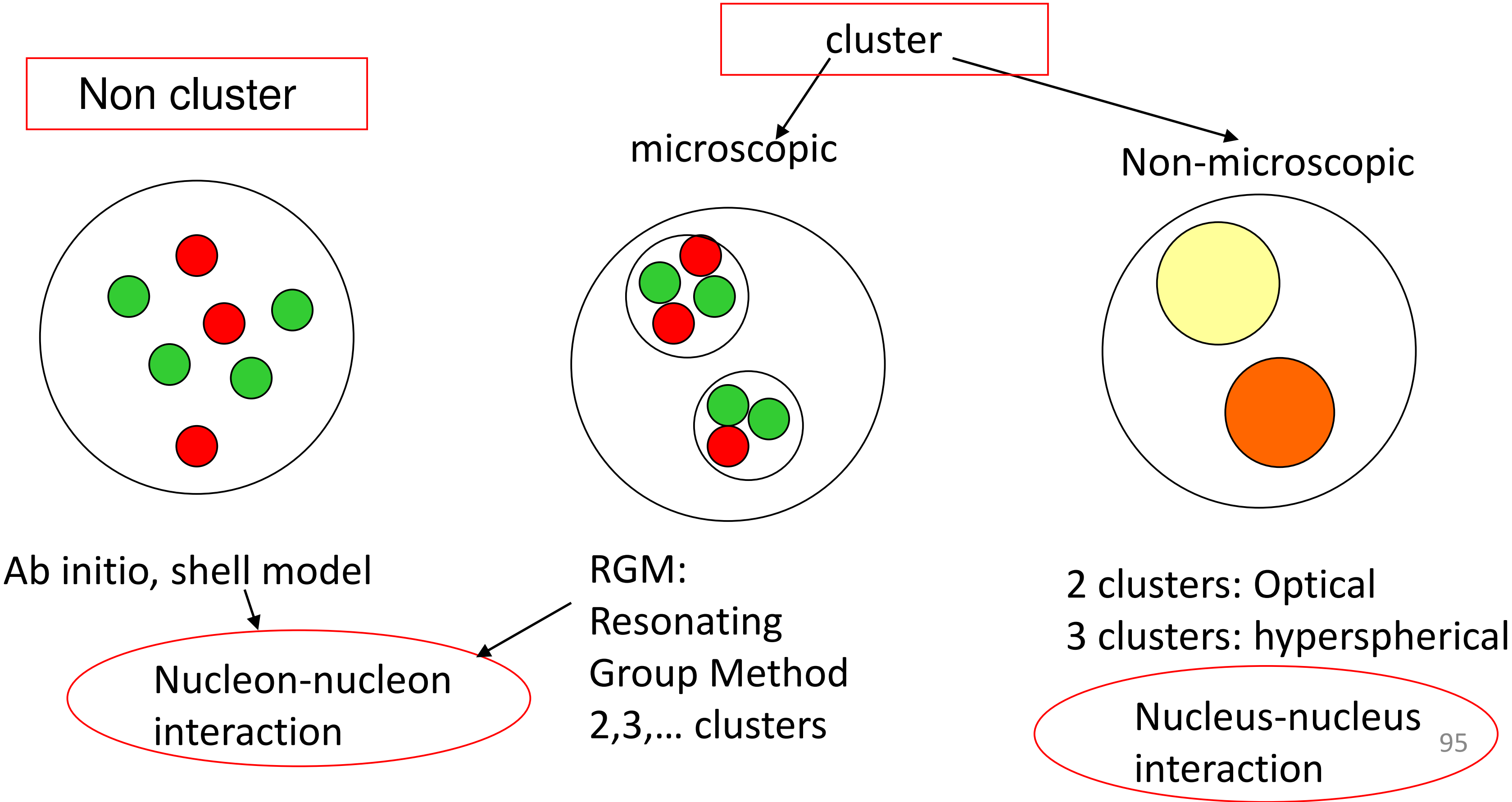
Description of the projectile

1. Introduction
2. Clustering in nuclei
3. Non-microscopic models
4. Microscopic cluster models
5. Applications of microscopic cluster models

1. Introduction

- Reaction processes:
  need for the wave function of the projectile (target very stable)
- From reaction theory:
  can we test the wave function of the projectile with reactions?

- Various types of structure models (stable and exotic nuclei)



Non cluster

cluster

microscopic

Non-microscopic

Ab initio, shell model

RGM:
Resonating
Group Method
2,3,… clusters

2 clusters: Optical
3 clusters: hyperspherical

Nucleon-nucleon interaction

Nucleus-nucleus interaction

1. Introduction

Hamiltonian of the nucleus:

$$H = \sum_{i} T_i + \sum_{j>i} V_{ij} + \sum_{k>j>i} V_{ijk} + \cdots$$

with

T_i = kinetic energy of nucleon *i*

V_{ij} = two-body nucleon-nucleon interaction

- Contains a nuclear part V_{ij}^N: short range
- Contains a coulomb part V_{ij}^C: long range $\sim e^2/r$

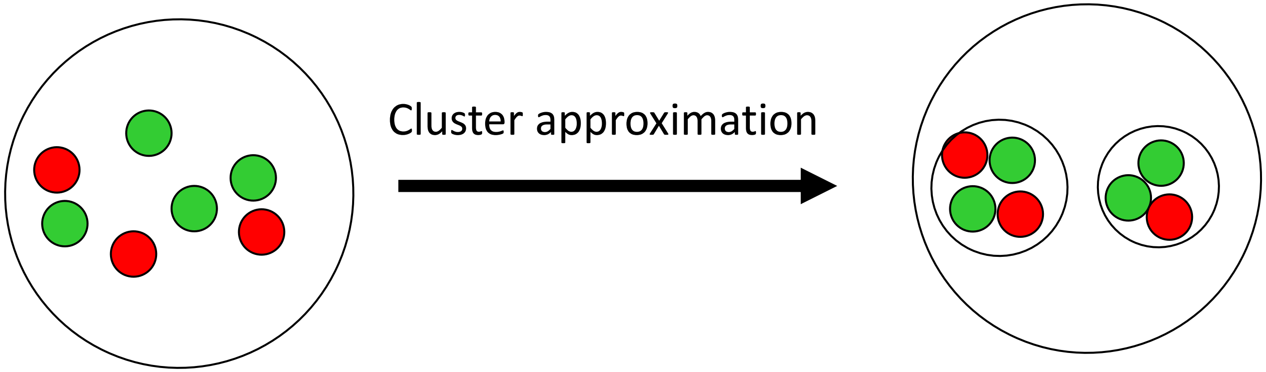
V_{ijk}=three-body interaction (often neglected)

Question: how to solve the Schrödinger equation?

Several techniques:

- Ab initio calculations: provide « exact » solutions
- Shell model
- Cluster approximation

2. Cluster models



- Clustering: well known effect in light nuclei
- Nucleons are grouped in “clusters”

  Best candidate: α particle (high binding energy, almost elementary particle)

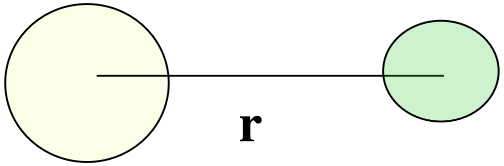
  → Ikeda diagram: cluster states near a threshold (^{8}Be, ^{20}Ne, etc.)

- Halo nuclei: special case of cluster states
- Beyond the nucleon level: hypernuclei
  quarks

- Well adapted to reactions (not true for the shell model, ab initio models, etc.)
- Possibility to have 3,4,… clusters (example: $^{12}C=\alpha+\alpha+\alpha$)

3. Non-microscopic models

Two clusters



Hamiltonian: $H = T_r + V(r)$

Wave function: $\Psi^{\ell m} = g^{\ell}(r) Y_{\ell}^{m}(\Omega)$

- Structure of the clusters is neglected
- Pauli principle: approximated (appropriate choice of the potential)
- Low relative energies: real potentials (in general)
  High relative energies: complex potentials (simulate absorption)

☺ Simple
→ Widely used for the description of projectiles

☺ Ex: ^{7}Li=α+t, ^{6}Li=α+d, ^{17}F=^{16}O+p

☹ Potential $V(r)$ in general not known
Fitted on data (binding energies, phase shifts)
Obtained from folding

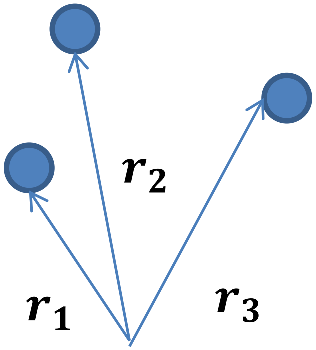
☹ Sometimes not adapted to the structure of the nucleus

3. Non-microscopic models

Many nuclei have a 3-body structure →Three-body problem must be solved

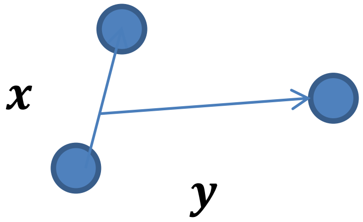
Hamiltonian: $H = T_1 + T_2 + T_3 + V_{12}(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) + V_{13}(|\boldsymbol{r}_1 - \boldsymbol{r}_3|) + V_{23}(|\boldsymbol{r}_2 - \boldsymbol{r}_3|)$

Absolute coordinates



Jacobi coordinates

(c.m. removed)



$$\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$$
$$\boldsymbol{y} = \boldsymbol{r}_3 - (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$$

Hyperspherical coordinates

- Ω_x, Ω_y
- $\rho = \sqrt{x^2 + y^2}$
- $\tan\alpha = \frac{y}{x}$

ρ =hyperradius

α= hyperangle

In hyperspherical coordinates: $H = T_\rho + V(\rho, \alpha, \Omega_x, \Omega_y)$

Eigenstates of T_ρ: **hypersphercial functions** $\mathcal{Y}^{L}_{K l_x l_y}(\alpha, \Omega_x, \Omega_y) = \mathcal{Y}^{L}_{K\gamma}(\Omega_5)$

known functions (analytical)

extension of spherical harmonics $Y_l^m(\Omega)$ in 2-body problems

K=hypermoment

3. Non-microscopic models

- Schrödinger equation :$H\Psi^{LM} = E\Psi^{LM}$

- The wave function is expanded in hyperspherical harmonics

$$\Psi^{LM}(\rho,\Omega_5) = \sum_{K=0}^{\infty}\sum_{\gamma} \underset{\substack{\uparrow \\ \boxed{\text{Known functions}}}}{\mathcal{Y}^{L}_{K\gamma}(\Omega_5)}\, \underset{\substack{\nwarrow \\ \boxed{\text{To be determined}}}}{\chi^{L}_{K\gamma}(\rho)}$$

- The radial functions are obtained from a set of coupled differential equations

$$-\frac{\hbar^2}{2\mathrm{m}_N}\left(\frac{d^2}{d\rho^2}-\frac{K(K+4)}{\rho^2}\right)\chi^{L}_{K\gamma}(\rho) + \sum_{K',\gamma'} V_{K\gamma,K'\gamma'}(\rho)\chi^{L}_{K\gamma}(\rho) = E\chi^{L}_{K\gamma}(\rho)$$

  - Potentials $V_{K\gamma,K'\gamma'}(\rho)$ are determined from $V_{12}+V_{13}+V_{23}$
  - Two-body potentials V_{ij} contains spurious Pauli forbidden states → must be removed
  - Equivalent to a standard coupled-channel problem (up to ~100-200 channels)
  - In practice: summation over K is limited to K_{max}

    $\chi^{L}_{K\gamma}(\rho)$ are expanded over a basis (Lagrange basis here)

- General form of the system: identical to all coupled-channel problems

3. Non-microscopic models

Number of channels in 3-body problems
example: ^{11}Li=^{9}Li+n+n (spin of the core neglected, $S_{nn} = 0\ or\ 1$)

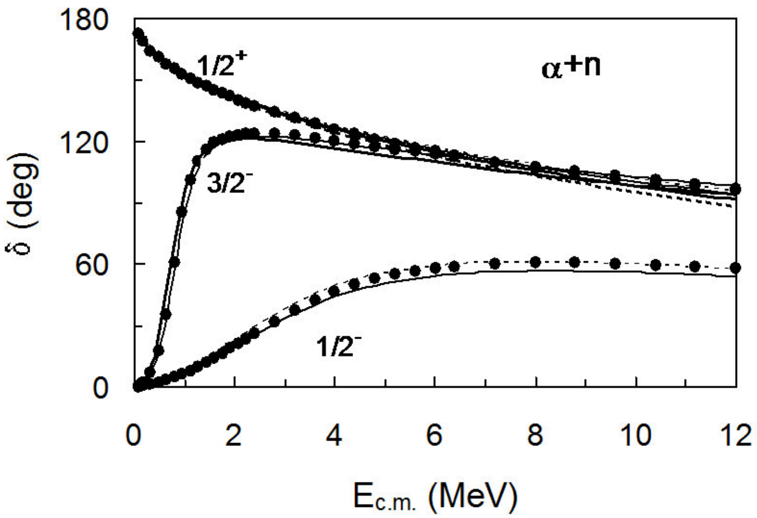
K_{max}	$J = 0^+$	K_{max}	$J = 1^-$
8	15	7	26
12	28	11	57
16	45	15	100
20	66	19	155
24	91	23	222
28	120		
32	153		

3. Non-microscopic models

Example: $^{6}He=\alpha+n+n$

- $V_{\alpha n}$: Kanada et al., Prog. Theor. Phys. 61 (1979) 1327

  fits the experimental α-p phase shifts (gaussians)



- V_{nn}: Minnesota, Nucl. Phys. A286 (1977) 53

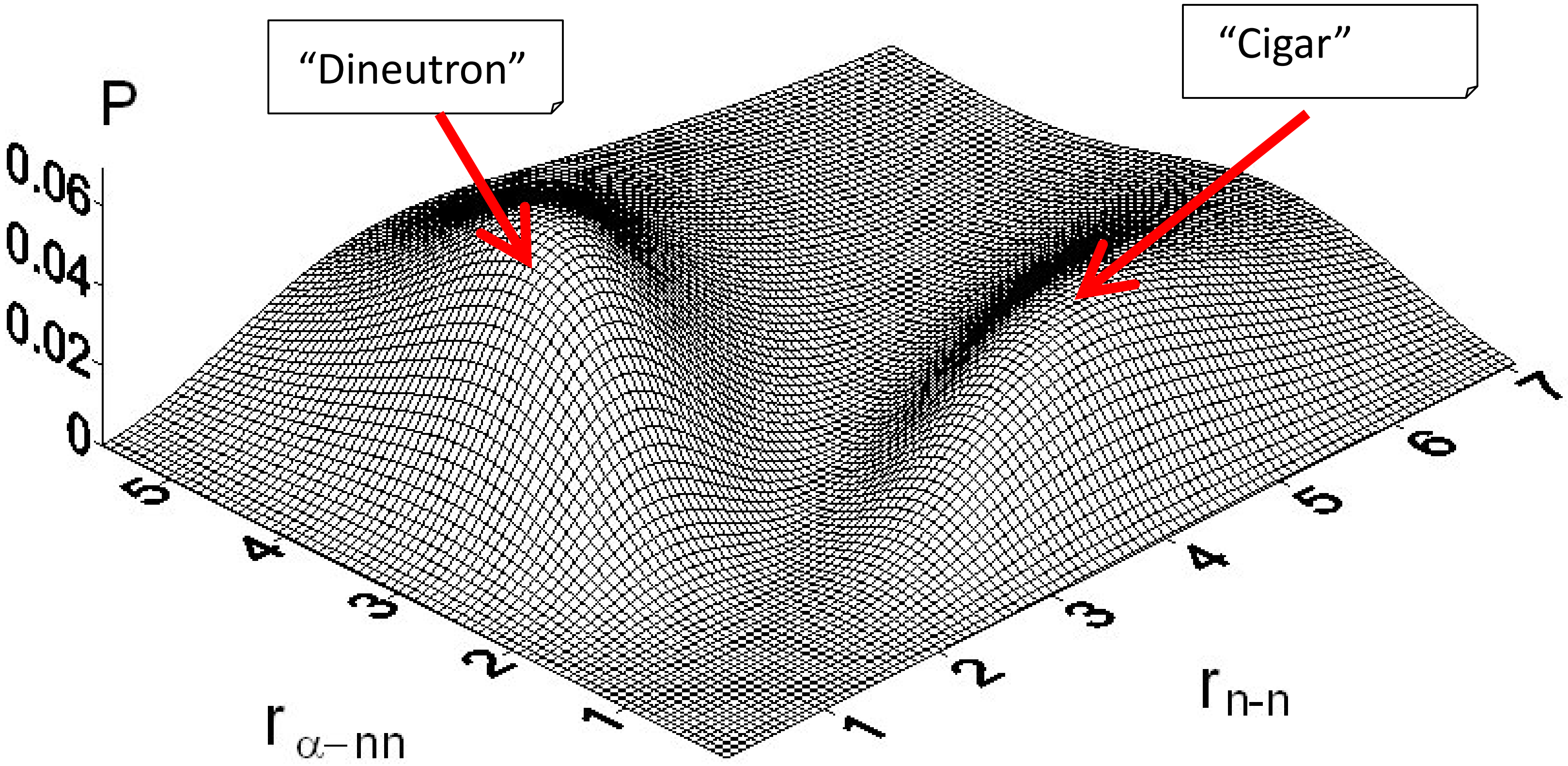
^{6}He	Theor.	Exp
Energy	-0.78 MeV	-0.97 MeV
$\sqrt{<r^2>}$	2.42 fm	2.33±0.04 fm

➔ 3 body effects?

➔ “effective” interactions?

3. Non-microscopic models

^{6}He wave functions (S=0)



“Dineutron”

“Cigar”

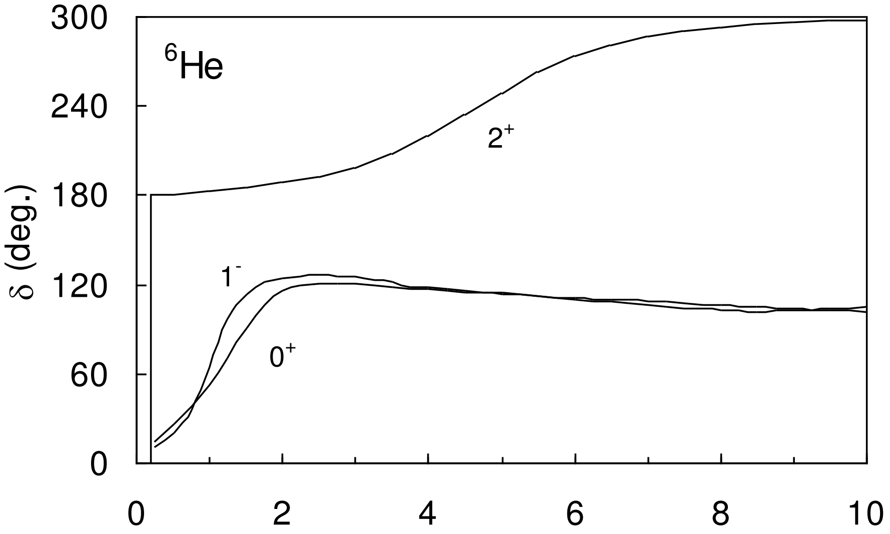
3. Non-microscopic models

Specific problems of 3-body scattering states ($E > 0$)

$$-\frac{\hbar^2}{2\mathrm{m}_N}\left(\frac{d^2}{d\rho^2}-\frac{K(K+4)}{\rho^2}\right)\chi^L_{K\gamma}(\rho)+\sum_{K',\gamma'}V_{K\gamma,K'\gamma'}(\rho)\chi^L_{K\gamma}(\rho)=E\chi^L_{K\gamma}(\rho)$$

- Many hypermomenta (K-values)→ large set for large Kmax (**slow convergence**)
- **Long range of the potentials**: behave as $\sim 1/\rho^3$
  → the asymptotic coulomb behaviour is not reached before (~500-1000 fm!)
  → propagation methods are necessary

R-matrix application to 3-body systems: P. D., E. Tursunov, D. Baye, *Nucl. Phys. A 765 (2006) 370*



^{6}He

3-body α+n+n phase shifts (+wave functions)
2^+ well known narrow resonance
0^+,1^-: « broad structures »

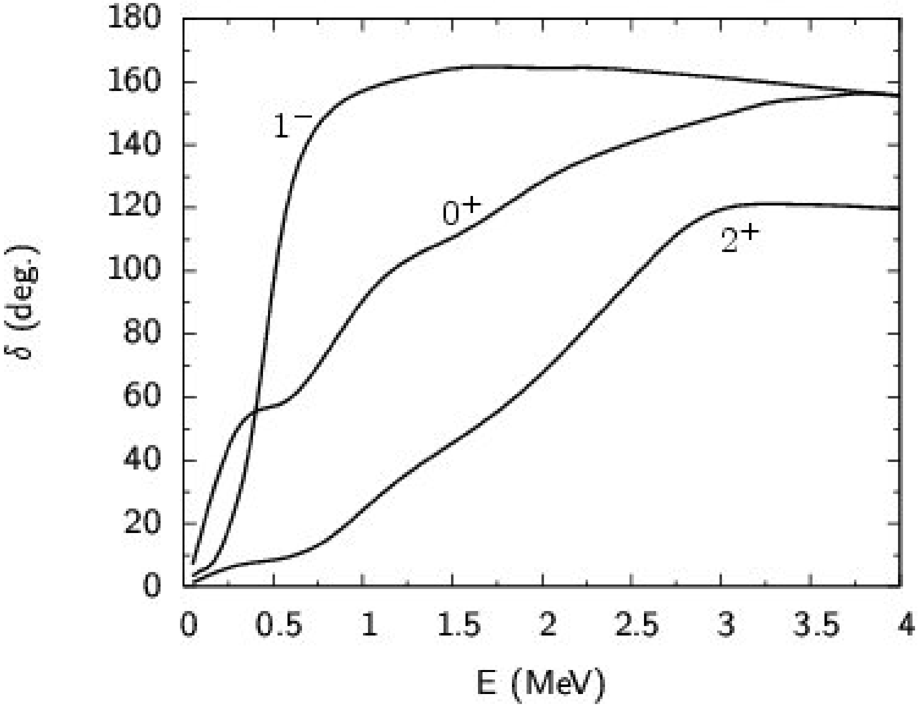
Break-up cross sections:
D. Baye, P. Capel, P. D., and Y. Suzuki: *Phys. Rev. C 79 (2009) 024607*

3. Non-microscopic models

Recent work on ^{11}Li

- Ref: E.C.Pinilla, P.D., D. Baye, *Phys. Rev. C 85 (2012) 054610*
- ^{11}Li described by a ^{9}Li+n+n structure (spin of ^{9}Li is neglected)

three-body phase shifts



^{11}Li

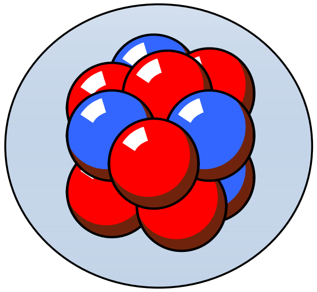
Narrow 1- resonance near 0.5 MeV

4. Microscopic models: overview

1. Overview of different models
2. Cluster models

4. Microscopic models: overview

Definition of a microscopic model



- **Wave functions**
  - fully antisymmetric
  - Depend on all nucleon coordinates → complicated many-body problem!
  - Exchange of particles i and j
    Pauli principle

$$P_{ij}\Psi(1,2,\ldots i,\ldots j,\ldots A) = -\Psi(1,2,\ldots j,\ldots i,\ldots A)$$

- **Hamiltonian**
  - given by

$$H = \sum_i T_i + \sum_{j>i} V_{ij} + \sum_{k>j>i} V_{ijk} + \cdots$$

with

T_i = kinetic energy of nucleon *i*

V_{ij} = two-body nucleon-nucleon interaction

- Contains a nuclear part V_{ij}^N: short range
- Contains a coulomb part V_{ij}^C: long range $\sim e^2/r$

V_{ijk}=three-body interaction (often neglected)

4. Microscopic models: overview

Main advantages

- Predictive power: in principle there is no parameter
- Coherent description of different processes:
  - spectroscopy (energies, radii, electromagnetic transitions, etc.)
  - scattering (elastic, transfer, radiative capture, etc.)

Main problems

- V_{ij}^{N} is not exactly known
  → approximations, **effective** NN interactions (adapted to the model)
- The Schrödinger equation may involve many terms (A(A-1)/2)
  → cannot be solved exactly
- Difficult to apply to scattering states

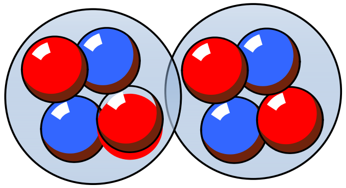
→ **various models**

- Shell model (and extensions: No-core shell model)
- Fermionic Molecular Dynamics (FMD), Antisymmetrized Molecular Dynamics (AMD)
- Cluster models: Resonating Group Method (RGM), Generator Coordinate Method (GCM)

4. Microscopic models: overview

Cluster models

- the A nucleons form « clusters » inside the nucleus
- origin: the α particle is strongly bound → keeps its own identity in the nucleus
- typical clusters: strongly bound nuclei (**alpha particle**)
- example : ^{8}Be=$\alpha+\alpha$ - formed of 4 neutrons and 4 protons grouped in 2 α



- **Cluster** approximation $\Psi = \mathcal{A}\phi_1\phi_2 g(\rho)$
  with

ϕ_1, ϕ_2 = internal wave functions (**input, shell-model**)

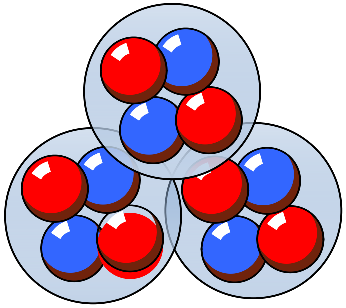
$g(\rho)$ =relative wave function (**output**)

$\mathcal{A}$ = antisymmetrization operator

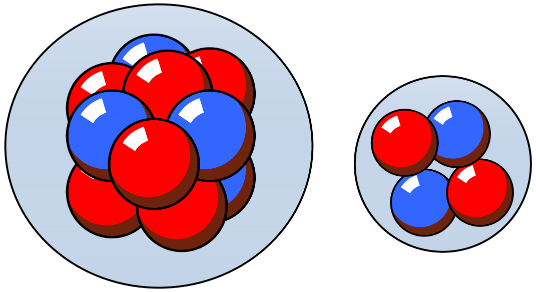
=**Resonating Group Method** (RGM)

- Describes spectroscopy and reactions
  → easy access to unbound states (+widths)

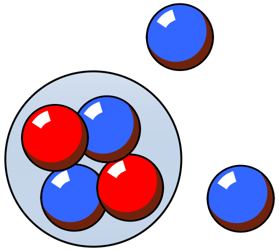
4. Microscopic models: overview



^{12}C described by 3 alphas



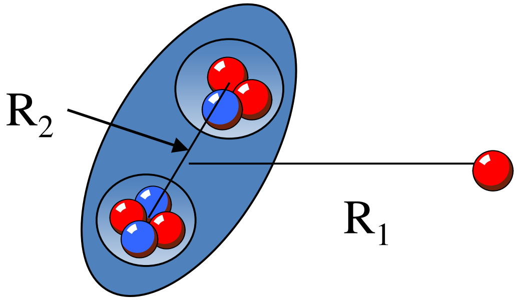
^{20}Ne described by ^{16}O+α



^{6}He described by α+n+n
nucleon=« particular » cluster, numerical techniques identical

4. Microscopic models: cluster models

Extensions



- 3 clusters (or more)
  projection more complicated (multidimension)
- p, sd orbitals: many Slater determinants
  → analytical calculations not possible
- Multichannel calculations: $\Psi = \mathcal{A}\phi_1\phi_2 g(\rho) + \mathcal{A}\phi_1^*\phi_2^* g^*(\rho) + \cdots$
  → core excitations (important in many nuclei)
  → better wave functions
  → inelastic scattering, transfer

5. Applications of microscopic cluster models

2-cluster models

- $^{17}C/^{17}Na$ (recent → exotic nuclei)

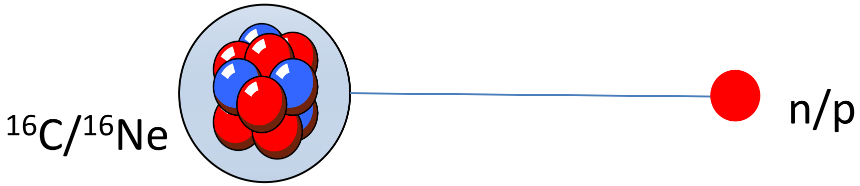
3-cluster models

- $^{6}He=\alpha+n+n$
- $^{12}C=\alpha+\alpha+\alpha$

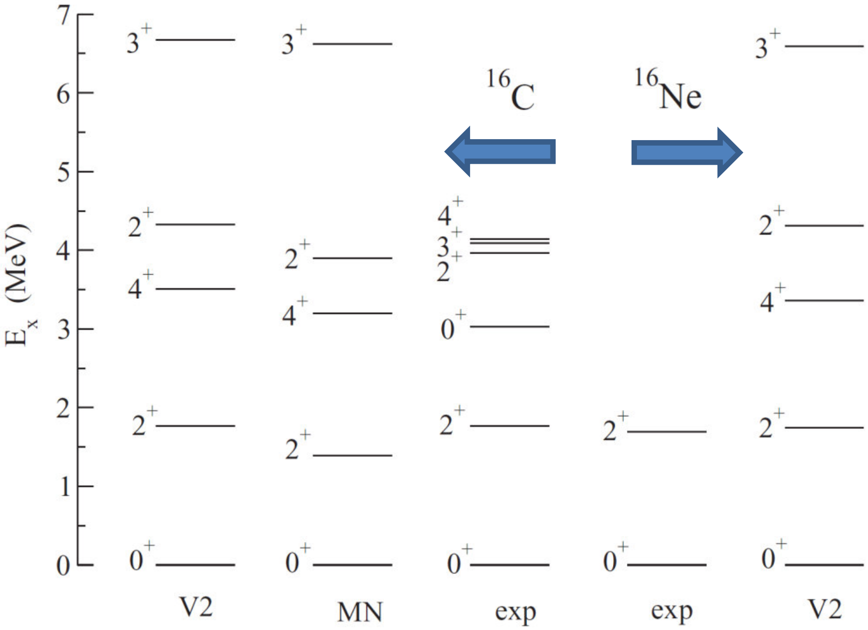
5. Applications of microscopic cluster models: ^{17}C / ^{17}Na

The ^{17}C and ^{17}Na mirror nuclei

- Ref: N. Timofeyuk, P.D. , Phys. Rev. C81 (2010) 051301
- ^{17}Na unstable (no experimental data but ^{19}Na unstable)
- The mirror ^{17}C nucleus is well known → test with charge symmetry
- Two-cluster systems: ^{16}C+n, ^{16}Ne+p



- ^{16}C/^{16}Ne wave functions: 6 protons (s^2p^4), 10 neutrons (s^2, p^6, sd^2) → 15x66=990 SD

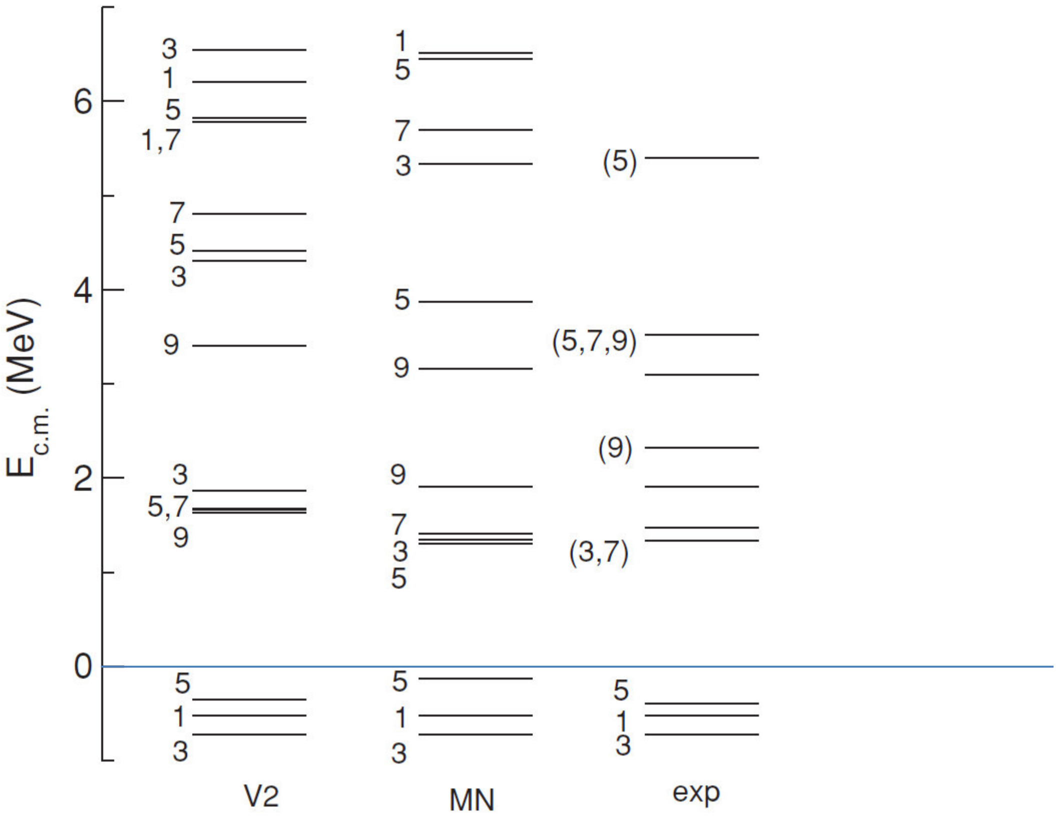


Two NN interactions: MN and V2

5. Applications of microscopic cluster models: ^{17}C / ^{17}Na

^{17}C spectrum (positive parity)

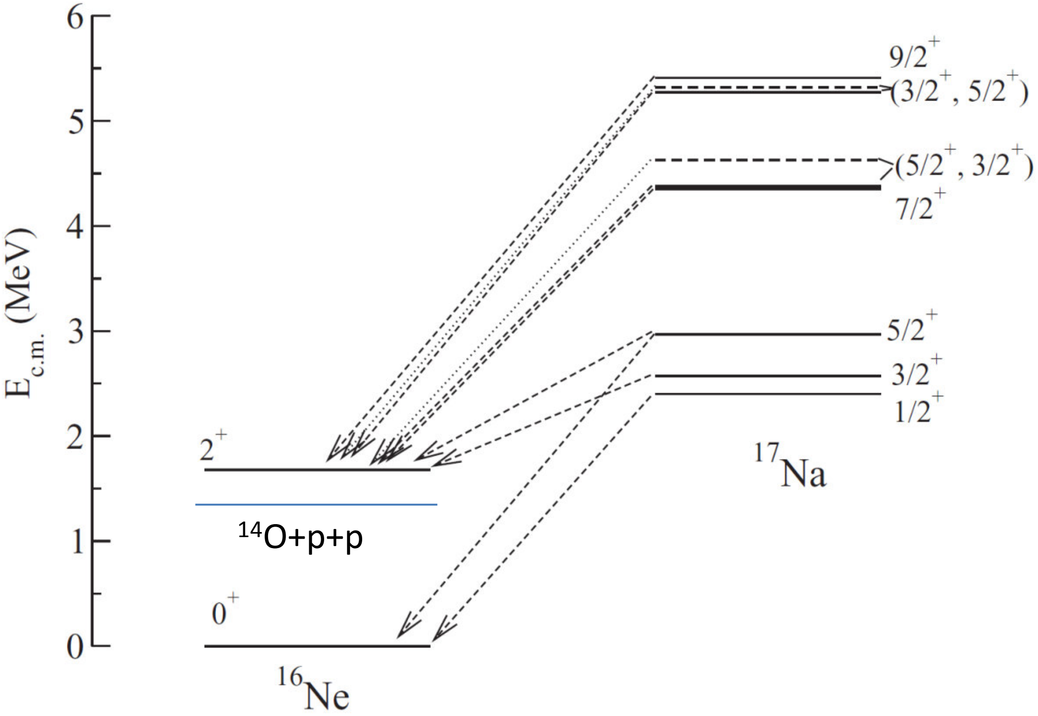
Two NN interaction V2 and MN (+spin-orbit)



^{16}C+n threshold

5. Applications of microscopic cluster models: ^{17}C / ^{17}Na

^{17}Na spectrum

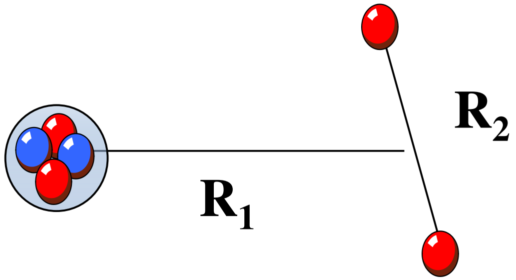


J	E	$\Gamma(0^+)$	$\Gamma(2^+)$
1/2+	2.40	1.36	
3/2+	2.57	0.001	0.024
5/2+	2.97	0.123	0.021
7/2+	4.35	$8\text{x}10^{-8}$	0.025

- all states are unbound → importance of continuum
- ground state: broad ($\ell = 0$) resonance
- excited states: narrow ($\ell = 2$), important decay to the ^{16}Ne(2^+)+p channel
  ➔ 3 proton emittors

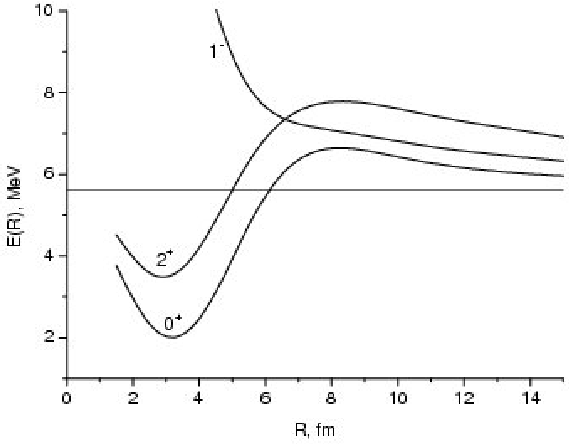
5. Applications of microscopic cluster models: ^{6}He and ^{6}Li

S. Korennov, P.D., Nucl. Phys. A740 (2004) 249

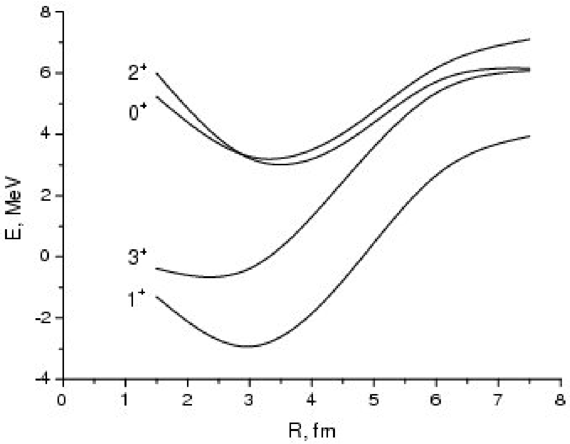


Hypershperical coordinate:

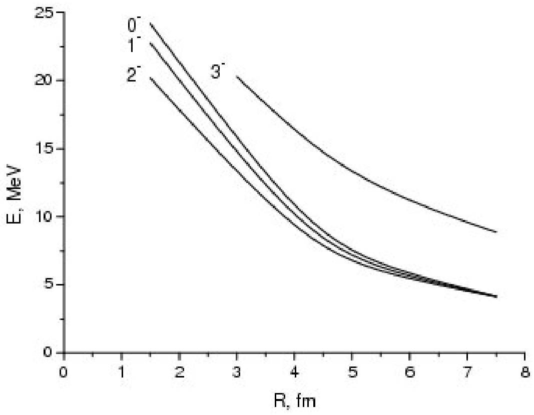
$$R \sim \sqrt{R_1^2 + R_2^2}$$



^{6}He

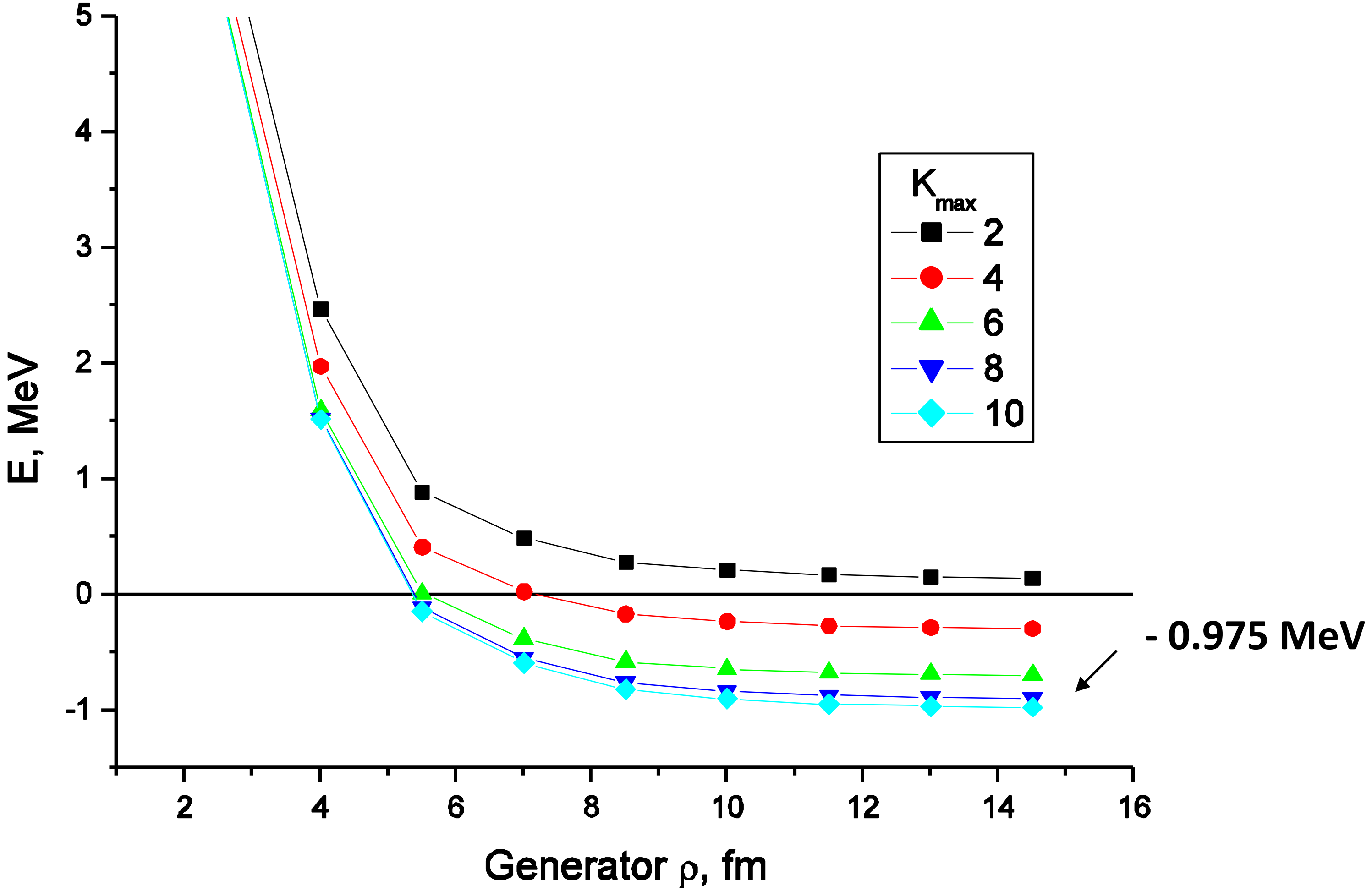


^{6}Li, π=+



^{6}Li, π=-

Energy convergence, 0^+



Applications

CDCC

- 2-body: ^{11}Be+^{64}Zn
- 3-body: ^{9}Be+^{208}Pb
- Microsopic CDCC: ^{7}Li+^{208}Pb

Eikonal:

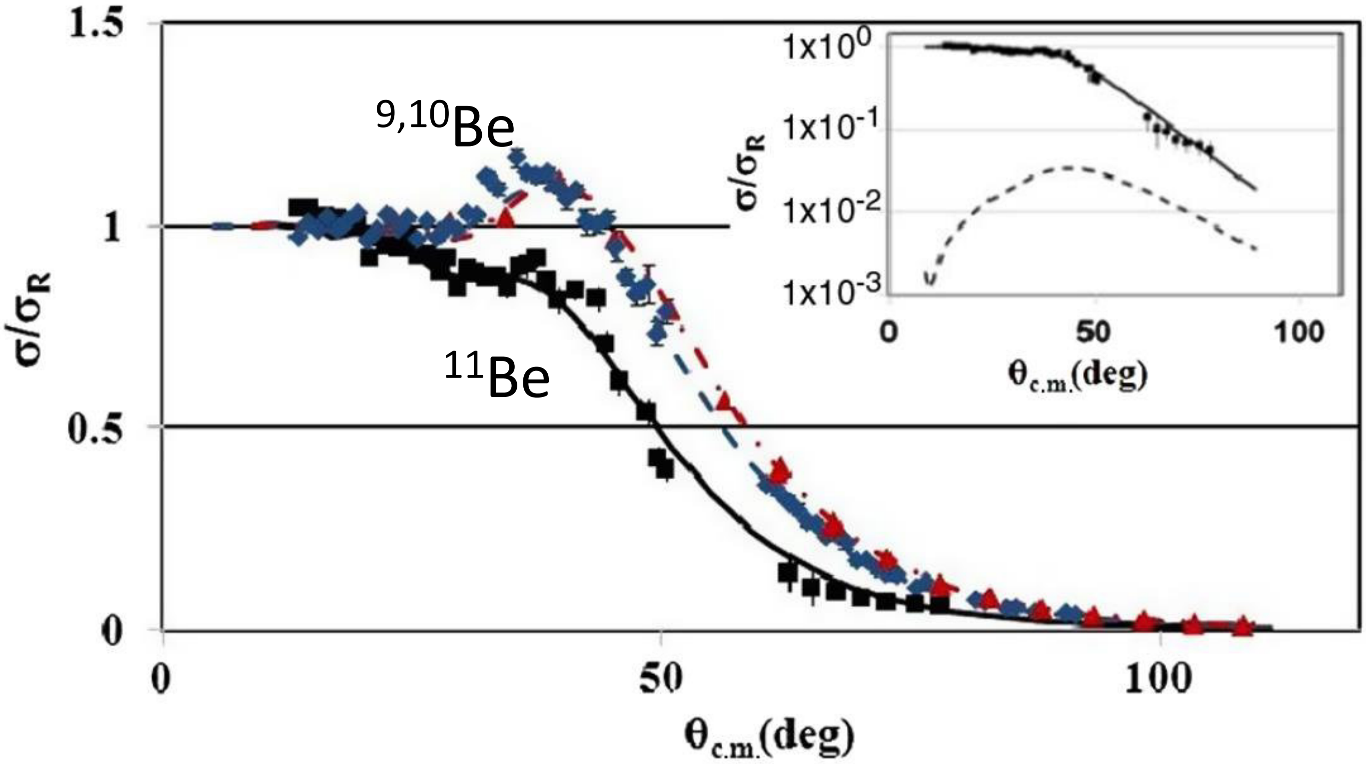
- Three-body breakup
- Microscopic eikonal (elastic scattering)

1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

^{9,10,11}Be+^{64}Zn at 25 MeV:

A. Di Pietro et al., Phys. Rev. Lett. 105, 022701 (2010)

A. Di Pietro et al., Phys. Rev. C85, 054607 (2012)



Important difference between ^{9,10}Be and ^{11}Be: role of the halo structure

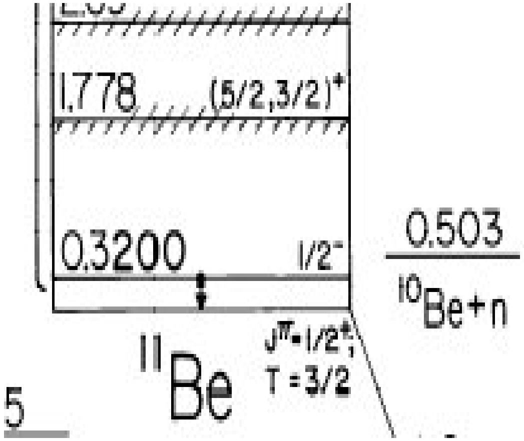
1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

Recent work: T. Druet, P.D., Eur. J. Phys. 48 (2012) 1

Main goal: to analyse the convergence of the cross sections (elastic, inelastic, breakup)

Conditions of the calculations: 3 potentials

- ^{10}Be+^{64}Zn: optical potential from experiment
- n+^{64}Zn: global parametrization of Koning-Delaroche
- n+^{10}Be: P. Capel et al., PRC 70 (2004) 064605: reproduces bound states and 5/2^{+} resonance

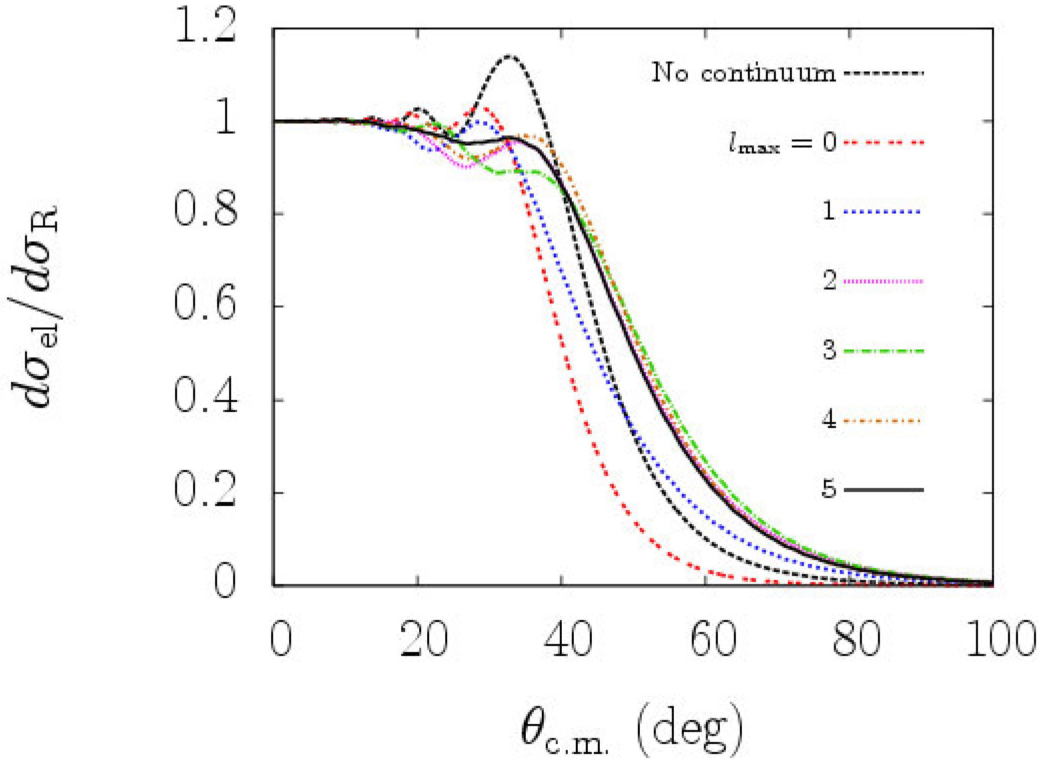


1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

Elastic cross section (data contain inelastic components)

First calculation : no spin-orbit force → reduces the size of the system by a factor 2

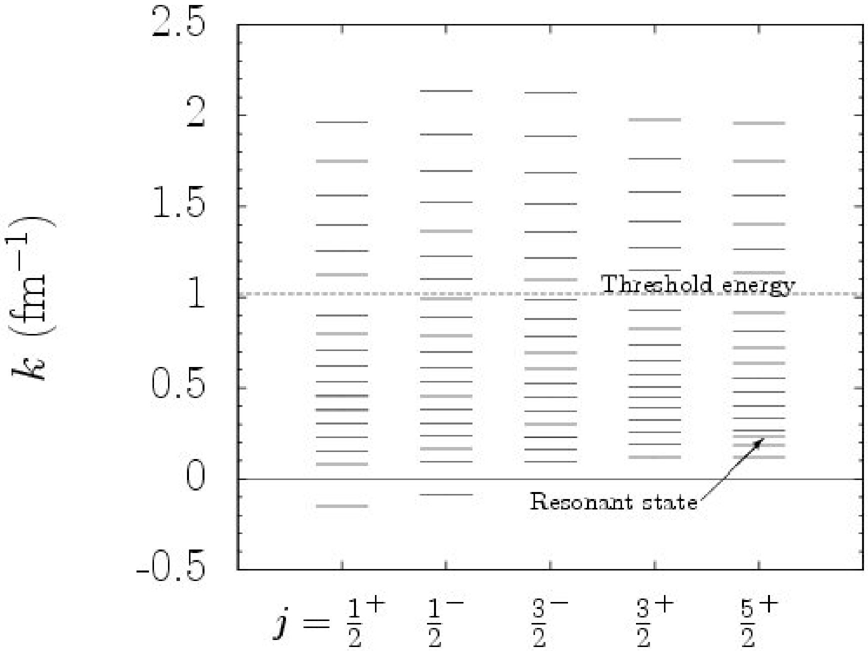
convergence with ℓ (n+^{10}Be angular momentum)

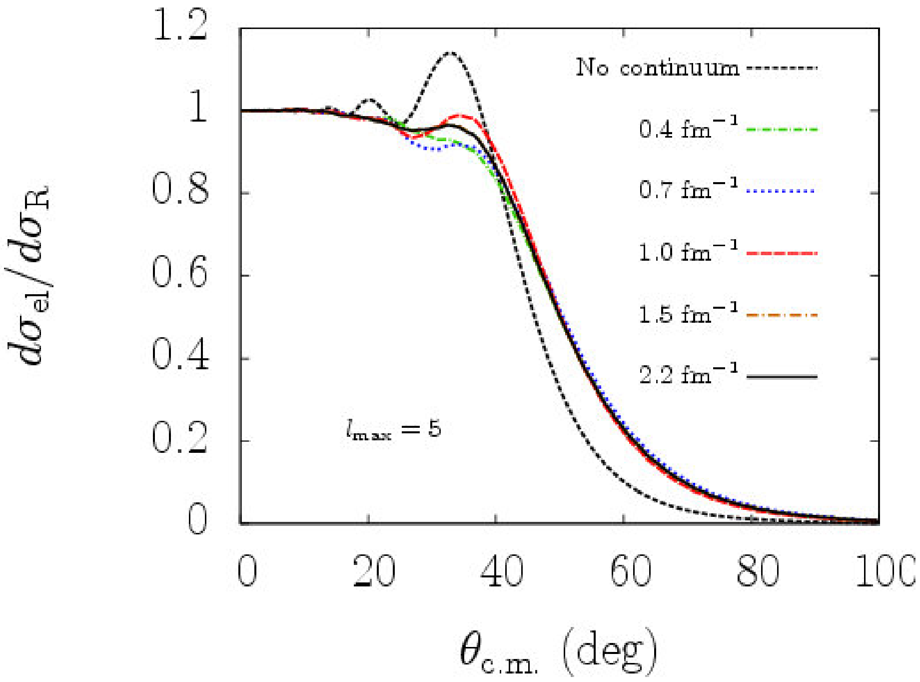


→ slow convergence

1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

Convergence with the pseudostates

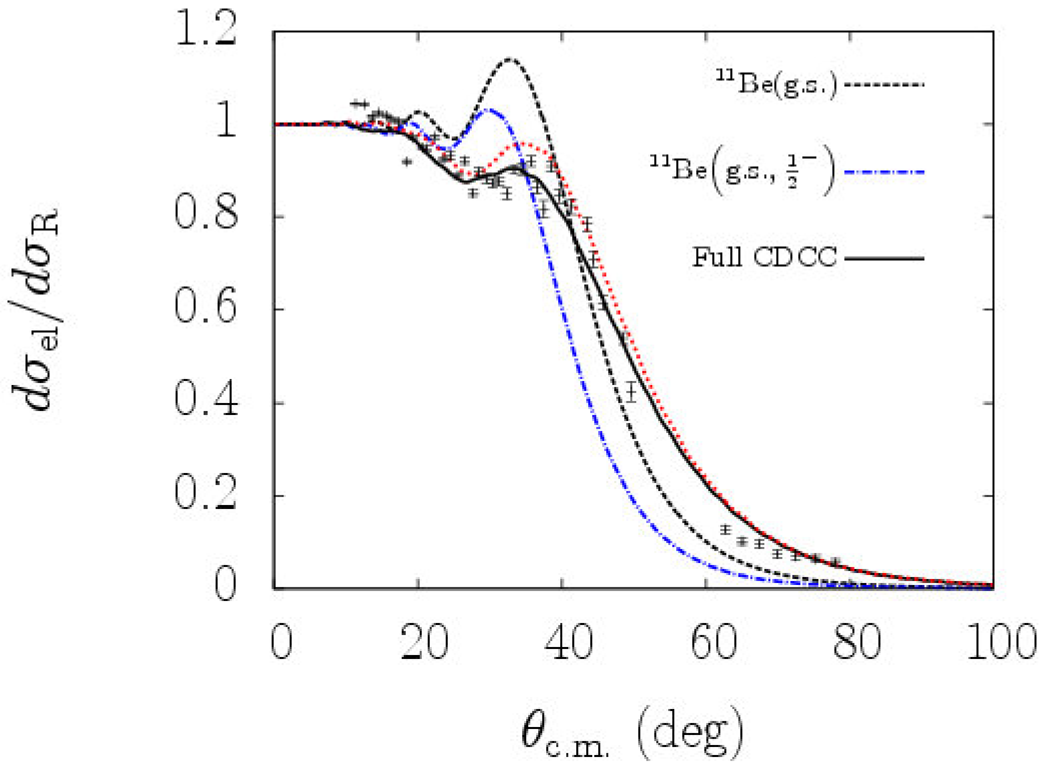




1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

Comparison with experiment

Calculation with lmax=2 (small inacurracy near $\theta \sim 40°$)

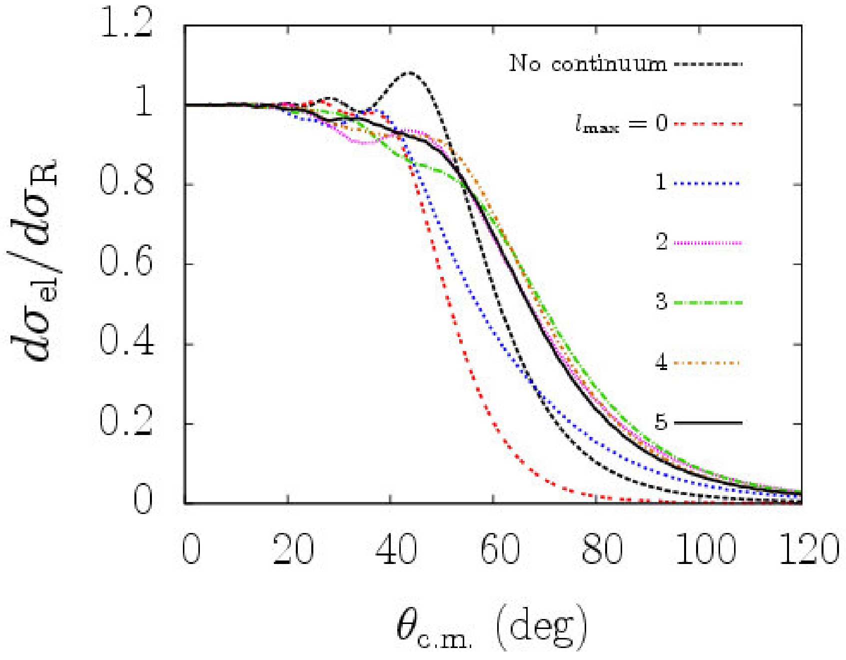


➔ importance of break-up channels!

1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

Influence of the collision energy

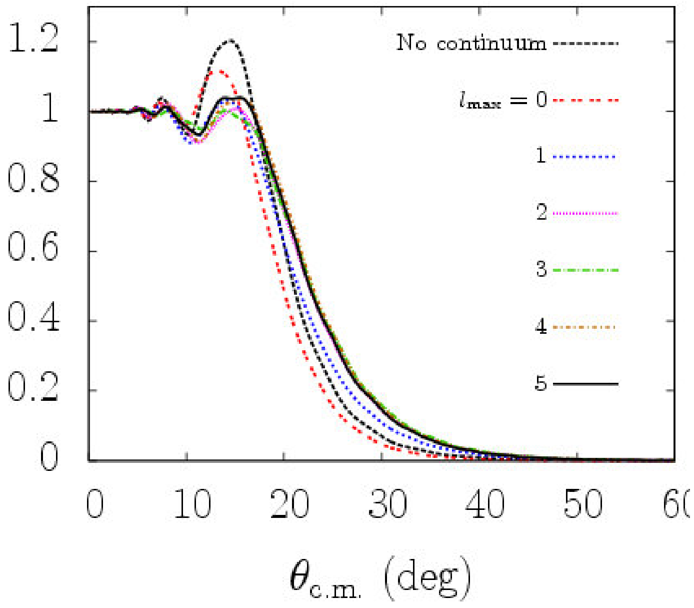
E_{cm}=20.5 MeV



Slow convergence

l_{max}=5 not sufficient

E_{cm}=44 MeV



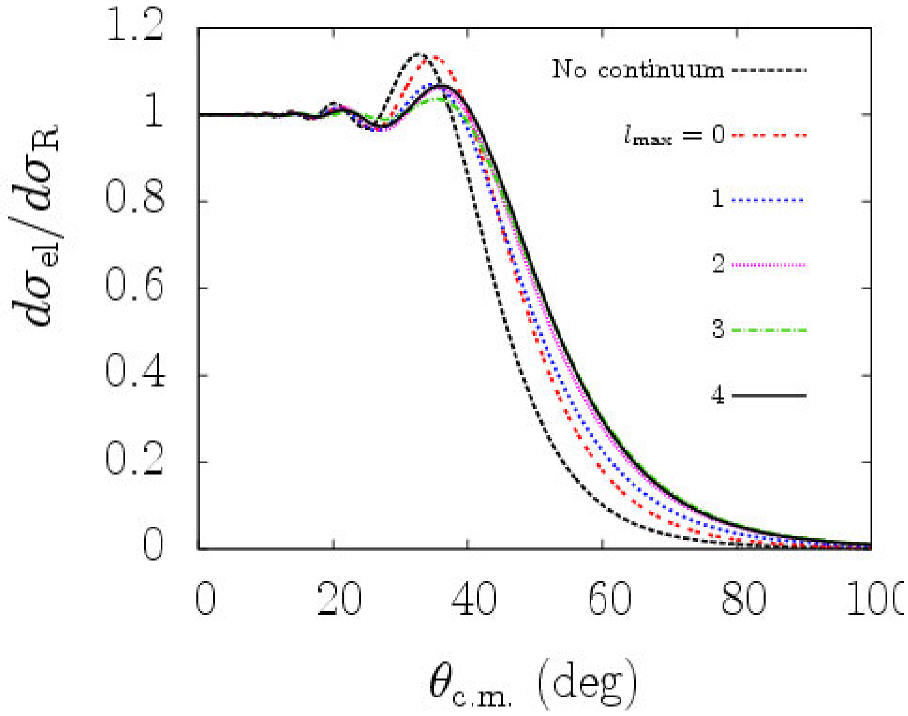
Fast convergence

l_{max}=2 sufficient

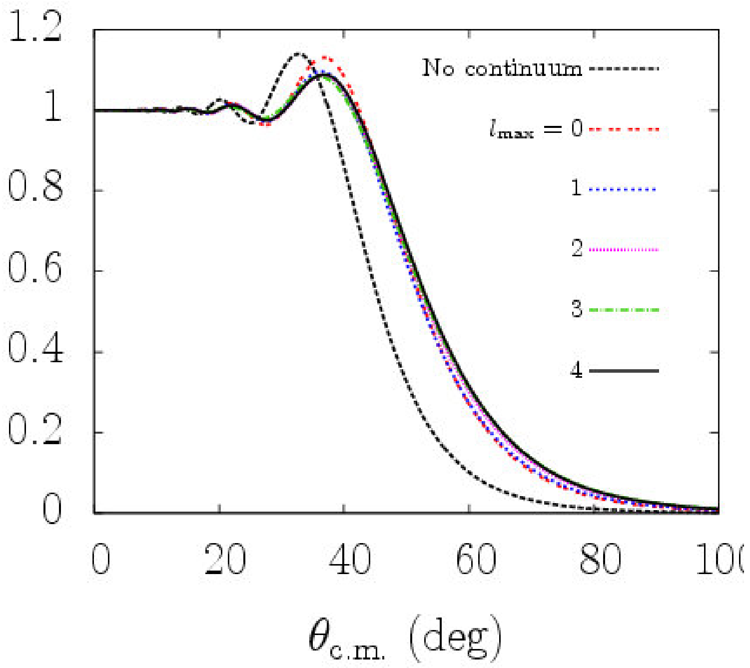
1. CDCC METHOD WITH 2-BODY PROJECTILES: ^{11}BE+^{64}ZN

Influence of the binding energy of the projectile (numerical simulation)
Original (experimental) energy: E_B=0.5 MeV

E_B=2 MeV

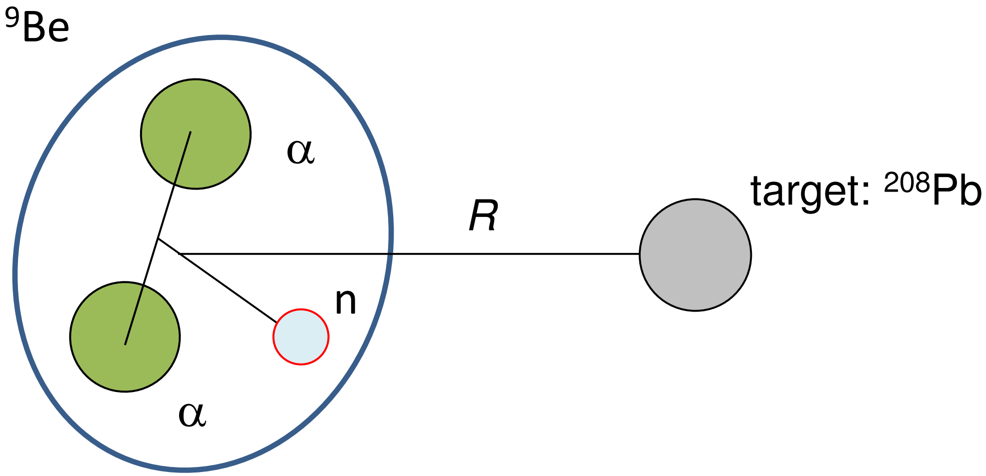


E_B=4 MeV



➔ faster convergence when the binding energy increases

2. CDCC METHOD WITH 3-BODY PROJECTILES: ^{9}BE+^{208}PB



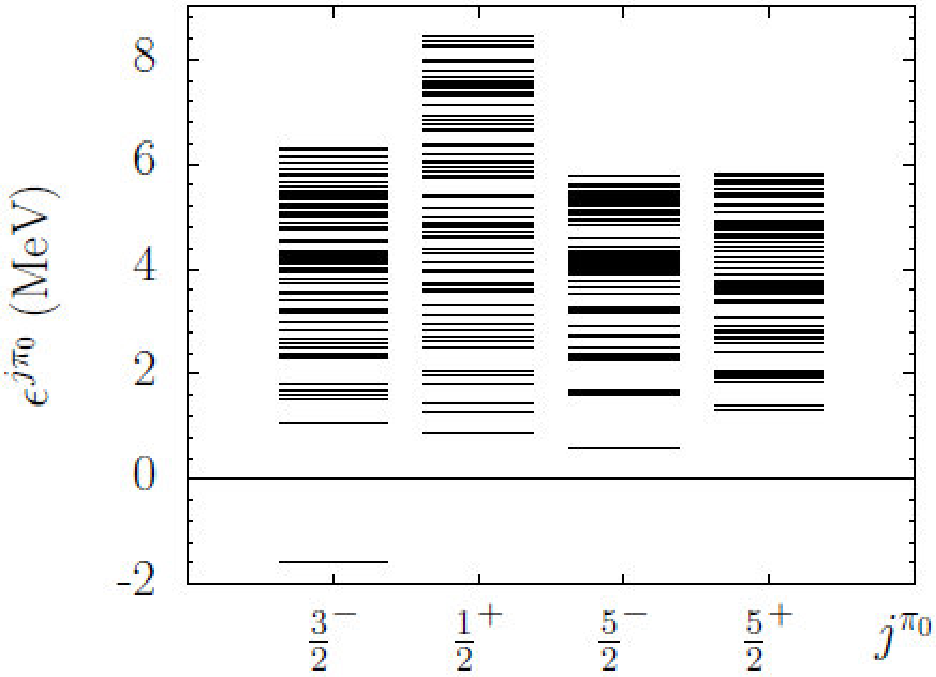
Description of ^{9}Be=α+α+n (preliminary results!)

- α+α potential: Buck et al.
- α+n: Kanada et al.

Both reproduce the experimental phase shifts

2. CDCC METHOD WITH 3-BODY PROJECTILES: ^{9}BE+^{208}PB

Discretization of the three-body α+α+n continuum



many pseudostates!

Ground-state of ^{9}Be J=3/2-

Fitted by renormalizing the α–α potential by 0.94

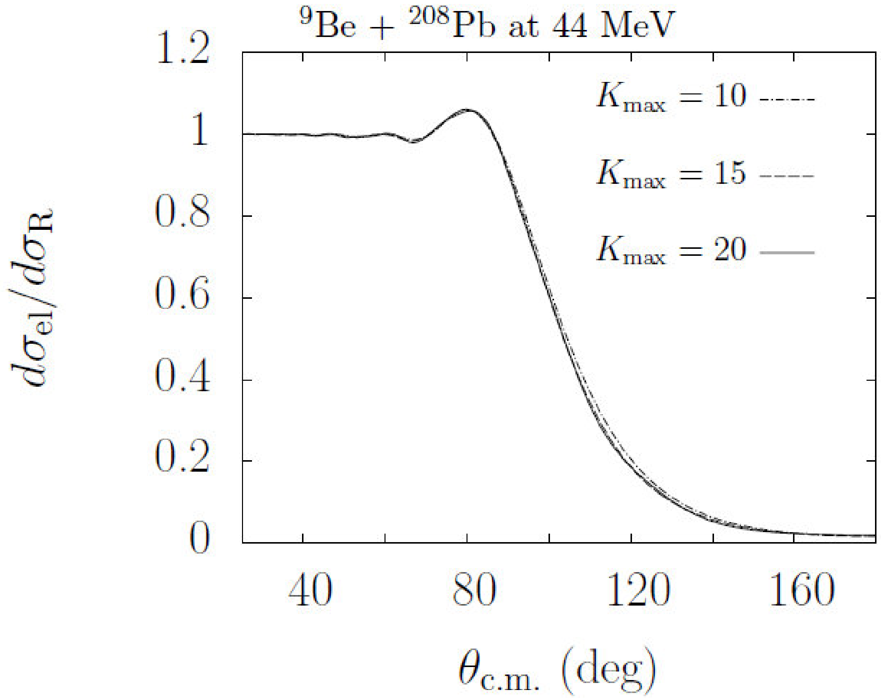
rms radius:

- theory: 2.41 fm
- exp: 2.45 ± 0.01 fm

2. CDCC METHOD WITH 3-BODY PROJECTILES: ^{9}BE+^{208}PB

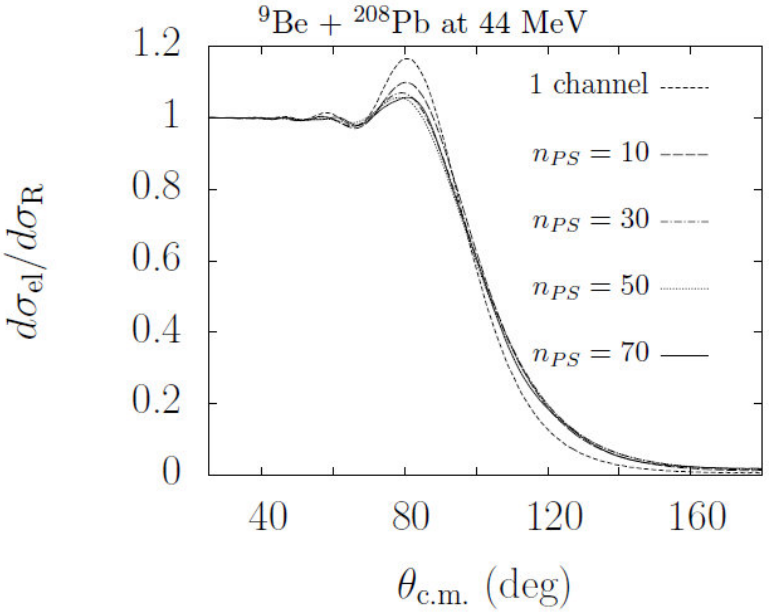
Elastic scattering ^{9}Be+^{208}Pb

- α+^{208}Pb and n+^{208}Pb optical potentials taken from literature
- Convergence with
  - Kmax (typical of 3-body problems)
  - jmax
  - discretization

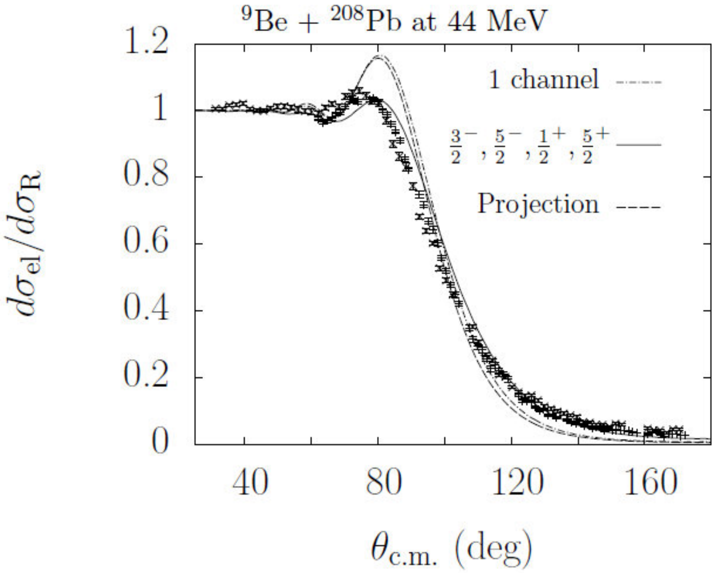


convergence with Kmax
Binding energy readjusted

2. CDCC METHOD WITH 3-BODY PROJECTILES: ^{9}BE+^{208}PB



discretization: number of pseudostates nps

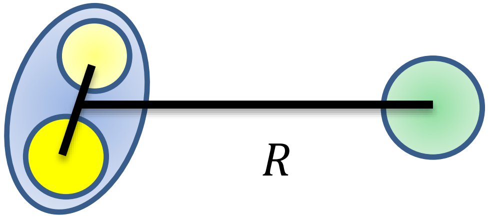


convergence with jmax

3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

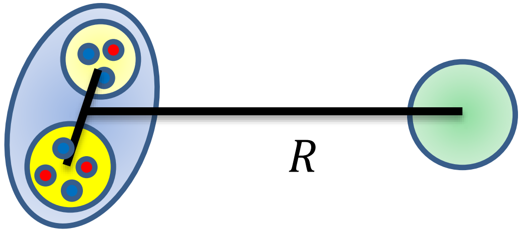
Comparison between both variants

Non-microscopic CDCC



- $H = H_0(r) - \frac{\hbar^2}{2\mu}\Delta_R + V_{ct}\left(-\frac{A_f}{A}r + R\right) + V_{ft}\left(\frac{A_c}{A}r + R\right)$
- Depends on **nucleus-target** interactions between the core/fragment and the target
- **Approximate wave functions** of the projectile
- Core excitations **difficult** (definition of the potentials?)

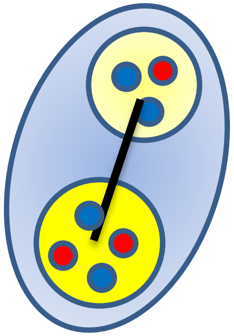
Microscopic CDCC



- $H = H_0(r_1 \dots r_A) - \frac{\hbar^2}{2\mu}\Delta_R + \sum_i v(r_i - R)$
- Depends on a **nucleon-target** interactions (in general well known)
- **Accurate wave functions** of the projectile
- Core excitations « **automatic** »

3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

First step: wave functions of the projectile



Solve $H_0\Phi_k^J = E_k\Phi_k^J$ for several J: ground-state but also additional J values

with $E_k < 0$: physical states

$E_k > 0$: pseudostates (approximation of the continuum)

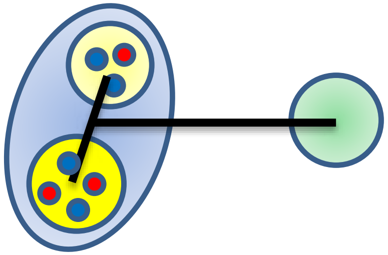
RGM: $\Phi_k^J = \mathcal{A}\Phi_1\Phi_2 g_k^J(r)$: combination of Slater determinants

RGM=Resonating Group Method (cluster approximation)

- 2 or 3 cluster models
- Core excitations: $\Phi^J = \sum_i \mathcal{A}\Phi_1^i\Phi_2 g^J(r)$, with Φ_1^i = excited states of cluster 1
  Example: ^{10}Be+n
- Well adapted to halo nuclei
- Many RGM wave functions are available

3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

Second step: wave function for projectile + target



$$H = H_0 - \frac{\hbar^2}{2\mu}\Delta_R + \sum_i v(r_i - R)$$

Expansion over projectile states: $\Psi(r_i, R) = \sum_{Jk} \Phi_k^J(r_i)\chi_k^J(R)$

→ Set of coupled equations $\left(-\frac{\hbar^2}{2\mu}\Delta_R + E_k^J - E\right)\chi_k^J(R) + \sum_{J'k'} V_{Jk,J'k'}(R)\,\chi_{k'}^{J'}(R) = 0$

$$V_{Jk,J'k'}(R) =< \Phi_k^J(r_i)\left|\sum_i v(r_i - R)\right|\Phi_{k'}^{J'}(r_i) >$$

- Matrix elements between Slater determinants: standard techniques
- Can be also computed with the densities and folding procedures → tests are possible

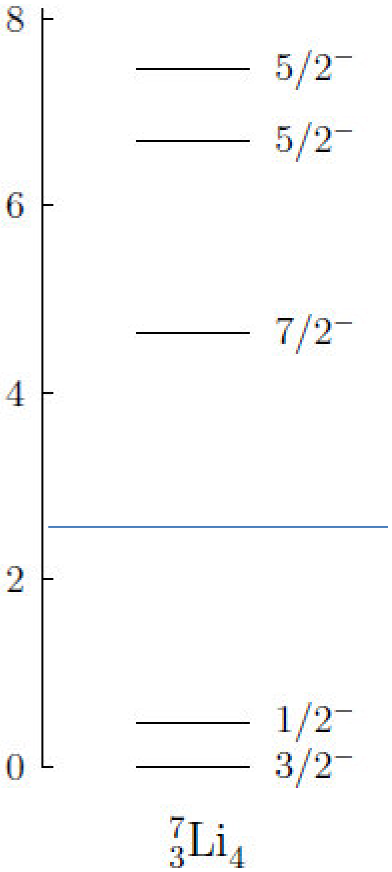
3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

- Data: E_{lab}=27 to 60 MeV (Coulomb barrier ~ 35 MeV)

- **Non-microscopic calculation** at 27 MeV:
  - Parkar et al, PRC78 (2008) 021601
  - uses $\alpha-{}^{208}\mathrm{Pb}$ and $\mathrm{t}-{}^{208}\mathrm{Pb}$ potentials renormalized by 0.6!

- **Microscopic calculation**
  - Ref.: P.D., M. Hussein, Phys. Rev. Lett. 111 (2013) 082701
  - $^7\mathrm{Li}$ wave functions: include gs,1/2$^-$,7/2$^-$,5/2$^-$ and pseudostates (E>0)
    Nucleon-nucleon potential: Minnesota interaction
    Reproduces ^{7}Li/^{7}Be, α+^{3}He scattering, ^{3}He(α,γ)^{7}Be cross section
  - Q(3/2-)=−37.0 e.mb (GCM), -40.6 ± 0.8 e.mb (exp.)
    B(E2, 3/2$^-$ → 1/2$^-$)=7.5 e^{2}fm^4 (GCM), 0.3 ± 0.5 e^{2}fm^4 (exp)
  - $\mathrm{n}-{}^{208}\mathrm{Pb}$ potential:
    local potential of Koning-Delaroche (Nucl. Phys. A 713 (2003) 231)
  - $\mathrm{p}-{}^{208}\mathrm{Pb}$ potential:
    only Coulomb (E_p=27/7 ~4 MeV, Coulomb barrier ~12 MeV)

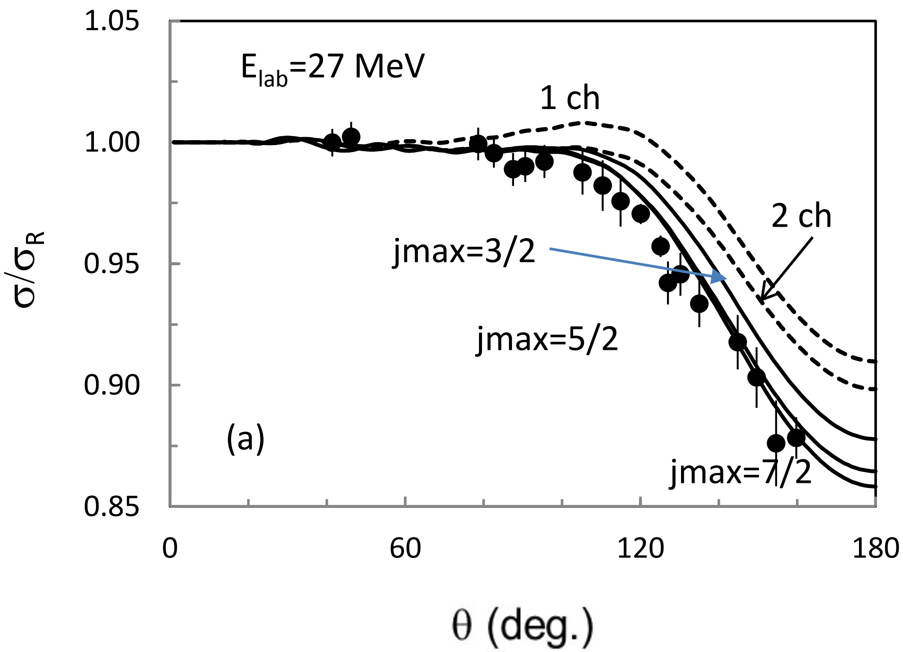
➔ NO PARAMETER



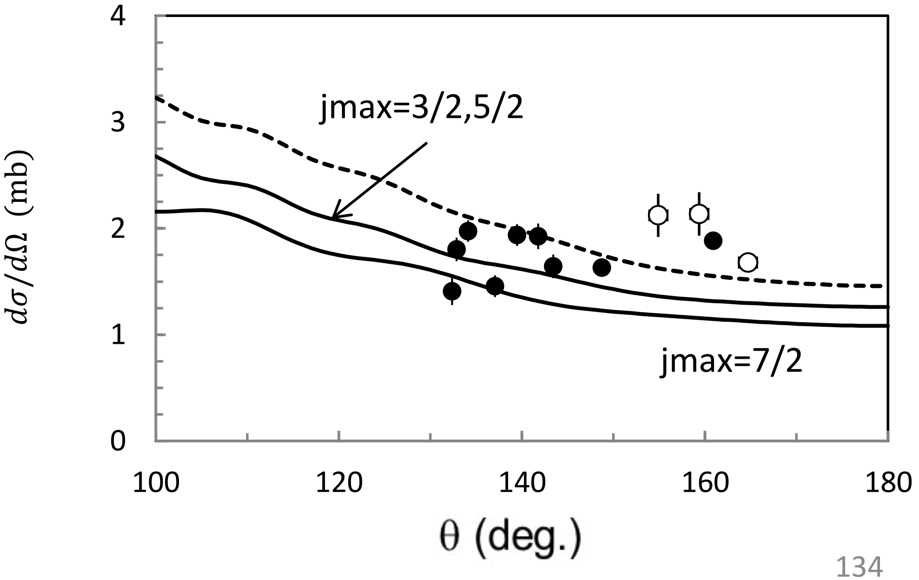
3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

- Convergence test: single-channel: $^{7}\mathrm{Li}(3/2^{-}) + {}^{208}\mathrm{Pb}$
  two channels: $^{7}\mathrm{Li}(3/2^{-}, 1/2^{-}) + {}^{208}\mathrm{Pb}$
  multichannel:
  $^{7}\mathrm{Li}(3/2^{-}, 1/2^{-}, \ldots) + {}^{208}\mathrm{Pb}$

  Elastic scattering at Elab=27 MeV

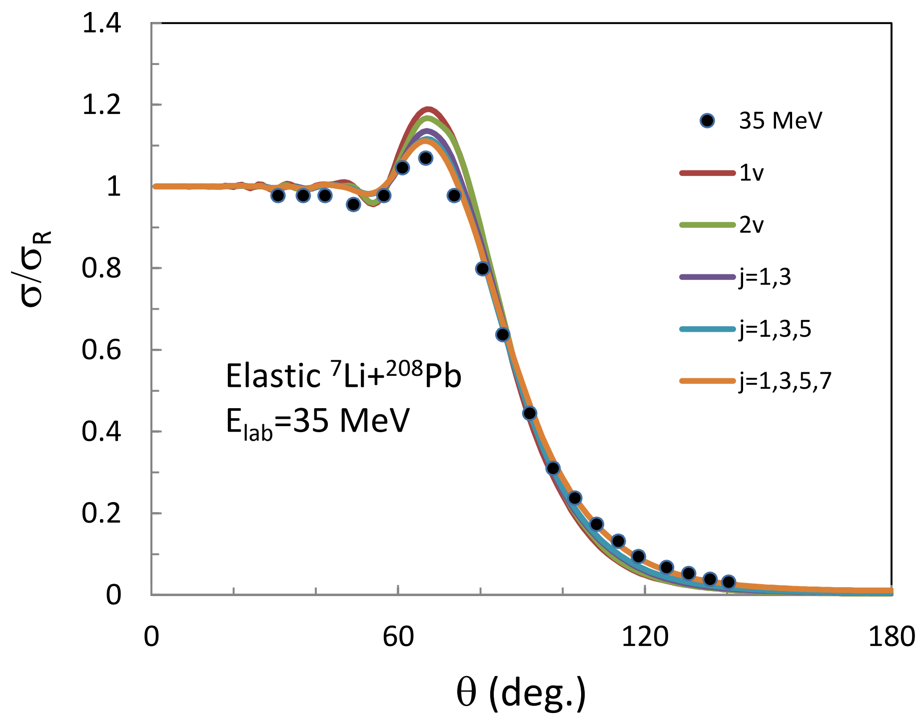


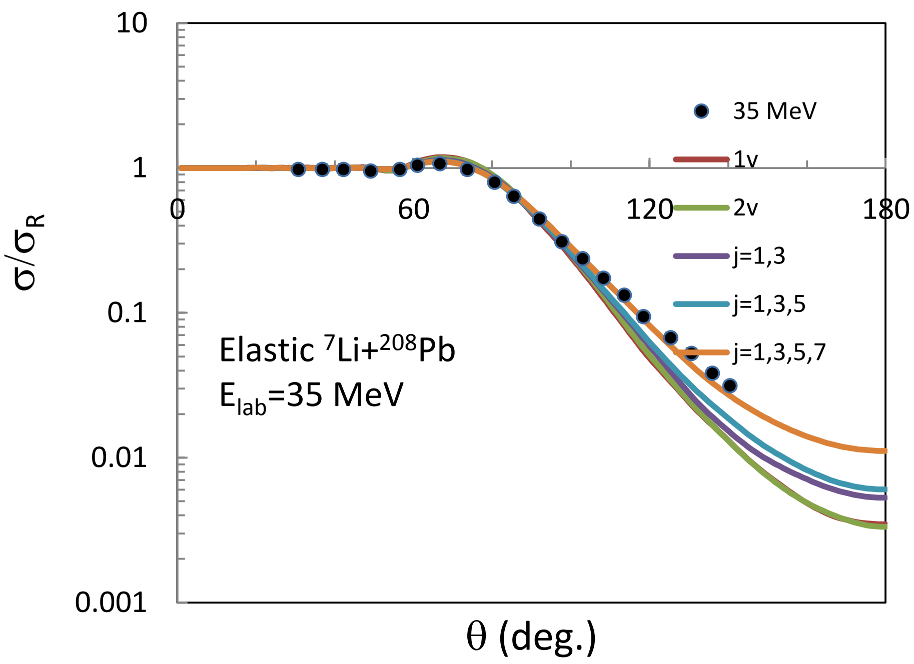
Inelastic scattering



3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

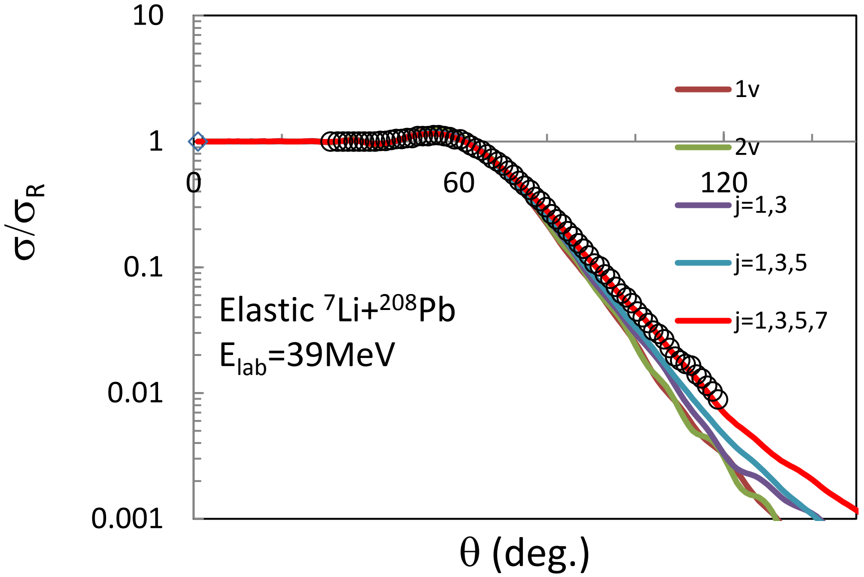
Elab=35 MeV

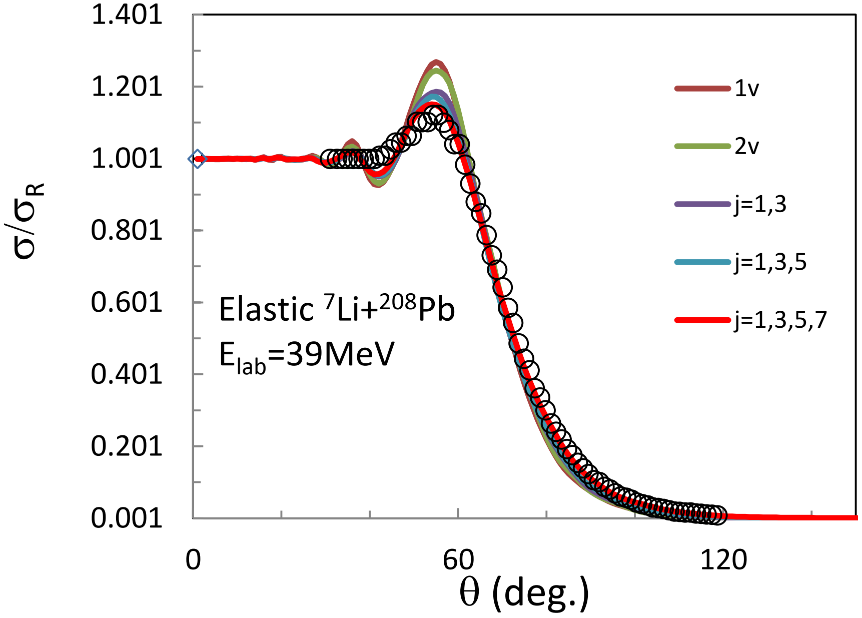




3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

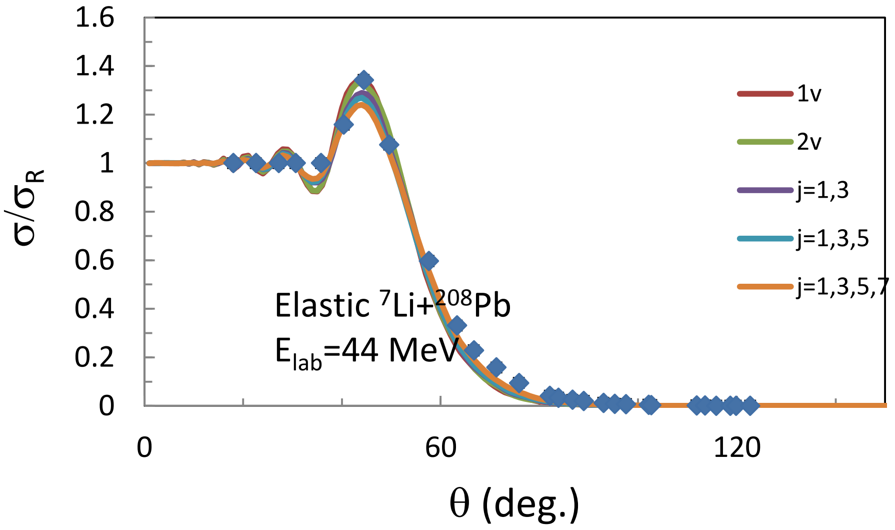
Elab=39 MeV

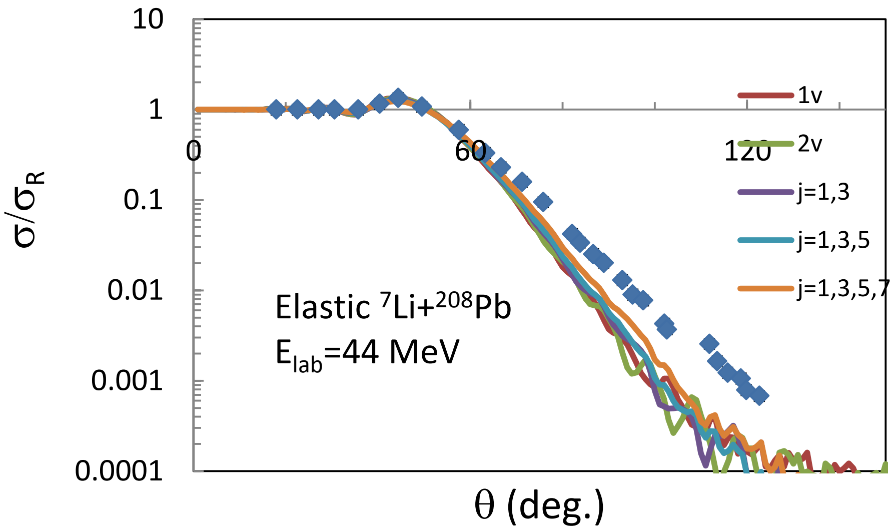




3. CDCC METHOD WITH MICROSCOPIC PROJECTILES: ^{7}LI+^{208}PB

Elab=44 MeV





Underestimation at large angles and high energies.

N. Timofeuyk and R. Johnson

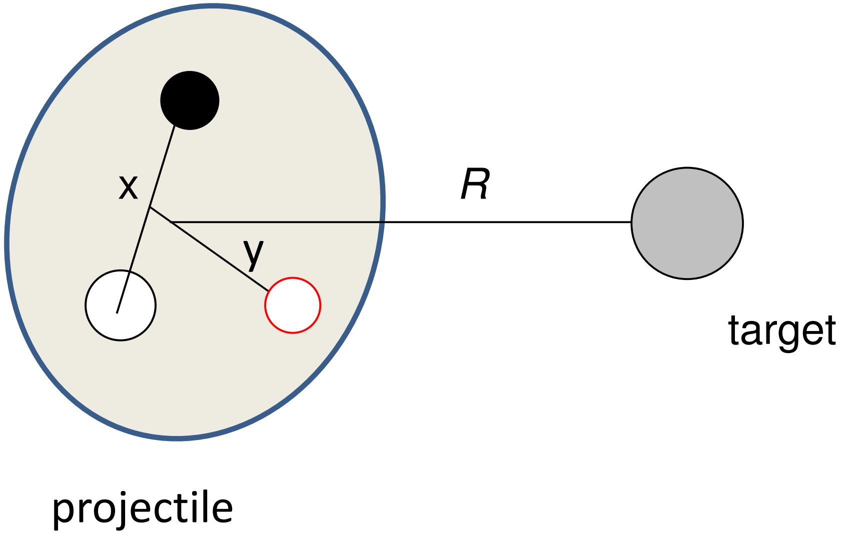
- Phys. Rev. Lett. **110**, 112501
- suggest that the nucleon energy in A(d,p) reaction mush be larger than Ed/2

→ Similar effect here?

Applications of the eikonal method

4. APPLICATION OF THE EIKONAL METHOD

Three-body breakup cross sections: D. Baye et al., Phys. Rev. C79 (2009) 024607



$H\Phi(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y}) = E\Phi(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y})$
with $H = H_0(\boldsymbol{x},\boldsymbol{y}) + T_R + V_{PT}(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y})$

$\boldsymbol{R} = (\boldsymbol{b}, Z)$, b=impact parameter

Eikonal approximation $\Phi(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y}) = e^{iKZ}\widehat{\Phi}\,(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y})$
with $\widehat{\Phi}\,(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y}) \approx \Psi_0(\boldsymbol{x},\boldsymbol{y})\exp[-\frac{i}{\hbar v}\int_{-\infty}^{Z} V_{PT}(\boldsymbol{b},\boldsymbol{Z}',\boldsymbol{x},\boldsymbol{y})dZ']$

Then: eikonal amplitude

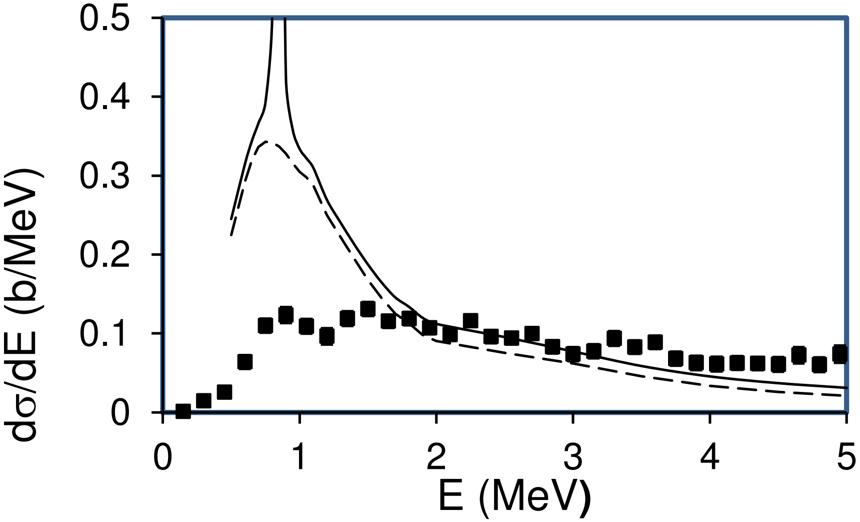
$$T_{fi} =< e^{i\boldsymbol{K}'.\boldsymbol{R}}\,\Psi^-(\boldsymbol{x},\boldsymbol{y}) \mid V_{PT} \mid \widehat{\Phi}\,(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y}) >$$

3-body scattering wave function
(expanded in $J\pi$)→ **heavy calculations**

From the eikonal amplitude → cross sections (breakup, elastic)

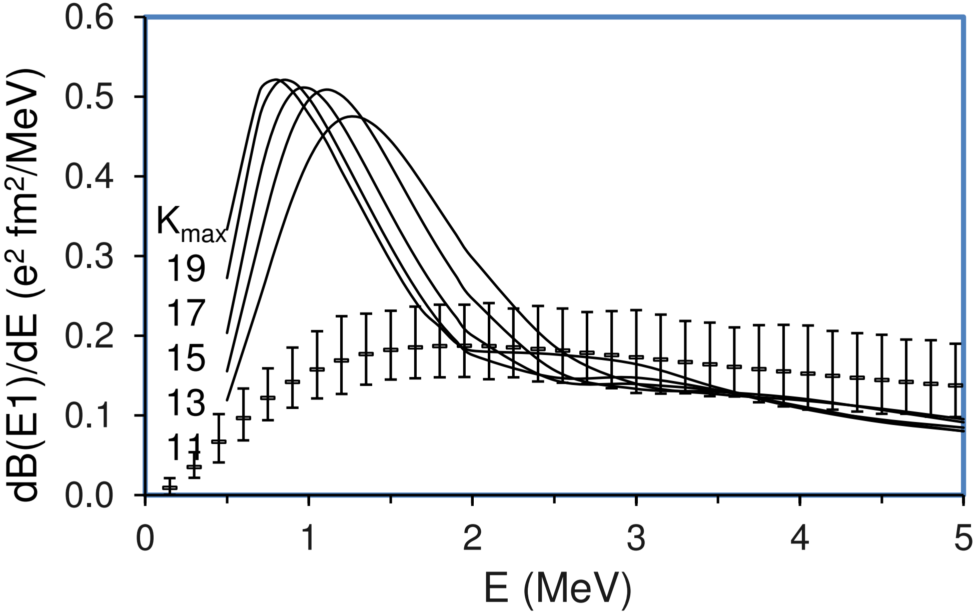
4. APPLICATION OF THE EIKONAL METHOD

Results for ^{6}He: D. Baye et al., Phys. Rev. C79 (2009) 024607



^{6}He+^{208}Pb breakup at 240 MeV/A
Data from T. Aumann et al., PRC59 (1999) 1252

E1 dominant (2$^+$ narrow resonance near 0.9 MeV)



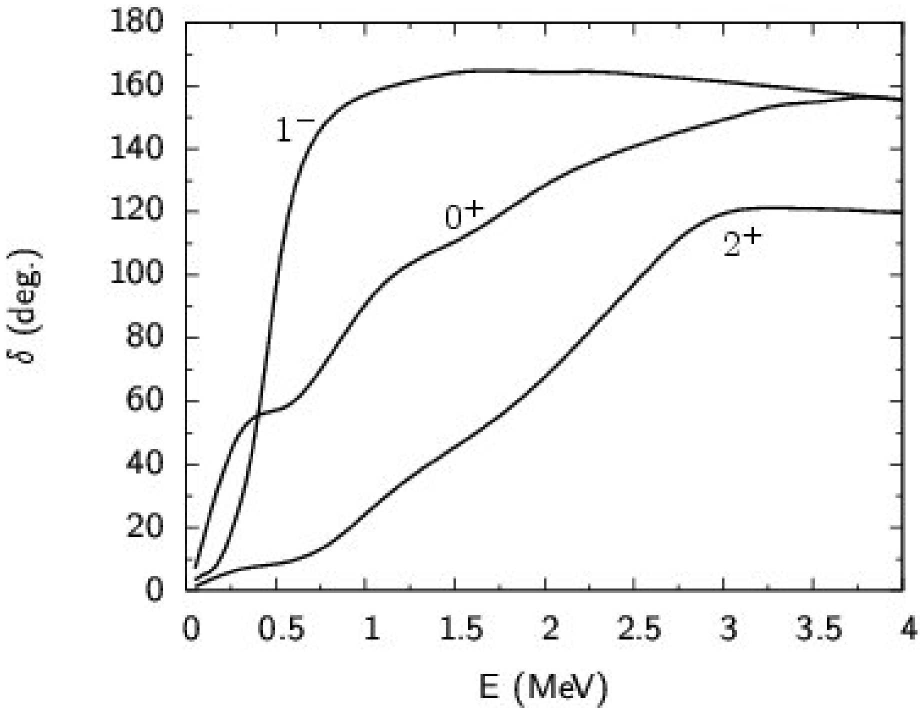
Convergence with Kmax:
Slow!

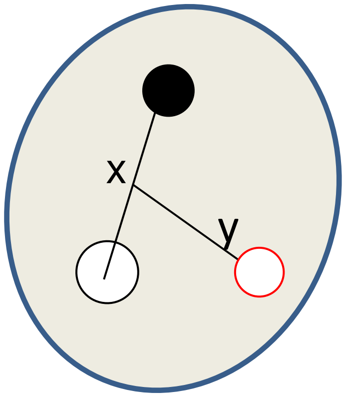
4. APPLICATIONS OF THE EIKONAL METHOD

Recent work on ^{11}Li

- Ref: E.C.Pinilla, P.D., D. Baye, *Phys. Rev. C 85 (2012) 054610*
- ^{11}Li described by a ^{9}Li+n+n structure (spin of ^{9}Li is neglected)

First step: three-body phase shifts and wave functions



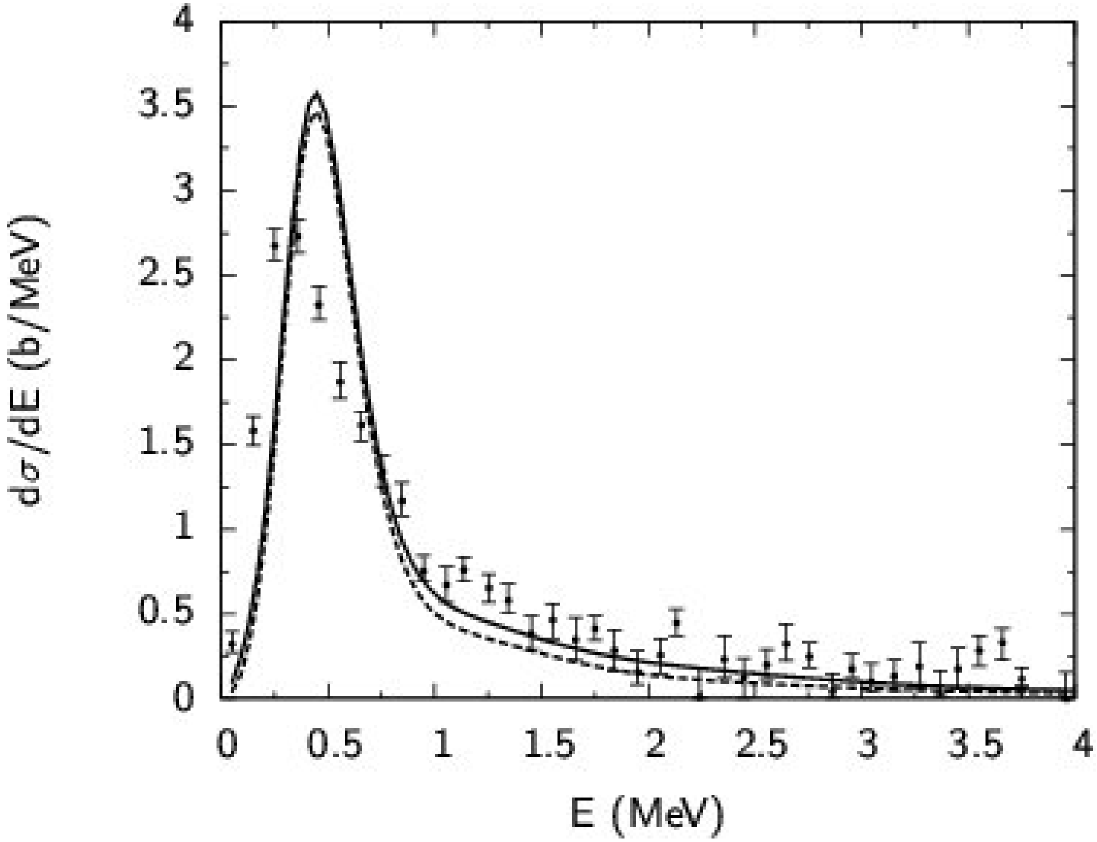


^{11}Li

Narrow 1- resonance near 0.5 MeV

Second step : Breakup of ^{11}Li on ^{208}Pb @ 70 MeV/A

- E.C.Pinilla, P.D., D. Baye, *Phys. Rev. C 85 (2012) 054610*
- Exp. data from T. Nakamura et. al, Phys. Rev. Lett. 252502 (2006).



- Direct calculation of the BU cross section (no E1 distribution is needed)
- Heavy calculation
- E1 contribution strongly dominant

Microscopic description of the projectile using the eikonal method

E. C. Pinilla and P. Descouvemont, Phys. Lett. B 686 , 124 (2010)

Total Hamiltonian $H = H_0(\boldsymbol{r}_i) + T_R + \underbrace{\sum_i v(\boldsymbol{r_i} - \boldsymbol{R})}_{\textbf{Nucleon-target interaction}}$

Hamiltonian of the projectile

$H_0\Phi_0(\boldsymbol{r}_i) = E_0\Phi_0(\boldsymbol{r}_i)$

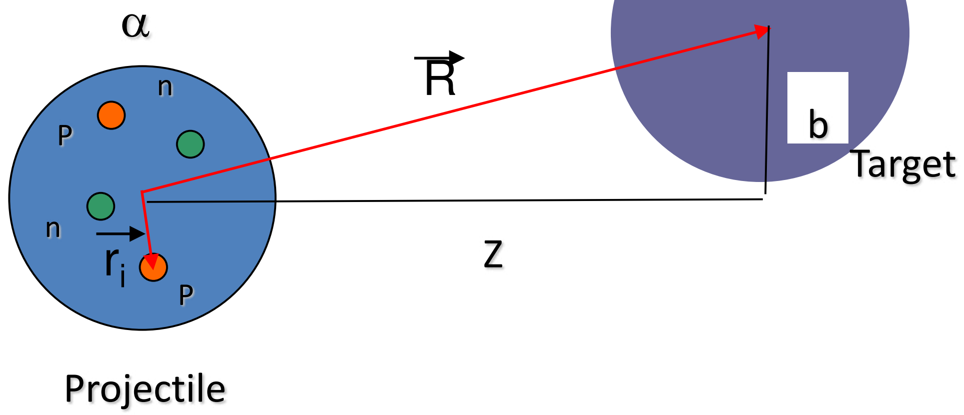
Eikonal approximation

$\Psi = \Phi_0(\boldsymbol{r}_i)e^{ikZ}\widehat{\Phi}(\boldsymbol{r}_i, \boldsymbol{b}, Z)$

Then

$$\widehat{\Phi}(\boldsymbol{r}_i, \boldsymbol{b}, Z) = \exp\left(-\frac{i}{\hbar}\sum_i \int_{-\infty}^{Z} v(\boldsymbol{r}_i - \boldsymbol{R})dZ'\right)$$

(symmetric→ Ψ remains antisymmetric)



General definition of the scattering amplitude

$$f(\theta) = ik\int_0^\infty db\, b\, J_0(qb)\, e^{i\chi_C(b)}[1 - \underbrace{< \Phi_0|\exp(-\frac{i}{\hbar}\sum_i \int_{-\infty}^{\infty} v(\boldsymbol{r}_i - \boldsymbol{R})dZ\ )|\Phi_0 >}_{S(b)}] \quad + f_C(\theta)$$

α particle: simple wave function (angular momentum 0+, all orbitals $\varphi_0(r_i)$ centred at the origin)

$$S(b) = \prod_{i=1}^{4} S_j(b) \qquad S_i(b) = < \varphi_0|\exp(-\frac{i}{\hbar v}\int_{-\infty}^{\infty} dZ V_{\mathrm{iT}}^{\mathrm{N}}(b, Z, \mathbf{r_i})|\varphi_0 >$$

With

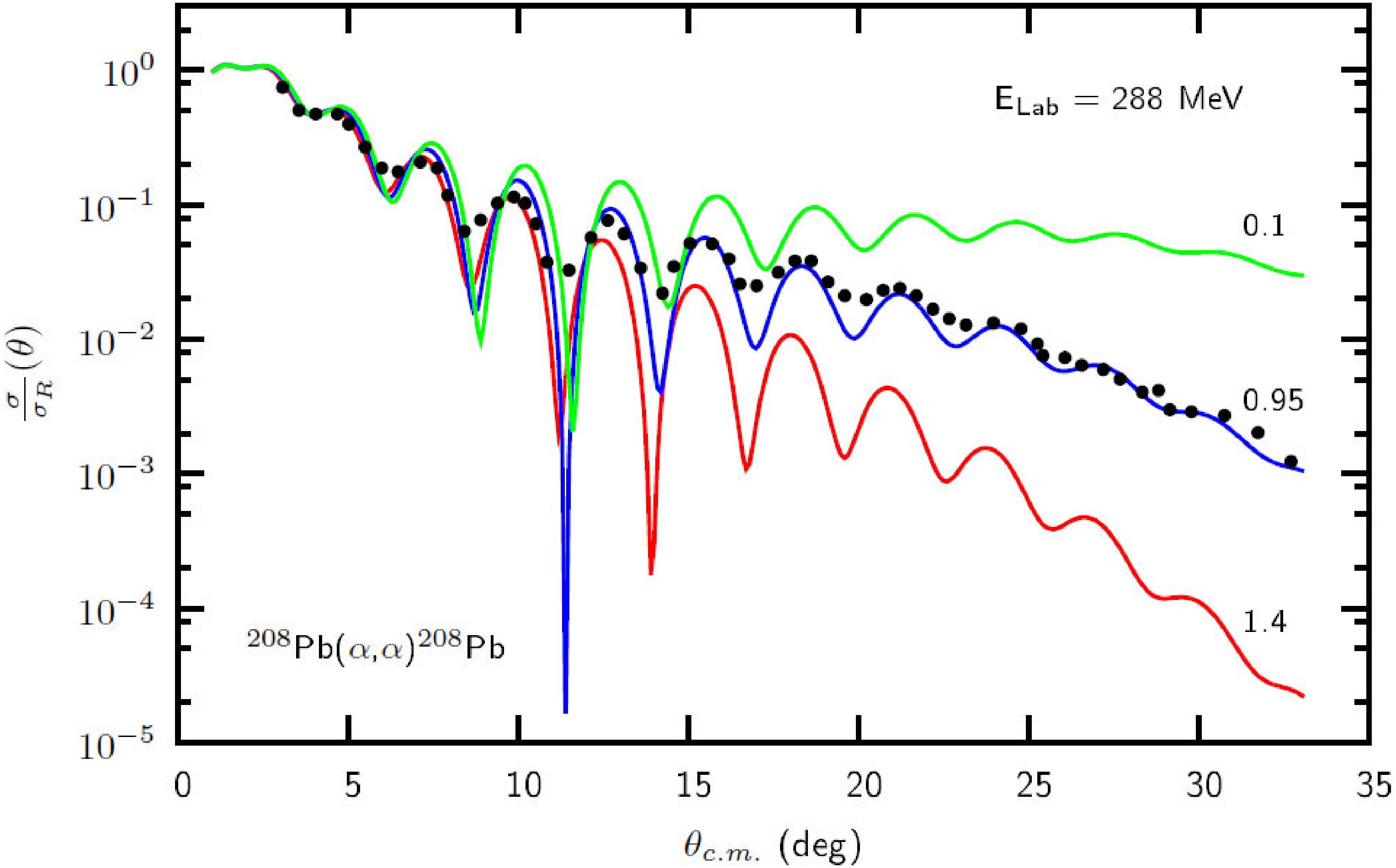
$$S_i(b) = (B^3\pi^{3/2})^{-1}\int_{-\infty}^{\infty} dx_i \int_{-\infty}^{\infty} dy_i \int_{-\infty}^{\infty} dz_i\, e^{-\frac{1}{B^2}(x_i^2+y_i^2+z_i^2)} e^{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} dZ V_{\mathrm{iT}}^{\mathrm{N}}(b,Z,\mathbf{r_i})}$$

Cluster wave functions: angular momentum projection of Φ_0 is necessary → multiple integrals

4. APPLICATIONS OF THE EIKONAL METHOD: microscopic eikonal

Elastic cross section of $\alpha + {}^{208}$Pb at 288 MeV for different oscillator parameters B

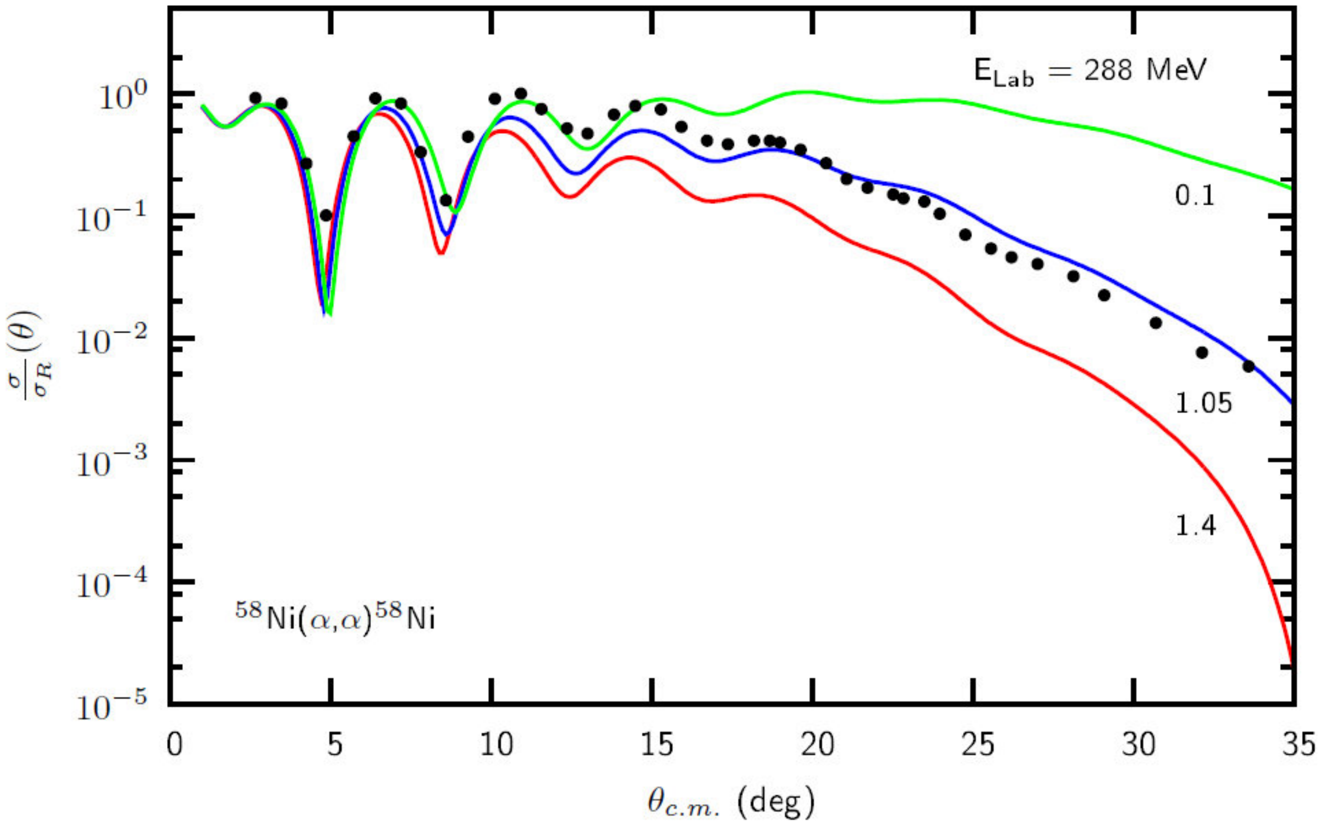
Nucleon-nucleus optical potentials: Koning and Delaroche, Nucl. Phys. A 713, 231 (2003).



- Strong dependence on the oscillator parameter.
- The form for *B* = 0.1 fm is far from the experimental data.
- A good *B* is 0.95 fm.

4. APPLICATIONS OF THE EIKONAL METHOD: microscopic eikonal

Elastic cross section of α+^{58}Ni at 288 MeV for different oscillator parameters B



4. APPLICATIONS OF THE EIKONAL METHOD: microscopic eikonal

Future projects:

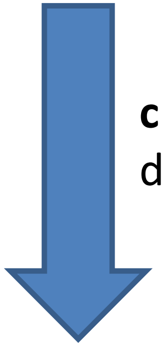
- cluster calculations $\Phi_0 = \mathcal{A}\Phi_1\Phi_2 g(\rho)$
- Must be projected on angular momentum
- $S(b) =< \Phi_0|\exp(-\frac{i}{\hbar}\sum_i \int_{-\infty}^{\infty} v(\boldsymbol{r}_i - \boldsymbol{R})dZ\ )|\Phi_0 >$ is more complicated

Nuclear astrophysics: brief overview

Nuclear astrophysics: brief overview

Types of reactions: general definitions valid for all models

Type	Example	Origin
Transfer	${}^{3}\mathrm{He}({}^{3}\mathrm{He},2p)\alpha$	Strong
Radiative capture	${}^{2}\mathrm{H}(p,\gamma){}^{3}\mathrm{He}$	Electromagnetic
Weak capture	$p+p \rightarrow d + e^{+} + \nu$	Weak



cross section decreases

Nuclear astrophysics: brief overview

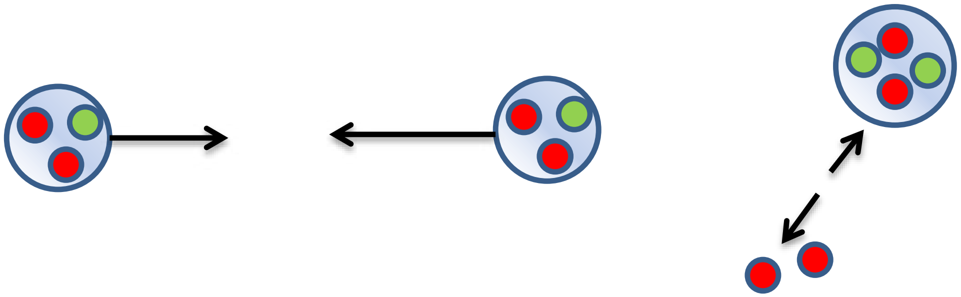
- Transfer: A+B → C+D (σ_t, strong interaction, example: ^{3}He(d,p)^{4}He)

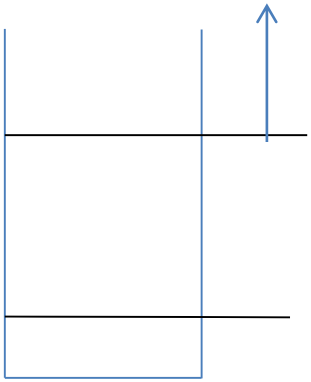
$$\sigma_{t,c\to c'}(E)=\frac{\pi}{k^2}\sum_{J\pi}\frac{2J+1}{(2I_1+1)(2I_2+1)}\left|U_{cc'}^{J\pi}(E)\right|^2$$

$U_{cc'}^{J\pi}(E)$ =collision matrix (obtained from scattering theory → various models)

c, c' =entrance and exit channels

Transfer reaction:
Nucleons are transfered





Scattering energy E

A+B threshold, ex: ^{3}He+d

C+D threshold, ex: ^{4}He+p

Compound nucleus, ex: ^{5}Li

Nuclear astrophysics: brief overview

- Radiative capture : A+B → C+γ (σ_C, electromagnetic interaction, example: ^{12}C(p,γ)^{13}N)

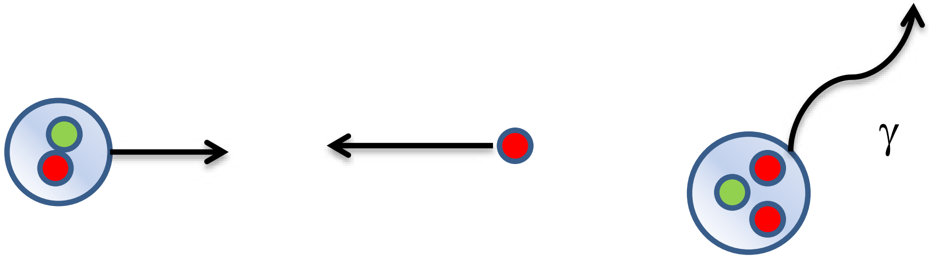
$$\sigma_C^{J_f\pi_f}(E) \sim \sum_{\lambda}\sum_{J_i\pi_i} k_\gamma^{2\lambda+1} \left|< \Psi^{J_f\pi_f} \| \mathcal{M}_\lambda \| \Psi^{J_i\pi_i}(E)\right|^2$$

$J_f\pi_f$=final state of the compound nucleus C

$\Psi^{J_i\pi_i}(E)$=initial scattering state of the system (A+B)

$\mathcal{M}_{\lambda\mu}$=electromagnetic operator (electric or magnetic): $\mathcal{M}_{\lambda\mu} \sim e\, r^\lambda Y_\lambda^\mu(\Omega_r)$

Capture reaction:
A photon is emitted



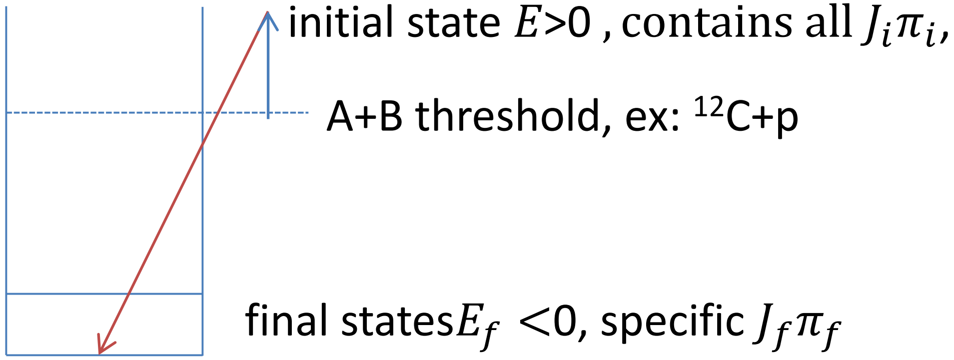
Long wavelength approximation:

Wave number $k_\gamma = E_\gamma/\hbar c$, wavelength: $\lambda_\gamma = 2\pi/k_\gamma$

Typical value: $E_\gamma = 1\ MeV, \lambda_\gamma \approx 1200$ fm >> typical dimensions of the system (R)

→ $k_\gamma R \ll 1$= Long wavelength approximation

Nuclear astrophysics: brief overview



initial state E>0 , contains all $J_i\pi_i$,

A+B threshold, ex: ^{12}C+p

final statesE_f <0, specific $J_f\pi_f$

$$\sigma_C^{J_f\pi_f}(E) \sim \sum_{J_i\pi_i}\sum_{\lambda} k_\gamma^{2\lambda+1}\left|< \Psi^{J_f\pi_f}\|\mathcal{M}_\lambda\|\Psi^{J_i\pi_i}(E) >\right|^2$$

- $k_\gamma = (E - E_f)/\hbar c$ = photon wave number
- In practice
    - Summation over λ limited to 1 term (often E1, or E2/M1 if E1 is forbidden)

      $$\frac{E2}{E1} \sim (k_\gamma R) \ll 1$$ (from the long wavelength approximation)
    - Summation over $J_i\pi_i$ limited by selection rules

      $$|J_i - J_f| \leq \lambda \leq J_i + J_f$$

      $\pi_i\pi_f = (-1)^\lambda$ for electric, $\pi_i\pi_f = (-1)^{\lambda+1}$ for magnetic

Nuclear astrophysics: brief overview

- **Weak capture** (p+p→ d+ν+e$^-$): tiny cross section → no measurement (only calc.)

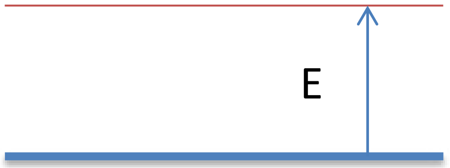
$$\sigma_W^{J_f\pi_f}(E) \sim \sum_{J_i\pi_i} \left| < \Psi^{J_f\pi_f} \| O_\beta \| \Psi^{J_i\pi_i}(E) > \right|^2$$

  - Calculations similar to radiative capture
  - O_β= Fermi ($\sum_i t_{i\pm}$) and Gamow-Teller ($\sum_i t_{i\pm}\sigma_i$) operators

- **Fusion**: similar to transfer, but with many output channels
  → statistical treatment
  → optical potentials

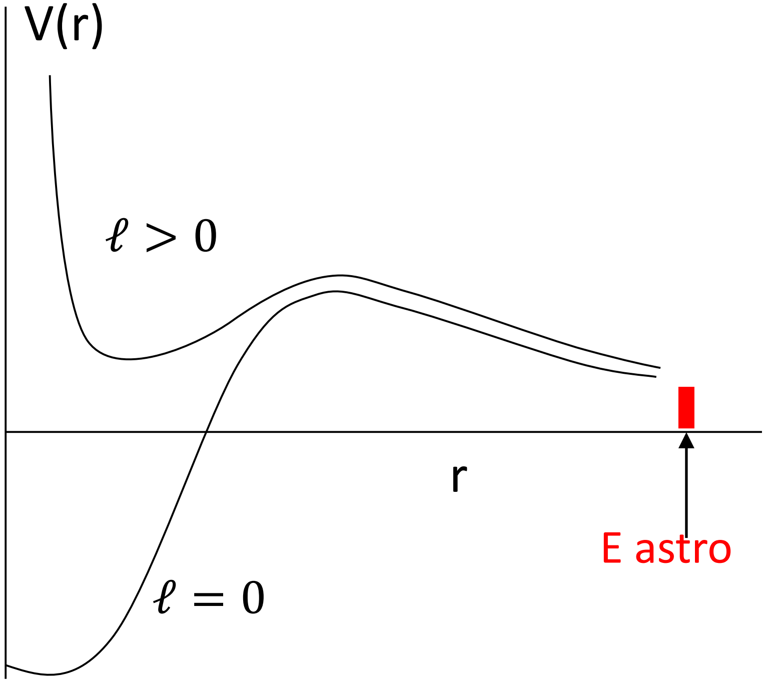
Nuclear astrophysics: brief overview

General properties



Scattering energy E: wave function $\Psi_i(E)$
common to all processes

Reaction threshold



- Cross sections dominated by Coulomb effects
  Sommerfeld parameter $\eta = Z_1 Z_2 e^2/\hbar v$

- Coulomb functions at low energies
  $$F_\ell(\eta,x) \rightarrow \exp(-\pi\eta)\,\mathcal{F}_\ell(x),$$
  $$G_\ell(\eta,x) \rightarrow \exp(\pi\eta)\,\mathcal{G}_\ell(x),$$

- Coulomb effect: strong E dependence : $\exp(2\pi\eta)$
  neutrons: $\sigma(E) \sim 1/v$

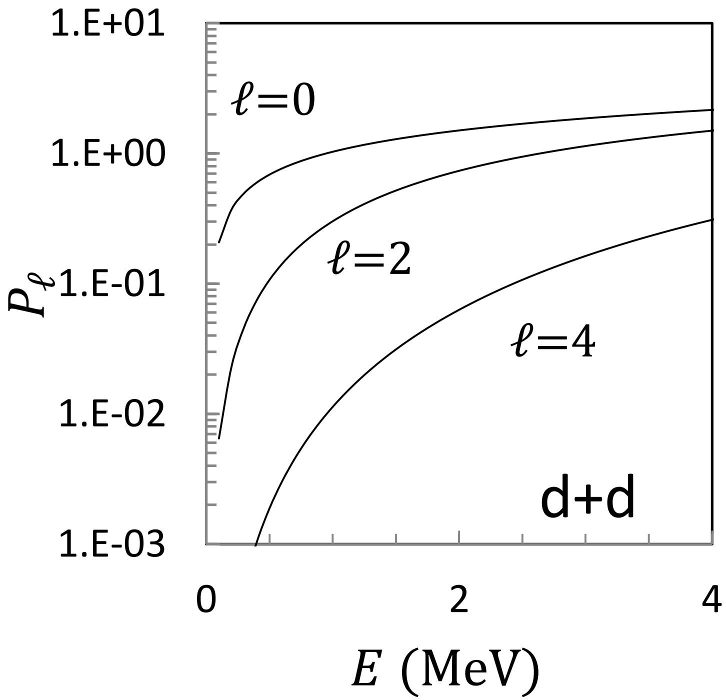
- Strong ℓ dependence
  Centrifugal term: $\sim \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}$
  → stronger for nucleons ($\mu \approx 1$) than for α ($\mu \approx 4$)

Nuclear astrophysics: brief overview

General properties: specificities of the entrance channel → common to all reactions

- All cross sections (capture, transfer) involve a summation over ℓ: $\sigma(E) = \sum_\ell \sigma_\ell(E)$
- The partial cross sections $\sigma_\ell(E)$ are proportional to the penetration factor

$$P_\ell(E) = \frac{ka}{F_\ell(ka)^2 + G_\ell(ka)^2}$$ (a =typical radius)



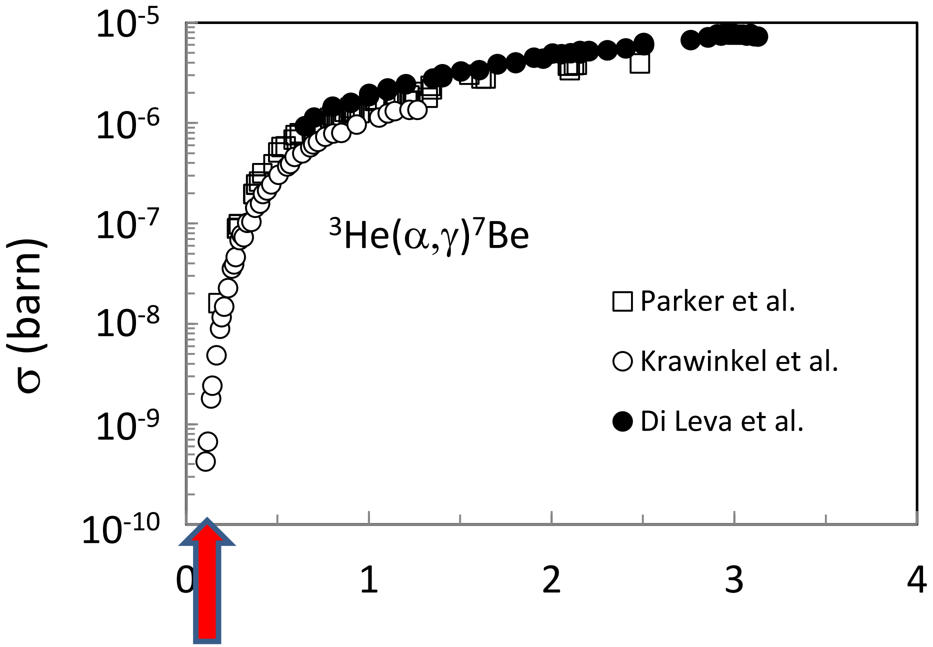
Consequences
- $\ell > 0$ are often negligible at low energies
- $\ell = \ell_{min}$ is dominant (often $\ell_{min} = 0$)
- For $\ell = 0$, $P_0(E) \sim \exp(-2\pi\eta)$

Astrophysical S factor: $S(E) = \sigma(E)E\exp(2\pi\eta)$ (Units: E*L^2: MeV-barn)

- removes the coulomb dependence → only nuclear effects
- weakly depends on energy →$\sigma(E) \approx S_0 \exp(-2\pi\eta)/E$ (any reaction at low E)

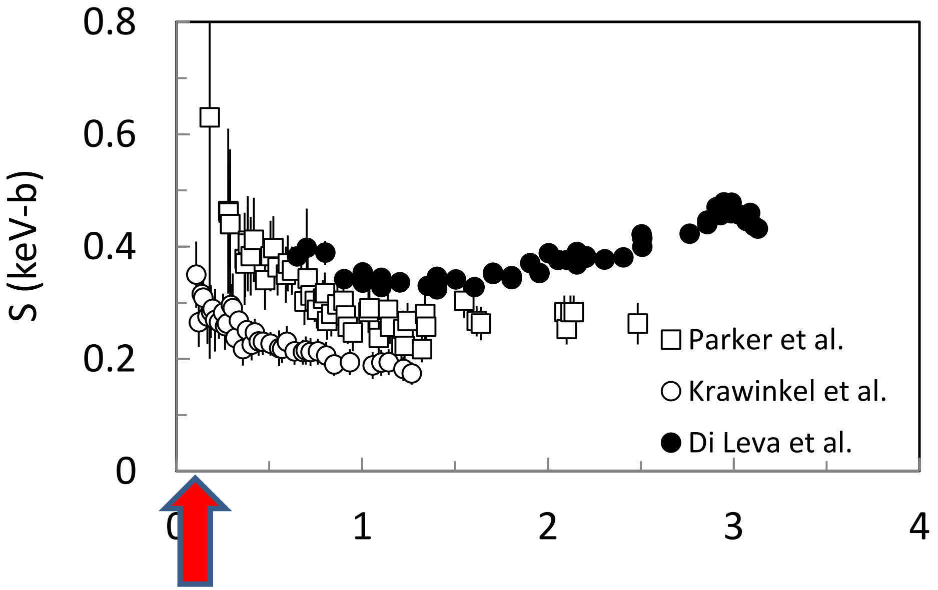
Nuclear astrophysics: brief overview

non resonant:$S(E) = \sigma(E)E\exp(2\pi\eta)$



Example: $^{3}He(\alpha,\gamma)^{7}Be$ reaction

- Cross section σ(E) Strongly depends on energy
- Logarithmic scale



S factor

- Coulomb effects removed
- Weak energy dependence
- Linear scale

Nuclear astrophysics: brief overview

Resonant cross sections: Breit-Wigner form

$$\sigma_R(E) \approx \frac{\pi}{k^2}\frac{(2J_R+1)}{(2I_1+1)(2I_2+1)}\frac{\Gamma_1(E)\Gamma_2(E)}{(E_R-E)^2+\Gamma^2/4}$$

- J_R, E_R=spin, energy of the resonance
- Valid for any process (capture, transfer)
- Valid for a single resonance → several resonances need to be added (if necessary)

- Γ_1=Partial width in the entrance channel (strongly depends on E, ℓ)
  $\Gamma_1(E) = 2\gamma_1^2 P_\ell(E)$ with γ_1^2=reduced width (does not depend on E)
  $P_\ell(E) \sim \exp(-2\pi\eta)$
  A resonance at low energies is always narrow (role of $P_\ell(E)$)

- Γ_2=Partial width in the exit channel (weakly depends on E, ℓ)
  - Transfer: $\Gamma_2(E) = 2\gamma_2^2 P_{\ell_f}(E+Q)$ (in general $Q \gg E$ → $P_{\ell_f}(E+Q)$ almost constant)
  - Capture: $\Gamma_2(E) \sim \left(E-E_f\right)^{2\lambda+1} B(E\lambda)$ → weak energy dependence

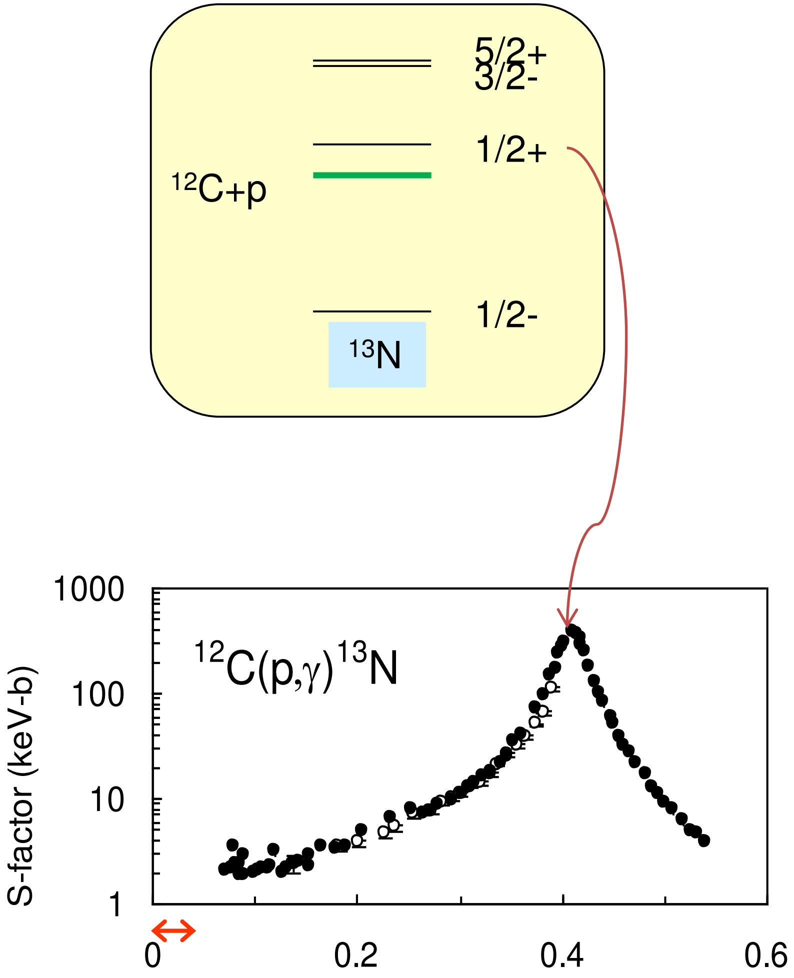
- S factor near a resonance $S(E) = \sigma(E)E\exp(2\pi\eta)$

$$S_R(E) \sim \frac{\gamma_1^2\Gamma_2}{(E_R-E)^2+\Gamma^2/4}\boxed{P_\ell(E)\exp(2\pi\eta)}$$

Almost constant

→ Simple estimate at low E (at the Breit-Wigner approximation)

Nuclear astrophysics: brief overview



$$S_R(E) \sim \frac{\gamma_1^2 \Gamma_2}{(E_R - E)^2 + \Gamma^2/4} P_\ell(E) \exp(2\pi\eta)$$
$$\sim \frac{\gamma_1^2 \Gamma_2}{(E_R - E)^2 + \Gamma^2/4}$$

- For $\ell = 0$: $P_0(E) \exp(2\pi\eta) \sim$ constant
- For $\ell > 0$, $P_\ell(E) \ll P_0(E)$
  → $\ell > 0$ resonances are suppressed

In $^{12}C(p,\gamma)^{13}N$:

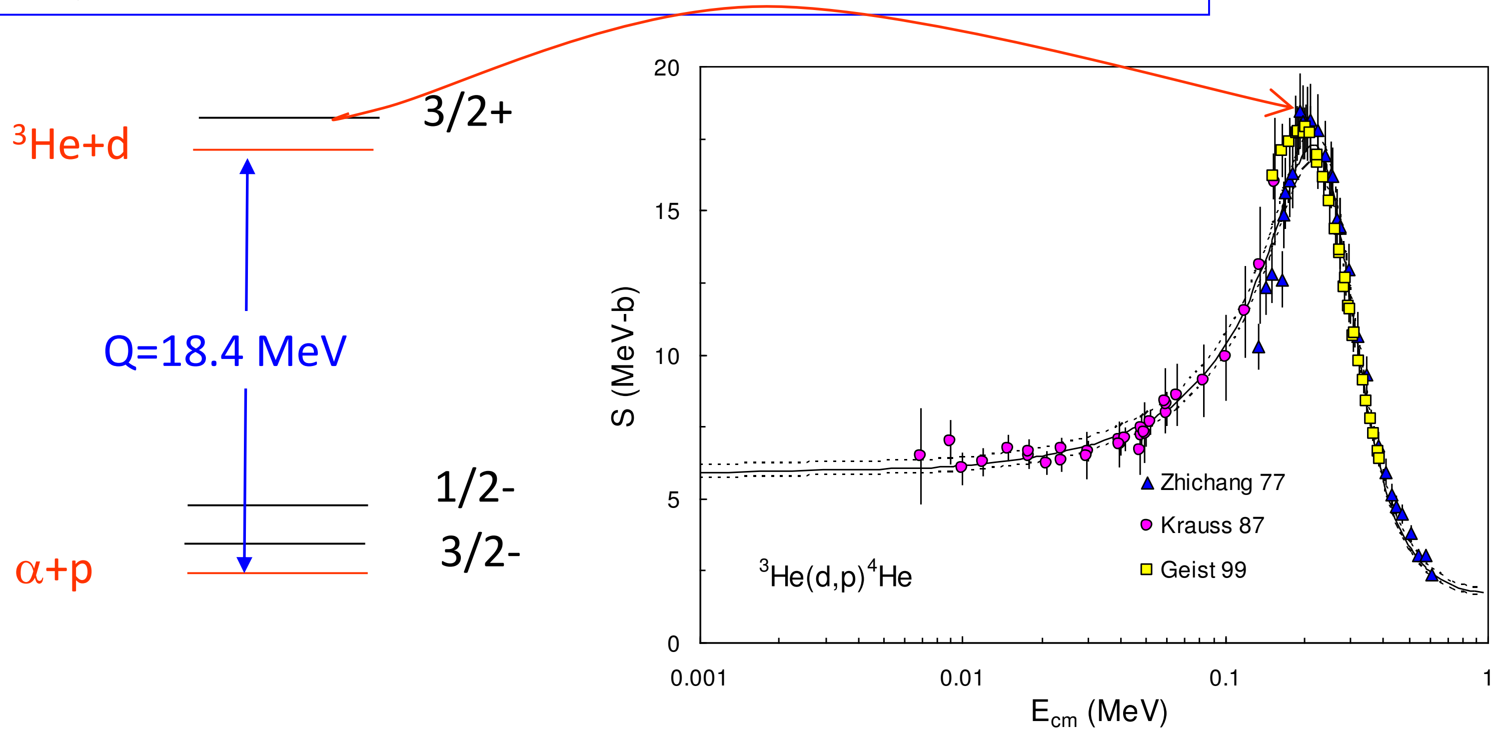
- Resonance $1/2^+$: $\ell = 0$
- Resonances $3/2^-$, $5/2^+$ $\ell = 1,2$ → negligible

Note: BW is an approximation

- Neglects background, external capture
- Assumes an isolated resonance
- Is more accurate near the resonance energy

Nuclear astrophysics: brief overview

^{3}He(d,p)^{4}He: isolated resonance in a transfer reaction



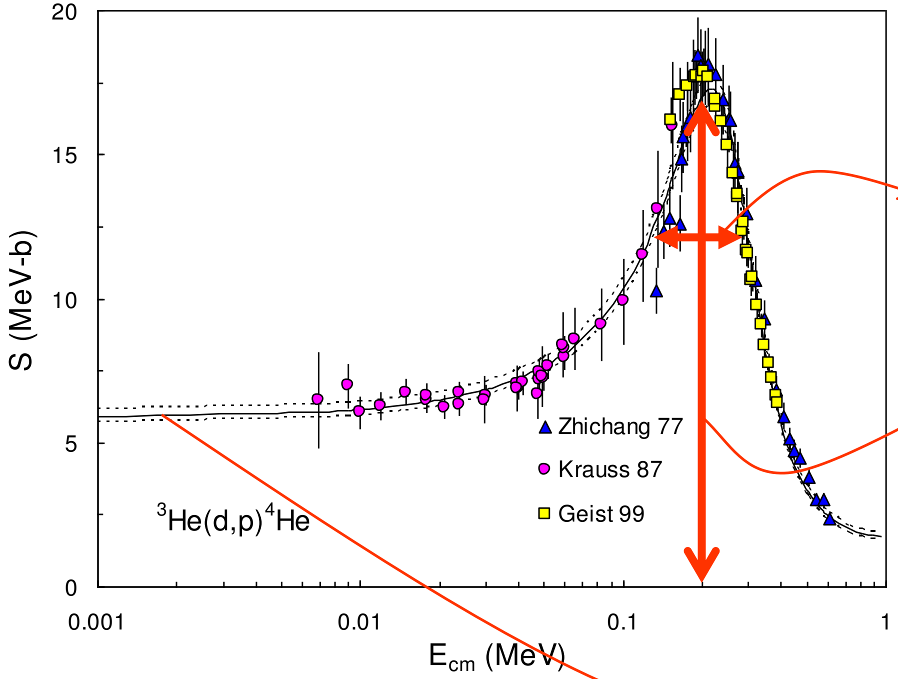
3/2+ resonance:

- Entrance channel: spin S=1/2,3/2, parity + → $\ell = 0{,}2$
- Exit channel: spin S=1/2, parity + → $\ell = 1$

Nuclear astrophysics: brief overview

Breit Wigner approximation

$$\sigma_{dp}(E) \approx \frac{\pi}{k^2} \frac{(2J_R+1)}{(2I_1+1)(2I_2+1)} \frac{\Gamma_d(E)\Gamma_p(E)}{(E_R-E)^2+\Gamma^2/4}$$



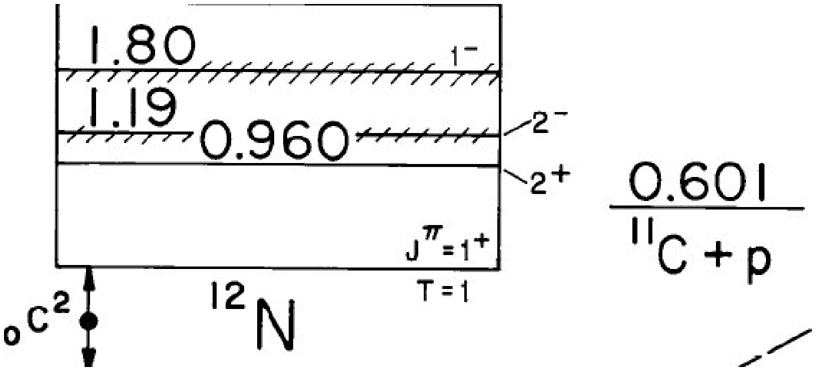
Width at half maximum=total width Γ

Amplitude: $\sim \Gamma_d\ \Gamma_p/\Gamma^2$

S(E) constant for $\ell = 0$

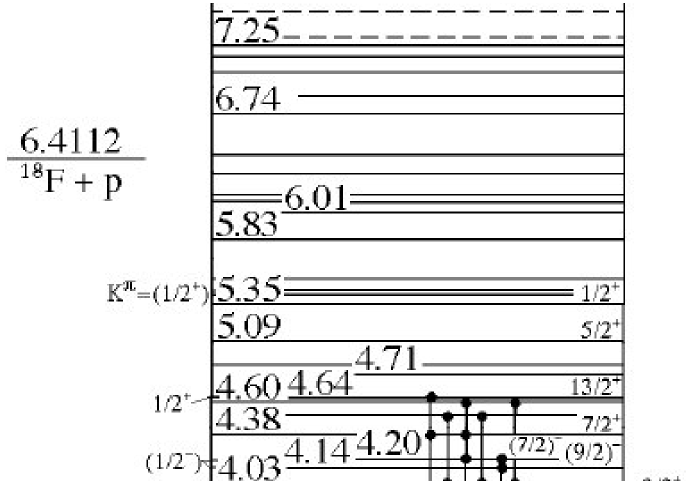
Nuclear astrophysics: brief overview

Selection of the main resonances



^{11}C(p,γ)^{12}N (spin ^{11}C=3/2$^-$)

- Resonance 2$^-$: $\ell = 0$, E1
- Resonance 2$^+$: $\ell = 1$, E2/M1
  → negligible



^{18}F(p,α)^{15}O (spin ^{18}F=1$^+$)

- Many resonances
- Only $\ell = 0$ resonances are important
  → $J = 1/2^+, 3/2^+$ only

→ In general a small number of resonances play a role

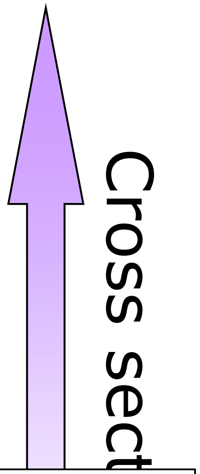
Nuclear astrophysics: brief overview

Many different situations

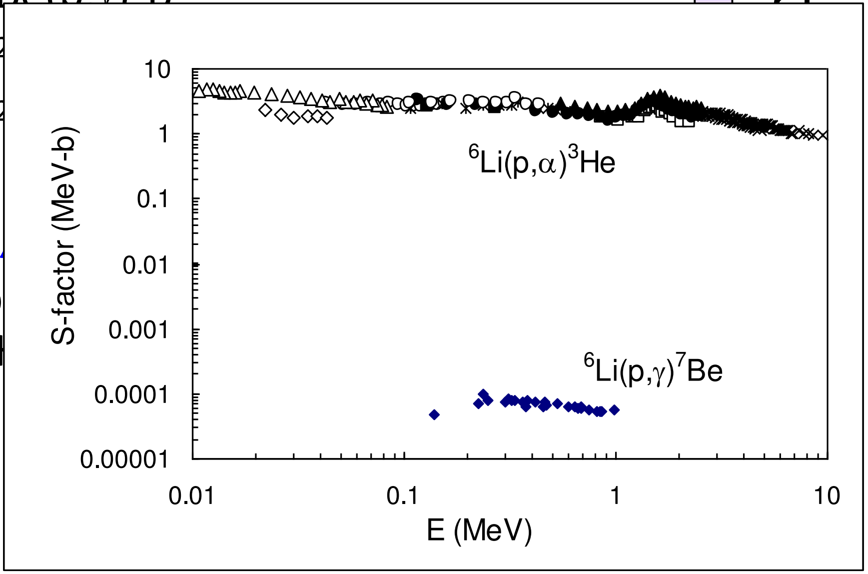
- *Transfer cross sections (**strong interaction**)*
  - Non resonant: ^{6}Li(p,α)^{3}He
  - Resonant, with $\ell_R=\ell_{min}$: ^{3}He(d,p)α
  - Resonant, with $\ell_R>\ell_{min}$: ^{11}B(p,α)^{8}Be
  - Multiresonance: ^{22}Ne(α,n)^{25}Mg

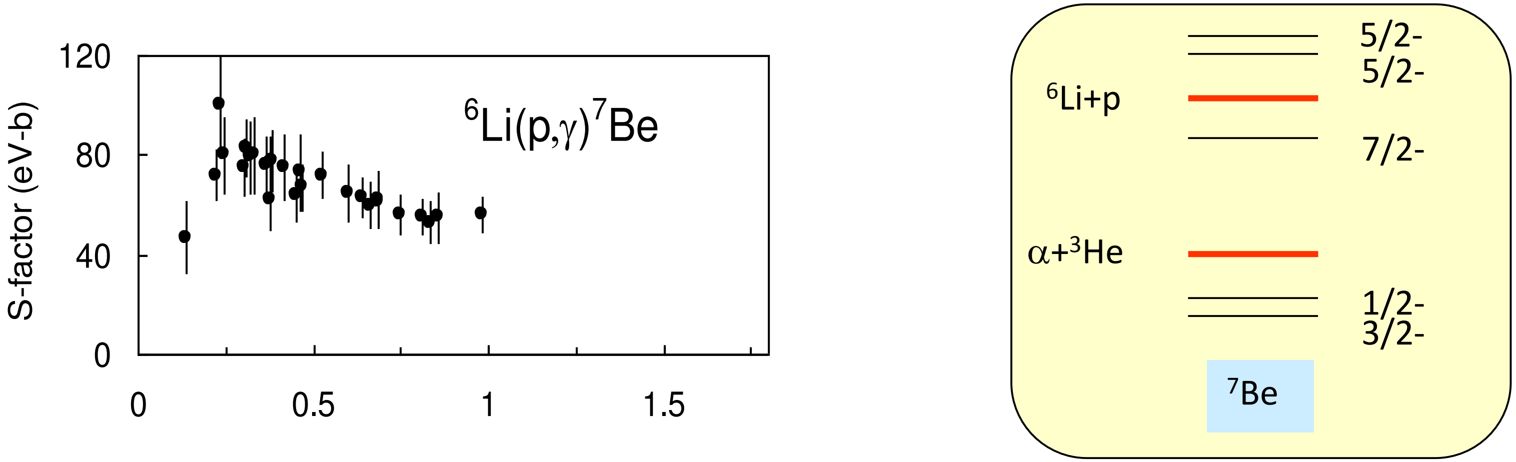
- *Capture cross sections (**electromagnetic interaction**)*
  - Non resonant: ^{6}Li(p,γ)^{7}Be
  - Resonant, with $\ell_R=\ell_{min}$: ^{12}C(p,γ)^{13}N
  - Resonant, with $\ell_R>\ell_{min}$: ^{7}Be(p,γ)^{8}B
  - Multiresonance: 22
  - Subthreshold state: 12

- *Weak capture cross sections (**weak interaction***
  - Non resonant p
    3

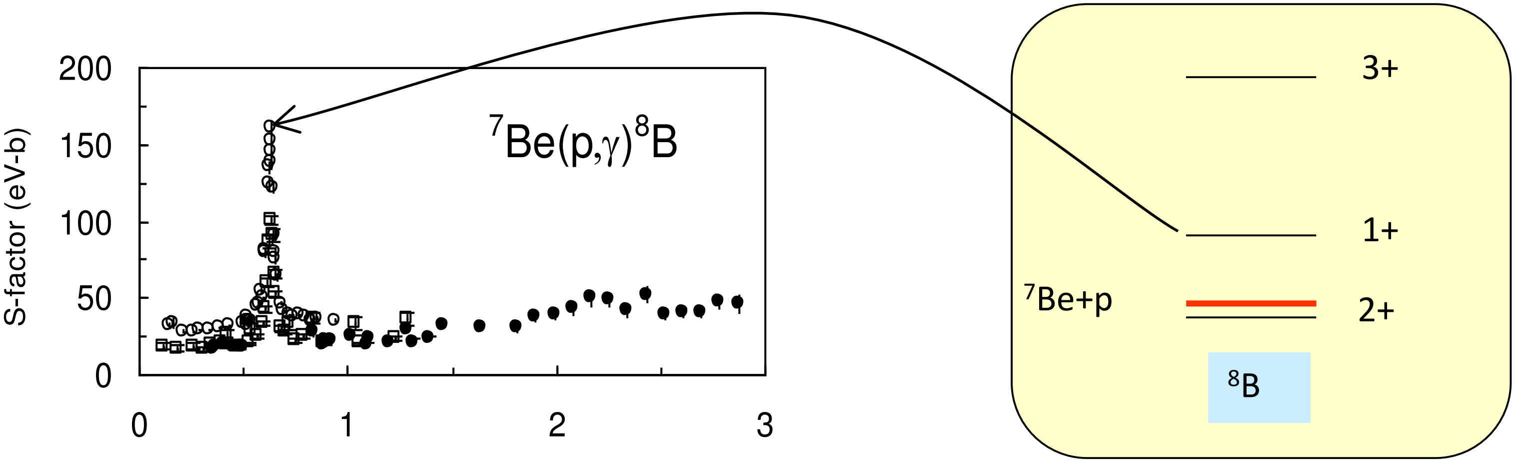


Cross sect





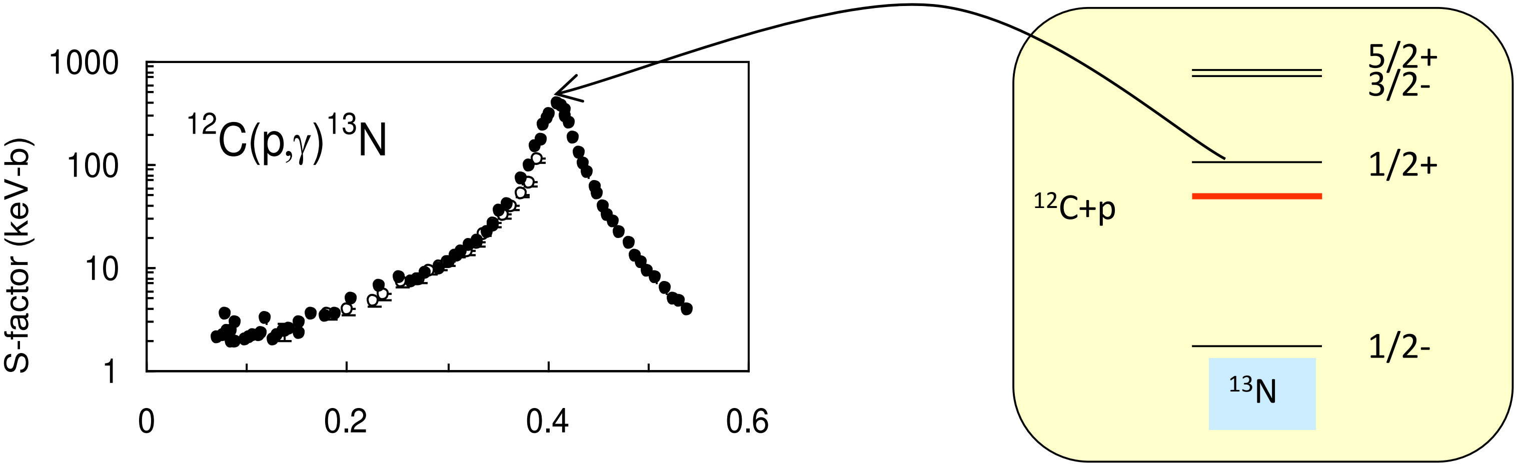
Non resonant



Resonant

ℓ_{min}=0, E1

ℓ_R=1, M1

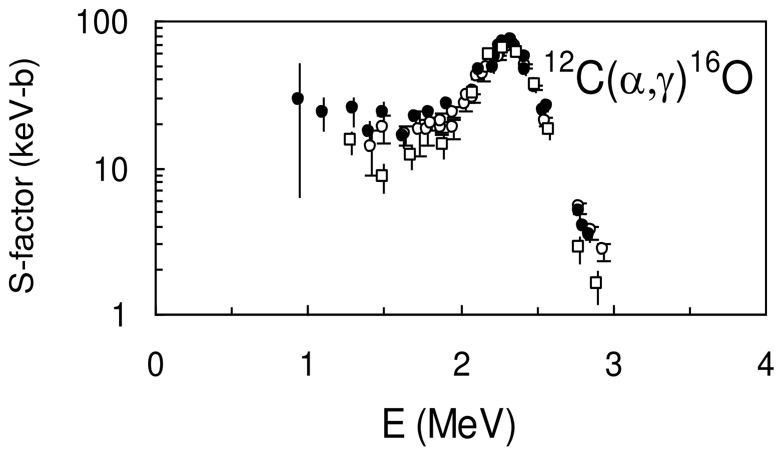


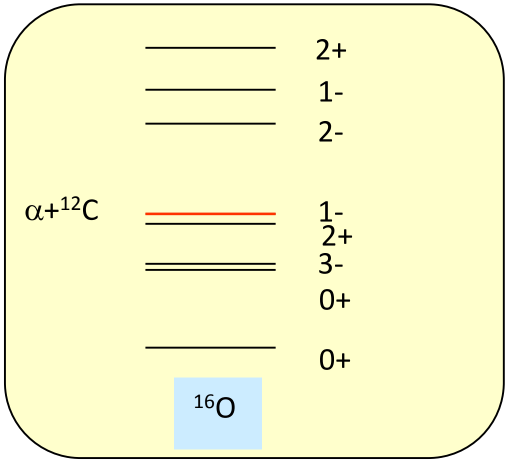
Resonant

ℓ_{min}=0

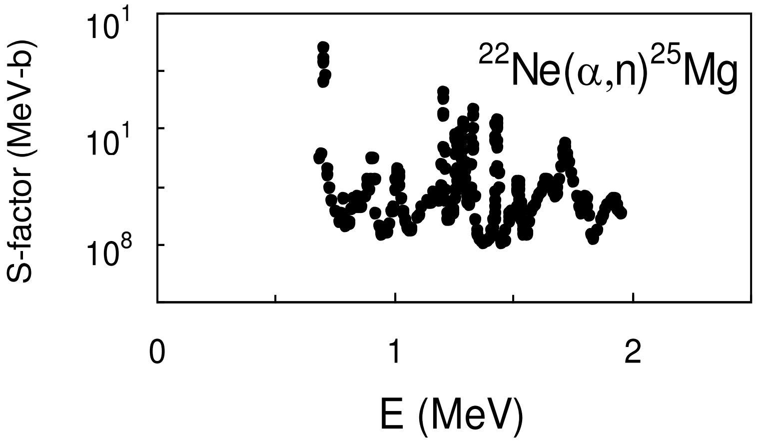
ℓ_R=0

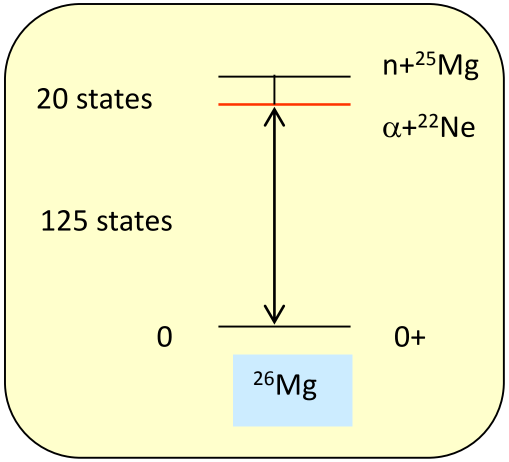
Nuclear astrophysics: brief overview





Subthreshlod states 2^+, 1^-





Multiresonant

General situation for heavy nuclei

Nuclear astrophysics: brief overview

Theoretical methods: Many different cases → no "unique" model!

Model	Applicable to	Comments	
Potential/optical model	Capture Fusion	• Internal structure neglected • Antisymetrization approximated	Light systems Low level densities
R-matrix	Capture Transfer	• No explicit wave functions • Physics simulated by some parameters	
DWBA	Transfer	• Perturbation method • Wave functions in the entrance and exit channels	
Microscopic models	Capture Transfer	• Based on a nucleon-nucleon interaction • *A*-nucleon problems • Predictive power	
Hauser-Feshbach	Capture Transfer	• Statistical model	Heavy systems
Shell model	Capture	• Only gamma widths	

Conclusion

Reactions with exotic nuclei require

- Accurate scattering theory: CDCC / eikonal
- Accurate description of the projectile →microscopic models

Open questions/outlook

- Predictive power?
- Reducing the number of channels in CDCC → stochastic methods?
- Excitations of the target?
- Choice of nucleus-target or nucleon-target interaction?
- Absorption?
- Probably many others!

Nuclear astrophysics

- Many reaction rates are needed
- Many different types of reactions!

→ No systematics!