Light as a probe for ultrasmall effects of General Relativity

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Foreword: General Relativity

- The gravitational field is geometry
- Studying the gravitational interaction amounts to explore the geometry of space time
- Locally the most common (approximate) symmetry is chiral about a time-like axis: steadily rotating massive object

Rotating sources of gravity

Among the classical weak effects of GR there are the ones due to rotating masses, often called gravito-magnetic effects.

The direct measurements of these effects is important *per se* and could evidence deviations addressing one or another extended or alternative theory of gravity

Axially symmetric stationary space-time

Remote inertial observer; "polar" space coordinates; (spherical symmetry in space)

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\varphi\varphi}d\varphi^2 + \boxed{2g_{t\varphi}dtd\varphi}$$

Gravito-magnetic effects

$$\boxed{2g_{ti}dx^i dt}$$

$$\vec{h}\cdot\vec{dl}$$

$g_{\mu\nu}$'s do not depend on t and φ

Line element for light in approximated geographic terrestrial coordinates

$$g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\varphi\varphi}d\varphi^2 + 2g_{t\varphi}dtd\varphi = 0$$

⇓

$$\left(1-2\frac{\mu}{r}-\frac{\Omega^2 r^2}{c^2}\sin^2\theta\right)d\tau^2-\left(1+2\frac{\mu}{r}\right)dr^2-r^2d\theta^2$$

$$-\left(1+2\frac{r^2\Omega^2}{c^2}\sin^2\theta\right)r^2\sin^2\theta d\varphi^2+2\left(2\frac{j}{r}-r^2\frac{\Omega}{c}-2\mu\frac{r\Omega}{c}\right)\sin^2\theta d\tau d\varphi=0$$

Laboratory coordinates

$$\left(1-\frac{\Omega^2R^2}{c^2}\sin^2\Theta-2\frac{\mu}{R}-2\frac{\Omega^2}{c^2}R\sin^2\Theta\left(\zeta\sin\Theta+y\cos\Theta\right)+2\frac{\zeta\mu}{R^2}\right)d\tau^2$$

$$-\left(1+2\frac{\mu}{R}-2\frac{\zeta\mu}{R^2}\right)d\zeta^2-\left(1+\frac{\zeta}{R}\right)^2dy^2$$

$$-\left(1+2\frac{R^2\Omega^2}{c^2}\sin^2\Theta+\frac{2}{R}\left(\zeta+y\frac{\cos\Theta}{\sin\Theta}\right)+8\frac{R\Omega^2}{c^2}\left(\zeta\sin\Theta+y\cos\Theta\right)\sin\Theta\right)dx^2$$

$$+2\left(2\frac{j}{R^2}\sin\Theta-\frac{R\Omega}{c}\sin\Theta-2\frac{\Omega}{c}\left(y\cos\Theta+\left(\zeta+\mu\right)\sin\Theta\right)\right)d\tau dx=0$$

General laboratory expression

$$
\begin{aligned}
&\left(1+2w\left(y,z;\Theta,\mu,\Omega,R\right)\right)d\tau^2 \\
&-\left(1+q\left(y,z;\Theta,\mu,\Omega,R\right)\right)dx^2 \\
&-\left(1+f\left(z;\Theta,\mu,\Omega,R\right)\right)^2 dy^2-dz^2 \\
&+2h\left(y,z;\Theta,\mu,j,\Omega,R\right)d\tau dx=0
\end{aligned}
$$

“Free” motion along a closed null path

Time of flight asymmetry:

$$\delta t = t_+ - t_- = -\frac{2}{c}\oint \frac{g_{0i}}{g_{00}} dx^i \neq 0$$

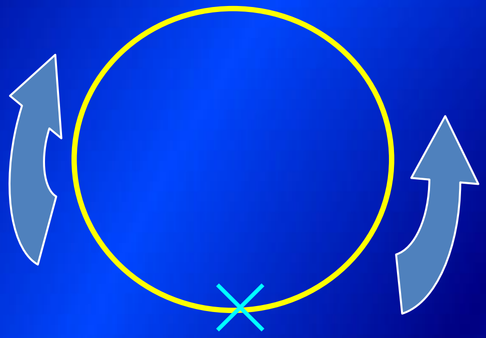
global coordinated time

$$\delta\tau = -2\sqrt{g_{00}}\oint \frac{g_{0i}}{g_{00}} dx^i$$

proper time of a fixed observer

Ring laser: light propagation

Closed loop (shape irrelevant)



Counterrotating light beams

Time of flight difference

$$\delta t = -2\oint \frac{g_{0i}}{g_{00}} dx^i$$

$$\delta\tau = -2\sqrt{g_{00}}\oint \frac{g_{0i}}{g_{00}} dx^i = -2\sqrt{g_{00}}\int_S \vec{\nabla} \wedge \vec{\mathrm{h}} \cdot \hat{n} dS$$

$$\mathrm{h}_i = \frac{g_{0i}}{g_{00}}$$

Weak field approximation

Relevant parameters: $\mu = G\frac{M}{c^2}$; $a = \frac{J}{Mc}$

Earth: μ = 4.43×10^{-3} m a = 3.95 m

$$ds^2 \cong \left(1-2\frac{\mu}{r}\right)c^2dt^2 - \left(1+2\frac{\mu}{r}\right)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2 + 4c\frac{j}{r}\sin^2\theta dtd\varphi$$

$$j = \mu a = G\frac{J}{c^3} \approx 1.75\times 10^{-2}\, m^2$$

$$\vec{h} = 2\frac{j}{r^2}\sin\theta\,\hat{\phi}$$

Earth-bound laboratory

$$ds^2 \cong \left(1 - 2\frac{\mu}{r} - \frac{r^2\Omega^2}{c^2}\sin^2\theta\right)c^2dt^2 - \left(1 + 2\frac{\mu}{r}\right)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2$$

$$+2\left(-\frac{\Omega r}{c} - 2\frac{\Omega\mu}{c} + 2\frac{j}{r^2}\right)cr\sin^2\theta dtd\varphi$$

$$\frac{r\Omega}{c} \approx 2.47\times10^{-7}$$

Earth-centered co-rotating axes

Local Cartesian

$$\left(1-\frac{\varOmega^2R^2}{c^2}\sin^2\varTheta-2\frac{\mu}{R}-2\frac{\varOmega^2}{c^2}R\sin^2\varTheta\left(\zeta\sin\varTheta+y\cos\varTheta\right)+2\frac{\zeta\mu}{R^2}\right)d\tau^2$$

$$-\left(1+2\frac{\mu}{R}-2\frac{\zeta\mu}{R^2}\right)d\zeta^2-\left(1+\frac{\zeta}{R}\right)^2dy^2$$

$$-\left(1+2\frac{R^2\varOmega^2}{c^2}\sin^2\varTheta+\frac{2}{R}\left(\zeta+y\frac{\cos\varTheta}{\sin\varTheta}\right)+8\frac{R\varOmega^2}{c^2}\left(\zeta\sin\varTheta+y\cos\varTheta\right)\sin\varTheta\right)dx^2$$

$$+2\left(2\frac{j}{R^2}\sin\varTheta-\frac{R\varOmega}{c}\sin\varTheta-2\frac{\varOmega}{c}\left(y\cos\varTheta+\left(\zeta+\mu\right)\sin\varTheta\right)\right)d\tau dx=0$$

Time of flight difference

$$\vec{\nabla}\wedge\vec{\mathrm{h}}=2\left(\frac{\Omega}{c}r+\frac{\Omega\mu}{c}+2\frac{j}{r^{2}}\right)\frac{\cos\theta}{cr}\hat{r}-2\left(\frac{\Omega r}{c}-\frac{\Omega\mu}{c}-\frac{j}{r^{2}}\right)\frac{\sin\theta}{cr}\hat{\theta}$$

$$\delta\tau=-2\sqrt{g_{00}}\int_{S}\vec{\nabla}\wedge\vec{\mathrm{h}}\cdot\hat{n}dA$$

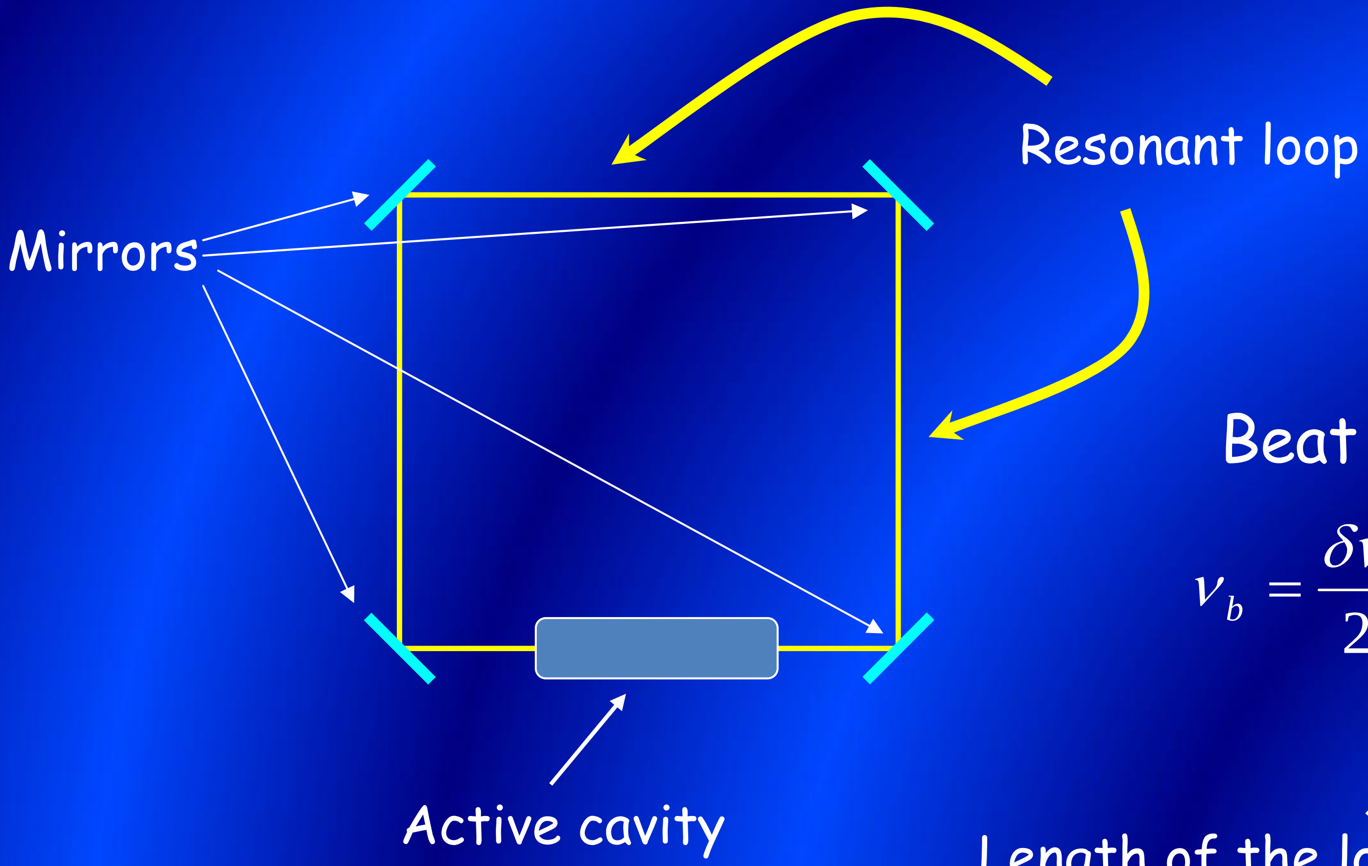
$$\delta\tau=-4\left(\frac{\Omega R}{c}+2\frac{j}{R^{2}}\right)\frac{\cos\theta}{cR}A\hat{r}\cdot\hat{n}+4\left(\frac{\Omega R}{c}+2\frac{\Omega\mu}{c}-\frac{j}{R^{2}}\right)\frac{\sin\theta}{cR}A\hat{\theta}\cdot\hat{n}$$

ToF difference in local Cartesian

$$h_x = \frac{h}{1+2w} \qquad \vec{\nabla} \wedge \vec{h} = \frac{\partial h_x}{\partial z}\hat{u}_y - \frac{\partial h_x}{\partial y}\hat{u}_z$$

$$\delta\tau = -2\frac{\partial h_x}{\partial z}\hat{u}_y \cdot \hat{n}S + 2\frac{\partial h_x}{\partial y}\hat{u}_z \cdot \hat{n}S$$

A ring laser



Mirrors

Resonant loop

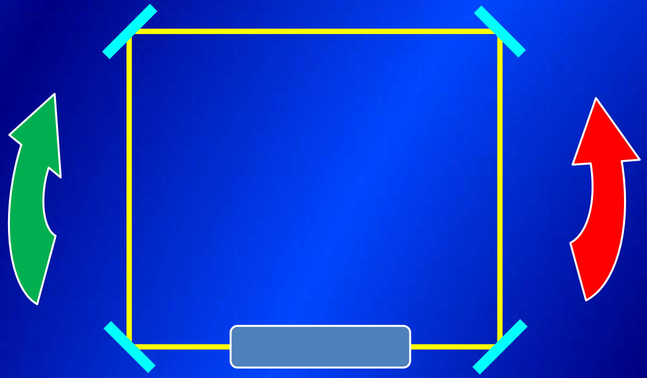
Active cavity

Beat frequency

$$\nu_b = \frac{\delta\nu}{2} = \frac{c^2}{\lambda P}\delta\tau$$

Length of the loop

Stationarity condition



$$N = \frac{\tau_+}{T_+} = \tau_+ \nu_+$$

$$N = \frac{\tau_-}{T_-} = \tau_- \nu_-$$

Single mode

⬇

$$N = \frac{P}{\lambda}$$

Length of the path

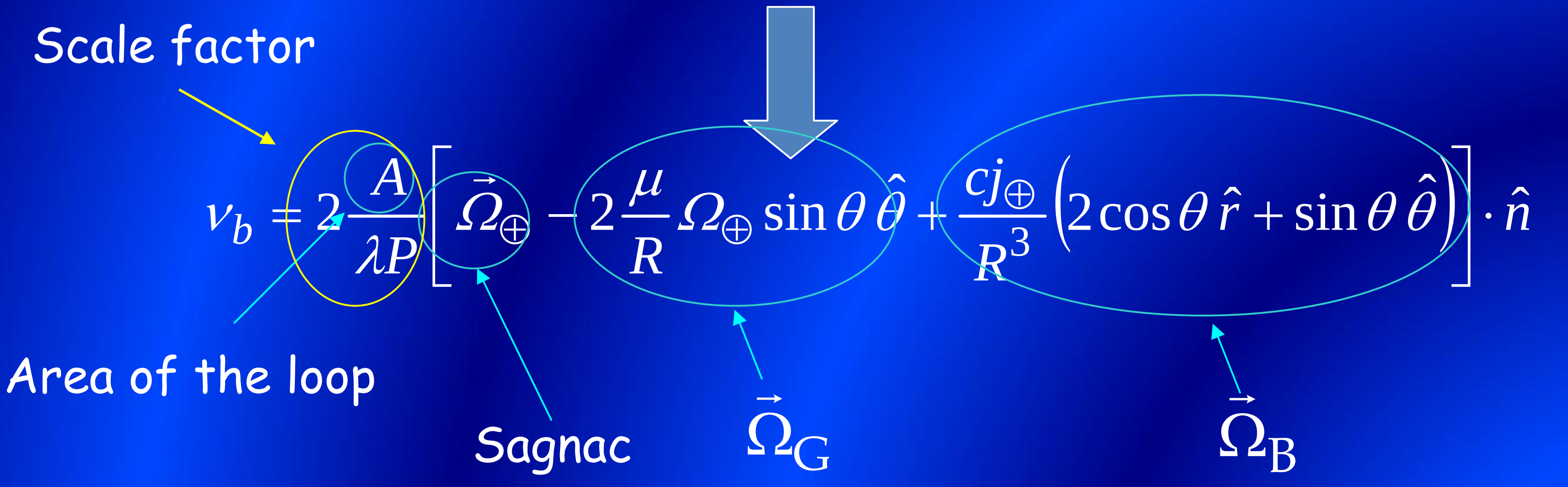
$$\nu_b = \frac{\nu_+ - \nu_-}{2} \cong \frac{c^2}{2\lambda P}\delta\tau$$

Beat frequency

Expected signal (GR)

Normal in the meridian plane

$$\nu_b = 2\frac{A}{\lambda P}\Omega_\oplus\left[\cos(\theta+\alpha) - 2\frac{\mu}{R}\sin\theta\sin\alpha + \frac{GI_\oplus}{c^2R^3}(2\cos\theta\cos\alpha + \sin\theta\sin\alpha)\right]$$



$$\nu_b = 2\frac{A}{\lambda P}\left[\vec{\Omega}_\oplus - 2\frac{\mu}{R}\Omega_\oplus\sin\theta\,\hat{\theta} + \frac{cj_\oplus}{R^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)\right]\cdot\hat{n}$$

Scale factor

Area of the loop

Sagnac

$\vec{\Omega}_G$

$\vec{\Omega}_B$

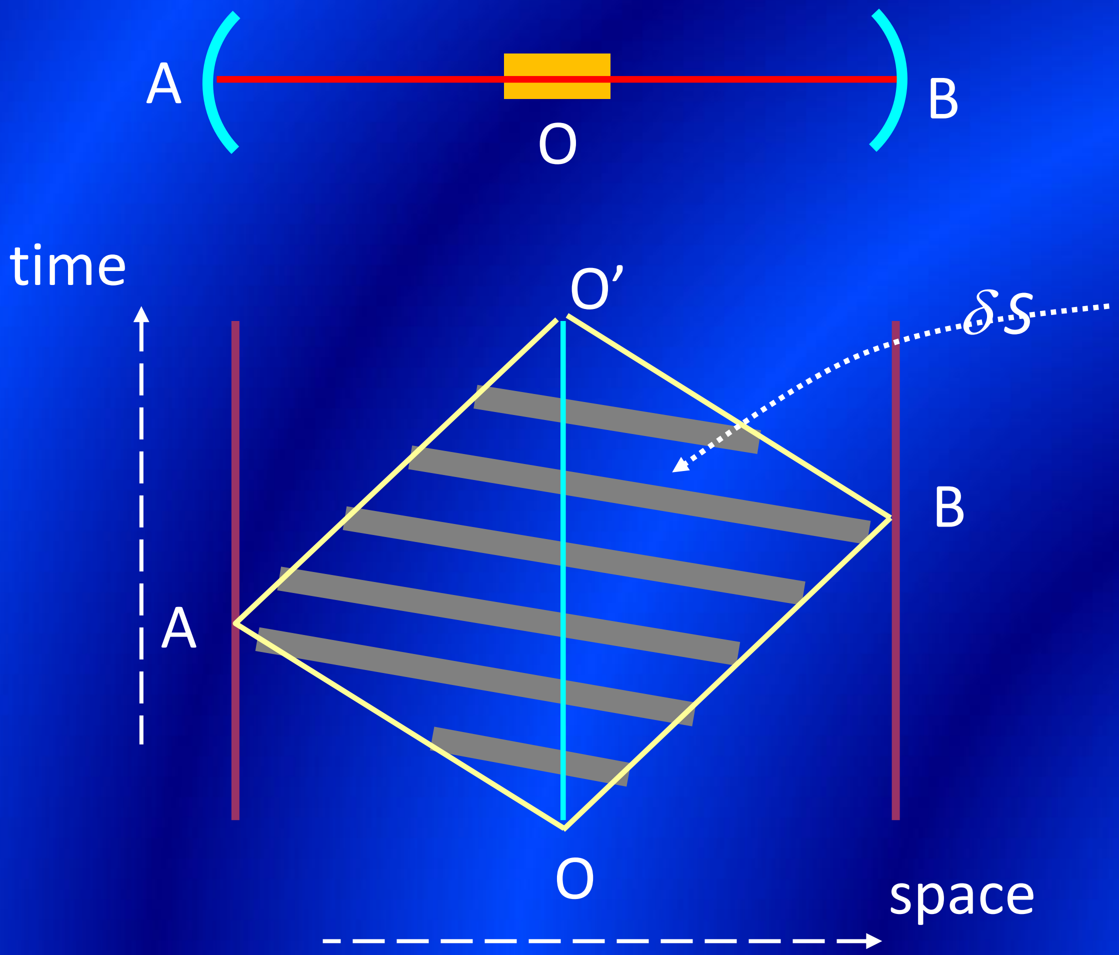
Orders of magnitude

$$\frac{\Omega_G}{\Omega} \approx \frac{\Omega_B}{\Omega} \approx 10^{-9}$$

$\nu_b \sim 10^2$ Hz

Required accuracy for g-magnetic effects: a few prad/s

A linear cavity



The effect of curvature and anisotropy

$$\delta F^{\mu\nu} = \left(R^{\mu}{}_{\varepsilon 0i}F^{\varepsilon\nu} + R^{\nu}{}_{\varepsilon 0i}F^{\mu\varepsilon}\right)\delta S^{0i}$$

Electromagnetic tensor

Riemann tensor

Space-time area spanned by the cavity

Depends on the orientation of the cavity

Linearized Riemann tensor. An example: cavity in the meridian plane

$R^{\phi}{}_{r0\theta}$

Length of the cavity

$$\delta F^{\vartheta\phi} \cong \left[\left(\frac{\mu\Omega}{cR^2} - 3\frac{j}{R^4}\right)\frac{\cos\vartheta}{\sin^2\vartheta}\right]\frac{l^2}{R}F^{\vartheta r}$$

Change in the radial component of the magnetic field

East-West component of the magnetic field

Rate of change of the EM field per bounce

Kinematic $$\frac{\delta F}{F} \approx 10^{-7}$$

Gravito-electric $$\frac{\delta F}{F} \approx 10^{-8}$$

Gravito-magnetic $$\frac{\delta F}{F} \approx 10^{-14}$$

Geometric effect in a cavity

k: null tangent vector

δ**k**: change while moving back and forth

$$\delta k^{\mu} = R^{\mu}{}_{\alpha\nu\beta} k^{\alpha} S^{\nu\beta}$$

$$\Downarrow$$

$$\delta k^{\mu} = R^{\mu}{}_{\alpha 0 i} k^{\alpha} S^{0i}$$

An example

East-West cavity

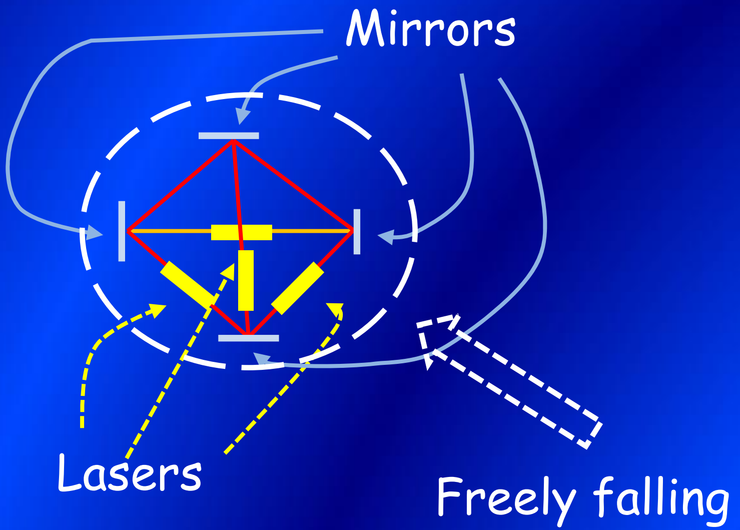
Frequency shift

$$\delta k^0 = R^0{}_{\phi 0 \phi} k^\phi S^{0\phi} \cong \left(\frac{\mu}{R} \sin^2 \Theta + \frac{\Omega^2 R^2}{c^2} \cos^2 \Theta \right) k^\phi S^{0\phi} = \left(\frac{\mu}{R} \sin^2 \Theta + \frac{\Omega^2 R^2}{c^2} \cos^2 \Theta \right) kl^2$$

$$\delta k^\phi = R^\phi{}_{\phi 0 \phi} k^\phi S^{0\phi} \cong \frac{\mu \Omega}{cR} \sin \Theta k^\phi S^{0\phi}; \qquad \delta k^x \cong \frac{\mu \Omega}{cR^2} kl^2$$

$$\frac{\delta k^x}{k^x} \approx 10^{-29} \text{ rad}$$

Ring lasers in space



Tetrahedron with four triangular ringlasers

Circular equatorial orbit (non-spinning device, spherical Earth)

$$\omega = \sqrt{G\frac{M}{R^3}}$$

$$ds^2 \cong \left(1-2\frac{\mu}{r}\right)c^2dt^2 - \left(1+2\frac{\mu}{r}\right)dr^2 - r^2 d\phi^2 + 4cr\left(\frac{j}{r^2} - \left(\frac{\mu}{r}\right)^{\frac{3}{2}}\right)dtd\phi$$

$$\nu_b = \frac{cA}{\lambda PR}\left(\left(\frac{\mu}{R}\right)^{\frac{3}{2}} - 2\frac{j}{R^2}\right)\hat{\theta}\cdot\hat{n}$$

$\sim 10^{-14}$

$\sim 10^{-16}$

Orbital gyro

Arrival times difference

Equatorial orbit

$$\delta\tau \cong -2\frac{A}{cR}\left(\left(\frac{\mu}{R}\right)^{\frac{3}{2}} - 2\frac{j}{R^2}\right) \approx 10^{-2}T$$

Pulses

Gaia and GAME

- GAIA: catalog of ~ billion stars, μas accuracy. Dynamic model of the gravitational field in the solar system needed. Areas close to the major objects (Sun, Jupiter, …) excluded.
- GAME: comparison of the apparent positions of stars on opposite sides of the sun or Jupiter. Look for asymmetries.

Experimental tests of General Relativity (historical)

- Precession of the perihelion of Mercury
  - 1915
    - Known value: 43"/Julian century
    - Value computed by Einstein: 43"/Julian century
- Lensing by the Sun
  - Theoretical value according to GR (1915)
  - Observed by Eddington (1919)
- Gravitational redshift
  - Recognized in the spectral lines of Sirius B (W. S. Adams, 1925)
  - Measured on Earth: Pound and Rebka experiment (1959)

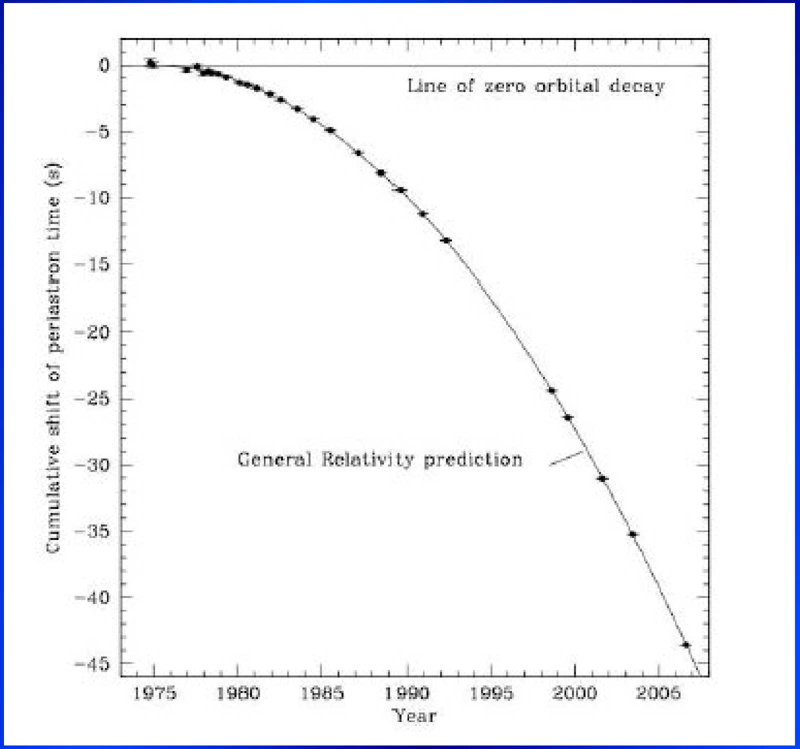
Observation of binaries

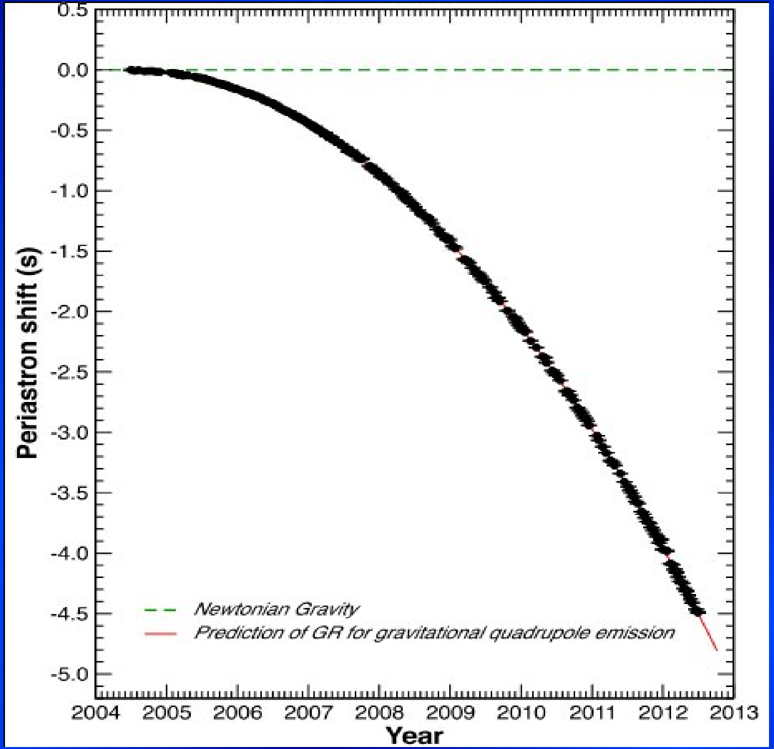
- Periapsis precession
  - PSR B1913+16 (Hulse and Taylor pulsar): 4.2°/year (compatible with GR)
  - PSR J0737-3029 (double pulsar): 16.9°/year
- Geodetic (de Sitter) precession
  - PSR J0737-3029B (double pulsar):
    - GR $(5.0734 \pm 0.0007)^\circ$/year
    - measured $(4.77^{+0.66}_{-0.65})^\circ$/year

A. Possenti and M. Burgay, private communication (2011)

Period decay (GW emission)

- 0.2% agreement with the quadrupole GW emission rate. PSR B1913+16 (Hulse and Taylor pulsar), after 33 years of data taking. Right: double pulsar, 9 years





Past and present tests

- Weak effects:
  - Geodetic precession
  - Inertial frame dragging
- Direct measurement in space
  - Gravity Probe B: 19%
  - LAGEOS I and II: 10%
  - LARES, under way → 1%
- Lunar laser ranging, pulsars in binary systems, double pulsar, ...

Extended or alternative theories: the PPN parameters

$$ds^2 = \left(1 - 2\frac{GM}{c^2 r}\right) d\tau^2 - \left(1 + 2\gamma\frac{GM}{c^2 r}\right) dr^2 - r^2 d\vartheta^2$$

$$- r^2 \sin^2 \vartheta \, d\phi^2 + \left(4 + 4\gamma + \alpha_1\right)\frac{GJ}{2c^3 r} \sin^2 \vartheta \, d\tau d\phi$$

$$2M \rightarrow (1+\gamma)M \qquad\qquad 2J \rightarrow \left(1 + \gamma + \frac{\alpha_1}{4}\right) J$$

Comparison with other techniques

- The experiments using mechanical gyroscopes in space sense a time and position depending gravitational field (along the orbit)
- With respect to matter waves, light has a local propagation velocity in vacuo which is a universal constant.
- Laser sources are far more stable and long lasting than the sources of matter waves

Light is a good probe of the structure of space-time

- In the case of GINGER the ring-lasers are fixed to the earth, so the gravitational field (both gravitoelectric and gravitomagnetic component) is constant.
- The side of the loops is far smaller than the local curvature radius of space-time: in practice the measurement is local.

Conclusion

- Closed paths of light in space (and space-time) evidence general relativistic effects either by
  - Interferometric techniques
    or
  - Beat frequency measurements
- The GINGER experiment is likely to verify the inertial frame dragging on Earth within a few years and with the 1% accuracy or better
- It would be useful and important to integrate different approaches and techniques in order to reveal possible deviations from GR.