

Primordial Nucleosynthesis

Alain Coc

CSNSM

(Centre de Sciences Nucléaires et de Sciences de la Matière, Orsay)

1. Standard Big-Bang Model and Nucleosynthesis
2. Nuclear Physics aspects
3. Beyond the Standard Model(s)



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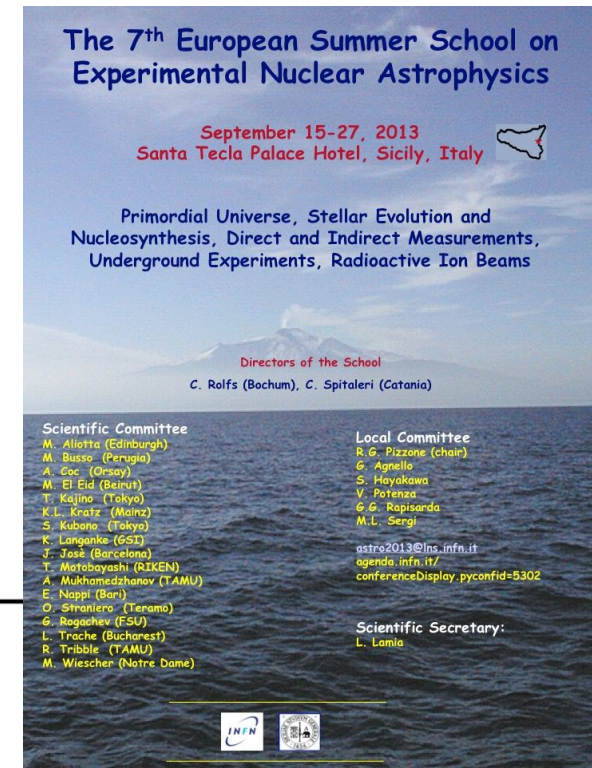
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Cosmology and Nuclear Physics


Alain Coc*

*Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), CNRS/IN2P3,
Université Paris Sud 11, UMR 8609, Bâtiment 104, F-91405 Orsay Campus, France
E-mail: Alain.Coc@csnsm.in2p3.fr*

There are important aspects of Cosmology, the scientific study of the large scale properties of the universe as a whole, for which nuclear physics can provide insights. Here, we will focus on the properties of early universe (Big-Bang nucleosynthesis during the first 20 mn) and on the *variation of constants* over the age of the universe.





The 7th European Summer School on Experimental Nuclear Astrophysics

September 15-27, 2013
Santa Tecla Palace Hotel, Sicily, Italy 

Primordial Universe, Stellar Evolution and Nucleosynthesis, Direct and Indirect Measurements, Underground Experiments, Radioactive Ion Beams

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Standard Big-Bang Model and Nucleosynthesis

1. The expansion of the Universe
2. Late time thermal history of the Universe
3. Primordial abundances deduced from observations
4. Standard Big Bang Nucleosynthesis
5. The lithium problem

Three observational evidences for the Big-Bang Model

1. The expansion of the Universe

Galaxies move away from each other according to Hubble's law:

$V = H_0 \times D$ with $H_0 \approx 72 \text{ km/s/Mpc}$, the Hubble parameter (or “constant”).

$D \propto a(t)$ (length scale parameter)

2. The Cosmic Microwave Background radiation (CMB)

A black body radiation at 2.7 K corresponding to the redshifted spectrum emitted when the universe became transparent

3. Primordial nucleosynthesis

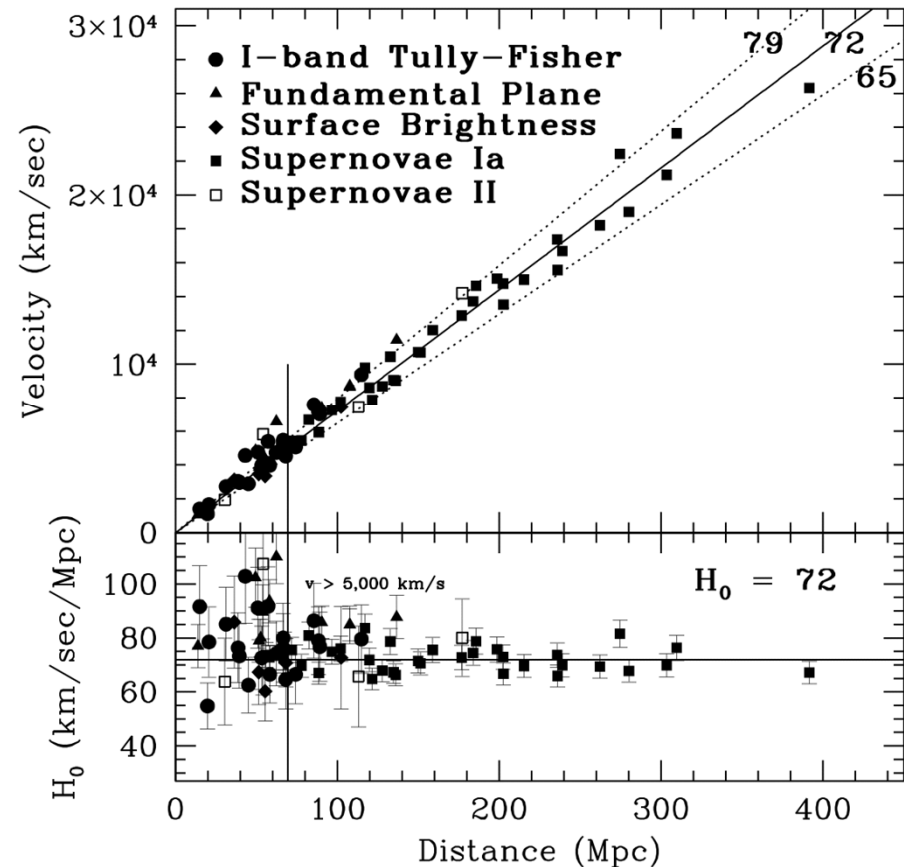
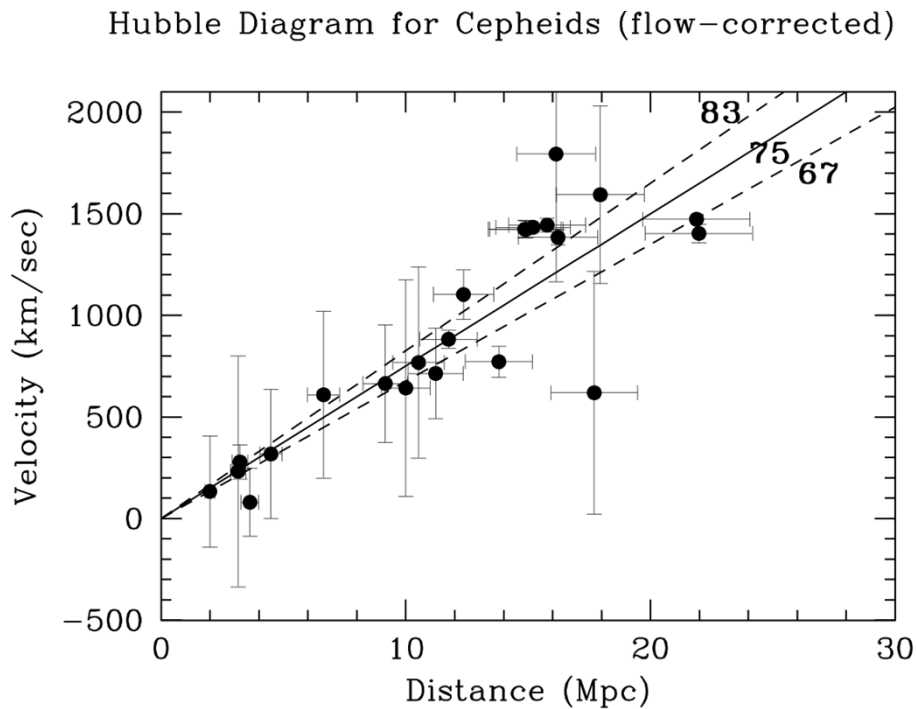
Reproduces the light-elements primordial abundances over a range of nine orders of magnitudes.

Hubble's law

$$V = H_0 \times D, H_0 \approx 72 \text{ km/s/Mpc}, h \equiv H_0 / (100 \text{ km/s/Mpc})$$

Where V is the recession velocity of a galaxy at a distance D

[Freedman et al. (2001)]



$$\text{Hubble's law: } V = H_0 \times D$$

Direct consequence of the expansion of the Universe

Mean distance between galaxies $\propto a(t)$ [scale factor]:

$$D(t) = \chi \times a(t)$$

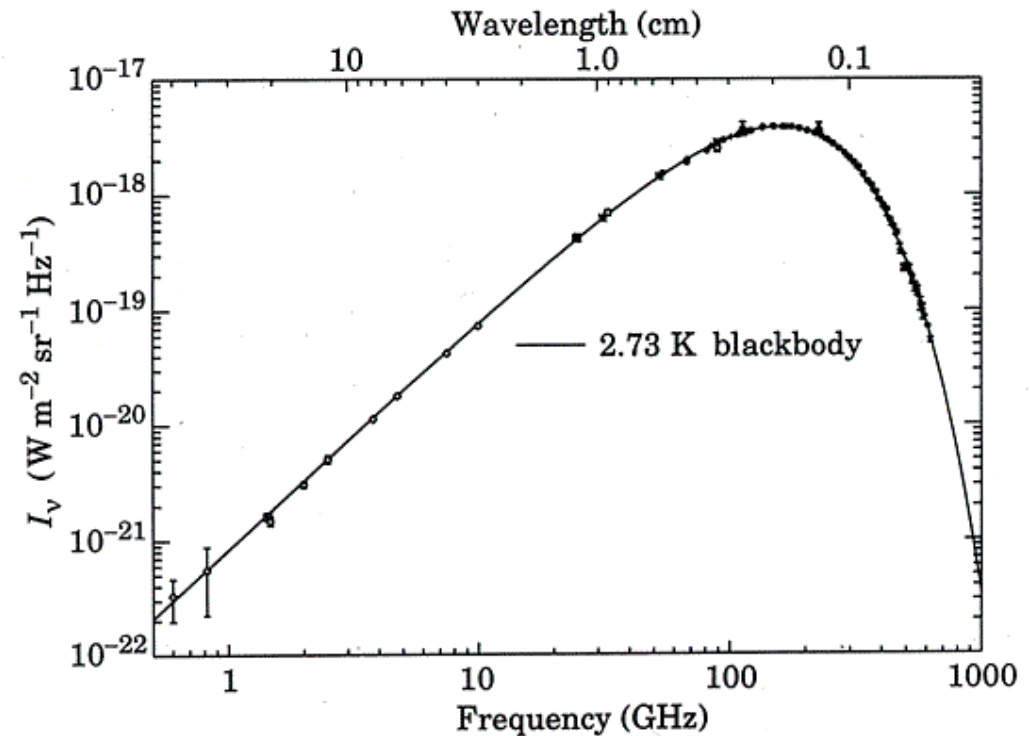
$$V \equiv \dot{D}(t) = \chi \times \dot{a}(t) = \overset{D}{\chi \times a} \times \dot{a}(t) / a(t)$$

$$H(t) \equiv \dot{a}(t) / a(t)$$

The Hubble parameter $H(t)$ or constant $H_0 = H(t=\text{now})$

Cosmic Microwave Background radiation (CMB)

Black body radiation spectrum
at $T = 2.728 \pm 0.004$ K



Redshifted spectrum of the photons released when the universe became transparent (electrons and nuclei recombination into neutral atoms)

Opacity caused by Compton scattering of photons on free electrons

As long as photoionization $H + \gamma \leftrightarrow p + e^-$ is effective i.e. until $\approx 300,000$ years after the Big-Bang when T dropped to ≈ 3000 K

$$T = 3000 \text{ K} \rightarrow 2.7 \text{ K}$$

Redshift (z)

$$T = 3000 \text{ K} \rightarrow 2.7 \text{ K}$$

$$\lambda(\text{present}) = \lambda(\text{recombination}) \times a(\text{present}) / a(\text{recombination})$$

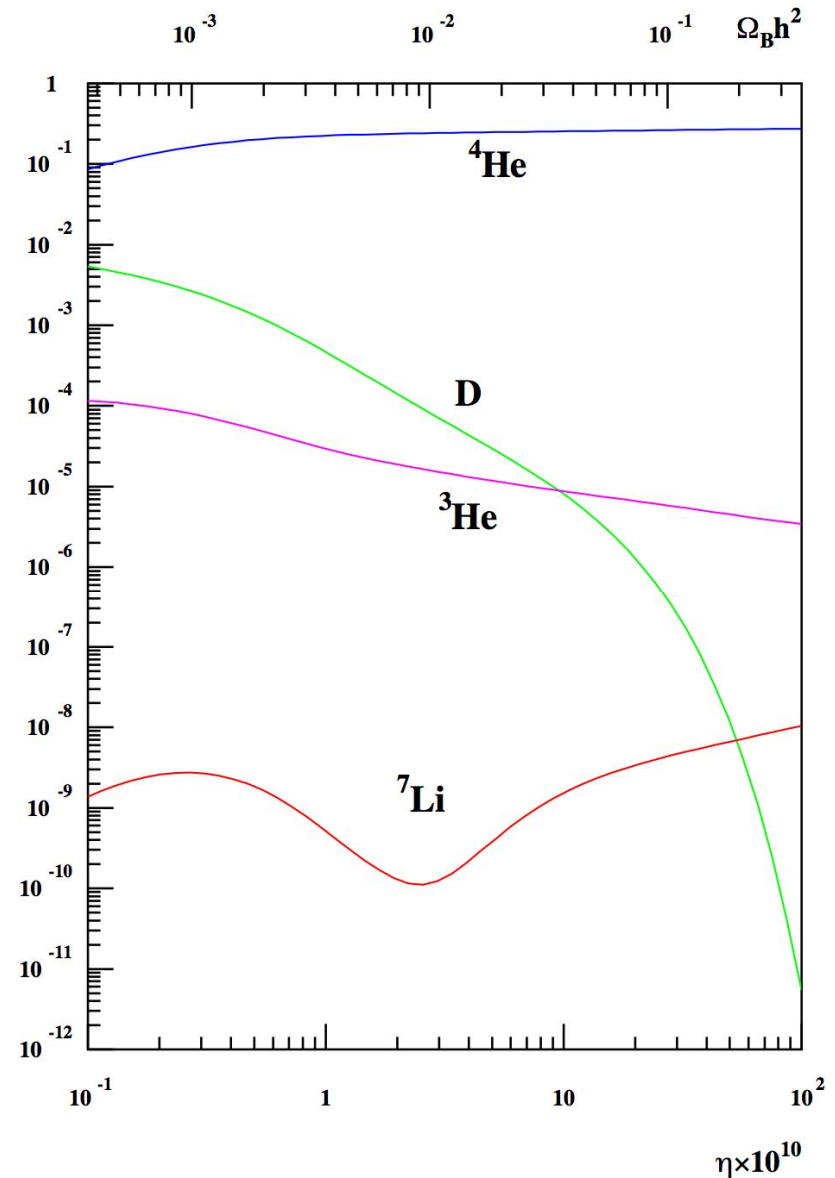
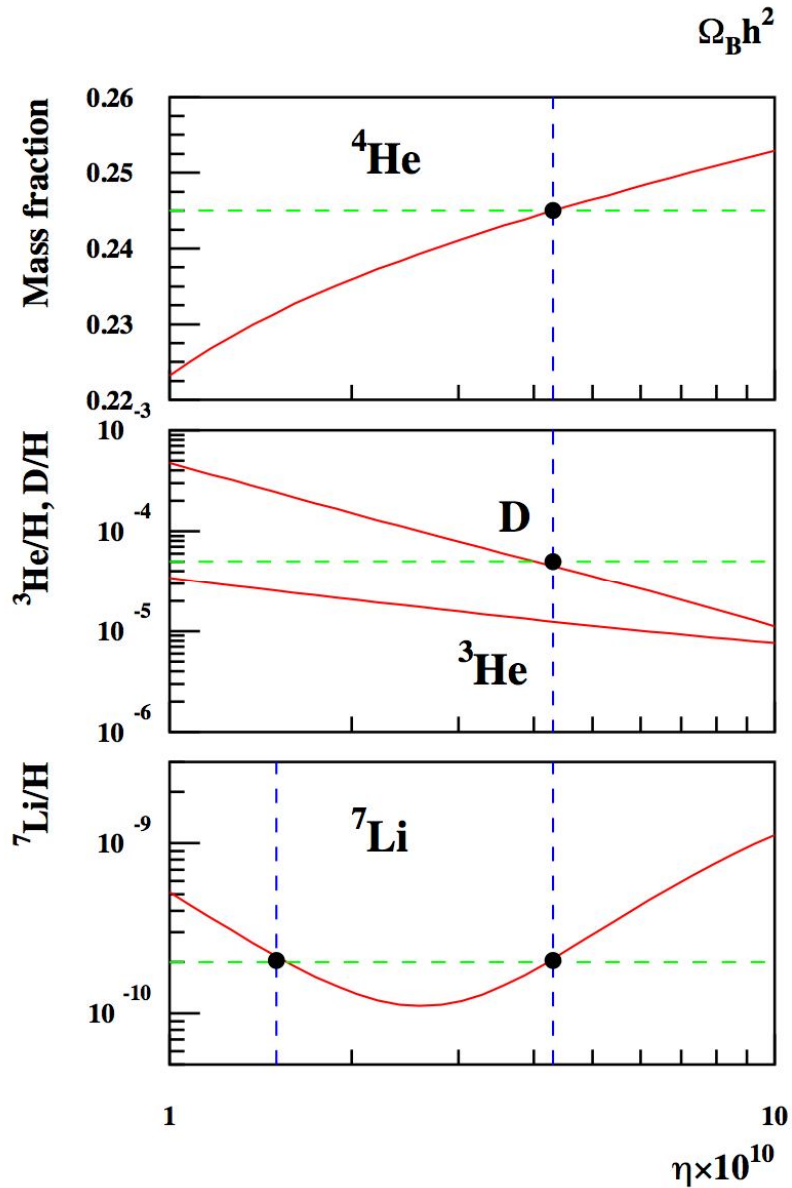
$$z \equiv \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

$$z + 1 \equiv \frac{a(t_0)}{a(t = t_{\text{emission}})}$$

$\lambda_{\text{emitted}} = \lambda$ as
measured in laboratory

0 \equiv present value

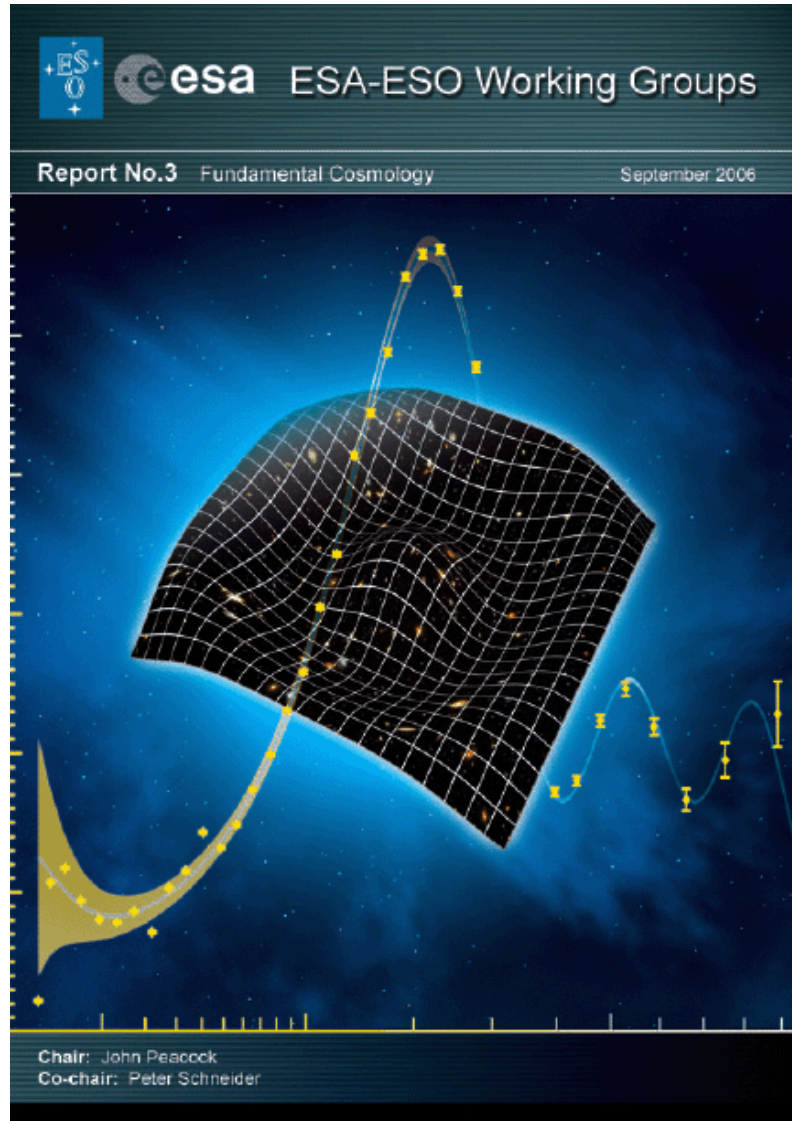
BBN determination of the baryonic density



Big-Bang Nucleosynthesis probe of new physics

- First determination of the baryonic density of the Universe, $(1-3) \times 10^{-31} \text{g/cm}^3$ [*Wagoner 1973*], need for baryonic dark matter
 - Baryonic density $\rho_B \approx 4.5 \times 10^{-31} \text{g/cm}^3$ from the anisotropies in the Cosmic Microwave Background radiation,
- First determination of the number of light neutrino families, $N_\nu \leq 3$ [*Yang, Schramm, Steigman, Rood 1979*]
 - Number of neutrino families $N_\nu = 2.984 \pm 0.008$ [*LEP experiments*]

“Key questions” in cosmology



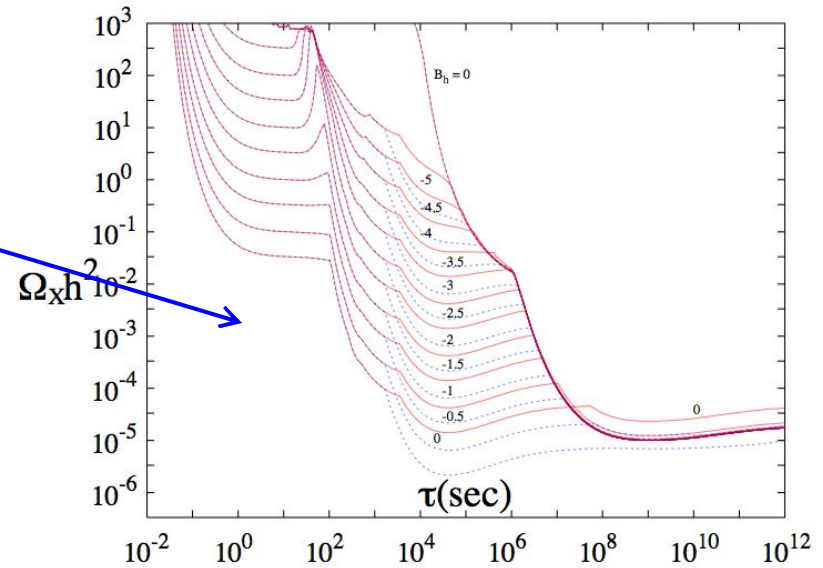
[arXiv:astro-ph/0610906]

- 1) What generated the **baryon asymmetry**? Why is there negligible antimatter, and what set the ratio of baryons to photons?
- 2) What is the **dark matter**? Is it a relic massive supersymmetric particle, or something (even) more exotic?
- 3) What is the **dark energy**? Is it Einstein's cosmological constant, or is it a dynamical phenomenon with an observable degree of evolution?
- 4) Did **inflation** happen? Can we detect relics of an early phase of vacuum-dominated expansion?
- 5) Is standard cosmology based on the correct physical principles? Are features such as dark energy artifacts of a **different law of gravity**, perhaps associated with extra dimensions? **Could fundamental constants actually vary**?

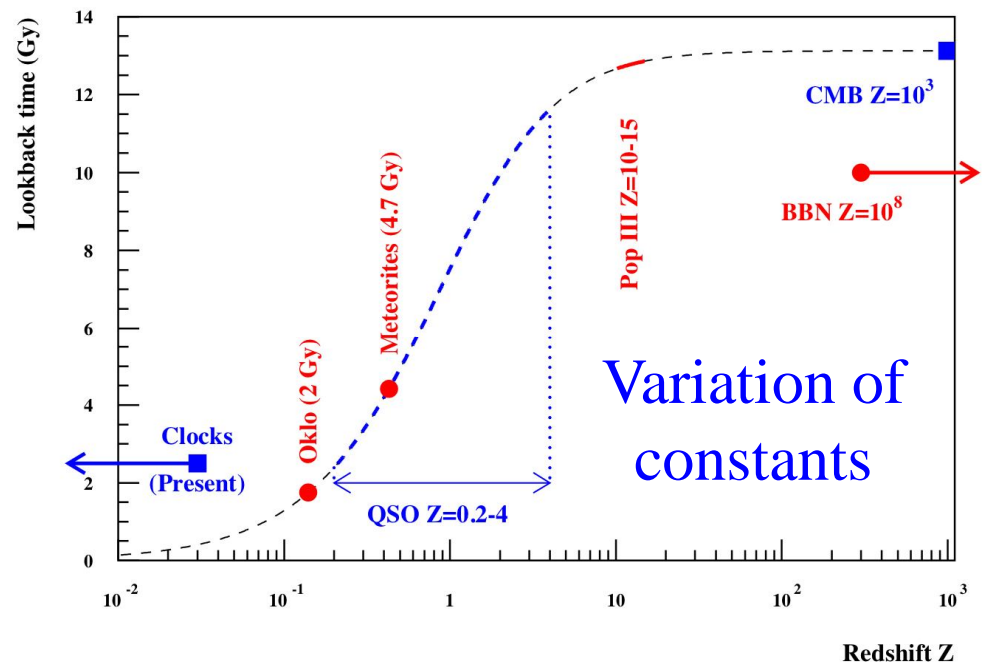
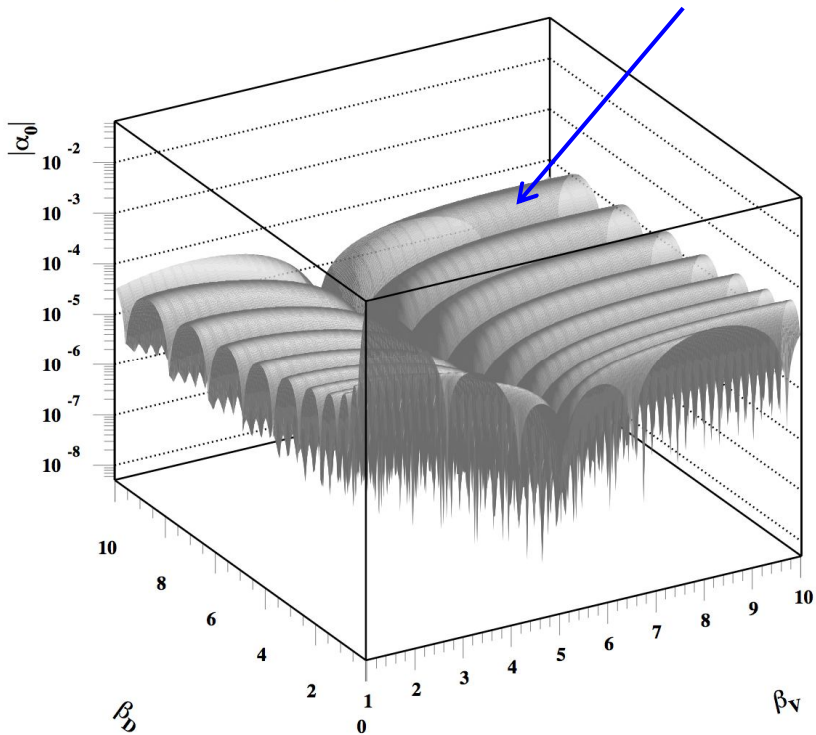
Beyond the Standard Models(s) ?

See review by *Iocco, Mangano, Miele, Pisanti & Serpico 2009*

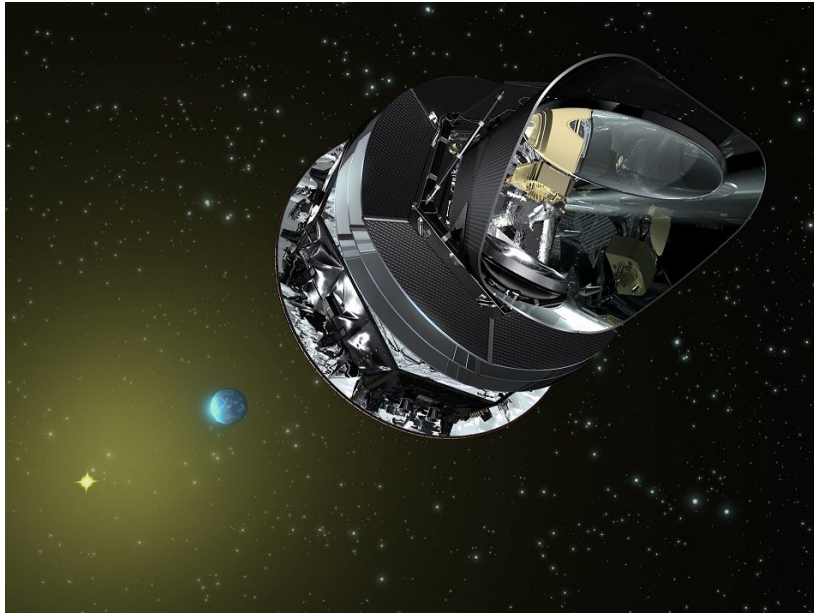
Relic Particle decay



Deviation from General Relativity

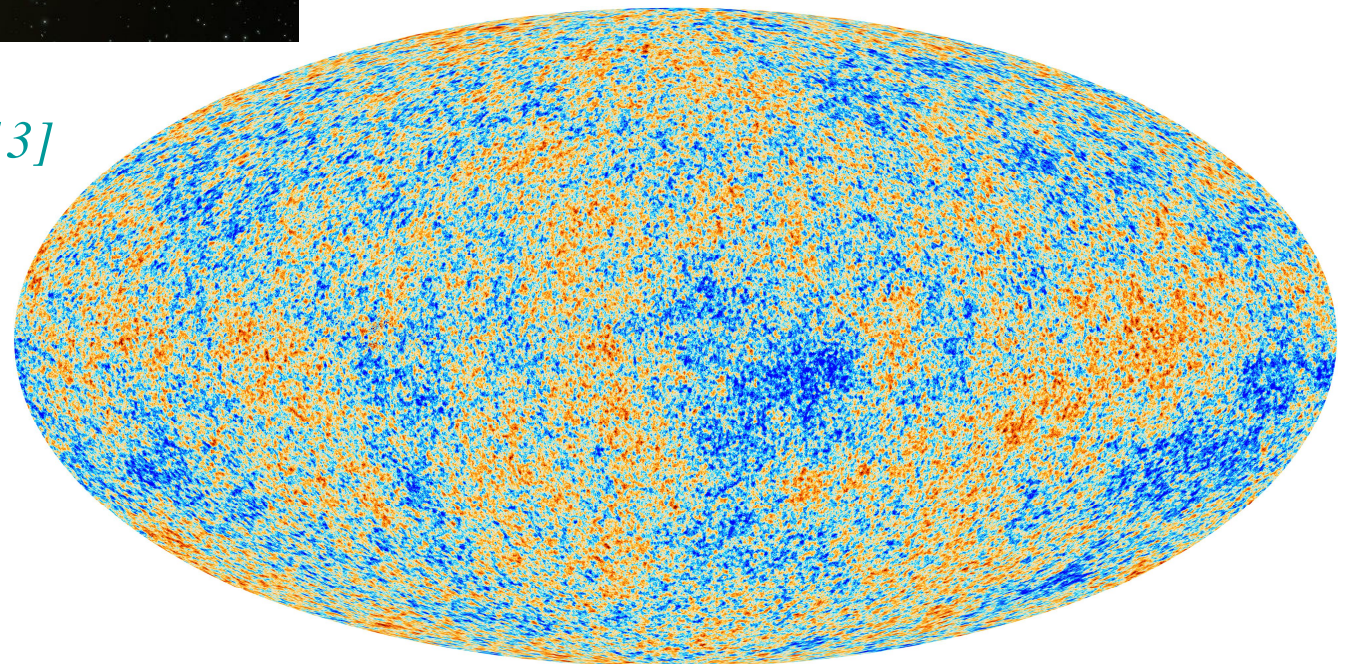


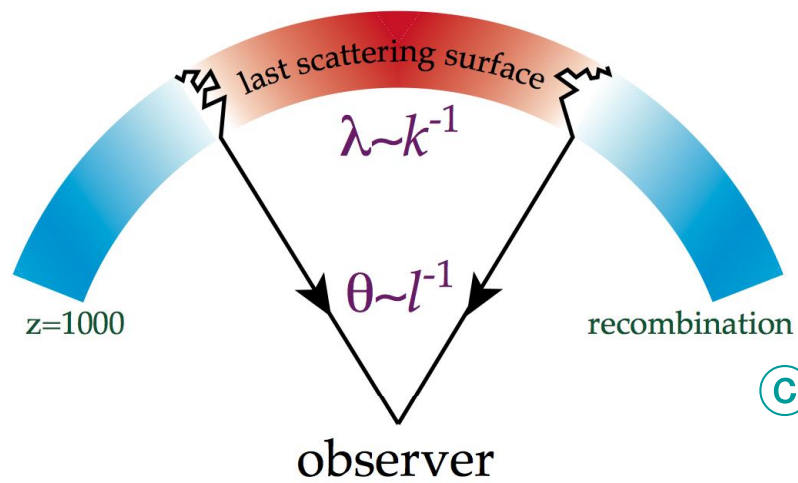
Anisotropies of the Cosmic Microwave Background



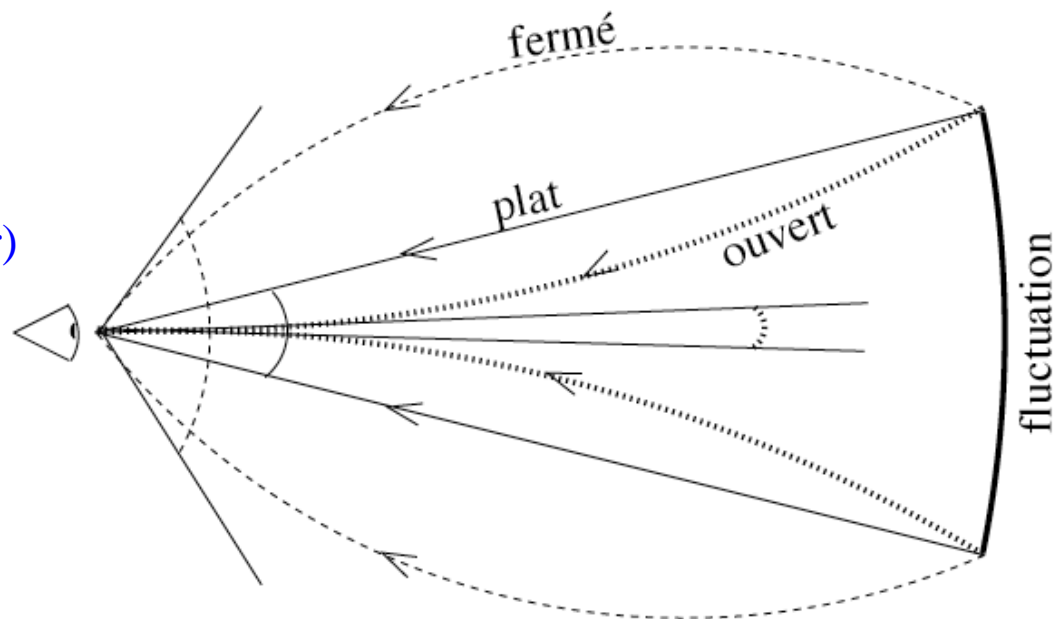
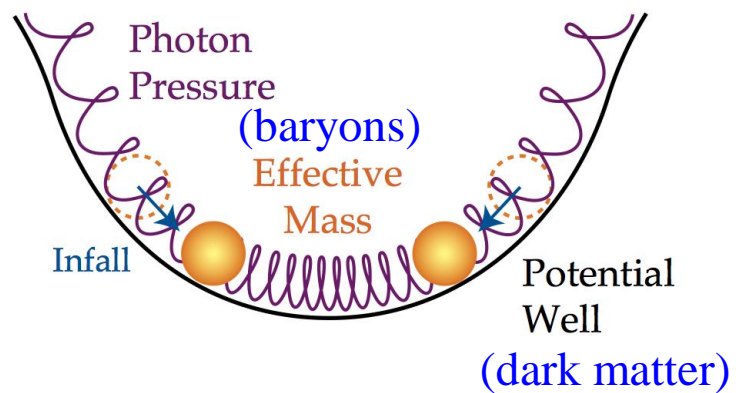
At $t \approx 0.38$ My, and $T \approx 3000$ K :
recombination, the Universe
becomes transparent
(presently 2.725 K)

WMAP [*Hinshaw+ 2013*]
Planck [*Ade+ 2013*]





© Wayne Hu

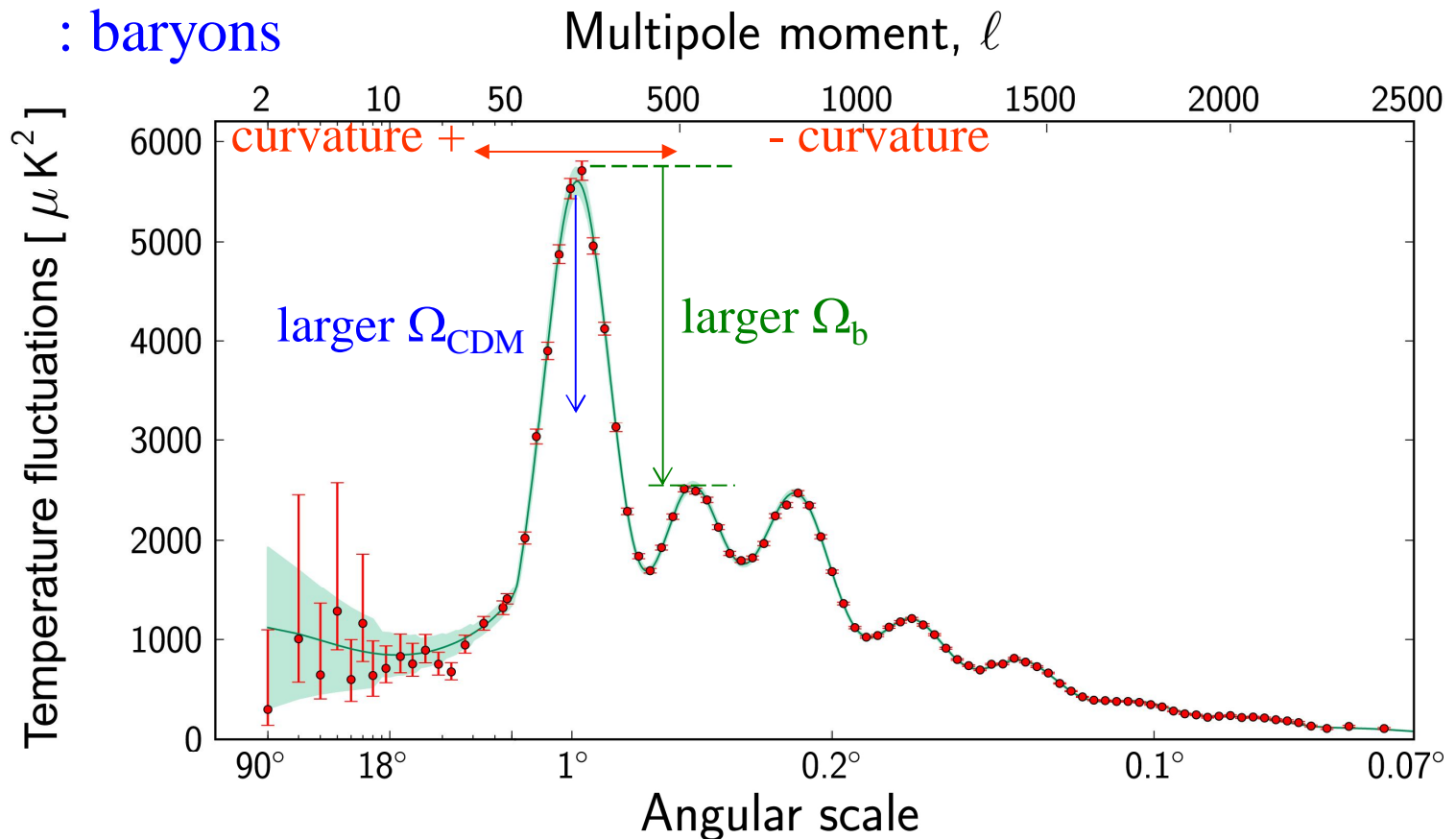


Anisotropies of the CMB

Spatial fluctuation spectrum of CMB
generated by acoustic oscillations

- Pressure : photons
- Inertia : baryons

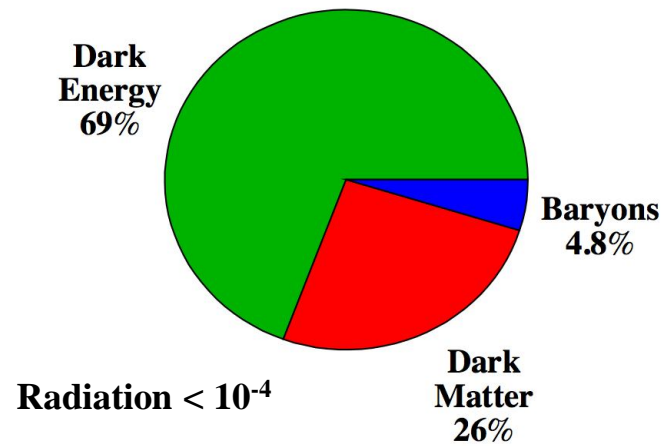
- Geometry ($\Omega_T \approx 1$), 1st peak
- Ω_b (2nd/1st peaks)



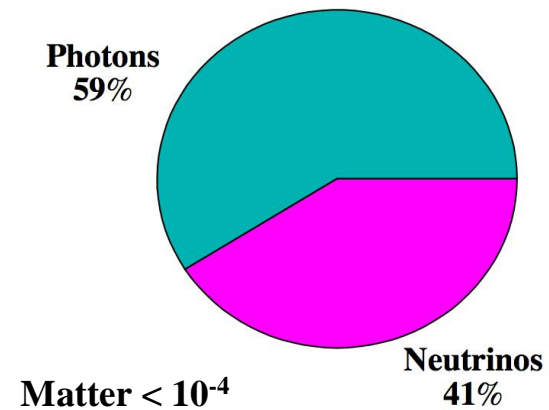
[Ade+ 2013+ (Planck)]

Density components of the Universe

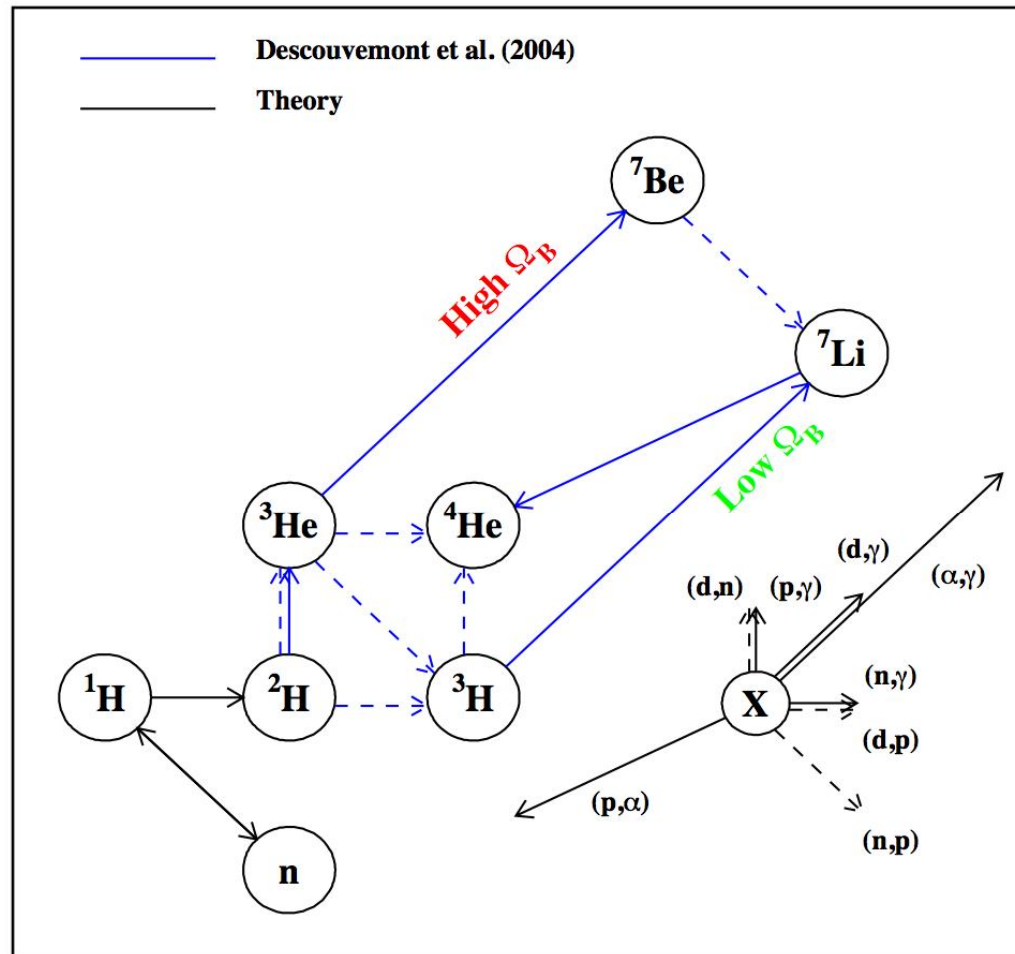
Now.... ($a=1$)



....and then ($a \approx 10^{-8}$)



Big Bang Nucleosynthesis calculations

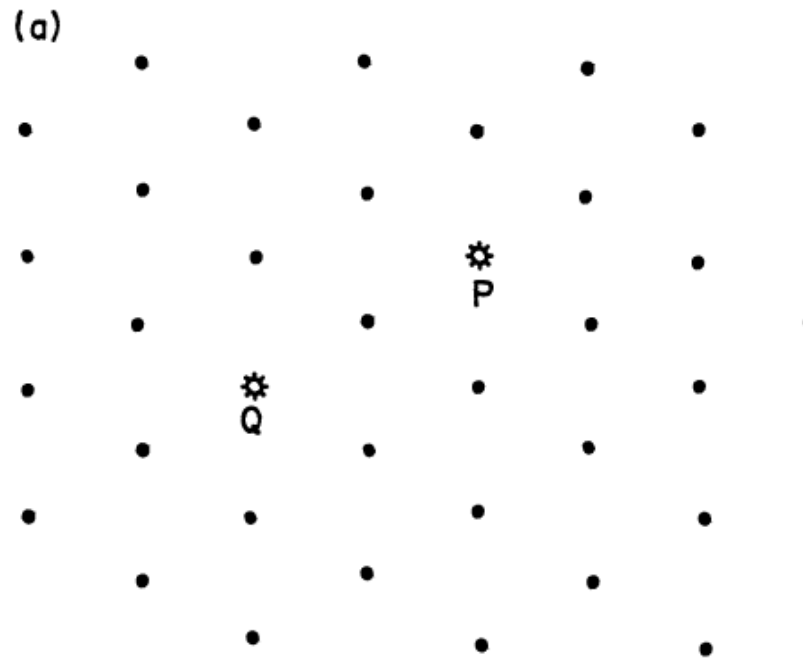


Needs:

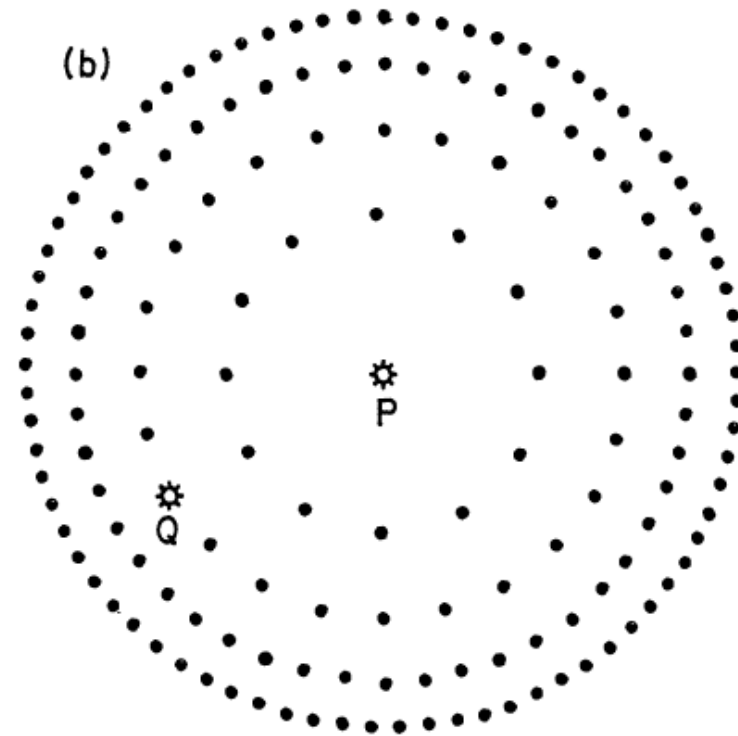
- Reaction rates
- Density $\rho_b(t)$, ions and photons $T(t)$ and neutrino $T_\nu(t)$ temperatures as a function of time

Dynamics of the expanding Universe

Cosmological principle : homogeneity and isotropy



Homogeneity



Isotropy

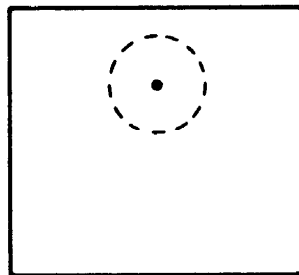
Dynamics of the expanding Universe

Friedmann-Lemaître-Robertson-Walker metrics:

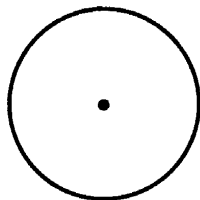
$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right)$$

scale factor

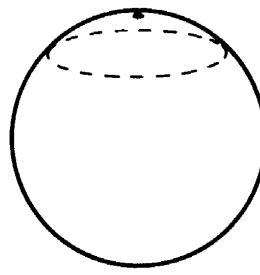
$k = 0, +1, -1$ for a null, positive or negative curvature



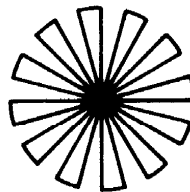
$=2\pi R$



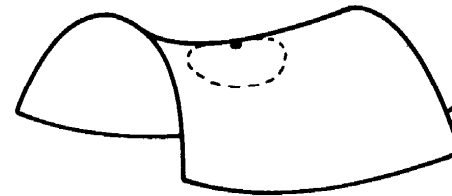
Zero curvature



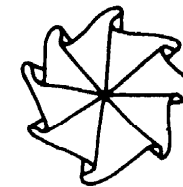
$<2\pi R$



Positive curvature



$>2\pi R$



Negative curvature

Dynamics of the expanding Universe

Einstein equation:
$$\underbrace{R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R}_{\text{curvature}} = \underbrace{8\pi G T_{\alpha\beta}}_{\propto \text{energy density}} (-\Lambda g_{\alpha\beta})$$

curvature \propto energy density (*-cste*)

Curvature tensor: $R_{\alpha\beta}$ (from derivative of $g_{\alpha\beta}$ metrics)

Energy-momentum tensor $T_{\alpha\beta}$:

$$T = \begin{pmatrix} \rho & & & \\ & p & & \\ & & \mathbf{0} & \\ & & & p \end{pmatrix}$$

ρ = density of energy
 p = pressure $\equiv w \times \rho$

Equation Of State: $p \equiv w \times \rho$

Reminders of thermodynamics

Volume density of states :

$$dn = \underbrace{\left(\frac{1}{\exp[(\varepsilon - \mu)/kT] \pm 1} \right)}_{\text{Fermi (+) / Bose-Einstein (-) statistics}} \times \underbrace{\frac{g}{2\pi^2 \hbar^3} p^2 dp}_{\text{Phase space}}$$

p : momentum

ε : energy

μ : chemical potential

g : spin factor ($2J+1$)

Fermi (+) / Bose-Einstein (-) statistics

Phase space

$$n(T) = \frac{g}{2\pi^2 \hbar^3} \int_m^\infty \frac{\sqrt{\varepsilon^2 - m^2} \varepsilon d\varepsilon}{\exp[(\varepsilon - \mu)/kT] \pm 1}$$

(Volume number density of particles)

$$\rho_E(T) = \frac{g}{2\pi^2 \hbar^3} \int_m^\infty \frac{\sqrt{\varepsilon^2 - m^2} \varepsilon^2 d\varepsilon}{\exp[(\varepsilon - \mu)/kT] \pm 1}$$

(Volume energy density)

$$p(T) = \frac{1}{3} \frac{g}{2\pi^2 \hbar^3} \int_m^\infty \frac{(\varepsilon^2 - m^2)^{3/2} d\varepsilon}{\exp[(\varepsilon - \mu)/kT] \pm 1}$$

(Pressure)

Relativistic limit : “radiation”

Relativistic limit (with $\mu=0$): $m \rightarrow 0$, $\varepsilon \rightarrow p$ e.g. photons

Photon gas energy density :
(with $g=2$, the spin factor and a_R the radiation constant)

$$\rho_\gamma = g \frac{k^2 \pi^2}{30 \hbar^3} T^4 \equiv a_R T^4$$

- Energy density for relativistic *bosons* ($kT \gg m$): identical except for the *spin factor g*
- Energy density for relativistic *fermions* : identical except for the *spin factor g* and an additional *7/8 factor*

$$\text{Energy density : } \rho_R = \frac{g_{\text{eff}}(T)}{2} a_R T^4$$

$$\text{Pressure : } p = \frac{1}{3} \rho_R$$

$$\text{Entropy density : } s = \frac{p + \rho_R}{T} \equiv \frac{4}{3} \frac{g_{\text{eff}}(T)}{2} a_R T^3$$

$$\text{EOS: } w = 1/3$$

Since ρ and s are the energy and entropy per unit volume:

$$E = \rho V$$

$$S = sV$$

$$dE = \rho dV + \frac{\partial \rho}{\partial T} dT$$

$$dS = s dV + \frac{\partial s}{\partial T} dT$$

$$dE = T dS - p dV$$

$$Ts = \rho + p$$

Classical limit

Classical limit : $\varepsilon \rightarrow m + p^2/2m$

$$n(T) = \frac{g}{2\pi^2\hbar^3} \int_0^\infty \exp\left[-\frac{(m + p^2/2m)}{kT}\right] p^2 dp = g \left(\frac{mkT}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{-\frac{m}{kT}}$$

Application to $p + e^- \rightarrow H + \gamma$ equilibrium

n_e, n_p, n_H, n_b = electron, proton, atomic, baryonic densities and

$$X_e \equiv \frac{n_e}{n_p + n_H} \equiv \frac{n_e}{n_b}, \text{ the ionized fraction } (n_b = n_p + n_H).$$

$$n_e = n_p = X_e n_b \text{ (neutrality) and } n_H = (1 - X_e) n_b$$

$$\frac{n_e n_p}{n_H} = \frac{X_e^2}{1 - X_e} n_b = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{-Q/kT} \quad \text{with } Q = m_e + m_p - m_H = E_I, \\ \text{the ionization potential}$$

Temperature of photons-electrons decoupling

Saha equation :

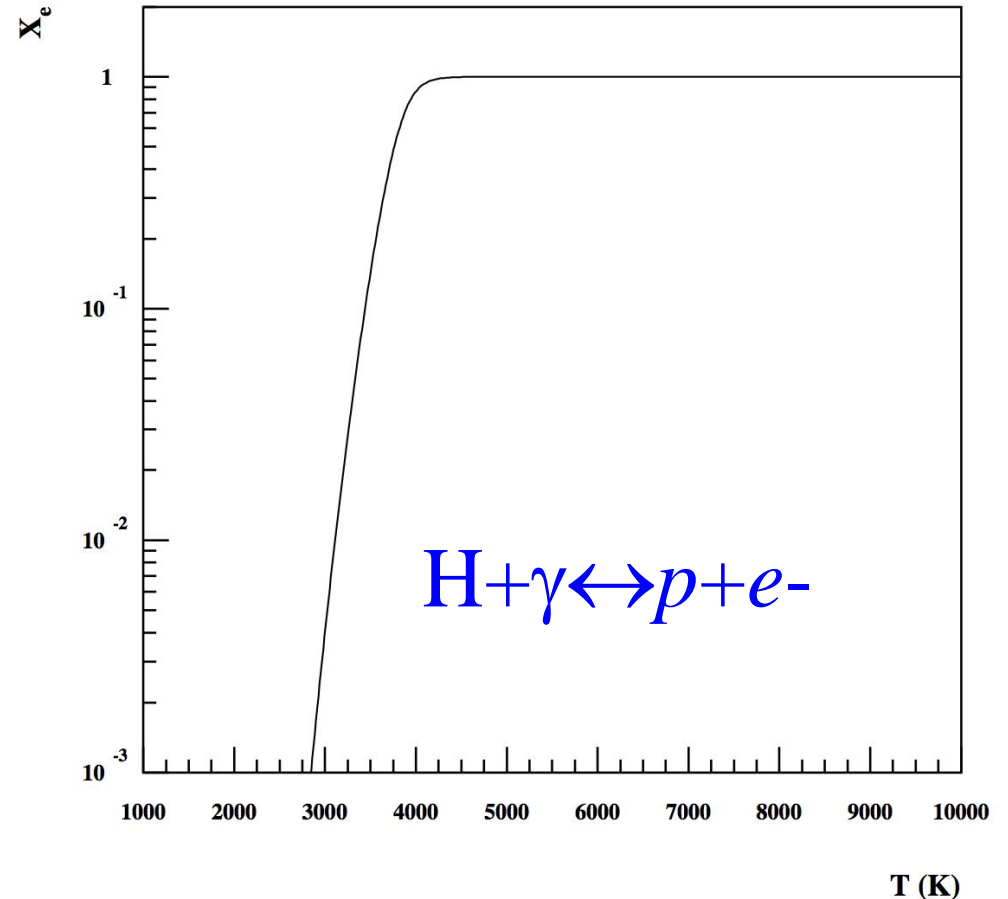
$$\frac{X_e^2}{1 - X_e} = \left(\frac{m_e c^2 k_B T}{2\pi(\hbar c)^2} \right)^{3/2} \frac{e^{-E_I/k_B T}}{n_b}$$

$$T = 2.725 \times (1+z) \text{ K}$$

$$n_b \approx \Omega_b h^2 \times 10^{-5} \times (1+z)^3$$

baryon / cm³

(z is the redshift)



Decoupling at $T \approx 3000 \text{ K} \ll E_I = 13.6 \text{ eV} (1.6 \cdot 10^5 \text{ K})$

Dynamics of the expanding Universe

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta} - \Lambda g_{\alpha\beta}$$

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right)$$

Friedmann equations :

Hubble “constant” :

$$\textcircled{1} \quad \left(\frac{\dot{a}}{a} \right)^2 \equiv H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

$$\textcircled{2} \quad \frac{\ddot{a}}{a} = \frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}$$

$$\Leftrightarrow \left\{ \begin{array}{l} \text{Matter conservation :} \\ \dot{\rho} + 3H(\rho + p) = 0 \\ \rho \propto a^{-3(1+w)} \end{array} \right.$$

$$T = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \quad \Lambda g = \begin{pmatrix} -\Lambda & & & \\ & +\Lambda & & \\ & & +\Lambda & \\ & & & +\Lambda \end{pmatrix}$$

$$w = \begin{array}{l} 0 \text{ (matter)} \\ 1/3 \text{ (radiation)} \\ 1 \text{ (\Lambda)} \end{array}$$

Critical density

In a flat universe ($k=0$), without cosmological constant ($\Lambda=0$) one should have ①, at present (t_0), the relation:

$$H_0^2 \equiv \left(\frac{\dot{a}}{a} \right)_0^2 = \frac{8\pi G \rho_{0,C}}{3}$$

The critical density corresponding to a flat universe :

$$\rho_{0,C} \equiv \frac{3H_0^2}{8\pi G}$$

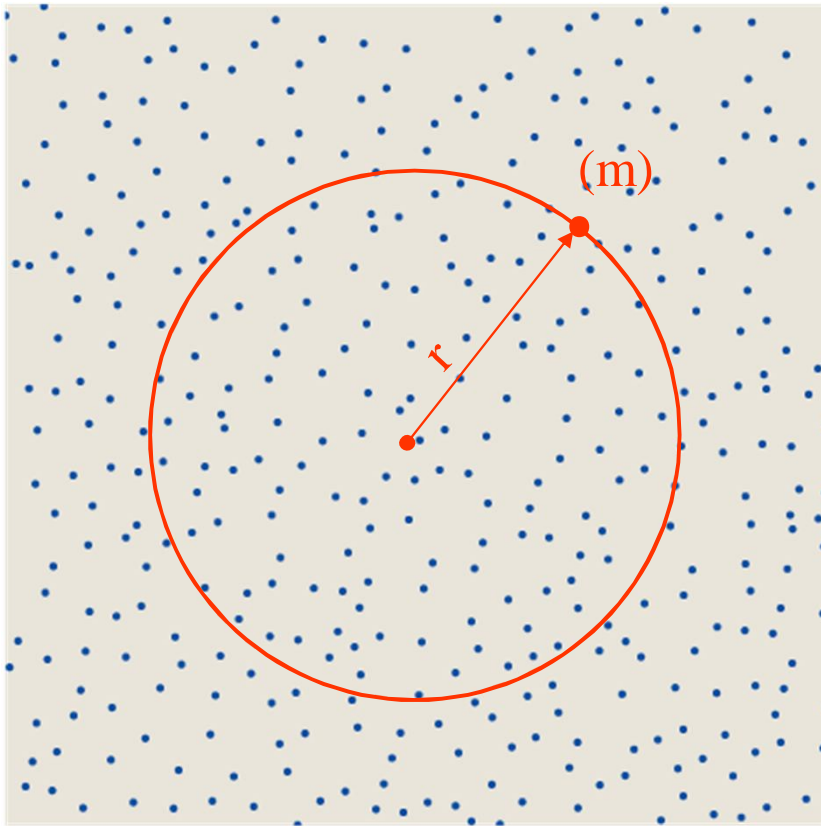
$$\Omega \equiv \frac{\rho}{\rho_{0,C}}$$

$$\rho_{0,C} = 1.87847 h^2 \times 10^{-29} \text{ g/cm}^3 \text{ or } 2.9 h^2 \times 10^{11} \text{ M}_\odot/\text{Mpc}^3$$

$$H_0 \equiv \text{Hubble "constant"} \quad (h = H_0/100 \text{ km/s/Mpc} \quad h \approx 0.72)$$

Critical density (Newtonian)

Birkoff's (Gauss in GR) theorem



$$E = \frac{1}{2} m \dot{r}^2 - \frac{GmM}{r}$$

$$\frac{1}{2} m \dot{r}^2 = \frac{Gm}{r} \left(\frac{4\pi r^3 \rho}{3} \right) + E$$

$$\left(\frac{\dot{r}}{r} \right)^2 = H_0^2 = \frac{8\pi G\rho}{3} + \frac{2E}{mr^2}$$

Number of baryons per photon (η)

Photon number density :

$$n_{\gamma,0} = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT_0}{\hbar c} \right)^3 = 410.73 \left(\frac{T_0}{2.7255} \right)^3 \quad (\text{number/cm}^3)$$

Number of baryons per photon : $\eta \equiv n_b/n_\gamma$

$$\eta \equiv \frac{\rho_b}{n_{\gamma,0} \bar{M}} = \frac{3H_0}{8\pi G\bar{M}} \frac{\pi^2}{2\zeta(3)} \left(\frac{\hbar c}{kT_0} \right)^3 \Omega_b$$

(using the mean baryon mass)

$$\bar{M} = M_p(1 - Y_p) + \frac{M_\alpha}{4} Y_p = (1.6735 - 0.0119 Y_p) \times 10^{-24} \quad (\text{g})$$

$$\eta = 2.7377 \times 10^{-8} \Omega_b h^2 \quad \text{or} \quad \Omega_b h^2 = 3.6528 \times 10^7 \eta$$

Density components of the Universe

$$\Omega \equiv \rho / \rho_C$$

Radiation (CMB)	Ω_R	$5 \cdot 10^{-5}$
Visible matter	Ω_L	≈ 0.003
Baryons	Ω_b	0.048
Dark Matter	Ω_c	0.257
Vacuum	Ω_Λ	0.691
Total	Ω_T	≈ 1.0

$$\Omega_b h^2 = 0.02264 \pm 0.00050$$

[WMAP: Hinshaw+ 2013]

$$\Omega_b h^2 = 0.02207 \pm 0.00033$$

[Planck: Ade+ 2013]

Dynamics of the expanding Universe

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}(\rho_R + \rho_M + \rho_\Lambda) - \frac{k}{a^2} \quad \rho \propto a^{-3(1+w)}$$

$w_R = 1/3, w_M = 1/3, w_\Lambda = -1$

Friedmann equation governs the rate of expansion:

$$H^2(z) = H_0^2 \left[\Omega_M (z+1)^3 + \Omega_R (z+1)^4 + \Omega_\Lambda + (1 - \Omega_T)(z+1)^2 \right]$$

a (a_0) is the (present) scale factor, $z \equiv a_0/a - 1$ the redshift, while $\Omega_M, \Omega_R, \Omega_\Lambda, \Omega_T$ are the present matter, radiation, vacuum energy and total density ratios

During BBN, $z \approx 10^8$ and the evolution is dominated by “radiation” (i.e. relativistic particles) while “matter”, curvature or a cosmological constant have no influence on the expansion rate

Expansion rate

$$\frac{\dot{a}}{a} = H_0 \left[\Omega_M \left(\frac{a_0}{a} \right)^3 + \Omega_R \left(\frac{a_0}{a} \right)^4 + \Omega_\Lambda + (1 - \Omega_T) \left(\frac{a_0}{a} \right)^2 \right]^{\frac{1}{2}}$$

For radiation (n=4), matter (n=3) and vacuum (n=0) dominated eras

$$\frac{1}{a} \frac{da}{dt} = H_0 \Omega_{x;0}^{\frac{1}{2}} a_0^{\frac{n}{2}} a^{-\frac{n}{2}}$$

$$dt = \frac{1}{H_0 \Omega_{x;0}^{1/2} a_0^{n/2}} a^{2-n} da$$

- Radiation dominated era :

$$a \propto t^{1/2},$$

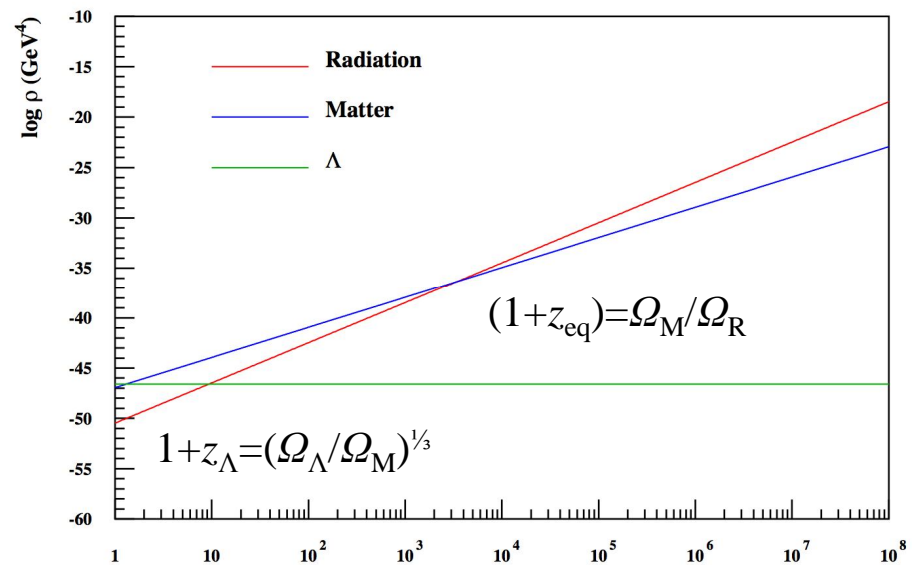
$$T \propto a \text{ and } \rho_B \propto a^{-3}$$

- Matter dominated era :

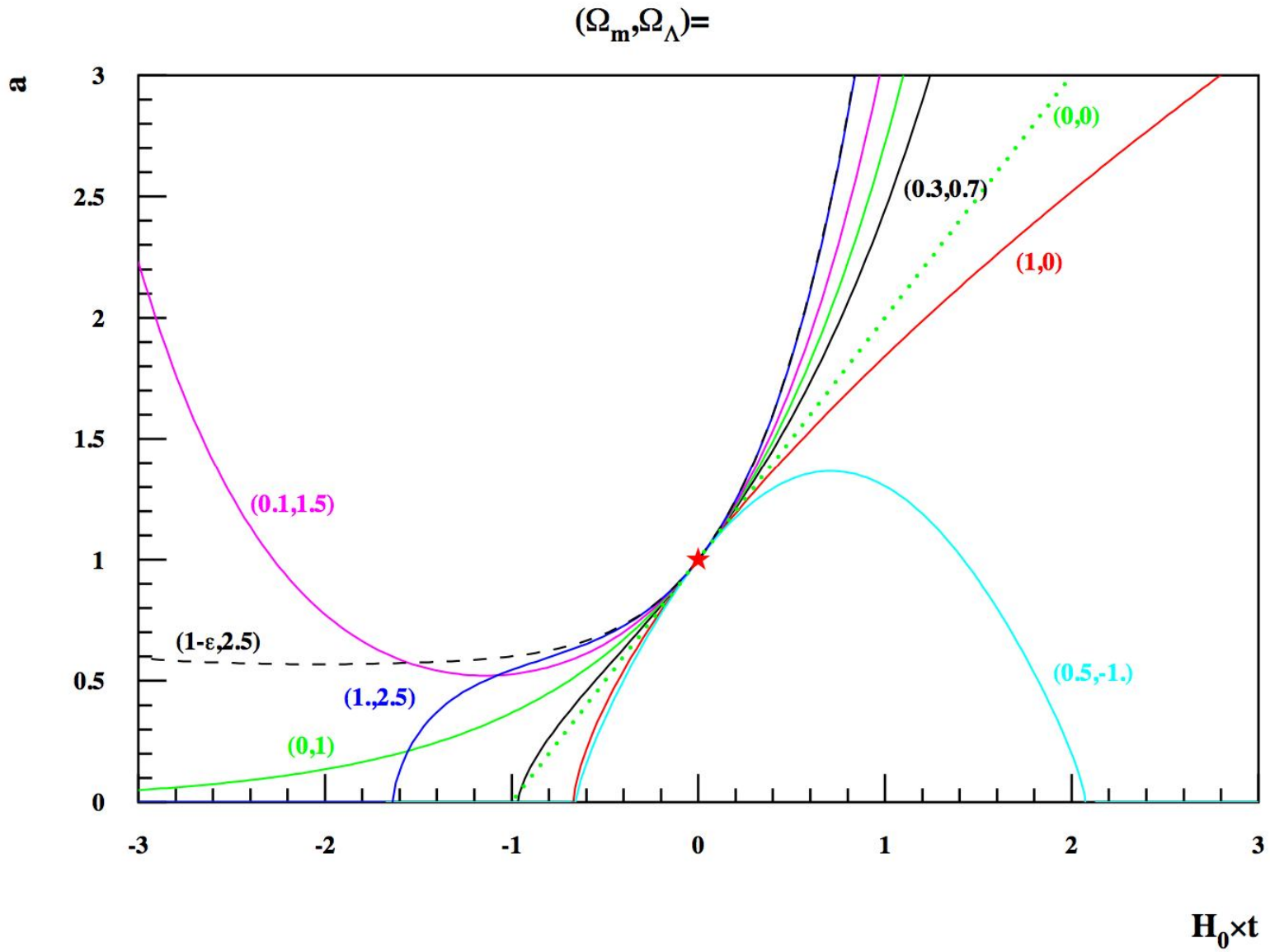
$$a \propto t^{2/3}$$

- Vacuum energy dominated era :

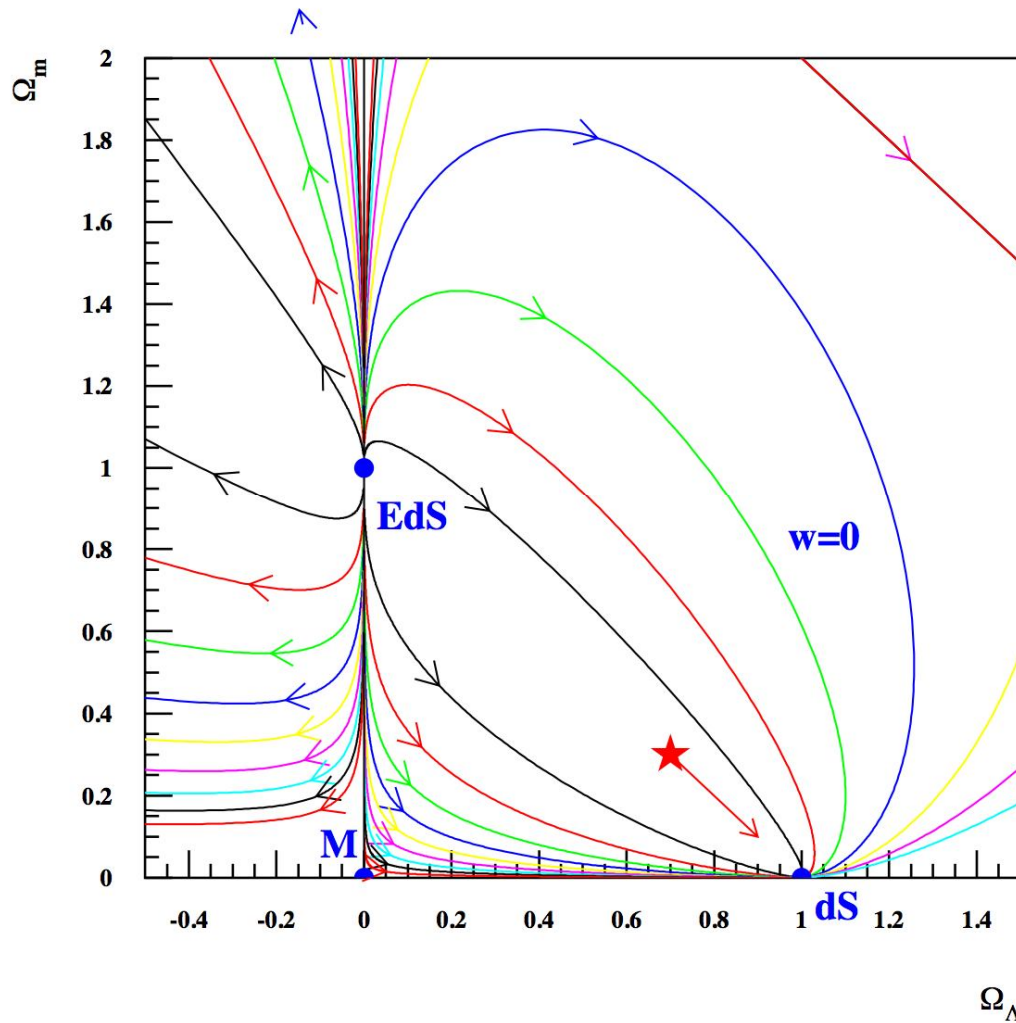
$$a \propto \exp(H_0 \Omega_\Lambda^{1/2} t)$$



Different types of Universe



Dynamics of the Universe



Fixed points :

$(\Omega_m, \Omega_\Lambda) = (0, 0)$: Milne

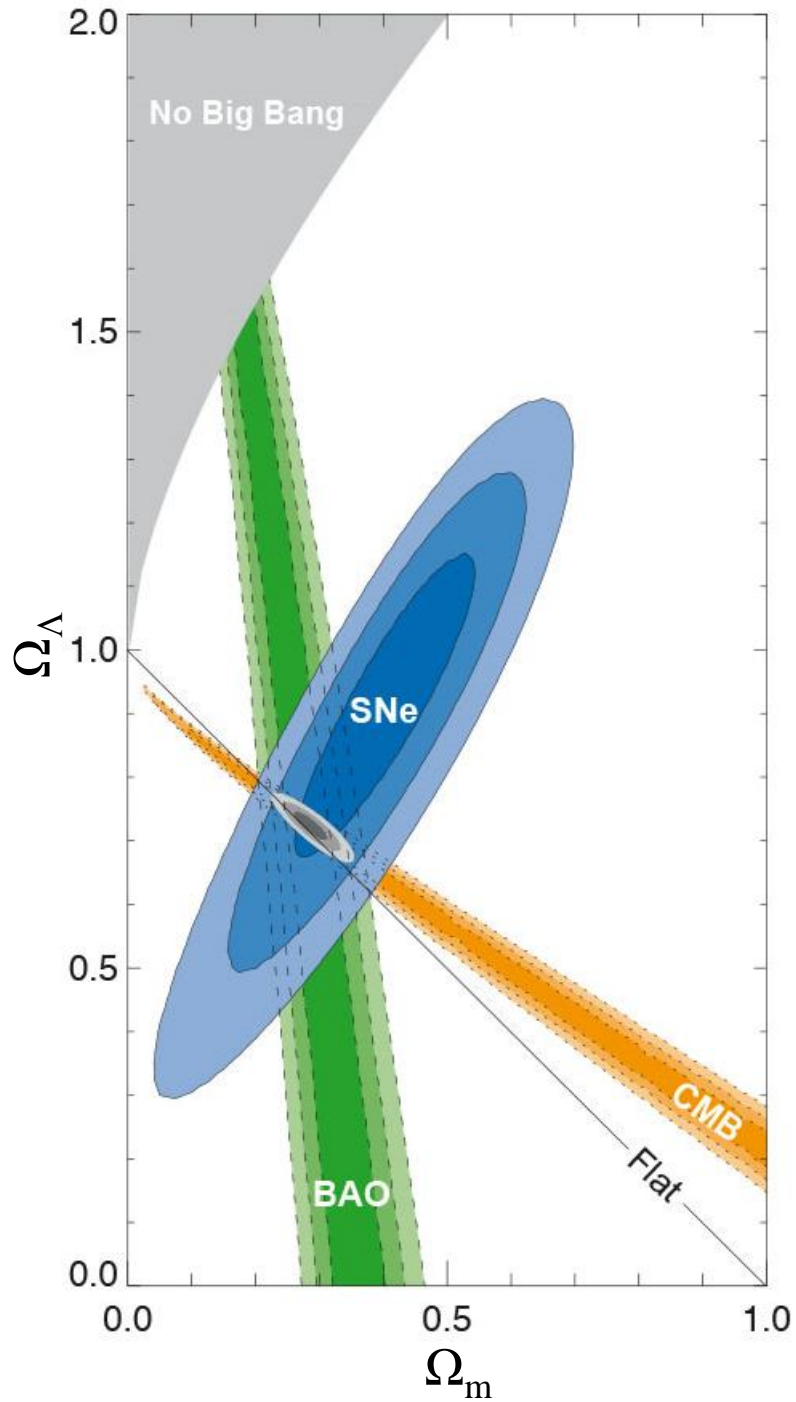
$(\Omega_m, \Omega_\Lambda) = (0, 1)$: de Sitter

$(\Omega_m, \Omega_\Lambda) = (1, 0)$: Einstein
de Sitter

$(\Omega_m, \Omega_\Lambda) \approx (0.3, 0.7)$: present
Universe

w=0 (matter dominated era)

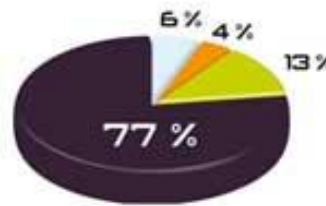
[Uzan & Lehoucq arXiv:physics/0108066]



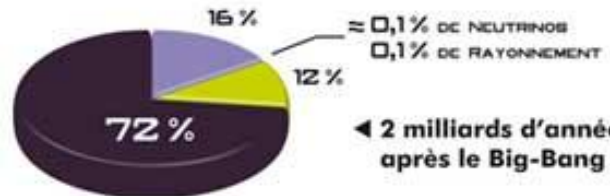
◀ Inflation
(pendant le Big-Bang)



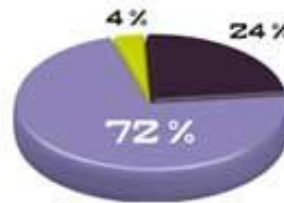
◀ 3 minutes
après le Big-Bang



◀ 400 000 ans
après le Big-Bang



◀ 2 milliards d'années
après le Big-Bang



◀ Aujourd'hui



◀ Dans 30 milliards
d'années



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Thermodynamics in the Standard Model

Cosmological distances $\propto a \equiv (1+z)^{-1}$ ($z = \text{redshift}$)

Rate of expansion $\propto (\text{radiation energy density})^{1/2}$

$$\textcircled{1} \quad \frac{1}{a} \frac{da}{dt} \propto \sqrt{\rho_{\text{rad}}(T)} \propto \sqrt{g_{*}^{\text{e}\nu}(T) T^2}$$

“Radiation”: γ , (e^-) , ν_x and antiparticles

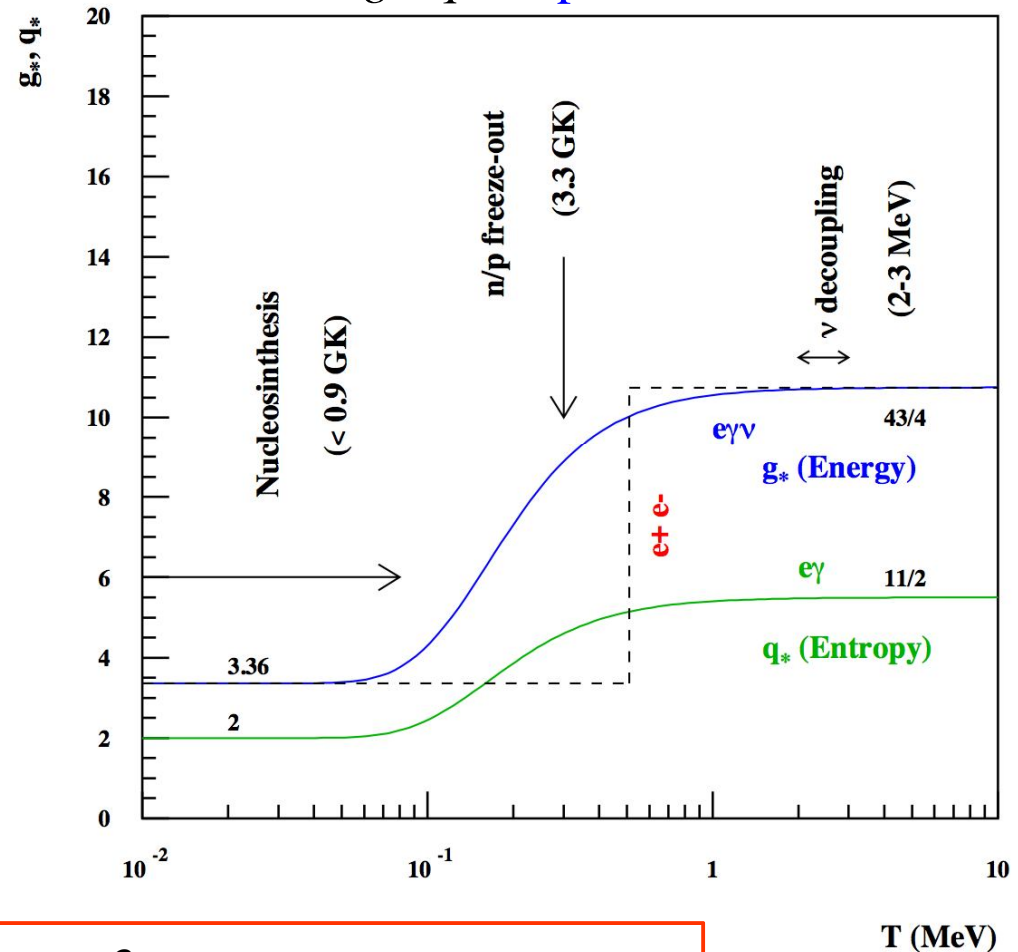
$T_\nu = T$ for $T \gg 1 \text{ MeV}$

$$\textcircled{2} \quad a^3 T_\nu^3 = \text{Cste}$$

Entropy constant

$$\textcircled{3} \quad a^3 q_{*}^{\text{e}\nu}(T) T^3 = \text{Cste}$$

$g_*, q_* = \text{spin factors}$



$$\textcircled{1} + \textcircled{2} + \textcircled{3} \Rightarrow \rho_b(t) \propto \Omega_b a^{-3}(t), T(t) \text{ and } T_\nu(t)$$

Radiation energy, entropy densities and pressure

$$\rho_R = \frac{g_{eff}(T)}{2} a_R T^4 \quad s = \frac{4}{3} \frac{g_{eff}(T)}{2} a_R T^3 \quad g_{eff}(T) = \sum_i g_i$$

During BBN, $i = \gamma, \nu_e, \nu_\mu, \nu_\tau, (e^+, e^-), \dots$

Particles	Spin factor g	T	g_{eff}
γ	2	$\ll m_e$	7.25 (\rightarrow 3.36)
ν_e, ν_μ, ν_τ	$N_\nu \times 2 \times 1 \times (7/8)$		
e^+e^-	$2 \times 2 \times (7/8)$	$\gg m_e$	10.75

Neutrino temperature (I)

Before *ν decoupling* ($T \approx 1\text{MeV}$) total entropy $S \equiv a^3 s$ in comoving volume a^3 remains constant and $a(t)T(t) = \text{cste}$.

At lower T , entropy of γ together with e^+e^- , and ν remain separately constant with possibly different T :

For neutrinos $a^3(t) \frac{1}{2} \frac{7}{8} (N_\nu \times 2 \times 1) \frac{4}{3} a_R T_\nu^3(t) \Rightarrow a(t) T_\nu(t) = \text{cste}$

For e^+ , e^- and γ $a^3(t_B) \frac{1}{2} \left(\frac{7}{8} 2 \times 2 + 2 \right) \frac{4}{3} a_R T_\gamma^3(t_B) = a^3(t_A) \frac{1}{2} (2) \frac{4}{3} a_R T_\gamma^3(t_A)$

Before (t_B) e^+e^-
annihilation

After (t_A) e^+e^-
annihilation

$$\frac{a(t_A) T_\gamma(t_A)}{a(t_B) T(t_B)} = \left(\frac{11}{4} \right)^{\frac{1}{3}}$$

$$\frac{a(t_A) T_\nu(t_A)}{a(t_B) T(t_B)} = 1$$

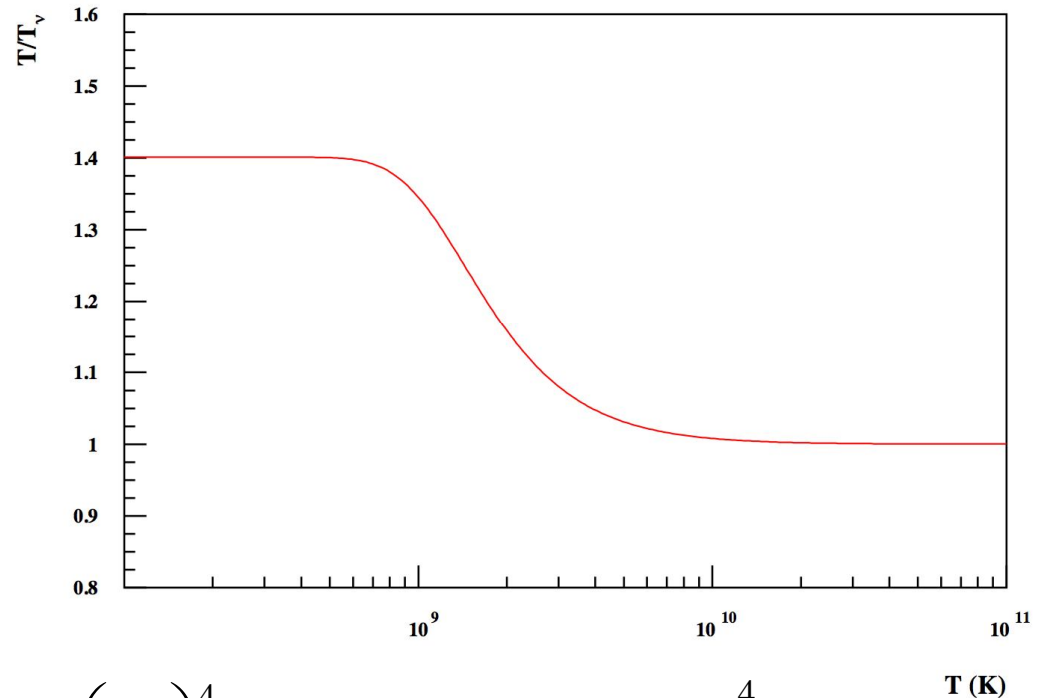
$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4} \right)^{\frac{1}{3}} \approx 1.4$$

Neutrino temperature (II)

Exact calculation
of T_γ/T_ν

$$\rho_R = \frac{g_*(T)}{2} a_R T^4$$

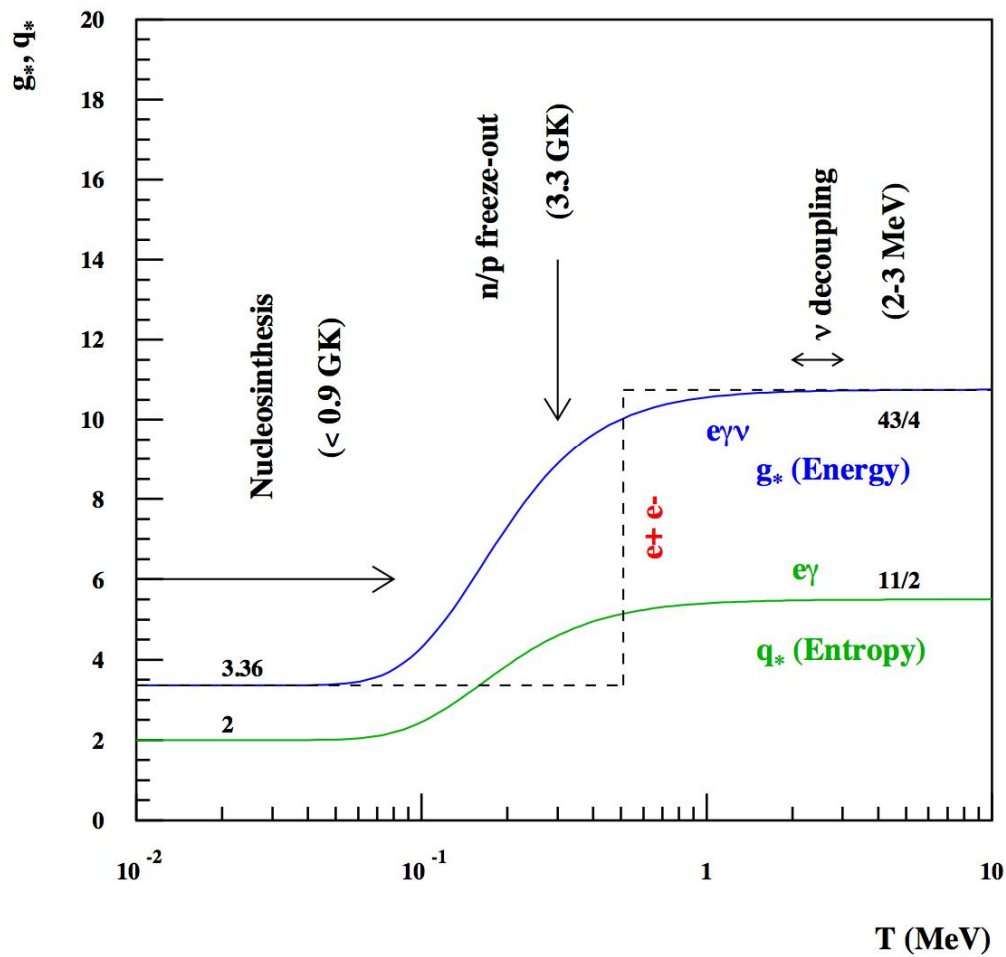
$$s = \frac{q_*(T)}{2} \frac{4}{3} a_R T^3$$



$$g_*^{e\nu}(T \ll m_e) = 2 + N_\nu \times 2 \times \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^4 = 2 + N_\nu \times 2 \times \frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} = 3.36$$

$$q_*^{e\nu}(T \ll m_e) = 2 + N_\nu \times 2 \times \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^3 = 2 + N_\nu \times 2 \times \frac{7}{8} \left(\frac{4}{11} \right) = 3.91$$

$$q_*^{e\gamma}(T \ll m_e) = 2$$

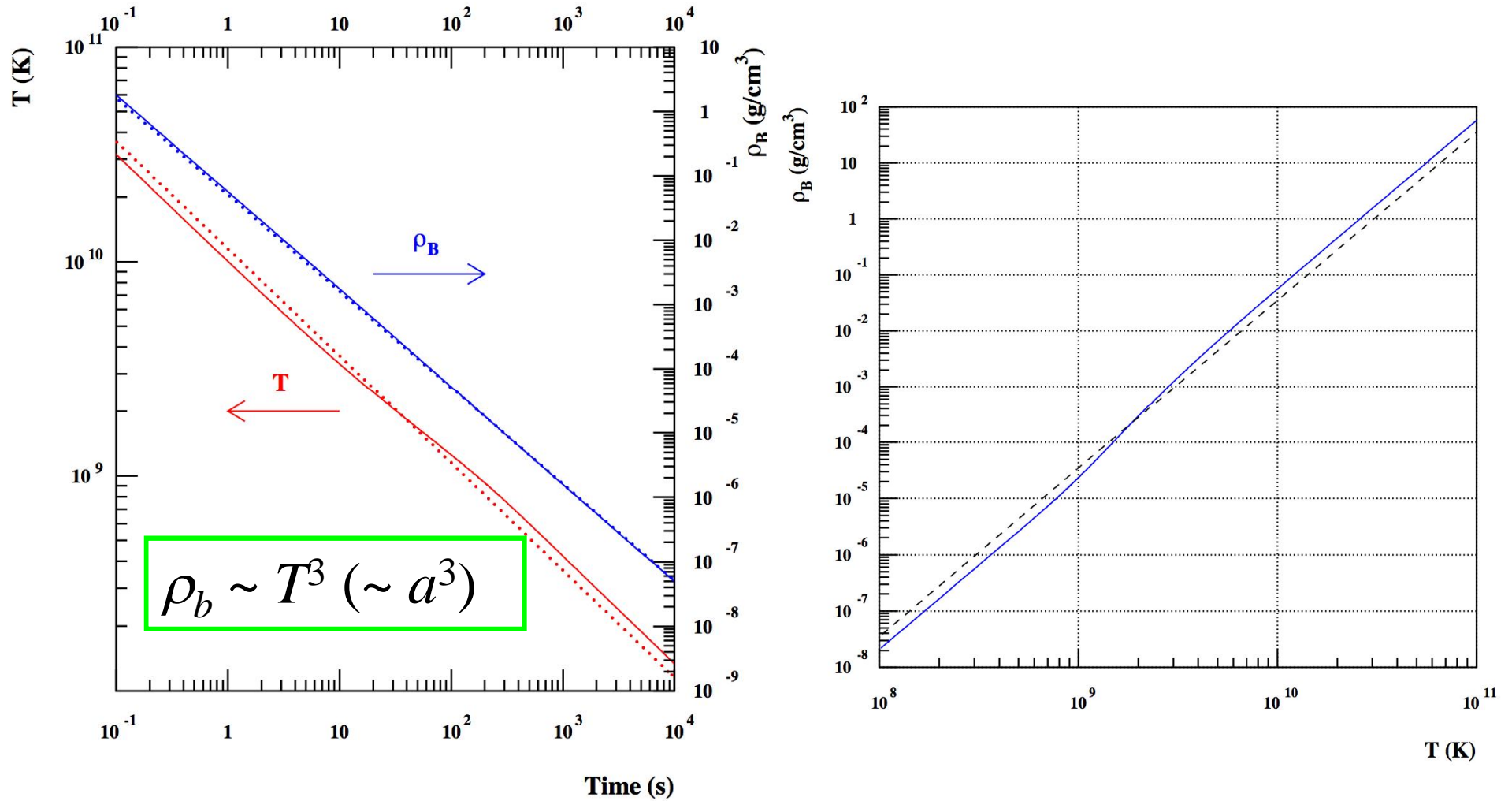


$$\rho_E^e(T) = \frac{2}{2\pi^2\hbar^3} \int_m^\infty \frac{\sqrt{\varepsilon^2 - m^2} \varepsilon^2 d\varepsilon}{\exp(\varepsilon/kT) + 1}$$

$$\rho^e(T) = \frac{1}{3} \frac{2}{2\pi^2\hbar^3} \int_m^\infty \frac{(\varepsilon^2 - m^2)^{3/2} d\varepsilon}{\exp(\varepsilon/kT) + 1}$$

$$S = \frac{\rho + \rho_E}{T}$$

Time evolution of baryonic density



Nucleosynthesis (I)

Equilibrium $p \leftrightarrow n$: $N_n/N_p = \exp(-Q_{np}/kT)$; $Q_{np} = 1.29 \text{ MeV}$



Followed by decoupling and freezeout

Equilibrium as long as the reaction rate is faster than the expansion rate:

$$\Gamma_{\leftrightarrow} \gg \frac{\dot{a}(t)}{a(t)} \quad (\equiv H(t))$$

Equilibrium breaks out when :

$$\Gamma_{n \leftrightarrow p} \sim G_F^2 T^5 \sim \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G \rho_R}{3}}$$

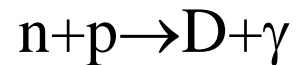
$$\rho_R = g_{\text{eff}}(T) \frac{k^2 \pi^2}{30 \hbar^3} T^4$$

$$(g_{\text{eff}}(T) = 4 \leftrightarrow 11)$$

Then $T \approx 3 \text{ GK}$ and $N_n/N_p \approx 1/6$

Nucleosynthesis (II)

Neutrons decay until T is low enough for :



becomes faster than deuterium photodisintegration



Then, $t = 3 \text{ mn}$, $T \approx 10^9 \text{ K}$ and N_n has decreased to $N_n/N_p \approx 1/7$

Nucleosynthesis starts to produce essentially ${}^4\text{He}$ together with traces of D, ${}^3\text{He}$, ${}^7\text{Li}$,

$$X({}^4\text{He}) \approx 2X(n) \approx 2/(1+7)=0.25$$

First steps in BBN

$n \leftrightarrow p$ decoupling (A)

n free decay (A-B)

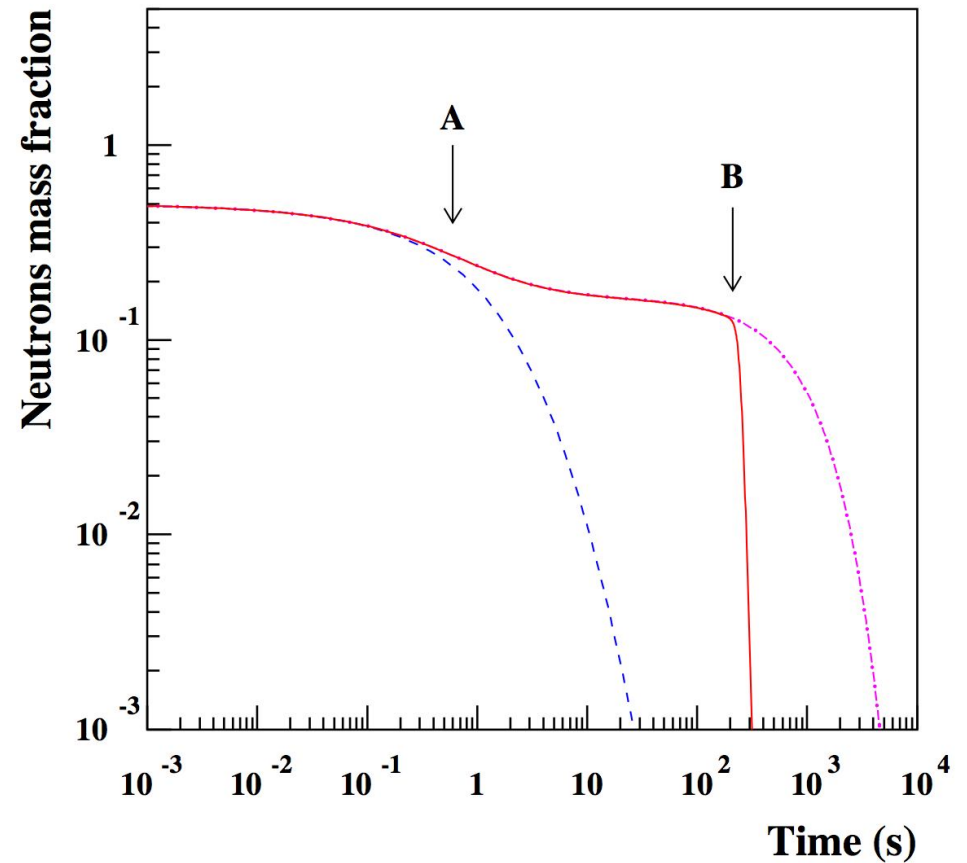
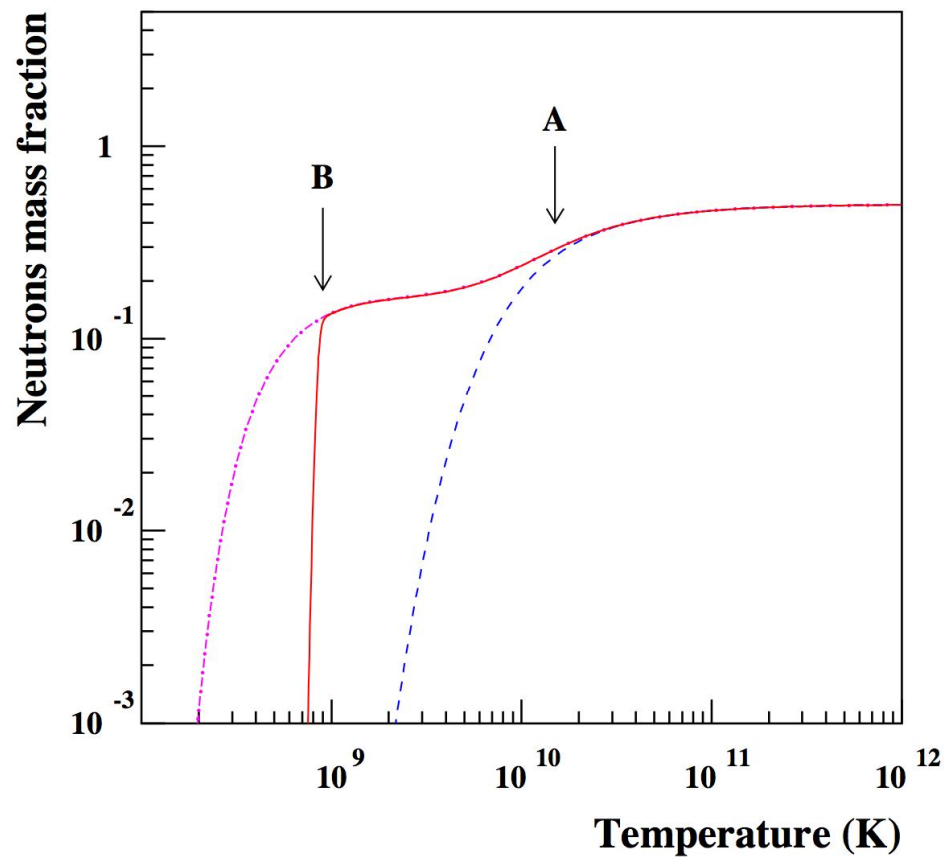
D formation (B)

--- $n \leftrightarrow p$ equilibrium

..... n free decay

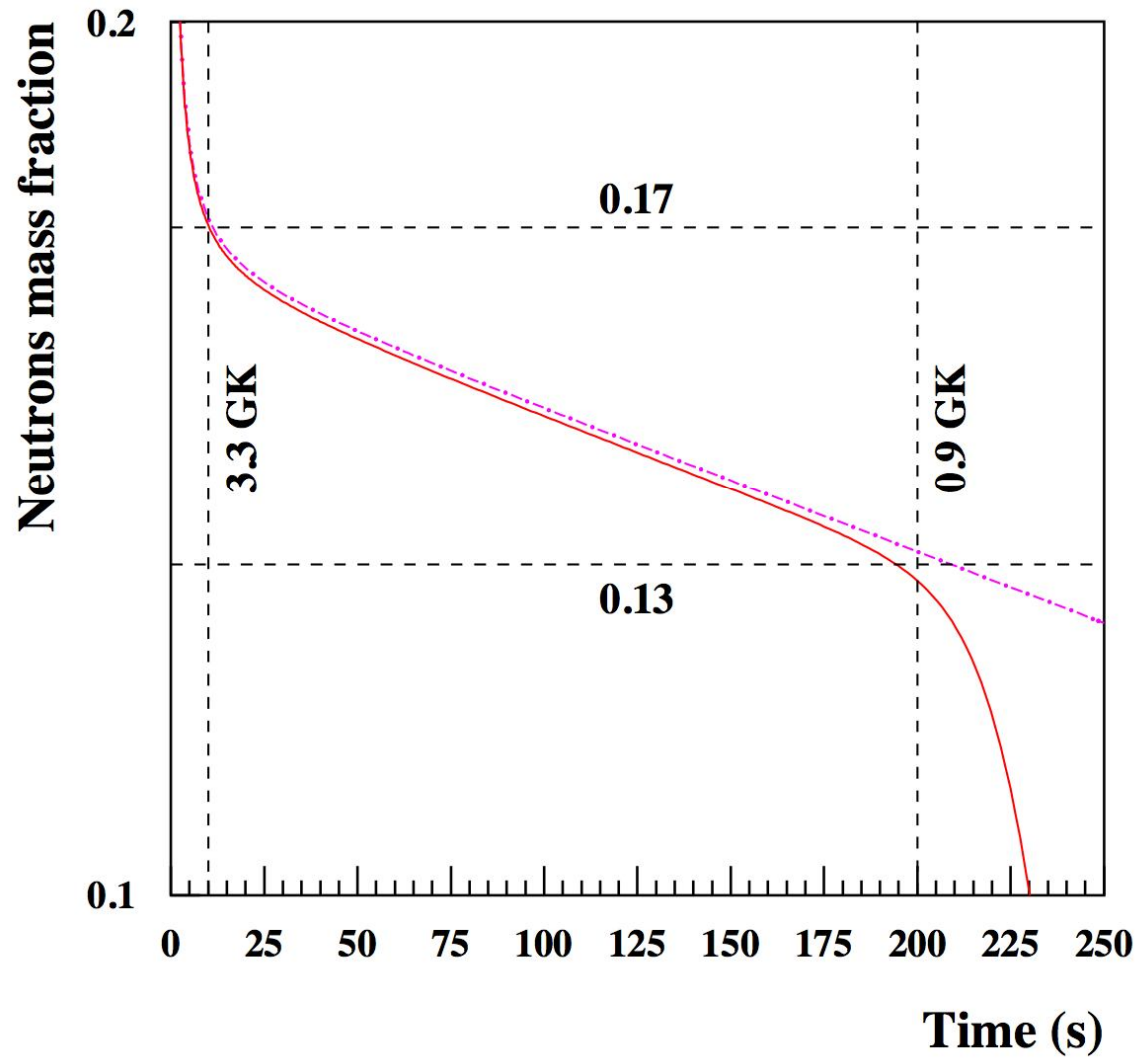
— exact calculation

$\Omega_b h^2 = \text{WMAP}/\Lambda\text{CDM}$



First steps in BBN

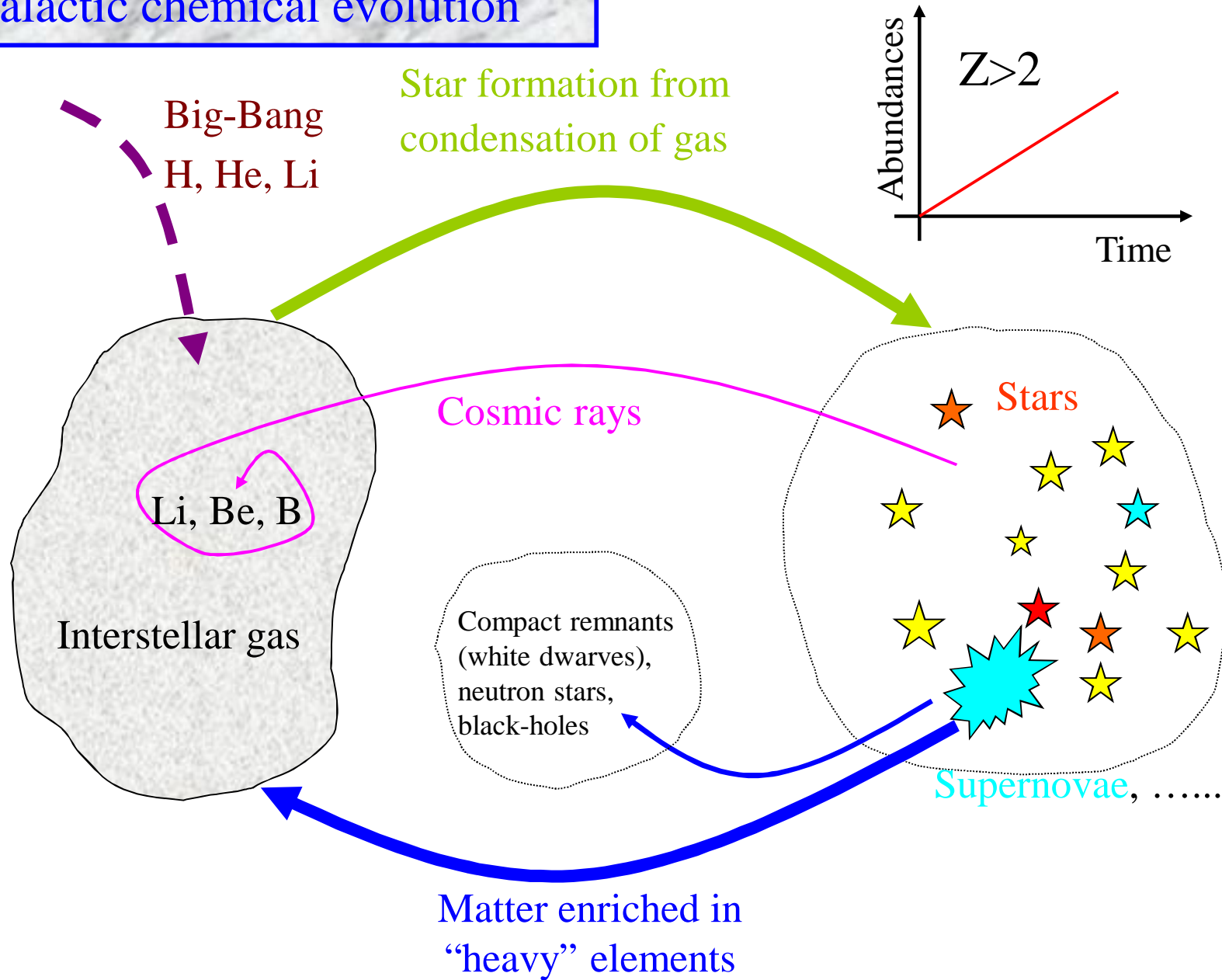
$$\Omega_{\text{B}} h^2 = \text{WMAP}/\Lambda\text{CMD}$$



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Galactic chemical evolution



“Metallicity”

In astrophysics:

“metals” = everything beyond helium

Metallicity:

- “metal” mass fraction (Z)
- $[Fe/H] \equiv \log(Fe/H) - \log(Fe_{\odot}/H_{\odot})$

Solar metallicity

$$Z_{\odot} = 0.0134$$

*[N. Grevesse, M. Asplund,
A.J. Sauval & P. Scott 2010]*

$$[Fe/H] \equiv 0$$

Determination of primordial abundances

Primordial abundances :

- 1) Observe a set of primitive objects born when the Universe was young
 - ^4He in H II (ionized H) regions of blue compact galaxies
 - ^3He in H II regions of *our* Galaxy
 - **D** in remote **cosmological clouds** (i.e. at high redshift) on the line of sight of quasars
 - ^7Li at the surface of low metallicity stars in the halo of our Galaxy
- 2) Extrapolate to zero metallicity : Fe/H, O/H, Si/H,.... $\rightarrow 0$

Notations des abondances

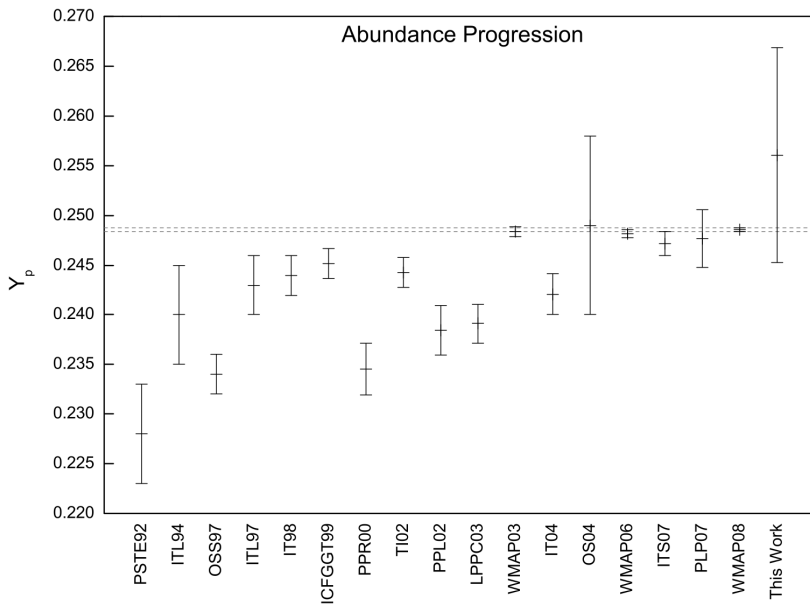
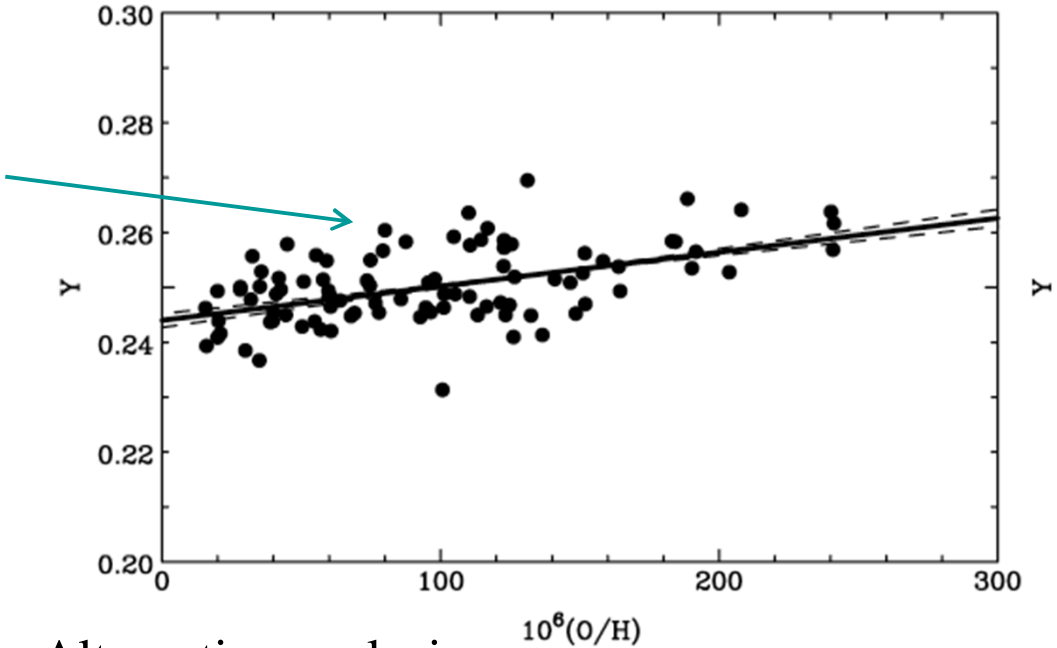
- ❑ Number of atoms : N (per unit volume : N)
 - $N_{\text{H}}(\text{U}) \equiv A(\text{U}) \equiv \varepsilon(\text{U}) \equiv \log_{10}(\text{N}(\text{U})/\text{N}(\text{H})) + 12$ (photosphere)
 - $N_{\text{Si}}(\text{U}) \equiv \log_{10}(\text{N}(\text{U})/\text{N}(\text{Si})) + 6$ (meteorites)
- ❑ Mass fraction : $X \equiv N \times A / N_A / \rho$
- ❑ Specific mole fraction: $Y = X/A = N/N_A / \rho$
- ❑ Notation $[\]$ (“dex”) with U_{\odot} = solar (\odot) abundance :
 - $[U] = \log_{10}(U/U_{\odot})$
 - $[U/W] = \log_{10}((U/W) / (U_{\odot}/W_{\odot}))$
- ❑ Metallicity : Z = Mass fraction of “heavy” elements (= beyond helium)

A = Nombre de masse; ρ = Masse volumique (« densité »), N_A = Nombre d' Avogadro

^4He observations in blue compact galaxies

^4He from a sample of 86 H II regions in 77 blue compact galaxies [Izotov, Thuan & Stasinska 2007]

➤ 0.2565 ± 0.0010 (st.) ± 0.0050 (sy.) [Izotov, Thuan & Stasinska 2010]



Alternative analysis

➤ ~~$0.232 < Y_p < 0.258$ [Olive & Skillman 2004]~~

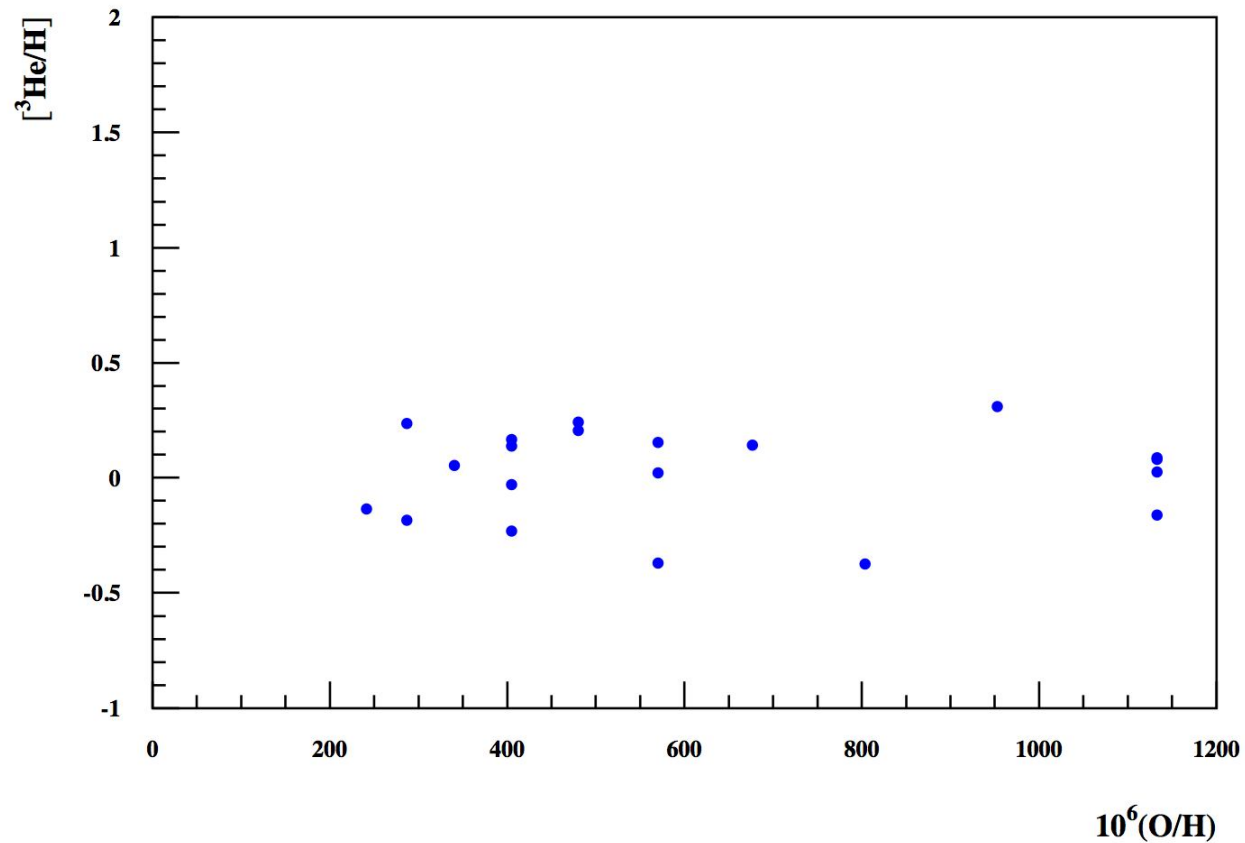
➤ ~~$0.245 < Y_p < 0.267$ [Aver, Olive & Skillman 2010] (new atomic and collisional emission data)~~

➤ $0.2368 < Y_p < 0.2562$ [Aver, Olive, Porter & Skillman 2013] (new He I emissivity)

^3He primordial abundance

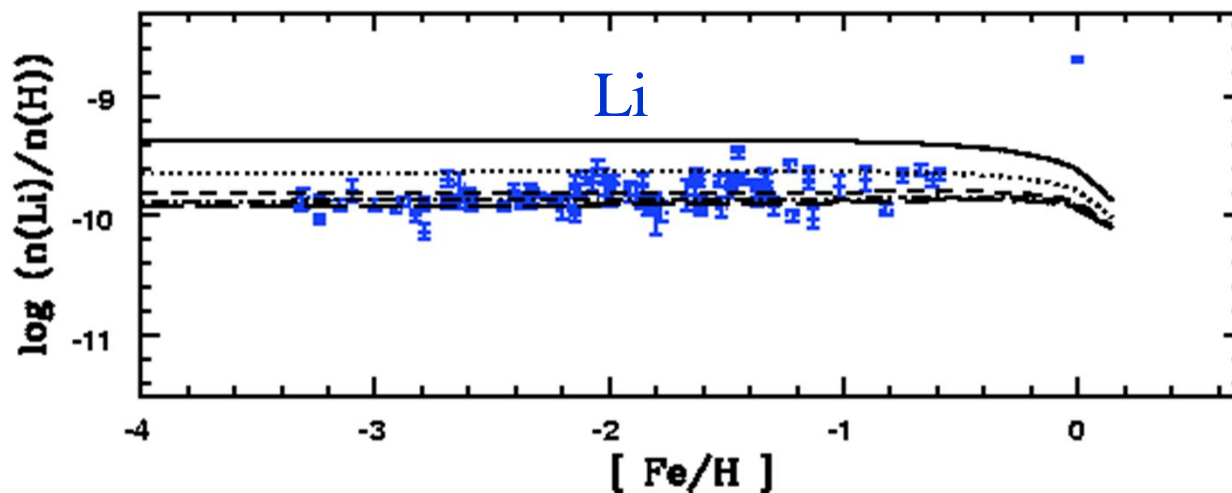
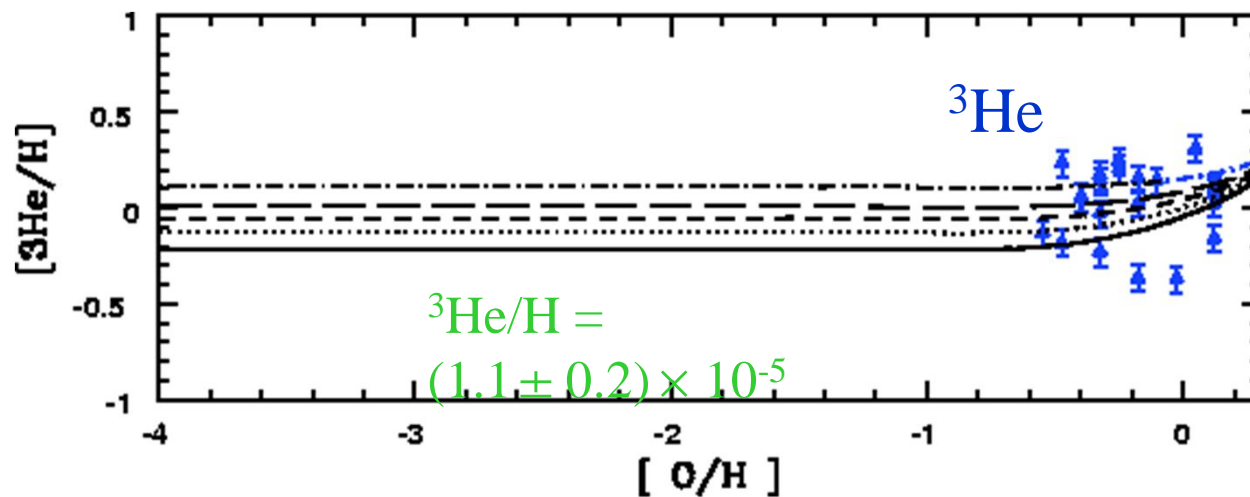
^3He from a sample of 21 local galactic H II regions; upper limit from best observation:

$$^3\text{He}/\text{H} \leq (1.1 \pm 0.2) \times 10^{-5} \quad [\text{Bania et al. 2002}]$$



^3He primordial abundance ?

New $^3\text{He}/\text{H}$ data [*Bania et al. 2002*] but at high metallicity : weak constrains on primordial value [*Vangioni-Flam et al. 2002*].



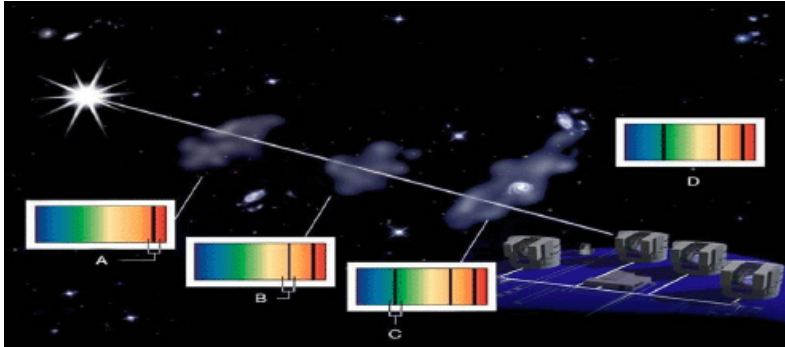
Deuterium primordial abundance

Fragile isotope : only destroyed after BBN

⇒ use highest observed value

1. Local interstellar medium (present) :
 $D/H \approx 10^{-5}$ [*FUSE: Hébrard & Moos 2003, Wood et al. 2004*]
2. Protosolar cloud (4.6 Gyr ago) :
 $D/H = (2.5 \pm 0.5) \times 10^{-5}$ [*Hersant et al. 2001*]
3. Remote **cosmological clouds** on the line of sight of quasars

D/H observations in a cosmological cloud

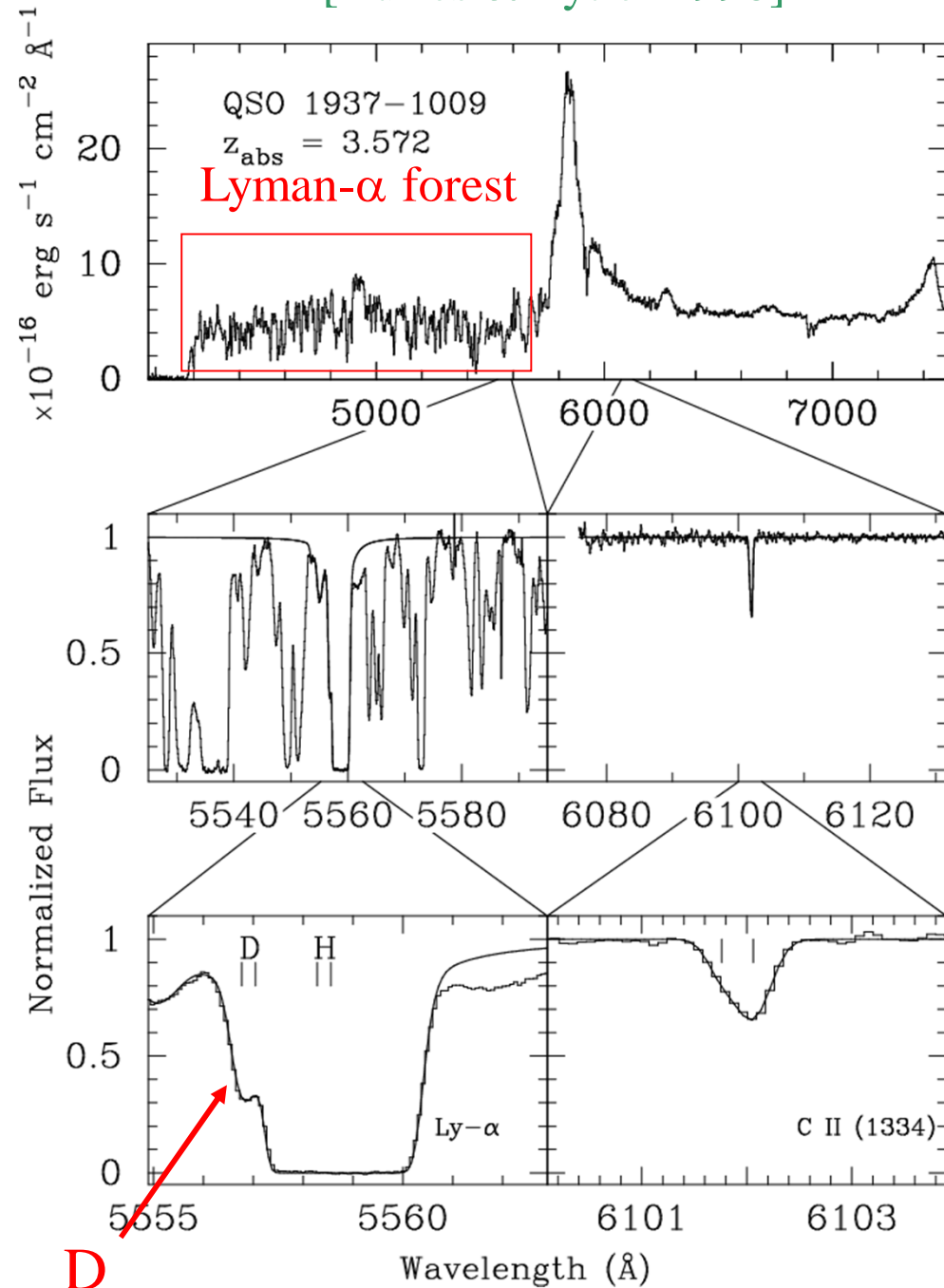


Cloud at redshift of $z = 3.6$
on the line of sight of
quasar QSO 1937-1009

Observations :

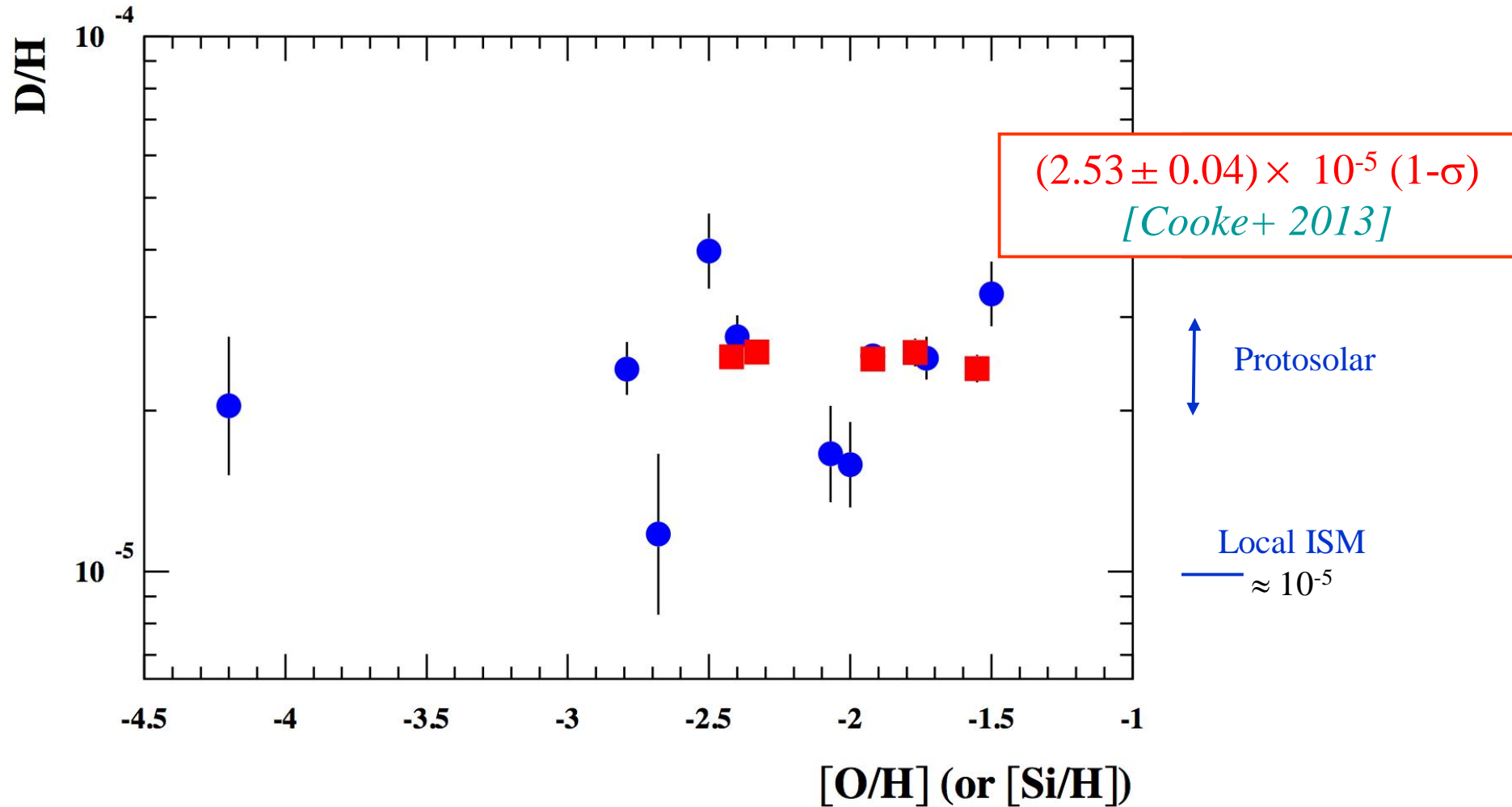
- D/H ratio at high redshift from the depth/width of absorption lines
- Baryonic density (Ω_b) from the census of the « Lyman- α Forest » lines

[Burles & Tytler 1998]



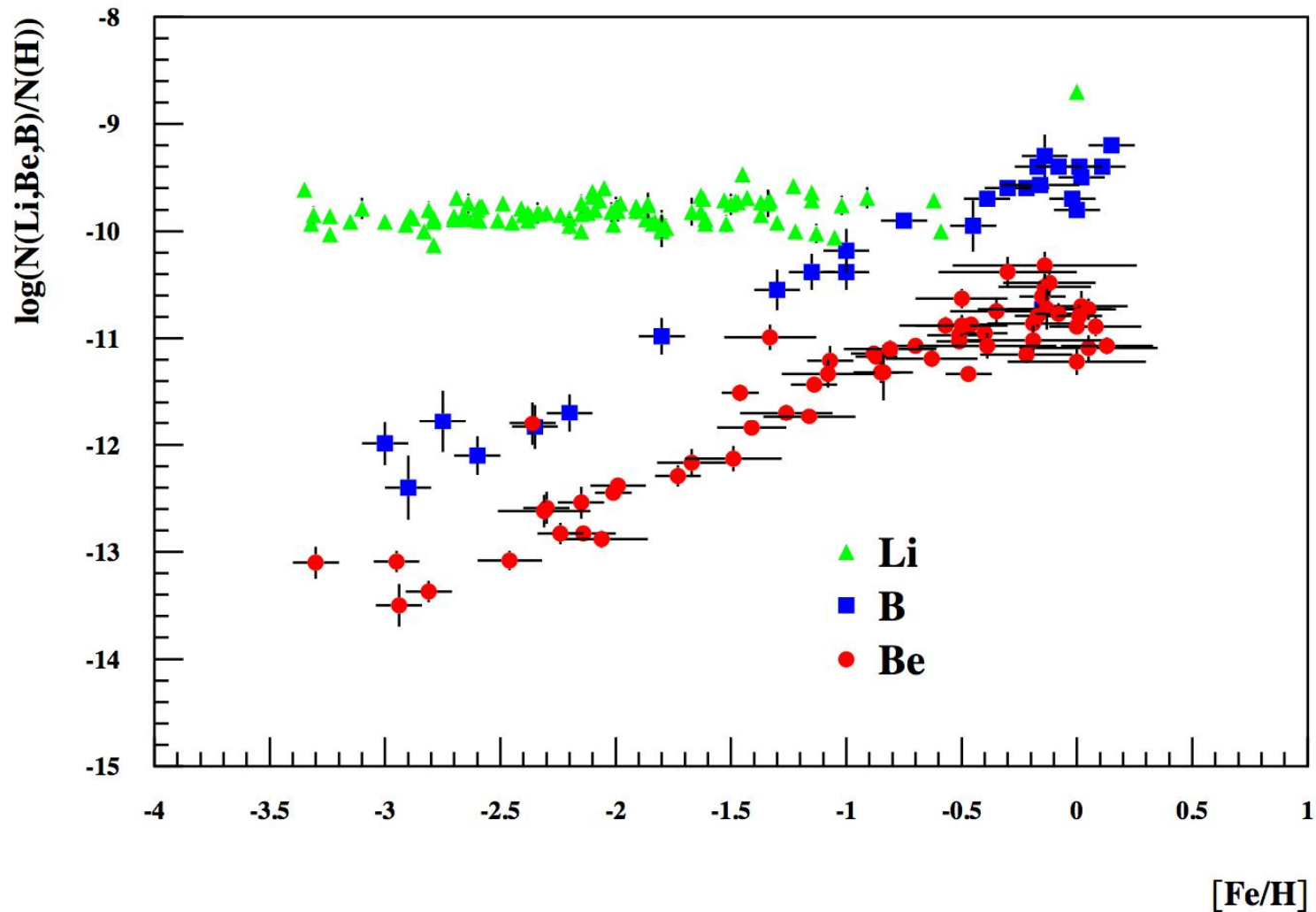
D

D/H observations in cosmological clouds



Burles & Tytler 1998; O'Meara+ 2001, 2006; Pettini+ 2001, 2008, 2012; Kirkman+ 2003, Crighton+ 2004; Srianand+ 2010; Cooke+ 2011; Fumagalli+ 2011; **Cooke+ 2013**

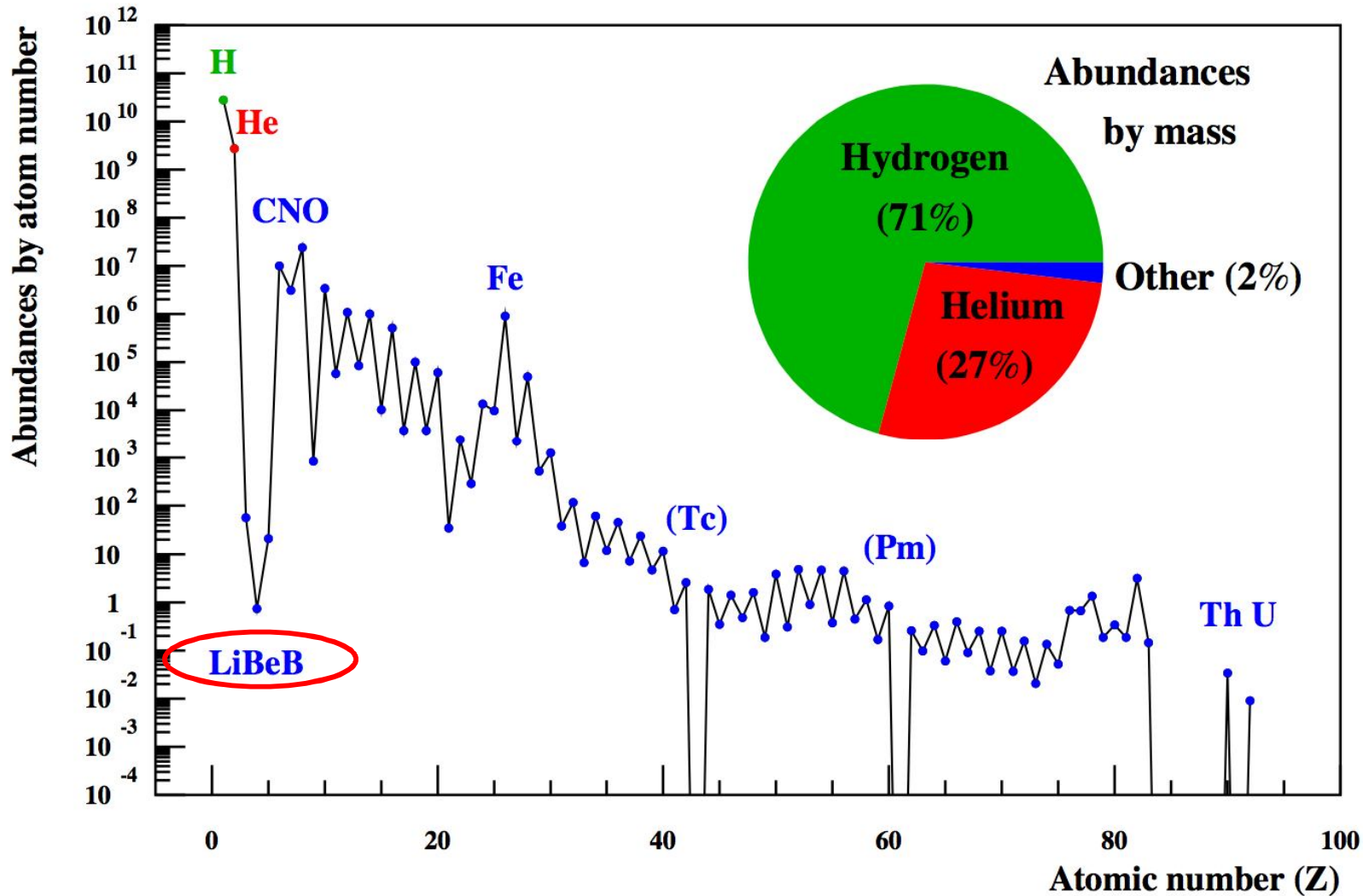
Galactic evolution of Li, Be and B abundances



Observation in halo stars, as a function of « metallicity »
 $\approx [Fe/H]$ increasing with time ($[Fe/H]=0$: 4.5 Gy ago)

Abundances of the elements

Elemental Abundances

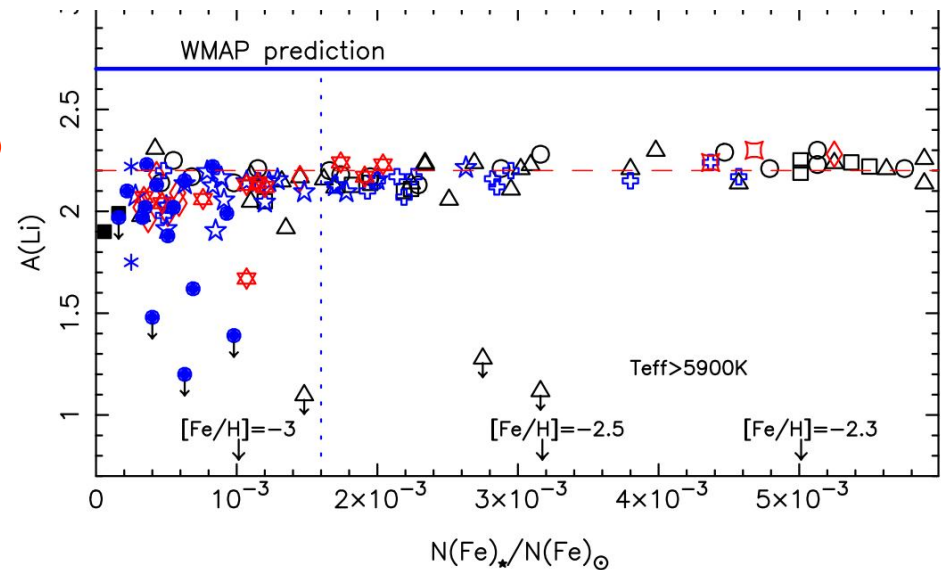
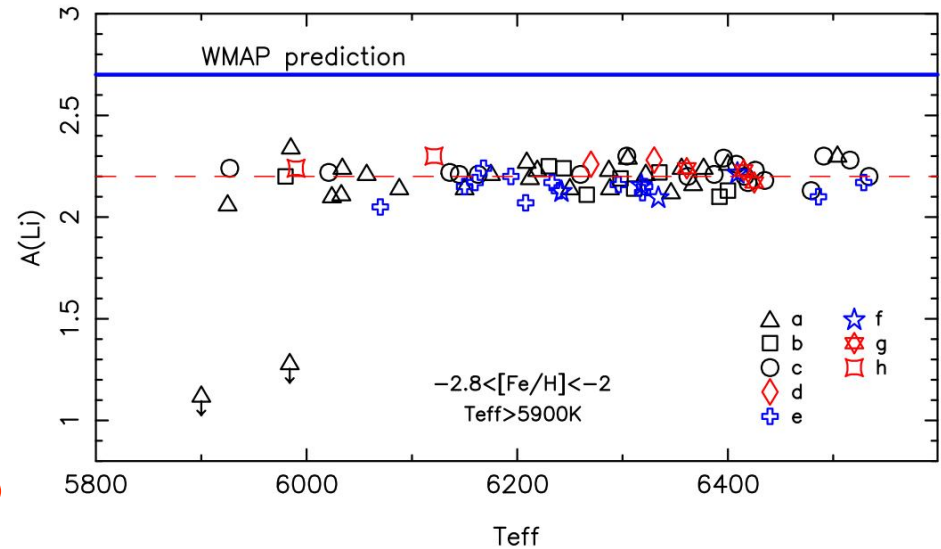


Primordial Li from observations

[Spite, Spite & Bonifacio 2012]

- Lifetime of $M < 0.9 M_{\odot}$ stars > 15 Gy
- Oldest (low metallicity) stars in galactic halo
- For $T_{eff} > 5900$ K, no deep convection and no Li surface depletion (?)
- $Li/H = (1.58 \pm 0.35) \times 10^{-10}$
[Sbordone et al. 2010]

(log scale : $\text{Log}(\text{Li}/\text{H})+12$) !



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Thermonuclear reaction rates

➤ Cross section :

$$\text{Cross section } (\sigma) = \frac{\text{Number of reactions / time}}{\text{Beam intensity} \times \text{Number of target nuclei}}$$

$$\sigma \text{ units : barn (b)} \\ \equiv 10^{-24} \text{ cm}^2$$

➤ Thermonuclear reaction rate :

$$N_A \langle \sigma v \rangle = N_A \int_0^{\infty} \sigma(v) v \varphi(v) dv$$

$\varphi(v)$ = Maxwell-Boltzmann distribution

N_A = Avogadro's number

Nuclear network equations

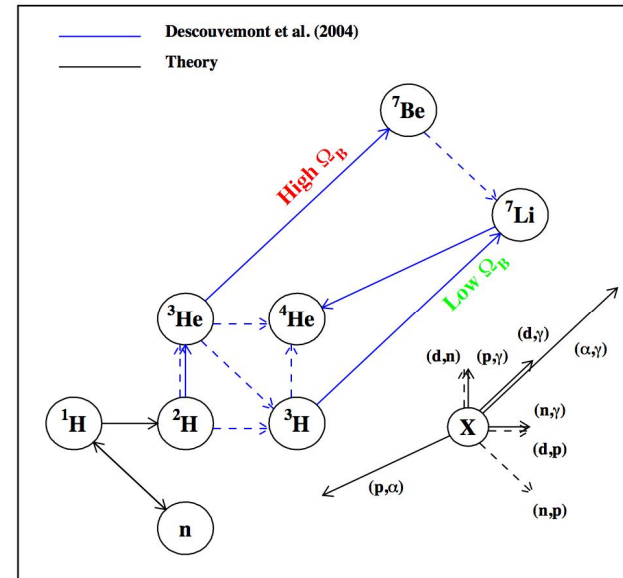
Set of coupled , stiff, non-linear differential equations solved numerically

Time evolution of e.g. ${}^7\text{Li}$ abundance :

$$\frac{dN_{{}^7\text{Li}}}{dt} = +N_{{}^7\text{Be}}\lambda_{{}^7\text{Be}}^{\beta^+} + N_{{}^7\text{Be}}N_n\langle\sigma v\rangle_{{}^7\text{Be}(n,p){}^7\text{Li}} + N_{{}^3\text{H}}N_\alpha\langle\sigma v\rangle_{{}^3\text{H}(\alpha,\gamma){}^7\text{Li}} \quad ({}^7\text{Li production})$$

$$\square - N_{{}^7\text{Li}}N_p\langle\sigma v\rangle_{{}^7\text{Li}(p,\alpha){}^4\text{He}} \quad ({}^7\text{Li destruction})$$

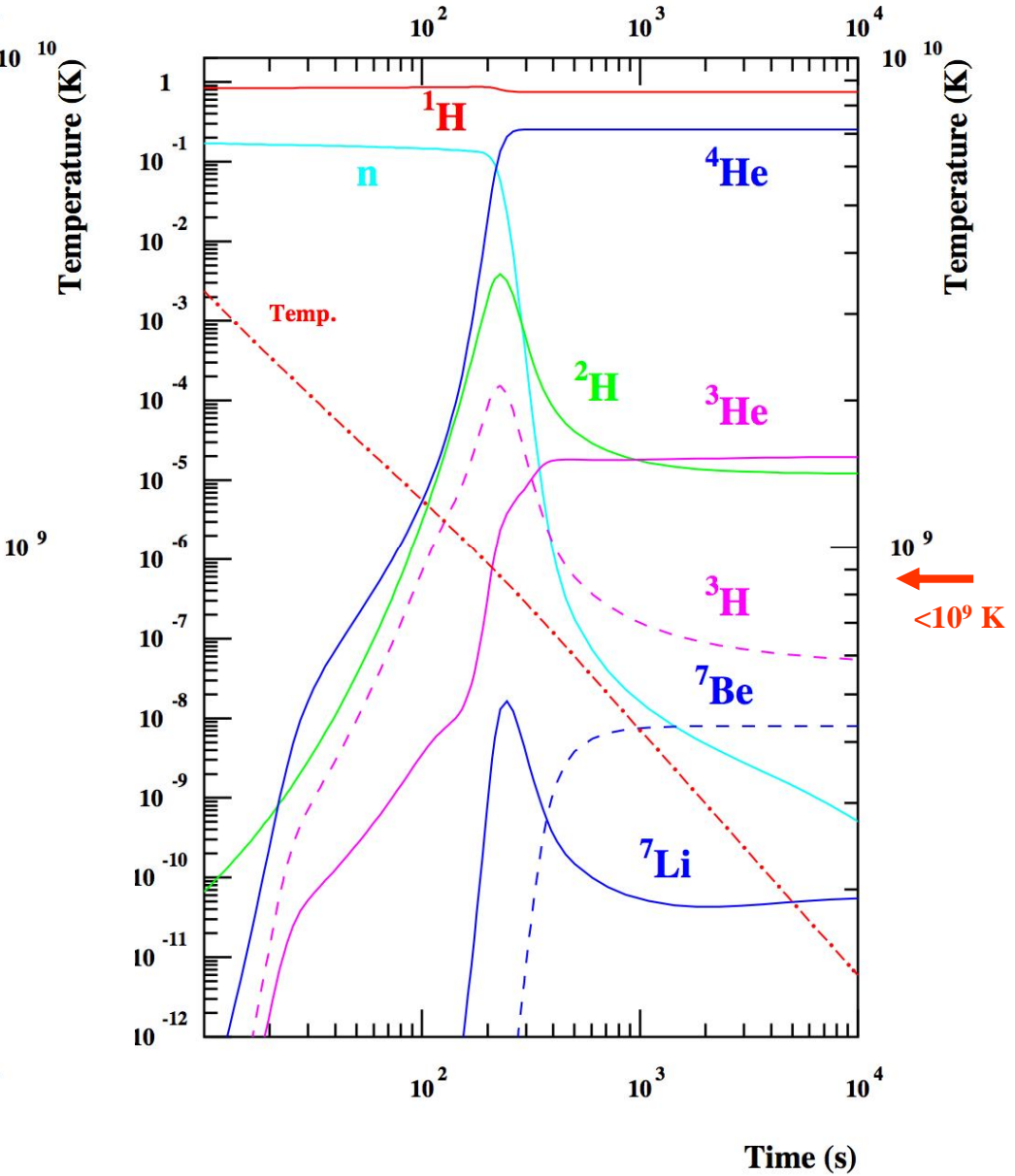
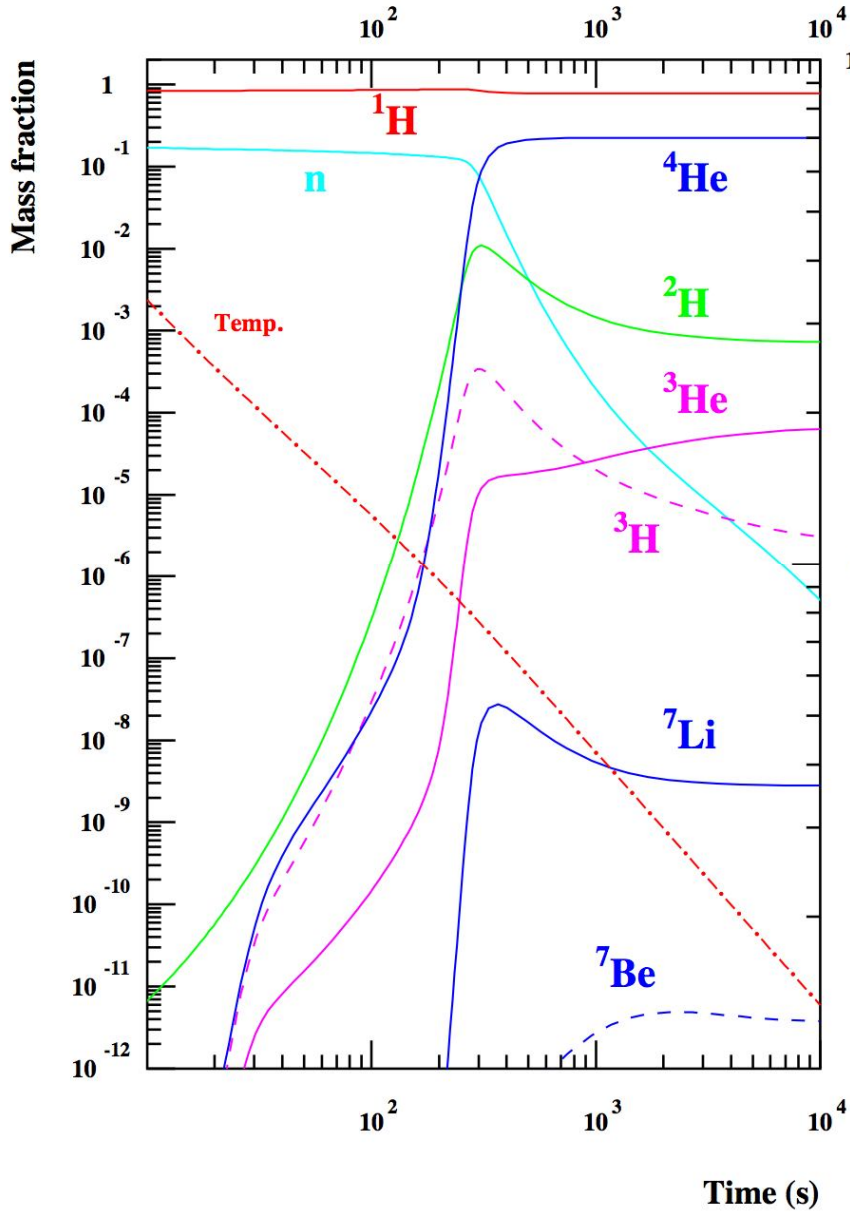
As many equations as isotopes



Primordial nucleosynthesis with $\eta_{10} = 1$ et 10

BBN $\eta_{10}=1$

BBN $\eta_{10}=10$

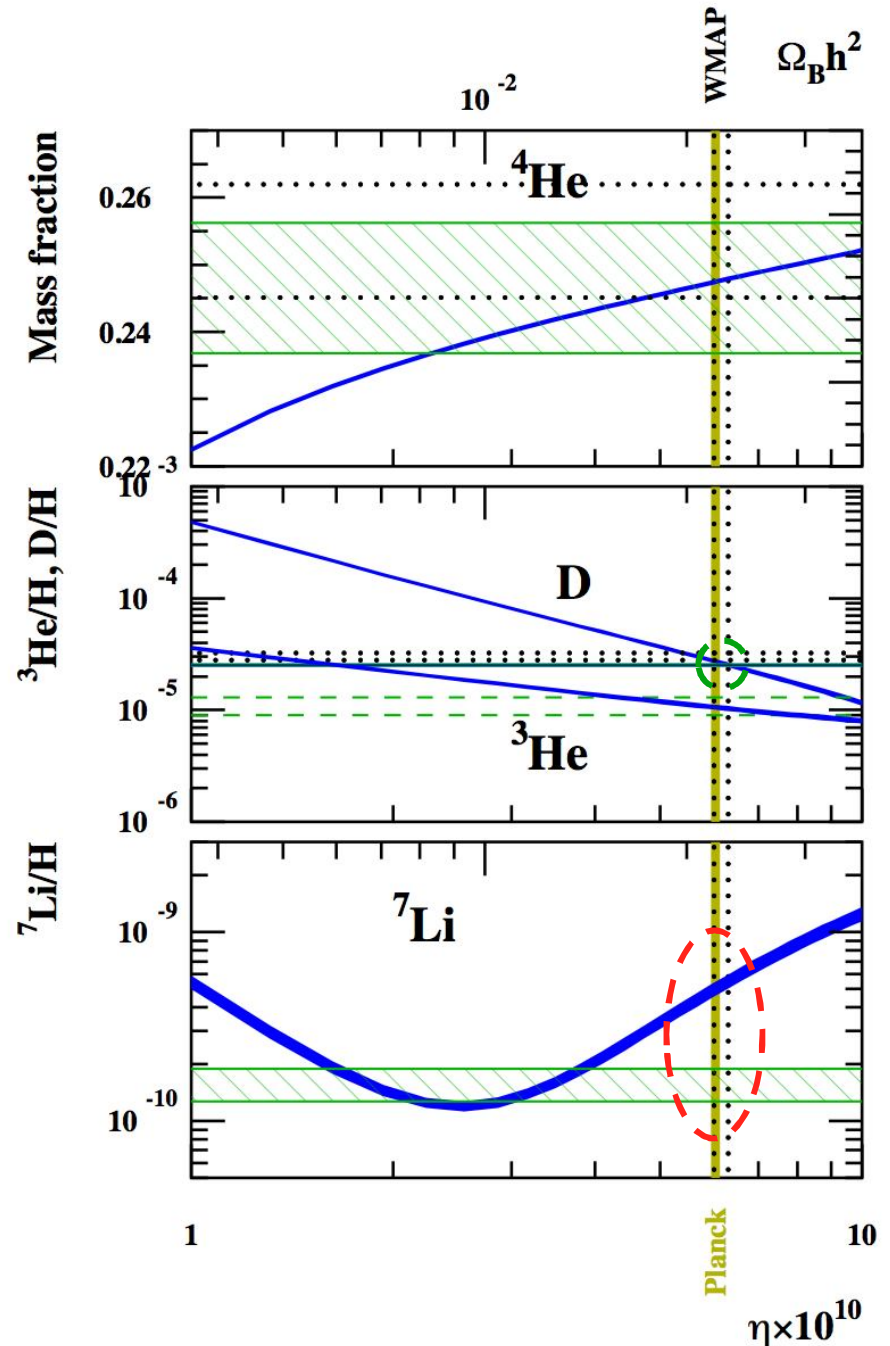


Comparison between observed and calculated abundances

Limits ($1-\sigma$) obtained by Monte-Carlo fusing *Descouvemont+ 2004; Ando+ 2006, Cyburt & Davids 2008* reaction rate uncertainties.

Concordance (?) BBN, spectroscopy and CMB

- $\Omega_B h^2$ [WMAP: Komatsu+ 2011; Planck: Ade+ 2013]
- ^4He [Aver+ 2011; 2013]
- D [Olive+ 2012; Cooke+ 2013]
- ^3He [Bania et al. (2002)]
- ^7Li [Sbordone+ 2010] : difference of a factor of 2-3 between calculated (BBN+CMB) and observed (Spite plateau) primordial lithium



BBN calculations versus observations

	BBN calculations			Observations	
	<i>Cybert et al.</i> 2008	<i>NPA IV</i> 2009	<i>ENA VII</i> 2013	*	
^4He	0.2486 ± 0.000 2	0.2476 ± 0.000 4	0.2463 ± 0.000 3	0.2368-0.2562	$\times 10^0$
D/H	2.49 ± 0.17	2.68 ± 0.15	2.67 ± 0.09	2.53 ± 0.04	$\times 10^{-5}$
$^3\text{He}/\text{H}$	1.00 ± 0.07	1.05 ± 0.04	1.05 ± 0.03	(0.9-1.3)	$\times 10^{-5}$
$^7\text{Li}/\text{H}$	$5.24^{+0.71}_{-0.62}$	5.14 ± 0.50	4.89 ± 0.40	1.58 ± 0.31	$\times 10^{-10}$

Changes from *NPA IV* to *ENA VII* due to:

- $\Omega_b h^2$ [*WMAP5*; 7; 9; *Planck 2013*]
- τ_n [*PDG 2008*; 2012]
- CNO network [*Coc+ 2012*], (neutron drains/sources)
- Sub-leading processes e.g. $^7\text{Be}(n,\alpha)^4\text{He}$

*[*Olive et al. 2013*;
Cooke et al. 2013;
Bania et al. 2002;
Sbordone et al. 2010]

i.e. systematics

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Astrophysical aspects

Extracting Li/H abundances from observed atomic spectra (*See Ryan et al. 2000*)

- Extrapolation to zero metallicity
- 1D versus 3D atmosphere model
- Surface gravity
- Non Local Thermodynamical Equilibrium
- Stellar depletion [*Richard, Michau & Richer, 2005; Korn et al. 2006*]
 - Diffusion and turbulent mixing depletes Li surface abundance
- Effective temperature scale [*Hosford et al. 2008*]

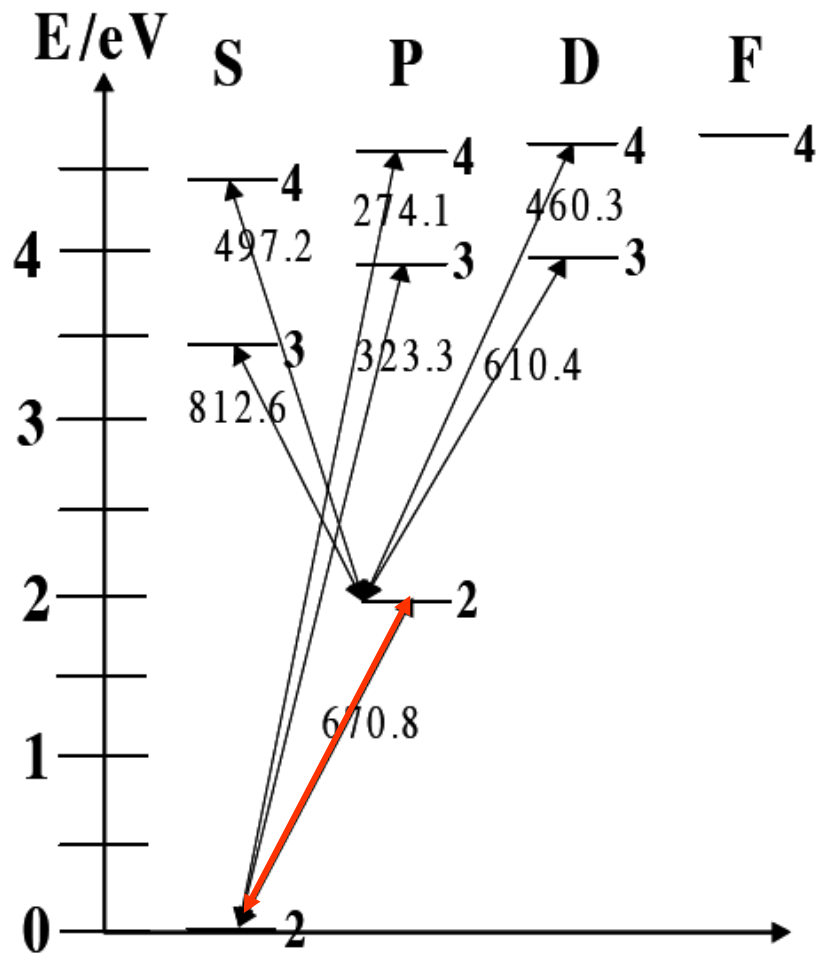
TABLE 1
 INFERRED PRIMORDIAL LITHIUM ABUNDANCE: OBSERVED (RNB) ABUNDANCE IS
 $\langle A(\text{Li}) \rangle_{-2.8} = 2.12 \pm 0.02$

Corrections to Apply Logarithmically	Value	Estimated Uncertainty
(1) GCE/GCR:		
Previous analyses (RNB)	-0.14 to -0.05	
Log data fit (eq. [1])	-0.20 to -0.09	
Linear data fit (eq. [2])	-0.12 to -0.04	
Linear data fit (eq. [3])	-0.16 to -0.05	
Model fits (eqs. [2]–[3])	-0.05 to -0.04	
Adopted (excludes model)		$-0.11^{+0.07}_{-0.09}$
(2) Stellar depletion		$+0.02^{+0.08}_{-0.02}$
(3a) T_{eff} -scale zero point		$+0.08 \pm 0.08$
(3b) One-dimensional atmosphere models		$+0.00^{+0.10}_{-0.00}$
(3c) Convective treatment		$+0.00^{+0.08}_{-0.00}$
(3d) Non-LTE		-0.02 ± 0.01
(3e) gf -values		$+0.00 \pm 0.04$
(4) Anomalous objects		$+0.00 \pm 0.01$
Total		$-0.03^{+0.19}_{-0.13}$
Inferred $A(\text{Li}_p)$		$+2.09^{+0.19}_{-0.13}$

NOTE.—The weighted mean and 95% CL uncertainty of observed Li abundances for a very metal-poor sample of halo main-sequence turnoff stars (RNB) with $\langle [\text{Fe}/\text{H}] \rangle = -2.8$ and the corrections required to deduce the primordial value.

Deduced primordial abundance : $\text{Li}/\text{H} = (0.91-1.91) \times 10^{-10}$
 after correction for systematic effects *[Ryan et al., 2000]*

Observational aspects



Abundances from the observation of the $\lambda=670.8$ nm *atomic* Li line

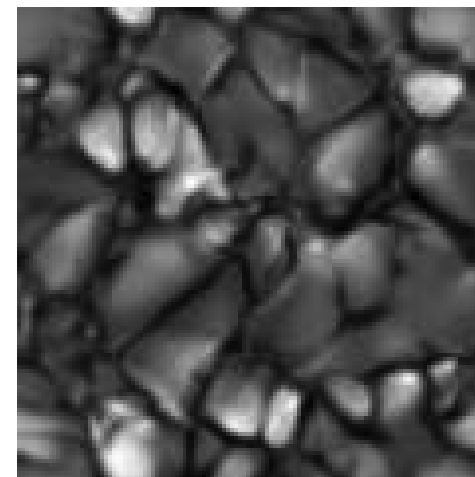
Ionized fraction ?

➤ Saha equation with T_{eff} ? “LTE”

➤ But no equilibrium \Rightarrow “Non-LTE”

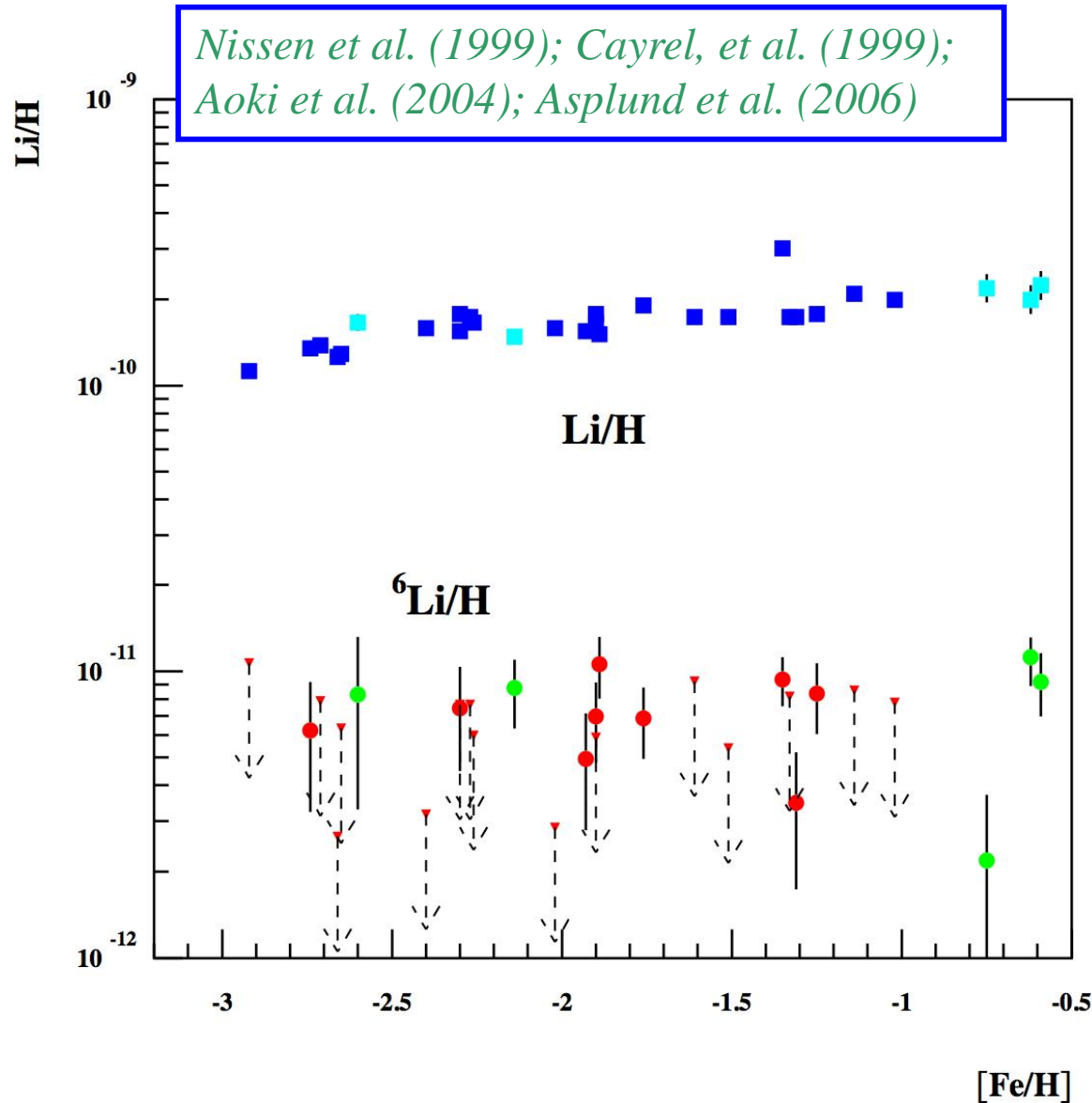
1D \rightarrow 3D

Fragile ${}^6\text{Li}$?



[Lind+ 2013]

${}^6\text{Li}$ observations in halo stars ?



Observed ratio in
halo stars :

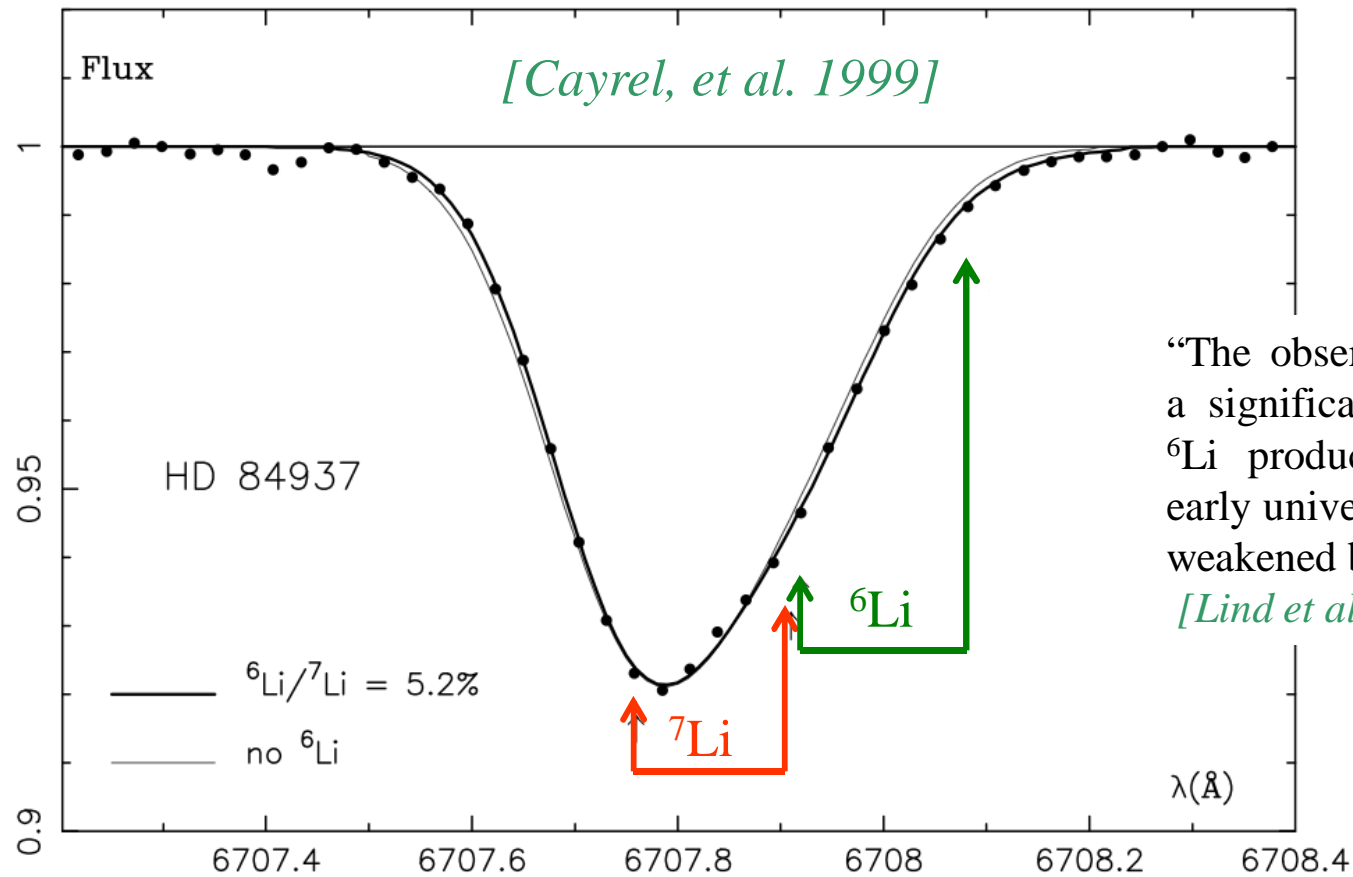
$${}^6\text{Li}/{}^7\text{Li} \approx 0.05$$

$$10^{-12} < {}^6\text{Li}/\text{H} < 10^{-11}$$

(conservative range)

${}^6\text{Li}$ observations in halo stars ?

Asymmetry in unresolved ${}^6\text{Li} + {}^7\text{Li}$ lines may result from up/down convection

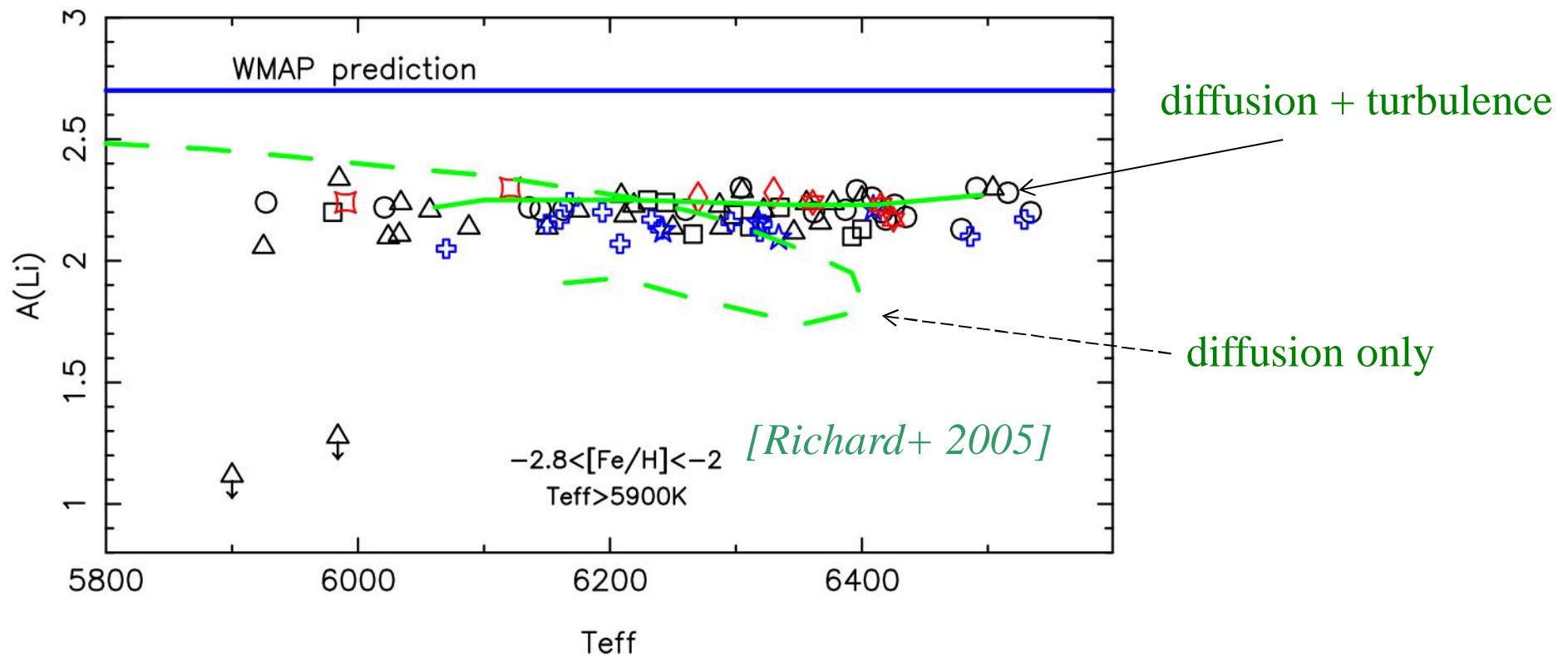


“The observational support for a significant and non-standard ${}^6\text{Li}$ production source in the early universe is substantially weakened by our findings.”

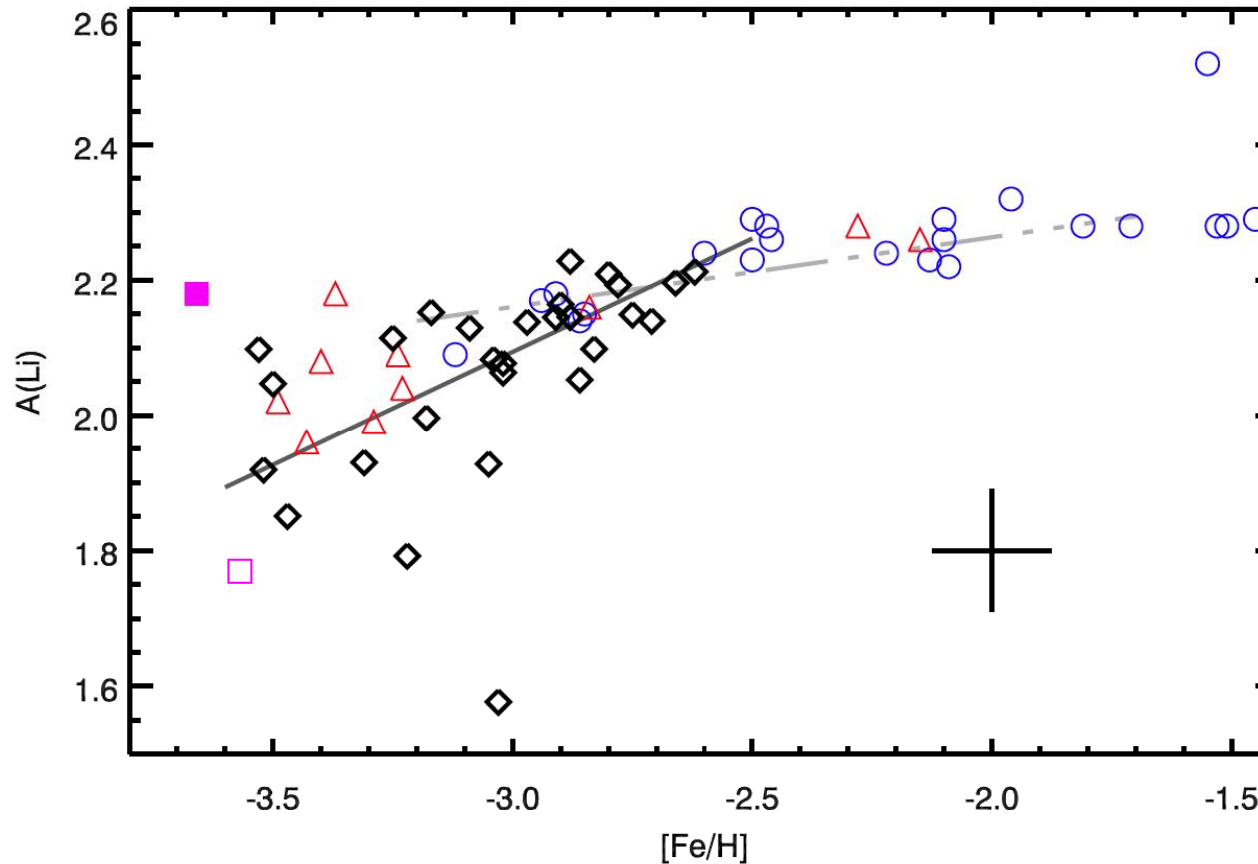
[Lind et al. (2013)]

Li stellar depletion ?

- “Astration” by a first generation of massive stars [*Piau+ 2006*] but heavy elements overproduction.
- In situ destruction (atomic diffusion + turbulence interplay) [*Michaud+ 1984; Richard+ 2005; Korn+ 2006*]



Low metallicity end of the Li plateau



“Meltingdown” of the plateau below
 $[\text{Fe}/\text{H}] \approx -2.5$ [Sbordone et al. 2010]
would need two depletion mechanism

$$\text{Li}/\text{H} = (1.58 \pm 0.34) \times 10^{-10} @ [\text{Fe}/\text{H}] > -2.8$$

Origin of CMB, SBBN and Li observations discrepancies

□ Stellar/Galactic ?

- Observational bias: 1D/3D, LTE/NLTE model atmospheres, effective temperature scale [*Ryan et al. 2000*]
- Li stellar destruction [*Richard, Michau & Richer, 2005; Korn et al. 2006*]

□ Nuclear ??? (Wednesday lecture)

□ Non Standard Model(s) ? (Friday lecture)

- Affecting expansion rate during BBN : Quintessence, Tensor-Scalar gravity, ν -degeneracy,....
- Variation of fundamental couplings [*Berengut+ 2010,2013; Coc+, 2012.....*]
- Massive particle decay [*Jedamzik 2004,2006;....Cyburt+2013.....*] or catalyst [*Pospelov 2006,.....Cyburt+ 2012; Kusakabe+ 2013*]
- Photon cooling by axion [*Sikivie & Yang 2009;.... Kusakabe+ 2013*]
- Mirror world [*Coc+ 2013*]

Conclusions

- ❑ SBBN calculations confirm good agreement for Ω_B values deduced from CMB, SBBN (D and ^4He).
- ❑ However disagreement between Li observations, SBBN and CMB (Wednesday and Friday)
- ❑ SBBN is now a parameter free model, that can be used to probe of the physics of the early Universe (Friday)

Compléments bibliographiques

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