

# How to stabilize the absolute length of the RLG diagonals

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## **OVERVIEW**

#### **1.** Motivation

2. A triple frequency-modulation technique for absolute length measurements

#### **3.** Preliminary results

4. Conclusions & future perspectives

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#### Sagnac frequency accuracy



The measurement of relativistic precessions requires to control the local geometry (i.e.  $k_s$ ) better than  $10^{-10}$ Corner mirror relative position and orientation should be accurate better than 1 nm and 1 nrad!



Napoli, 26th November, 2013

#### **Degrees of freedom**

Thanks to mirror spherical symmetry, the radiation path inside a 4-spherical mirror cavity is univocally defined by the positions of the four centers of curvature (CC).

Then, the whole optical cavity, which we suppose having a shape very near to a plane square, can be defined by 12 degrees of freedoms which are linear combinations of linear CC displacements (or, equivalentely, of mirror displacements).



#### **Motivation**

## **Numerical simulations**

Tracing out rigid body movements (6 d.o.f.) of the whole apparatus, a set of linear combinations of the CC movements can be chosen that produces:

 relative motions of the two diagonals, keeping fixed their lengths (no rotation)

2 d.o.f. horizontal + 1 d.o.f. vertical

 relative rotations of the diagonals keeping fixed their lengths

1 d.o.f.

stretching of the square diagonals
 2 d.o.f.





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Fig.1 Comparison of the relative changes induced in the scale factor of 1 m side ring laser by the 6 d.o.f



Fig.2 Total relative change in the scale factor i) with no constraint (continous line), and ii) by fixing the diagonal length (dashed line)

#### **Motivation**

#### How to reach a perfect square configuration?

- ♦ the ring cavity perimeter can be known with very high accuracy (>10<sup>11</sup>) by the beat note of the laser emission and a reference laser
- in a near-square 4-mirror cavity, the opposite mirrors define two Fabry-Pérot resonators whose length can be locked to the reference laser

Keeping the diagonal lengths locked, the other d.o.f. can be optimized one by one





## GP2

- Square optical cavity 1.60 m x 1.60 m on granite slab
- Access to FP diagonal cavities
- 6 PZT actuators
  - 3 axial on 1 mirror tower
  - 1 axial, in diagonal direction, on the other 3

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A triple frequency-modulation technique for absolute cavity length measurements

#### Aim of esperiment

The aim of this first experiment (University of Pisa, Department of Physics & INFN Section of Pisa) is to lock the absolute length of two linear cavities to a frequency standard by use of frequency-modulated light



Cavity length

 $L = \frac{c}{2f_n}(n + \varepsilon)$ 

 $f_n$  Resonance frequencies for TEM00 mode

$$0 < \varepsilon = \frac{1}{\pi} \cos^{-1} \sqrt{\left[ \left( 1 - \frac{L}{R_1} \right) \left( 1 - \frac{L}{R_2} \right) \right]} < 1$$

To measure the cavity length L with  $10^{-10}$  accuracy we have to:

- 1. measure  $f_n$  with the same accuracy
- 2. define univocally the value of n

 $\rightarrow$  to measure FSR=c/2L better than 1/n

3.  $f_{n+q} - f_n = q FSR$   $\longrightarrow$  the accuracy on Free Spectral Range increases as q

#### **Detection scheme**

The diagonal lengths can be measured with respect to an interrogating high-stability laser by a two frequency Pound-Drever-Hall circuit



## Locking laser to cavity: why?



Acoustics: unavoidable uncertainty on the measurement of the instantaneous length

- AOM compensates perfectly this noise by shifting the laser frequency
- This noise is reduced by a factor **n** on the measurement of FSR

$$\Delta n = \frac{n}{L} \sqrt{\frac{S_L}{\tau}}$$

In order to determine n, one needs to integrate for a time  $\tau$  (GP2):

$$n = 5 \times 10^{6} \qquad \Delta n < 1$$
  

$$S_{L} = 10^{-12} m/Hz \qquad \Delta n < 1$$
  

$$L = 2.26 m \qquad T > 4 s$$

#### A triple frequency-modulation technique for absolute cavity length measurements

## **Frequency Modulation**





#### FM signal spectrum

The spectrum of a FM wave contains the carrier and in the sidebands an infinite set of spectral lines symmetrically arranged on both sides of the carrier, at a distance of  $f_m$ ,  $2f_m$ ,  $3f_m$ , .... from it.



#### When the modulation index is small almost all the power

is in the carrier and the first order sidebands

Napoli, 26th November, 2013

## Triple modulation model with a single EOM

The incident electric field in the experimental set-up that we propose is triple frequency-modulated:

$$E_{i}(t) = E_{0} \exp\left\{i\left[\omega_{0}t + \alpha \sin\left(\omega_{A}t\right) + \beta \sin\left(\omega_{B}t + \gamma \sin\left(\omega_{C}t\right)\right)\right]\right\}$$



Using the Bessel functions of the 1<sup>st</sup> kind identity:

$$E_{i}(t) = E_{0} \exp\left\{i \omega_{0} t \left[\sum_{n} (-1)^{n} J_{n}(\alpha) \exp\left(i n \omega_{A} t\right)\right] \left[\sum_{m} (-1)^{m} J_{m}(\beta) \exp\left(i m \omega_{B} t\right) \exp\left[i m \gamma \sin\left(\omega_{C} t\right)\right]\right]\right\}$$

#### Incident and reflected electric field

For small modulation indices (  $\alpha,\beta,\gamma<2$  ) the laser electric field can be expanded using the first Bessel functions (n=m=0,-1, +1):

$$E_{i}(t) = E_{0} \exp\left\{i\omega_{0}t\left[\sum_{n}(-1)^{n}J_{n}(\alpha)\exp(i\,n\,\omega_{A}t)\right]\left[\sum_{m}(-1)^{m}J_{m}(\beta)\exp(i\,m\,\omega_{B}t)\exp[i\,m\,\gamma\sin(\omega_{C}t)]\right]\right\}$$
$$= E_{0} \exp\left\{i\omega_{0}t\left[J_{0}(\alpha)-J_{1}(\alpha)\exp(i\,\omega_{A}t)-J_{1}(\alpha)\exp(-i\,\omega_{A}t)\right]\right\}$$
$$\left[J_{0}(\beta)-J_{1}(\beta)\exp(i\,\omega_{B}t)\exp(i\,\gamma\sin(\omega_{C}t))+J_{-1}(\beta)\exp(-i\,\omega_{B}t)\exp(-i\,\gamma\sin(\omega_{C}t))\right]\right\}$$

The reflected beam is obtained by multiplying the incident beam with the cavity transfer function given by the mirror reflection coefficient (symmetric cavity; no losses):

$$(\omega) = \frac{r(\exp(i\omega/FSR) - 1)}{1 - r^2 \exp(i\omega/FSR)}$$

*r* amplitude reflection coefficient

 $\varphi = \frac{\omega}{FSR} = \frac{\omega}{c/2L}$  phase the light picks up in one round trip inside the cavity

$$\blacktriangleright E_r(t) = \sum_n F(\tilde{\omega}_n) E_i(\tilde{\omega}_n, t) \qquad n \text{ monochromatic components}$$

F

## **Reflected intensity**

The reflected intensity (components oscillating at the optical frequency are averaged out by the detector) is:

$$\begin{split} I_{r} &= E_{r}(t) E^{*}_{r}(t) \\ &= J_{0}^{2}(\alpha) J_{0}^{2}(\beta) \left[ F(\omega_{0}) F^{*}(\omega_{0}) \right] \\ &+ J_{1}^{2}(\alpha) J_{0}^{2}(\beta) \left[ F(\omega_{0}+\omega_{A}) F^{*}(\omega_{0}+\omega_{A}) \right] + \left[ \omega_{0}+\omega_{A} \Rightarrow \omega_{0}-\omega_{A} \right] \\ &+ J_{0}^{2}(\alpha) J_{1}^{2}(\beta) J_{0}^{2}(\gamma) \left[ F(\omega_{0}+\omega_{B}) F^{*}(\omega_{0}+\omega_{B}) \right] + \left[ \omega_{0}+\omega_{B} \Rightarrow \omega_{0}-\omega_{B} \right] \\ &+ J_{1}^{2}(\alpha) J_{1}^{2}(\beta) J_{0}^{2}(\gamma) \left[ F(\omega_{0}+\omega_{A}-\omega_{B}) F^{*}(\omega_{0}+\omega_{A}-\omega_{B}) + F(\omega_{0}+\omega_{A}+\omega_{B}) F^{*}(\omega_{0}+\omega_{A}+\omega_{B}) \right] + \left[ \omega_{0}+\omega_{A} \Rightarrow \omega_{0}-\omega_{A} \right] \\ &+ J_{0}^{2}(\alpha) J_{1}^{2}(\beta) J_{1}^{2}(\gamma) \left[ F(\omega_{0}+\omega_{B}-\omega_{C}) F^{*}(\omega_{0}+\omega_{B}-\omega_{C}) + F(\omega_{0}+\omega_{B}+\omega_{C}) F^{*}(\omega_{0}+\omega_{B}+\omega_{C}) \right] + \left[ \omega_{0}+\omega_{B} \Rightarrow \omega_{0}-\omega_{B} \right] \\ &+ J_{1}^{2}(\alpha) J_{1}^{2}(\beta) J_{1}^{2}(\gamma) \left[ F(\omega_{0}+\omega_{A}+\omega_{B}+\omega_{C}) F^{*}(\omega_{0}+\omega_{A}+\omega_{B}+\omega_{C}) + F(\omega_{0}+\omega_{A}-\omega_{B}-\omega_{C}) F^{*}(\omega_{0}+\omega_{A}-\omega_{B}-\omega_{C}) \right] \\ &+ F(\omega_{0}+\omega_{A}+\omega_{B}-\omega_{C}) F^{*}(\omega_{0}+\omega_{A}+\omega_{B}-\omega_{C}) + F(\omega_{0}+\omega_{A}-\omega_{B}+\omega_{C}) F^{*}(\omega_{0}+\omega_{A}-\omega_{B}+\omega_{C}) \right] + \left[ \omega_{0}+\omega_{A} \Rightarrow \omega_{0}-\omega_{A} \right] \end{split}$$



21 frequency components symmetrically distributed around the carrier

#### **Reference signals for the carrier resonance**

The mixer pulls out the term that is proportional to  $\sin(\omega_A t)$ , so the carrier error signal is:

$$S_{carrier}(\omega_0, \omega_A) = i a_0(\alpha, \beta) a_1(\alpha, \beta) \left\{ \left[ -F(\omega_0)F^*(\omega_0 + \omega_A) + F(\omega_0 + \omega_A)F^*(\omega_0) \right] + \left[ + \Rightarrow - \right] \right\}$$

with

$$\begin{cases} a_0(\alpha,\beta) = J_0(\alpha)J_0(\beta) \\ a_1(\alpha,\beta) = J_1(\alpha)J_0(\beta) \end{cases}$$



#### **Reference signals for the sidebands resonance**

The mixer pulls out the term that is proportional to  $sin(\omega_c t)$ , so the sidebands error signal is:

$$S_{side}(\omega_{0},\omega_{B},\omega_{C})=ia_{2}(\alpha,\beta,\gamma)a_{4}(\alpha,\beta,\gamma)\left\{\left[F(\omega_{0}-\omega_{B})F(\omega_{0}-\omega_{B}+\omega_{C})-F(\omega_{0}+\omega_{B})F(\omega_{0}-\omega_{B}+\omega_{C})+F(\omega_{0}+\omega_{B}+\omega_{C})F(\omega_{0}+\omega_{B})\right]+[+\Rightarrow-]\right\}$$

$$+F(\omega_{0}+\omega_{B}-\omega_{C})F(\omega_{0}+\omega_{B})+F(\omega_{0}+\omega_{B}+\omega_{C})F(\omega_{0}+\omega_{B})\left]+[+\Rightarrow-]\right\}$$
with
$$\begin{cases}a_{2}(\alpha,\beta,\gamma)=J_{0}(\alpha)J_{1}(\beta)J_{0}(\gamma)\\a_{4}(\alpha,\beta,\gamma)=J_{0}(\alpha)J_{1}(\beta)J_{1}(\gamma)\end{cases}$$





# Effect of carrier detuning on sidebands resonance:

- It causes signal deformation and change in the sign of the slope
- This criticity could be enhanced for an high Finesse!

#### Compatibility between FM theory and

our experimental set-up!

## **Modulation index optimization**

We have to determine the proper  $(\alpha, \beta, \gamma)$  to obtain the *best* signal for the sidebands resonance reference signal as well as for the carrier reference signal.

A good criterion is to maxime the two product

$$A_{carrier}(\alpha,\beta) = a_0(\alpha,\beta)a_1(\alpha,\beta) \qquad A_{side}(\alpha,\beta,\gamma) = a_2(\alpha,\beta,\gamma)a_4(\alpha,\beta,\gamma)$$
$$= J_0(\alpha)J_1(\alpha)J_0^2(\beta) \qquad = J_0^2(\alpha)J_1^2(\beta)J_0(\gamma)J_1(\gamma)$$

to obtain the maximum Signal to Noise Ratio on the two signals.



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## **Experimental set-up**



#### Resonance frequencies

$$f_n = f_{iodine} + f_{het} + f_{AOM}$$

Description	Frequency	Purpose	Accuracy
f <sub>iodine</sub>	473.61235412(3) THz	frequency standard	10-11
f <sub>het</sub>	5 GHz	PLL offset frequency	10 <sup>-6</sup>
f <sub>AOM</sub>	200 MHz	Carrier locking frequency shift	10-5
f <sub>sb</sub>	m FSR	Sideband locking frequency modulation	5/m *10 <sup>-7</sup>

#### **Experimental set-up**



#### He-Ne/I2 frequency standard



The master frequency is locked so that  $f_M-f_S=8$  MHz (SNR optimization); than the slave frequency is locked to the center of a transition as seen from master: in this way the master frequency is driven by slave

The hyperfine components are from 11-5 R(127) transition; the separation between components "d" and "g" is approximately 40 MHz





### **Experimental set-up**



Procedure flow chart

Accuracy in mirrors positioning  $\delta L \simeq 30 \mu m$ 

$$\delta n = \frac{2 \,\delta L}{\lambda} \simeq 100$$
$$\delta f_n = \frac{\delta n}{n} \times f_n \simeq 1 \,GHz$$

AOM tuning range  $f_{AOM} \approx 10 MHz$ 

Length measurement (and stabilization) can be done without unlocking the carrier only approximately within a given value of n



## "GP2ino"



### Locking the laser to cavity (feedback to ECDL)



Spectral density of the cavity length with the laser locked to the cavity by slow (to cavity PZT) and broad-band correction (to laser current)

#### Locking the laser to cavity (feedback to AOM)



To eliminate beam deflections due to dependence of the laser beam diffraction angle on the modulation frequency we made a double-pass AOM system (October 2013)





On a second pass through AOM the beam, with its polarization rotate by 90°, is deflected back such that it counterpropagates the incident laser beam and it can be separated from the input beam with a PBS

To eliminate beam deflections due to dependence of the laser beam diffraction angle on the modulation frequency we made a double-pass AOM system (October 2013)



#### **Specifications**



AO Medium	TeO2	
Active Aperture	2.5 x 1.5 mm	
Center Frequency	200 MHz	
RF Bandwith	50 MHz @ -10 dB return loss	
Wavelength	470–690 nm	
Bragg angle range (mr)	11.2 - 16.4	

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## **AOM locking to cavity**

To test locking performance we record the spectrogram trace of the laser frequency shift (with respecto to frequency reference 200 MHz) due to an artificial change of the cavity length (i.e. change induced by us acting on cavity PZT)



#### **Double lock (AOM & cavity)**

We report a preliminary locking of the interrogating laser to our diagonal prototype using AOM double-pass and giving the correction to cavity PZT by @LabVIEW





$$f_{C} = f_{Iodine} + f_{AOM}$$

3 hours measurement time, November 2013







#### **Double lock (AOM & cavity) - Electronics**



**Rosa Santagata** 

#### Double lock (AOM & cavity) – AOM Loop Filter



## **Room temperature fluctuations**



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## **Room temperature fluctuations**



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#### **Conclusions & future perspectives**

We have reported the first results concerning an experimental technique for the absolute RLG diagonal lengths control using a triple-frequency modulation scheme.

• The triple-frequency modulation model elaborated shows the compatibility between the modulation theory and the error signals that we want to use in our detection scheme

optimization of the model using experimental validation

- The preliminary locking of the interrogating laser to one diagonal cavity proves the feasibility of the experiment, but to achieve our goal we have to stabilize the laser source
  - -----
- He-Ne/I2 frequency standard
- -----
- location with less environmental noise (INFN lab, Polo Fibonacci) and a more mechanical stability of experimental set-up

## Thank you for your attention!