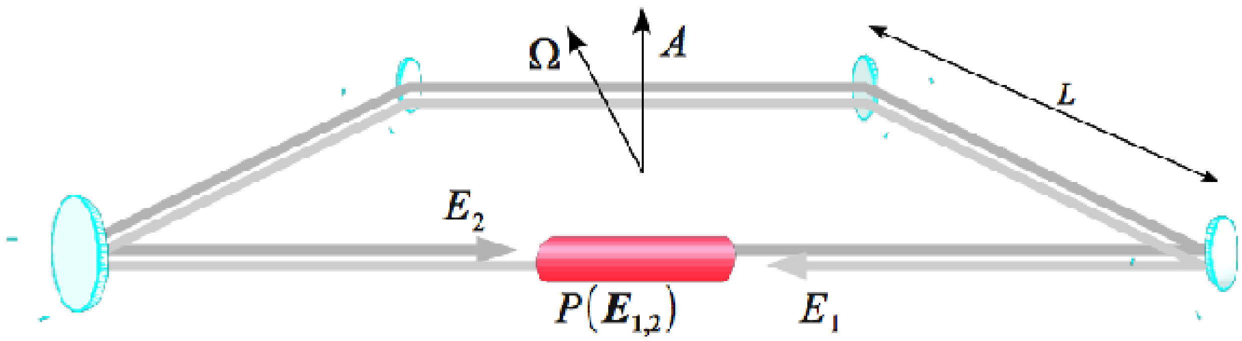
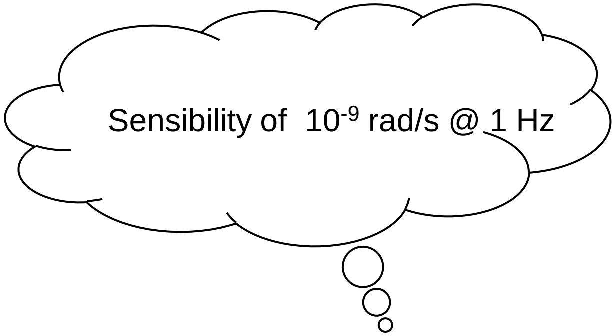
Ring Laser Gyroscopes



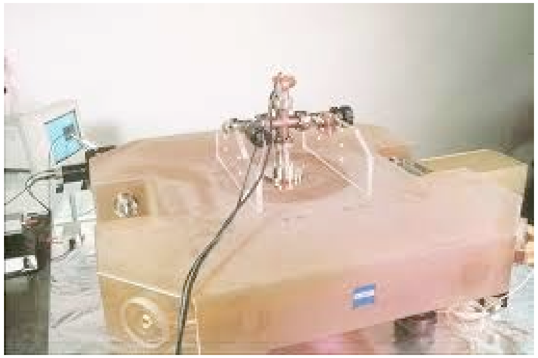


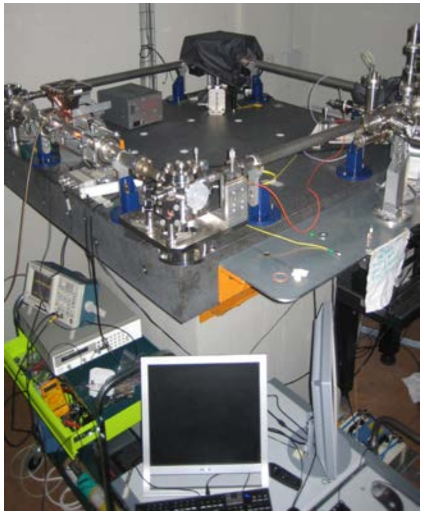
- *Large Size: (5-10 m) Geodesy, Astronomy,*

G.R. Tests.

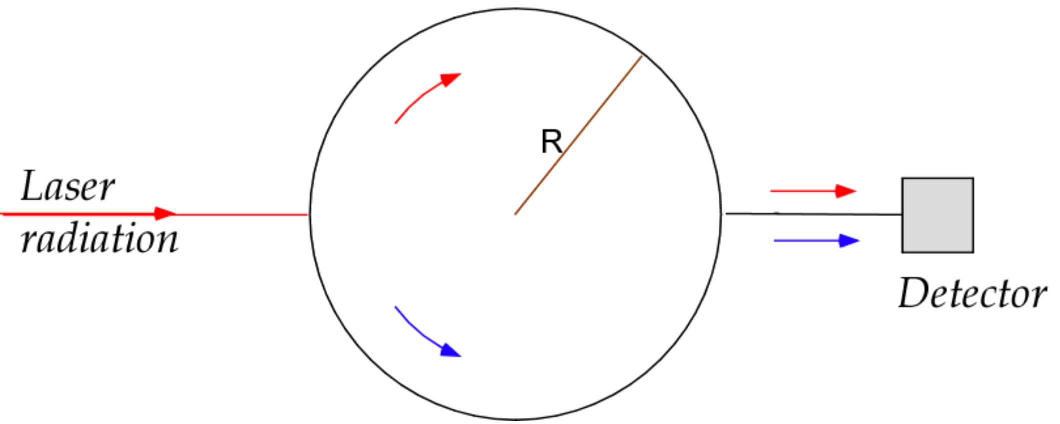
- *Medium Size: (1-5 m) Geophysics, Sismology, metrology.*

- *Small Size: (5-50 cm) Inertial Guidance.*

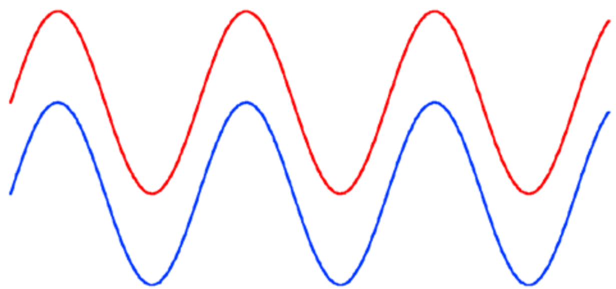


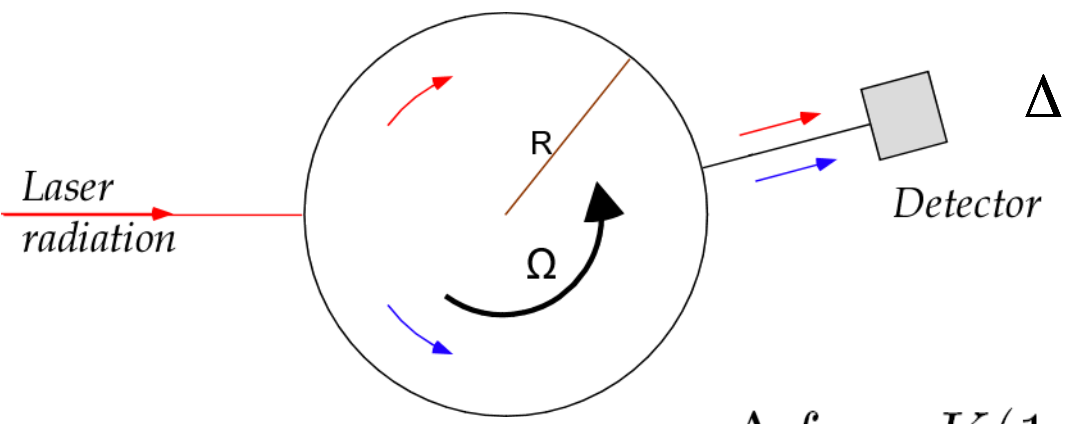


Sagnac Effect

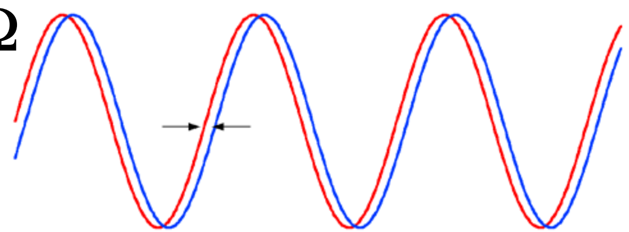


$$\Delta\phi = 0$$

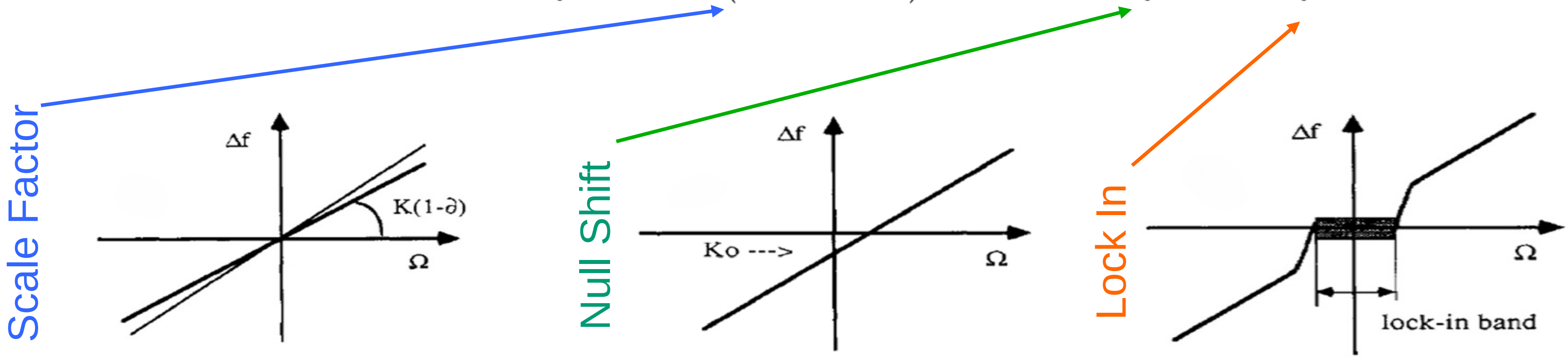




$$\Delta\phi = \frac{8\pi A}{\lambda c}\vec{n}\cdot\vec{\Omega}$$



$$\Delta f = K(1 + K_A)n \cdot \Omega + \Delta f_0 + \Delta f_{NL}$$



Ring Laser Equations: Complex Form

Dynamics expressed using complex field amplitudes

2 coupled Stuart-Landau Oscillators (SLO)

$$\begin{aligned}\dot{E}_1(t) &= \left(\mathcal{A}_1 - \mathcal{B}_1|E_1(t)|^2 - \mathcal{C}_{21}|E_2(t)|^2\right) E_1(t) + \mathcal{R}_2 E_2 + q_1 \\ \dot{E}_2(t) &= \left(\mathcal{A}_2 - \mathcal{B}_2|E_2(t)|^2 - \mathcal{C}_{12}|E_1(t)|^2\right) E_2(t) + \mathcal{R}_1 E_1 + q_2\end{aligned}$$

in matrix formalism separation of linear and non-linear parts

$$\dot{\mathbf{E}} = \left[\mathbf{A} - \mathscr{D}(\mathbf{E}).\mathbf{B}.\mathscr{D}(\mathbf{E}^*)\right]\mathbf{E}$$

$$\mathbf{A} \equiv \begin{pmatrix} \mathcal{A}_1 & \mathcal{R}_2 \\ \mathcal{R}_1 & \mathcal{A}_2 \end{pmatrix} \quad \mathbf{B} \equiv \begin{pmatrix} \mathcal{B}_1 & \mathcal{C}_{21} \\ \mathcal{C}_{12} & \mathcal{B}_2 \end{pmatrix} \quad \mathscr{D}(\mathbf{E}) \equiv \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

Operation conditions of large rings (G-PISA, G-WETTZELL , etc.) biased by Earth rotation

$$\begin{aligned}\mathcal{B}_1 &\simeq \mathcal{B}_2 = \mathcal{B} \\ \mathcal{C}_{21} &\simeq \mathcal{C}_{12} = \mathcal{C}\end{aligned}$$

Split of parameters: «passive cavity»/«active medium»

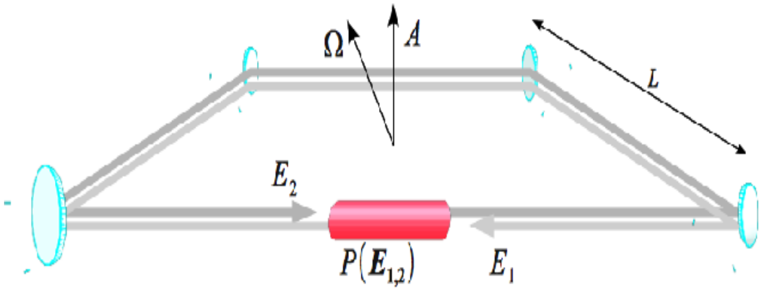
$$\mathbf{A} \equiv \frac{c}{L}\mathbf{P}^{(0)} - \mathbf{M}$$

LINEAR in the fields $E_{1,2}$

$$\mathbf{B} \equiv \frac{c}{L}\mathbf{P}^{(2)}$$

NON-LINEAR in the fields $E_{1,2}$

$$A = \begin{pmatrix} \alpha_1 + i\omega_s & r_2 e^{i\epsilon} \\ r_1 e^{i\epsilon} & \alpha_2 - i\omega_s \end{pmatrix}$$



$$B = \begin{pmatrix} \beta & \theta + i\tau \\ \theta + i\tau & \beta \end{pmatrix}$$

$$P^{(3)}(E_{1,2}) = \frac{-2i\,\mu_{ab}^2}{\gamma_{ab}} \int_{-\infty}^{\infty} \chi_{1,2}(v)\,\rho^{(2)}(v, E_{1,2})\,dv$$

Polarizability of active medium (isotopic mixture He-Ne)
predictable but hard to be identified (non linear and ill-posed)
(single pass gain G and plasma dispersion function Z(ξ))

Dissipative phenomena (M Matrix) are not predictable but
they can be IDENTIFIED (if polarizability P is given!)
(mirror losses, backscattering,etc.)

$$\mathbf{P}^{(0)} = \frac{G}{2}\begin{pmatrix} z^{(0)}(\xi_1) & 0 \\ 0 & z^{(0)}(\xi_2) \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} \frac{c}{L}\frac{\mu_1}{2} + i\omega_1 & -\frac{c}{L} r_2 e^{i\epsilon_2} \\ -\frac{c}{L} r_1 e^{i\epsilon_1} & \frac{c}{L}\frac{\mu_2}{2} + i\omega_2 \end{pmatrix}$$

$$\mathbf{P}^{(2)} = G\begin{pmatrix} z_s^{(2)}(\xi_1) & z_c^{(2)}(\xi_{1,2}) \\ z_c^{(2)}(\xi_{1,2}) & z_s^{(2)}(\xi_2) \end{pmatrix}$$

Lamb parameters & Lamb units

$$\alpha_{1,2} = \frac{G}{Z_I(0)}[kZ_I(\xi_{1,2}) + k'Z_I(\xi'_{1,2})] - \mu_{1,2},$$

$$\beta_{1,2} = \alpha_{1,2} + \mu_{1,2},$$

$$\sigma_{1,2} = \frac{f_0}{2}\frac{G}{Z_I(0)}[kZ_R(\xi_{1,2}) + k'Z_R(\xi'_{1,2})],$$

$$\theta_{12} = \frac{\Gamma G}{Z_I(0)}\left[k\frac{Z_I(\xi_{1,2})}{1+(\xi_m/\eta)^2} + k'\frac{Z_I(\xi'_{1,2})}{1+(\xi'_m/\eta)^2}\right],$$

$$\tau_{12} = \frac{\Gamma f_0}{2}\frac{G}{Z_I(0)}\left[k\frac{Z_I(\xi_{1,2})\xi_m/\eta}{1+(\xi_m/\eta)^2} + k'\frac{Z_I(\xi'_{1,2})\xi'_m/\eta}{1+(\xi'_m/\eta)^2}\right]$$

Plasma dispersion function

$$Z_I(\xi) \simeq \sqrt{\pi}e^{-\xi^2} - 2\eta,$$

$$Z_R(\xi) \simeq -2\xi e^{-\xi^2},$$

$$\xi_{1,2} = (\omega_{1,2} - \omega_0)/\Gamma_D$$

s_b	$18 \cdot 10^{-6}$ m^2
P_{out}	1–10 nW
p	5.25 mbar
γ_a	12 MHz
γ_b	127 MHz
γ_{ab}	234 MHz
T_p	450 K
μ_{ab}	$3.2 \cdot 10^{-30}$ Cm

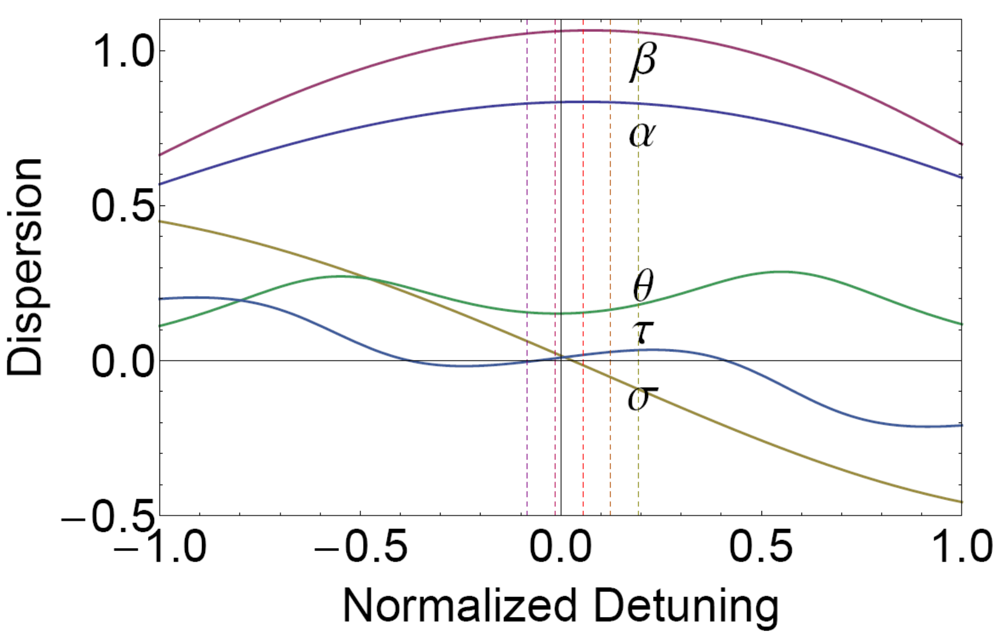
Note: Lamb parameters scale with the laser (single pass) gain G

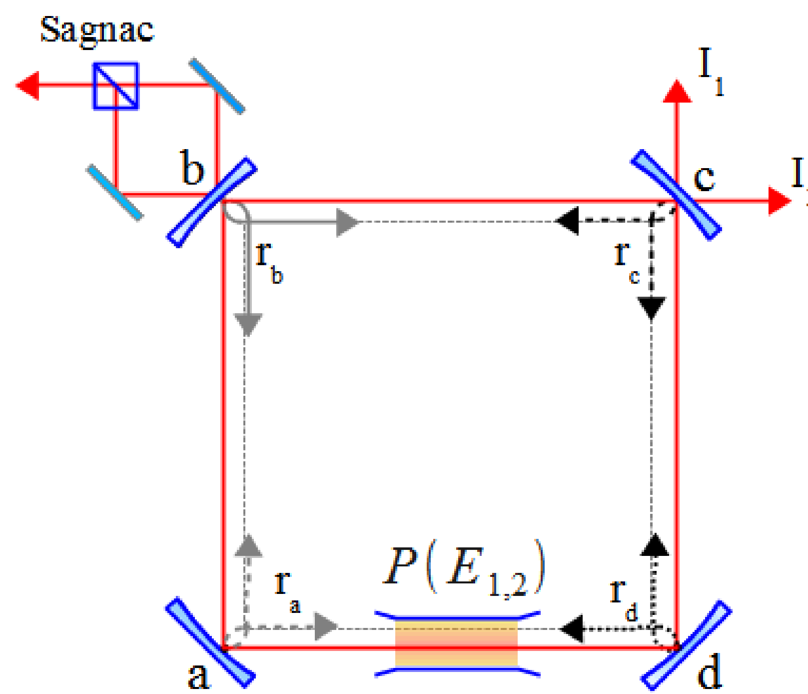
$$I_{1,2} = \frac{|\mu_{ab}|^2(\gamma_a + \gamma_b)}{4\hbar^2\gamma_a\gamma_b\gamma_{ab}} \quad E^2_{1,2} = \frac{|\mu_{ab}|^2(\gamma_a + \gamma_b)}{4\hbar^2\gamma_a\gamma_b\gamma_{ab}} \cdot \frac{P_{\text{out}1,2}}{2c\epsilon_0 s_b T} \equiv c_{\text{Lamb}}P_{\text{out}1,2},$$

Experimental calibration: the multimode threshold allows one to estimate c_{Lamb}

Ring Laser equations in terms of ring observables

$$\dot{I}_1 = \alpha_1 I_1 - \beta I_1^2 - \theta_2 I_2 I_1 + r_2 \sqrt{I_1 I_2} \cos(\psi - \epsilon_2) \,,$$
$$\dot{I}_2 = \alpha_2 I_2 - \beta I_2^2 - \theta_1 I_2 I_1 + r_1 \sqrt{I_1 I_2} \cos(\psi + \epsilon_2) \,,$$
$$\dot{\psi} = \omega_s + \tau_1 I_1 - \tau_2 I_2 - r_2 \sqrt{\frac{I_2}{I_1}} \sin(\psi - \epsilon_2) - r_1 \sqrt{\frac{I_1}{I_2}} \sin(\psi + \epsilon_1)$$



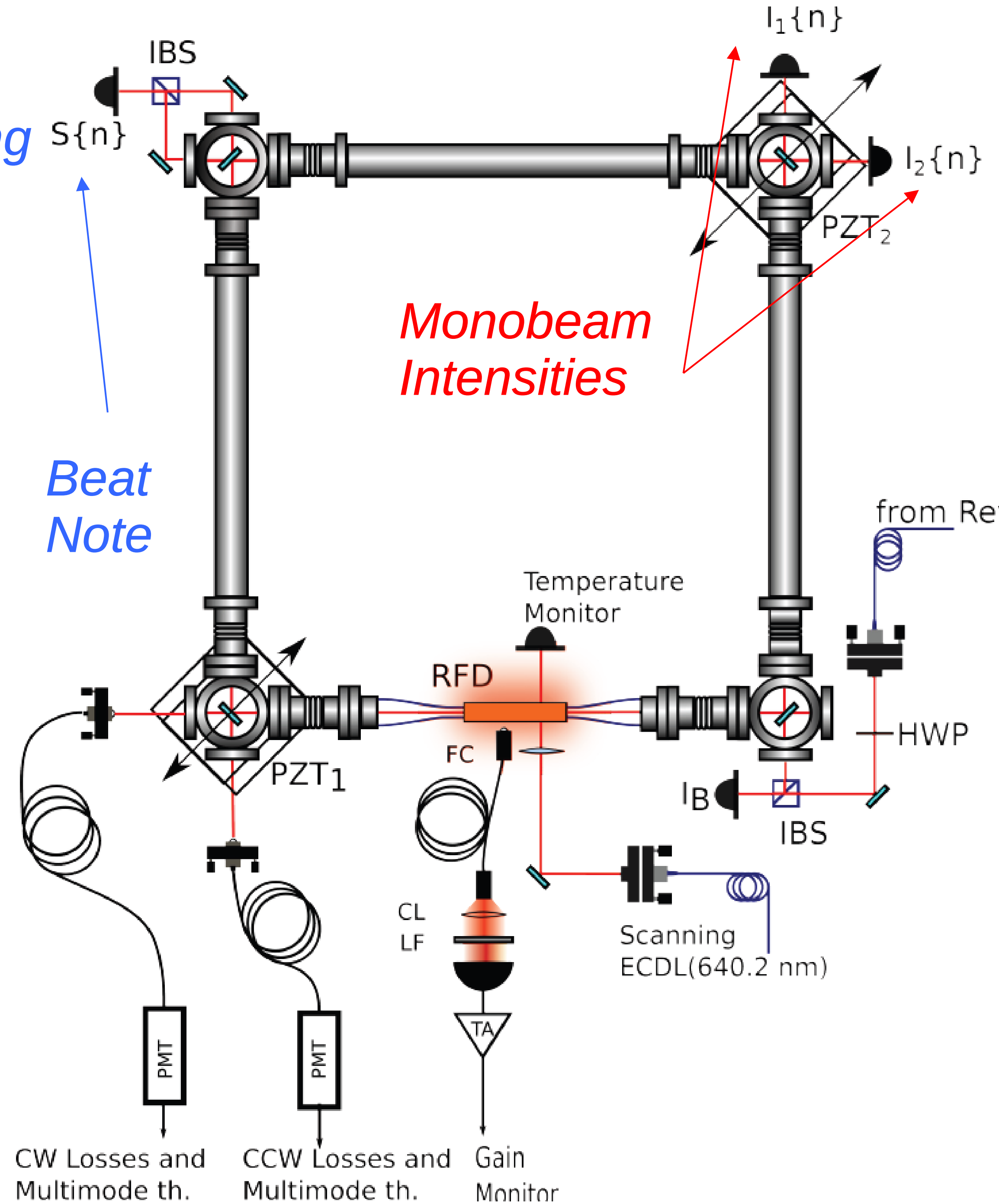


Real Coordinates For System Study

$$\begin{cases} \dfrac{E_1^* E_2 + E_2^* E_1}{2} = x_1 \\ \dfrac{E_1^* E_2 - E_2^* E_1}{2i} = x_2 \\ \dfrac{E_1^* E_1 + E_2^* E_2}{2} = x_3 \\ \dfrac{E_1^* E_1 - E_2^* E_2}{2} = x_4 \end{cases}$$

Oscillating Parts (x_1, x_2)

Absolute values (x_3, x_4)



$$\dot{\boldsymbol{x}} = \left(A - B(\boldsymbol{x})\right)\boldsymbol{x}$$

Trajectiories stay on the $\mathbb{R}^4$ *cone:*

$$x_1^2 + x_2^2 = x_3^2 - x_4^2$$

Parameters Positions

Accounting for asymmetry:

$$
\begin{cases}
\alpha = \alpha_1 + \alpha_2 \\
r = r_1 + r_2 \\
s = 2(\beta + \theta) \\
c = 2(\beta - \theta) \\
\delta_\alpha = \alpha_1 - \alpha_2 \\
\delta_r = r_1 - r_2
\end{cases}
$$

$$
B(\boldsymbol{x}) = \begin{pmatrix} s\,x_3 & 2\tau x_4 & 0 & 0 \\ -2\tau x_4 & s\,x_3 & 0 & 0 \\ 0 & 0 & s\,x_3 & c\,x_4 \\ 0 & 0 & c\,x_4 & s\,x_3 \end{pmatrix}
$$

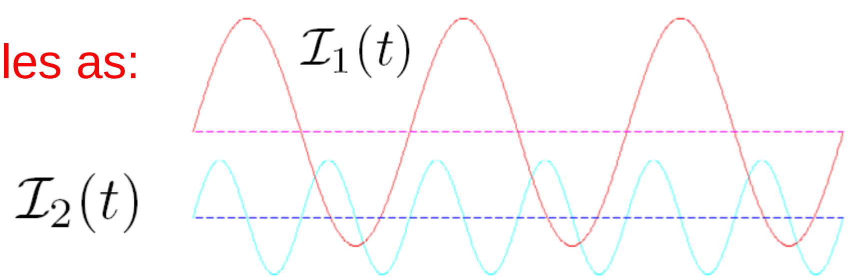
$$
A = \begin{pmatrix} \alpha & -\omega_s & r\cos\epsilon & \delta_r \sin\epsilon \\ \omega_s & \alpha & \delta_r\cos\epsilon & r\sin\epsilon \\ r\cos\epsilon & \delta_r\cos\epsilon & \alpha & \delta_\alpha \\ -\delta_r\sin\epsilon & -r\sin\epsilon & \delta_\alpha & \alpha \end{pmatrix}
$$

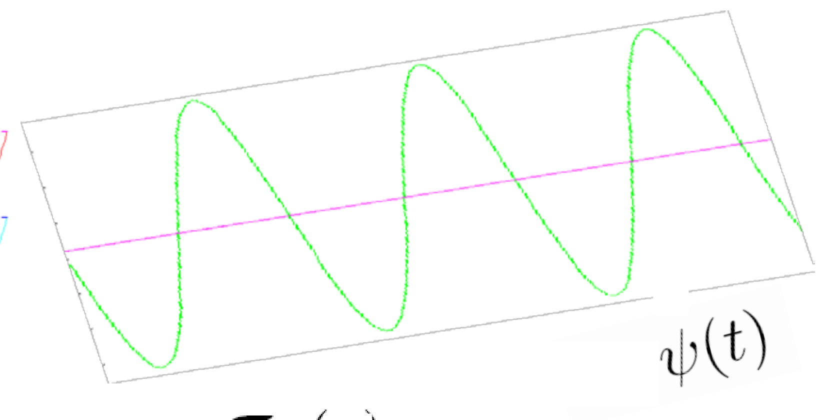
Usually $\sim \omega$

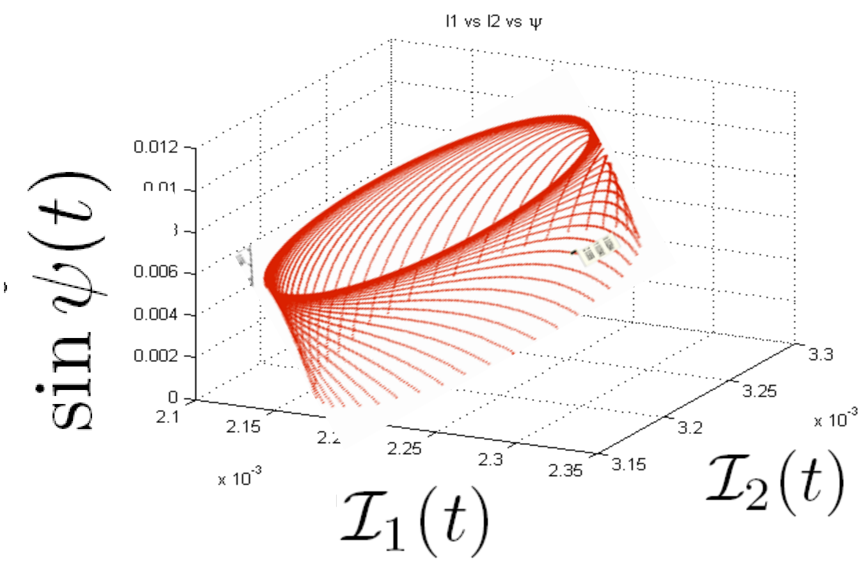
Usually $\ll \omega$

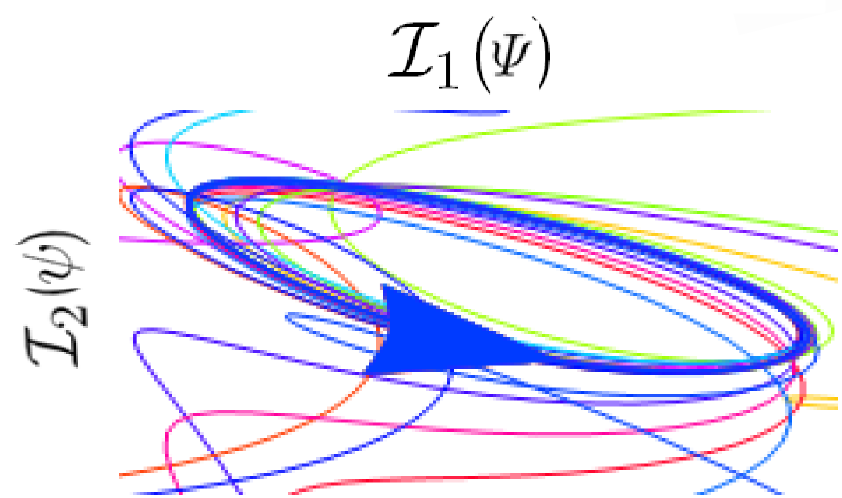
Asymptotic stationarity solutions

- Reciprocal rings exhibit the symmetry→ $(E_1,\ E_2\ ,\Omega_1,\Omega_2)\longrightarrow(E_2,\ E_1\ ,\Omega_2,\Omega_1)$
- Topology of phase space is a **«clifford torus»**
- Slightly different parameters for beams 1 and 2 preserve topology
  - Two kinds of orbits (depending on average values of I_1 e I_2)
    - Orbits that can't be shrunk to a point (Sagnac)
    - Orbits shrinkable to a point (constant I_1, I_2 e ψ : laser locked-in or switched off)
- Stable limit cycles as:





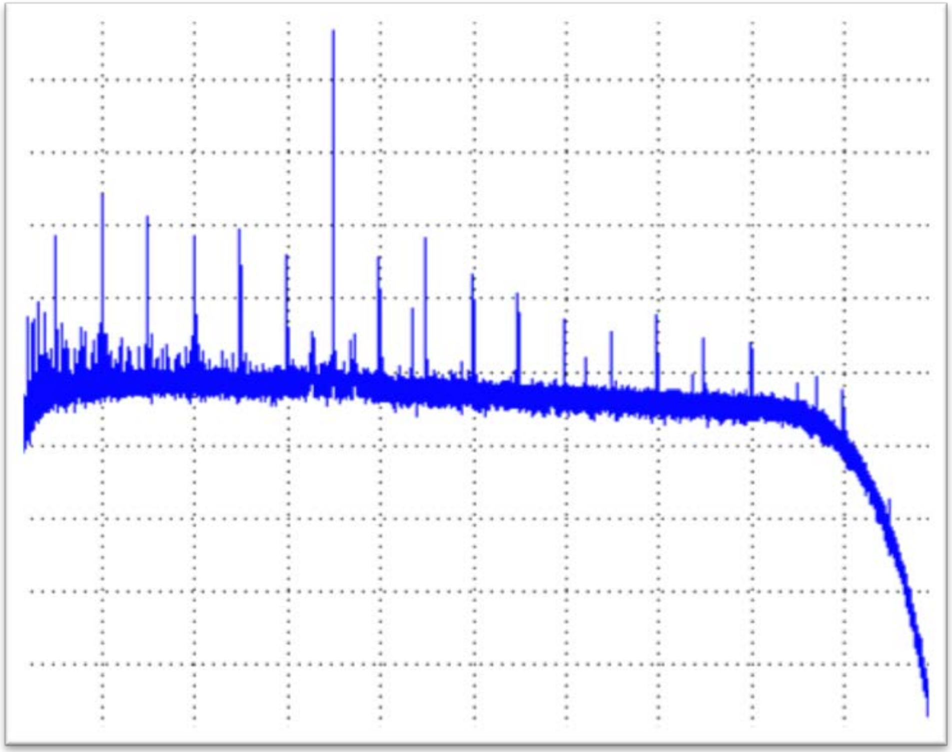




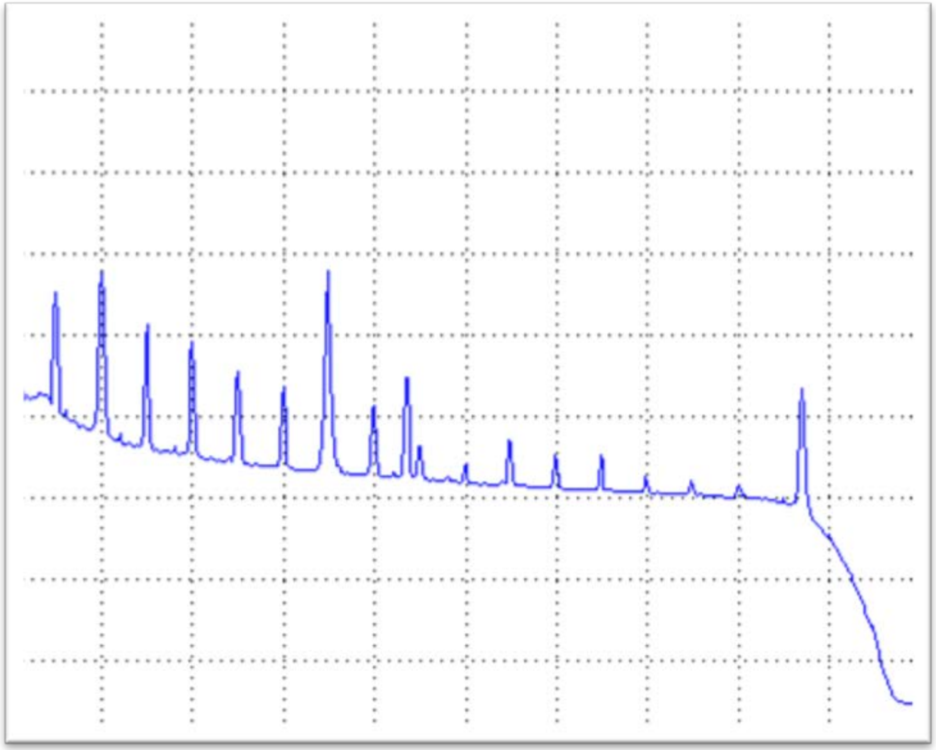
Losses can be identified while system goes through the limit cycle

Power spectrum of G-Wettzell data

Interferogram

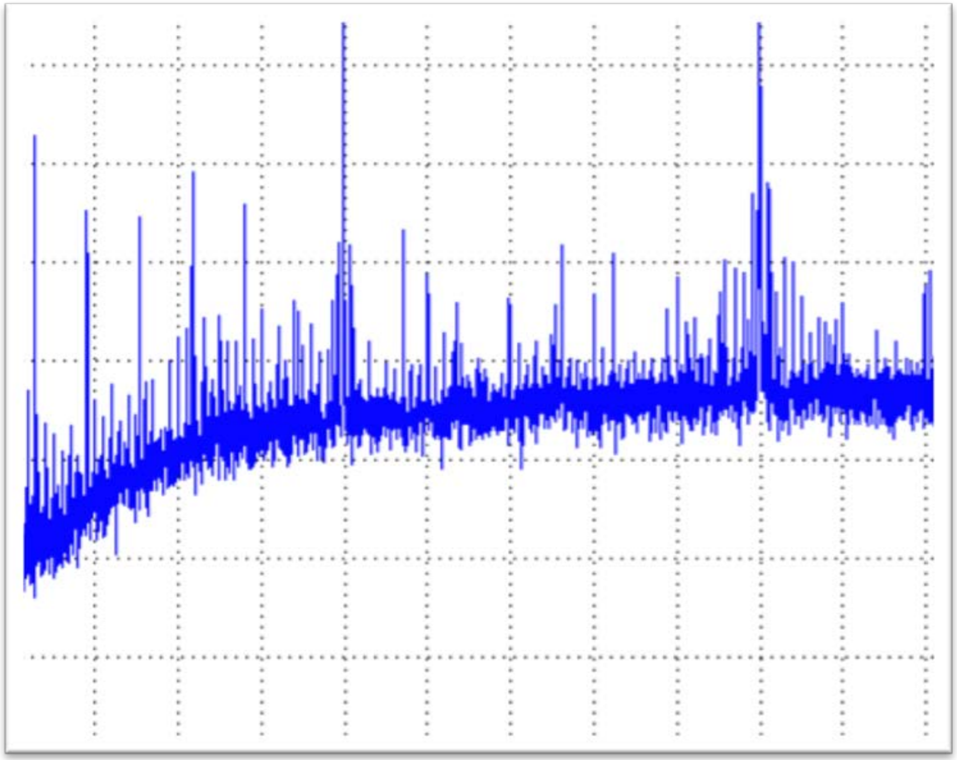


CW Intensity



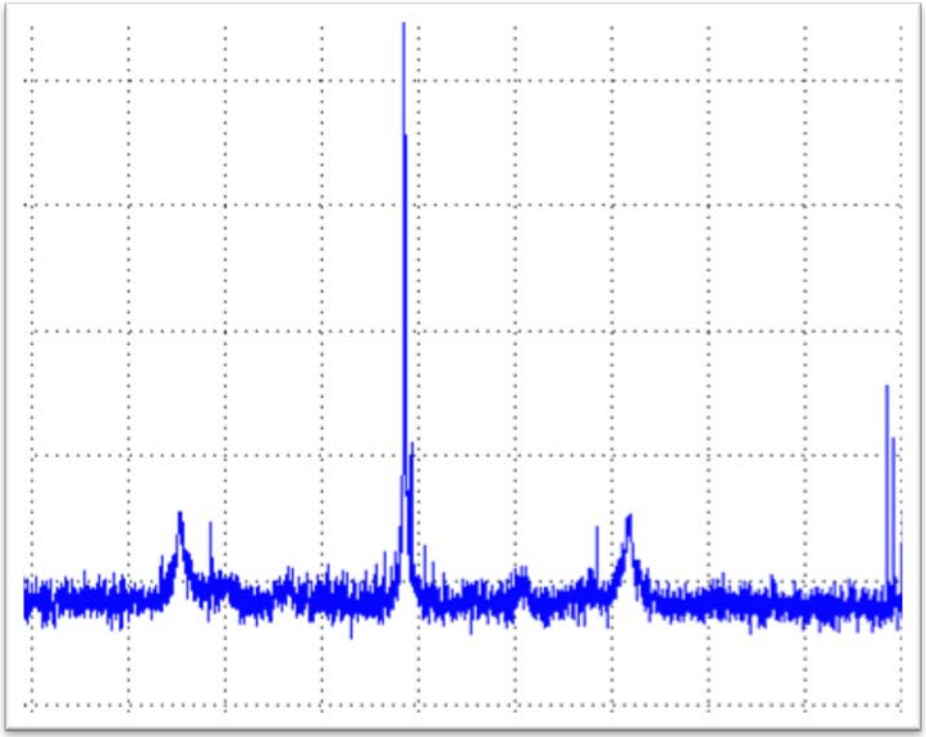
Power spectrum of G-Wettzell Interferogram (and around Sagnac Frequency)

Low Frequency



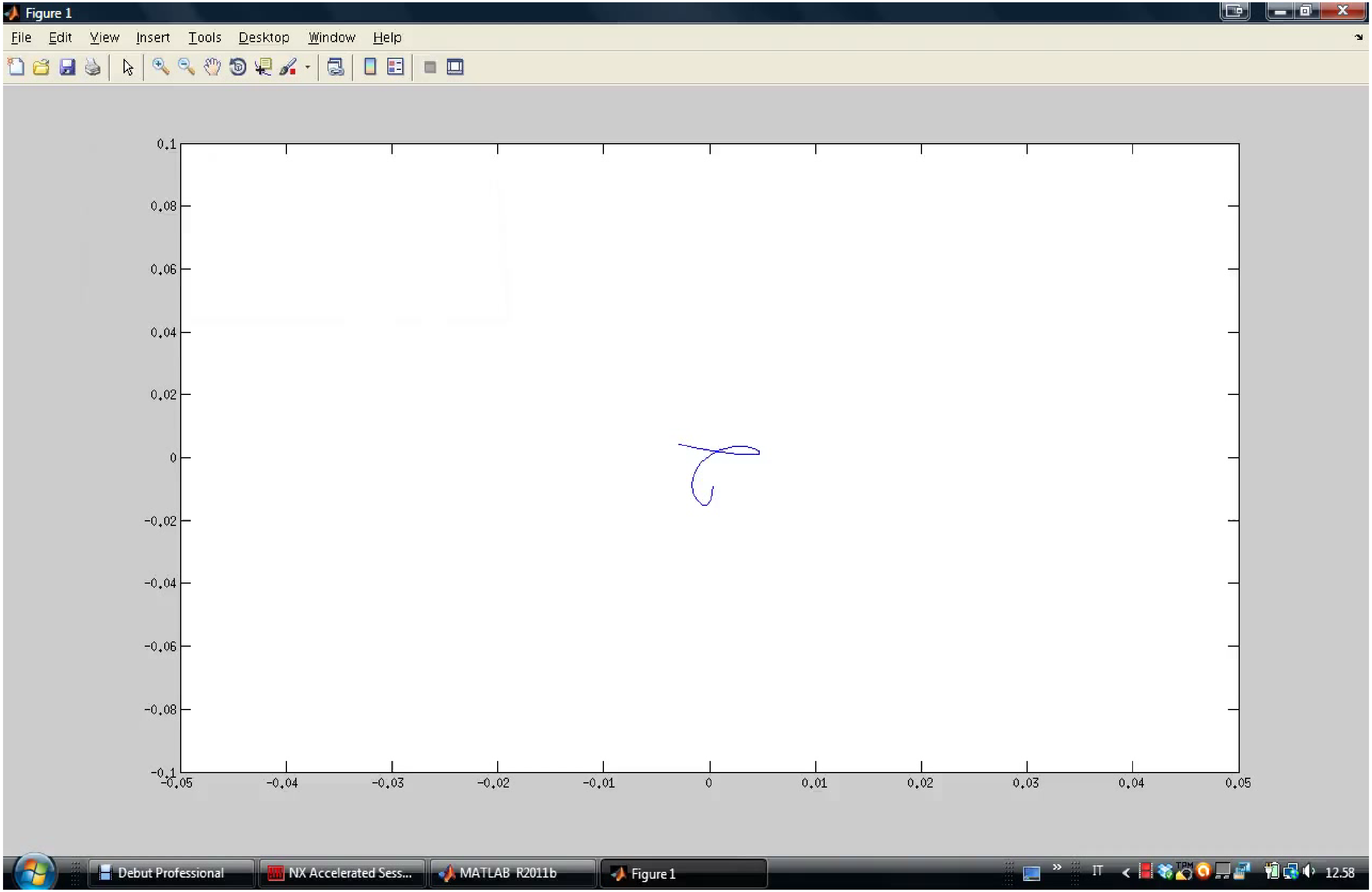
0 Hz 120 Hz

Around Sagnac Frequency



310 Hz 400 Hz

Limit Cycle of G-Pisa



Identification of Lamb parameters associated to losses of optical cavity

Limit cycle of the system described by an ellipse (5 numbers)

Statistics of the 5 Lamb parameters associated to cavity losses

$$\widehat{\varepsilon} = \frac{\phi_1 - \phi_2}{2}$$

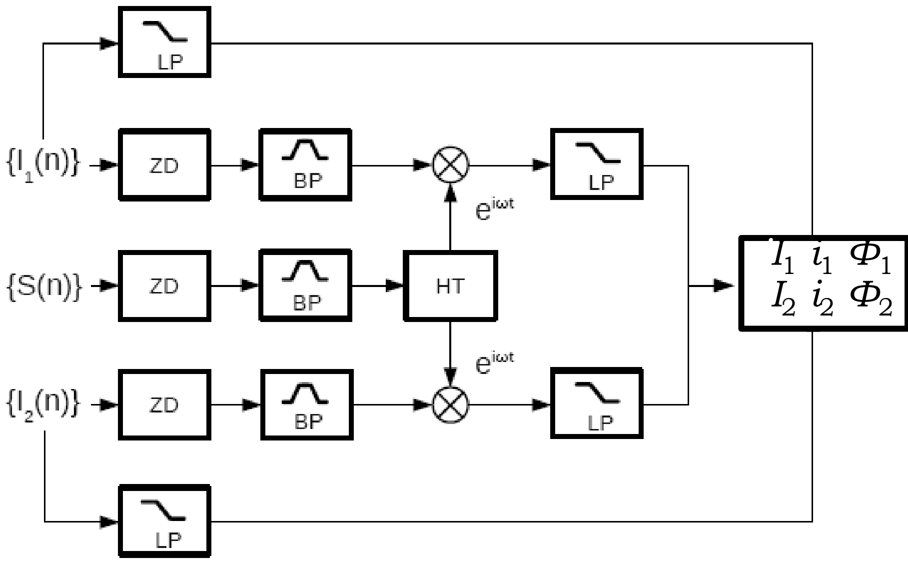
$$\begin{cases} \hat{\mu}_{1,2} & = \alpha_0 - \beta \left(I_{1,2} + \frac{i_{1,2}^2}{I_{1,2}} \right) - \frac{i_1 i_2 I_{2,1} (L\omega/c) \cos\widehat{\epsilon}}{4 I_{1,2}^2} - \\ & -\theta \left(\frac{i_{2,1}^2 + 4 I_{2,1}^2}{4 I_{1,2}} - \frac{i_{1,2}^2 I_{2,1}^2}{2 I_{1,2}^3} + \frac{i_1 i_2 I_{2,1} \cos\widehat{\epsilon}}{I_{1,2}^2} + \frac{i_{2,1}^2 \cos 2\widehat{\epsilon}}{4 I_{1,2}} \right) \\ \hat{r}_{1,2} & = \frac{i_{2,1} (L\omega/c)}{2\sqrt{I_1 I_2}} \mp i_{1,2} \sqrt{\frac{I_{1,2}}{I_{2,1}}} \theta \sin\widehat{\epsilon} \end{cases}$$

Self- and cross-saturation parameters β and θ must be measured
Intensity $I_{1,2}$ must be calibrated in Lamb units

Key points of losses identification and Kalman estimation scheme:

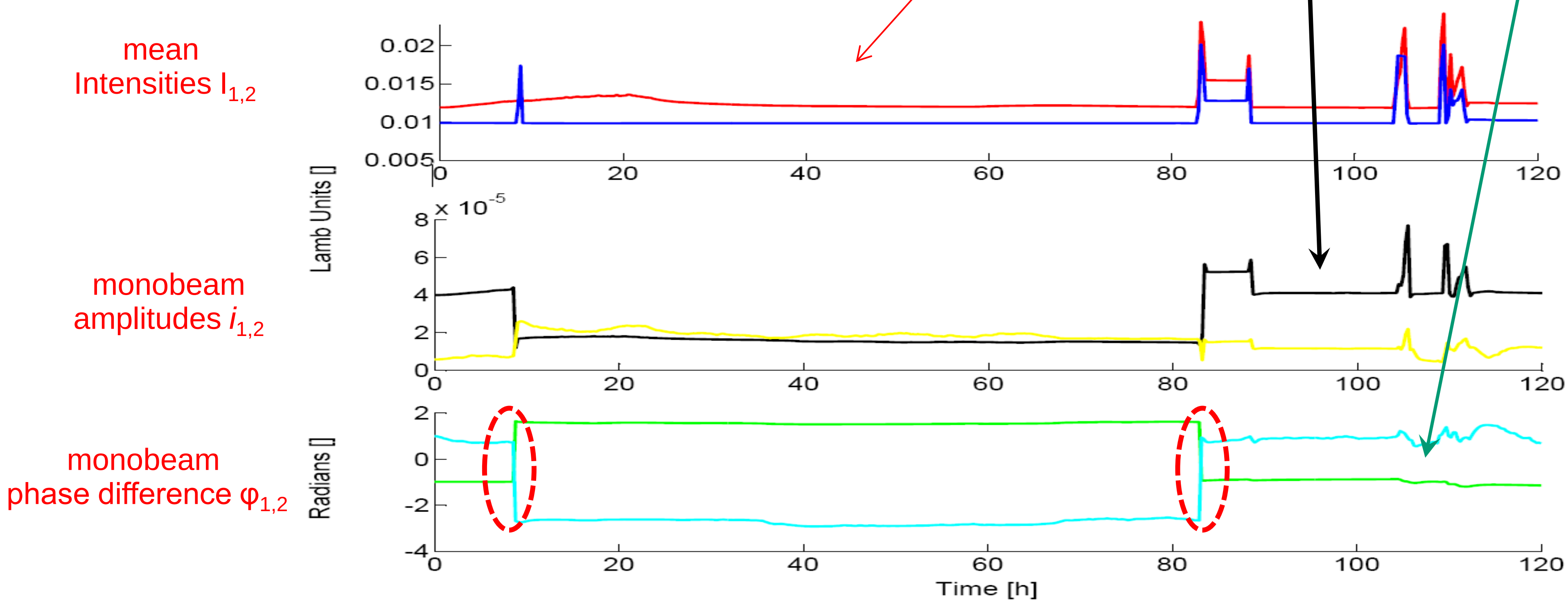
- Cavity losses are unpredictable → their variations in time must be identified
- Contribution to Sagnac frequency due to losses drift can be removed by Kaman filter
- The aim is improving the Allan variance of the rotation estimate for long times (ring time stability)
- Accuracy depends on non-linear laser parameters

Schematic of the identification of Lamb parameters associated to losses of optical cavity

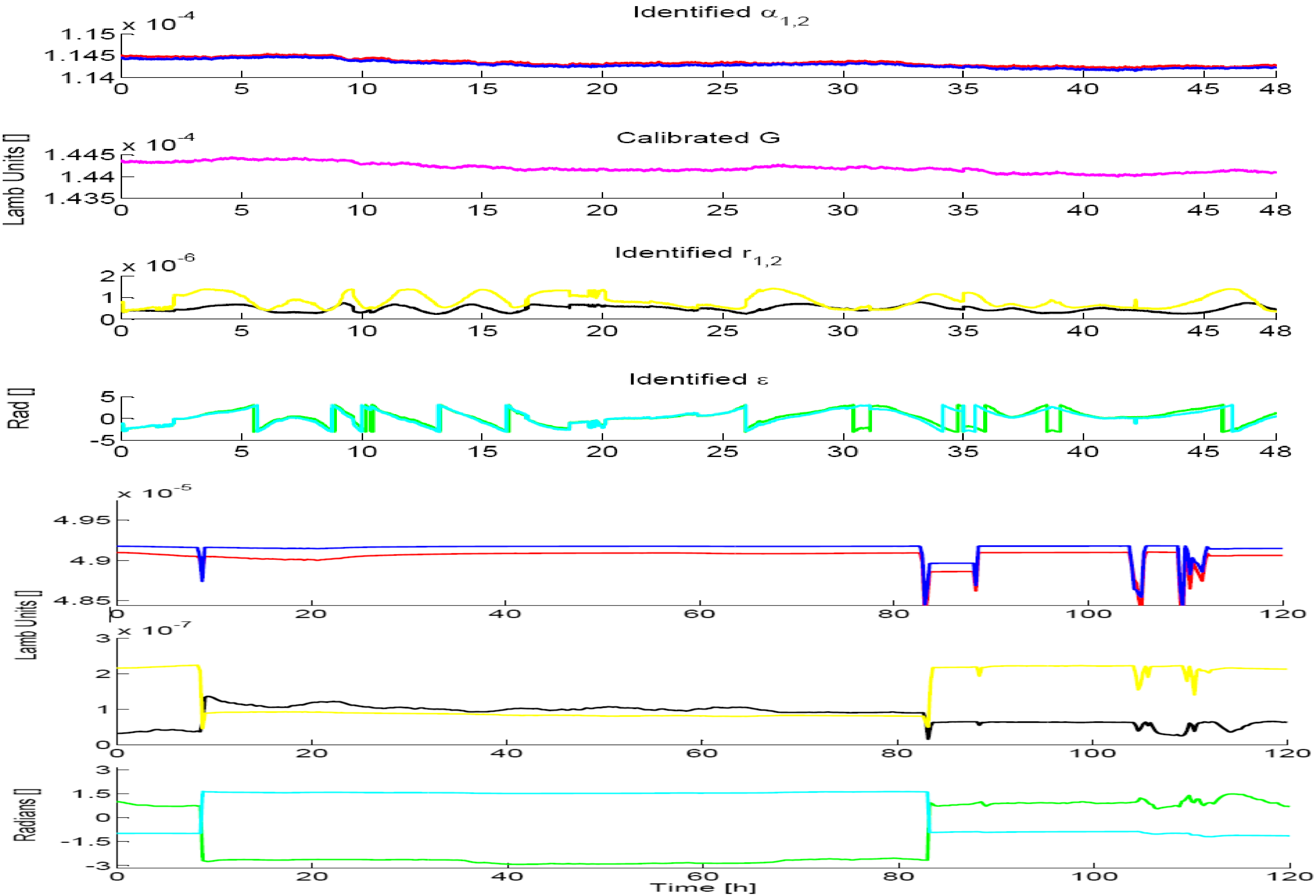


$$
\begin{cases}
\mathcal{I}_1(t) = I_1 + i_1 \sin(\omega t + \phi_1) \\
\mathcal{I}_2(t) = I_2 + i_2 \sin(\omega t + \phi_2) \\
\Psi(t) = \omega t
\end{cases}
$$

(Sagnac phase)



Identification of Lamb Parameters of G-WETTZELL and G-PISA



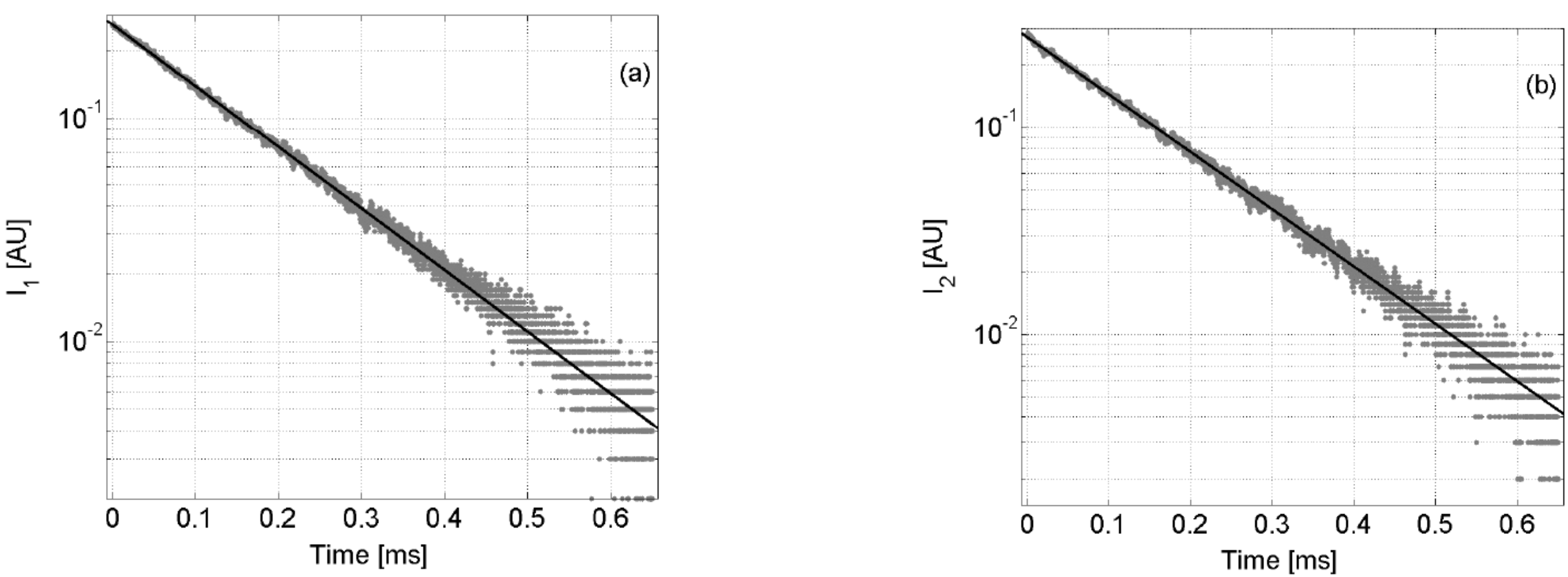
Ring down time: losses measurements with G=0

Problem: how to fix the single pass gain G at the begining of the run

From ring laser Eqs. → We must estimate losses independently from G

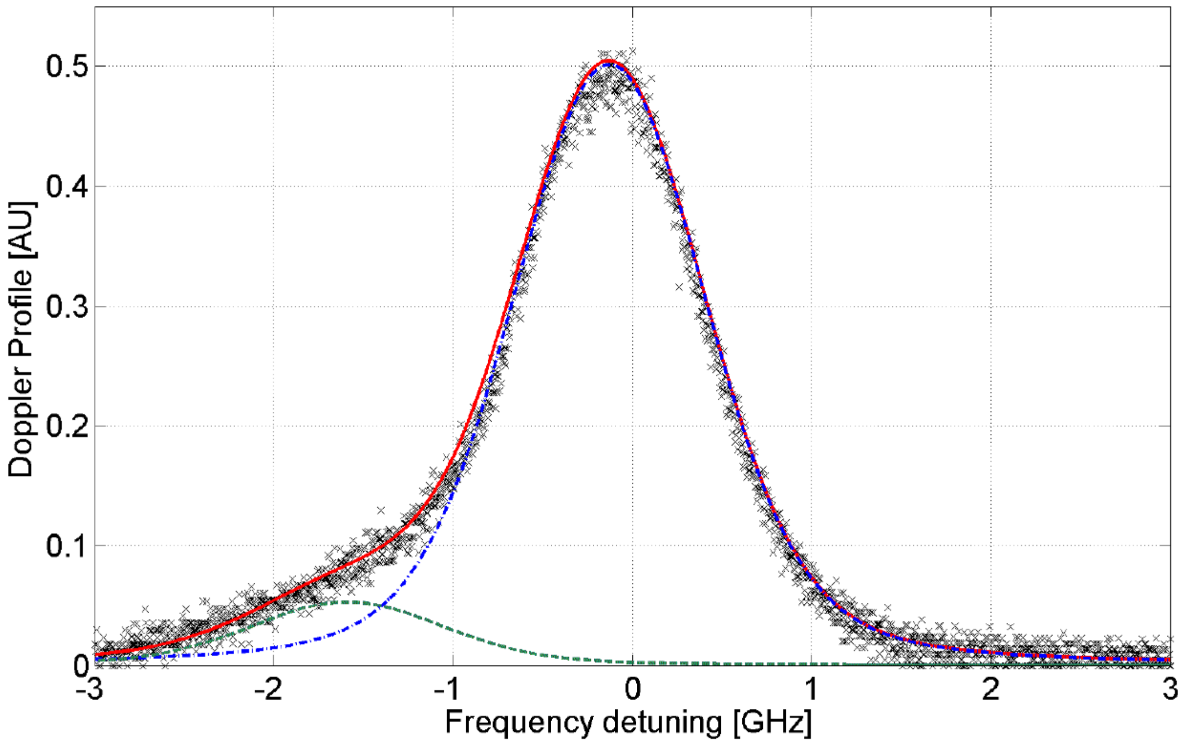
Switch off the RF excitation → put G=0 in ring Eqs.

$$\dot{\mathbf{E}} = \mathbf{M}.\mathbf{E} \quad \xrightarrow{\text{solution}} \quad \begin{cases} I_1(t) = & I_1(0)\, e^{-\frac{c}{L}\mu_1 t} \\ I_2(t) = & I_2(0)\, e^{-\frac{c}{L}\mu_2 t} \end{cases}$$



Plasma Dispersion Function Fit (Voigth Curve)

Natural Ne mixture 90-10



$$i\sqrt{\pi}e^{-(\eta-iy)^2}\operatorname{erfc}(\eta-iy)$$

$$T_{Ne}=\sqrt{\frac{\Gamma_{20}^2 * m_{20}}{\lambda \ln 2 K_B}} \sim 360\,K$$

$\eta=\Gamma/\Gamma_D=0.4$

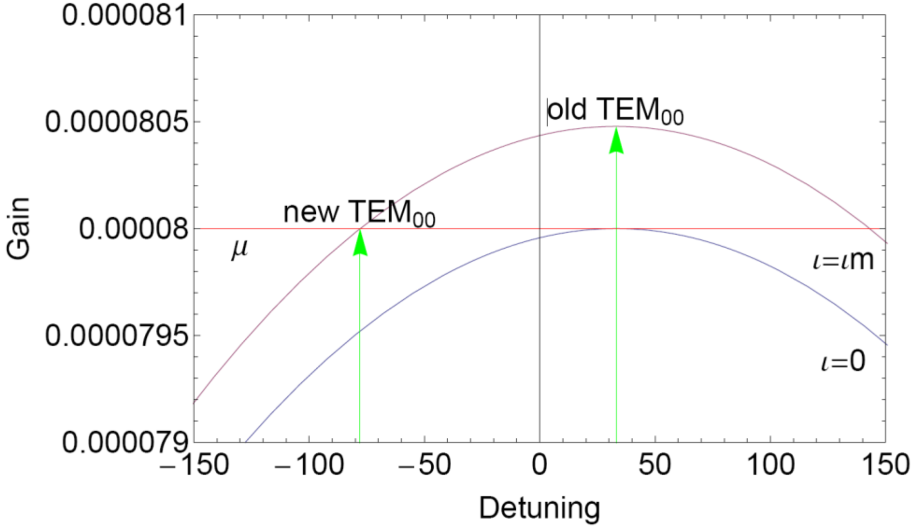
$$z^{(0)}(\xi_{1,2}) = \quad k' z^{(0)}(\xi'_{1,2}) + k'' z^{(0)}(\xi''_{1,2})|$$

ForAronowitz η <0.1
per G-PISA η =0.4 → z(ξ) without approximations

Standard Isotopic Mixture 50-50 di G-PISA k'/k'' = sqrt(20/22)

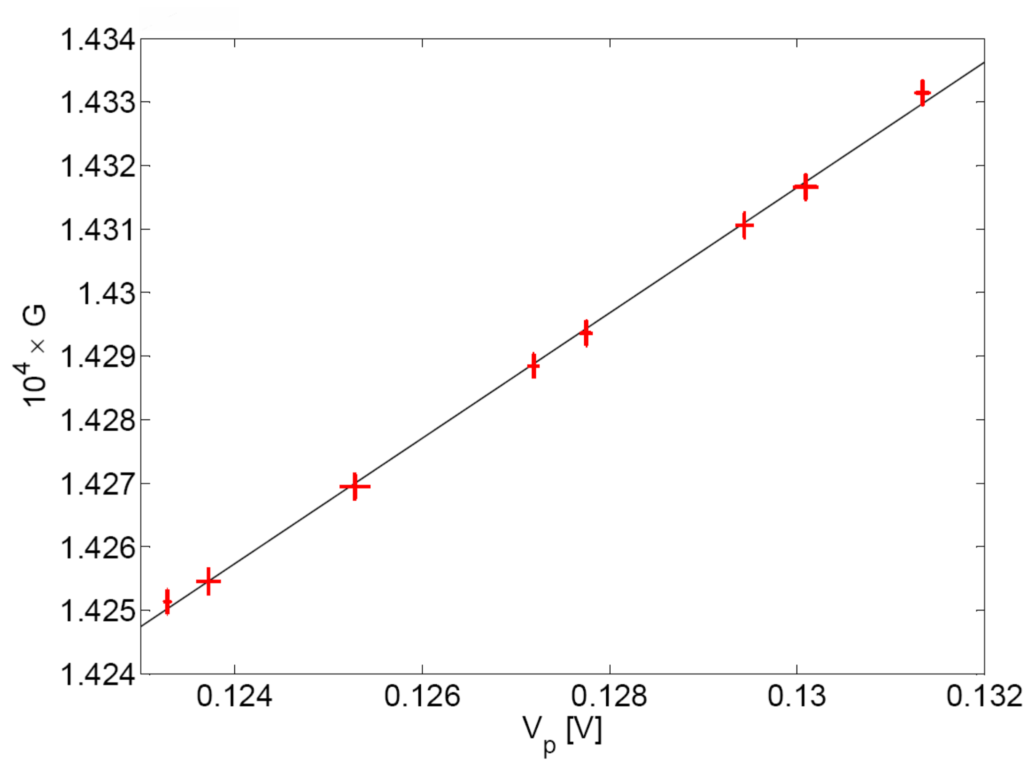
Calibration of Gain Monitor (monomode to multimode transition)

Transition caused by increasing gain, to measure gain and monobeams at the multimode thresold



Analysis of the system stability → theorical prediction of (I_{th},G_{th}) :

1) Calibration of I from volt to Lamb units
2) Gain at multimode (μ from RDT)

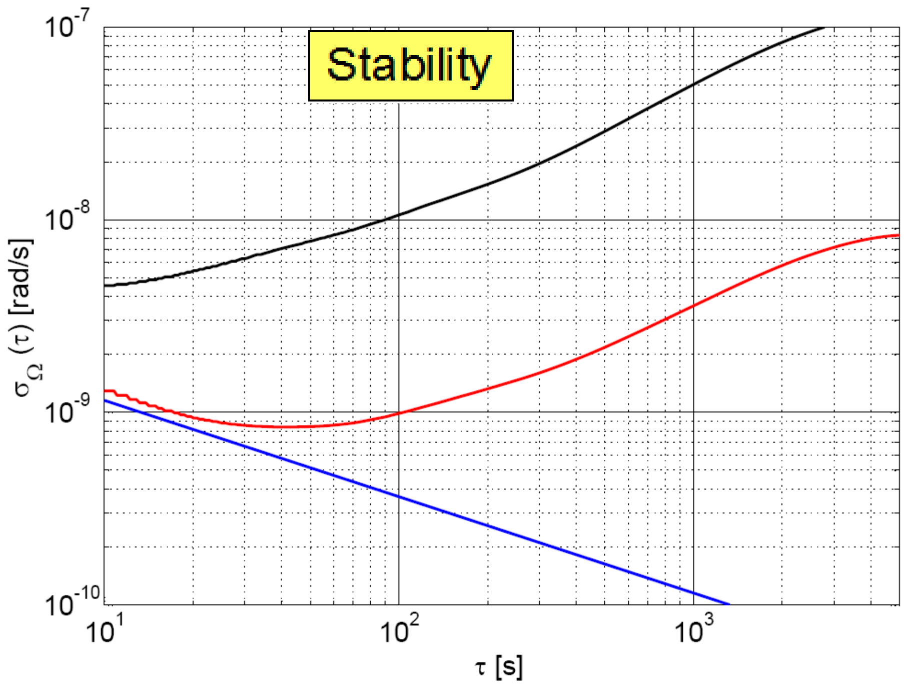


OLD: The new mode exists but dosent have enough energy to balance losses

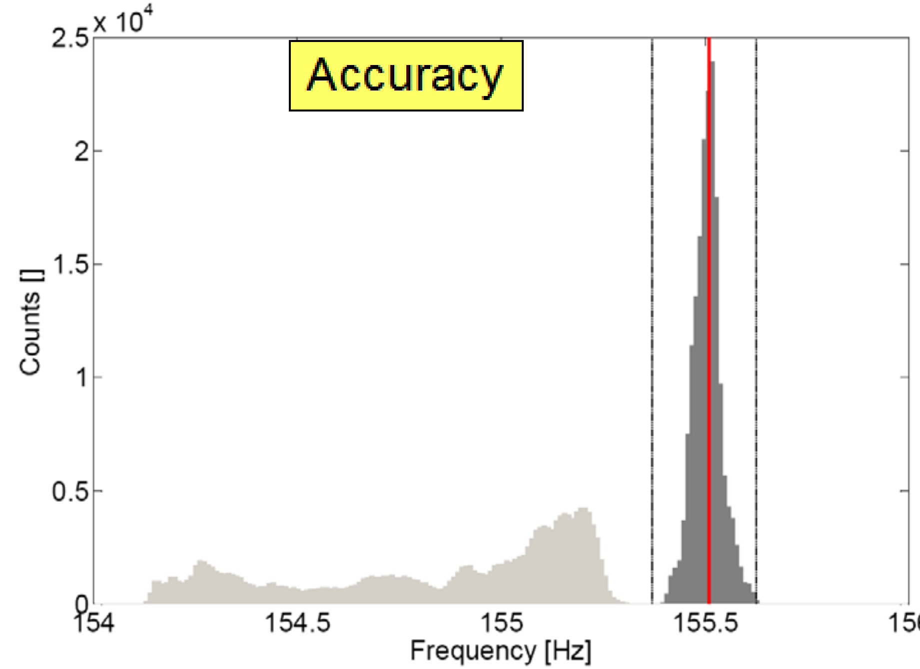
NEW: increasing H the new curve permits the birth of a new mode for μ=G

Afterwards the Gain Monitor channel is itself calibrated by means of steps of the input power

G-PISA as an inertial rotation sensor

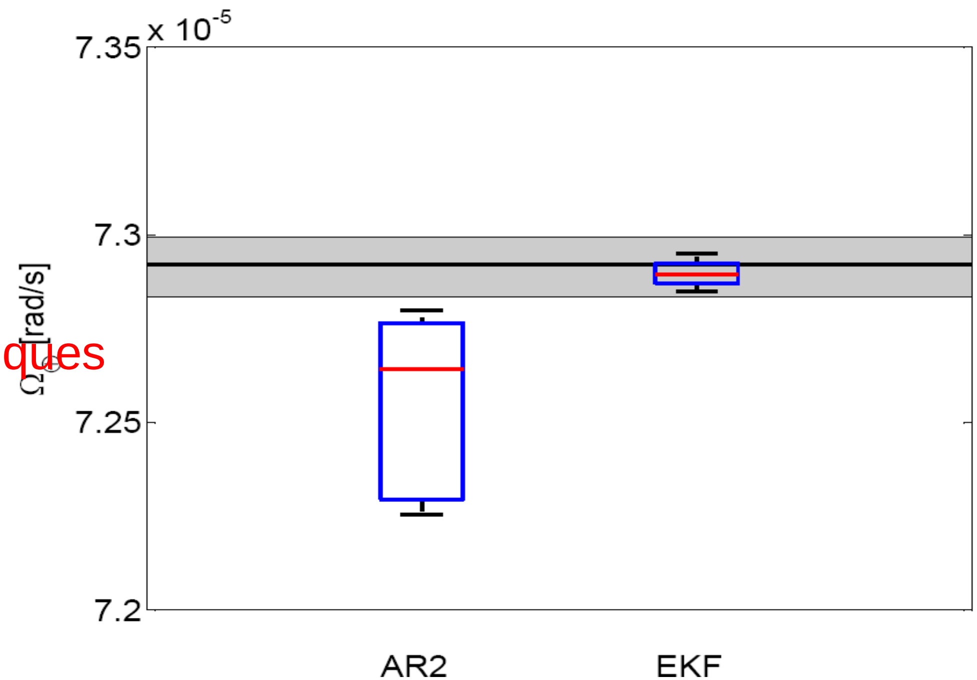


St. Dev. of the measured Sagnac frequency reduced by a factor of 10



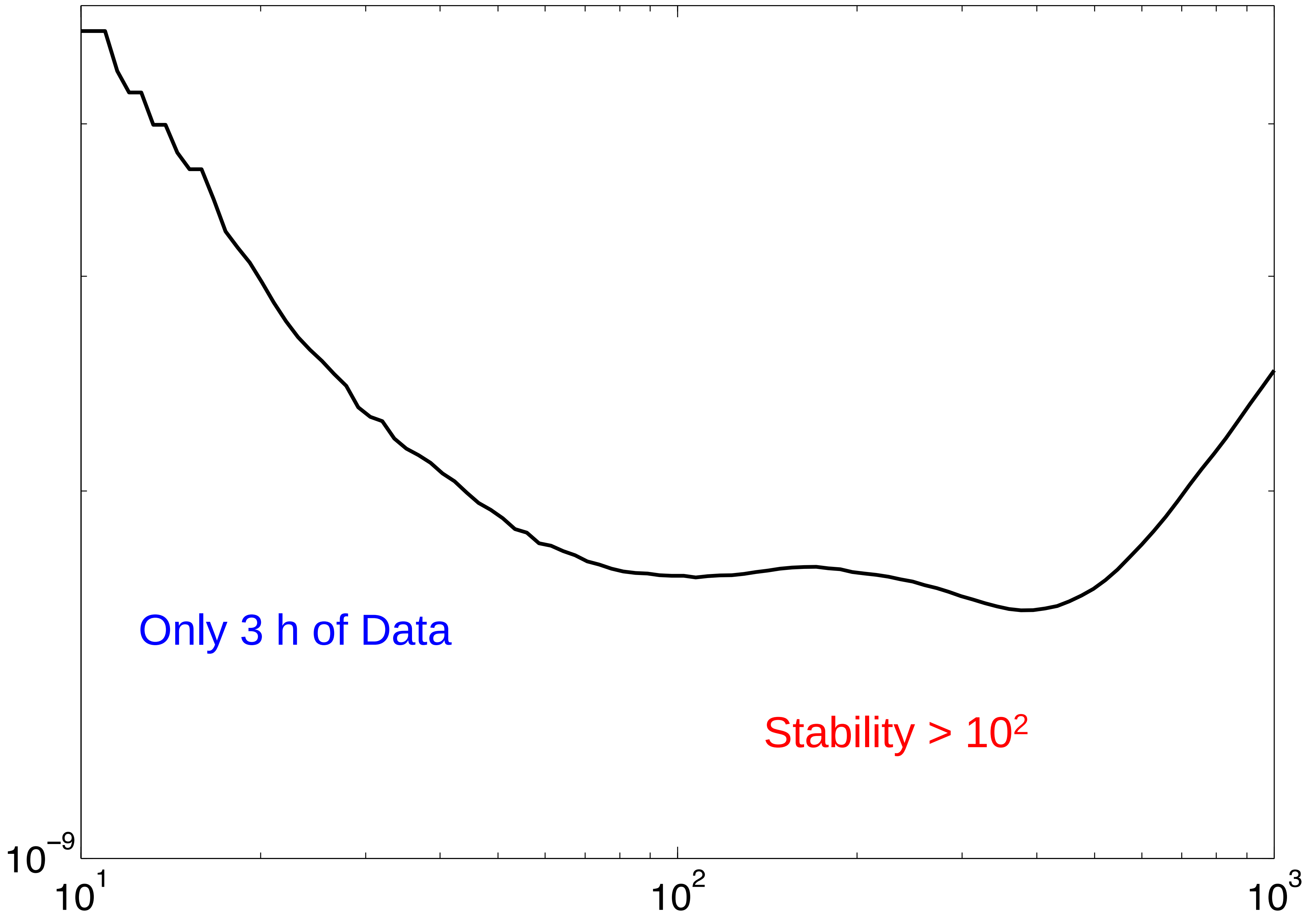
Earth rotation rate estimation falls inside the Confidence bounds arising from uncertaintes on scale factor A/P and Orientation:

« frequency null shift» Estimated via EKF tecniques



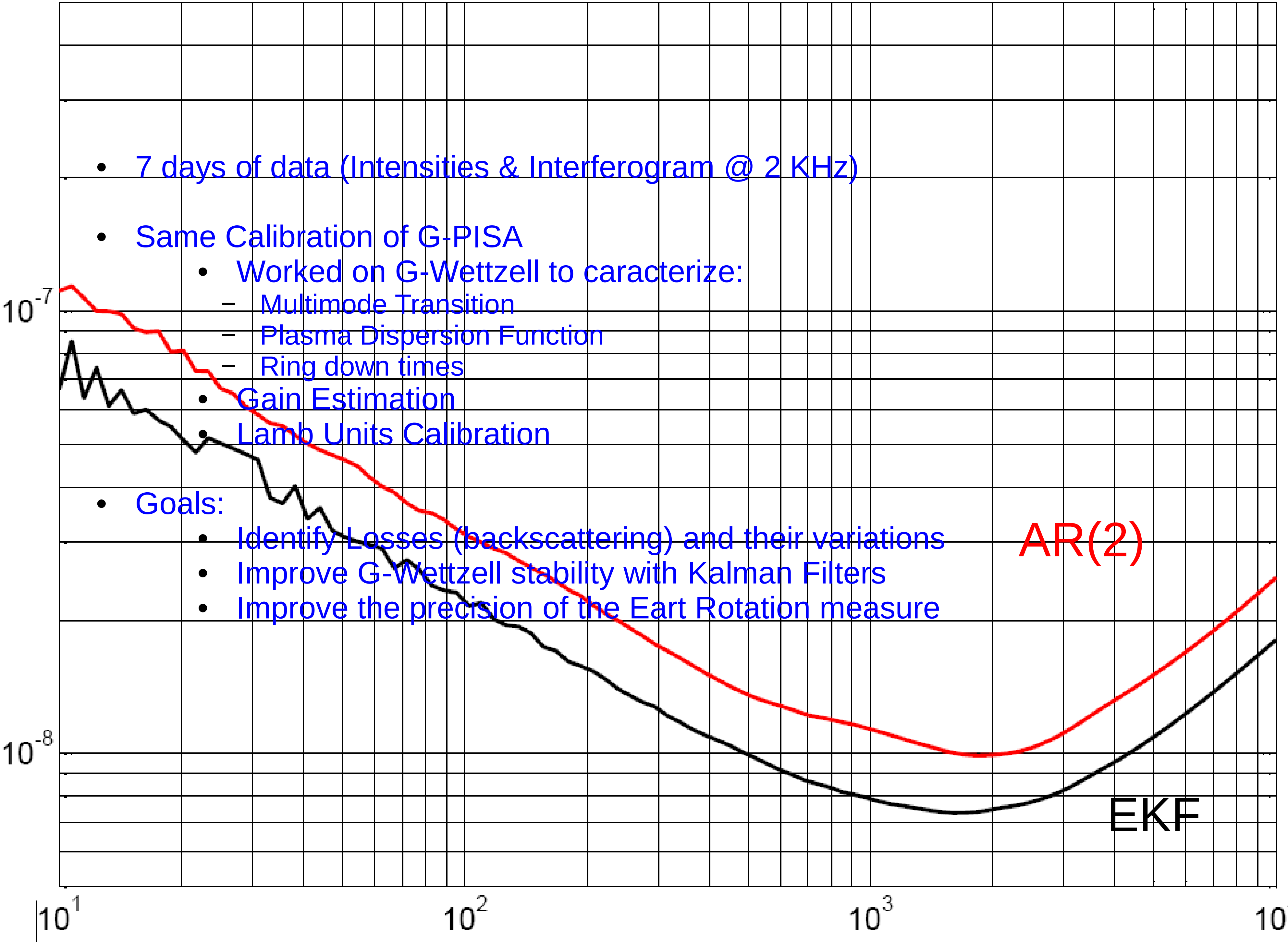
Error Source	Freq. error
Back-scattering $\mathcal{R}_2 e^{-X} + \mathcal{R}_1 e^{X}$ + c.c.	0.4695 Hz
Null Shift $\tau\,(I_1 - I_2)$	-8.7×10^{-4} Hz
Atomic Scale Factor $\sigma_1 - \sigma_2$	5.56×10^{-6} Hz
Cross Dispersion $I\,(\tau_{21} - \tau_{12})$	1.75×10^{-6} Hz

Preliminary Results LNGS

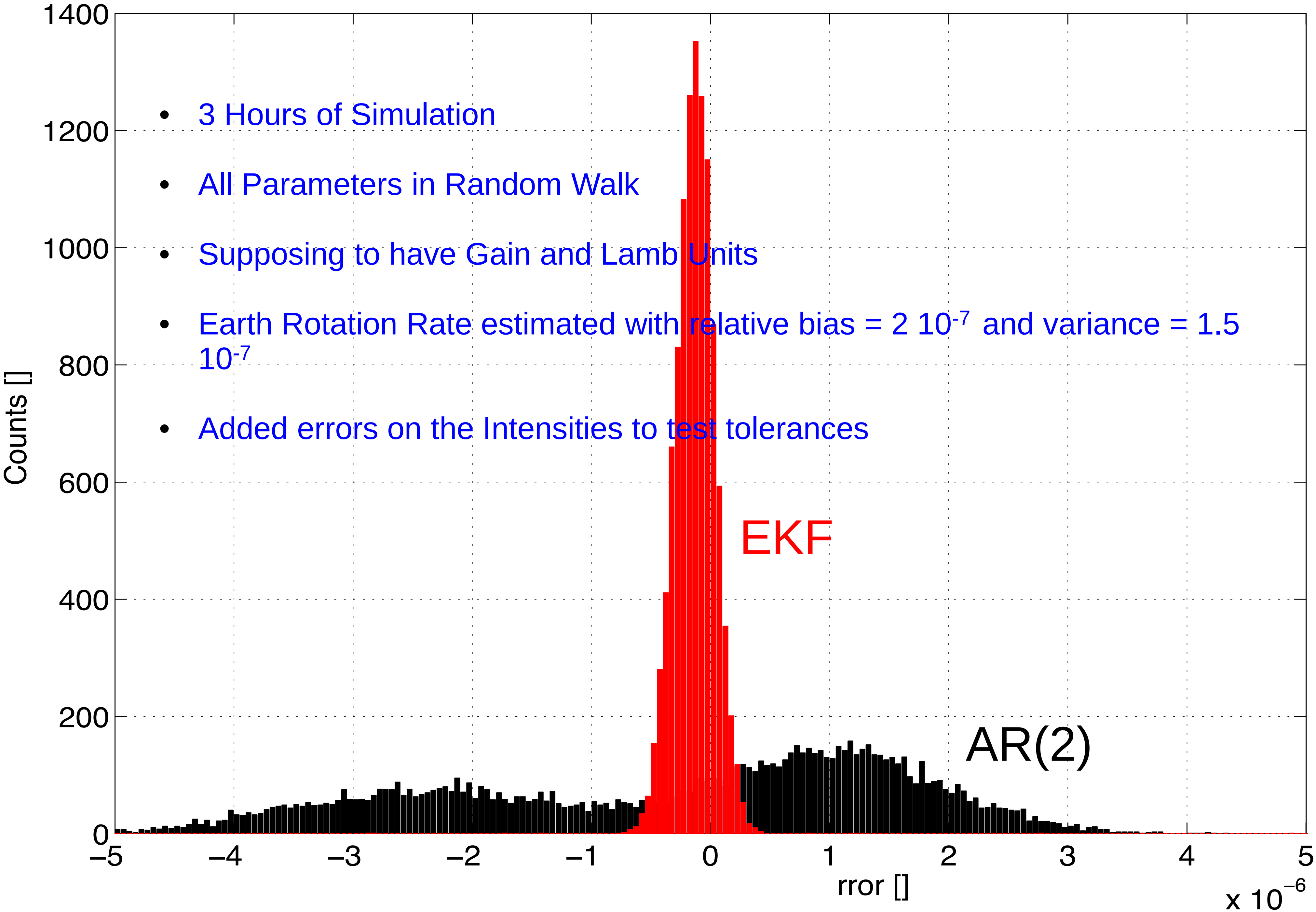


Currently working on G-Wettzell…

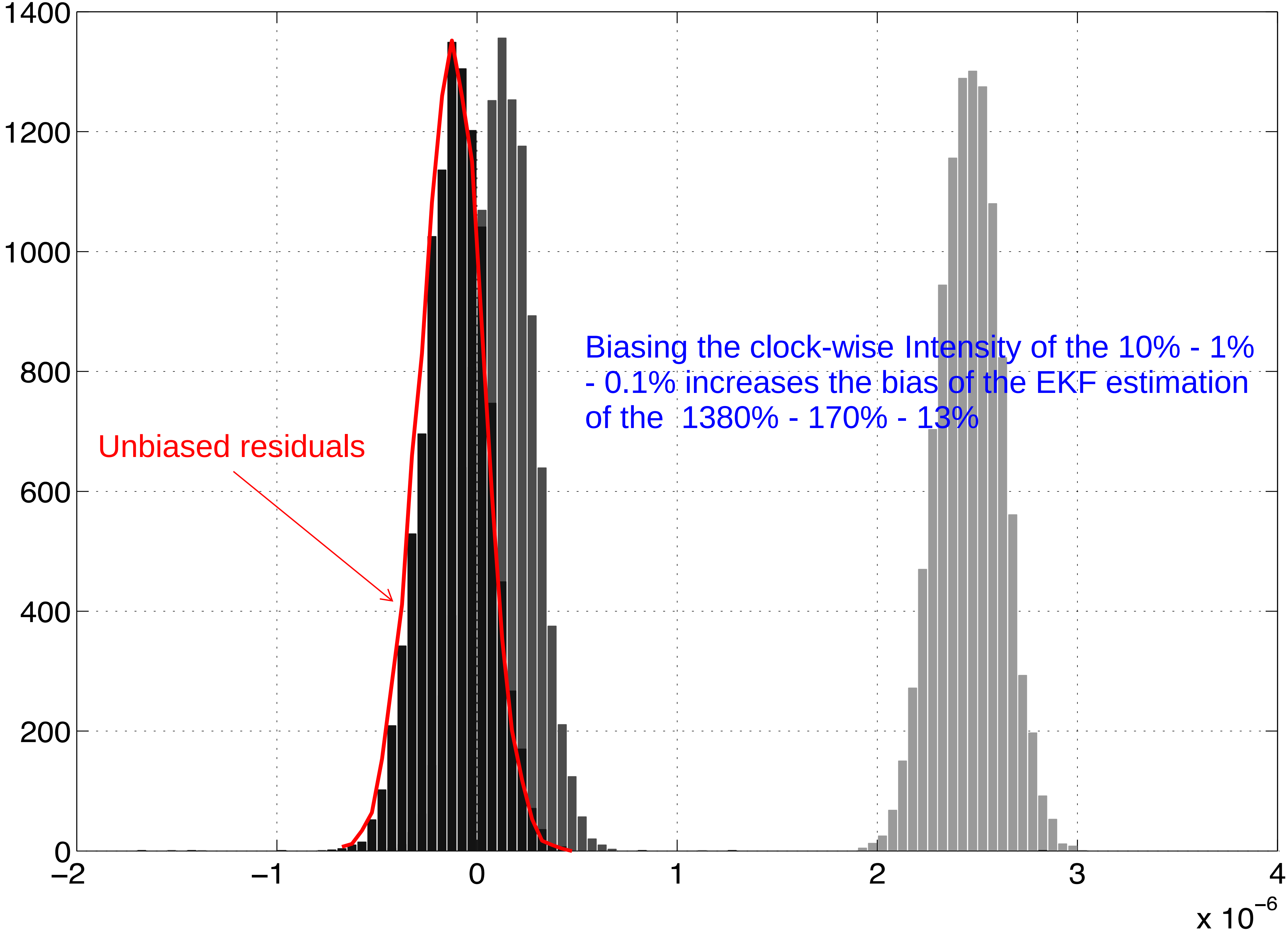
- 7 days of data (Intensities & Interferogram @ 2 KHz)
- Same Calibration of G-PISA
    - Worked on G-Wettzell to caracterize:
        - Multimode Transition
        - Plasma Dispersion Function
        - Ring down times
    - Gain Estimation
    - Lamb Units Calibration
- Goals:
    - Identify Losses (backscattering) and their variations
    - Improve G-Wettzell stability with Kalman Filters
    - Improve the precision of the Eart Rotation measure



Simulation Tolerances of G-Wettzell



Simulation Tolerances of G-Wettzell



Simulation Tolerances of G-Wettzell