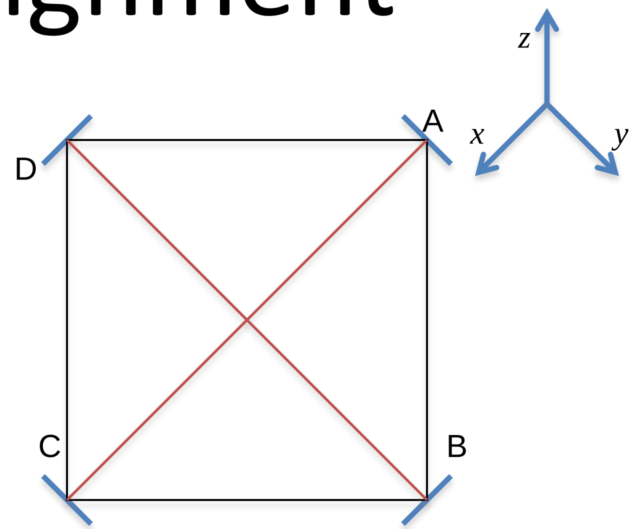
Square cavity alignment



- Each mirror has a 3-axial PZT movement.
- Globally there are 12 degrees of freedom, which can be expressed as linear combination of:

  $X_A, X_B, X_C, X_D, Y_A, Y_B, Y_C, Y_D, Z_A, Z_B, Z_C, Z_D$.
- The 3 rigid translations can be expressed by linear combinations of the motion of the 4 mirrors:

  $(X_A + X_B) + (X_C + X_D)$

  $(Y_A + Y_B) + (Y_C + Y_D)$

  $(Z_A + Z_B) + (Z_C + Z_D)$
- The 3 rigid rotations can be expressed as:

  $(X_B - X_D) + (Y_A - Y_C)$

  $(Y_A - Y_C) + (Z_B - Z_D)$

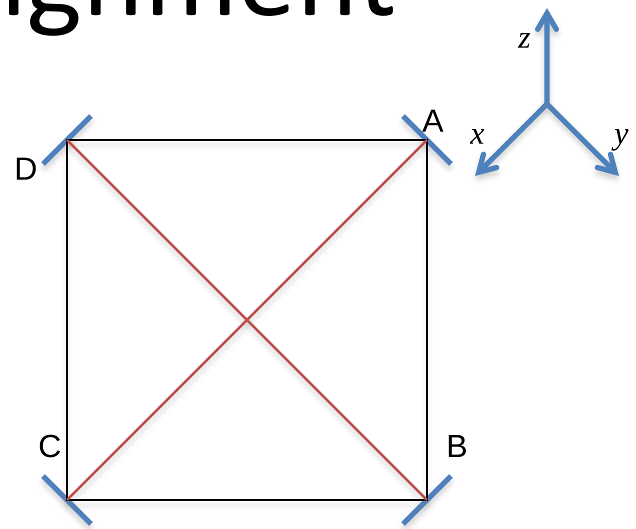
  $(Z_A - Z_C) + (X_B - X_D)$
- 2 degrees of freedom are freezed assuming that the length of the two Fabry-Pérot (FP1 and FP2) between the opposite mirrors are measured at 10^{-10}

  $X_A - X_C = Y_B - Y_D = d$

- the residual 4 degrees of freedom :
- $(X_A + X_C) - (X_B + X_D)$: FP1 orthogonal to FP2; the centre of FP2 is translated by δ_1 along FP1
- $(Y_B + Y_D) - (Y_A + Y_C)$: FP1 orthogonal to FP2; the centre of FP1 is translated by δ_2 along FP2
- $(Y_A - Y_C) - (X_B - X_D)$: FP1 axis is rotated of $\pi/4 + \varepsilon$ with respect to FP2 axis
- $(Z_A + Z_C) - (Z_B + Z_D)$: FP1 orthogonal and centred to FP2, but with a vertical displacement ζ .

Square cavity alignment

- For all the kind of deformations, keeping fixed the diagonals, perimeter and area values approach quadratically the case of perfect square. The value of the perimeter when optimized must be $2d\sqrt{2}$.
- Around the best geometry, the perimeter is a quadratic function of the deviations. The motion, combined as indicated above, are mutually orthogonal.
- It is then possible to optimize one degree of freedom at time. It can be achieved by a parabolic fit or, as an error signal by modulating the PZT. In principle, it should be possible to modulate each degree of freedom at different frequencies, and to detect on-line the error signals.



- If we neglect the 6 rigid body degrees:

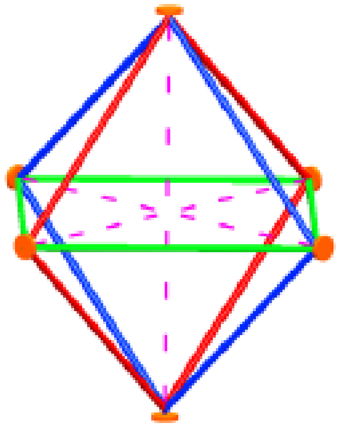
  6 PZT are sufficient: 3 in A, 1 in B, C, D(along the diagonal)
- Diagonal length: $X_A - X_C$ and $Y_B - Y_D$
- Planarity control: Z_A
- Angle between FP1 and FP2 axis: Y_A
- Crossing point: $(X_A + X_C)$ and $(Y_B + Y_D)$

Octahedron

- Each mirror has a 3-axial PZT movement.
- Globally there are 18 degrees of freedom, which can be expressed as linear combination of:

  $X_A, X_B, X_C, X_D, X_E, X_F;$
  $Y_A, Y_B, Y_C, Y_D, Y_E, Y_F;$
  $Z_A, Z_B, Z_C, Z_D, Z_E, Z_F$.

$$\Omega_{obs}^2 = \Omega^2 \cdot \sum_{i=1}^{3} \left(\hat{\Omega} \cdot \hat{n}_i \right)^2$$



- Diagonal deplacements (fixed length; no rotation) 3x3 degrees of freedom:
- Rotations of one diagonal with respect to the other two (fixed length; no deplacements) 3x2 degrees of freedom
- Diagonal stratching 3 degrees of freedom

- Diagonal deplacements (fixed length; no rotation) 3x3 degrees of freedom
  - 3: Rigid body translations along *x,y,z*
- Do not affect the orientation of the rings
  - The effect on the scale factor has been already calculated to be quadratic

Octahedron

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  $Y_A, Y_B, Y_C, Y_D, Y_E, Y_F;$
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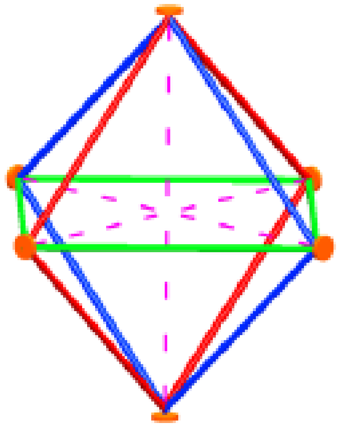
$$\Omega_{obs}^2 = \Omega^2 \cdot \sum_{i=1}^{3} \left(\hat{\Omega} \cdot \hat{n}_i \right)^2$$



- Diagonal deplacements (fixed length; no rotation) 3x3 degrees of freedom:
- Diagonal stratching 3 degrees of freedom
- Rotations of one diagonal with respect to the other two (fixed length; no deplacements) 3x2 degrees of freedom

- Diagonal stratching
  - Fixed by servos!

Octahedron

- Each mirror has a 3-axial PZT movement.
- Globally there are 18 degrees of freedom, which can be expressed as linear combination of:

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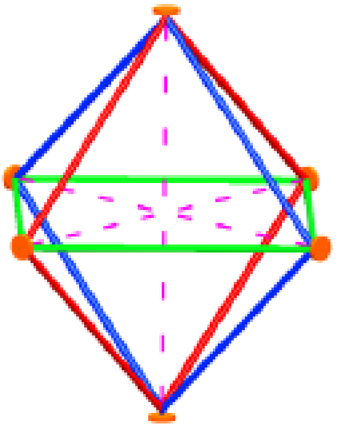


$$\Omega_{obs}^2 = \Omega^2 \cdot \sum_{i=1}^{3} \left(\hat{\Omega} \cdot \hat{n}_i \right)^2$$

- Diagonal deplacements (fixed length; no rotation) 3x3 degrees of freedom:
- Diagonal stratching 3 degrees of freedom
- Rotations of one diagonal with respect to the other two (fixed length; no deplacements) 3x2 degrees of freedom

- Rotations (6):
  - *around x-axis:* $R_x^z ; R_y^z$
  - *around y-axis:* $R_x^y ; R_z^y$
  - *around z-axis:* $R_y^x ; R_z^x$
- Rigid rotations :

$$\begin{cases} R_y^x + R_z^x \\ R_x^y + R_z^y \\ R_x^z + R_y^z \end{cases}$$

- Angular deformation

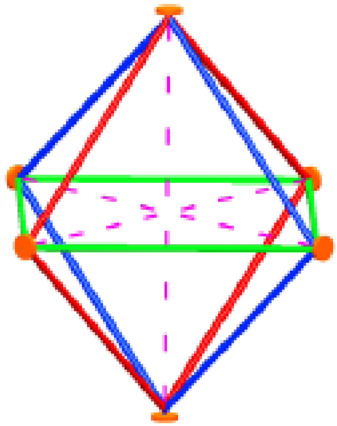
$$\begin{cases} R_y^x - R_z^x \\ R_x^y - R_z^y \\ R_x^z - R_y^z \end{cases}$$

Octahedron

$$\Omega_{obs}^2 = \sum_{i=1}^{3}\left(\vec{\Omega}\cdot\hat{n}_i'\right)^2 = \sum_{i=1}^{3}\left(\vec{\Omega}\cdot\left(\hat{n}_i+\lambda_{ij}\hat{n}_j\right)\right)^2 = \sum_{i=1}^{3}\left(\vec{\Omega}\cdot\hat{n}_i+\lambda_{ij}\vec{\Omega}\cdot\hat{n}_j\right)^2$$

$$= \sum_{i=1}^{3}\left(\vec{\Omega}\cdot\hat{n}_i\right)^2 + \sum_{i=1}^{3}\left(\lambda_{ij}\vec{\Omega}\cdot\hat{n}_j\right)^2 + 2\sum_{i=1}^{3}\left(\vec{\Omega}\cdot\hat{n}_i\right)\left(\lambda_{ij}\vec{\Omega}\cdot\hat{n}_j\right)$$

$$= \Omega^2 + 2\lambda_{ij}\Omega_i\Omega_j$$



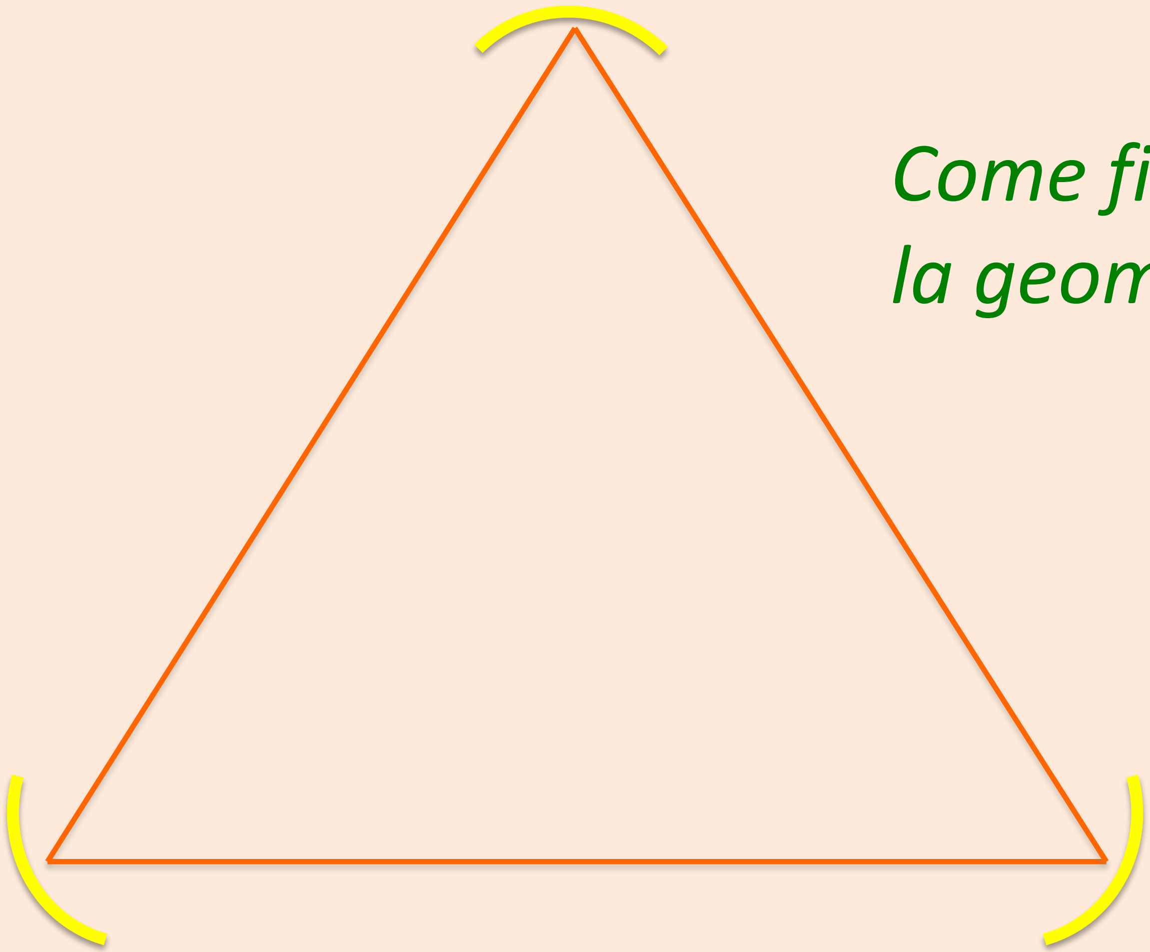
- Angular motion $R_y^x - R_z^x$ deforms the geometry of the ring *yz* and rotates the direction of the normal to the other two rings.

$$\hat{n}_x' = \hat{n}_x$$
$$\hat{n}_y' = \hat{n}_y + \lambda\hat{n}_z$$
$$\hat{n}_z' = \hat{n}_z + \lambda\hat{n}_y$$

$$\Omega_{obs}^2 = \Omega^2 + 4\lambda\Omega_y\Omega_z$$

ELUCUBRAZIONI DI FANTASIA

Cavità ottiche triangolari



Come fissare la geometria