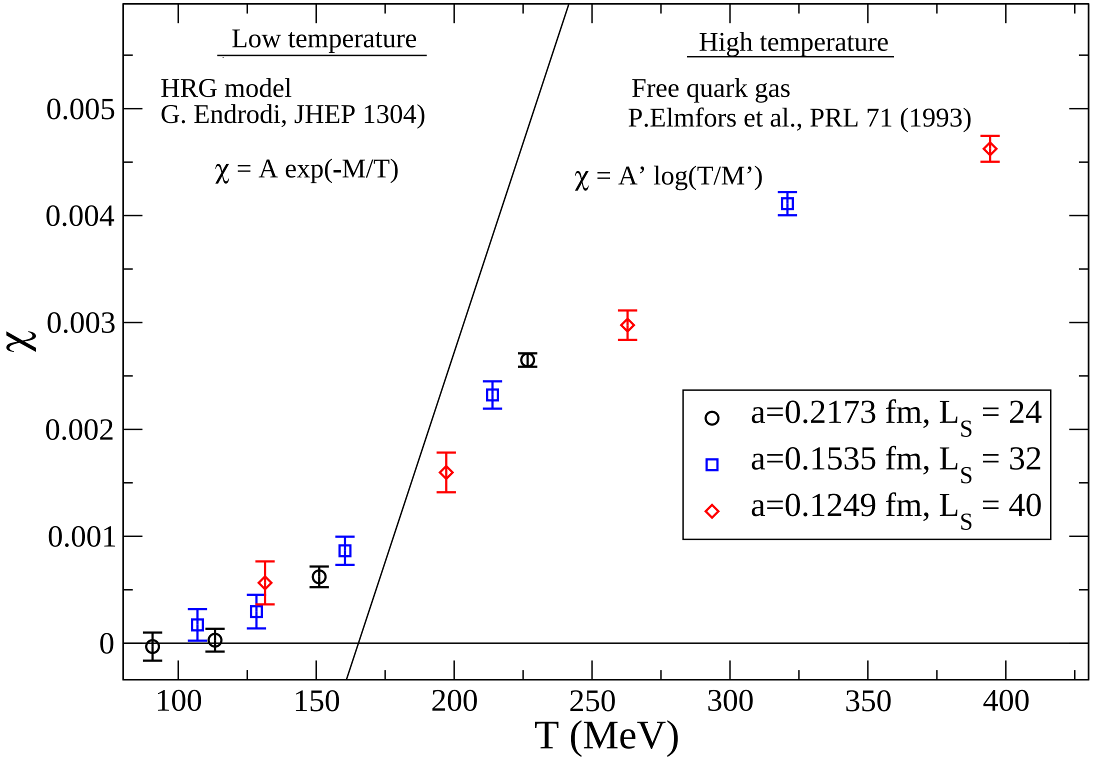




Intermediate results for $\partial f/\partial b$ and for the finite free energy differences show the same qualitative behavior as for unimproved fermions

It is interesting to notice that the linear response region of strongly interacting matter seems to extend to $eB \sim 0.1 - 0.2$ GeV^2, which is the region relevant for heavy ion collisions.

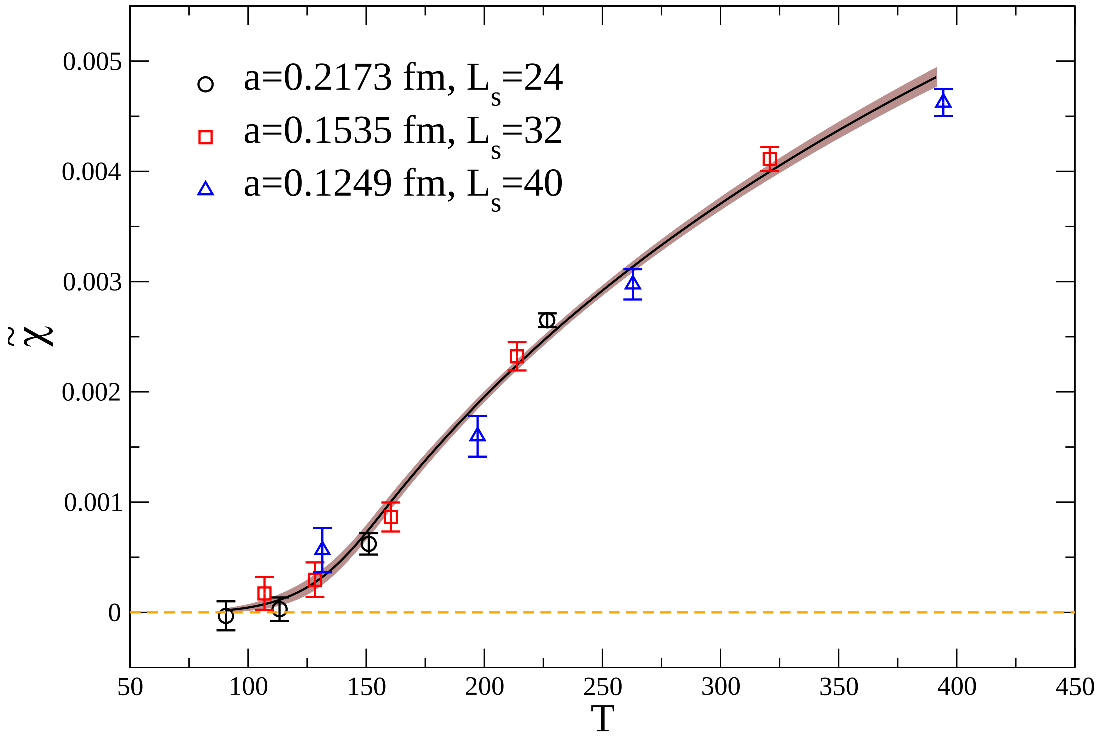
Magnetic susceptibility



Results for χ do not change qualitatively with respect to unimproved results

- **there is a slight increase, partly due to the inclusion of the strange quark**
- **UV cutoff effects seem well under control**
- **The system is paramagnetic also in the region around and below $T_c \simeq 155$ MeV**
- **Low T and high T regions are well described by HRG or free quark predictions**

Magnetic susceptibility

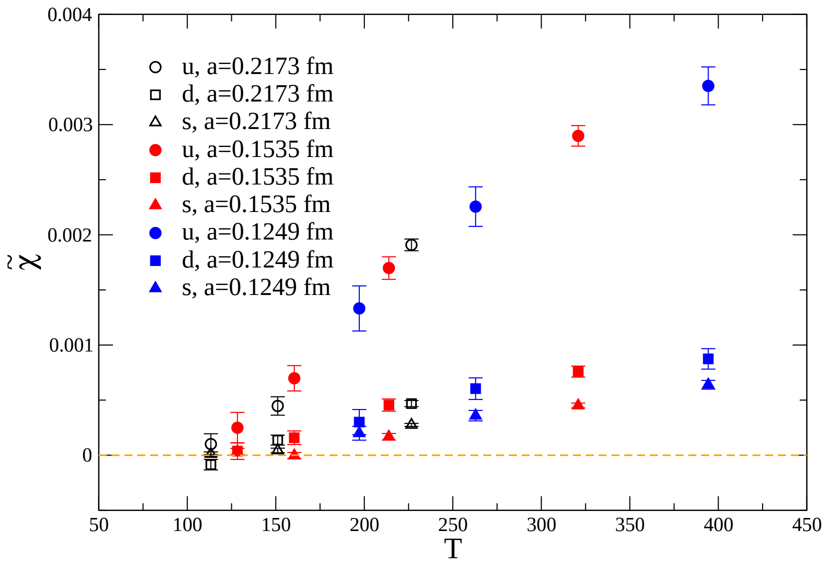


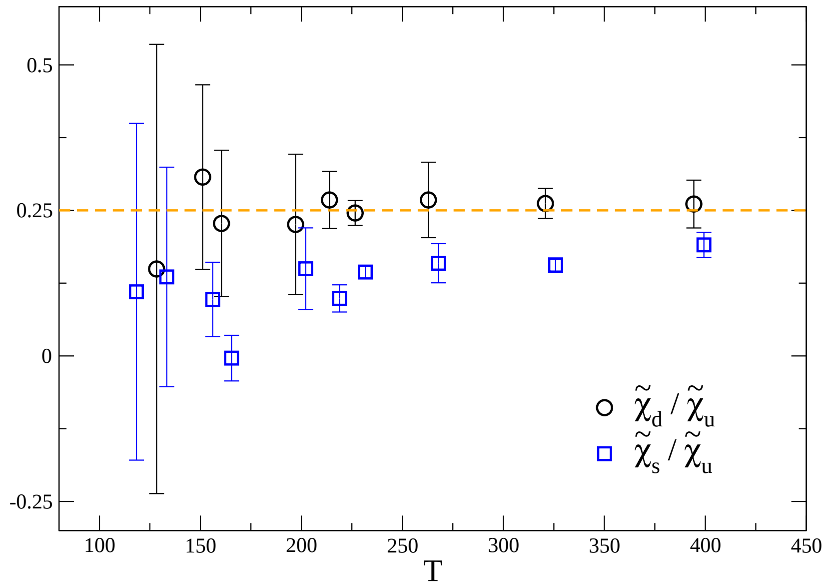
Remarkably, we are able to fit data in the whole T range by the mentioned predictions:

- $\tilde{\chi} = A\ \exp(-M/T)$ **for low T**
- $\tilde{\chi} = A'\ \log(T/M')$ **for high T**

with a differentiable matching at $T \sim T_c$. $M \sim 900$ MeV, in agreement with the lightest hadrons carrying a non-trivial magnetic moment, and $M' \sim T_c$.

Flavor contributions

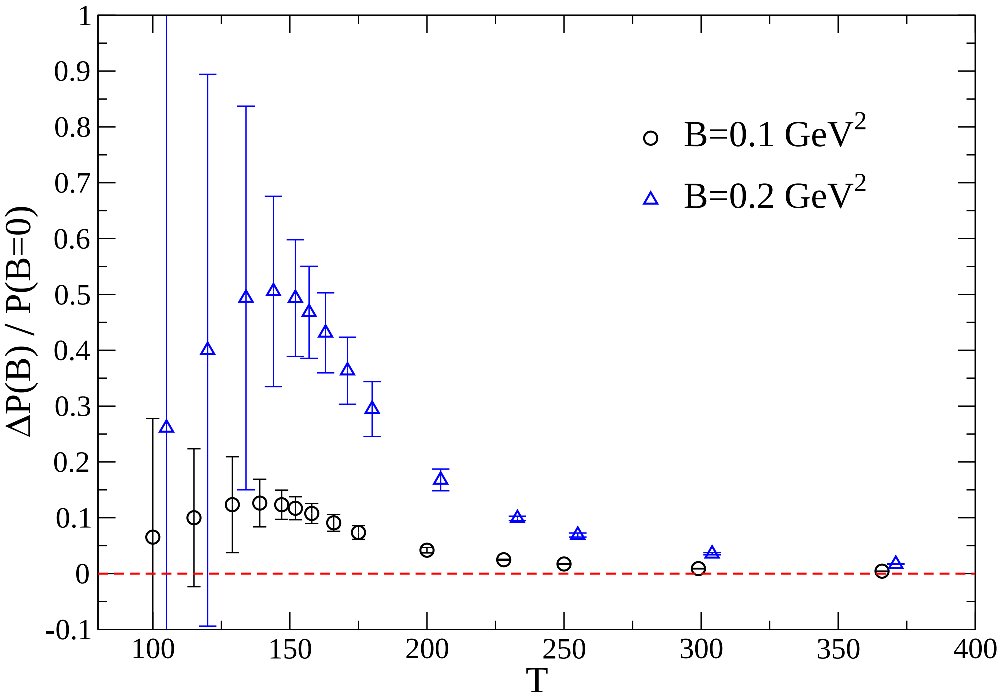




$M \equiv a^4 \frac{\partial f}{\partial b} = \frac{1}{4L_t L_s^3} \sum_f \left\langle \left\{ \frac{\partial D^{(q_f)}}{\partial b} D^{(q_f)^{-1}} \right\} \right\rangle \quad \Longrightarrow$ **we can distinguish the different contributions, $\tilde{\chi} = \tilde{\chi}_u + \tilde{\chi}_d + \tilde{\chi}_s$ even if the separation is not strict, because of quark loop mixings.**

- $\tilde{\chi}_d/\tilde{\chi}_u \simeq (q_d/q_u)^2 = 0.25$ **over the whole T range**
- $\tilde{\chi}_s/\tilde{\chi}_u \to 0.25$ **only for high T: strangeness thermally suppressed for lower T.**

Magnetic contributions to the QCD pressure



For homogeneous systems $\Delta P(B) = -\Delta f_R$. We plot $\Delta P(B)/P(B=0)$ for two different values of B **(data at $B=0$ taken from S. Borsanyi it et al, arXiv:1309.5258)**

- **$\Delta P/P$ is larger around the transition and already in the range 10-50% for the typical fields produced in heavy ion collisions at the LHC, $eB \sim 0.1 - 0.2$ GeV^2.**
- **In the high T regime $\Delta P/P \to 0$ as expected, since $P(B=0) \propto T^4$**

4 – Conclusions and perspectives

- **We have determined the response of strongly interacting matter to external magnetic fields**

- **Strongly interacting matter is a paramagnetic medium, with a linear response for fields up to $eB \sim O(0.1)$ Gev^2 and a magnetic susceptibility which steeply rises above deconfinement, and apparently like $\log(T)$ for high T**

- **The relative increase in the pressure may be significant, around T_c, already for the magnetic fields produced in heavy ion collisions.**

- **Future studies should compute the non-linear contributions, which become significant for $eB \sim 1$ GeV^2, and which could be important for cosmological models**

- **The c quark contribution could also be non-negligible at moderately high T (mass suppression but $(q_c/q_s)^2 = 4$) and should be taken into account**