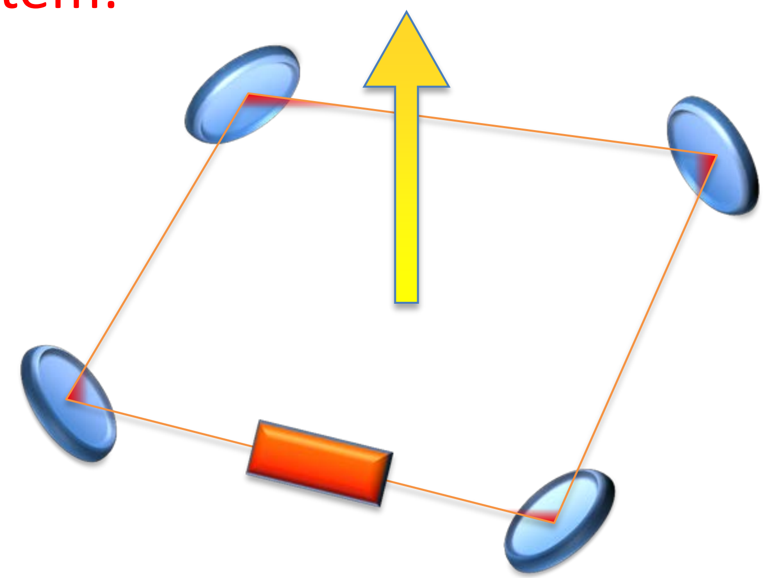
Ring Laser Optical Cavity

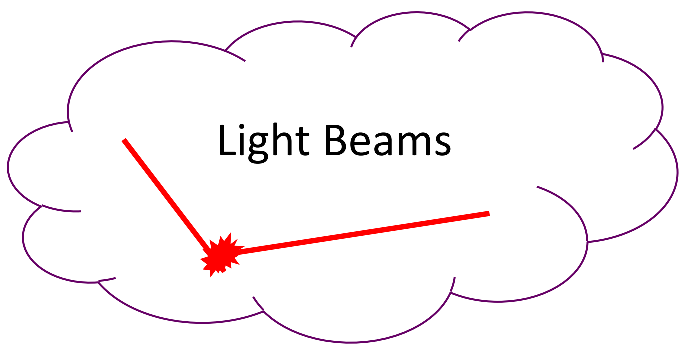
Model and Control Requirements

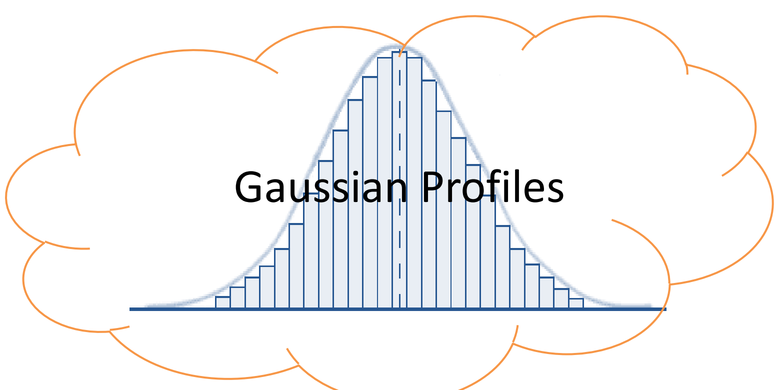
RLG optical cavity

Different Formalisms to describe the system:

- Geometric Optics
- Gaussian Optics
- Wave Optics



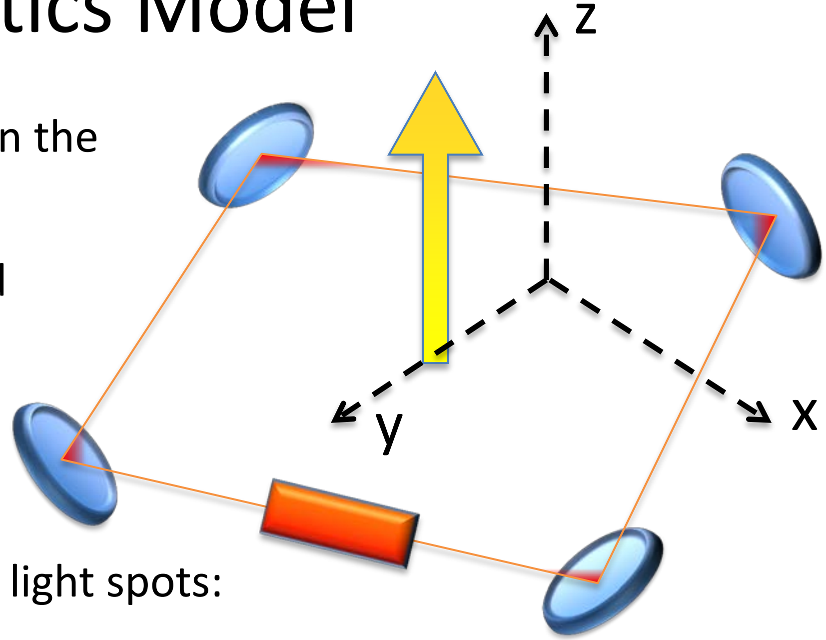




2D System Space-Time

$$\frac{\partial^2 E}{\partial z^2} - \epsilon_0\mu_0 \frac{\omega}{Q}\frac{\partial E}{\partial t} - \epsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} = -\mu_0\omega^2 P.$$

Geometric Optics Model



12 State Variables: 3D coordinates of light spots on the mirrors.

Parameters: Position of the mirror C.O.C (4x3) and value of R.O.C. (4)

Spherical mirrors –> each one invariant for rotations!

One to one correspondence between C.O.C.'s and light spots: M. Valentini Master Thesis, 2012

Beam Forming:
(100 MHz -1 MHz)

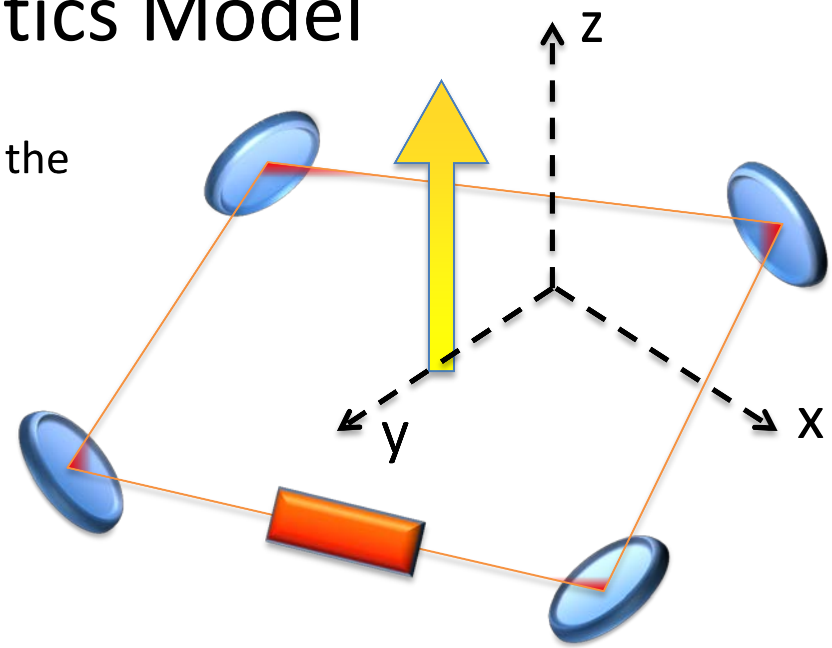
Characteristic timescales of the System

Geometric Deformations:
(0.1 Hz – 10^{-6} Hz)

Beam Amplitude Stabilization:
(100 KHz -1 KHz)

To find the path of laser light given the mirror displacement: → Fermat Principle → Snell Law

Geometric Optics Model



Fermat Principle: among all possible closed path the light will follow the shortest one

$$\mathbf{x} = \arg\min_{\xi} P(\xi, \mathbf{u}, \mathbf{R})$$

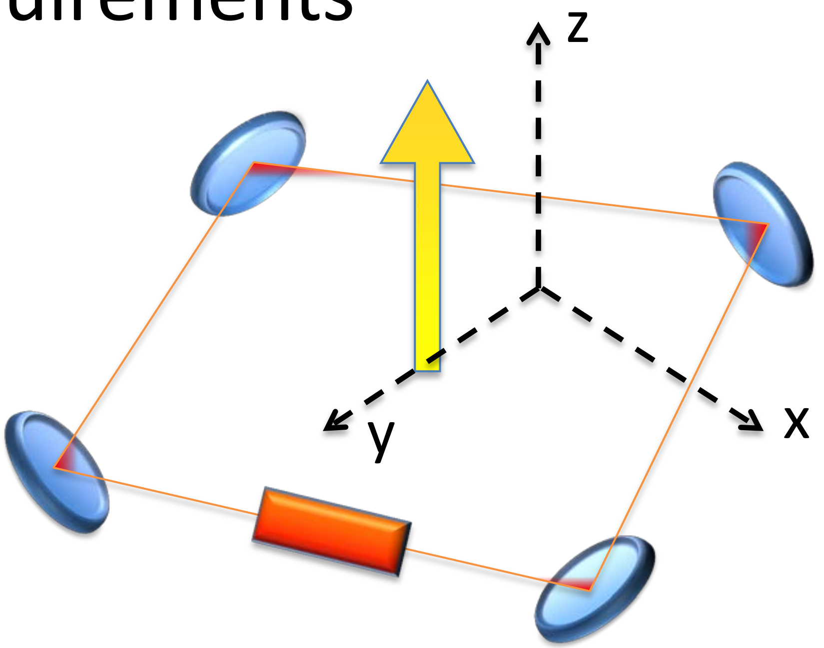
Is it a stable closed path?

- We can use 5 points and impose the constraint 5=1
- For small deviations from the perfect square geometry 4 points are enough

The spots positions are computed using a 2nd order approximation of $P(\xi, \mathbf{u}, \mathbf{R})$ around the perfect square and then searching for the minimum

The accuracy of the steering within the approximations is sufficient for movements of the C.O.C. $\leq 10^{-4}$

Control Requirements



- Stabilize the Perimeter of the Ring
  Necessary for the laser continuous operation

- Stabilize the Area of the Ring
  Necessary for suppressing variations in the Scale Factor of the instrument

- Stabilize the relative orientation of rings (3D problem)
  Necessary for obtaining accurate measurements of the projections of the earth rotation vector

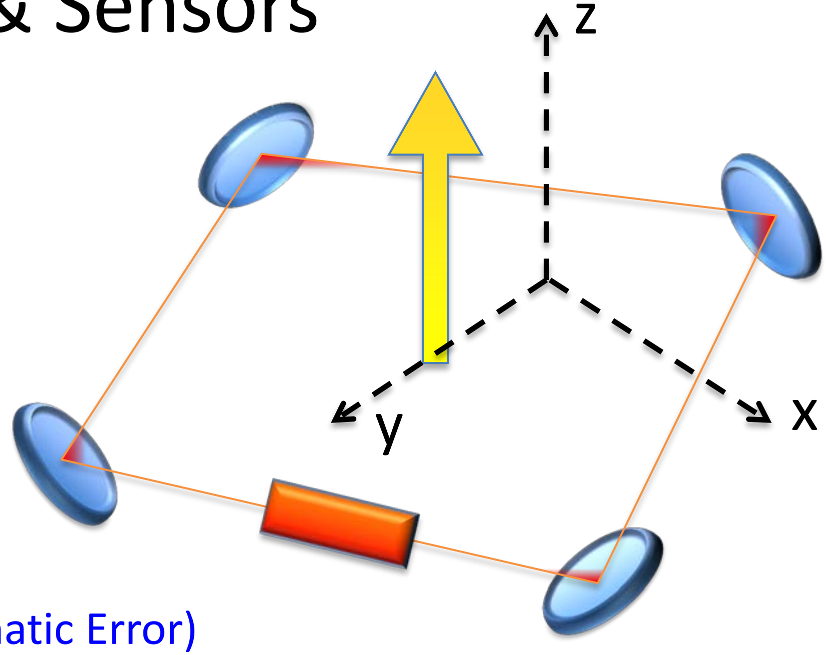
Control the Optical cavity as a Rigid Body!

In Addition:

- Rotate rigidly the Optical Cavity of a known Angle
  Active Calibration of the Instrument

- Stabilize the positions of the spots on the mirrors
  Active Rejection of Backscattering effects

Observables & Sensors



Few Observables !!!

Accurate Value of Perimeter from the measure of the mean optical frequency of the two travelling beams

Precise Value of the Diagonals from a support diagnostic (Michelson -Moore interferometer).
Their measure is referred to the injector! (Systematic Error)

Relative Variations of the 4 Sides from the same detectors used for the timeseries analysis: Monobeam Intensities & Interferogram.

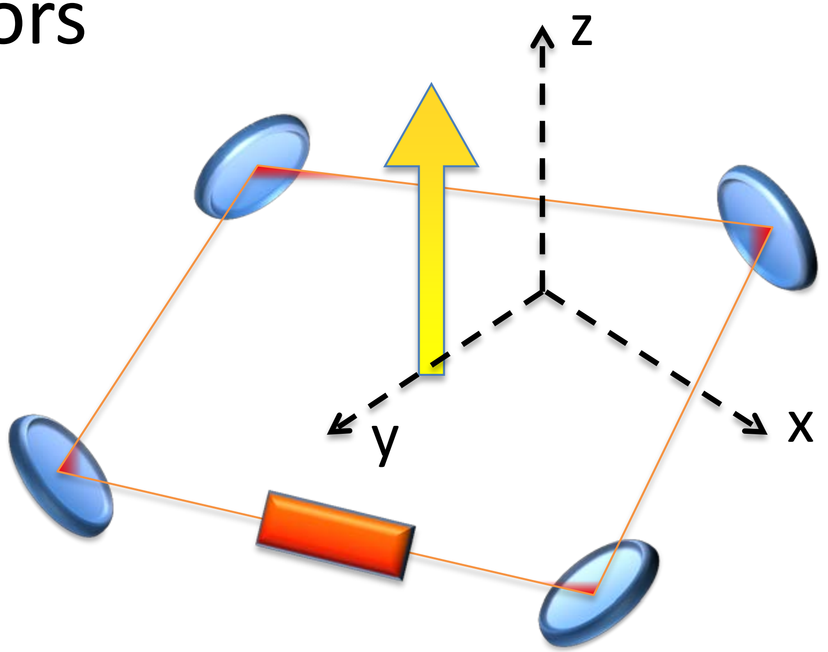
Tilt of the Diagonals using information contained in the higher cavity modes (different from TEM 00). SNR? Injector tilts?

Global Bandwidth (5 kHz – 0.1 Hz)

Actuators

How many actuators we need?

To stabilize the relative distances of the spots, 6 independent translations of the C.O.C (not the rigid body d.o.f.) are sufficient.



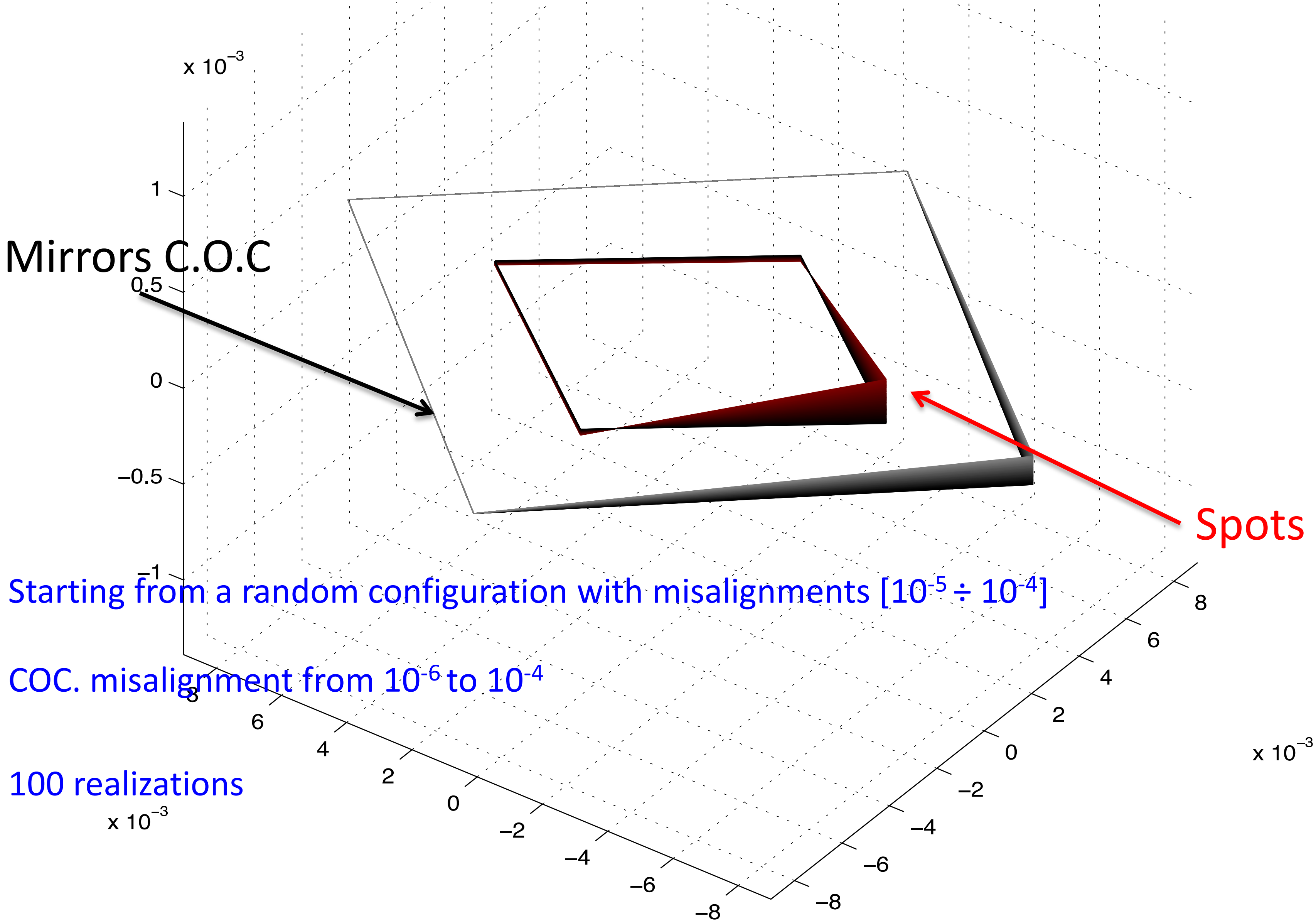
Piezoelectric Actuators able to translate the position of the C.O.C. in a neighborhood of its rest position

Accurate PZTs may represent a way to calibrate the instrument by rotation of a known angle. Suitable PI Piezos ensure nanoradians precision. Very Expensive!

Actuators Bandwidth
$\leq$ 100 Hz

Standard PZTs have uncertainties in the direction of the induced translation, as well hysteresis problems that must be overcome with a calibration / identification procedure. Limited by the Sensor Precision!

Case Study: 1 Spot Out of Plane

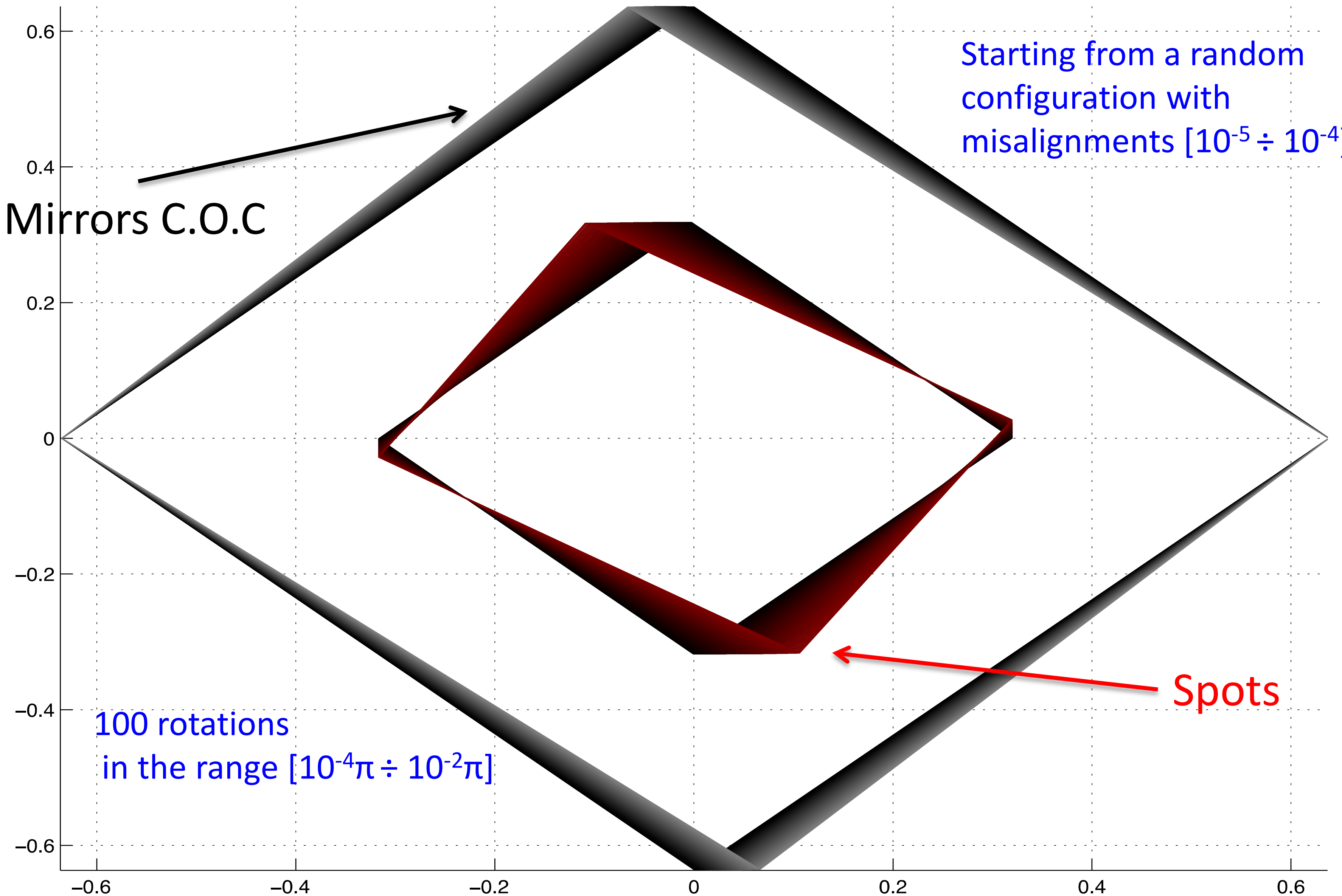


Starting from a random configuration with misalignments $[10^{-5} \div 10^{-4}]$

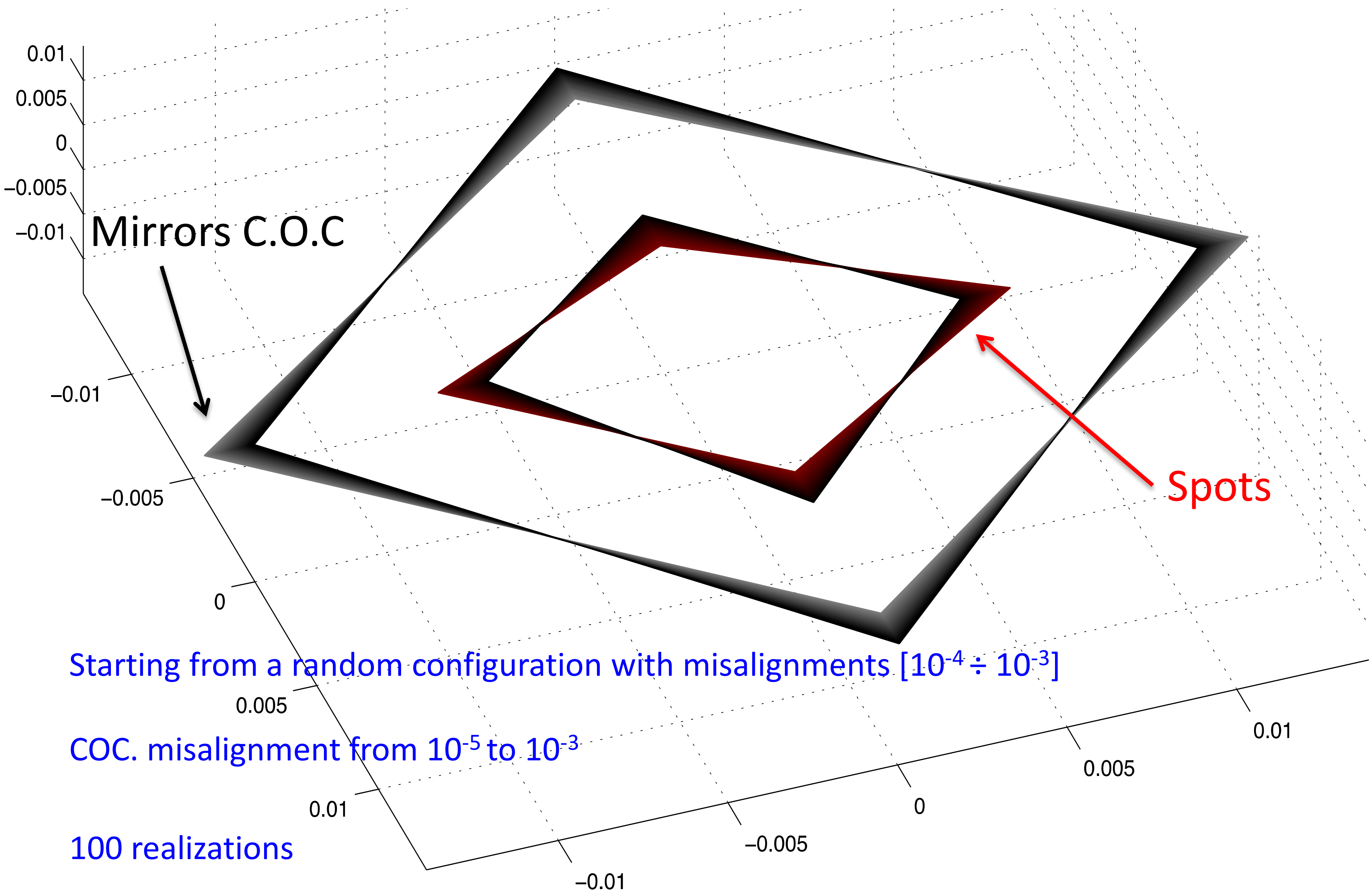
COC. misalignment from 10^{-6} to 10^{-4}

100 realizations

Case Study: Diagonal Rotation



Case Study: Diagonals Dilation

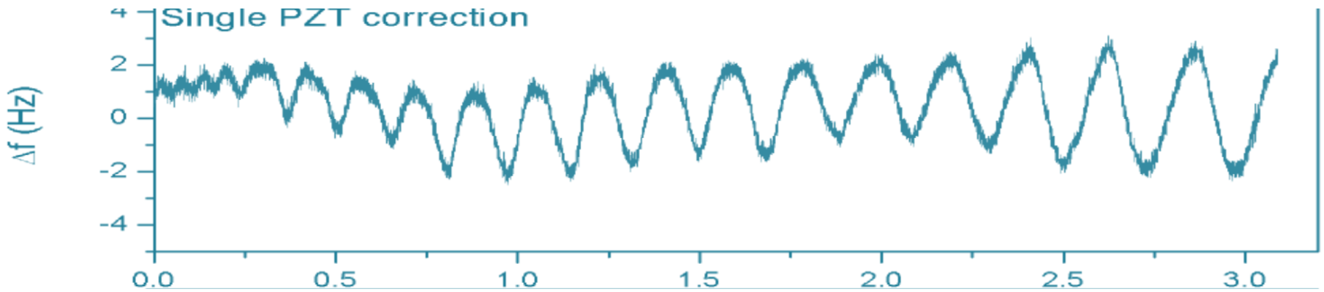


Starting from a random configuration with misalignments [$10^{-4} \div 10^{-3}$]

COC. misalignment from 10^{-5} to 10^{-3}

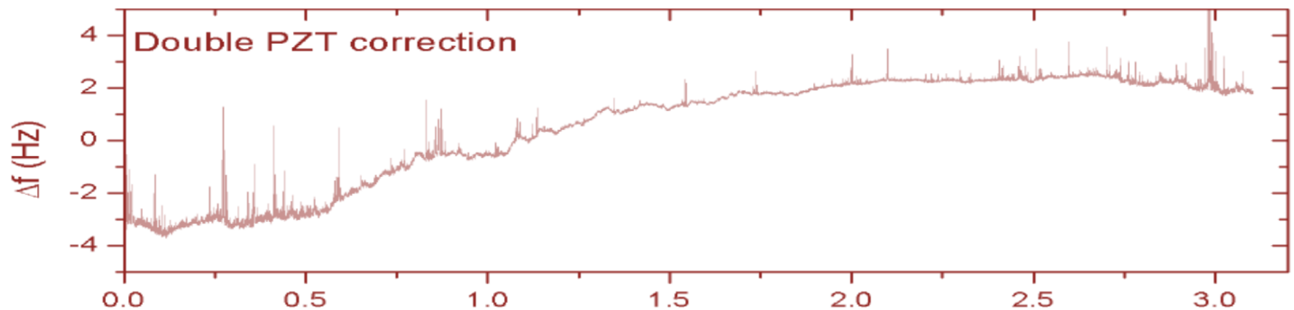
100 realizations

Conclusions



- 1 Piezo

Stabilize the perimeter without simmetry

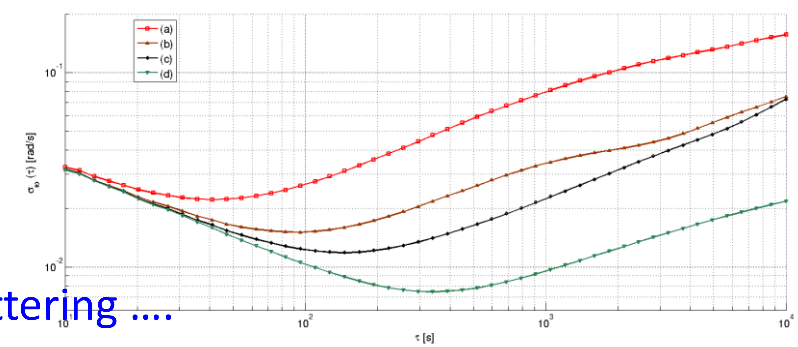


- 2 Piezos

Stabilize the perimeter with simmetry

- 6 Piezos

Rigid Body with active control (Zerodur).



- 12 Piezos

Actively minimize backscattering ....