

Primordial Nucleosynthesis

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1. Standard Big-Bang Model and Nucleosynthesis
2. Nuclear Physics aspects
3. Beyond the Standard Model(s)

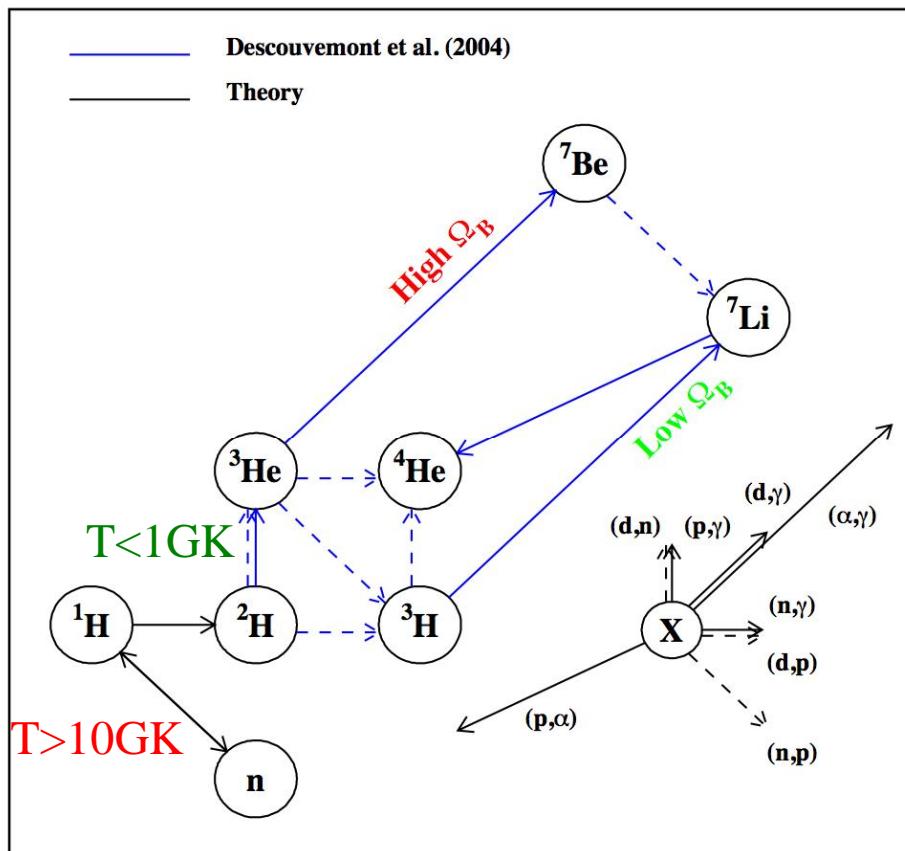


Nuclear Physics aspects

1. The main reactions for H, He and Li BBN production
2. Search for nuclear solutions to the lithium problem
3. Be, B and CNO production in BBN
4. Monte-Carlo evaluation of reaction rates
5. Monte-Carlo investigation of BBN

The 12 reactions of standard BBN

- 10 thermonuclear reaction rates deduced from experimental data:
[Serpico+ 2004, Descouvemont+2004, Cyburt 2004] evaluations
- First 2 from theory: weak $n \leftrightarrow p$ rate and $p(n,\gamma)d$ (i.e. $p+n \rightarrow d+\gamma$)



Standard BBN

- No convection
- No diffusion
- No mixing
- No screening
- Known physics
- 11/12 reactions measured at BBN energies



Simple problem (?)

Sensitivity to thermonuclear reaction rates

$$\frac{\Delta Y}{Y} = \left. \frac{\partial \ln(Y)}{\partial \ln(N_A \langle \sigma v \rangle)} \right|_{\eta=\eta_{WMAP}} \times \frac{\Delta N_A \langle \sigma v \rangle}{N_A \langle \sigma v \rangle}$$

At WMAP baryonic density

| Reaction | $\partial \ln(Y) / \partial \ln(N_A \langle \sigma v \rangle)$ | | | | $E_0(\Delta E_0/2)$ (MeV @ 1GK) |
|---|--|-------|-----------------|-----------------|------------------------------------|
| | ${}^4\text{He}$ | D | ${}^3\text{He}$ | ${}^7\text{Li}$ | |
| $\tau_n(n \leftrightarrow p)$ | 0.73 | 0.42 | 0.15 | 0.40 | |
| ${}^1\text{H}(n,\gamma){}^2\text{H}$ | 0 | -0.20 | 0.08 | 1.33 | 0.025 |
| ${}^2\text{H}(p,\gamma){}^3\text{He}$ | 0 | -0.32 | 0.37 | 0.57 | 0.11(0.11) |
| ${}^2\text{H}(d,n){}^3\text{He}$ | 0 | -0.54 | 0.21 | 0.69 | 0.12(0.12) |
| ${}^2\text{H}(d,p){}^3\text{H}$ | 0 | -0.46 | -0.26 | 0.05 | 0.12(0.12) |
| ${}^3\text{H}(d,n){}^4\text{He}$ | 0 | 0 | -0.01 | -0.02 | 0.13(0.12) |
| ${}^3\text{H}(\alpha,\gamma){}^7\text{Li}$ | 0 | 0 | 0 | 0.03 | 0.23(0.17) |
| ${}^3\text{He}(n,p){}^3\text{H}$ | 0 | 0.02 | -0.17 | -0.27 | |
| ${}^3\text{He}(d,p){}^4\text{He}$ | 0 | 0.01 | -0.75 | -0.75 | 0.21(0.15) |
| ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ | 0 | 0 | 0 | 0.97 | 0.37(0.21) |
| ${}^7\text{Li}(p,\alpha){}^4\text{He}$ | 0 | 0 | 0 | -0.05 | 0.24(0.17) |
| ${}^7\text{Be}(n,p){}^7\text{Li}$ | 0 | 0 | 0 | -0.71 | |

n↔p weak reaction rates

$$\lambda_{n \leftrightarrow p} = \tau_n^{-1} \times \\ \sum \text{(phase space)} \times \text{(e distribution)} \times \text{(v}_e \text{ distribution)} dE \\ + \text{small corrections}$$

$$\lambda_{n \rightarrow p} = \lambda_{n \rightarrow p + e^- + \bar{\nu}_e} + \lambda_{n + e^+ \rightarrow p + \bar{\nu}_e} + \lambda_{n + \nu_e \rightarrow p + e^-}$$

$$\lambda_{p \rightarrow n} = \lambda_{p + e^- + \bar{\nu}_e \rightarrow n} + \lambda_{p + \bar{\nu}_e \rightarrow n + e^+} + \lambda_{p + e^- \rightarrow n + \nu_e}$$

[Dicus et al. (1982), Lopez & Turner (1999)]

$$\lambda_{n \rightarrow p \nu} = C \int_1^q \frac{\varepsilon (\varepsilon - q)^2 (\varepsilon^2 - 1)^{1/2} d\varepsilon}{[1 + \exp(-\varepsilon z)] \{1 + \exp[(\varepsilon - q) z_\nu]\}} \quad T \rightarrow 0 \quad \frac{1}{\tau_n} = C \int_1^q \varepsilon (\varepsilon - q)^2 (\varepsilon^2 - 1)^{1/2} d\varepsilon$$

$$(q \equiv Q_{np}/m_e, \varepsilon \equiv E_e/m_e, z \equiv m_e/T_\gamma, z_\nu \equiv m_e/T_\nu)$$

The neutron lifetime is still a matter of debate (but not essential to BBN)

- Weak rate change mostly affects n/p ratio at freeze out and hence ${}^4\text{He}$ abundance
- Change in expansion rate gives similar effect (n/p freezeout when weak rate ≈ expansion rate)

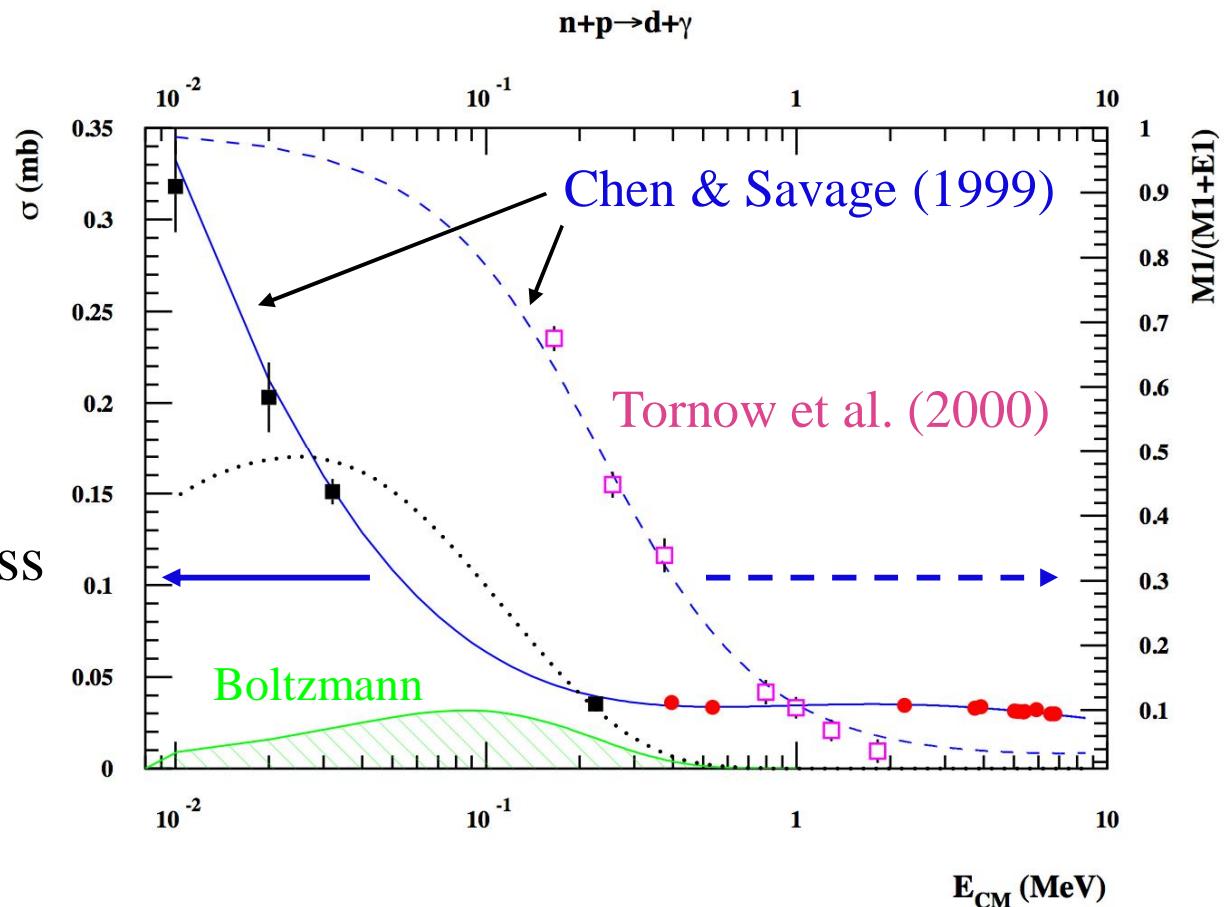
$^1\text{H}(\text{n},\gamma)\text{D}$: theory versus experiments

Rate calculated from Effective Field theory with (theoretical) uncertainties of 4% [Chen & Savage (1999)] or 1% [Rupak (2000)] compared to experiments [Arenhovel & Sanzone (1991) review]

BBN energy ~ 25 keV

Additional check with polarized beam $E1$ and $M1$ measurements
[Tornow et al. (2000)]

... and new (>1991) cross section measurements
[Suzuki et al. (1995),
Tomoya et al. (2003)]

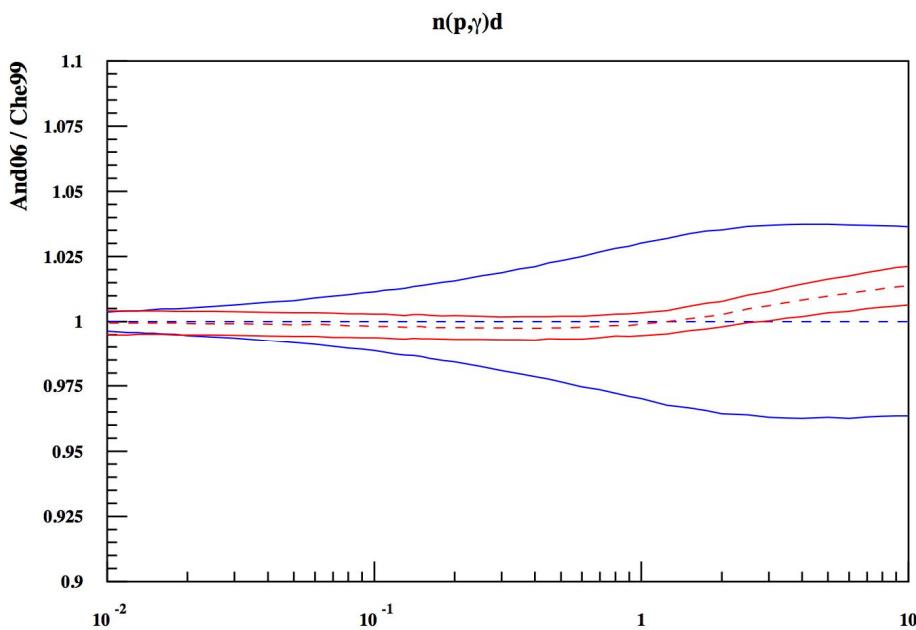


The ${}^1\text{H}(\text{n},\gamma){}^2\text{H}$ reaction

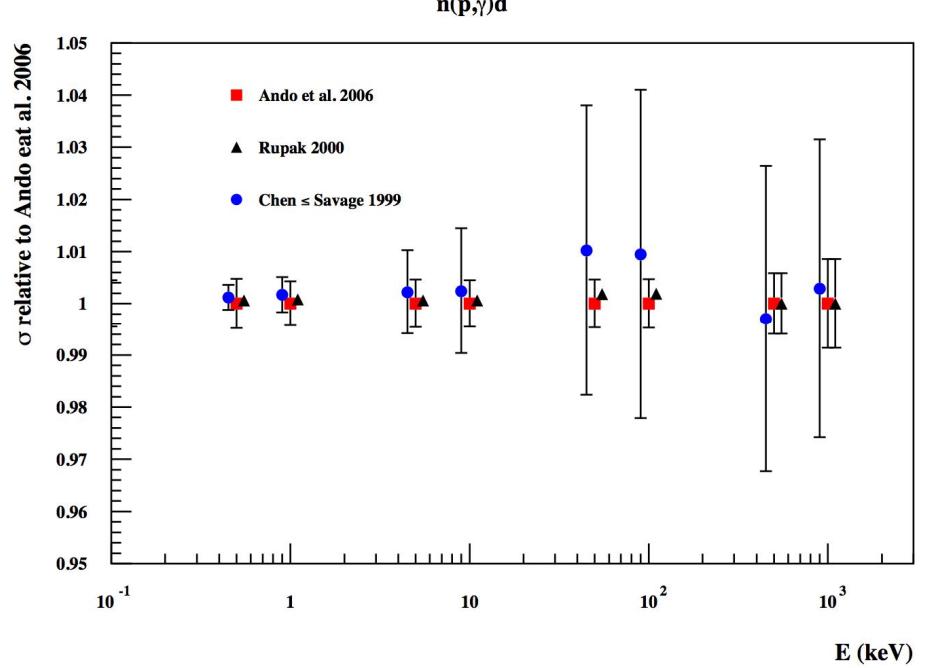
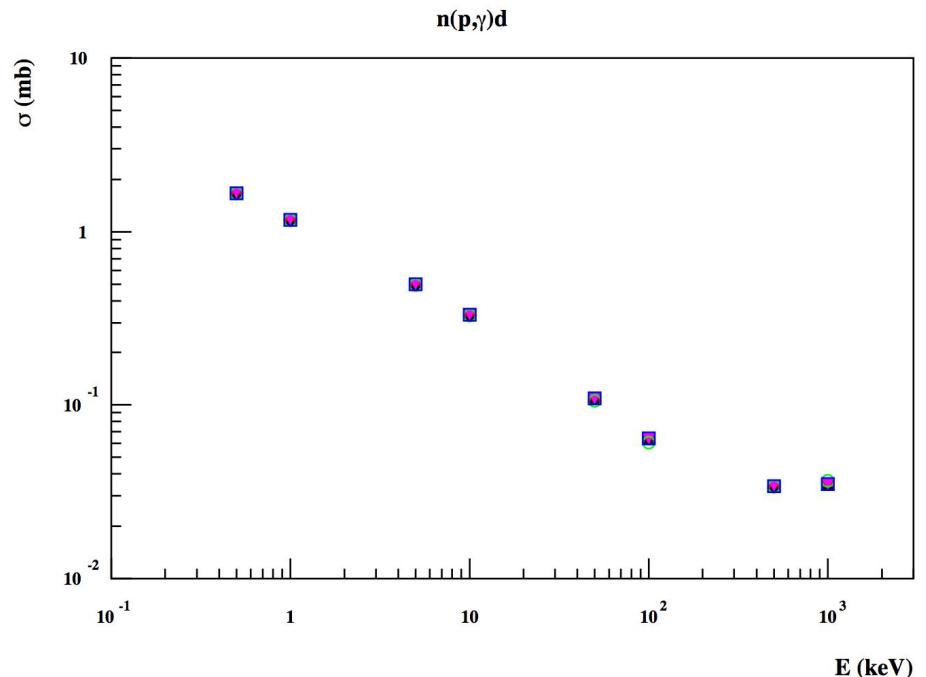
Sensitivity = 1.33

$E \sim 25 \text{ keV}$

New precise $n(p,\gamma)d$ EFT cross section
and rate calculation [Ando et al. 2006]



T9



Reactions with charged particles

➤ Below the Coulomb barrier!

➤ Very weak cross sections :

$\sigma(E) \searrow \searrow \searrow$ when $E \searrow$

Coulomb and et centrifugal
penetrability :

$$P_L = \frac{\rho}{F_L^2(\eta, \rho) + G_L^2(\eta, \rho)}$$

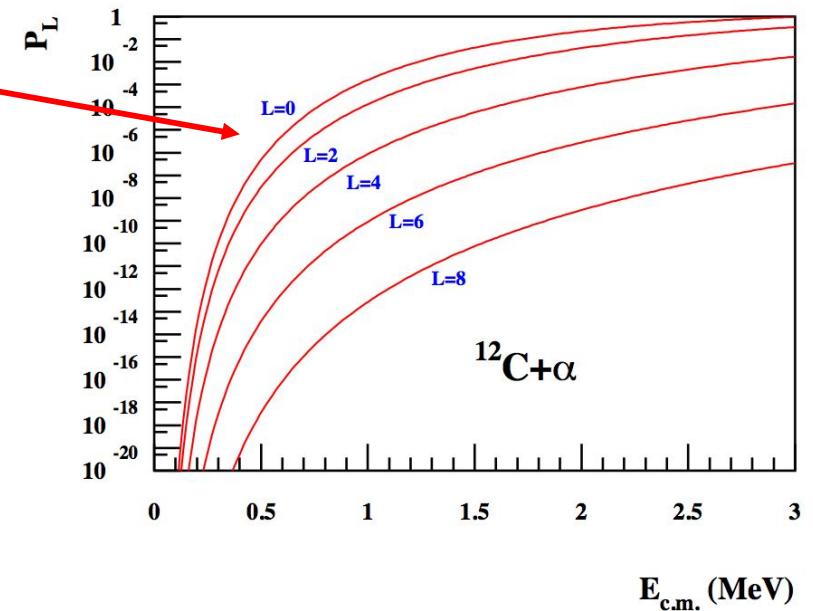
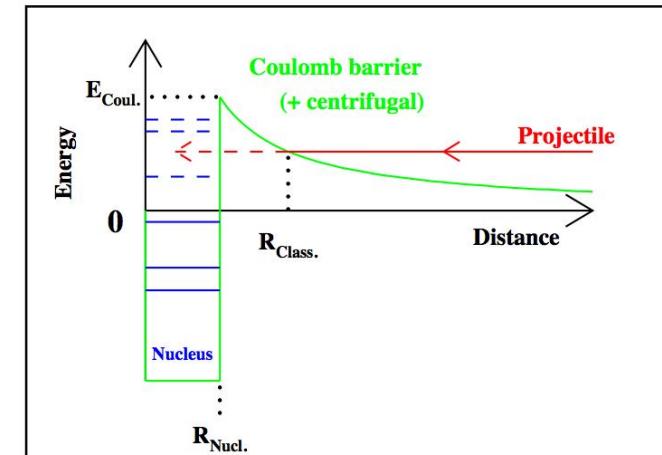
F, G = Coulomb functions

η = Sommerfeld parameter

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

$$\rho = \frac{R}{\lambda} = \frac{\sqrt{2\mu E}}{\hbar} R$$

Penetrability factor



Astrophysical S-factor : $S(E)$

WKB Approximation : $P_{L=0} \propto \exp(-2\pi\eta)$

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

Astrophysical *S-factor*:

$$S(E) = \sigma(E) \cdot E \cdot \exp(2\pi\eta) = \sigma(E) \cdot E \cdot \exp\left(-\sqrt{\frac{E_G}{E}}\right)$$

Corrects for the $\sigma(E) \sim \lambda^2$ effect where $\lambda \propto E^{-1/2}$ is the de Broglie wavelength

Corrects for the effect of penetrability ($L=0$)

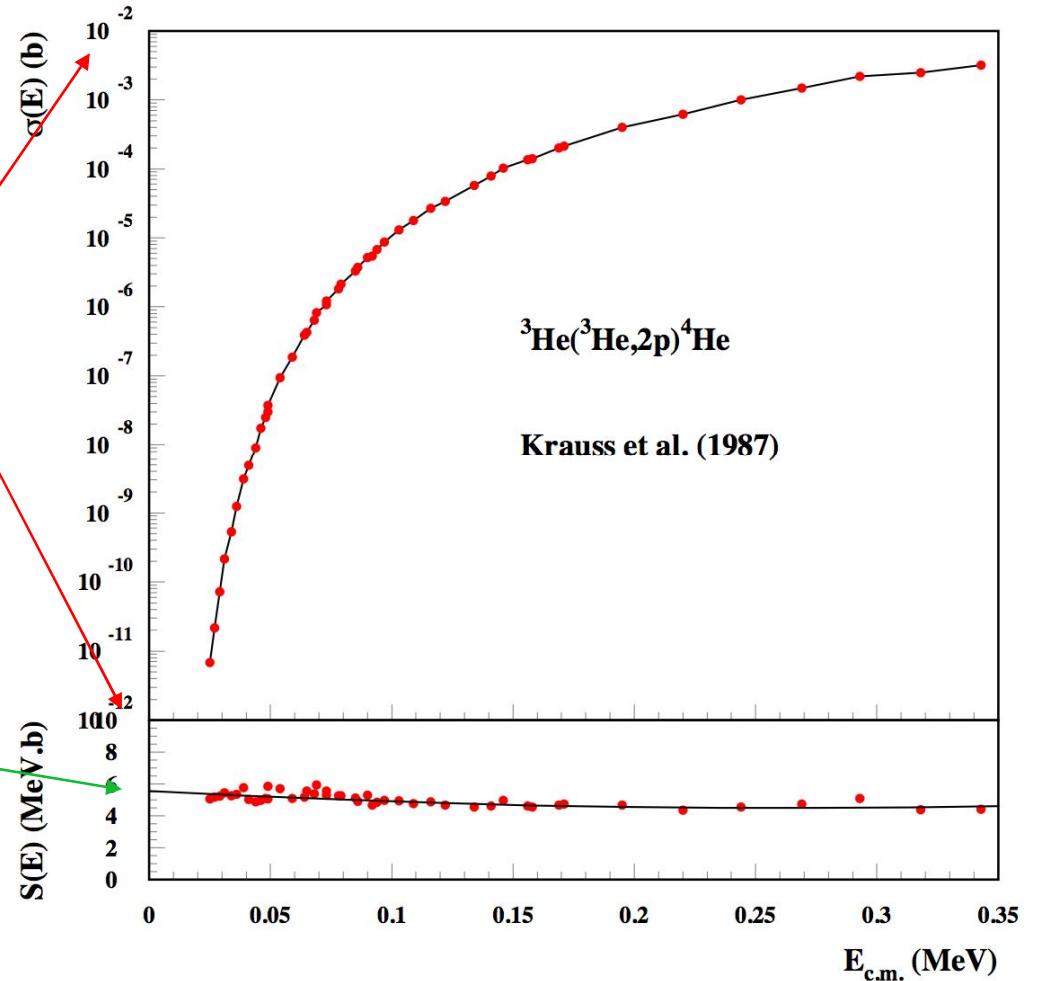
Gamow energy : $E_G = \frac{1}{2}(2\pi\alpha Z_1 Z_2)^2 \mu c^2 = (0.989 \cdot Z_1 Z_2 A^{1/2})^2$ [MeV]

⇒ Much weaker variation of $S(E)$ with energy as compared to $\sigma(E)$

Astrophysical S-factor : $S(E)$

Variation of cross section $\sigma(E)$:
9 orders of magnitude
between 25 and 340 keV

Variation of the
astrophysical S-factor
 $S(E)$: 50%



⇒Extrapolation to low energy

10 rates deduced from experimental data

Compilations and evaluations for BBN thermonuclear rates

- *Smith, Kawano & Malaney 1999* (with uncertainties)
- *NACRE, Angulo et al. 1999* (7/10, tabulated rates and uncertainties)
- *Nollett & Burles 2000* (no rates provided)
- *Cyburt, Fields & Olive 2003* (revaluation of NACRE)
- *Serpico et al. 2004* (rates and uncertainties provided)
- *Descouvemont, Adahchour, Angulo, Coc & Vangioni-Flam 2004 [DAACV]*
 - « R-Matrix » formalism: S-factors fits of data constrained by theory
 - Provide also reaction rate uncertainties
- *Cyburt 2004* (rates provided, not the uncertainties)
- *Cyburt, Matthews,... 2014 (?)*

R-Matrix formalism

(P. Descouvemont lectures next week)

Parametrization (interpolation/extrapolation) of nuclear cross sections with a reduced set of parameters $\{p_i\}$ *related* to nuclear quantities

- Two regions of space ($a \sim$ nuclear radius)
 - External ($r \geq a$) : nuclear force negligible, Coulomb wave functions
 - Internal ($r \leq a$) : complicated
- R-matrix
 - Connects internal and external wave functions at $r=a$
 - Introducing poles *related* to resonant and non resonant contributions

Rate uncertainties

- Solve for $\{p_i\}$: $\chi^2(\{p_i\}) = \chi^2(\{p_i^{min}\}) + \Delta\chi^2$

where $\{p_i^{min}\}$ is the set of parameters that minimizes the χ^2 and $\Delta\chi^2$ is a function of the confidence level (68% or 1- σ here)

- Calculate $\pm 1\text{-}\sigma$ rate limits by spanning the parameter space limited by the above condition;
- Special treatments

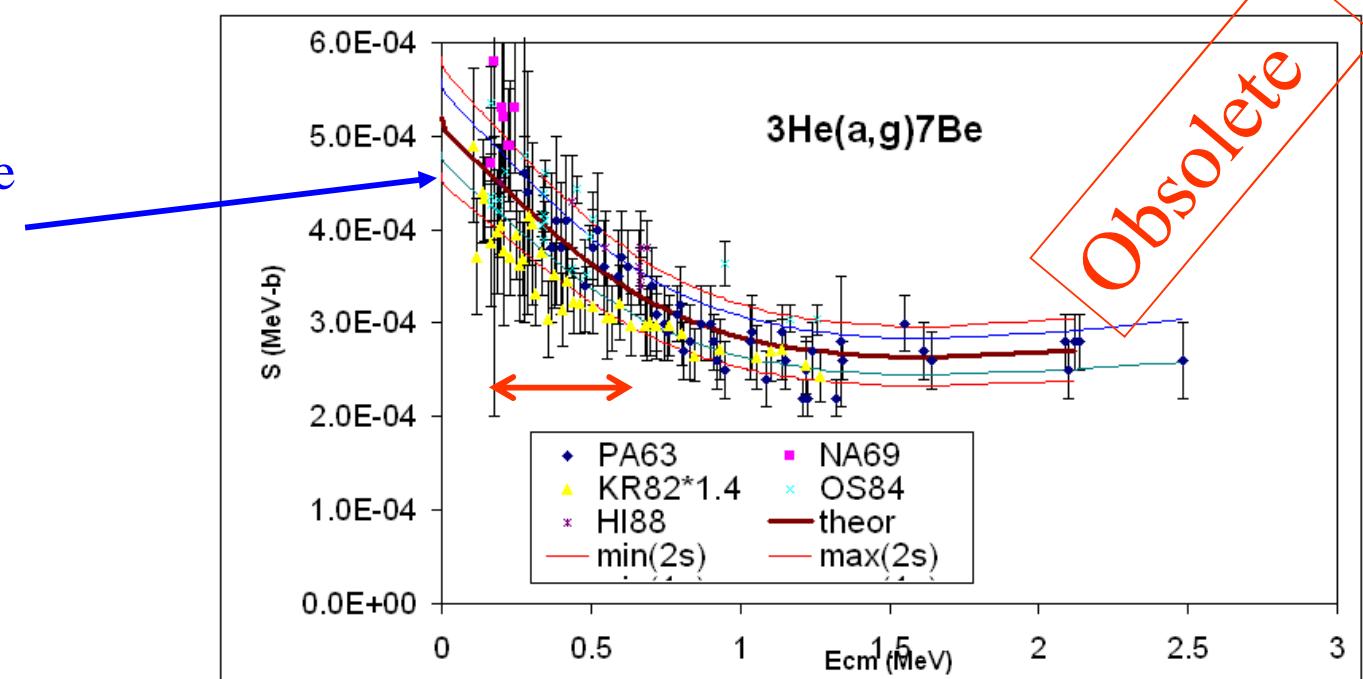
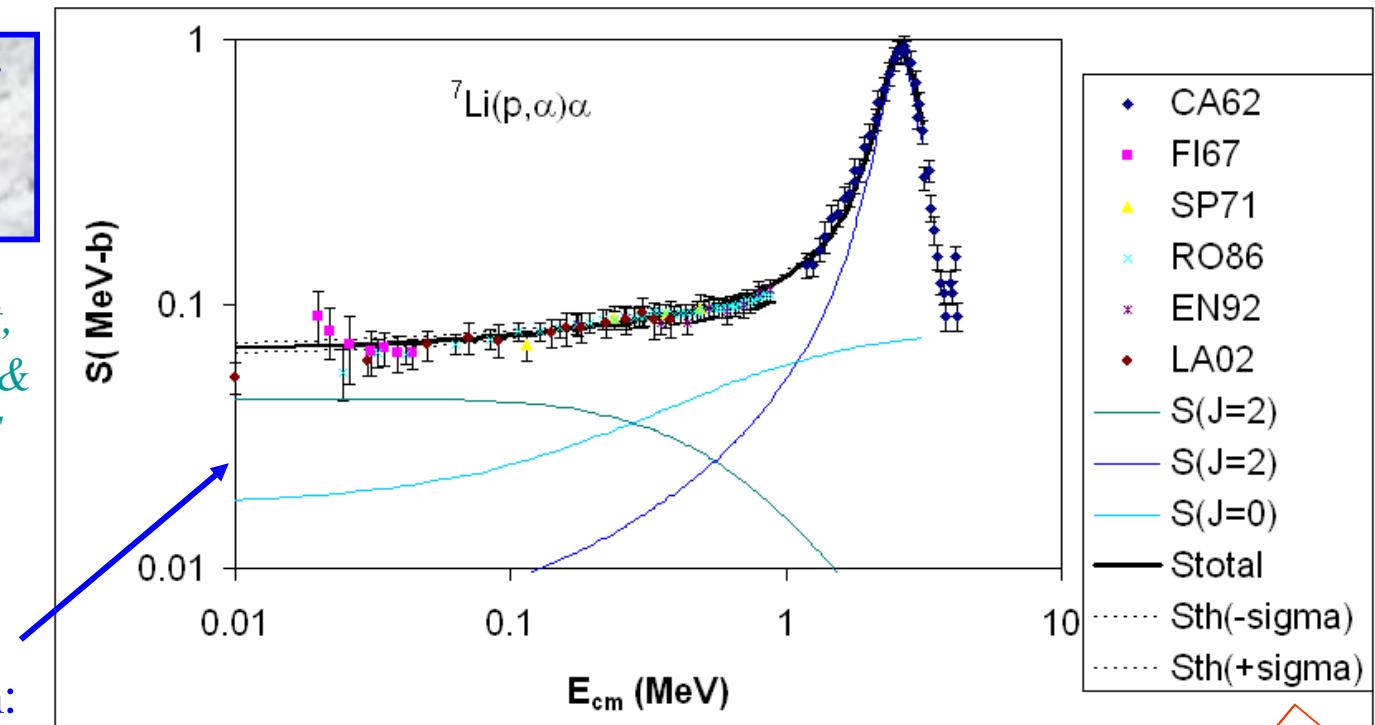
- $\chi^2 > 1$: errors are scaled by $(\chi^2)^{1/2}$
- Incompatible data sets (normalization) : individual fits of data sets

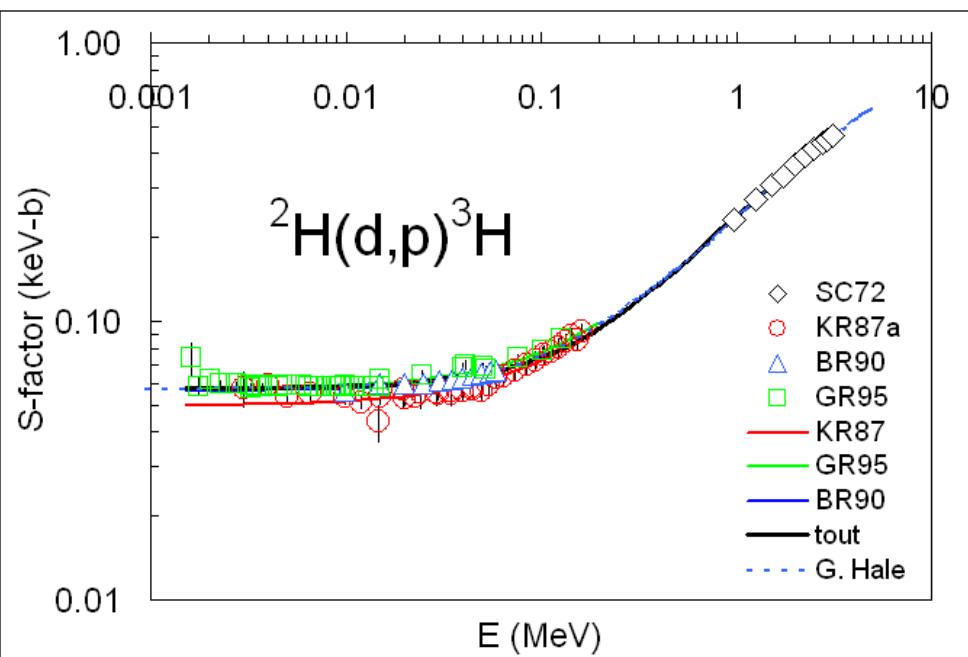
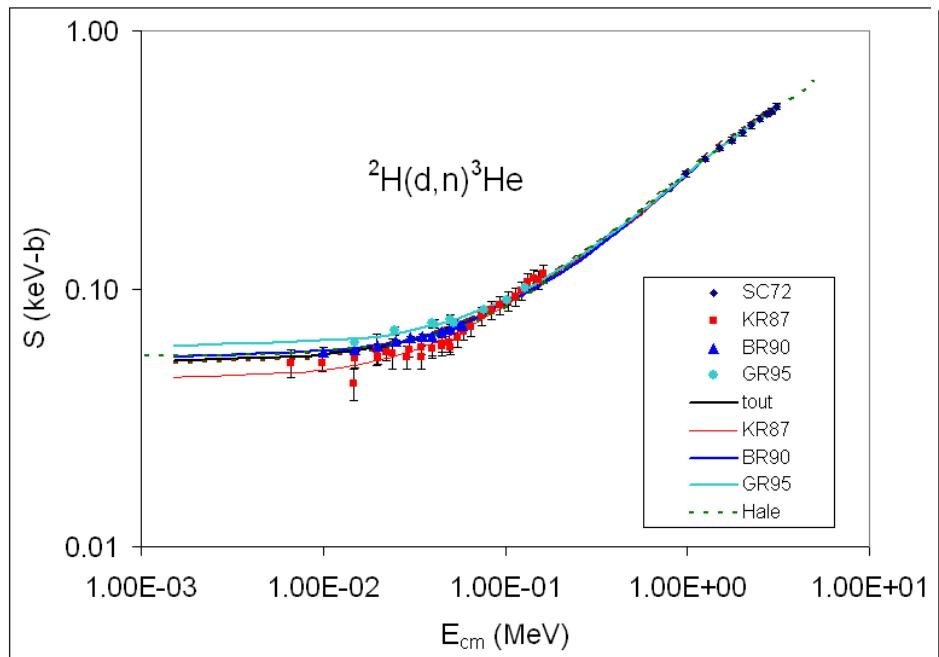
New analysis of BBN rates

[DAACV] Descouvemont,
Adahchour, Angulo, Coc &
Vangioni-Flam ADNDT
(2004)

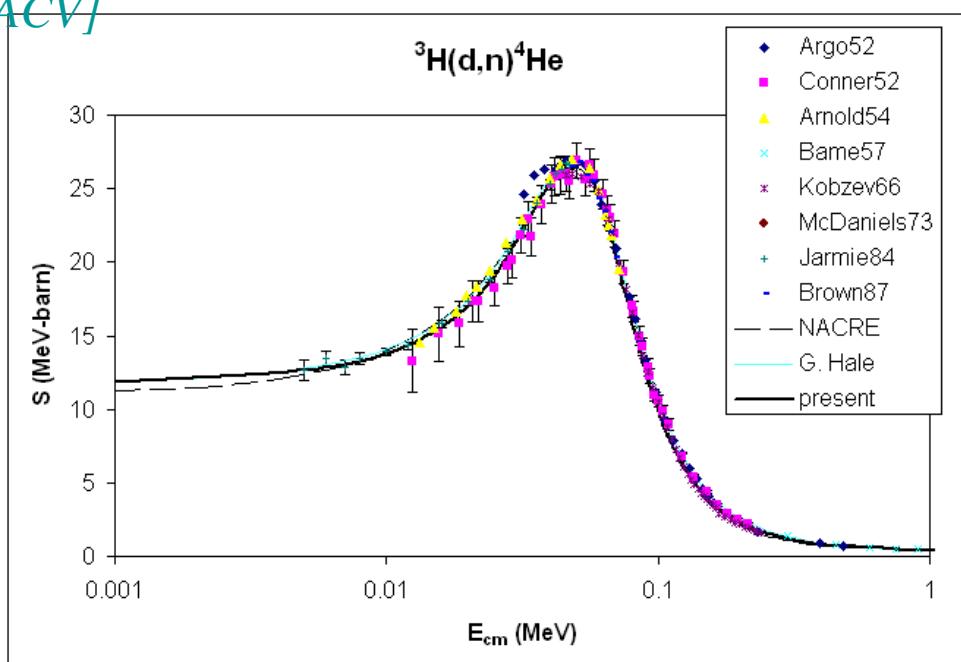
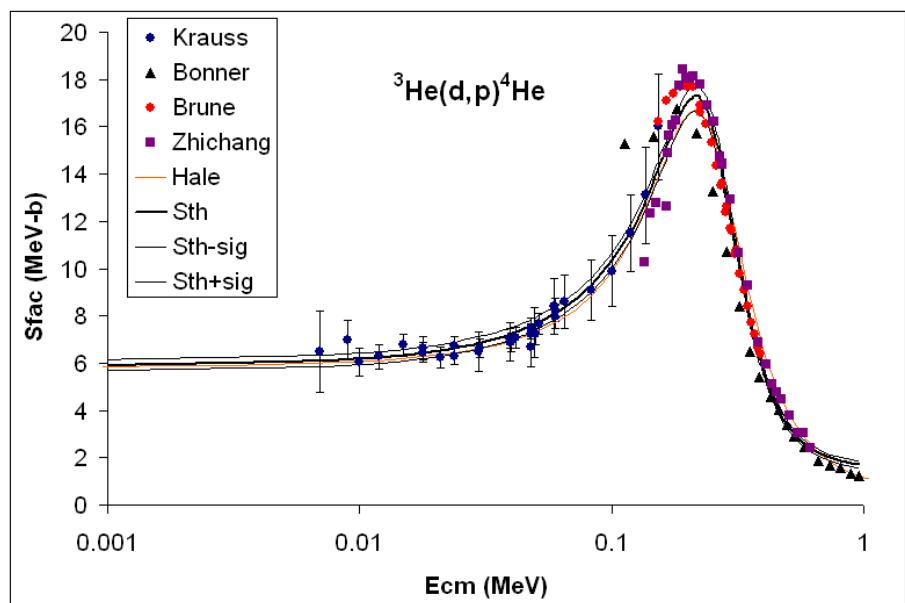
« R-Matrix » formalism:
S-factors fits of data
constrained by theory

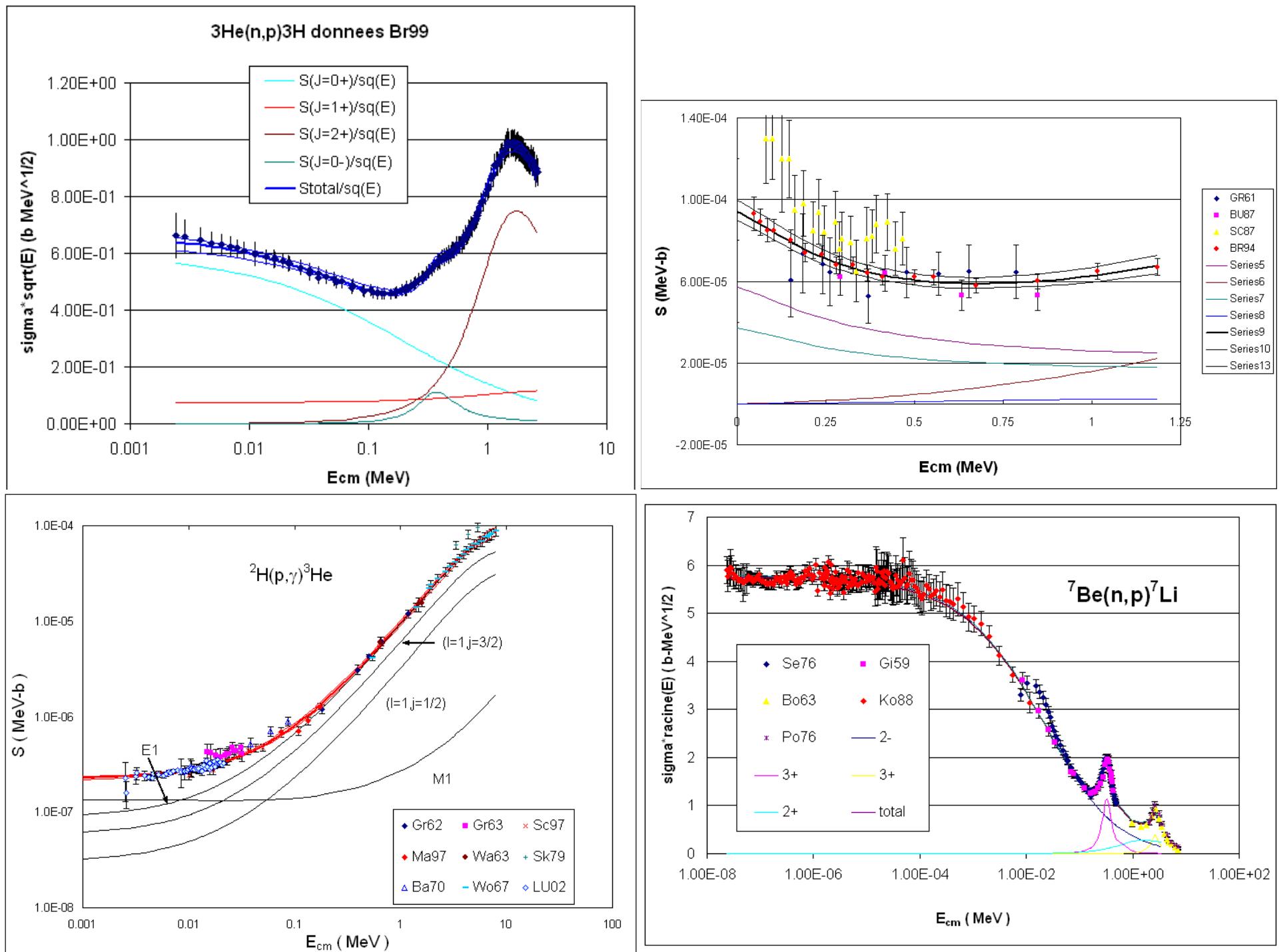
Provide also reaction rate
uncertainties



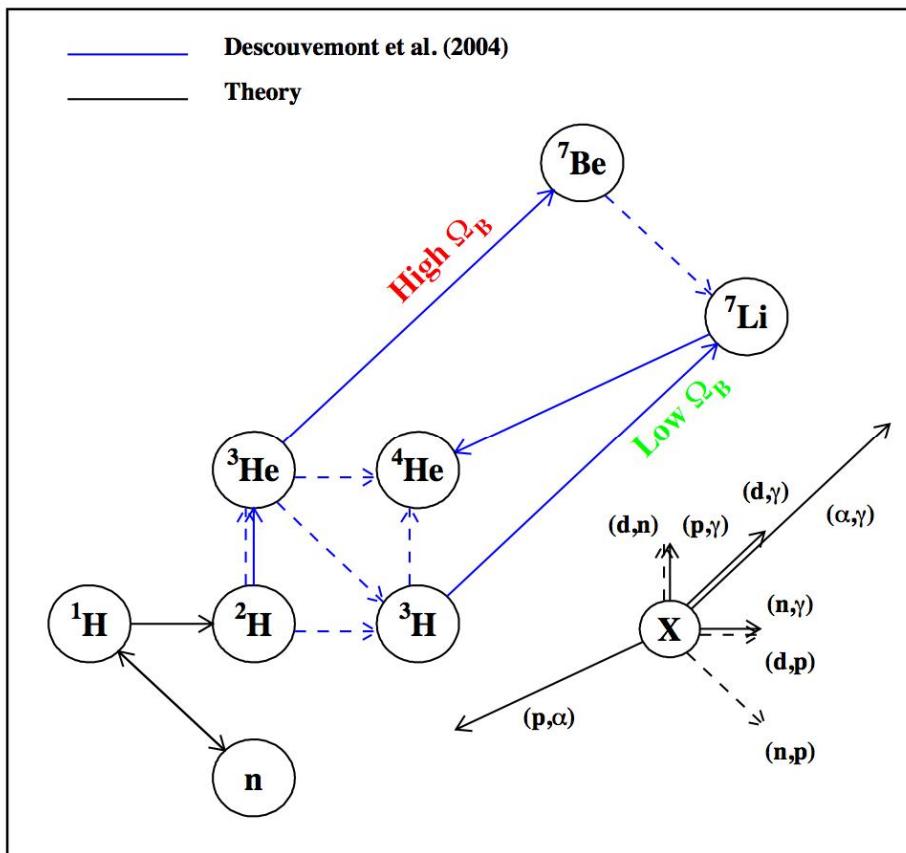


[DAACV]





Nuclear physics experiments since 2004

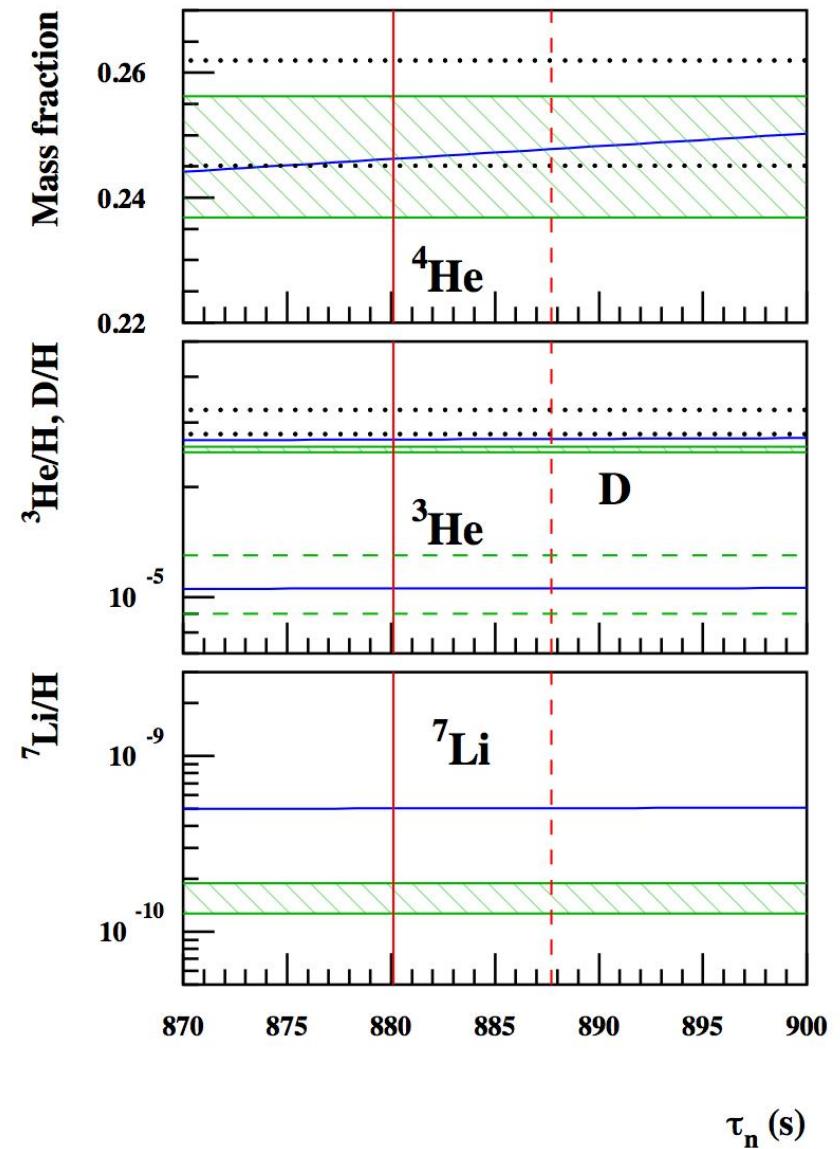


Concerning the main reactions

- Neutron lifetime
- ${}^1\text{H}(n,\gamma){}^2\text{H}$ reaction
- ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ [*Many....*]
- D(d,p) ${}^3\text{H}$ and D(d,n) ${}^3\text{He}$ [*Leonard et al. (2006)*]

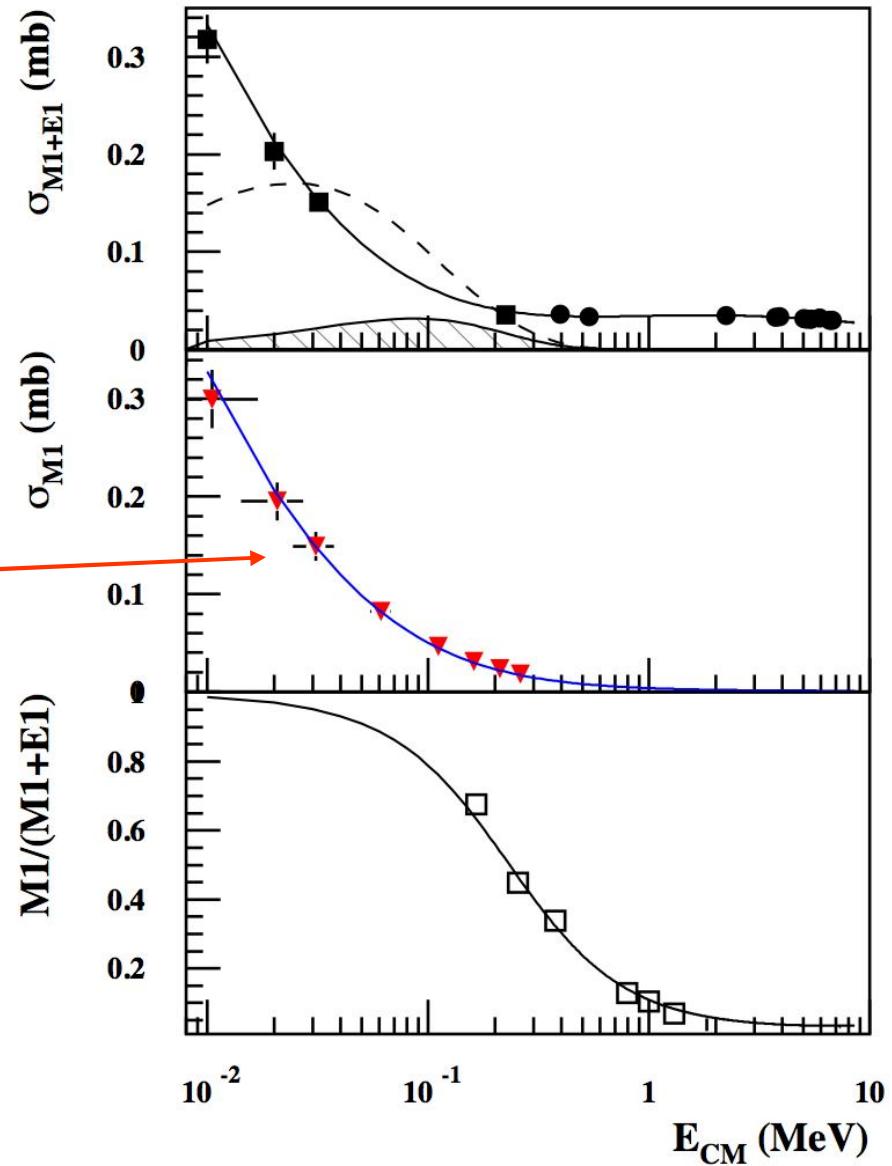
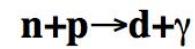
Influence of the neutron lifetime

- $\tau_n = 885.7 \pm 0.8$ s [PDG 2008]
- $\tau_n = 878.5 \pm 0.7 \pm 0.3$ s [Serebrov+ 2005]
- $\tau_n = 880 - 884$ s [Wietfeldt & Greene 2011]
- $\tau_n = 880.1 \pm 1.1$ s [PDG 2013]
- $\tau_n = 887.7 \pm 1.2 \pm 1.9$ s [Yue+ 2013]



The ${}^1\text{H}(\text{n},\gamma){}^2\text{H}$ reaction

New measurement of the M1 contribution [*Ryezaveva et al. 2006*] by inelastic electron scattering off D

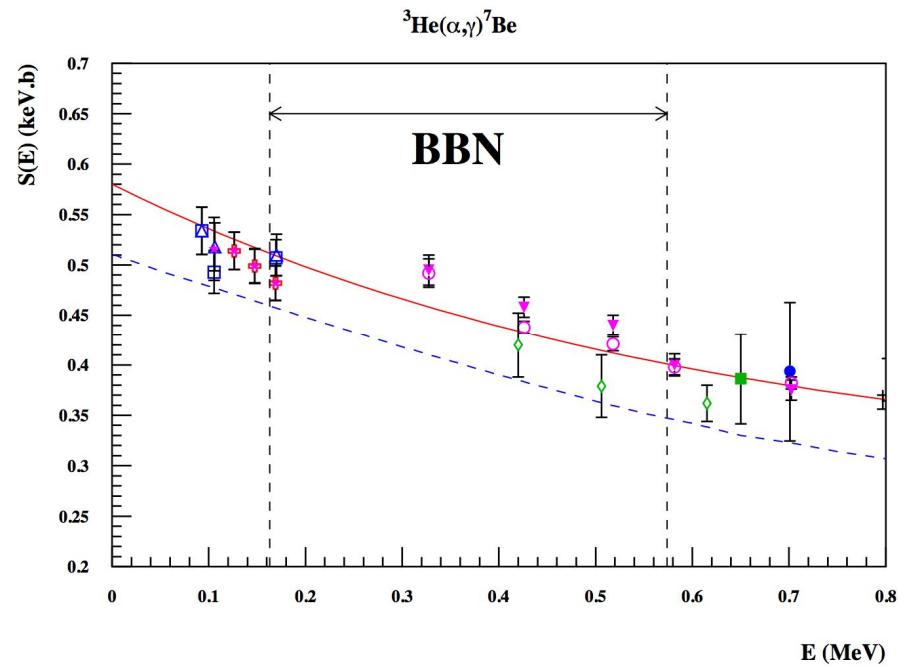


The ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ reaction

□ ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ (${}^7\text{Li}$ sensitivity 0.97)

New precise (*prompt, activation, recoil*) measurements to resolve previous systematic uncertainties [*Brown+ 2007; Confortola+ 2007; Costantini+ 2008; Nara Singh+ 2005, 2013; Brown+ 2007; Gyürky+ 2007; Di Leva+ 2009; Carmona-Gallardo+ 2012; Bordeanu+ 2013;....(?).. NPA VI*]

Reanalysis of ${}^3\text{He}(\alpha,\gamma){}^7\text{Be}$ rate [*Cyburt & Davids 2008*]: $S(0) = 0.580 \pm 0.043$



The $^2\text{H}(\text{d},\text{n})^3\text{He}$ reaction

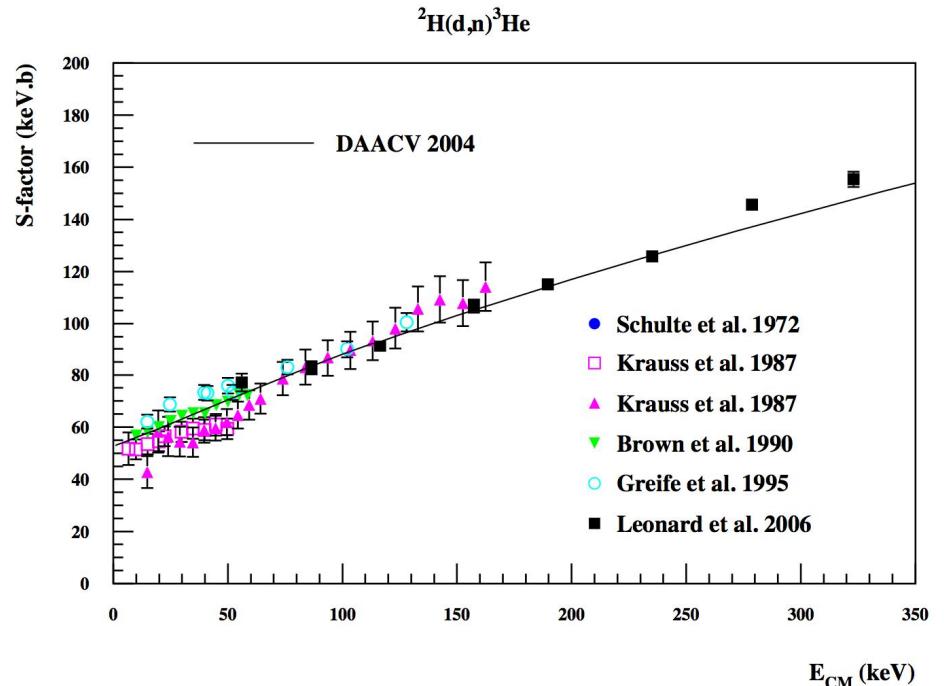
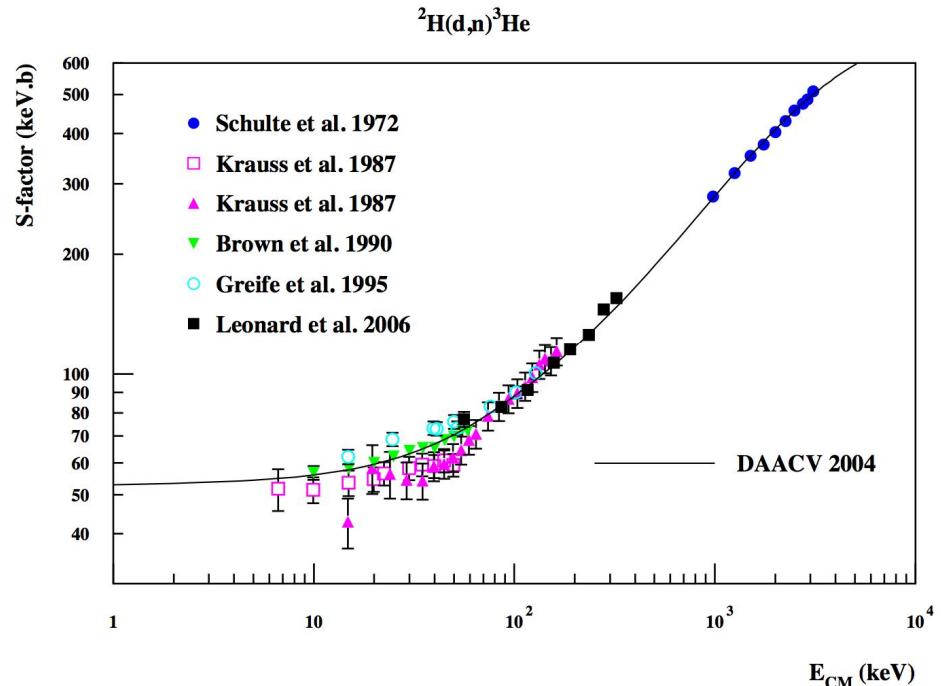
Sensitivity = 0.61

$$E_0(\Delta E_0/2) = 0.12(0.12) \text{ MeV}$$

New precise measurements of
 $^2\text{H}(\text{d},\text{n})^3\text{He}$ (and $^2\text{H}(\text{d},\text{p})^3\text{H}$) reaction
at TUNL [Leonard et al. 2006]

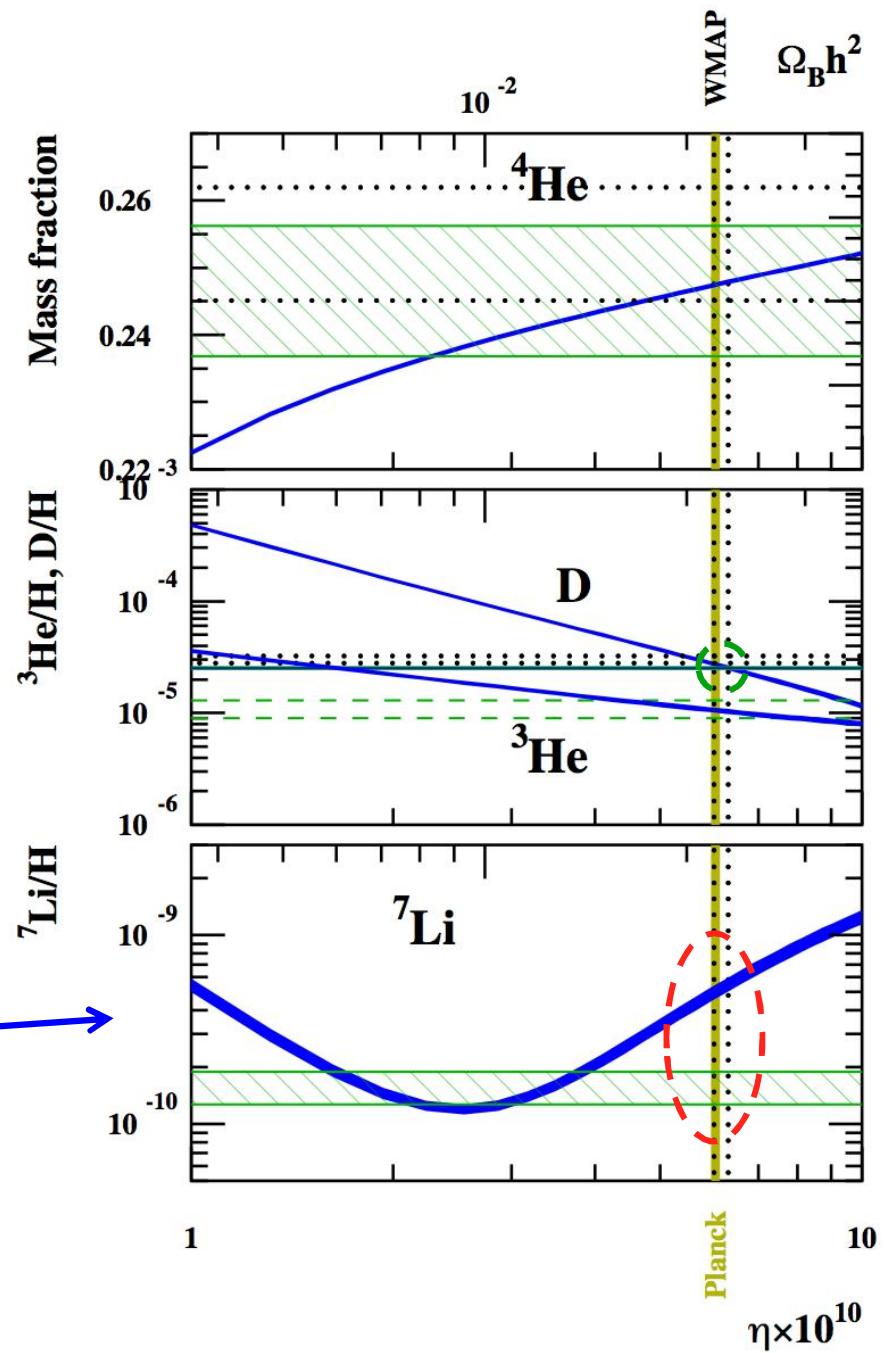
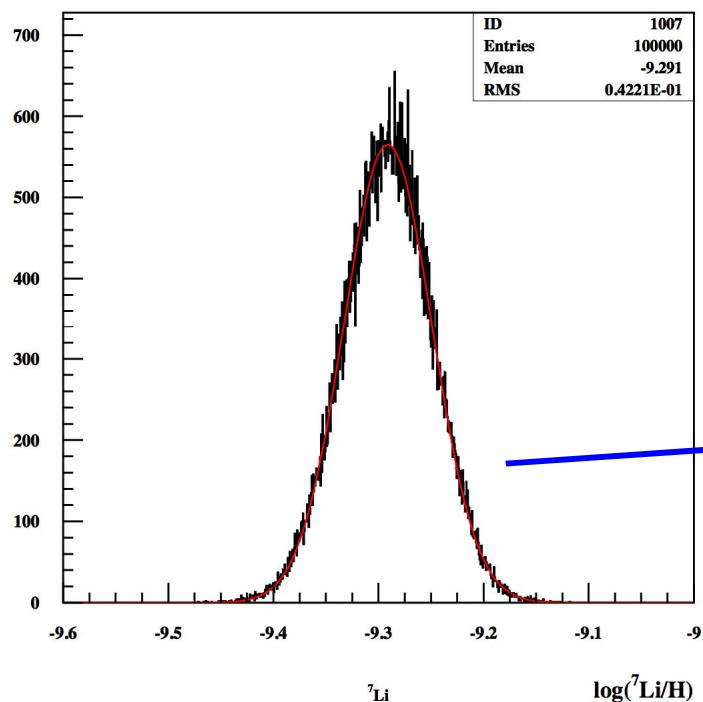
Excellent agreement with DAACV
2004 fit within Gamow window

- No change in central Li/H value
- Reduced uncertainty
- R-matrix fit reliability



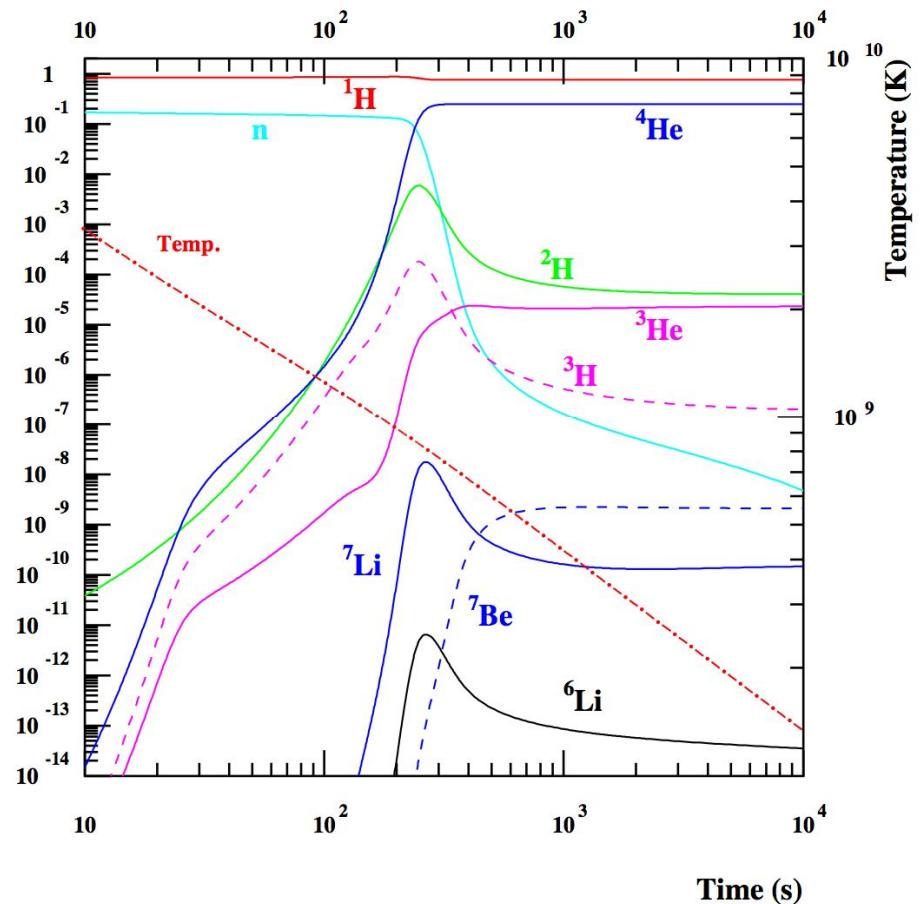
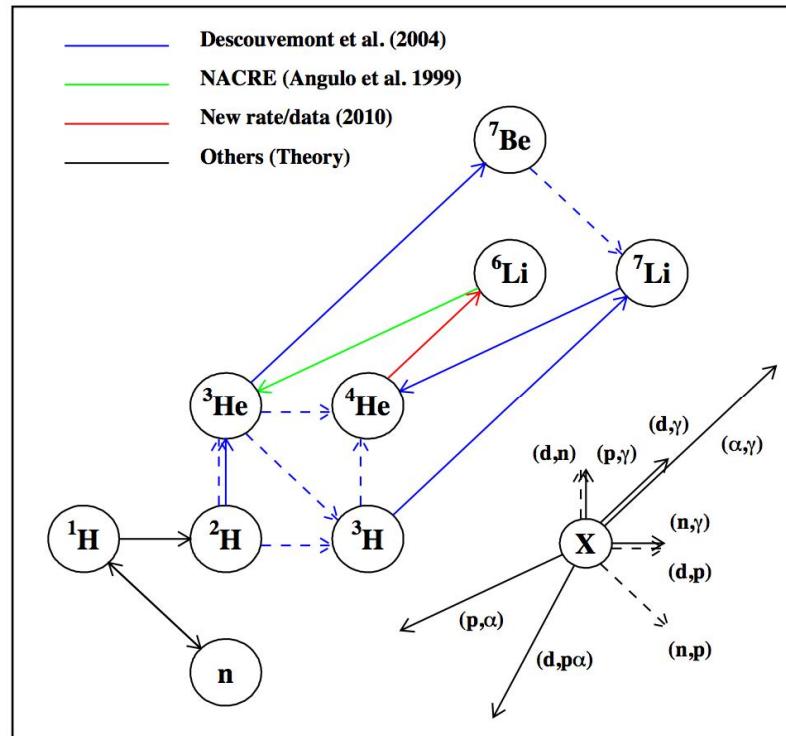
Monte Carlo BBN calculations

Using log-normal distribution
 [Iliadis+ 2010] for the reaction rates
 from DAACV, Ando et al. 2006,
 Leonard et al. 2006, and Cyburt &
 Davids 2008.



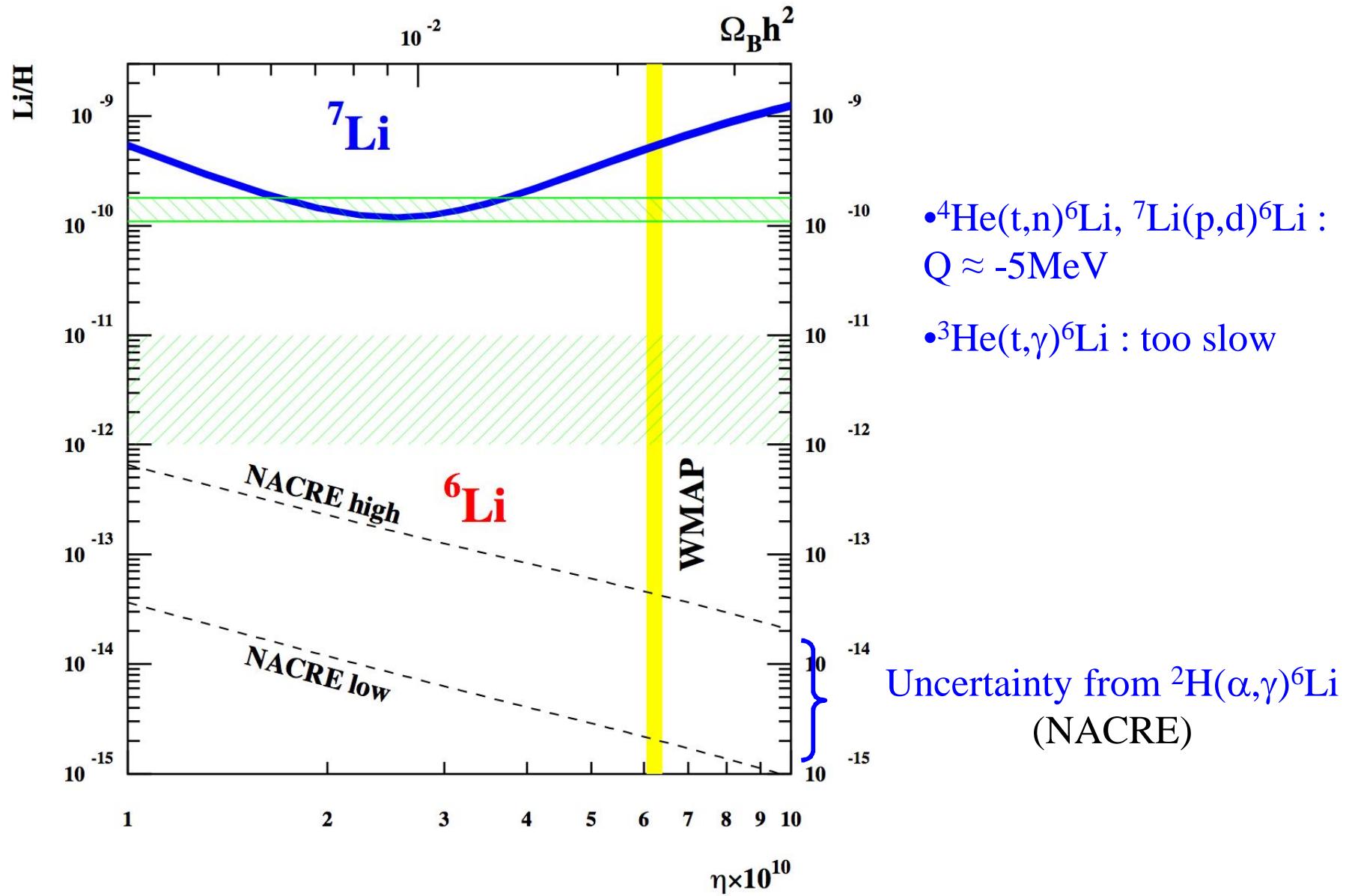
^6Li nucleosynthesis

At WMAP baryonic density

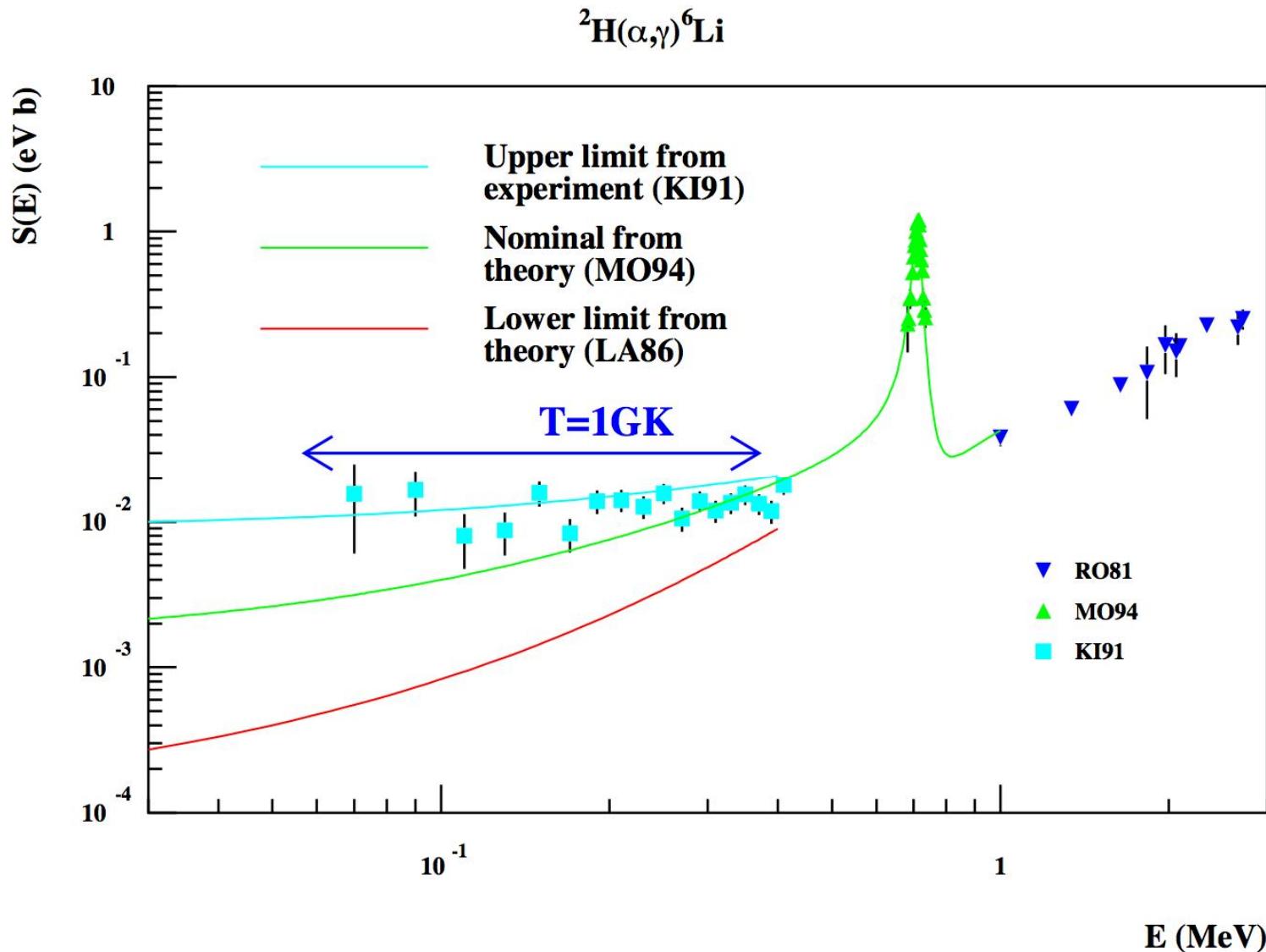


12 main reactions for ^4He , D, ^3He , ^7Li (+2 for ^6Li) nucleosynthesis:
 10 (+2) from experiments and 2 from theory

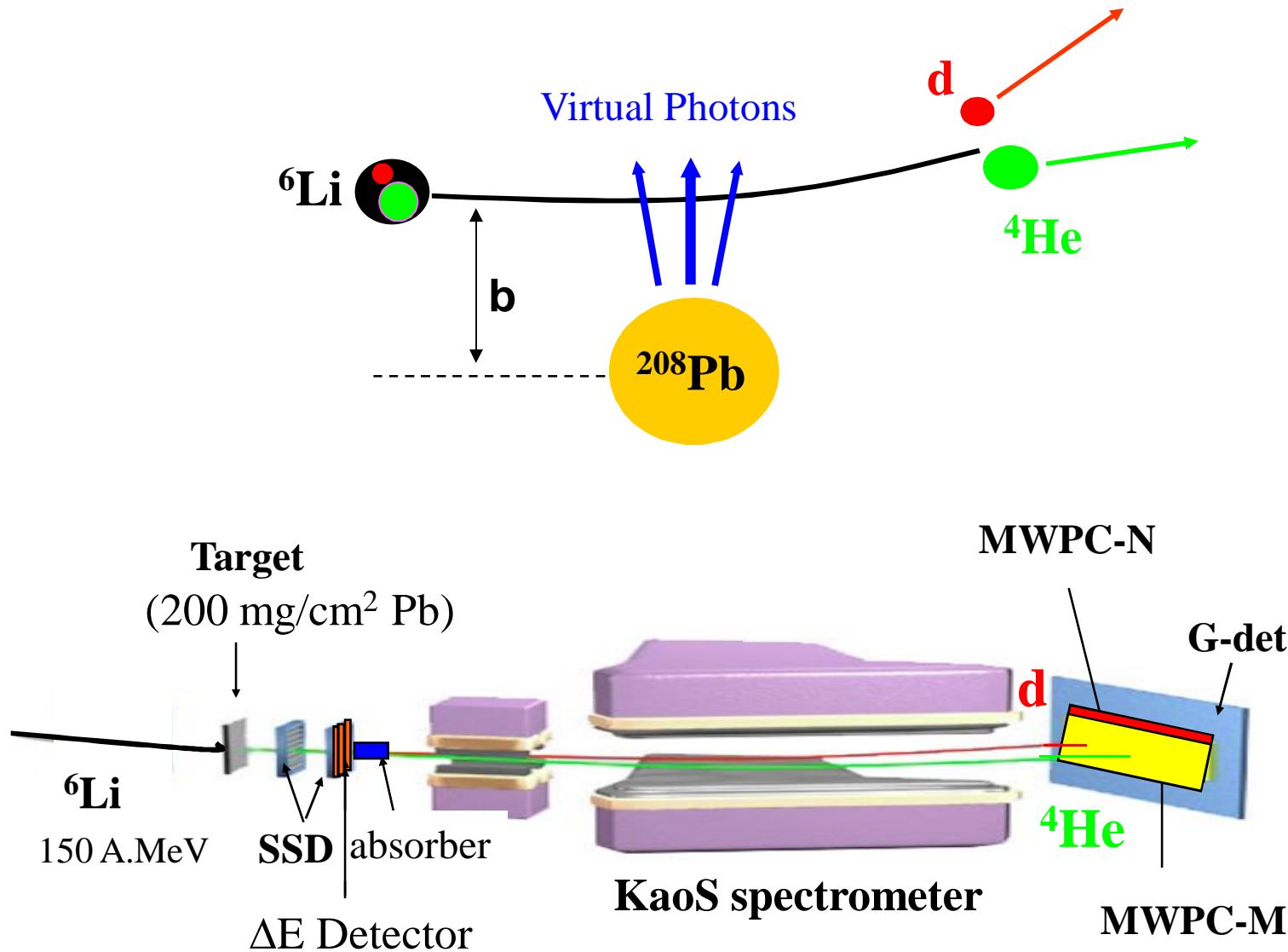
The ${}^6\text{Li}$ problem vs nuclear physics



The $^2\text{H}(\alpha,\gamma)^6\text{Li}$ rate in NACRE



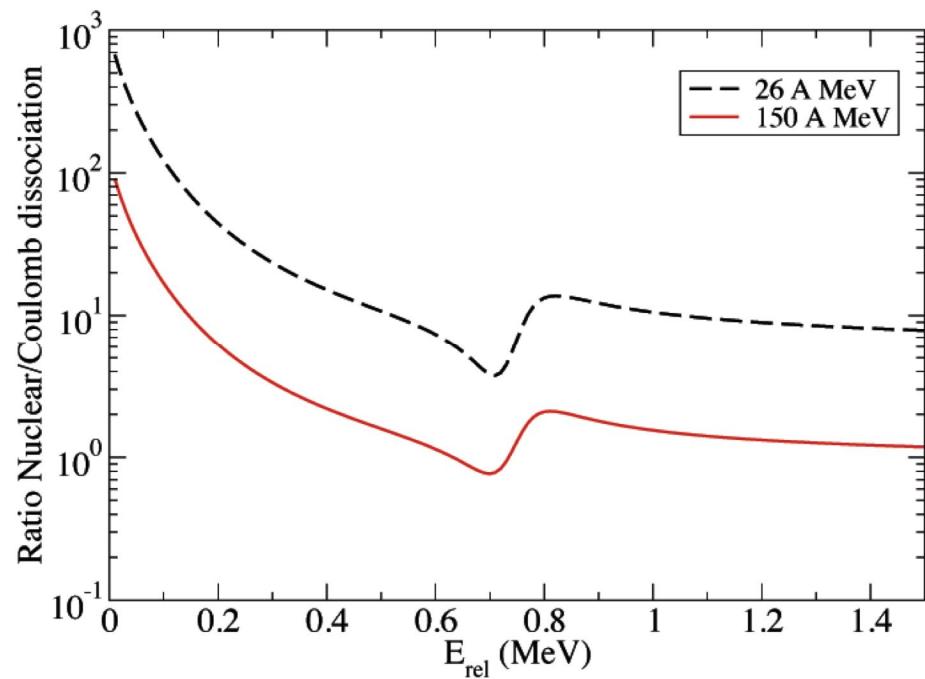
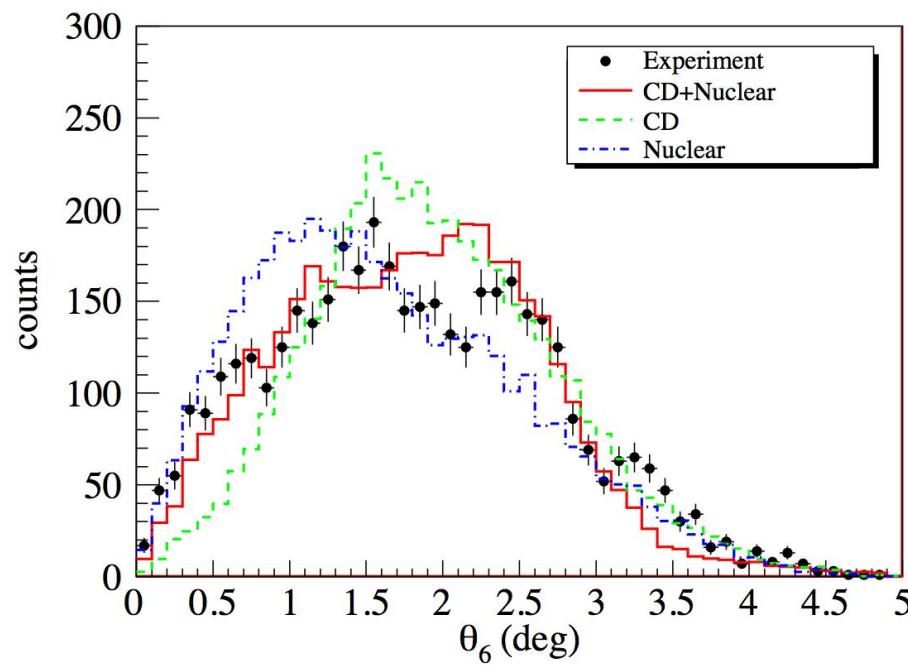
Coulomb dissociation of ${}^6\text{Li}$ at GSI



Coulomb dissociation of ${}^6\text{Li}$ at GSI

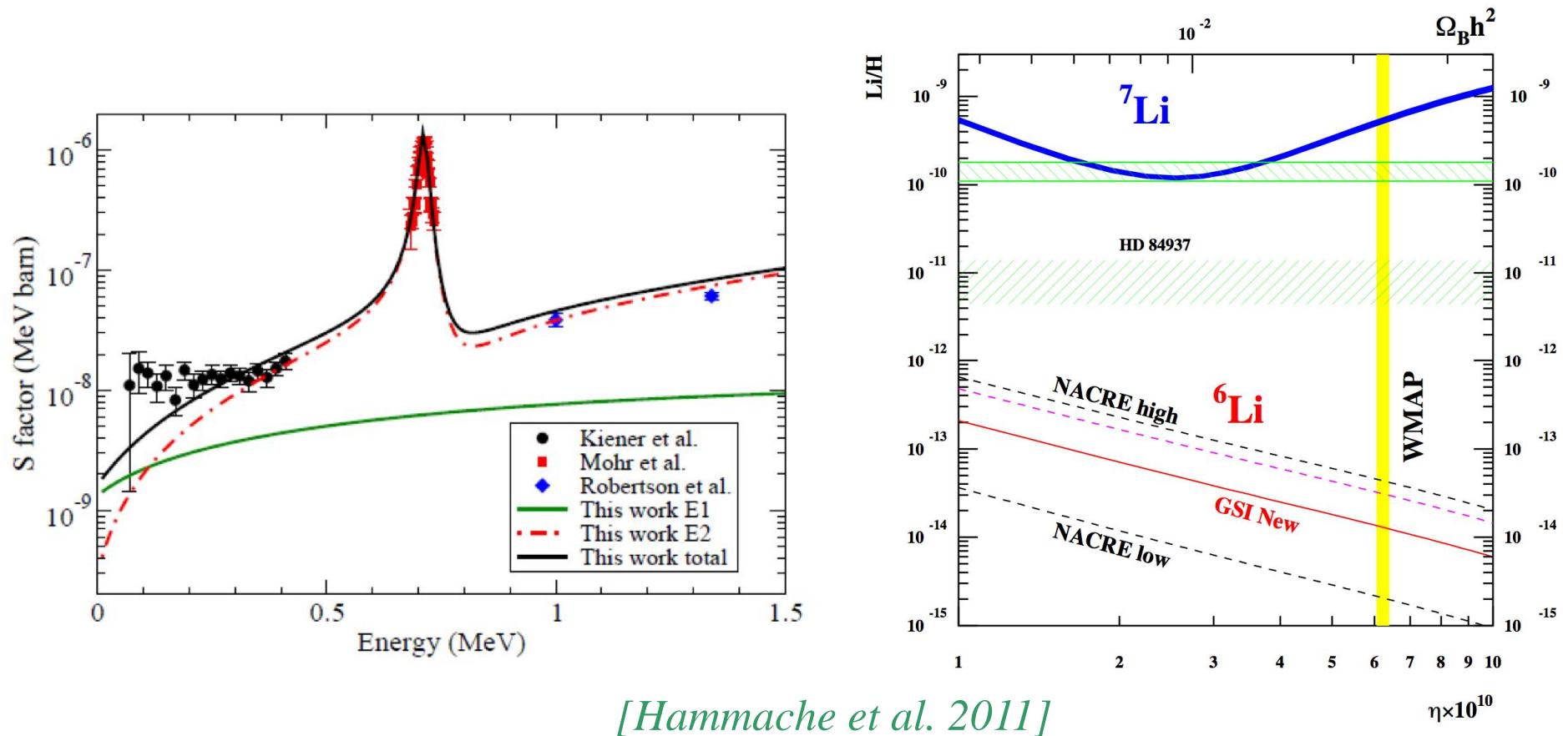
□ From *S. Typel's* calculations :

- At 150 A MeV (GSI), the nuclear break-up contribution is important
- At 26 A MeV [*Kiener et al. 1991*], the nuclear break-up contribution *is dominant*



[*Hammache et al. 2010*]

Reduction of the uncertainty on ${}^6\text{Li}$ yield



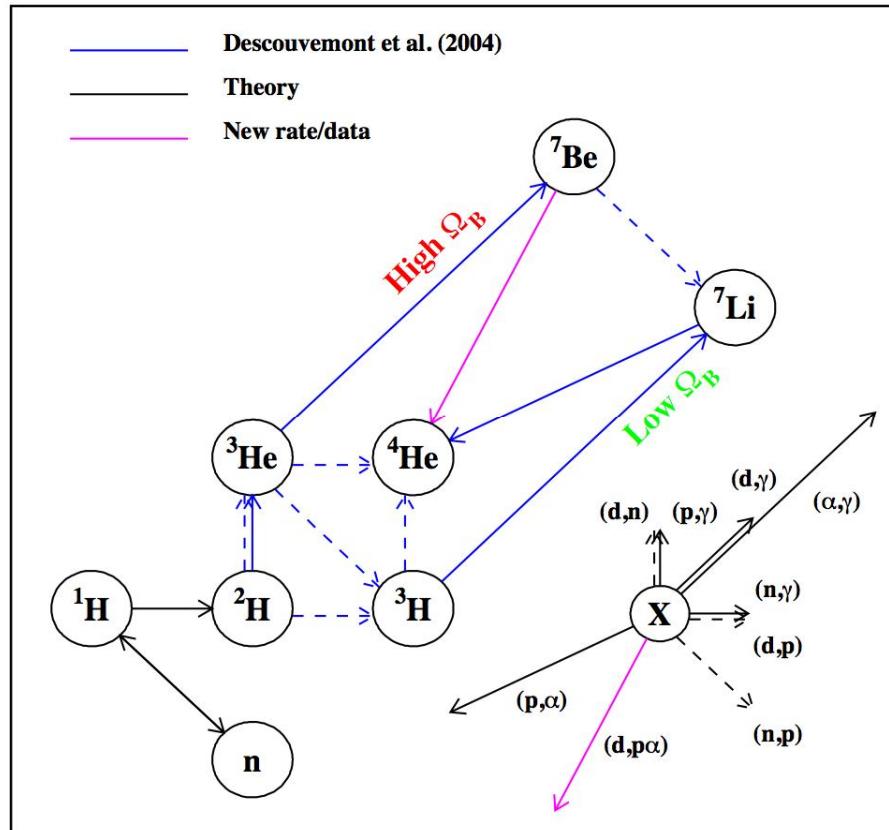
[Hammache et al. 2011]

Coming next : results from LUNA [Gustavino et al. NIC XII; NPA 6]

Nuclear Physics aspects

1. The main reactions for H, He and Li BBN production
2. **Search for nuclear solutions to the lithium problem**
3. Be, B and CNO production in BBN
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Nuclear solution to the Li problem ?



At η_{WMAP} ^7Li from ^7Be post BBN decay

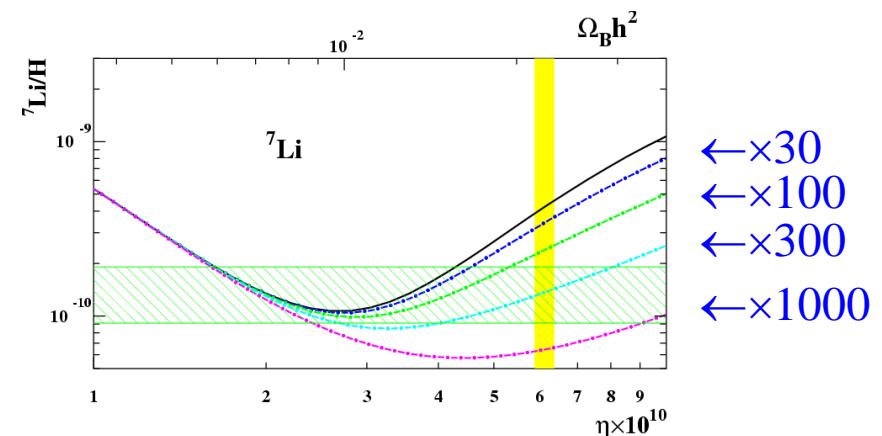
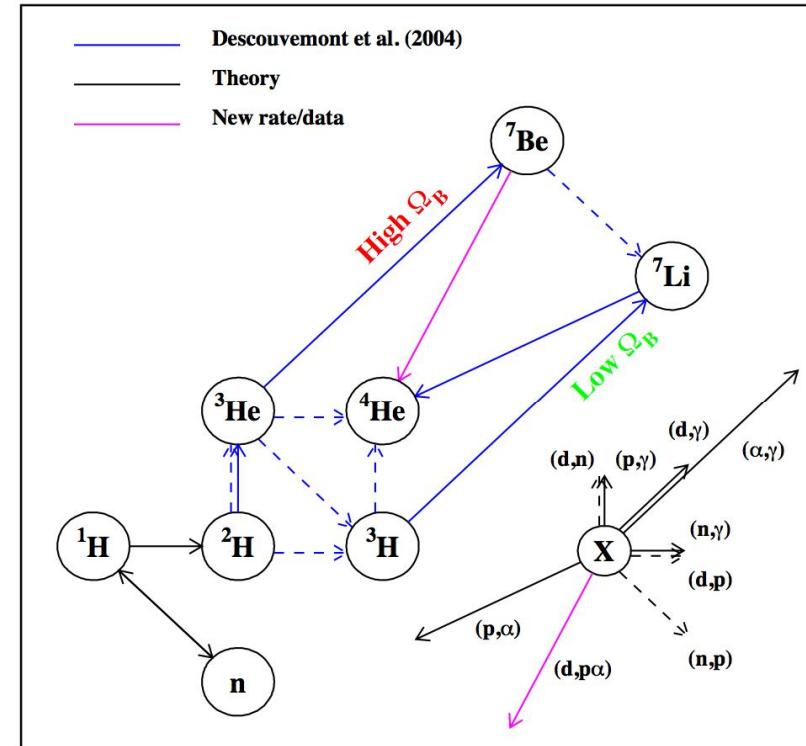
Tentative nuclear solutions:
 ^7Be destruction by:

1. Supplementary reactions]
e.g. $^7\text{Be}(d,p)^8\text{Be}^* \rightarrow 2\alpha$
2. Extra neutron sources in
 $^7\text{Be}(n,p)^7\text{Li}(p,\alpha)^4\text{He}$
destruction channel

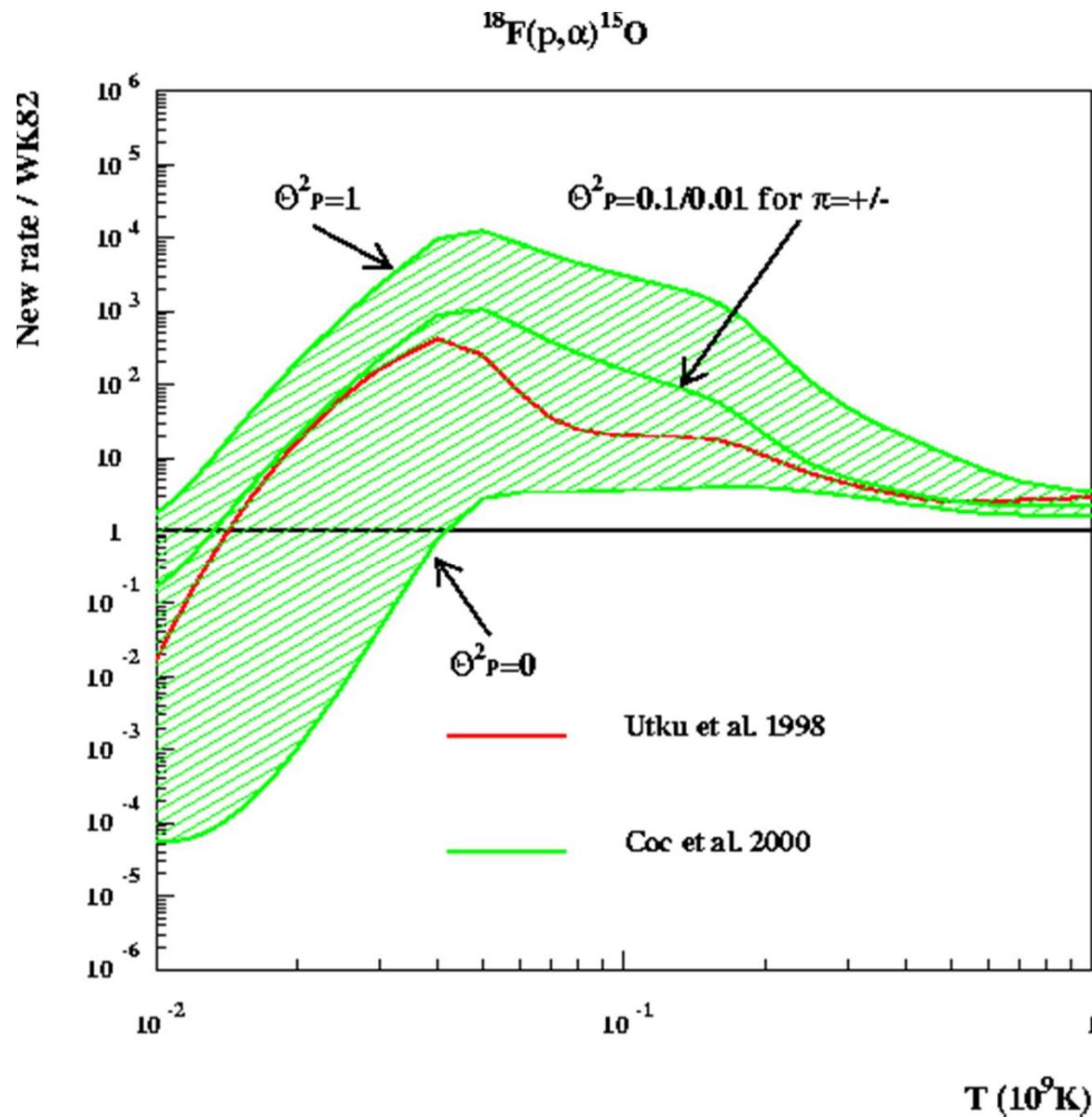
Other nuclear reaction rates

- About 100 other reactions involved in SBBN from H to B
- Among them ≈ 40 remain whose uncertainty on rate is not available
 - Systematic check by varying the rates by factors of 10, 100, 1000.

An interesting case :
the $^{7}\text{Be}(\text{d},\text{p})2\alpha$ reaction



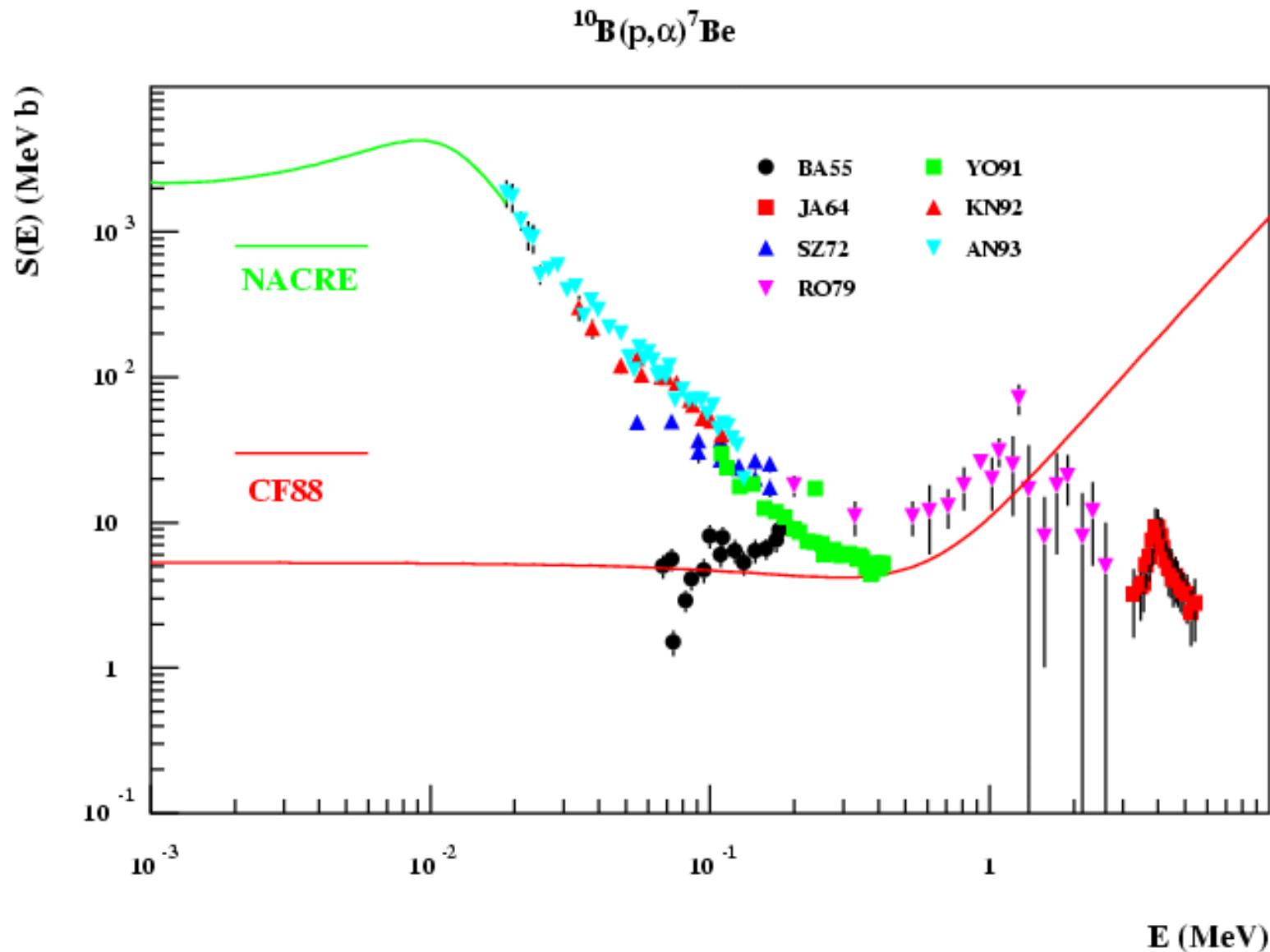
Example: uncertainties on $^{18}\text{F}(\text{p},\alpha)^{15}\text{O}$ reaction



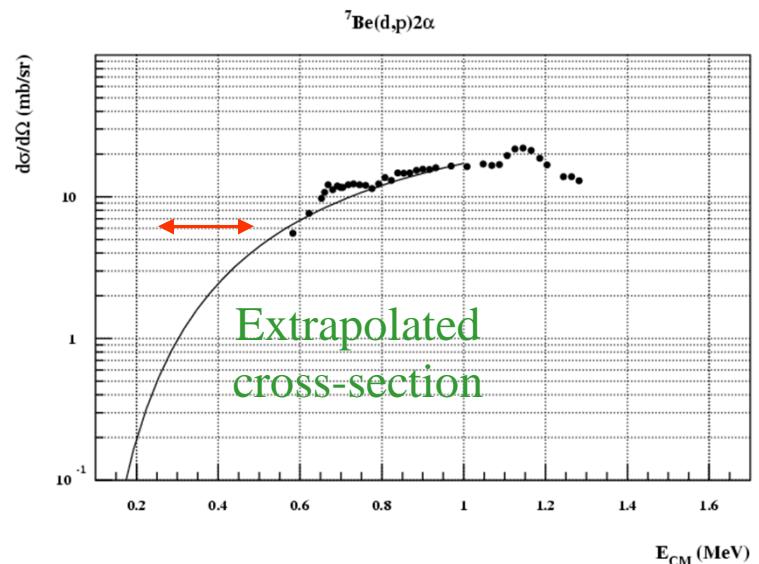
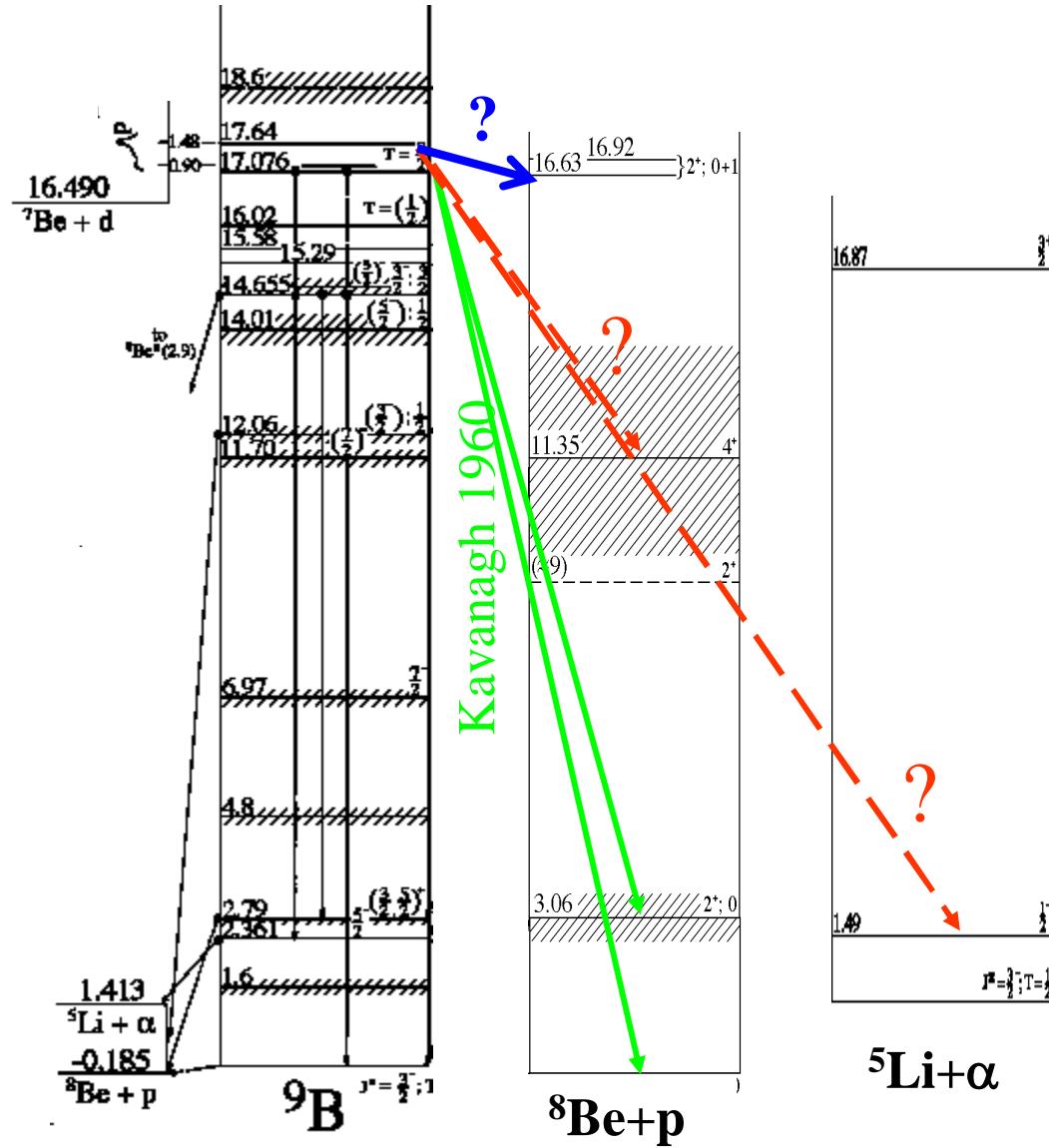
Comparison w.r.t. CF88

- A factor of 300 uncertainty in 2000
- After many experiments, the situation is still confused in 2014

Example: experimental data on $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$ reaction



The ${}^7\text{Be}(\text{d},\text{p})2\alpha$ and ${}^7\text{Be}(\text{d},\alpha){}^5\text{Li}$ reactions

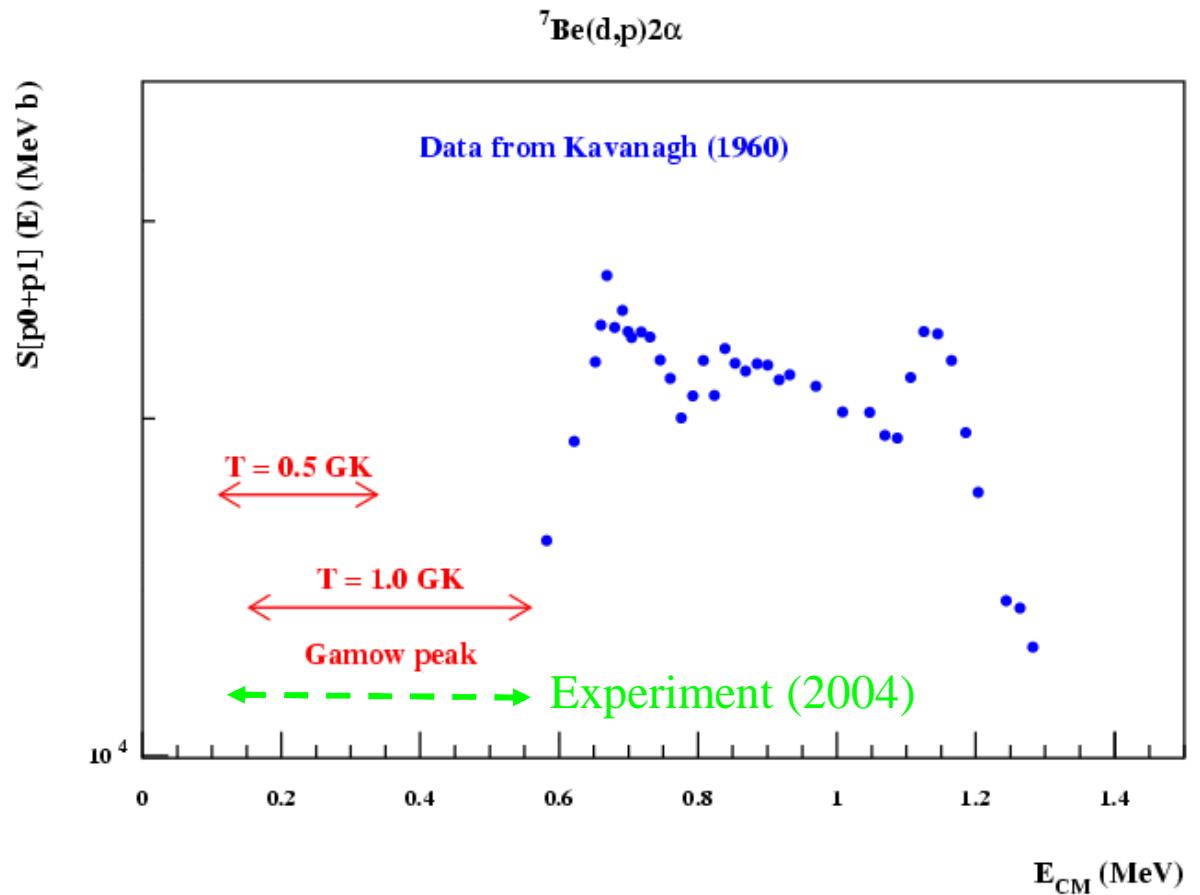


Experimental data on ${}^7\text{Be}(\text{d},\text{p})2\alpha$ reaction



Reaction rate [CF88] from
an estimate [Parker, 1972]
based on partial experimental
data [Kavanagh, 1960]

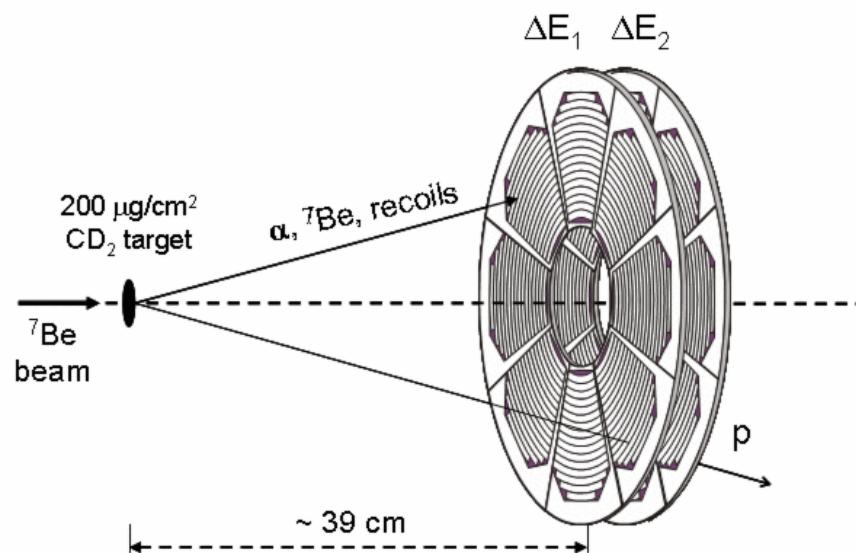
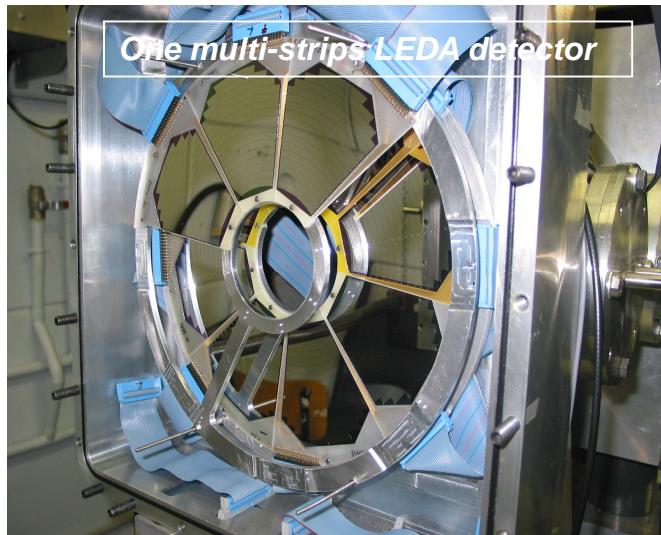
No data at BBN energies!



Reverse kinematics : ${}^7\text{Be}$ beam (Louvain la Neuve) and CD_2 target.
(CRC (Louvain LN), CSNSM, Edinburgh, ULB)

The ${}^7\text{Be}(\text{d},\text{p})2\alpha$ experiment

Centre de Recherche du Cyclotron, Louvain-la-Neuve



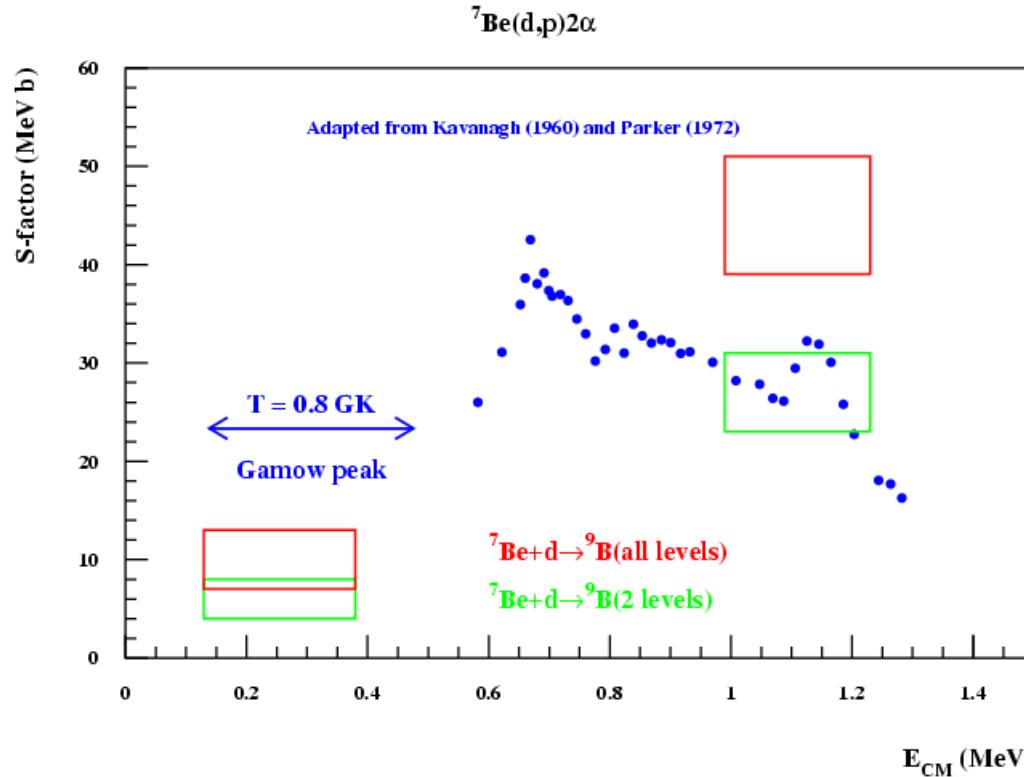
Beam : 0.2-1. 10^7 pps of ${}^7\text{Be}$ at 5.8 MeV,
degraded to 1.8 MeV (**0.4 MeV c.m.**)

Target : 200 $\mu\text{g}/\text{cm}^2$ CD_2 polyethylene

Detectors : two (LEDA) multistrips
(8×16) Si detectors (300 and 500 μm)

$^{7}\text{Be}(\text{d},\text{p})2\alpha$ cross-section

Integrated ($\Delta E \approx 0.23$ MeV) cross section [Angulo et al. , ApJL (2005)]



- ^{9}B ground state and first excited level contribution (comparison with Kavanagh)
 - All ^{9}B level contribution
 - No cross section enhancement

Section efficace résonnante

Section efficace de Breit-Wigner

Mesure directe

$$\sigma_{BW}(E) = \pi \lambda^2 \omega \frac{\Gamma_e(E) \Gamma_s(E+Q)}{(E - E_R)^2 + \Gamma_T^2(E)/4}$$

Mesures indirecte

- ω = le facteur statistique de spin
- Γ_x = la largeur partielle pour la voie $x = p, n, \alpha, \gamma, \beta, \dots$, (avec un moment angulaire transféré L)

➤ $\Gamma(E) \propto P_L(E)$ pour des particules chargées

➤ $\Gamma(E) \propto E^{1/2}$ pour des neutrons ($L=0$)

➤ $\Gamma(E) \propto E^{2L+1}$ pour des photons

Avec $\Gamma_T = \sum_x \Gamma_x$ la largeur totale et $\tau = \hbar \Gamma^{-1}$

Largeur à une particule

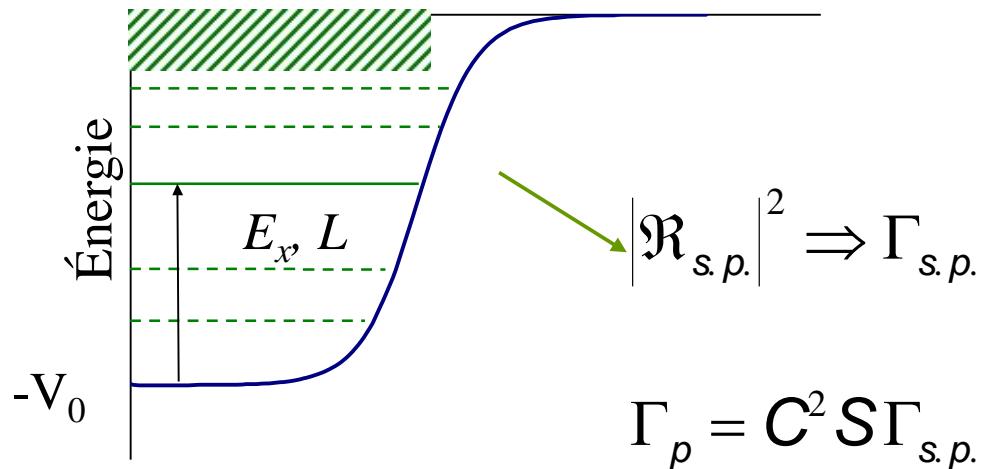
P_L : Facteur de pénétrabilité

s : rayon de la voie
 μ : masse réduite

$$\Gamma_p(E) = \frac{\hbar^2 s}{\mu} |\mathcal{R}(s)|^2 P_L(E, s)$$

Rayon (R, s)

Fonction d' onde radiale



Une particule dans un puis de potentiel de Woods-Saxon

$$\frac{-V_0}{1 + \exp\left(\frac{R - R_0}{a}\right)}$$

$$\Gamma_p = C^2 S \Gamma_{s.p.}$$

avec $C^2 S \leq 1$ (facteur spectroscopique)

$$S = \left| \langle A+1 | (|A\rangle \otimes |p\rangle) \right|^2$$

Approximation de la limite de Wigner (Γ_W):

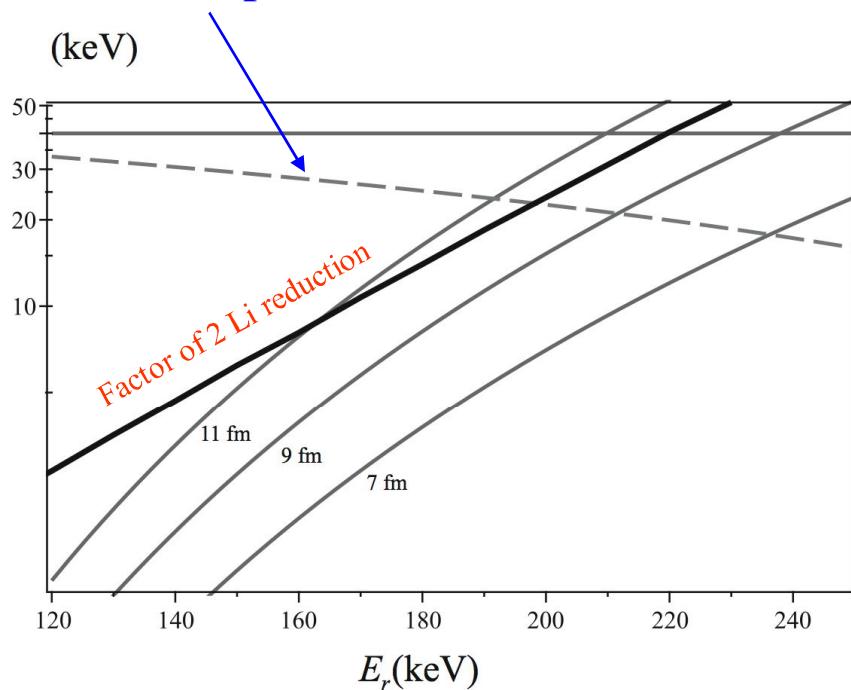
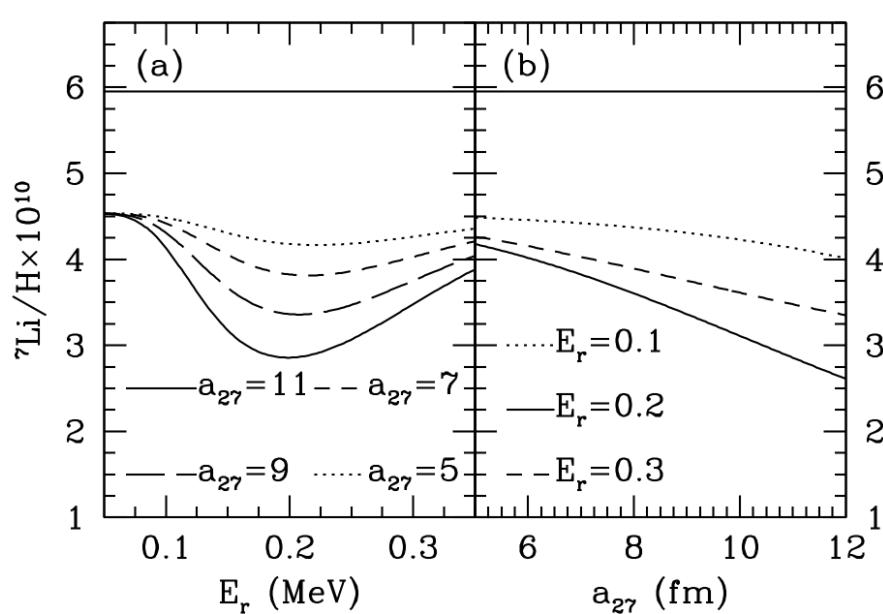
Densité de probabilité uniforme \Rightarrow

$$|\mathcal{R}_W|^2 = \frac{3}{s^3} \quad \Gamma_p = \theta^2 \Gamma_W \quad \theta^2 < 1$$

A new resonance in ${}^7\text{Be}(\text{d},\text{p})2\alpha$???

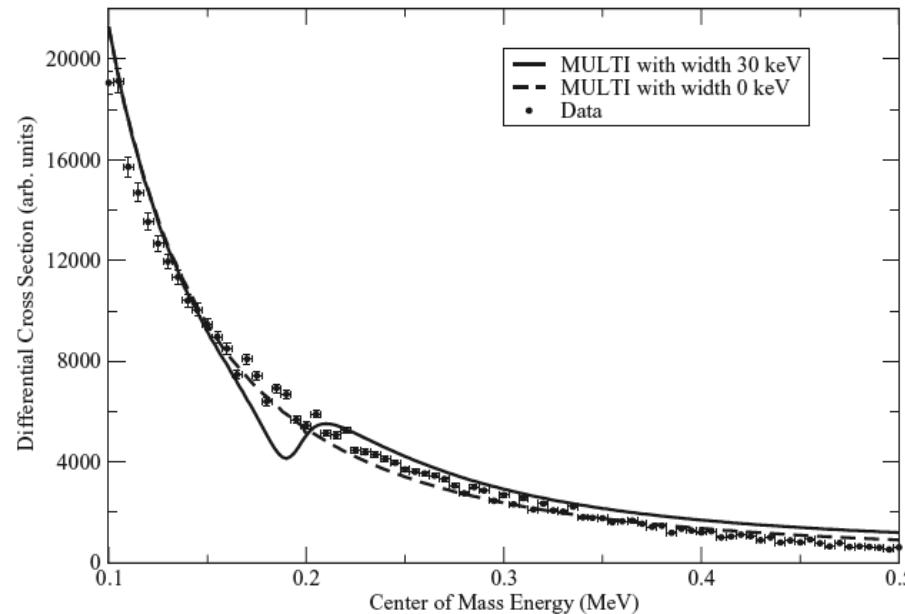
□ Hypothetical resonance at $E_R = 200 \pm 100$ keV with $\Gamma \leq 40$ keV [Cyburt & Pospelov 2009]

- corresponding to a ${}^9\text{B}$ level analog of the 16.7 MeV $5/2^+$ one in ${}^9\text{Be}$
- extreme Γ_d value at “Wigner limit” with very large interaction radius
- within limits given by Louvain-la-Neuve experiment

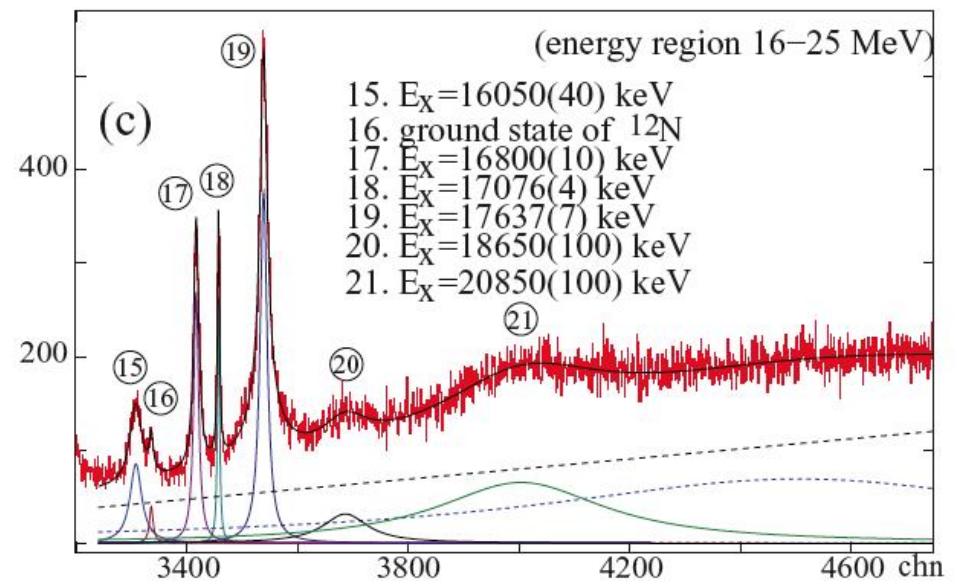


Search for new resonances in ${}^7\text{Be}(\text{d},\text{p})2\alpha$

Search for resonance in ${}^2\text{H}({}^7\text{Be},\text{d}){}^7\text{Be}$
with a ${}^7\text{Be}$ beam at Oak Ridge
[O'Malley+ 2011] shows no sign of large
 Γ_d



High resolution ${}^9\text{Be}({}^3\text{He},\text{t}){}^9\text{B}$ analysis at
Osaka *[Scholl+ 2011]* gives
 $E_X=16.800(10)$ MeV and $\Gamma_T=81(5)$ keV



Effect on ${}^7\text{Li} < 4\%$ in BBN *[Kirsebom & Davids 2011]*

Other resonances ?

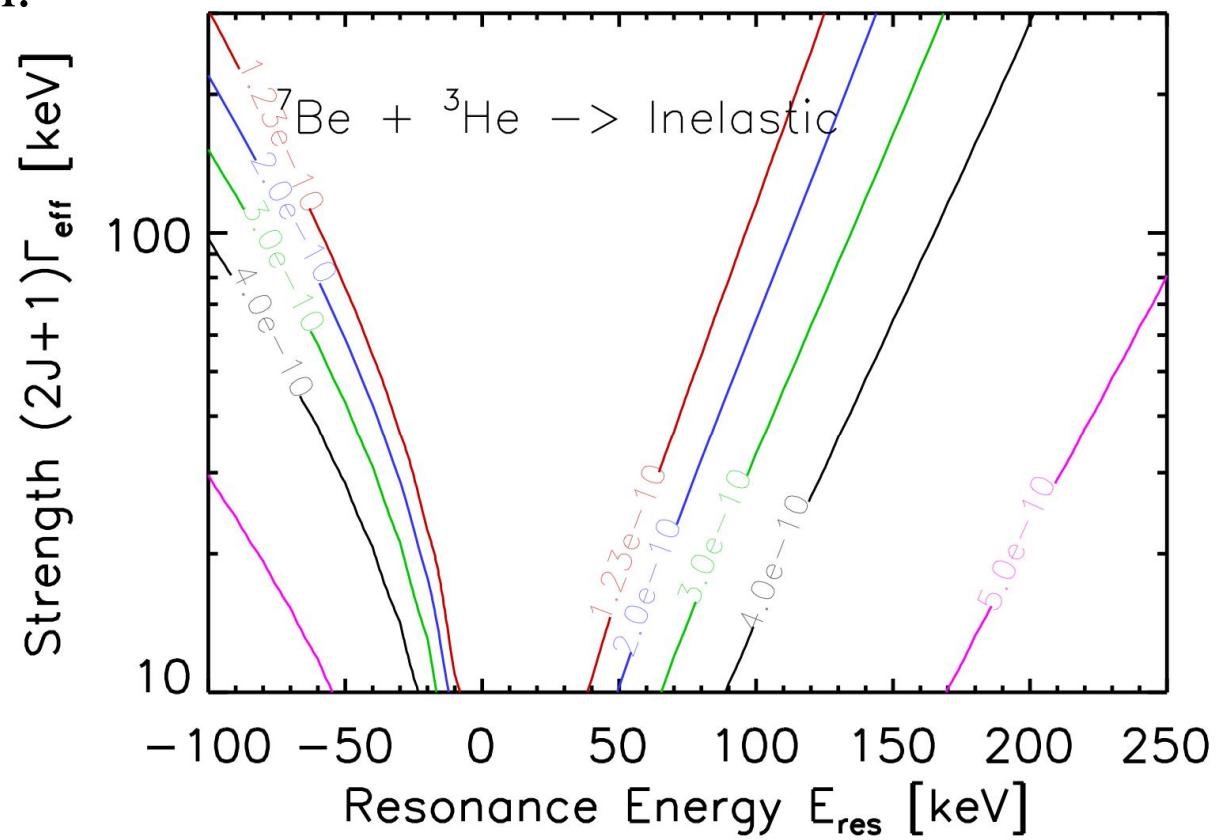
[Chakraborty, Fields & Olive (2011)]

Unknown Resonances in ${}^7\text{Be} + \text{n}, \text{p}, \text{d}, \text{t}, {}^3\text{He}$ and α ?

${}^8\text{Be}, {}^{9,10,11}\text{B}, {}^{10,11}\text{C}$ c.n.

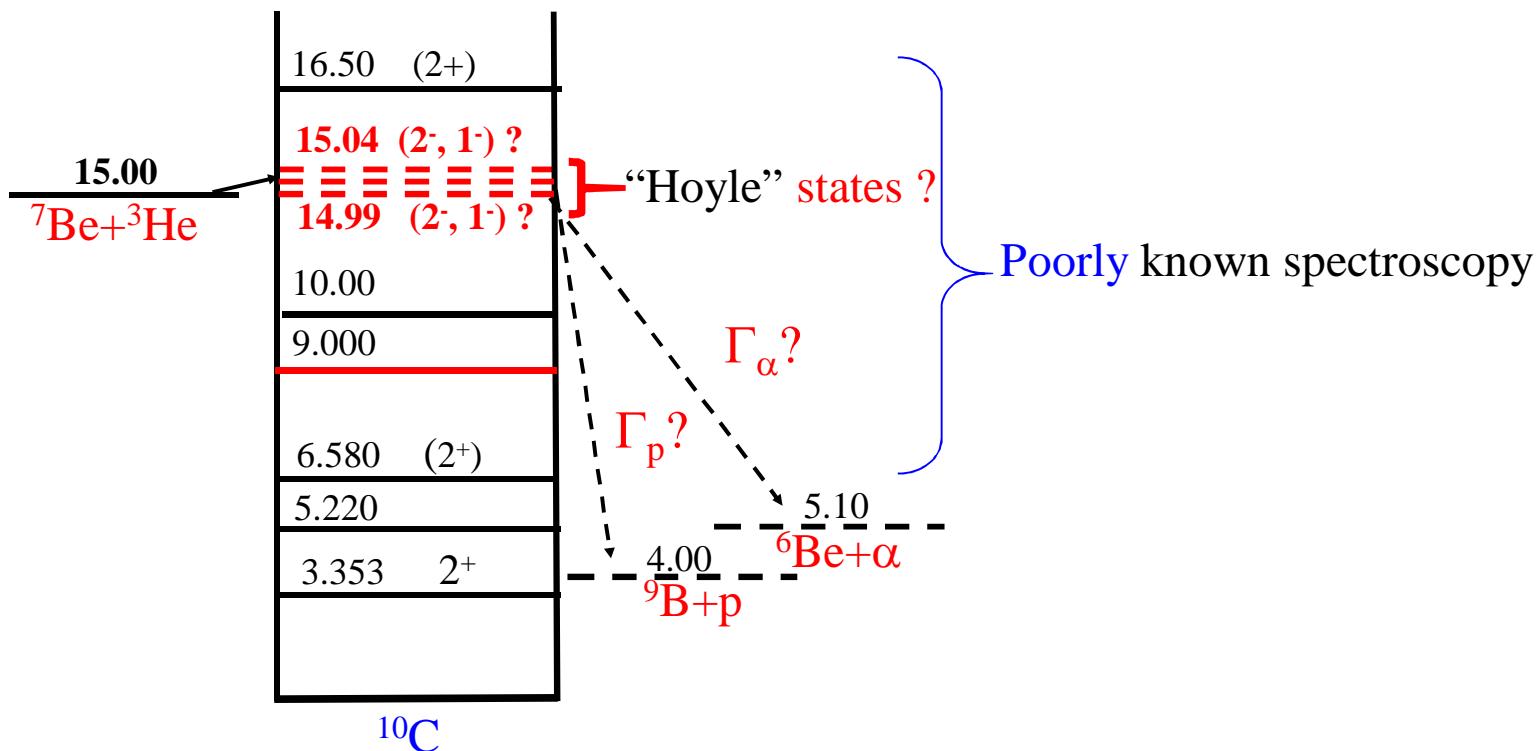
Unknown level properties in ${}^{9,11}\text{B}$ and unknown levels in ${}^{10}\text{C}$

- ${}^7\text{Be}(\text{t,p}){}^9\text{Be}$
- ${}^7\text{Be}(\text{t,}{}^3\text{He}){}^7\text{Li}$
- ${}^7\text{Be}(\text{t,}\alpha){}^6\text{Li}$
- ${}^7\text{Be}({}^3\text{He,2p}) 2\alpha$



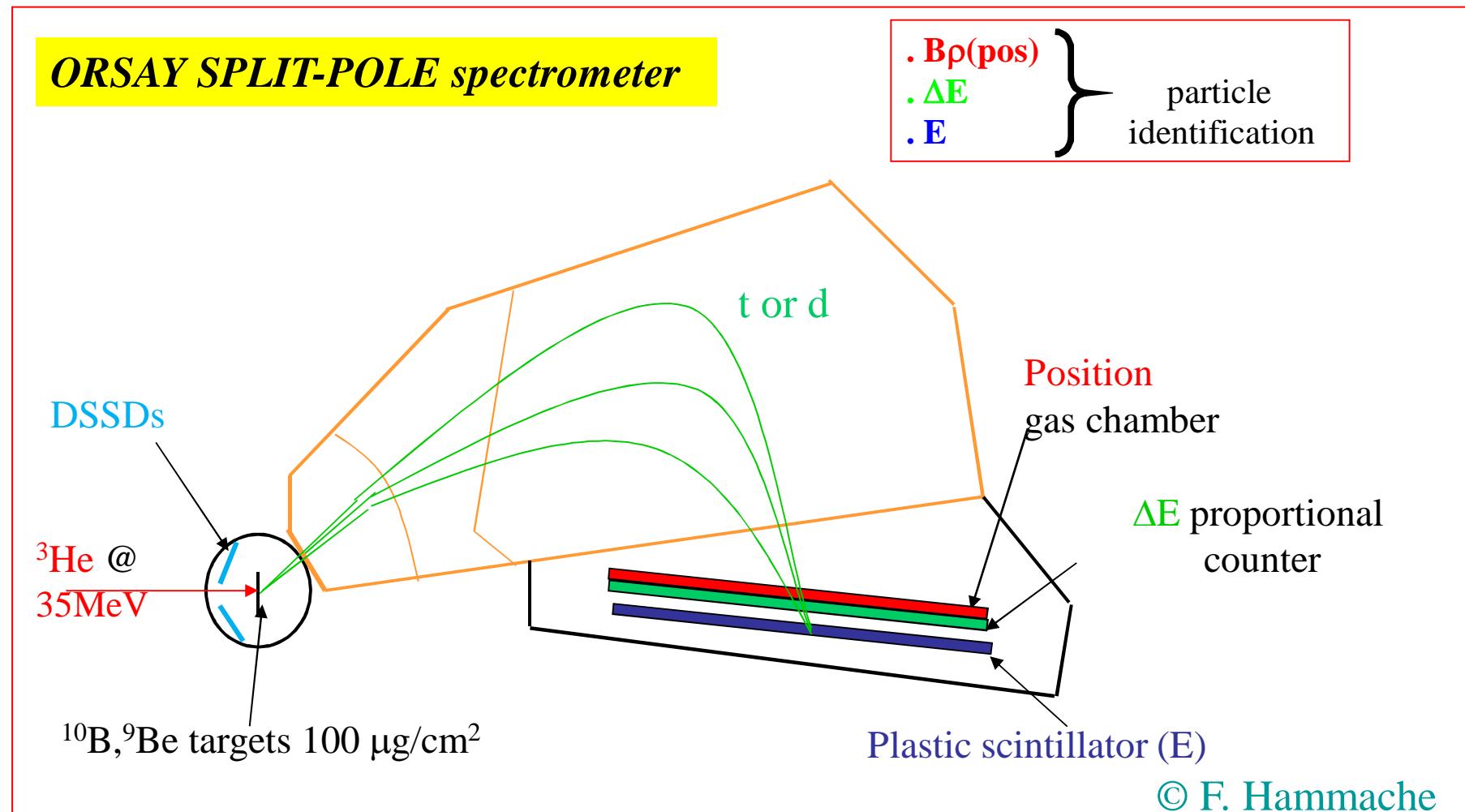
Spectroscopy Status of ^{10}C , ^9B & ^{10}B

^{10}C case



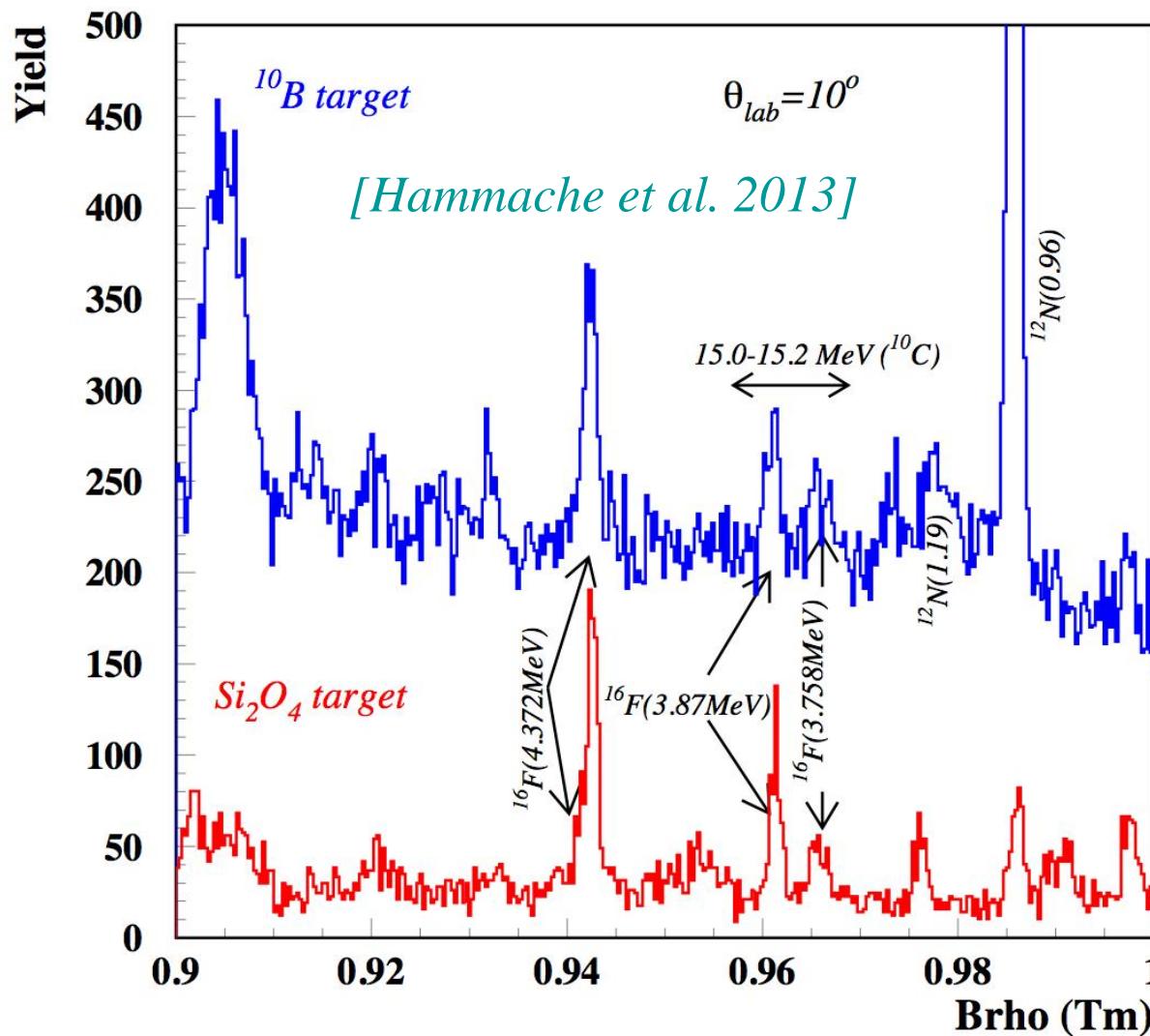
Recent Orsay Tandem Experiment

Indirect study of ^{10}C , ^9B & ^{10}B states via $(^3\text{He},t)$, $(^3\text{He},d)$ reactions
on ^{10}B and ^9Be targets

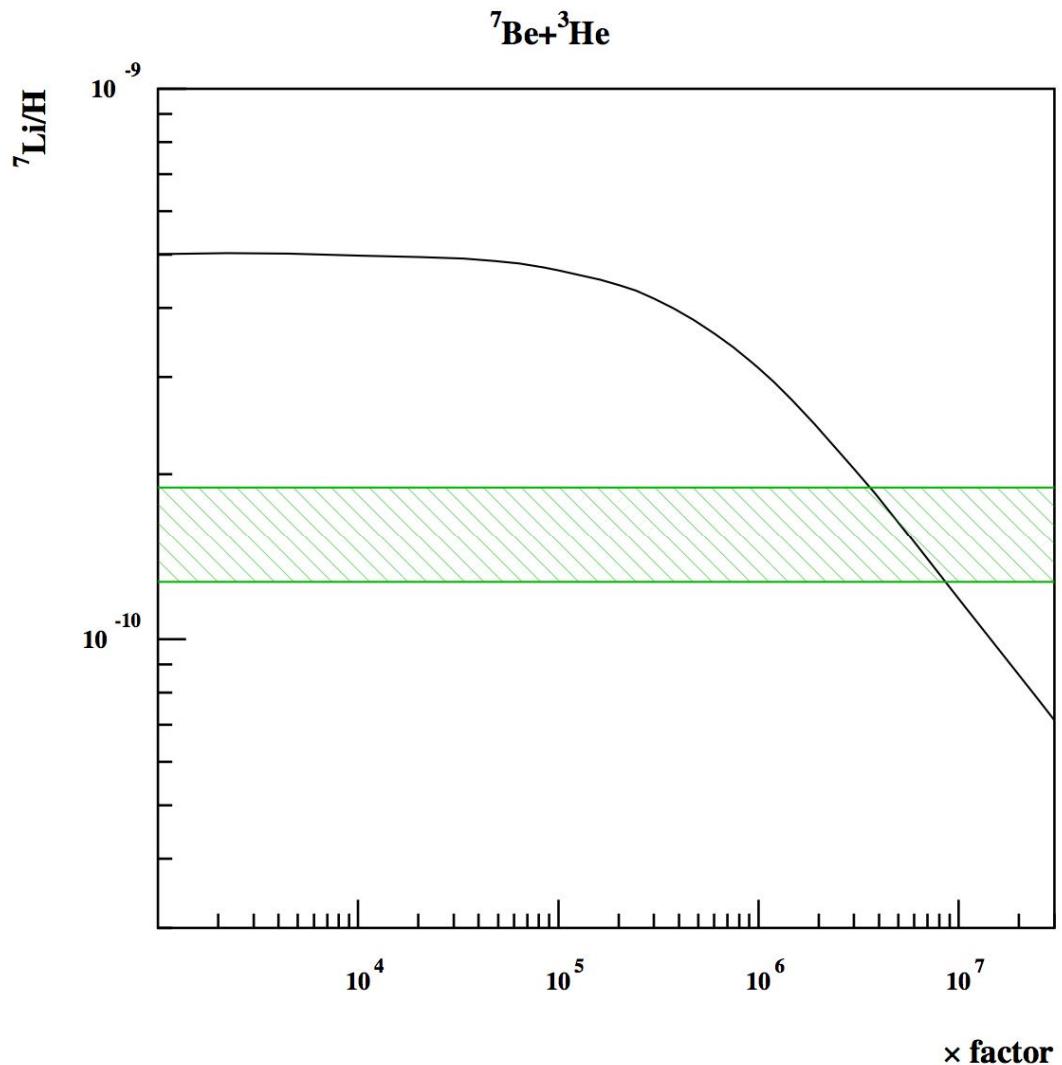


Results from Orsay Tandem experiment

In search of new ^{10}C levels: $^{10}\text{B}({}^3\text{He}, \text{t})^{10}\text{C}$



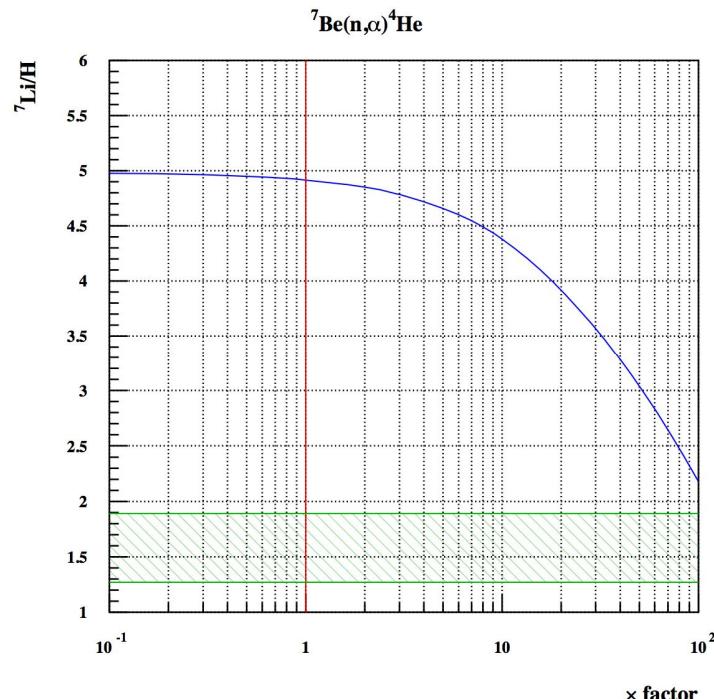
- No new ^{10}C level (^{12}C ^{16}O contaminants in spectra) nor in ^{11}C [${}^7\text{Be} + \alpha$]
 - Broad levels?
 - No ${}^7\text{Be} + {}^3\text{He}$ or ${}^7\text{Be} + \alpha$ rate increase
- [Hammache et al. 2013]*



Rate from experimental upper limits too low by $\sim 10^6$ factor !
Unlikely to contribute (Coulomb barrier)
[Broggini+ 2012] and F. Villante seminar tomorrow

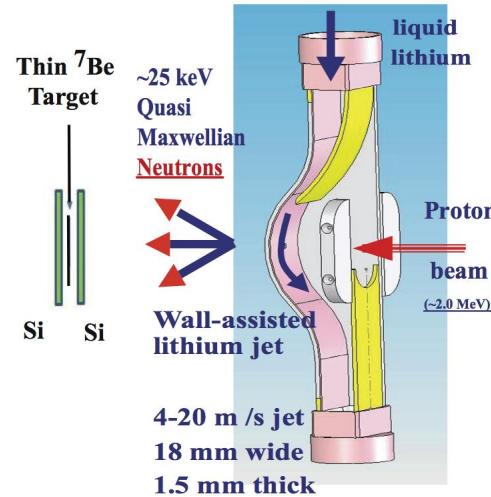
The ${}^7\text{Be}(\text{n},\alpha){}^4\text{He}$ reaction

- If $l=0$, ${}^7\text{Be}+\text{n} \rightarrow 2\alpha$ (forbidden) rate from *Wagoner 1969*, much slower than ${}^7\text{Be}(\text{n},\text{p}){}^7\text{Li}$ (data from *Bassi et al. 1962; 1963*)
- If $l=1$, could contribute to ${}^7\text{Be}$ destruction [*Serpico+ 2004*]



The LiLiT at the SARAf facility in the
Soreq Nuclear Center, Israel

For the Study of the ${}^7\text{Be}(\text{n},\alpha)\alpha$ Reaction



- Experimental project with n beam from Liquid Lithium Target at the Soreq Accelerator Facility [*M. Gai, M. Paul priv. comm.*]

Nuclear Physics aspects

1. The main reactions for H, He and Li BBN production
2. Search for nuclear solutions to the lithium problem
3. Be, B and CNO production in BBN
4. Monte-Carlo evaluation of reaction rates
5. Monte-Carlo investigation of BBN

Extension of the network up to CNO

➤ Applications:

- Potential neutron sources for ^7Be destruction by $^7\text{Be}(\text{n},\text{p})^7\text{Li}(\text{p},\alpha)^4\text{He}$ in BBN (the lithium problem)
- CNO seeds for first stars
- Standard CNO primordial abundances versus exotic production (e.g. “variation of fundamental constants”)
- Future observations ?

➤ Involves many (>400), $^A\text{Z} + \text{n}$, p, d, t, ^3He and α , reactions

➤ *Mostly unknown rates* hence possibly high yield uncertainty

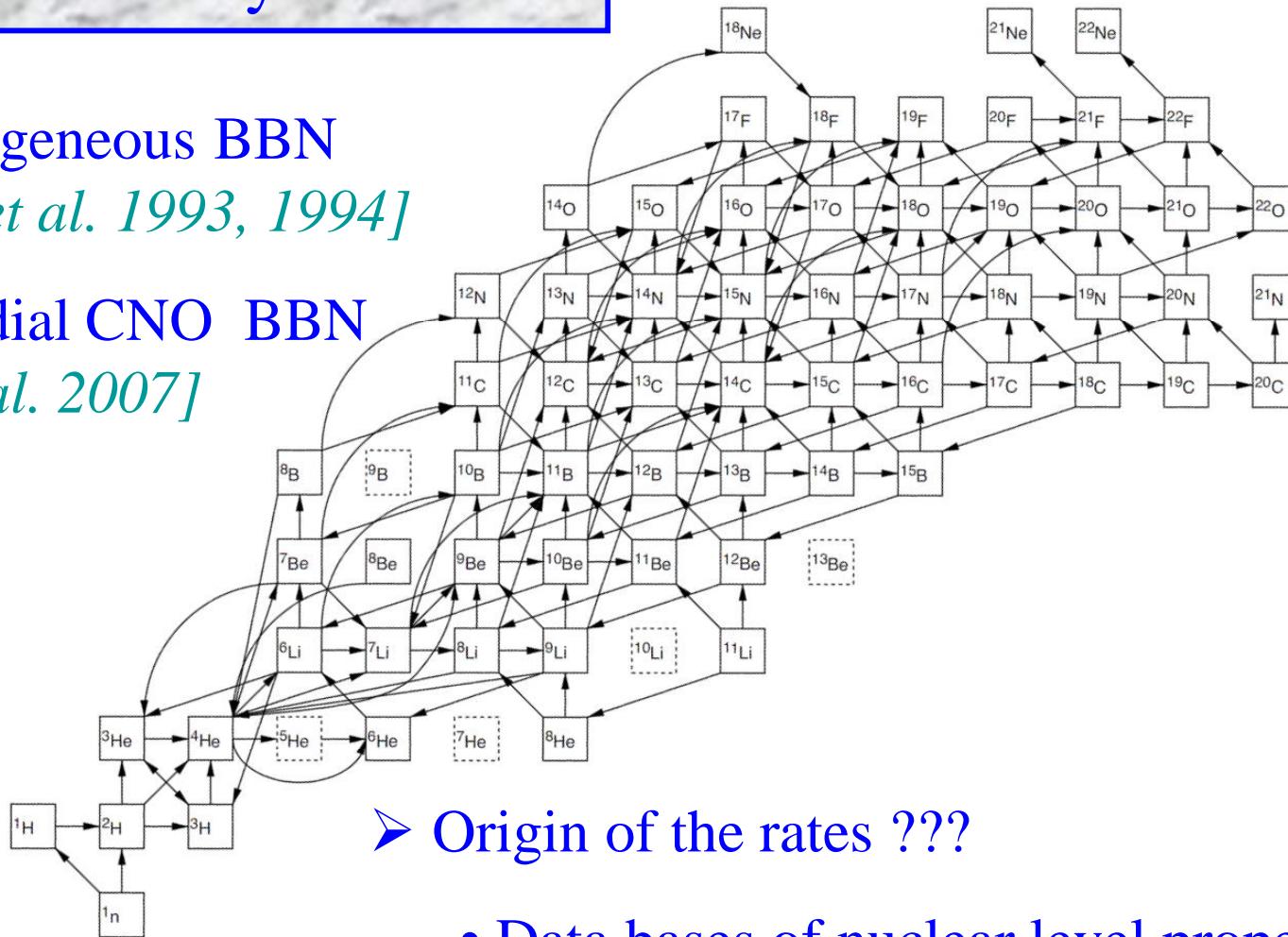
CNO nucleosynthesis

➤ Inhomogeneous BBN

[Thomas et al. 1993, 1994]

➤ Primordial CNO BBN

[Iocco et al. 2007]



➤ Origin of the rates ???

- Data bases of nuclear level properties
- Estimates following *Fowler & Hoyle 1964; Wagoner 1967* prescriptions

©Fields & Olive 2006

Sensitivity study (I)

1. Start from “best guess” set of reaction rates
2. Systematic, one by one, reaction rate variation within evaluated rate uncertainty or guessed uncertainty factor e.g. 0.001, 0.01, 0.1, 10., 100. & 1000.
3. Identification of important reactions e.g. variation >20% on selected yields
4. Reevaluation of selected rates following re-analyses of available experimental and theoretical data
5. Calculations with set of improved reaction rates (and uncertainties)

CNO nucleosynthesis updated network

➤ 59 isotopes :

| Z | A |
|----|---------|
| n | 1 |
| H | 1-3 |
| He | 3,4,6 |
| Li | 6-9 |
| Be | 7,9-12 |
| B | 8,10-15 |
| C | 9-16 |
| N | 12-17 |
| O | 13-20 |
| F | 17-20 |
| Ne | 18-23 |
| Na | 20-23 |

➤ 391 reaction rates ${}^A Z + n, p, d, t, {}^3 He$ and α , *mostly unknown* hence possibly high yield uncertainty

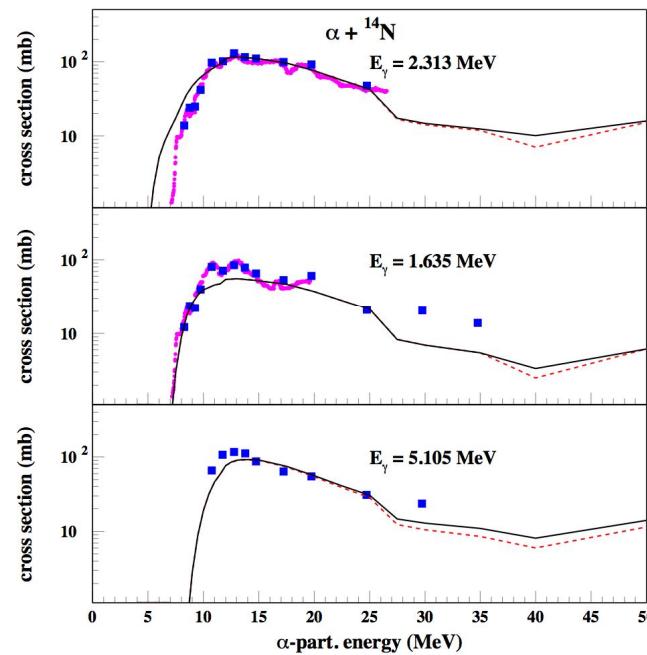
- ✓ *Descouvemont et al. 2004 (DAACV)*
- ✓ *Angulo et al. 1999 (NACRE I)*
- ✓ *Iliadis et al. 2010*
- ✓ *Talys (271 rates)* within 3 orders of magnitude, at $T \approx 1$ GK, compared with experiments!
- ✓

➤ 33 decay rates
[Audi et al. 2003]

Talys code [Koning et al. 2005; Goriely et al. 2008]

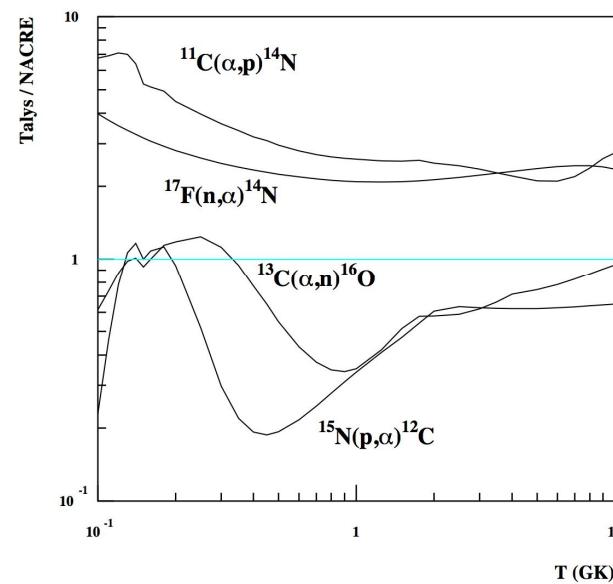
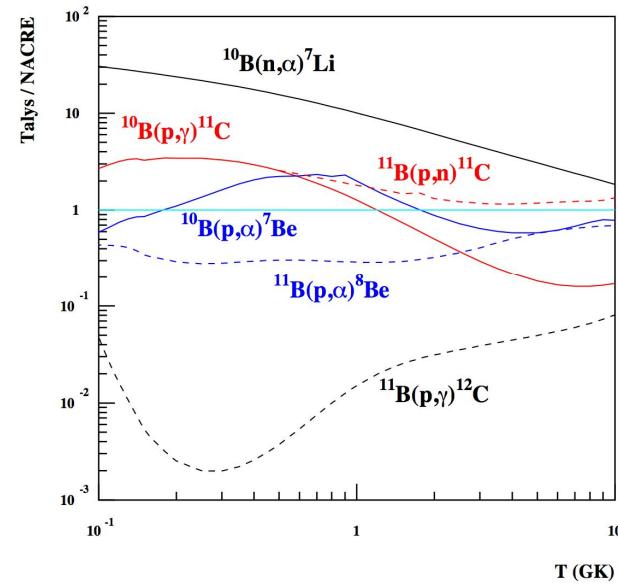
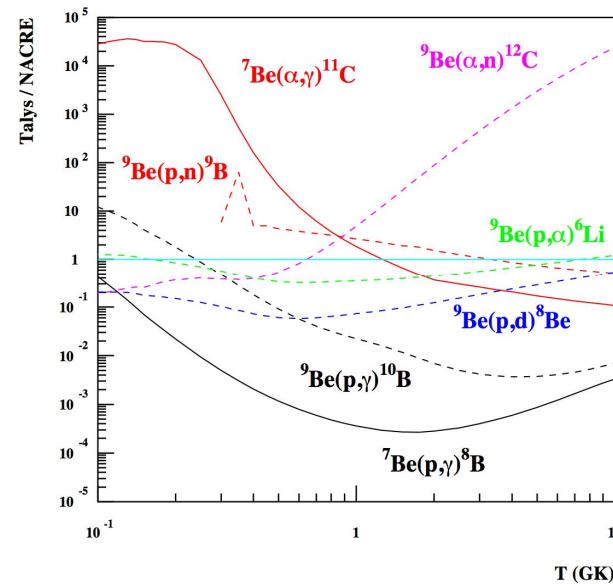
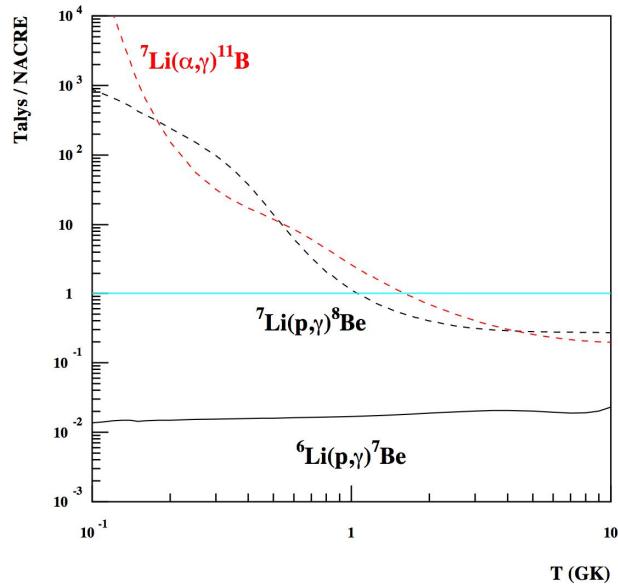
- Based on Hauser-Feshbach statistical model
 - Compound nucleus mechanism assuming high level densities
 - $n, p, d, t, {}^3\text{He}$ and α capture and emission on $Z \geq 3$ nuclei
 - A priori not adapted to low mass nuclei but...

Comparison Orsay experiment with Talys
[Benhabiles-Mezhoud, Kiener et al. 2011]



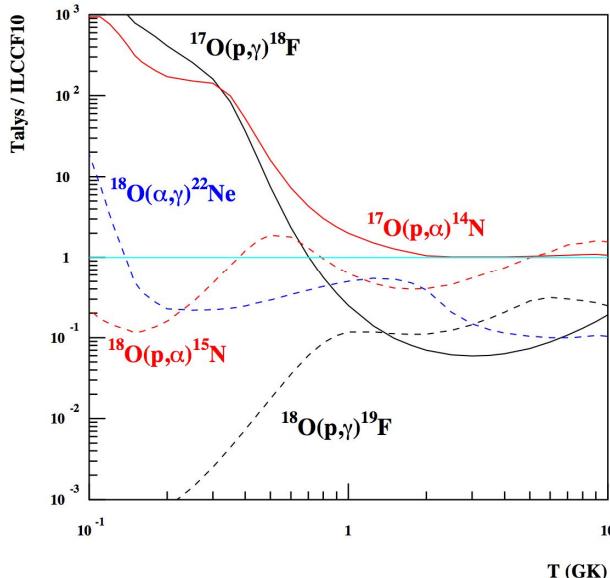
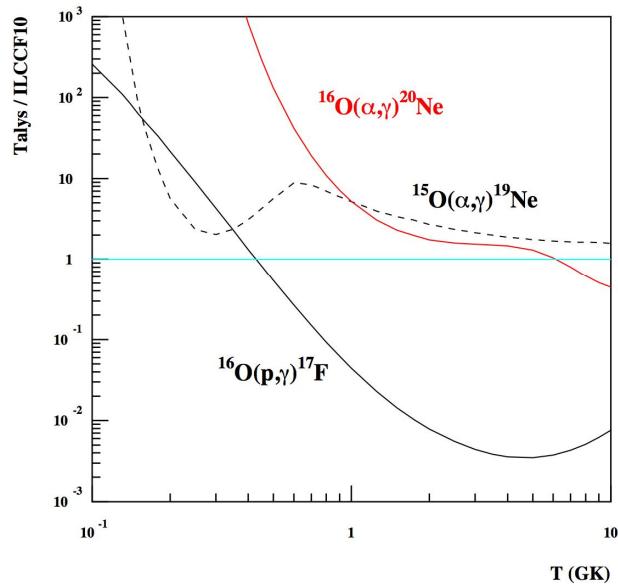
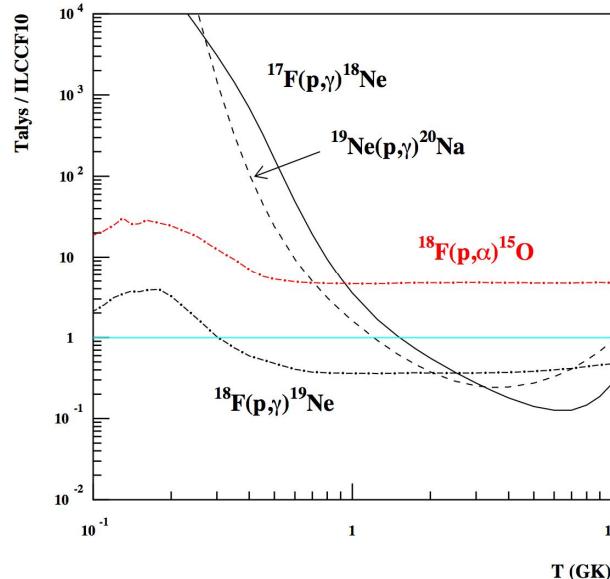
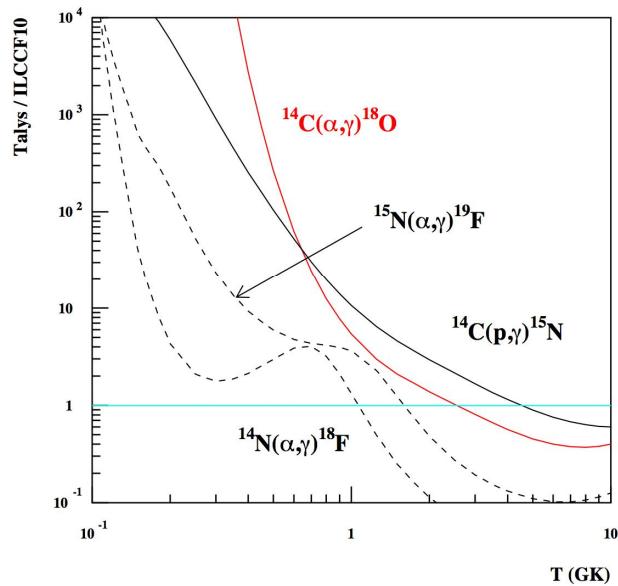
Comparison between Talys and experiments

[NACRE 1999]



Comparison between Talys and experiments

[Iliadis et al. 2010]



Most important reactions for CNO nucleosynthesis

| Reaction | Fractional change in CNO abundance | | | | | | Test rate reference |
|---|------------------------------------|------|------|------|------|-------|---------------------|
| Rate factor | 0.001 | 0.01 | 0.1 | 10. | 100. | 1000. | |
| $^7\text{Li}(\text{d},\gamma)^9\text{Be}$ | 1.00 | 1.00 | 1.00 | 1.01 | 1.11 | 2.10 | TALYS |
| $^7\text{Li}(\text{d},\text{n})2\alpha$ | 1.66 | 1.65 | 1.55 | 0.28 | 0.06 | 0.02 | Boy93 |
| $^7\text{Li}(\text{t},\text{n})^9\text{Be}$ | 0.99 | 0.99 | 0.99 | 1.10 | 2.14 | 11.7 | Bru90 |
| $^7\text{Li}(\text{t},2\text{n})2\alpha$ | 1.00 | 1.00 | 1.00 | 0.99 | 0.91 | 0.53 | MF89 |
| $^8\text{Li}(\text{n},\gamma)^9\text{Li}$ | 1.00 | 1.00 | 1.00 | 1.01 | 1.06 | 1.62 | Rau94 |
| $^8\text{Li}(\text{t},\text{n})^{10}\text{Be}$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.23 | TALYS |
| $^8\text{Li}(\alpha,\gamma)^{12}\text{B}$ | 1.00 | 1.00 | 1.00 | 1.01 | 1.11 | 2.15 | TALYS |
| $^8\text{Li}(\alpha,\text{n})^{11}\text{B}$ | 0.89 | 0.89 | 0.90 | 1.97 | 11.2 | 78.1 | Miz01 |
| $^9\text{Li}(\alpha,\text{n})^{12}\text{B}$ | 1.00 | 1.00 | 1.00 | 1.01 | 1.08 | 1.73 | TALYS |
| $^{10}\text{Be}(\alpha,\text{n})^{13}\text{C}$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.03 | 1.28 | TALYS |
| $^{11}\text{B}(\text{n},\gamma)^{12}\text{B}$ | 0.91 | 0.91 | 0.92 | 1.81 | 9.91 | 87.7 | Rau94 |
| $^{11}\text{B}(\text{d},\text{n})^{12}\text{C}$ | 0.70 | 0.71 | 0.73 | 3.67 | 30.2 | 280. | TALYS |
| $^{11}\text{B}(\text{d},\text{p})^{12}\text{B}$ | 0.99 | 0.99 | 0.99 | 1.08 | 1.83 | 9.33 | TALYS |
| $^{11}\text{B}(\text{t},\text{n})^{13}\text{C}$ | 1.00 | 1.00 | 1.00 | 1.01 | 1.12 | 2.17 | TALYS |
| $^{11}\text{C}(\text{n},\gamma)^{12}\text{C}$ | 1.00 | 1.00 | 1.00 | 1.01 | 1.08 | 1.75 | Rau94 |
| $^{11}\text{C}(\text{d},\text{p})^{12}\text{C}$ | 0.99 | 0.99 | 0.99 | 1.05 | 1.55 | 5.67 | TALYS |
| $^{12}\text{C}(\text{t},\alpha)^{11}\text{B}$ | 1.00 | 1.00 | 1.00 | 1.00 | 0.97 | 0.75 | TALYS |
| $^{13}\text{C}(\text{d},\alpha)^{11}\text{B}$ | 1.00 | 1.00 | 1.00 | 0.96 | 0.84 | 0.75 | TALYS |

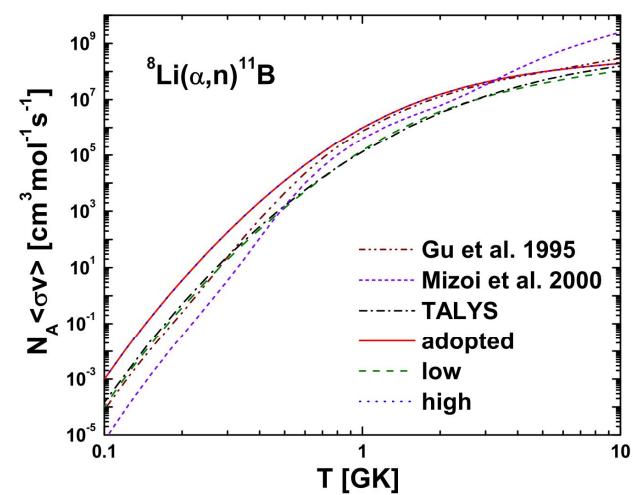
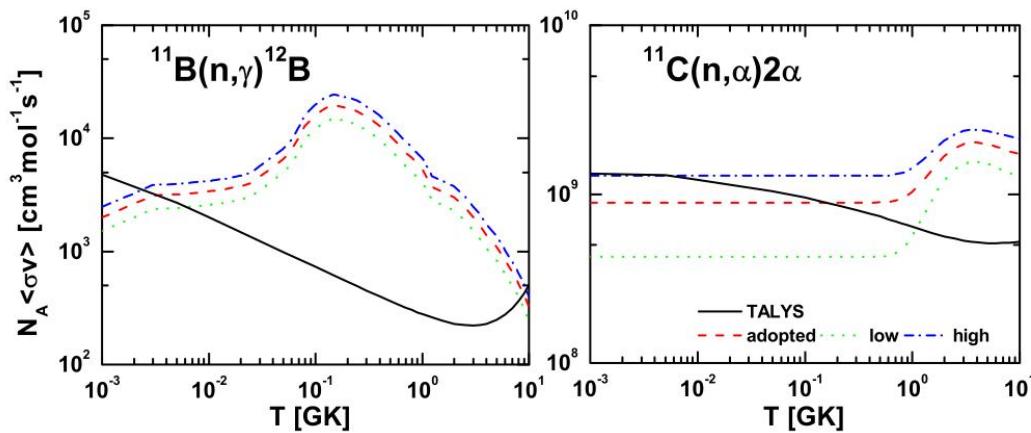
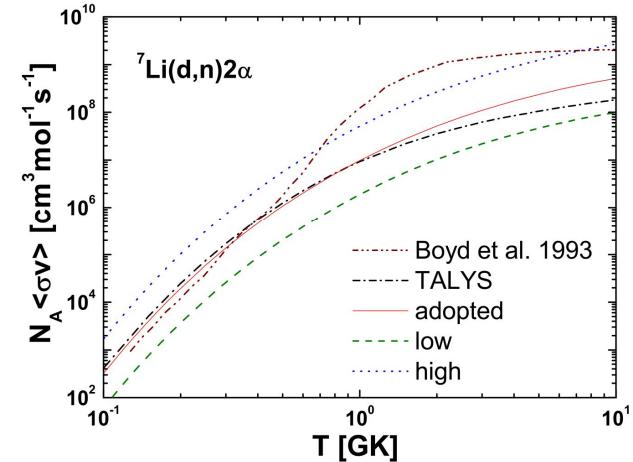
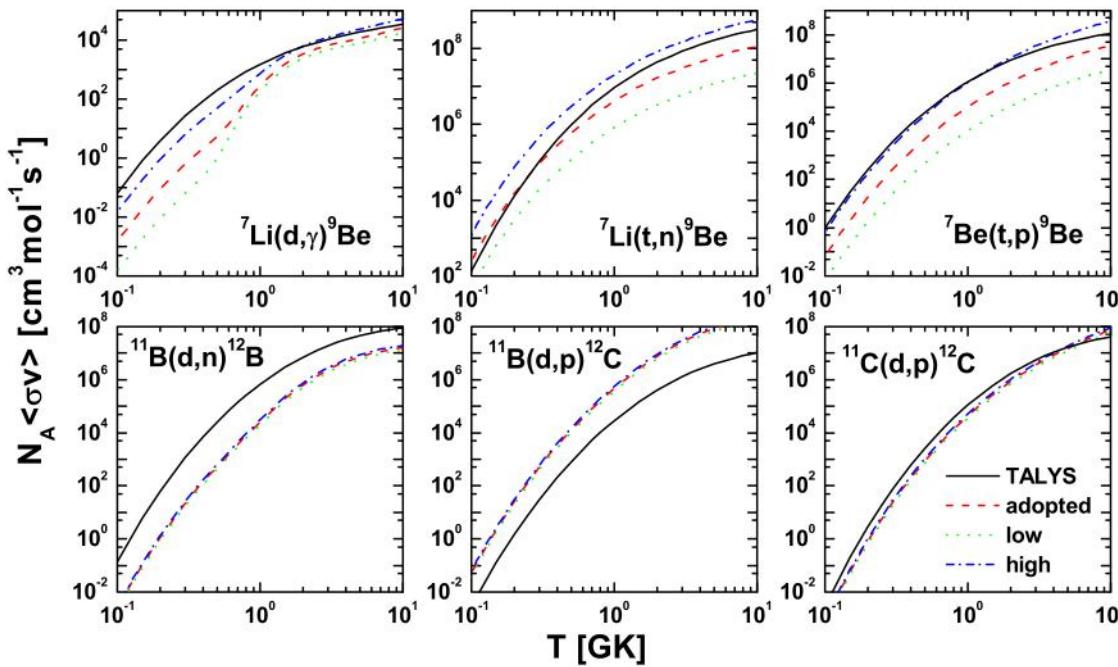
Most important reactions for ${}^6\text{Li} + {}^{11}\text{B}$ nucleosynthesis

| Reaction | Fractional change in ${}^6\text{Li}$ abundance | | | | | | Test rate reference |
|--|--|-------|-------|------|------|-------|---------------------|
| | 0.001 | 0.01 | 0.1 | 10. | 100. | 1000. | |
| ${}^3\text{He}(t,\gamma){}^6\text{Li}$ | 1.00 | 1.00 | 1.00 | 1.03 | 1.31 | 4.11 | FK90 |
| ${}^4\text{He}(d,\gamma){}^6\text{Li}$ | 0.004 | 0.013 | 0.010 | 9.97 | 99.7 | 995. | Ham10 |

| Reaction | Fractional change in ${}^9\text{Be}$ abundance | | | | | | Test rate reference |
|---|--|------|------|------|------|-------|---------------------|
| | 0.001 | 0.01 | 0.1 | 10. | 100. | 1000. | |
| ${}^7\text{Li}(d,\gamma){}^9\text{Be}$ | 0.83 | 0.83 | 0.85 | 2.52 | 17.7 | 170. | TALYS |
| ${}^7\text{Li}(t,n){}^9\text{Be}$ | 0.52 | 0.53 | 0.57 | 5.29 | 48.2 | 477. | Bru90 |
| ${}^7\text{Li}({}^3\text{He},p){}^9\text{Be}$ | 1.00 | 1.00 | 1.00 | 1.04 | 1.45 | 5.49 | TALYS |
| ${}^7\text{Be}(d,p)2\alpha$ | 1.01 | 1.01 | 1.01 | 0.95 | 0.67 | 0.38 | CF88 |
| ${}^7\text{Be}(t,p){}^9\text{Be}$ | 0.65 | 0.65 | 0.69 | 4.15 | 35.6 | 345. | TALYS |

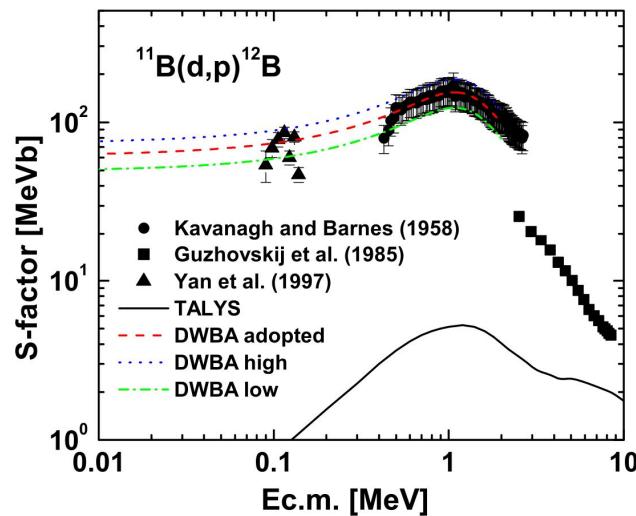
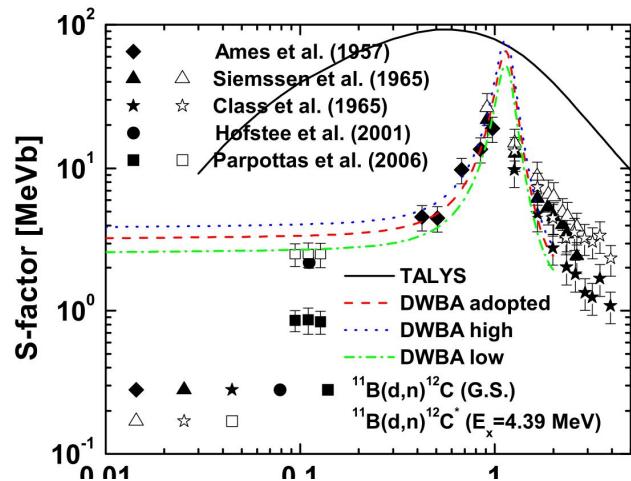
| Reaction | Fractional change in ${}^{11}\text{B}$ abundance | | | | | | Test rate reference |
|-------------------------------------|--|------|------|------|------|--------|---------------------|
| | 0.001 | 0.01 | 0.1 | 10. | 100. | 1000. | |
| ${}^3\text{He}(t,np){}^4\text{He}$ | 1.00 | 1.00 | 1.00 | 1.00 | 0.97 | 0.79 | CF88 |
| ${}^7\text{Be}(d,p)2\alpha$ | 1.01 | 1.01 | 1.01 | 0.93 | 0.55 | 0.11 | CF88 |
| ${}^{11}\text{C}(n, \alpha)2\alpha$ | 1.16 | 1.16 | 1.15 | 0.40 | 0.01 | 0.0001 | Rau94 |

Reevaluated reaction rates

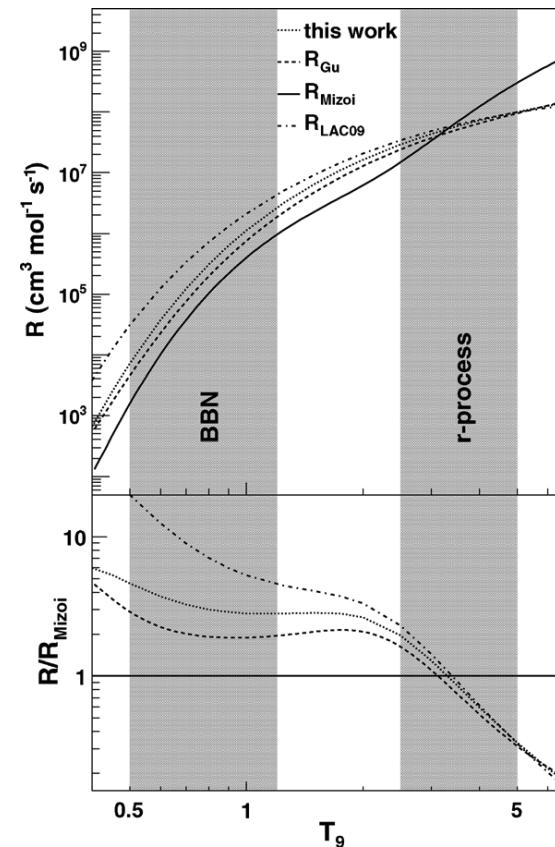


Stability of results with re-evaluated reaction rates

Changes in $^{11}\text{B}(\text{d},\text{n})$ by $^{11}\text{B}(\text{d},\text{p})$ cancel each other

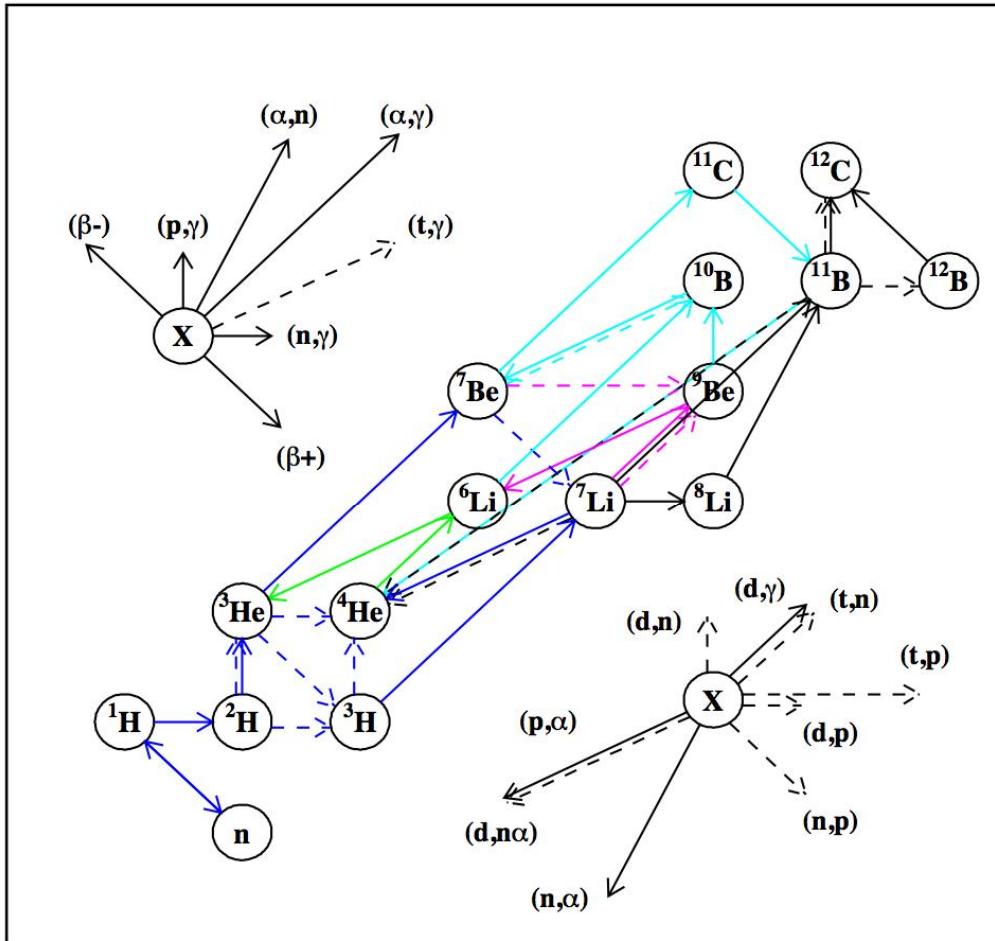


Independent re-evaluation of
 $^{8}\text{Li}(\alpha,\text{n})^{11}\text{B}$ by
La Cognata & Del Zoppo 2011



Changes CNO by 1.5%

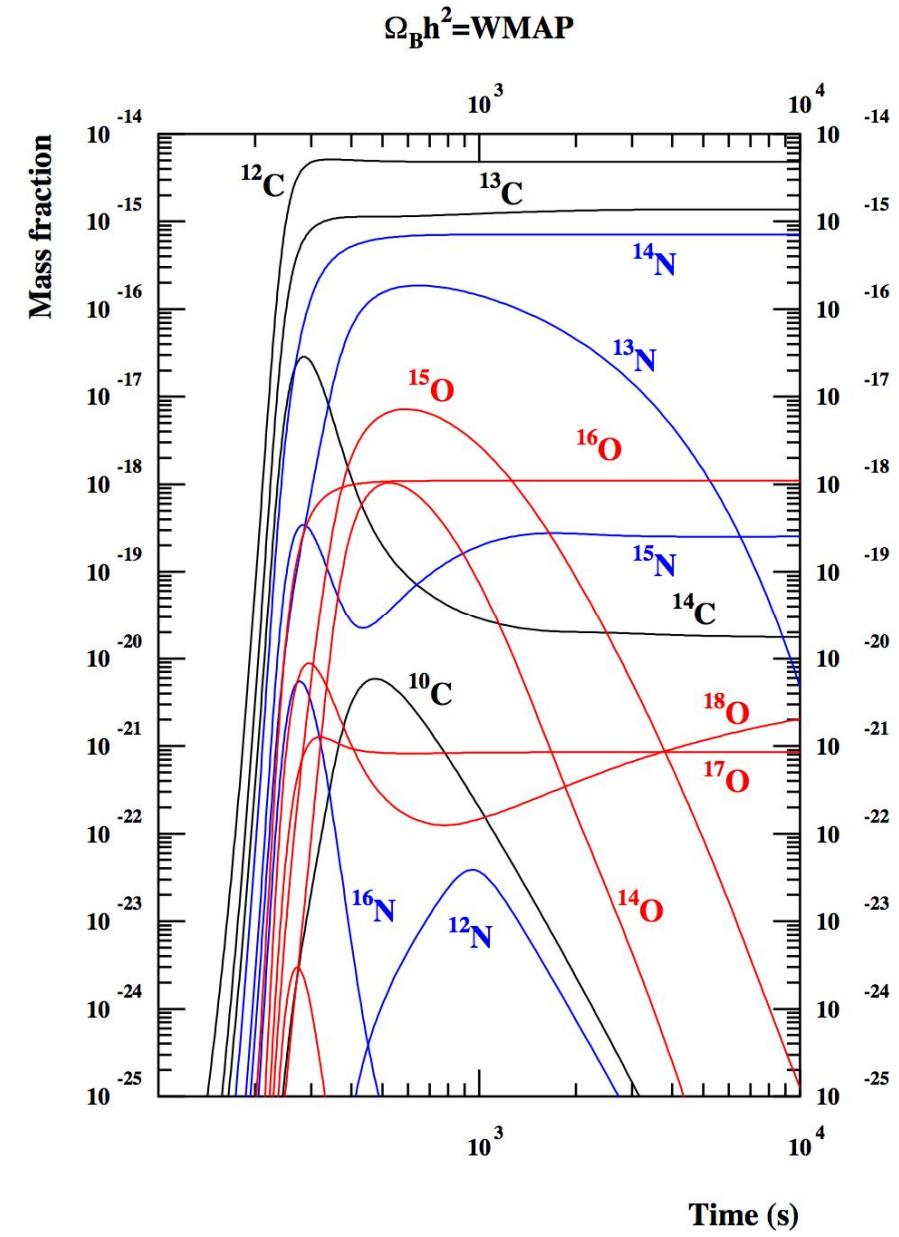
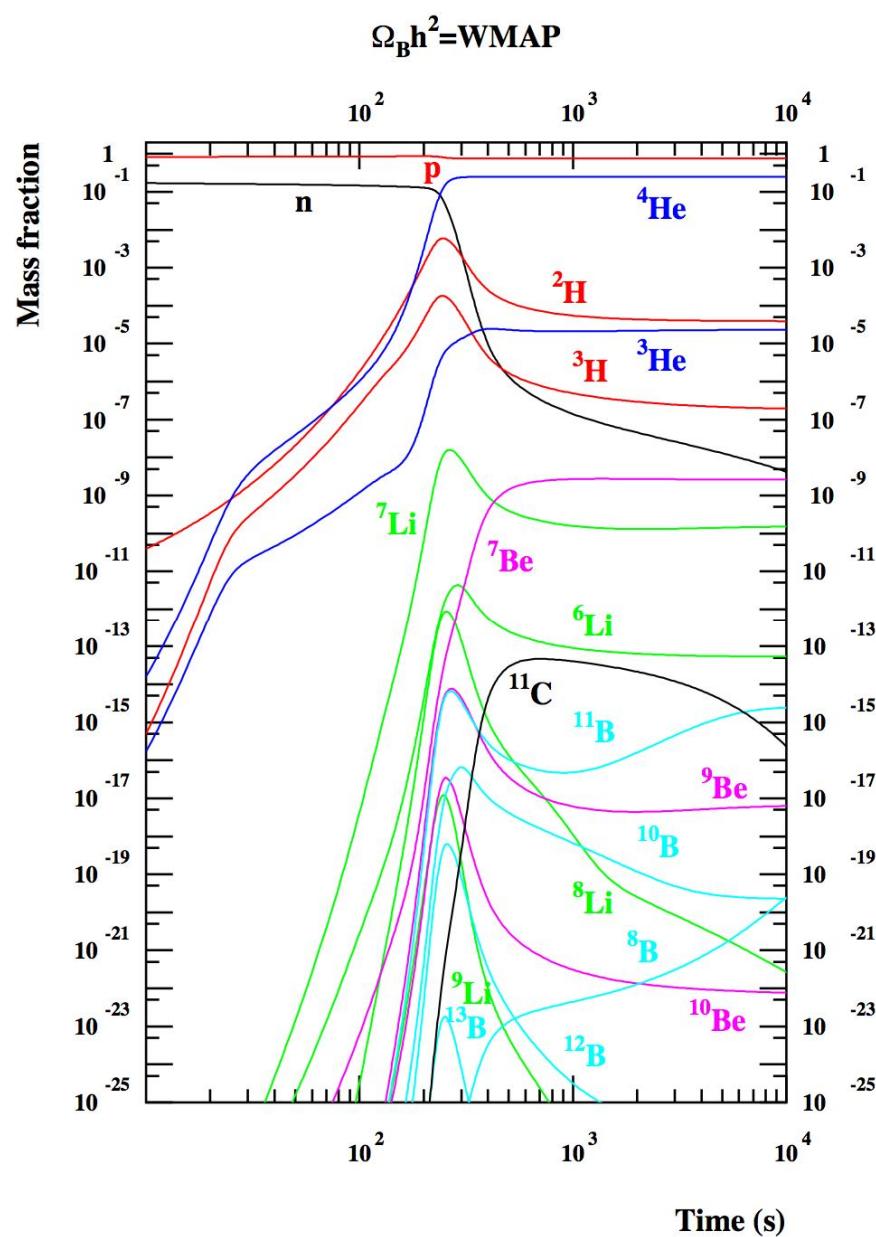
Main paths of BB nucleosynthesis



Main path for:

- H, D, ^3He , ^4He , ^7Li
- ^6Li
- ^9Be
- $^{10,11}\text{B}$
- CNO (^{12}C)

CNO nucleosynthesis



CNO nucleosynthesis

Stability of results:

| Number of atoms | [Iocco et al. 2007] | Initial Network | Updated Network |
|--|------------------------|--------------------|--------------------|
| ${}^6\text{Li}/\text{H} (\times 10^{-14})$ | | 1.23* | 1.23* |
| ${}^9\text{Be}/\text{H} (\times 10^{-19})$ | 2.5 | 2.24 | 9.60 |
| $({}^{10}\text{B} + {}^{11}\text{B})/\text{H} (\times 10^{-16})$ | 3.9 | 5.86 | 3.00 |
| $({}^{12}\text{C} + {}^{13}\text{C})/\text{H} (\times 10^{-16})$ | 5.5 | 4.43 | 6.75 |
| $({}^{14}\text{C} + {}^{14}\text{N})/\text{H} (\times 10^{-17})$ | 5.0 | 3.98 | 6.76 |
| ${}^{16}\text{O}/\text{H} (\times 10^{-20})$ | 2.7 | 5.18 | 9.13 |
| CNO/H ($\times 10^{-16}$) | 6.0 | 4.83 | 7.43 |

*With the ${}^2\text{H}(\alpha, \gamma){}^6\text{Li}$ rate from Hammache et al. 2010

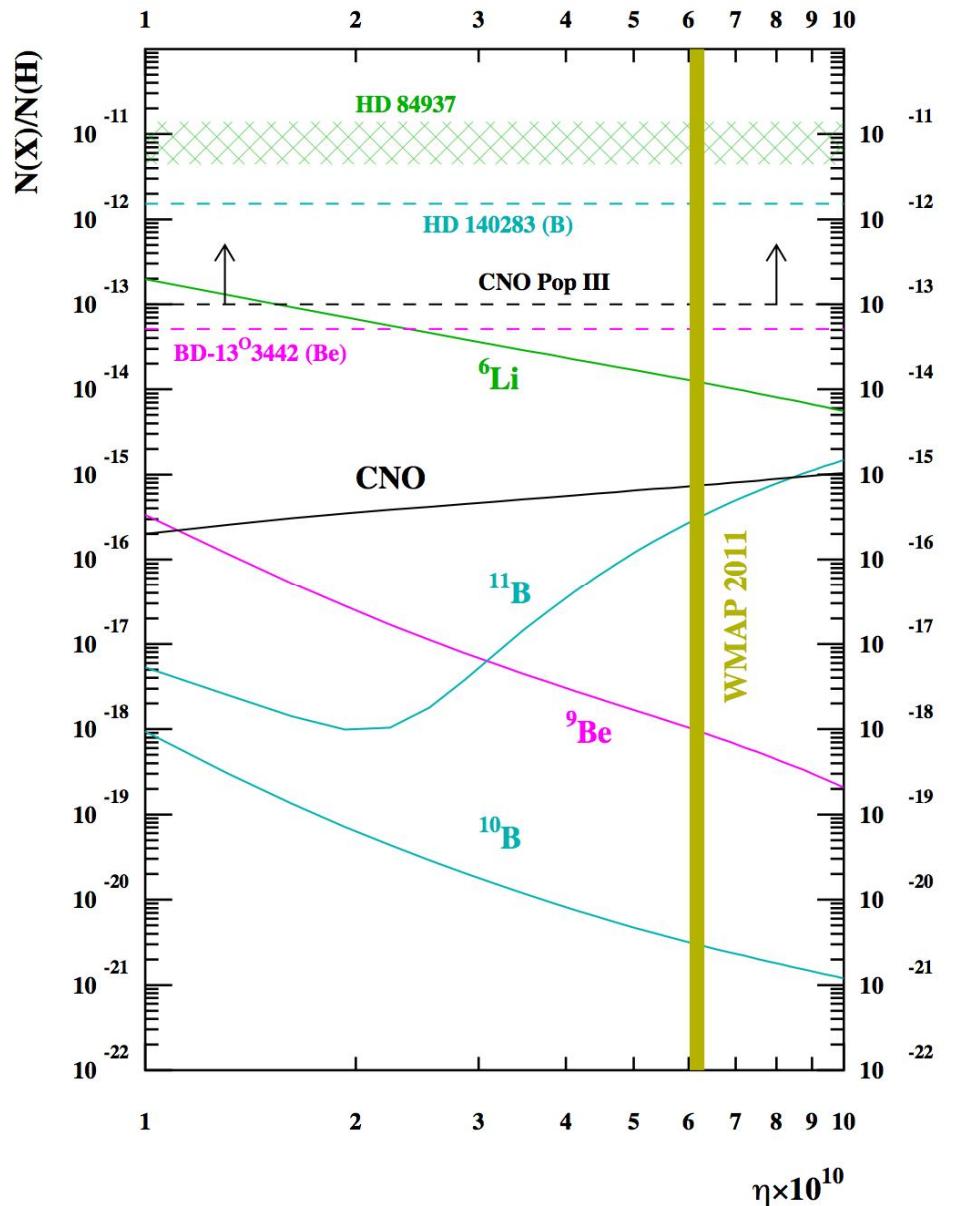
Even though important rates (Updated vs Initial)
or isotopic ratio (vs Iocco et al.) differ

Nucleosynthesis with extended network

The lithium problem:

- No important reaction besides already known [e.g. $^7\text{Be}(\text{d},\text{p})2\alpha$]
- No extra neutron source

| Number of atoms | [Iocco et al. 2007] | [Coc et al. 2012] |
|--|------------------------|----------------------|
| $(^{12}\text{C}+^{13}\text{C})/\text{H}$ ($\times 10^{-16}$) | 5.5 | 6.75 |
| $(^{14}\text{C}+^{14}\text{N})/\text{H}$ ($\times 10^{-17}$) | 5.0 | 6.76 |
| $^{16}\text{O}/\text{H}$ ($\times 10^{-20}$) | 2.7 | 9.13 |
| CNO/H ($\times 10^{-16}$) | 6.0 | 7.43 |



Importance of “brute force” sensitivity studies

Standard BBN

- No convection
- No diffusion
- No mixing
- No screening
- Known physics
- Reactions measurable at BBN energies

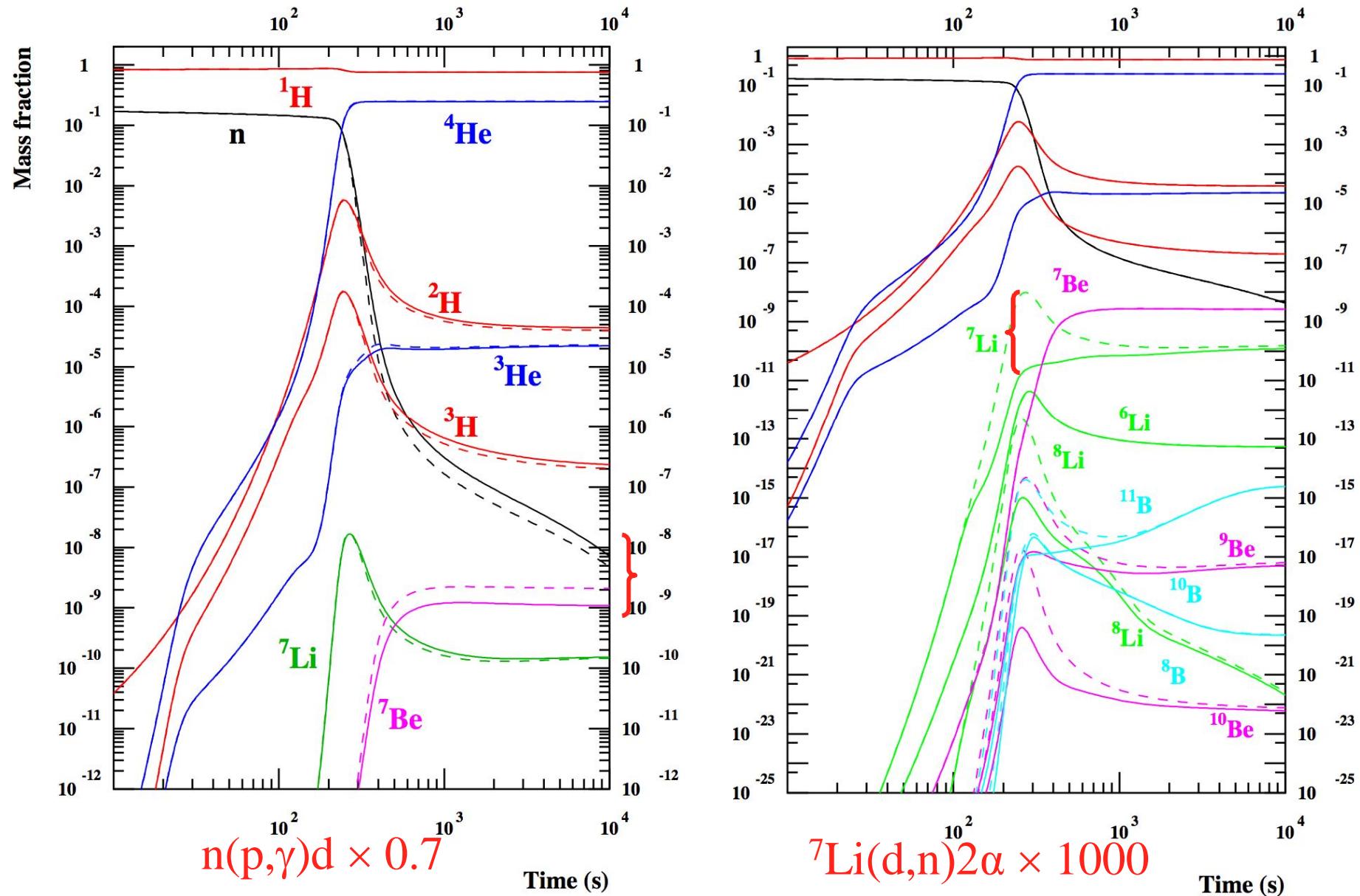
↓

Simple problem (?)

But totally unexpected effects:

- The $^1\text{H}(n,\gamma)\text{D}$ reaction affects mostly ^7Li
- The $^7\text{Li}(\text{d},\text{n})^2\text{He}$ reaction affects strongly the CNO but leaves ^7Li (and other isotopes) unchanged!

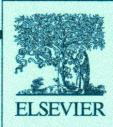
Influence of $^1\text{H}(n,\gamma)\text{D}$ & $^7\text{Li}(d,n)2^4\text{He}$ reactions



Nuclear Physics aspects

1. The main reactions for H, He and Li BBN production
2. Search for nuclear solutions to the lithium problem
3. Be, B and CNO production in BBN
4. **Monte-Carlo evaluation of reaction rates**
5. Monte-Carlo investigation of BBN

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Topical Issue

The 2010 Evaluation of Monte Carlo based Thermonuclear Reaction Rates

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COMPLETE VOLUME

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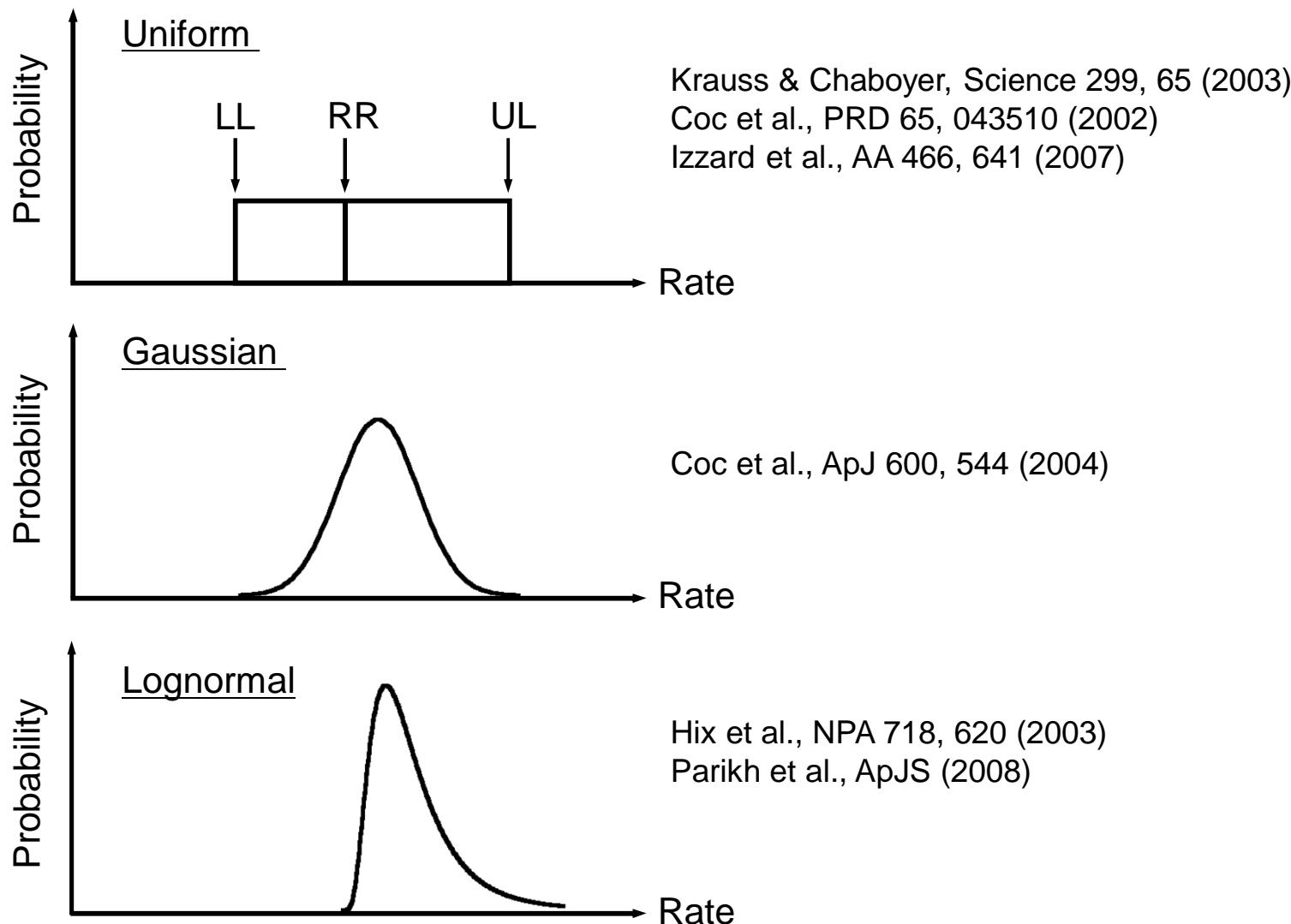
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The need for a new evaluation

- Almost ten years since the last evaluations
 - The series of papers by Fowler and collaborators ended with *Caughlan and Fowler (1988)*, including 128, mostly proton and α induced reactions on up to Si nuclei
 - The NACRE evaluation [*Angulo et al. 1999*] followed with 86 charged-particle induced reactions (also up to Si) including uncertainties
 - Extended from Ne to Ca and to unstable targets (55 reactions) by *Iliadis et al. (2001)* also including uncertainties
 - More specific evaluations for solar physics [*Adelberger et al. 1998, 2010*] or BBN [*e.g. Descouvemont et al. 2004*]
- Need of statistically derived uncertainties
 - Sensitivity study to nuclear uncertainties (BBN, novae, AGB, XRB,...)
 - Yield uncertainties from Monte-Carlo calculations (BBN, XRB,...)

Classical interpretation of rate PDF



The 1999 NACRE evaluation

| Reaction | Notation in refs. | Reaction | Notation in refs. | Reaction | Notation in refs. |
|---|----------------------|---|----------------------|---|----------------------|
| $^1\text{H}(\text{p},\nu\text{e}^+)^2\text{H}$ | ppnu | $^{11}\text{B}(\text{p},\alpha)^8\text{Be}$ | b11pa | $^{20}\text{Ne}(\text{p},\alpha)^{17}\text{F}$ | ne20pa |
| $^2\text{H}(\text{p},\gamma)^3\text{He}$ | dpg | $^{12}\text{C}(\text{p},\gamma)^{13}\text{N}$ | c12pg | $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$ | ne20ag |
| $^2\text{H}(\text{d},\gamma)^4\text{He}$ | ddg | $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ | c12ag | $^{21}\text{Ne}(\text{p},\gamma)^{22}\text{Na}$ | ne21pg |
| $^2\text{H}(\text{d,n})^3\text{He}$ | ddn | $^{13}\text{C}(\text{p},\gamma)^{14}\text{N}$ | c13pg | $^{21}\text{Ne}(\alpha,\text{n})^{24}\text{Mg}$ | ne21an |
| $^2\text{H}(\text{d,p})^3\text{H}$ | ddp | $^{13}\text{C}(\text{p,n})^{13}\text{N}$ | c13pn | $^{22}\text{Ne}(\text{p},\gamma)^{23}\text{Na}$ | ne22pg |
| $^2\text{H}(\alpha,\gamma)^6\text{Li}$ | dag | $^{13}\text{C}(\alpha,\text{n})^{16}\text{O}$ | c13an | $^{22}\text{Ne}(\alpha,\gamma)^{26}\text{Mg}$ | ne22ag |
| $^3\text{H}(\text{d,n})^4\text{He}$ | tdn | $^{13}\text{N}(\text{p},\gamma)^{14}\text{O}$ | n13pg | $^{22}\text{Ne}(\alpha,\text{n})^{25}\text{Mg}$ | ne22an |
| $^3\text{H}(\alpha,\gamma)^7\text{Li}$ | tag | $^{14}\text{N}(\text{p},\gamma)^{15}\text{O}$ | n14pg | $^{22}\text{Na}(\text{p},\gamma)^{23}\text{Mg}$ | na22pg |
| $^3\text{He}(^3\text{He},2\text{p})^4\text{He}$ | he3he3 | $^{14}\text{N}(\text{p,n})^{14}\text{O}$ | n14pn | $^{23}\text{Na}(\text{p},\gamma)^{24}\text{Mg}$ | na23pg |
| $^3\text{He}(\alpha,\gamma)^7\text{Be}$ | he3ag | $^{14}\text{N}(\text{p},\alpha)^{11}\text{C}$ | n14pa | $^{23}\text{Na}(\text{p,n})^{23}\text{Mg}$ | na23pn |
| $^4\text{He}(\alpha\text{n},\gamma)^9\text{Be}$ | aang | $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}$ | n14ag | $^{23}\text{Na}(\text{p},\alpha)^{20}\text{Ne}$ | na23pa |
| $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$ | aaag | $^{14}\text{N}(\alpha,\text{n})^{17}\text{F}$ | n14an | $^{23}\text{Na}(\alpha,\text{n})^{26}\text{Al}^{\text{g}}$ | na23an |
| $^6\text{Li}(\text{p},\gamma)^7\text{Be}$ | li6pg | $^{15}\text{N}(\text{p},\gamma)^{16}\text{O}$ | n15pg | $^{23}\text{Na}(\alpha,\text{n})^{26}\text{Al}^{\text{m}}$ | na23an |
| $^6\text{Li}(\text{p},\alpha)^3\text{He}$ | li6pa | $^{15}\text{N}(\text{p,n})^{15}\text{O}$ | n15pn | $^{23}\text{Na}(\alpha,\text{n})^{26}\text{Al}^{\text{t}}$ | na23an |
| $^7\text{Li}(\text{p},\gamma)^8\text{Be}$ | li7pg | $^{15}\text{N}(\text{p},\alpha)^{12}\text{C}$ | n15pa | $^{24}\text{Mg}(\text{p},\gamma)^{25}\text{Al}$ | mg24pg |
| $^7\text{Li}(\text{p},\alpha)^4\text{He}$ | li7pa | $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ | n15ag | $^{24}\text{Mg}(\text{p},\alpha)^{21}\text{Na}$ | mg24pa |
| $^7\text{Li}(\alpha,\gamma)^{11}\text{B}$ | li7ag | $^{16}\text{O}(\text{p},\gamma)^{17}\text{F}$ | o16pg | $^{25}\text{Mg}(\text{p},\gamma)^{26}\text{Al}^{\text{g}}$ | mg25pg |
| $^7\text{Li}(\alpha,\text{n})^{10}\text{B}$ | li7an | $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ | o16ag | $^{25}\text{Mg}(\text{p},\gamma)^{26}\text{Al}^{\text{m}}$ | mg25pg |
| $^7\text{Be}(\text{p},\gamma)^8\text{B}$ | be7pg | $^{17}\text{O}(\text{p},\gamma)^{18}\text{F}$ | o17pg | $^{25}\text{Mg}(\text{p},\gamma)^{26}\text{Al}^{\text{t}}$ | mg25pg |
| $^7\text{Be}(\alpha,\gamma)^{11}\text{C}$ | be7ag | $^{17}\text{O}(\text{p},\alpha)^{14}\text{N}$ | o17pa | $^{25}\text{Mg}(\alpha,\text{n})^{28}\text{Si}$ | mg25an |
| $^9\text{Be}(\text{p},\gamma)^{10}\text{B}$ | be9pg | $^{17}\text{O}(\alpha,\text{n})^{20}\text{Ne}$ | o17an | $^{26}\text{Mg}(\text{p},\gamma)^{27}\text{Al}$ | mg26pg |
| $^9\text{Be}(\text{p,n})^9\text{B}$ | be9pn | $^{18}\text{O}(\text{p},\gamma)^{19}\text{F}$ | o18pg | $^{26}\text{Mg}(\alpha,\text{n})^{29}\text{Si}$ | mg26an |
| $^9\text{Be}(\text{p,d})^8\text{Be}$ | be9pd | $^{18}\text{O}(\text{p},\alpha)^{15}\text{N}$ | o18pa | $^{26}\text{Al}^{\text{gs}}(\text{p},\gamma)^{27}\text{Si}$ | al26pg |
| $^9\text{Be}(\text{p},\alpha)^6\text{Li}$ | be9pa | $^{18}\text{O}(\alpha,\gamma)^{22}\text{Ne}$ | o18ag | $^{26}\text{Al}^{\text{ms}}(\text{p},\gamma)^{27}\text{Si}$ | al26pg |
| $^9\text{Be}(\alpha,\text{n})^{12}\text{C}$ | be9an | $^{18}\text{O}(\alpha,\text{n})^{21}\text{Ne}$ | o18an | $^{27}\text{Al}(\text{p},\gamma)^{28}\text{Si}$ | al27pg |
| $^{10}\text{B}(\text{p},\gamma)^{11}\text{C}$ | b10pg | $^{19}\text{F}(\text{p},\gamma)^{20}\text{Ne}$ | f19pg | $^{27}\text{Al}(\text{p},\alpha)^{24}\text{Mg}$ | al27pa |
| $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$ | b10pa | $^{19}\text{F}(\text{p,n})^{19}\text{Ne}$ | f19pn | $^{27}\text{Al}(\alpha,\text{n})^{30}\text{P}$ | al27an |
| $^{11}\text{B}(\text{p},\gamma)^{12}\text{C}$ | b11pg | $^{19}\text{F}(\text{p},\alpha)^{16}\text{O}$ | f19pa | $^{28}\text{Si}(\text{p},\gamma)^{29}\text{P}$ | si28pg |
| $^{11}\text{B}(\text{p,n})^{11}\text{C}$ | b11pn | $^{20}\text{Ne}(\text{p},\gamma)^{21}\text{Na}$ | ne20pg | | |

86 charged-particle
induced reactions on
stable targets + ^3H , ^7Be ,
 ^{22}Na and ^{26}Al

10 institutes (Brussels×2,
Orsay, Bochum, Stuttgart,
Athens, Ferrara, Lisboa,
Bucharest and Edinburgh)

Directed by C. Rolfs
and M. Arnould and
coordinated by
C. Angulo and P.
Descouvemont

The 1999 NACRE evaluation

❑ Improvement w.r.t. CF88

- References to the original data provided
- Information on the evaluation procedure given for each reaction
- Systematic estimation of uncertainties leading to “lower and upper bounds” for the rates
- Tabulated recommended and lower and upper bounds for the rates

❑ Limitations and shortcomings

- Uniformity of evaluations within the (10 institutes) collaboration
- No definition of “lower and upper bounds” (and even rec. rate) renders precise statistical interpretation meaningless
- Only partial documentation on specific reaction evaluation and input data

The 1999 NACRE evaluation

□ Examples of lower, recommended and upper rates evaluations

➤ Upper limit with $\omega\gamma < X$

- “low” with $\omega\gamma = 0$
- “rec.” with $\omega\gamma = 0.1 \times X$ as in Fowler’s papers
- “high” with $\omega\gamma = X$

➤ Sharp resonance at $E_R \pm \Delta E_R$ and with $\omega\gamma \pm \Delta\omega\gamma$ strength:

- “low” with $E_R + \Delta E_R$ and $\omega\gamma - \Delta\omega\gamma$
- “rec.” with E_R and $\omega\gamma$
- “high” with $E_R - \Delta E_R$ and $\omega\gamma + \Delta\omega\gamma$

Our evaluation

□ Improvements w.r.t. NACRE (and most other) evaluations

- Better uniformity by using the *same code* for rate calculations
- Precise definitions of *low*, *median* and *high* rates, obtained from Monte-Carlo calculations
- All input data given and evaluation better documented
- Short lived radioactive targets added (^{14}C , $^{17,18}\text{F}$, ^{19}Ne , ^{21}Na , $^{22,23}\text{Mg}$, $^{23-25}\text{Al}$, $^{26,27}\text{Si}$, $^{27,29}\text{P}$, $^{30,31}\text{S}$,...)

□ However

- For technical reasons, important reactions not included as $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$, some CNO reactions and $A < 14$ targets

Table 1

Information on reaction rates evaluated in the present work.

| Reaction ^a | Q^b (keV) | Some nuclear data sources ^c |
|---|-------------------------------|--|
| $^{14}\text{C}(\text{p}, \gamma)^{15}\text{N}$ | 10207.42 ± 0.00 | Görres et al. [102] |
| $^{14}\text{C}(\alpha, \gamma)^{18}\text{O}$ | 6226.3 ± 0.6 | Gai et al. [97], Lugaro et al. [151] |
| $^{14}\text{N}(\alpha, \gamma)^{18}\text{F}$ | 4414.6 ± 0.5 | Tilley et al. [226], Görres et al. [104] |
| $^{15}\text{N}(\alpha, \gamma)^{19}\text{F}$ | 4013.74 ± 0.07 | de Oliveira et al. [56], Wilmes et al. [246] |
| $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}$ | 3529.1 ± 0.6 | Mao et al. [160], Fisker et al. [89] |
| $^{16}\text{O}(\text{p}, \gamma)^{17}\text{F}$ | 600.27 ± 0.25 | Iliadis et al. [129] |
| $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ | 4729.85 ± 0.00 | Tilley et al. [227], Mohr [165] |
| $^{17}\text{O}(\text{p}, \gamma)^{18}\text{F}$ | 5606.5 ± 0.5 | Fox et al. [93], Chafa et al. [37] |
| $^{17}\text{O}(\text{p}, \alpha)^{14}\text{N}$ | 1191.82 ± 0.11 | Chafa et al. [37], Newton et al. [175] |
| $^{18}\text{O}(\text{p}, \gamma)^{19}\text{F}$ | 7994.8 ± 0.6 | Wiescher et al. [244], Becker et al. [25] |
| $^{18}\text{O}(\text{p}, \alpha)^{15}\text{N}$ | 3981.09 ± 0.62 | Lorentz-Wirzba et al. [148], La Cognata et al. [144] |
| $^{18}\text{O}(\alpha, \gamma)^{22}\text{Ne}$ | 9668.1 ± 0.6 | Giesen et al. [99], Dababneh et al. [58] |
| $^{17}\text{F}(\text{p}, \gamma)^{18}\text{Ne}$ | 3923.5 ± 0.4 | Bardayan et al. [13] |
| $^{18}\text{F}(\text{p}, \gamma)^{19}\text{Ne}$ | 6411.2 ± 0.6 | Adekola [1], Bardayan et al. [14] |
| $^{18}\text{F}(\alpha, \gamma)^{15}\text{O}$ | 2882.15 ± 0.73 | Adekola [1], Bardayan et al. [14] |
| $^{19}\text{Ne}(\text{p}, \gamma)^{20}\text{Na}$ | 2193 ± 7 | Vancraeynest et al. [235], Couder et al. [49] |
| $^{20}\text{Ne}(\text{p}, \gamma)^{21}\text{Na}$ | $2431.69 \pm 0.14^{\text{h}}$ | Rolfs et al. [196] |
| $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ | 9316.55 ± 0.01 | Schmalbrock et al. [202] |
| $^{21}\text{Ne}(\text{p}, \gamma)^{22}\text{Na}$ | 6739.6 ± 0.4 | Iliadis et al. [128] |
| $^{22}\text{Ne}(\text{p}, \gamma)^{23}\text{Na}$ | 8794.11 ± 0.02 | Görres et al. [101], Hale et al. [110] |
| $^{22}\text{Ne}(\alpha, \gamma)^{26}\text{Mg}$ | 10614.78 ± 0.03 | Wolke et al. [247], Ugalde et al. [232] |
| $^{22}\text{Ne}(\alpha, \text{n})^{25}\text{Mg}$ | -478.29 ± 0.04 | Jaeger et al. [131], Koehler [140] |
| $^{21}\text{Na}(\text{p}, \gamma)^{22}\text{Mg}$ | $5504.18 \pm 0.34^{\text{e}}$ | D'Auria et al. [54], Ruiz et al. [200] |
| $^{22}\text{Na}(\text{p}, \gamma)^{23}\text{Mg}$ | 7580.3 ± 1.4 | Stegmüller et al. [215], Jenkins et al. [13] |
| $^{23}\text{Na}(\text{p}, \gamma)^{24}\text{Mg}$ | 11692.68 ± 0.01 | Hale et al. [111], Rowland et al. [199] |
| $^{23}\text{Na}(\text{p}, \alpha)^{20}\text{Ne}$ | 2376.13 ± 0.00 | Hale et al. [111], Rowland et al. [199] |
| $^{22}\text{Mg}(\text{p}, \gamma)^{23}\text{Al}$ | 122 ± 19 | Caggiano et al. [33], He et al. [115] |
| $^{23}\text{Mg}(\text{p}, \gamma)^{24}\text{Al}$ | 1872 ± 3 | Visser et al. [238], Lotay et al. [149] |
| $^{24}\text{Mg}(\text{p}, \gamma)^{25}\text{Al}$ | 2271.6 ± 0.5 | Powell et al. [184], Engel et al. [81] |
| $^{24}\text{Mg}(\alpha, \gamma)^{28}\text{Si}$ | 9984.14 ± 0.01 | Strandberg et al. [216] |
| $^{25}\text{Mg}(\text{p}, \gamma)^{26}\text{Al}^{\text{f}}$ | 6306.45 ± 0.05 | Iliadis et al. [125], Iliadis et al. [128] |
| $^{25}\text{Mg}(\text{p}, \gamma)^{26}\text{Al}^{\text{g}}$ | 6306.45 ± 0.05 | Iliadis et al. [125], Iliadis et al. [128] |
| $^{25}\text{Mg}(\text{p}, \gamma)^{26}\text{Al}^{\text{m}}$ | 6078.15 ± 0.05 | Iliadis et al. [125], Iliadis et al. [128] |
| $^{26}\text{Mg}(\text{p}, \gamma)^{27}\text{Al}$ | 8271.05 ± 0.12 | Iliadis et al. [122], Iliadis et al. [128] |
| $^{23}\text{Al}(\text{p}, \gamma)^{24}\text{Si}$ | 3304 ± 27 | Herndl et al. [116], Schatz et al. [204] |
| $^{24}\text{Al}(\text{p}, \gamma)^{25}\text{Si}$ | 3408 ± 10 | Herndl et al. [116] |
| $^{25}\text{Al}(\text{p}, \gamma)^{26}\text{Si}$ | $5513.7 \pm 0.5^{\text{d}}$ | Parpottas et al. [182], Wrede [251] |
| $^{26}\text{Al}^{\text{g}}(\text{p}, \gamma)^{27}\text{Si}$ | 7462.96 ± 0.16 | Vogelaar et al. [239], Lotay et al. [150] |
| $^{27}\text{Al}(\text{p}, \gamma)^{28}\text{Si}$ | 11585.11 ± 0.12 | Iliadis et al. [128], Harissopoulos et al. [113] |
| $^{27}\text{Al}(\text{p}, \gamma)^{24}\text{Mg}$ | 1600.96 ± 0.12 | Endt [77], Iliadis et al. [128] |
| $^{26}\text{Si}(\text{p}, \alpha)^{27}\text{P}$ | 861 ± 27 | Caggiano et al. [33], Gade et al. [96] |
| $^{27}\text{Si}(\text{p}, \gamma)^{28}\text{P}$ | 2063 ± 3 | Iliadis et al. [127] |
| $^{28}\text{Si}(\text{p}, \gamma)^{29}\text{P}$ | 2748.8 ± 0.6 | Endt [76], Graff et al. [107] |
| $^{29}\text{Si}(\text{p}, \gamma)^{30}\text{P}$ | 5594.5 ± 0.3 | Iliadis et al. [128] |
| $^{30}\text{Si}(\text{p}, \gamma)^{31}\text{P}$ | 7296.93 ± 0.19 | Iliadis et al. [128] |
| $^{27}\text{P}(\text{p}, \gamma)^{28}\text{S}$ | $2460 \pm 30^{\text{g}}$ | Herndl et al. [116] |
| $^{29}\text{P}(\text{p}, \gamma)^{30}\text{S}$ | 4399 ± 3 | Iliadis et al. [128], Bardayan et al. [20] |
| $^{31}\text{P}(\text{p}, \gamma)^{32}\text{S}$ | 8863.78 ± 0.21 | Iliadis et al. [128] |

List of reactions

Table 1 (Continued)

| Reaction ^a | Q^b (keV) | Some nuclear data sources ^c |
|--|---------------------------|---|
| $^{31}\text{P}(\alpha, \gamma)^{28}\text{Si}$ | 1915.97 ± 0.18 | Iliadis et al. [128] |
| $^{30}\text{S}(\text{p}, \gamma)^{31}\text{Cl}$ | 290 ± 50 | Iliadis et al. [128], Wrede et al. [249] |
| $^{31}\text{S}(\text{p}, \gamma)^{32}\text{Cl}$ | 1574 ± 7 | Iliadis et al. [127] |
| $^{32}\text{S}(\text{p}, \gamma)^{33}\text{Cl}$ | 2276.7 ± 0.4 | Iliadis et al. [128] |
| $^{31}\text{Cl}(\text{p}, \gamma)^{32}\text{Ar}$ | 2420 ± 50 | Herndl et al. [116] |
| $^{32}\text{Cl}(\text{p}, \gamma)^{33}\text{Ar}$ | 3343 ± 7 | Herndl et al. [116], Schatz et al. [205] |
| $^{35}\text{Cl}(\text{p}, \gamma)^{36}\text{Ar}$ | 8506.97 ± 0.05 | Iliadis et al. [128], Röpke et al. [189] |
| $^{35}\text{Cl}(\text{p}, \alpha)^{32}\text{S}$ | 1866.21 ± 0.13 | Iliadis et al. [128], Röpke et al. [189] |
| $^{34}\text{Ar}(\text{p}, \gamma)^{35}\text{K}$ | $84.5 \pm 0.7^{\text{f}}$ | Herndl et al. [116], Trinder et al. [231] |
| $^{35}\text{Ar}(\text{p}, \gamma)^{36}\text{K}$ | 1668 ± 8 | Iliadis et al. [127] |
| $^{36}\text{Ar}(\text{p}, \gamma)^{37}\text{K}$ | 1857.63 ± 0.09 | Iliadis et al. [128] |
| $^{35}\text{K}(\text{p}, \gamma)^{36}\text{Ca}$ | $2556 \pm 40^{\text{f}}$ | Herndl et al. [116], Doornenbal et al. [68] |
| $^{39}\text{Ca}(\text{p}, \gamma)^{40}\text{Sc}$ | 538 ± 3 | Iliadis et al. [127], Hansper et al. [112] |
| $^{40}\text{Ca}(\text{p}, \gamma)^{41}\text{Sc}$ | 1085.09 ± 0.08 | Iliadis et al. [128] |

Principle of the new evaluation

1. Compilation and evaluation of available original data [*paper II*] with all evaluated data published [*paper III*]
2. Monte-Carlo sampling of input data according to relevant statistical distributions [*paper I*], allowing for 2 level interferences, better treatment of upper limits,... to obtain the distribution of rates on a temperature grid [*R. Longland's RatesMC code*].
3. In general, matching of the “experimental” rate to Hauser-Feschbach theoretical rate above a calculated [*Newton, Longland & Iliadis 2008*] temperature
4. Production of tables for the statistically calculated low, median and high rates and of the log-normal parameters μ and σ of the rate distribution [*paper II*]
5. Comparison with previous rates [*paper IV*]

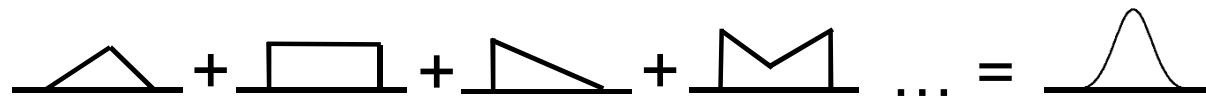
PDF of a resonance energy

$E_r, \Delta E_r$ → 50% of thick - target yield curves

$E_r = E_x - Q$ → γ - ray spectroscopy

Central limit theorem of statistics:

sum of n independent random variables x_i becomes a Gaussian random variable in the limit of large n, independent of the form of the individual PDFs of the x_i



E_r is Gaussian distributed :

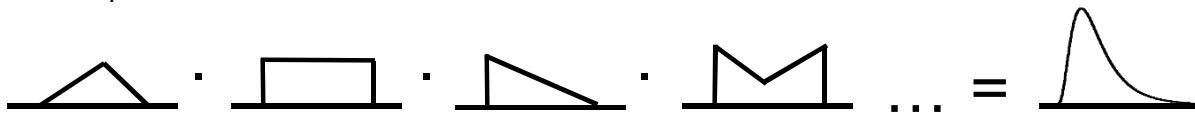
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$E[x] = \mu, \quad V[x] = \sigma^2$$

PDF of a resonance strength or partial width

Central limit theorem of statistics:

product of n independent random variables x_i becomes a lognormal random variable in the limit of large n , independent of the form of the individual PDFs of the x_i



$\omega\gamma, \Gamma_i$ are lognormally distributed :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{x} e^{-(\ln x - \mu)^2/(2\sigma^2)}$$

$$x > 0!$$

$$E(x) = \exp(\mu + \sigma^2/2) \quad V(x) = E^2(x)(\exp(\sigma^2) - 1)$$

If y is Gaussian distributed, then $x = e^y$ will be lognormally distributed

The lognormal probability density function

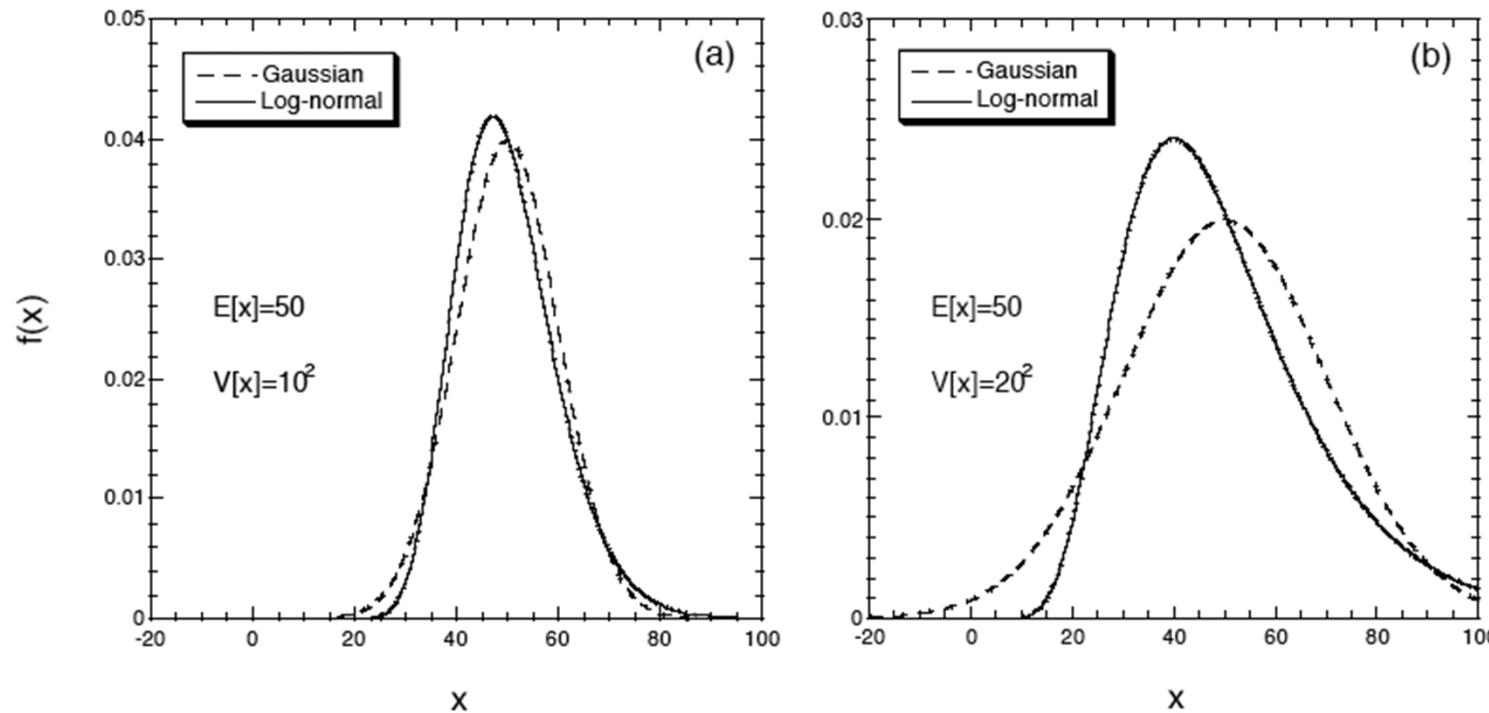


Fig. 1. Comparison of Gaussian (dashed lines) and lognormal (solid lines) probability density functions; (a) $E[x] = 50$ and $V[x] = 10^2$, and (b) $E[x] = 50$ and $V[x] = 20^2$. See text.

- For small variance, Gaussian PDF \approx Lognormal PDF
- Lognormal MC sampling ensures that variable remains positive

Upper limits resonance strength or partial width

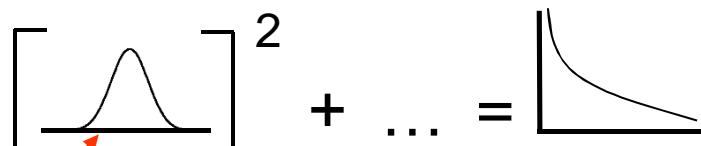
θ_a^2 is Porter - Thomas distributed :

$$f(\theta^2) = \frac{c}{\sqrt{\theta^2}} e^{-\theta^2/(2\langle\theta^2\rangle)}$$
$$\theta^2 > 0!$$

Porter and Thomas, PR 104, 483 (1956)

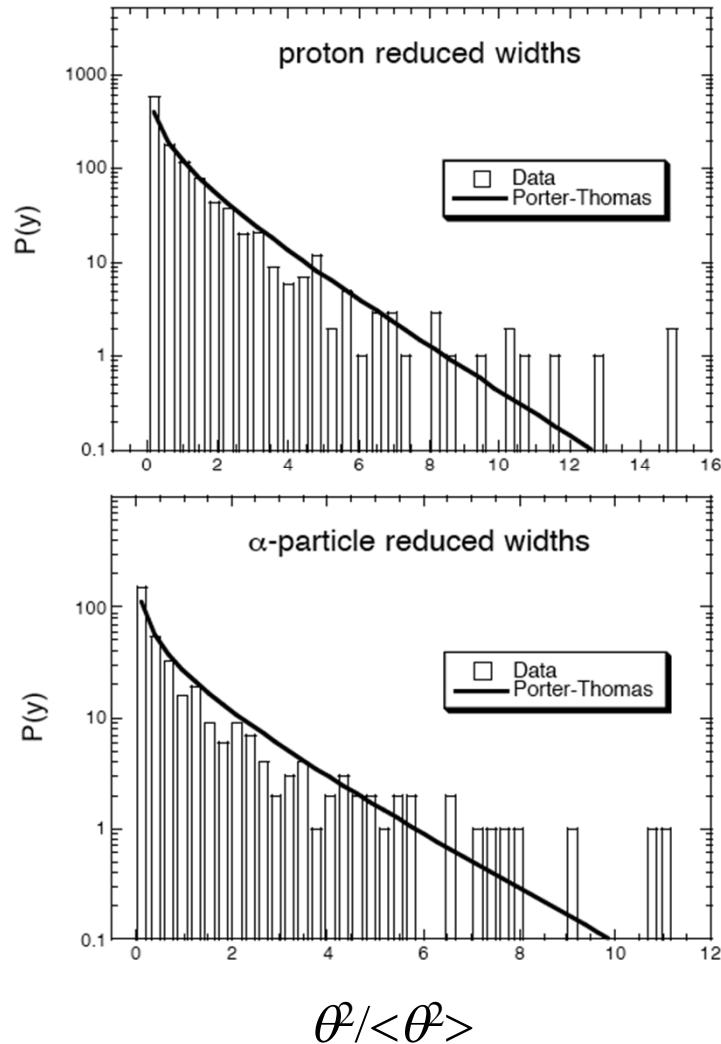
Porter - Thomas distribution = chi - squared distribution with
one degree of freedom

If x is Gaussian distributed, then x^2 is chi-squared distributed



Sum of amplitudes expected to follow a normal PDF centered at zero

Upper limits of partial widths



Charged particles

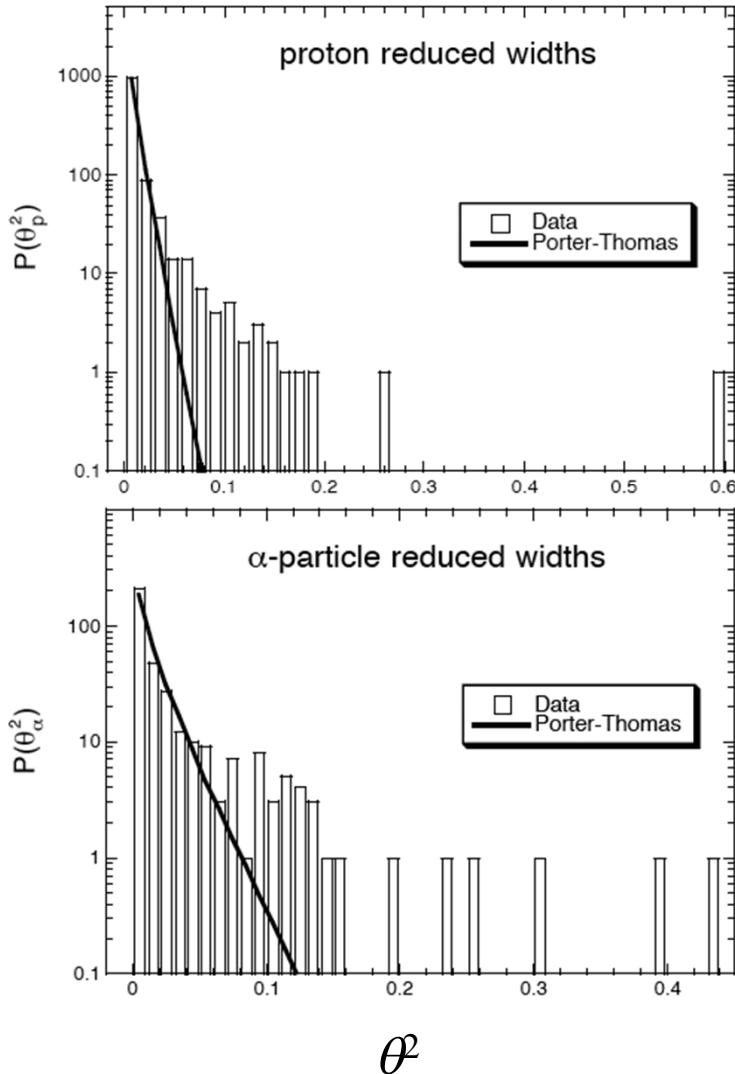
Theory: Reduced width, θ^2 or spectroscopic factors, are expected to follow a Porter-Thomas PDF.

Experiment: While experimental bias favors high θ^2 measured values, statistics shows that $\theta^2/\langle\theta^2\rangle$ locally follows a Porter-Thomas PDF.

Experimental distribution of $\theta^2/\langle\theta^2\rangle$ for 1127 p and 360 α , (after nucleus and l grouping) and for ^{24}Mg , ^{28}Si , ^{30}P , ^{32}S , ^{36}Ar , ^{40}Ca levels

Data from TUNL [Mitchell et al. 1975-1995]

Upper limits of partial widths



$$f_{PT}(\theta^2) \propto \frac{1}{\sqrt{\theta^2}} \exp\left(-\frac{\theta^2}{2\langle\theta^2\rangle}\right) \quad (\text{P.-T. PDF})$$

However, in general, *local value* of $\langle\theta^2\rangle$ is not known!

The best fits of experimental data give
 $\langle\theta_p^2\rangle = 0.0045$ for protons and
 $\langle\theta_\alpha^2\rangle = 0.01$ for α -particles

- Different from the often assumed $\theta_p^2 = 0.1$ value
- $\langle\theta_\alpha^2\rangle$ larger than $\langle\theta_p^2\rangle$

Upper limits: $\theta^2 < \theta_{\text{upper}} \rightarrow f(\theta^2) = f_{PT}(\theta^2)$
 below θ_{upper} and 0 above.

The Porter-Thomas probability density function

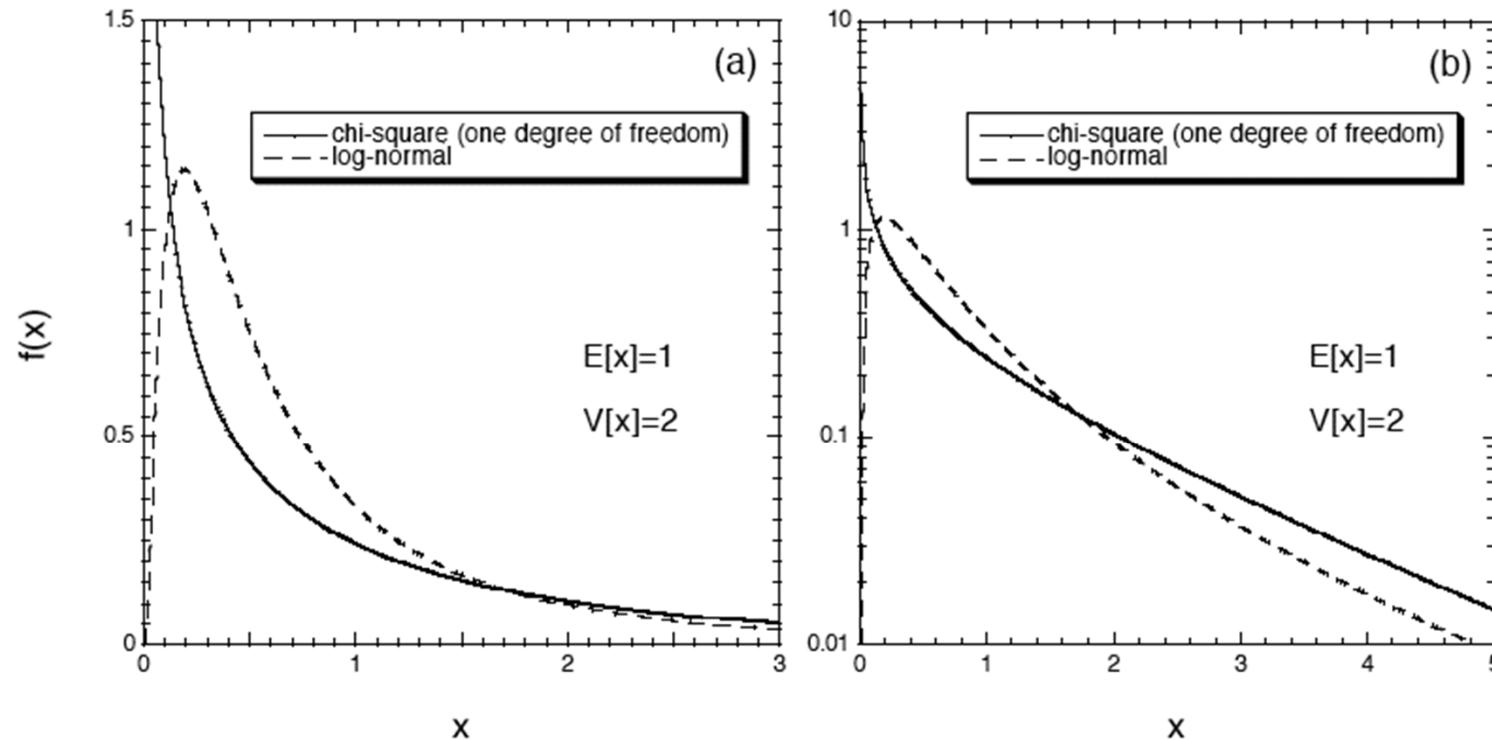


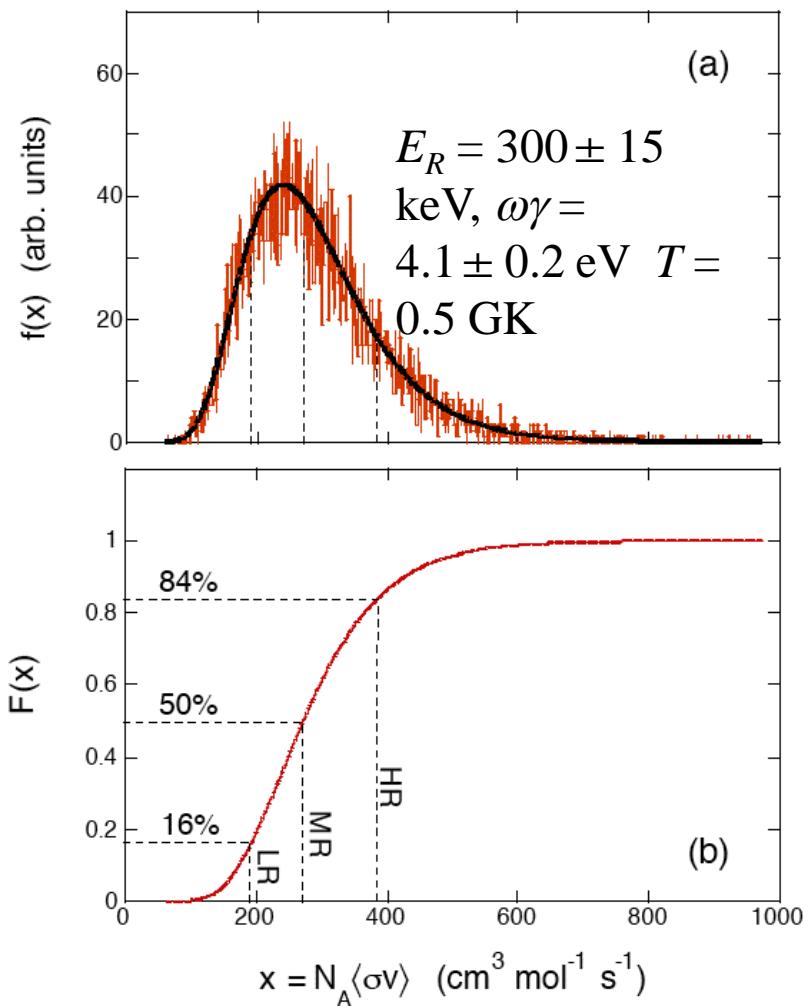
Fig. 2. Chi-square distribution of one degree of freedom (solid line) on (a) a linear vertical axis scale, and (b) a logarithmic vertical axis scale. For comparison, a lognormal distribution (dashed line) of same expectation value and variance as the chi-squared distribution is shown. The lognormal parameters amount to $\mu = -0.54931$ and $\sigma = 1.0482$, according to Eq. (27).

Summary of MC sampling

Monte-Carlo rate calculations assume

- Normal PDF for resonance energies
- Lognormal PDFs for resonance strengths and partial widths
- Truncated Porter-Thomas PDF for p and α upper limits of partial widths
- Lognormal PDF for spectroscopic factors with a 40% uncertainty
- “ “ for radiative widths with a 50% uncertainty
- “ “ direct capture with a 40% uncertainty
- Numerical integration: in general, no other non-resonant contribution
- Two level interference : “+”, “-”, or unknown “+/-” sampling
- Correlations, e.g. energy in penetrability and Boltzmann factor

Output of MC sampling



Distribution of rates (10^4 trials) at a selected temperature (schematic, not real example)

The low, median and high rates are then obtained as:

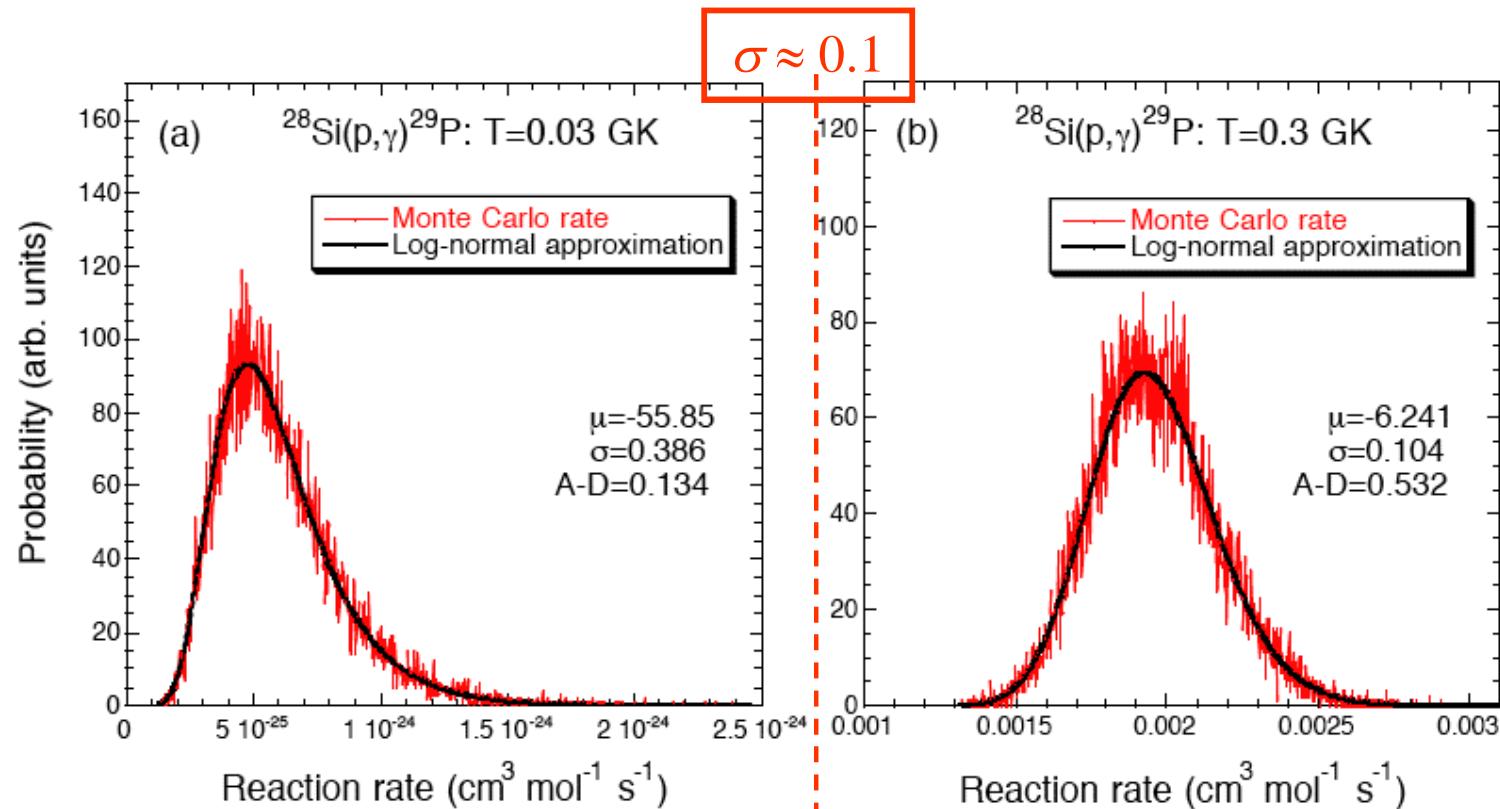
- Low Rate $(e^{\mu-\sigma}) = 0.16$ quantile
- Median Rate $(e^\mu) = 0.5$ quantile
- High Rate $(e^{\mu+\sigma}) = 0.84$ quantile

from the cumulative distribution.

Lognormal distribution with μ and σ calculated from the distribution

- Tabulated : T , LR, MR, HR, μ , σ , (and AD)

A real output of MC sampling



Lognormal DC dominates
→ lognormal PDF

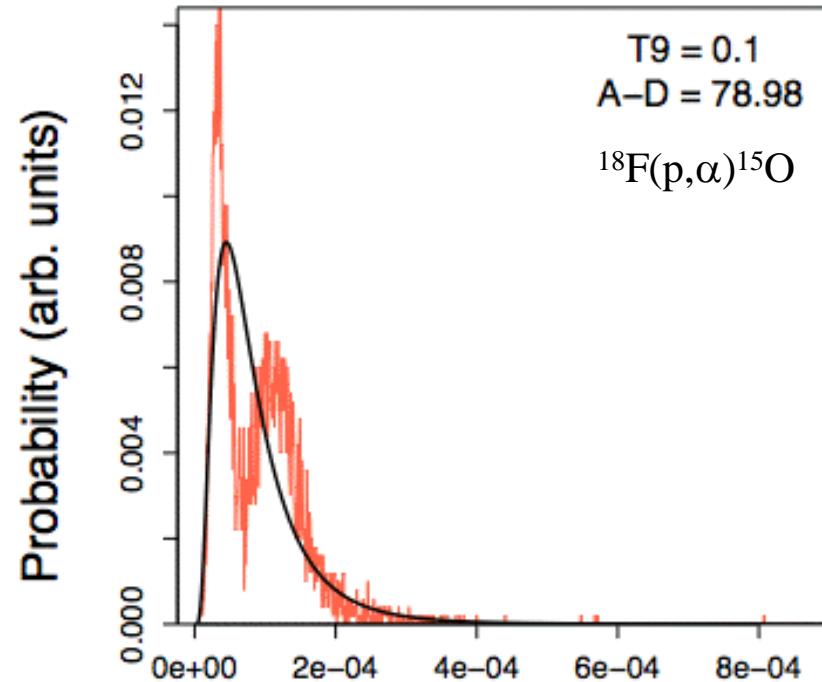
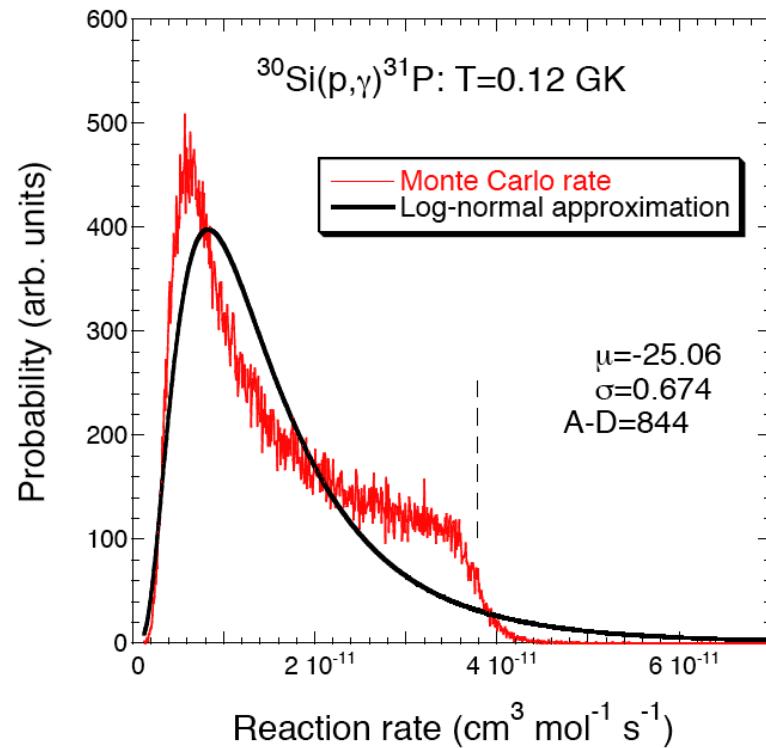
Small ($\sigma \leq 0.1$) $\omega\gamma$ uncertainty
dominates → \approx normal PDF

“A-D” = Anderson–Darling statistical test that a sample follows a PDF

99% c.l. rejection if $A-D > 1.092$ but still acceptable up to $A-D \approx 30$

Most (but not all) rates lognormally distributed

Rate dominated by:



➤ Upper limit on θ_p^2

➤ Two level interferences with unknown sign

A real example of MC input [Paper III]

```

180(p,a)15N
*****
1 ! Zproj
8 ! Ztarget
2 ! Zexitparticle (=0 when only 2 channels open)
1.0078 ! Aproj
17.999 ! Atarget
4.0026 ! Aexitparticle (=0 when only 2 channels open)
0.5 ! Jproj
0.0 ! Jtarget
0.0 ! Jexitparticle (=0 when only 2 channels open)
7994.8 ! projectile separation energy (keV)
4013.74 ! exit particle separation energy (=0 when only 2 channels open)
1.25 ! Radius parameter R0 (fm)
3 ! Gamma-ray channel number (<2 if ejectile is a g-ray; >3 otherwise)
*****
1.0 ! Minimum energy for numerical integration (keV)
5000 ! Number of random samples (>5000 for better statistics)
0 ! =0 for rate output at all temperatures; >NT for rate output at selected temperatures
*****
Non-Resonant Contribution
S(keVb) S'(b) S''(b/keV) fraccErr Cutoff Energy (keV)
0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0
*****
Resonant Contribution
Note: G1 = entrance channel, G2 = exit channel, G3 = spectator channel !! Ecm, Exf in (keV); wg, Gs in (eV) !!
Note: if Er<0, theta^2=C2S*theta_sp/2 must be entered instead of entrance channel partial width
Ecm DEcm wg Dwg Jr G1 DG1 L1 G2 DG2 L2 G3 DG3 L3 Exf In
19.3 0.7 0 0 2.5 2.3e-19 0.5e-19 2 2.5e3 1.0e3 3 2.3 1.0 1 0.0 1
142.9 0.1 0 0 0.5 1.67e-1 0.12e-1 0 1.23e2 0.24e2 1 0.72 0.15 1 0.0 1
315.2 1.3 0 0 2.5 1.9e-2 0.3e-2 2 4.7e1 1.9e1 3 0.78 0.34 1 0.0 1
597.1 1.2 0 0 1.5 1.4e2 0.7e2 1 2.0e3 0.1e3 2 0.71 0.39 1 0.0 1
931.9 2.8 0 0 1.5 7.6e1 0.7e1 1 3.5e3 0.3e3 2 0.34 0.06 1 0.0 1
1106.2 4.0 0 0 3.5 4.7 0.6 4 5.6e2 0.76e2 3 0 0 0 0.0 1
1172.2 1.5 0 0 0.5 3.8e2 0.3e2 0 5.4e3 0.38e3 1 1.4 1.0 1 0.0 1
1326.2 1.2 0 0 0.5 2.2e2 0.2e2 0 4.7e3 0.4e3 1 3.4 1.7 1 0.0 1
1672.7 1.6 0 0 1.5 2.0e3 0.6e3 2 1.4e3 0.4e3 1 1.0 0.4 1 0.0 1
1825.2 1.2 0 0 2.5 9.0e1 3.0e1 3 7.0e1 2.0e1 2 0 0 0 0.0 1
1892.0 3.0 0 0 0.5 1.1e4 0.3e4 0 1.8e4 0.54e4 1 0.36 0.20 1 0.0 1
2167.0 3.0 0 0 0.5 2.2e3 0.7e3 0 0.9e3 0.3e3 1 0 0 0 0.0 1
2237.0 3.0 0 0 0.5 2.7e3 0.8e3 0 1.6e3 0.5e3 1 0 0 0 0.0 1
2259.0 3.0 0 0 0.5 1.0e4 3.0e3 0 12.0e3 4.0e3 1 0 0 0 0.0 1
2313.0 4.0 0 0 1.5 4.9e3 1.5e3 2 4.3e3 1.3e3 1 0 0 0 0.0 1
2501.5 1.4 0 0 1.5 2.3e3 0.7e3 2 0.95e3 0.3e3 1 0 0 0 0.0 1
2619.5 1.7 0 0 2.5 0.66e3 0.20e3 2 1.0e3 0.3e3 3 0 0 0 0.0 1
2768.5 2.6 0 0 0.5 4.3e3 1.3e3 1 1.1e3 0.3e3 0 0 0 0.0 1
2864.9 2.0 0 0 2.5 12.3e3 3.7e3 2 5.4e3 1.6e3 3 0 0 0 0.0 1
2980.2 2.6 0 0 1.5 4.7e3 1.4e3 2 4.3e3 1.3e3 1 0 0 0 0.0 1
3291.0 7.0 0 0 2.5 4.07e3 0.95e3 2 7.7e3 4.8e3 3 0 0 0 0.0 1
3355.0 25.0 0 0 0.5 228.3e3 1.9e3 0 43.0e3 31.0e3 1 0 0 0 0.0 1
3455.0 3.5 0 0 0.5 16.1e3 2.8e3 1 22.0e3 7.0e3 0 0 0 0.0 1
3507.0 5.0 0 0 1.5 11.4e3 1.9e3 1 16.0e3 6.0e3 2 0 0 0 0.0 1
3545.0 7.0 0 0 2.5 3.5e3 1.0e3 2 18.3e3 4.8e3 3 0 0 0 0.0 1
3608.0 12.0 0 0 1.5 26.0e3 8.0e3 1 43.0e3 16.0e3 2 0 0 0 0.0 1
3658.0 4.0 0 0 1.5 11.2e3 1.8e3 2 19.0e3 8.0e3 1 0 0 0 0.0 1

```

Basic reaction data
for $^{18}\text{O}(\text{p},\alpha)^{15}\text{N}$

(No DC/NR)

| | | | | | | | | | | | | | | | |
|---------|-------|---|---|-----|---------|--------|---|---------|---------|---|---|---|---|-----|---|
| 4045.0 | 20.0 | 0 | 0 | 0.5 | 70.0e3 | 60.0e3 | 1 | 64.0e3 | 16.0e3 | 0 | 0 | 0 | 0 | 0.0 | 1 |
| 4141.0 | 8.0 | 0 | 0 | 1.5 | 61.0e3 | 15.0e3 | 1 | 51.0e3 | 9.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 4227.0 | 12. | 0 | 0 | 1.5 | 39.0e3 | 10.0e3 | 2 | 36.0e3 | 9.0e3 | 1 | 0 | 0 | 0 | 0.0 | 1 |
| 4527.0 | 7.0 | 0 | 0 | 0.5 | 2.6e3 | 0.9e3 | 1 | 13.4e3 | 4.4e3 | 0 | 0 | 0 | 0 | 0.0 | 1 |
| 4582.0 | 10.0 | 0 | 0 | 2.5 | 4.3e3 | 1.6e3 | 2 | 44.4e3 | 7.8e3 | 3 | 0 | 0 | 0 | 0.0 | 1 |
| 4585.0 | 25.0 | 0 | 0 | 0.5 | 112.0e3 | 28.0e3 | 1 | 226.0e3 | 33.0e3 | 0 | 0 | 0 | 0 | 0.0 | 1 |
| 4785.0 | 10.0 | 0 | 0 | 2.5 | 12.3e3 | 6.2e3 | 2 | 82.0e3 | 33.0e3 | 3 | 0 | 0 | 0 | 0.0 | 1 |
| 4865.0 | 30.0 | 0 | 0 | 1.5 | 118.0e3 | 25.0e3 | 2 | 161.0e3 | 24.0e3 | 1 | 0 | 0 | 0 | 0.0 | 1 |
| 4945.0 | 25.0 | 0 | 0 | 2.5 | 11.0e3 | 8.0e3 | 2 | 76.0e3 | 14.0e3 | 3 | 0 | 0 | 0 | 0.0 | 1 |
| 4985.0 | 50.0 | 0 | 0 | 0.5 | 18.0e3 | 10.0e3 | 1 | 105.0e3 | 33.0e3 | 0 | 0 | 0 | 0 | 0.0 | 1 |
| 5095.0 | 75.0 | 0 | 0 | 1.5 | 71.0e3 | 27.0e3 | 1 | 213.0e3 | 56.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 5322.0 | 8.0 | 0 | 0 | 3.5 | 9.1e3 | 2.1e3 | 3 | 22.0e3 | 10.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 5365.0 | 25.0 | 0 | 0 | 1.5 | 1.9e3 | 1.2e3 | 1 | 36.0e3 | 18.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 5737.0 | 11.0 | 0 | 0 | 3.5 | 11.6e3 | 2.2e3 | 3 | 43.0e3 | 9.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 6045.0 | 20.0 | 0 | 0 | 2.5 | 11.4e3 | 2.8e3 | 2 | 129.0e3 | 29.0e3 | 3 | 0 | 0 | 0 | 0.0 | 1 |
| 6105.0 | 21.0 | 0 | 0 | 1.5 | 7.6e3 | 2.8e3 | 1 | 76.0e3 | 29.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 6335.0 | 20.0 | 0 | 0 | 1.5 | 8.5e3 | 2.8e3 | 1 | 67.0e3 | 29.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 6705.0 | 20.0 | 0 | 0 | 1.5 | 19.9e3 | 4.7e3 | 1 | 103.0e3 | 38.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 6745.0 | 50.0 | 0 | 0 | 0.5 | 95.0e3 | 24.0e3 | 0 | 265.0e3 | 70.0e3 | 1 | 0 | 0 | 0 | 0.0 | 1 |
| 6785.0 | 20.0 | 0 | 0 | 2.5 | 29.9e3 | 5.7e3 | 2 | 179.0e3 | 48.0e3 | 3 | 0 | 0 | 0 | 0.0 | 1 |
| 6925.0 | 30.0 | 0 | 0 | 3.5 | 19.0e3 | 3.8e3 | 3 | 178.0e3 | 29.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 7365.0 | 20.0 | 0 | 0 | 0.5 | 5.7e3 | 1.9e3 | 1 | 61.0e3 | 10.0e3 | 0 | 0 | 0 | 0 | 0.0 | 1 |
| 7405.0 | 30.0 | 0 | 0 | 2.5 | 6.6e3 | 1.9e3 | 2 | 73.0e3 | 24.0e3 | 3 | 0 | 0 | 0 | 0.0 | 1 |
| 7775.0 | 21.0 | 0 | 0 | 1.5 | 7.6e3 | 2.8e3 | 1 | 89.0e3 | 24.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 8205.0 | 40.0 | 0 | 0 | 1.5 | 15.2e3 | 3.8e3 | 2 | 155.0e3 | 29.0e3 | 1 | 0 | 0 | 0 | 0.0 | 1 |
| 8235.0 | 30.0 | 0 | 0 | 3.5 | 12.3e3 | 3.8e3 | 3 | 209.0e3 | 38.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 8285.0 | 20.0 | 0 | 0 | 1.5 | 12.3e3 | 3.8e3 | 1 | 154.0e3 | 29.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 9055.0 | 40.0 | 0 | 0 | 1.5 | 37.0e3 | 7.6e3 | 1 | 293.0e3 | 67.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 9165.0 | 40.0 | 0 | 0 | 3.5 | 28.4e3 | 7.5e3 | 3 | 294.0e3 | 67.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 9455.0 | 30.0 | 0 | 0 | 1.5 | 2.8e3 | 1.9e3 | 1 | 29.0e3 | 19.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 9655.0 | 60.0 | 0 | 0 | 3.5 | 4.7e3 | 2.8e3 | 3 | 90.0e3 | 57.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 9935.0 | 40.0 | 0 | 0 | 1.5 | 21.8e3 | 4.7e3 | 1 | 232.0e3 | 57.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 10035.0 | 40.0 | 0 | 0 | 3.5 | 30.3e3 | 6.6e3 | 3 | 333.0e3 | 57.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |
| 11075.0 | 60.0 | 0 | 0 | 1.5 | 20.8e3 | 6.6e3 | 1 | 532.0e3 | 142.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 11835.0 | 150.0 | 0 | 0 | 2.5 | 12.3e3 | 5.7e3 | 3 | 355.0e3 | 57.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 11895.0 | 30.0 | 0 | 0 | 1.5 | 37.0e3 | 7.6e3 | 1 | 435.0e3 | 57.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 12815.0 | 50.0 | 0 | 0 | 0.5 | 30.3e3 | 4.7e3 | 1 | 381.0e3 | 57.0e3 | 0 | 0 | 0 | 0 | 0.0 | 1 |
| 12935.0 | 50.0 | 0 | 0 | 1.5 | 11.4e3 | 3.8e3 | 1 | 305.0e3 | 48.0e3 | 2 | 0 | 0 | 0 | 0.0 | 1 |
| 13055.0 | 50.0 | 0 | 0 | 3.5 | 23.7e3 | 4.7e3 | 3 | 423.0e3 | 29.0e3 | 4 | 0 | 0 | 0 | 0.0 | 1 |

Upper Limits of Resonances
Note: enter partial width upper limit by choosing non-zero value for PT, where PT=<theta^2> for particles and...
Note: # upper limits <# open channels!
Ecm DEcm Jr G1 DG1 L1 PT G2 DG2 L2 PT G3 DG3 L3 PT Exf Int
89.0 3.0 1.5 8.0e-8 2.5e-8 1 0.0 3.0e3 0.0 2 0.010 0.6 0.25 1 0.0 0.0 1
204.2 1.0 2.5 7.7e-4 2.0e-4 2 0.0 0.8e3 0.0 3 0.010 0.0 0.0 0 0.0 0.0 1

Interference between Resonances [numerical integration only]
Note: + for positive, - for negative interference; +- if interference sign is unknown
Ecm DEcm Jr G1 DG1 L1 PT G2 DG2 L2 PT G3 DG3 L3 PT Exf
886.0 30.0 0.5 5.6e3 1.0e3 0.0 0.0 2.065 1.1e5 1 0.0 0.0 0 0.0 0.0 0.0
798.4 1.6 0.5 2.46e4 0.14e4 0.0 0.0 2.0e4 1.0e4 1 0.0 0.25 0.4 1 0.0 0.0 0.0

Reaction Rate and PDF at NT selected temperatures only
Note: default values are used for reaction rate range if Min-Max=0.0

2-level interferences with unknown signs

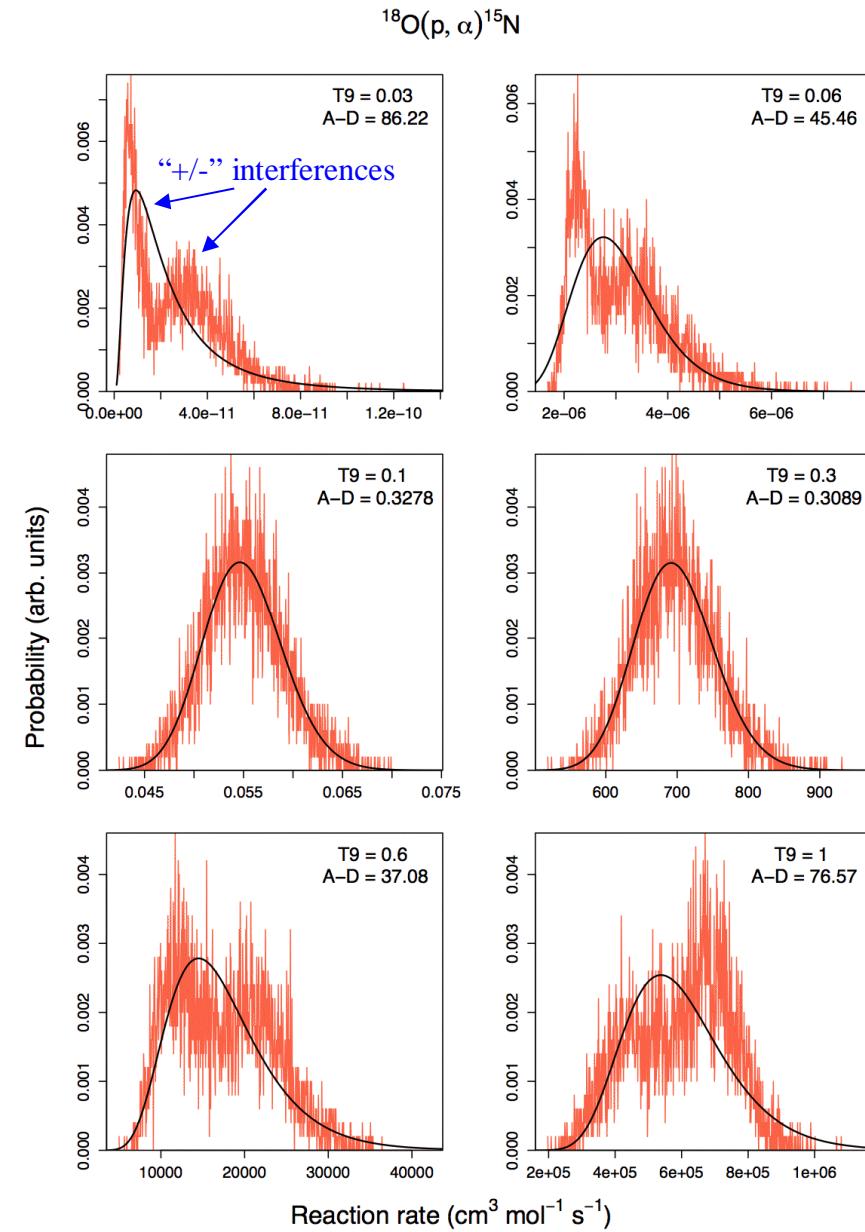
Two upper limits on Γ_α

$$\frac{\Gamma_R}{\Delta \Gamma_p} \pm \frac{\Gamma_\gamma}{\Delta \Gamma_\alpha}$$

A real example of MC output [Paper II]

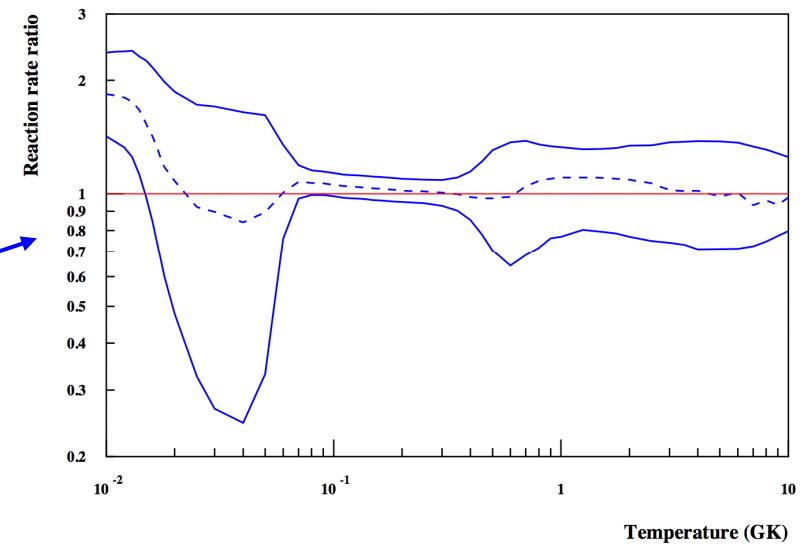
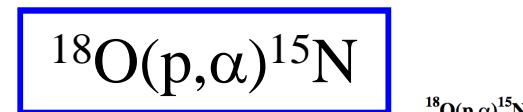
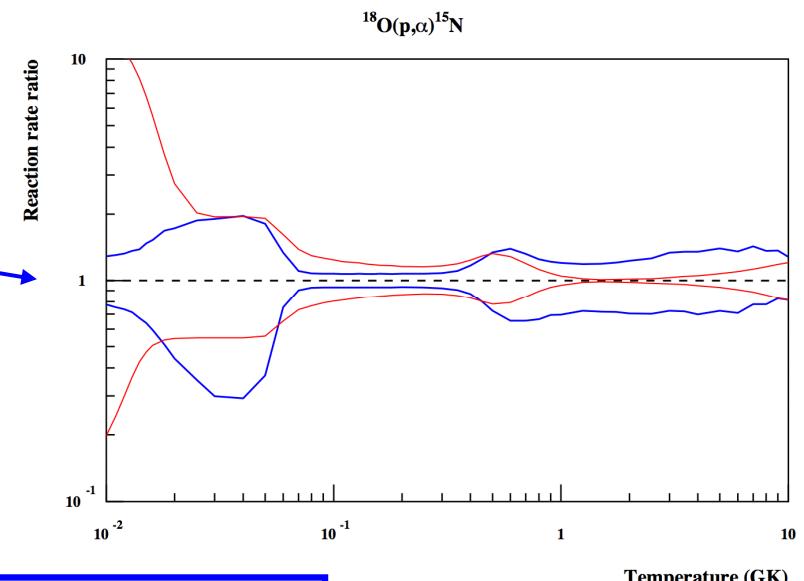
Table B.11: Total thermonuclear reaction rates for $^{18}\text{O}(\text{p},\alpha)^{15}\text{N}$.

| T (GK) | Low rate | Median rate | High rate | lognormal μ | lognormal σ | A-D |
|--------|------------------------|------------------------|------------------------|--------------------------|------------------------|------------------------|
| 0.010 | 1.52×10^{-20} | 1.97×10^{-20} | 2.51×10^{-20} | $-4.538 \times 10^{+01}$ | 2.51×10^{-01} | $1.30 \times 10^{+00}$ |
| 0.011 | 1.18×10^{-19} | 1.56×10^{-19} | 2.01×10^{-19} | $-4.332 \times 10^{+01}$ | 2.70×10^{-01} | $1.33 \times 10^{+00}$ |
| 0.012 | 6.79×10^{-19} | 9.27×10^{-19} | 1.23×10^{-18} | $-4.154 \times 10^{+01}$ | 2.97×10^{-01} | $3.61 \times 10^{+00}$ |
| 0.013 | 3.21×10^{-18} | 4.53×10^{-18} | 6.15×10^{-18} | $-3.995 \times 10^{+01}$ | 3.30×10^{-01} | $5.71 \times 10^{+00}$ |
| 0.014 | 1.28×10^{-17} | 1.88×10^{-17} | 2.65×10^{-17} | $-3.853 \times 10^{+01}$ | 3.62×10^{-01} | $5.54 \times 10^{+00}$ |
| 0.015 | 4.48×10^{-17} | 7.02×10^{-17} | 1.03×10^{-16} | $-3.722 \times 10^{+01}$ | 4.07×10^{-01} | $9.91 \times 10^{+00}$ |
| 0.016 | 1.42×10^{-16} | 2.41×10^{-16} | 3.66×10^{-16} | $-3.600 \times 10^{+01}$ | 4.55×10^{-01} | $1.34 \times 10^{+01}$ |
| 0.018 | 1.14×10^{-15} | 2.23×10^{-15} | 3.68×10^{-15} | $-3.379 \times 10^{+01}$ | 5.48×10^{-01} | $2.34 \times 10^{+01}$ |
| 0.020 | 7.45×10^{-15} | 1.64×10^{-14} | 2.88×10^{-14} | $-3.182 \times 10^{+01}$ | 6.29×10^{-01} | $3.09 \times 10^{+01}$ |
| 0.025 | 3.80×10^{-13} | 1.01×10^{-12} | 1.89×10^{-12} | $-2.775 \times 10^{+01}$ | 7.53×10^{-01} | $5.13 \times 10^{+01}$ |
| 0.030 | 8.51×10^{-12} | 2.45×10^{-11} | 4.78×10^{-11} | $-2.457 \times 10^{+01}$ | 7.98×10^{-01} | $5.47 \times 10^{+01}$ |
| 0.040 | 9.11×10^{-10} | 2.86×10^{-09} | 5.56×10^{-09} | $-1.985 \times 10^{+01}$ | 8.46×10^{-01} | $5.68 \times 10^{+01}$ |
| 0.050 | 3.81×10^{-08} | 9.60×10^{-08} | 1.76×10^{-07} | $-1.627 \times 10^{+01}$ | 6.93×10^{-01} | $5.17 \times 10^{+01}$ |
| 0.060 | 2.27×10^{-06} | 3.05×10^{-06} | 4.17×10^{-06} | $-1.269 \times 10^{+01}$ | 2.82×10^{-01} | $3.77 \times 10^{+01}$ |
| 0.070 | 7.81×10^{-05} | 8.60×10^{-05} | 9.59×10^{-05} | $-9.356 \times 10^{+00}$ | 1.04×10^{-01} | $3.87 \times 10^{+00}$ |
| 0.080 | 1.17×10^{-03} | 1.26×10^{-03} | 1.36×10^{-03} | $-6.679 \times 10^{+00}$ | 7.63×10^{-02} | 2.45×10^{-01} |
| 0.090 | 9.57×10^{-03} | 1.03×10^{-02} | 1.10×10^{-02} | $-4.577 \times 10^{+00}$ | 7.26×10^{-02} | 5.48×10^{-01} |
| 0.100 | 5.12×10^{-02} | 5.48×10^{-02} | 5.89×10^{-02} | $-2.903 \times 10^{+00}$ | 7.19×10^{-02} | 4.82×10^{-01} |
| 0.110 | 2.00×10^{-01} | 2.14×10^{-01} | 2.29×10^{-01} | $-1.542 \times 10^{+00}$ | 7.17×10^{-02} | 4.96×10^{-01} |
| 0.120 | 6.14×10^{-01} | 6.58×10^{-01} | 7.06×10^{-01} | -4.184×10^{-01} | 7.19×10^{-02} | 5.00×10^{-01} |
| 0.130 | $1.57 \times 10^{+00}$ | $1.69 \times 10^{+00}$ | $1.81 \times 10^{+00}$ | 5.235×10^{-01} | 7.18×10^{-02} | 5.41×10^{-01} |
| 0.140 | $3.50 \times 10^{+00}$ | $3.76 \times 10^{+00}$ | $4.02 \times 10^{+00}$ | $1.323 \times 10^{+00}$ | 7.18×10^{-02} | 4.22×10^{-01} |
| 0.150 | $6.95 \times 10^{+00}$ | $7.46 \times 10^{+00}$ | $8.00 \times 10^{+00}$ | $2.010 \times 10^{+00}$ | 7.18×10^{-02} | 3.70×10^{-01} |
| 0.160 | $1.26 \times 10^{+01}$ | $1.35 \times 10^{+01}$ | $1.45 \times 10^{+01}$ | $2.605 \times 10^{+00}$ | 7.17×10^{-02} | 4.66×10^{-01} |
| 0.180 | $3.35 \times 10^{+01}$ | $3.59 \times 10^{+01}$ | $3.85 \times 10^{+01}$ | $3.581 \times 10^{+00}$ | 7.20×10^{-02} | 4.49×10^{-01} |
| 0.200 | $7.20 \times 10^{+01}$ | $7.73 \times 10^{+01}$ | $8.29 \times 10^{+01}$ | $4.348 \times 10^{+00}$ | 7.18×10^{-02} | 3.20×10^{-01} |
| 0.250 | $2.74 \times 10^{+02}$ | $2.95 \times 10^{+02}$ | $3.16 \times 10^{+02}$ | $5.686 \times 10^{+00}$ | 7.31×10^{-02} | 2.53×10^{-01} |
| 0.300 | $6.46 \times 10^{+02}$ | $6.98 \times 10^{+02}$ | $7.56 \times 10^{+02}$ | $6.549 \times 10^{+00}$ | 7.90×10^{-02} | 2.52×10^{-01} |



Comparison with previous work [Paper IV]

- Comparison between ranges of uncertainties i.e. LR/MR and HR/MR for previous (often NACRE) and new rates
- Origin of the differences w.r.t. NACRE:
 - $E_R = 19$ keV resonance
 - $\Delta\Gamma_\alpha$ for $E_R = 656$ keV resonance
 - Interferences
- Comparison between new LR, MR and HR and previous mean or recommended rate



Conclusions (I)

- 62 reactions from $^{14}\text{C}(\text{p},\gamma)^{15}\text{O}$ to $^{40}\text{Ca}(\text{p},\gamma)^{41}\text{Sc}$
- Better uniformity by using the *same code* for rate calculations
- Numerical integration whenever possible
- Precise definitions of *low*, *median* and *high* rates, obtained from Monte-Carlo calculations
- All input data given and evaluation better documented
- Short lived radioactive targets added (^{14}C , $^{17,18}\text{F}$, ^{19}Ne , ^{21}Na , $^{22,23}\text{Mg}$, $^{23-25}\text{Al}$, $^{26,27}\text{Si}$, $^{27,29}\text{P}$, $^{30,31}\text{S}$, ...)
- Tables of parameters to be used in Monte Carlo for astrophysical applications

<http://starlib.physics.unc.edu/>

Nuclear Physics aspects

1. The main reactions for H, He and Li BBN production
2. Search for nuclear solutions to the lithium problem
3. Be, B and CNO production in BBN
4. Monte-Carlo evaluation of reaction rates
5. **Monte-Carlo investigation of BBN**

Sensitivity study (II)

1. Start from “best guess” set of reaction rates and evaluated or guessed uncertainties
2. Perform Monte-Carlo calculations storing for each event sampled reaction rates and yields
3. Calculate correlations between reaction rates and yields
4. Identification of important reactions e.g. >10% correlation with selected yields
5. Reevaluation of selected rates following re-analyses of available experimental and theoretical data
6. Calculations with set of improved reaction rates

Monte-Carlo sampling

Let x be the short notation for a reaction rate: $x \equiv N_A \langle \sigma v \rangle$

Assumed to follow a
Log-Normal distribution:

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \frac{1}{x} e^{-(\ln x - \mu)^2 / (2\sigma^2)}$$

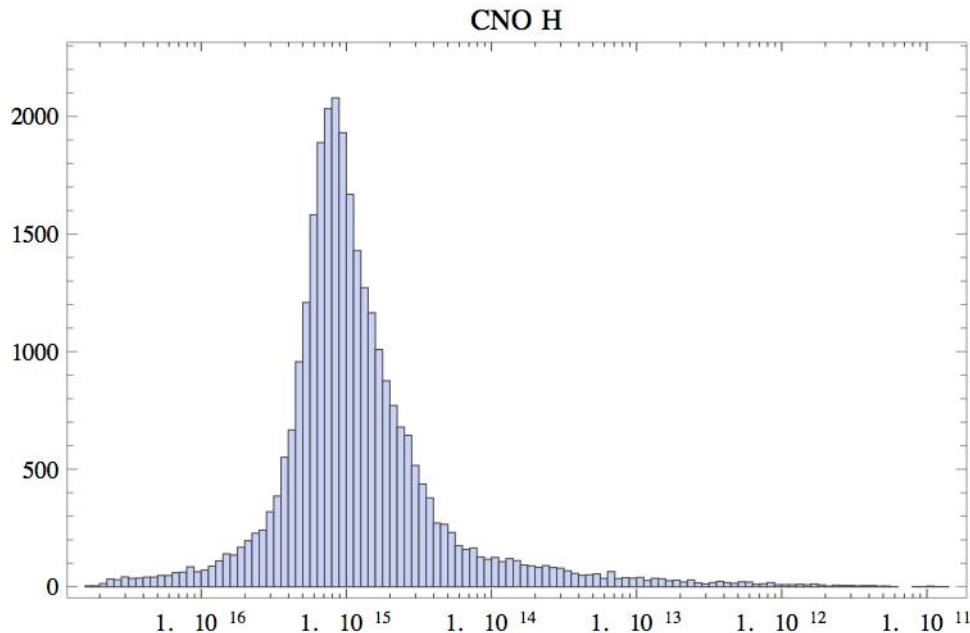
i.e $\ln(x)$ follows a Normal distribution with mean μ and variance σ^2

Monte-Carlo sampling : p follows a Normal
distribution with mean 0 and variance 1
(i = trial number)

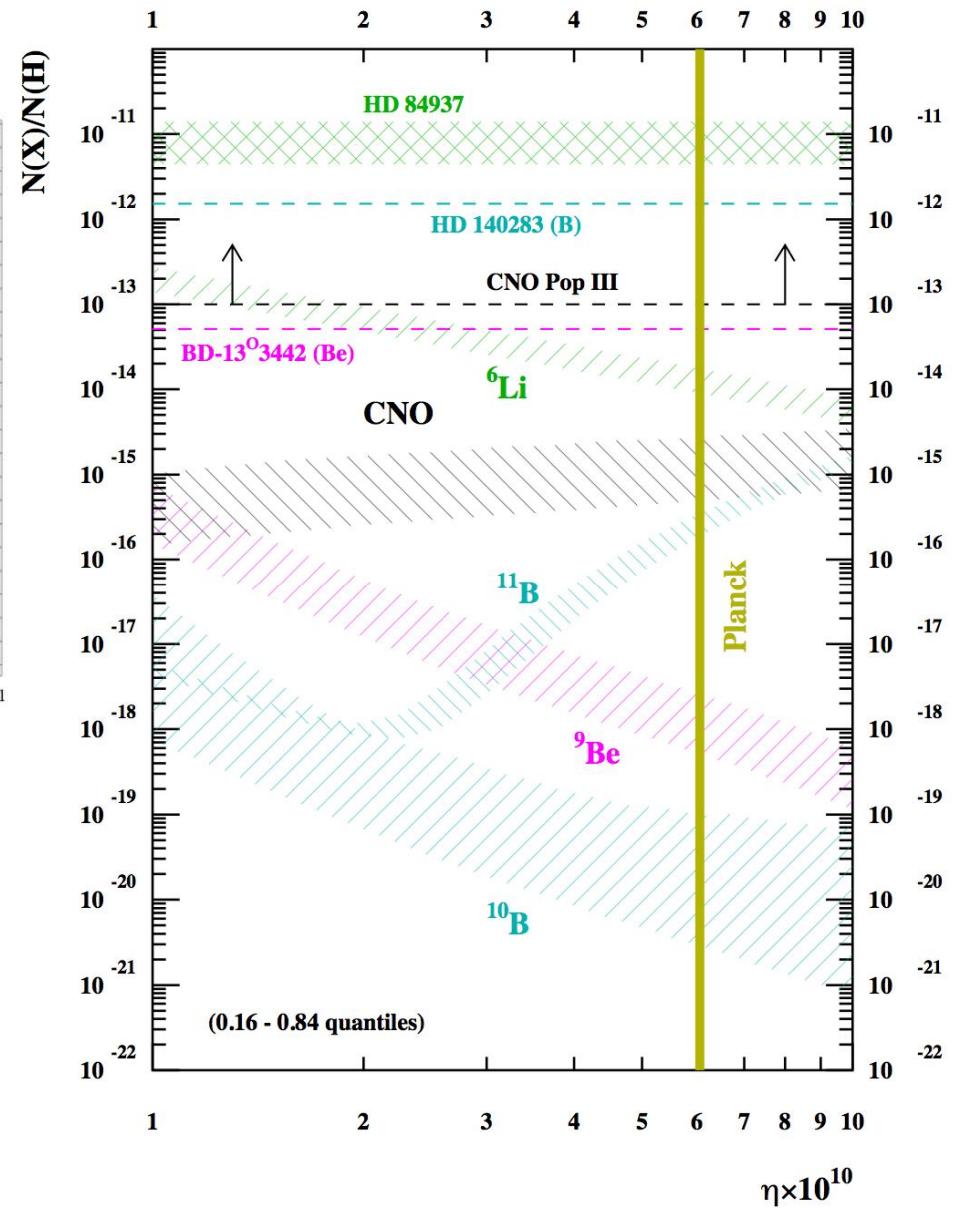
$$x_{\text{med}} \equiv \exp(\mu)$$
$$f.u \equiv \exp(\sigma)$$

$$x_i = \exp(\mu + p_i \sigma) \equiv x_{\text{med}} (f.u)^{p_i} \rightarrow y_{j;i} \quad (\text{yield of isotope } j)$$

Monte-Carlo results: improved uncertainties



| Number of atoms | [Coc et al. 2012] | [Coc et al. 2014] |
|--|----------------------|----------------------|
| ${}^6\text{Li}/\text{H} (\times 10^{-14})$ | 1.23 | 0.90-1.77 |
| ${}^9\text{Be}/\text{H} (\times 10^{-19})$ | 9.60 | 5.10-26.3 |
| ${}^{11}\text{B}/\text{H} (\times 10^{-16})$ | 3.05 | 1.85-3.56 |
| $\text{CNO}/\text{H} (\times 10^{-16})$ | 7.43 | 4.94-28.5 |



Monte-Carlo results: correlations

Correlation coefficient between yield of isotope j (y_j) and rate of reaction k (x_k) rate:

$$C_{j,k} = \frac{\frac{1}{N} \sum_{i=1}^N y_{j;i} p_{k;i} - \bar{y}_j \bar{p}_k}{\sigma_j \sigma'_k}$$

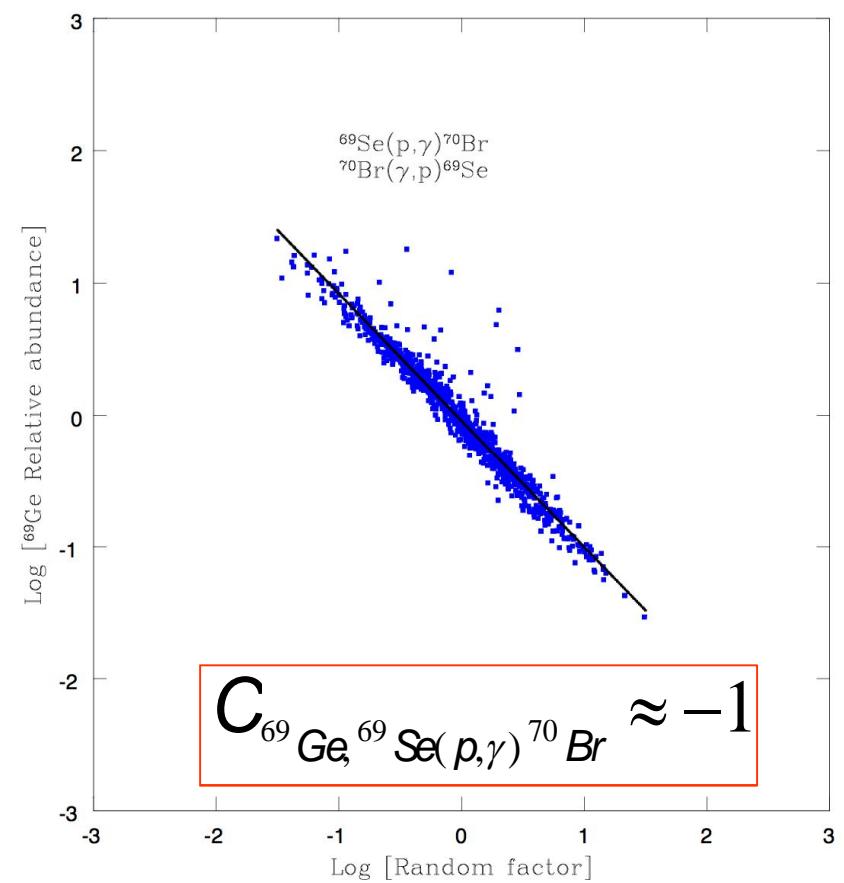
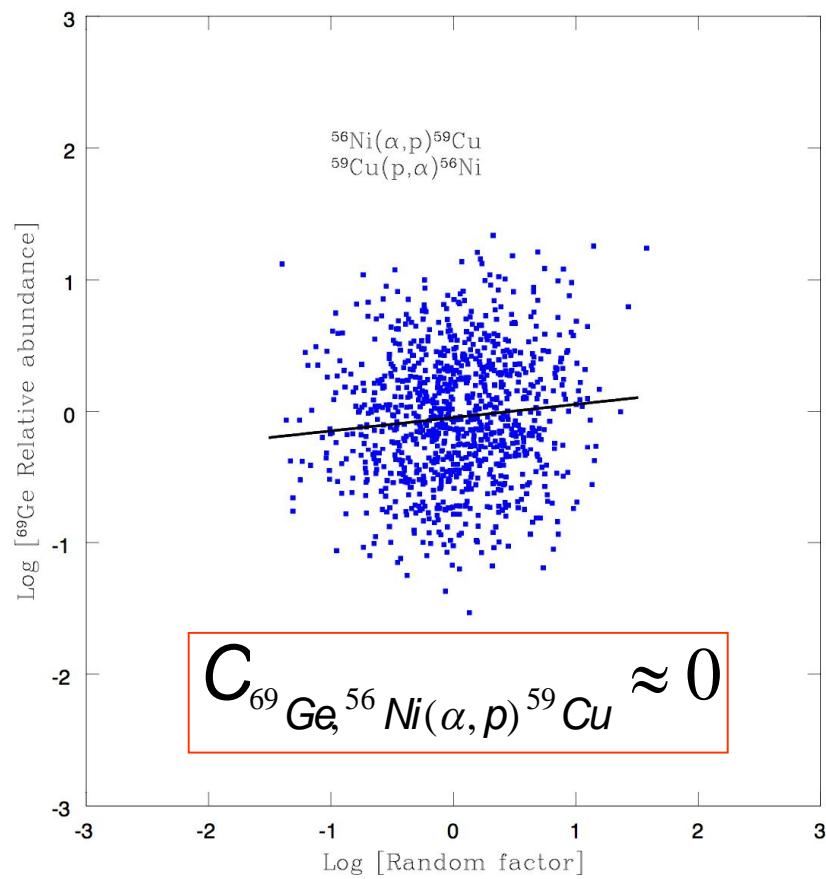
$$\bar{y}_j = \frac{1}{N} \sum_{i=1}^N y_{j;i} \quad \text{Mean and variance of } y_j \quad (\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (y_{j;i} - \bar{y}_j)^2)$$

$\bar{p}_k = 0$ and $\sigma'_k = 1$

Monte-Carlo: example of correlations

« The Effects of Variations in Nuclear Processes
on Type I X-Ray Burst Nucleosynthesis »

[Parikh, Jose, Moreno & Iliadis 2008]



Correlations > 10%

| ${}^4\text{He}$ | |
|------------------------------------|----------|
| Reaction | C_{jk} |
| $1/\tau_n$ | -0.9677 |
| ${}^3\text{He}(t,np){}^4\text{He}$ | 0.1151 |
| $D(d,n){}^3\text{He}$ | 0.1282 |
| $D(d,p){}^3\text{H}$ | 0.1296 |

| D | |
|----------------------------|----------|
| Reaction | C_{jk} |
| $D(p,\gamma){}^3\text{He}$ | -0.7790 |
| $D(d,n){}^3\text{He}$ | -0.4656 |
| $D(d,p){}^3\text{H}$ | -0.4082 |

| ${}^3\text{He}+T$ | |
|-----------------------------------|----------|
| Reaction | C_{jk} |
| $D(p,\gamma){}^3\text{He}$ | 0.6699 |
| $D(d,n){}^3\text{He}$ | 0.1640 |
| $D(d,p){}^3\text{H}$ | -0.1897 |
| ${}^3\text{He}(d,p){}^4\text{He}$ | -0.6841 |

| ${}^7\text{Be}+{}^7\text{Li}$ | |
|---|----------|
| Reaction | C_{jk} |
| $D(p,\gamma){}^3\text{He}$ | 0.4043 |
| $D(d,n){}^3\text{He}$ | 0.1547 |
| ${}^3\text{He}(d,p){}^4\text{He}$ | -0.2232 |
| ${}^3\text{He}(\alpha,\gamma){}^7\text{Li}$ | 0.7107 |
| ${}^7\text{Be}(n,\alpha){}^4\text{He}$ | -0.3057 |
| ${}^7\text{Be}(d,p){}^2{}^4\text{He}$ | -0.2079 |

Correlations > 10%

| ^6Li | |
|--|----------|
| Reaction | C_{jk} |
| $\text{D}(\alpha,\gamma)^6\text{Li}$ | 0.9835 |
| $^6\text{Li}(\text{p},\alpha)^3\text{He}$ | -0.1333 |
| ^9Be | |
| Reaction | C_{jk} |
| $^7\text{Li}(^3\text{He},\text{p})^9\text{Be}$ | 0.2820 |
| $^7\text{Li}(\text{t},\text{n})^9\text{Be}$ | 0.8910 |
| $^{11}\text{B} + ^{11}\text{C}$ | |
| Reaction | C_{jk} |
| $\text{D}(\text{p},\gamma)^3\text{He}$ | 0.1081 |
| $^3\text{He}(\alpha,\gamma)^7\text{Li}$ | 0.1621 |
| $^7\text{Be}(\alpha,\gamma)^{11}\text{C}$ | 0.2188 |
| $^{11}\text{C}(\text{n},\alpha)^{24}\text{He}$ | -0.9069 |

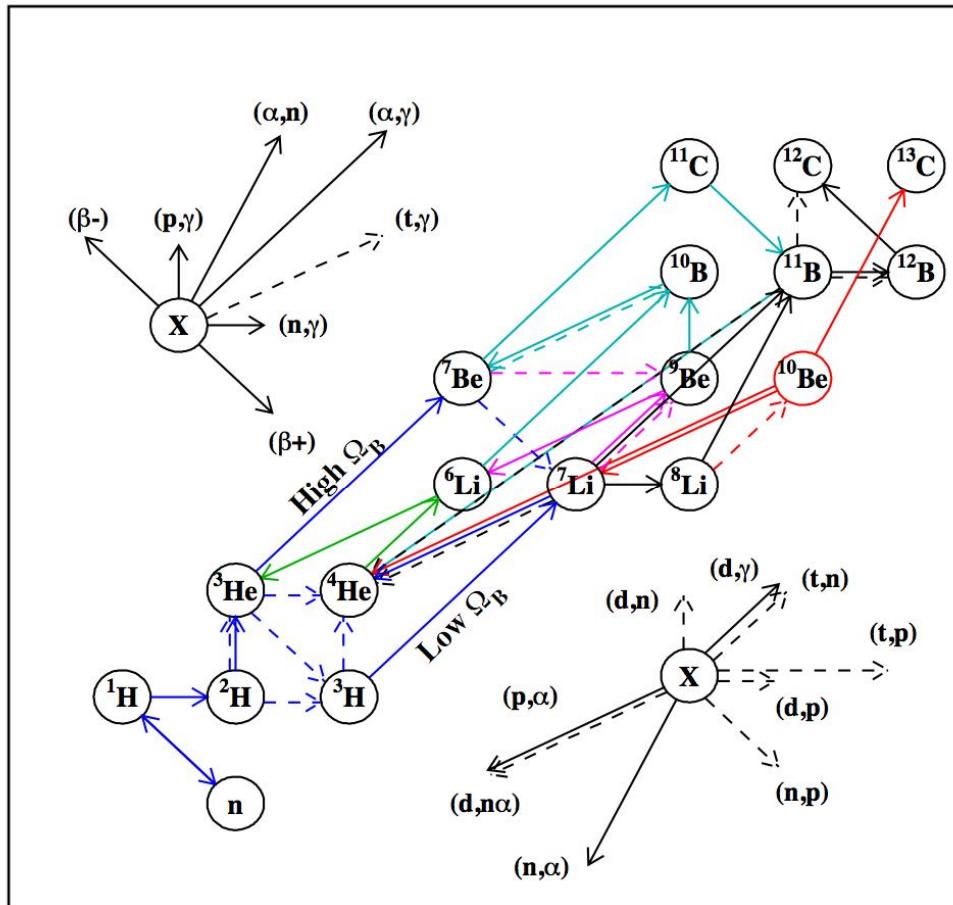
| CNO | |
|---|----------|
| Reaction | C_{jk} |
| $^{12}\text{B}(\text{p},\alpha)^9\text{Be}$ | -0.1030 |
| $^{10}\text{Be}(\text{p},\alpha)^7\text{Li}$ | -0.2479 |
| $^{10}\text{Be}(\alpha,\text{n})^{13}\text{C}$ | 0.2175 |
| $^7\text{Li}(\text{t},\gamma)^{10}\text{Be}$ | 0.1363 |
| $^8\text{Li}(\alpha,\text{n})^{11}\text{B}$ | 0.1449 |
| $^8\text{Li}(\text{t},\text{n})^{10}\text{Be}$ | 0.1958 |
| $^{10}\text{Be}(\text{t},\text{n})^{12}\text{B}$ | 0.1139 |
| $^{13}\text{C}(\text{d},\alpha)^{11}\text{B}$ | -0.2378 |
| $^{10}\text{Be}(\text{p},\text{t})^{24}\text{He}$ | -0.1297 |
| $^7\text{Li}(\text{d},\text{n})^{24}\text{He}$ | -0.1806 |

Known from the previous analysis

Combinatorial analysis

For the 6 reactions change the rate by factor 10^{-n} , 10^0 and 10^{+n}
(e.g. $n=2$), i.e. $3^6=729$ combinations

New path of BB nucleosynthesis to CNO?



Shopping list for nuclear physicists

Precision cosmology ?

- ^4He : Neutron lifetime controversy
- D : D/H at $\pm 1.6\%$ (!?!?) [Cooke+ 2014] what about $^2\text{H}(\text{p},\gamma)^3\text{He}$,
 $^2\text{H}(\text{d},\text{n})^3\text{He}$, $^2\text{H}(\text{d},\text{p})^3\text{H}$ rates (sensitivity: -0.32 to -0.54) ?
- ^7Li : The unsolved lithium problem
 - New conventional ^7Be destruction channel or neutron source ???
 - Reduce uncertainties on $^7\text{Be}(\text{n},\alpha)^4\text{He}$, $^3\text{He}(\alpha,\gamma)^7\text{Be}$ rates
 - Spallation reactions for relic decay (Friday)
- CNO: Explore new paths
 - $^{10}\text{Be}(\text{p},\alpha)^7\text{Li}$, $^{10}\text{Be}(\alpha,\text{n})^{13}\text{C}$, $^8\text{Li}(\text{t},\text{n})^{10}\text{Be}$, $^{10}\text{Be}(\text{p},\text{t})^2\text{He}$