The magnetic properties of strongly interacting matter

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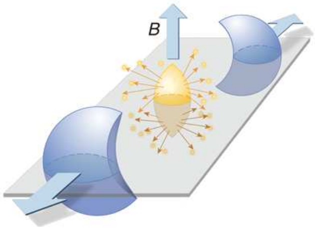
Outline

Why QCD in external magnetic field?

External magnetic fields can be relevant for the phenomenology of

- primordial universe and cosmological EWSM, $B \sim 10^{16}$Tesla
  Vachaspati, Grasso & Rubinstein
- neutron star and magnetars, $B \sim 10^{10}$Tesla
  Duncan & Thompson
- non central heavy-ion collision, $B \sim 10^{15}$ Tesla
  Skokov & Illarionov & Toneev

$10^{15}\, e\,\mathrm{Tesla} \sim 0.06\mathrm{GeV}^2 \sim 3.3 m_\pi^2$
Possible modifications of the strong interactions dynamics.



What could happen?

Model computations (like NJL-model) and effective field theories (like χPT) predict a rich phenomenology:

- Effects on the QCD vacuum structure (*e.g.* chiral symmetry breaking and condensates)
- Effects on the QCD phase diagram (*e.g.* changes in the location and nature of the phase transitions, decoupling of χSB and confinement, new phases)
- Effects on the QCD equation of state (*e.g.* magnetic contribution to the pressure)

Need for a first principles non-perturbative study of QCD in background e.m. fields.

Lattice QCD (LQCD) is an ideal tool to study such questions, at least in the limit of vanishing density where no algorithmic problems are present (*i.e.* no magnetars!)

LQCD crash course

The starting point is the Feynman path integral approach in the Euclidean space-time and the basic ideas are the following

- Vacuum expectation values of T-ordered products are written as expectation values with respect to the path measure
- The continuum space-time is approximated by a (finite) number of points and the path integration by standard integration
- The integration is performed numerically by Monte-Carlo techniques

In order to maintain gauge invariance the natural variables are not the gauge fields $A_\mu \in \mathfrak{su}(N)$ but the elementary parallel transports $U_\mu \in SU(N)$, $U_\mu \sim e^{iaA_\mu}$, where a is the lattice spacing.

In finite temperature studies the temperature is related to the temporal extent of the lattice: $T = 1/(N_t a)$.

External e.m. fields are introduced by means of an additional $u_\mu \in U(1)$ gauge field coupled to fermions.

LQCD crash course (2)

The theory discretized on a lattice of linear extent L and lattice size a is a non-perturbative IR and UV regularization of the original theory.
To extract the physical result we have to

- check for finite size effects: is L large enough? This means that L is "much larger" than the typical length scale of the process we are interested in. Usually this requires $L \gtrsim 3m_\pi^{-1}$.
- perform the continuum limit: we regularized the theory, we have to renormalize it. The lattice spacing is related to some "standard physical observable" (like *e.g.* the $q\bar{q}$ potential) and when everything is rewritten in term of physical length scales the results are of the form

$$R(a) = R(a = 0) + \mathcal{O}(a^\alpha) \qquad \alpha > 0$$

and by studying different values of the lattice spacing we can extrapolate the continuum result.

Lattice results

Effects on the QCD vacuum structure

- At zero temperature a magnetic field increases the χSB (magnetic catalysis)
  Buividovich *et al*, Phys. Lett. **B** 682, 484 (2010), Nucl. Phys. B **826**, 313 (2010)
  D'Elia and Negro, Phys. Rev. D **83**, 114028 (2011)
  Bali, Bruckmann, Endrodi, Fodor, Katz and Schafer, Phys. Rev. D **86**, 071502 (2012)
- An external magnetic field induces anisotropies in the gluon action
  Ilgenfritz *et al*, Phys. Rev. D **85**, 114504 (2012)
  Bali, Bruckmann, Endrodi, Gruber and Schaefer, JHEP **1304**, 130 (2013)
- An external *CP*-odd e.m. field ($\vec{E} \cdot \vec{B} \neq 0$) induces an effective θ term
  D'Elia, Mariti and Negro, Phys. Rev. Lett. **110**, 082002 (2013)

Lattice results (2)

Effects on the QCD phase diagram

- A magnetic field *does not* induce a split between χSB and deconfinement.
- Near T_c a magnetic field decrease the amount of χSB (inverse magnetic catalysis).
- A magnetic field decreases the value of T_c (still some controversy).

D'Elia, Mukherjee and Sanfilippo, Phys. Rev. D **82**, 051501 (2010)

Bali *et al*, JHEP **1202**, 044 (2012)

Ilgenfritz *et al*, Phys. Rev. D **85**, 114504 (2012)

Bali, Bruckmann, Endrodi, Gruber and Schaefer, JHEP **1304**, 130 (2013)

Bruckmann, Endrodi and Kovacs, JHEP **1304**, 112 (2013)

Ilgenfritz, Muller-Preussker, Petersson and Schreiber, arXiv:1310.7876 [hep-lat]

The magnetic properties of the QCD medium

We are interested in the magnetic properties of QCD at finite temperature.

The free energy can be expanded in B as

$$F(B,T) = F(B=0,T) + F_1(T)B - \frac{1}{2}\chi(T)B^2 + \mathcal{O}(B^3)$$

$F_1 \equiv 0$ if there is no ferromagnetism
$\chi > 0$ for paramagnetic media and $\chi < 0$ for diamagnetic media.

Our aims are:

- check that F_1 is compatible with zero
- study $\chi(T)$
- check for which B values the linear approximation $F \sim F_0 - \frac{1}{2}\chi B^2$ is reliable

The standard way and a no-go

The determination of magnetic susceptibilities is a standard problem in statistical physics. An estimator for χ is obtained by using the relation

$$\chi(T) = -\left.\frac{\partial^2 F(B,T)}{\partial B^2}\right|_{B=0}$$

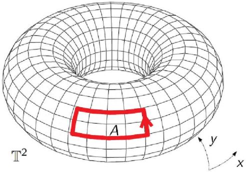
and it is enough to compute the mean value of some well defined lattice observable at $B = 0$.

In LQCD this is not possible: to reduce finite size effects simulations are performed on compact manifold without boundary and as a consequence the possible values of the *homogeneous* magnetic field are quantized.

$\frac{\partial}{\partial B}$ on the lattice is not well defined!

The magnetic field on the lattice

On a compact manifold with no boundary the Stokes theorem can be applied to each of the two connected component separated by the continuous closed path.



For an homogeneous magnetic field $B\hat{z}$ we have

$$\oint A_\mu \mathrm{d}x_\mu = \mathcal{A}B \qquad \oint A_\mu \mathrm{d}x_\mu = -(\ell_x\ell_y - \mathcal{A})B$$

This does not affect the motion of a particle of charge q if we impose

$$\exp(iqB\mathcal{A}) = \exp(iqB(\mathcal{A} - \ell_x\ell_y)) \quad \Rightarrow \quad \boxed{qB = \frac{2\pi b}{\ell_x\ell_y} \quad b \in \mathbb{Z}}$$

(the ℓ_μ's are the lengths in physical units)

The magnetic field on the lattice (2)

A simple choice of the lattice discretization is

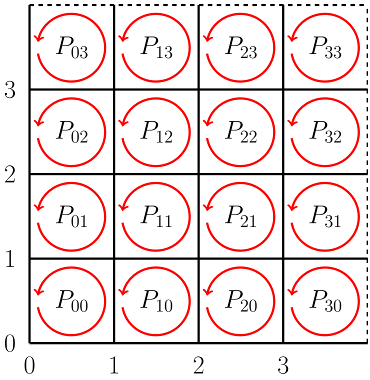
$$u_y(n) = e^{ia^2 qBn_x} \quad u_x(L_x - 1) = e^{-ia^2 qBL_x n_y} \quad \text{otherwise} \quad u_j(n) = 1$$

An example for $L_x = L_y = 4$.

The e.m. plaquettes are given by

- $P_{ij} = e^{ia^2 qB}$ for $(i, j) \neq (3, 3)$
- $P_{33} = \exp(ia^2 qB + ia^2 qBL_x L_y)$

Everything is ok if $a^2 qBL_x L_y = 2\pi b$ with $b \in \mathbb{Z}$. The idea is the same as the Dirac quantization condition for monopoles (*i.e.* “invisible” string).



How to compute χ

By now three different ways exist to avoid the "derivative problem"

- Using anisotropies:
  G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber, A. Schaefer
  JHEP **1304**, 130 (2013) [arXiv:1303.1328 [hep-lat]] &
  arXiv:1311.2559 [hep-lat].

- Using finite differences of the free energy:
  C. B., M. D'Elia, M. Mariti, F. Negro, F. Sanfilippo
  Phys. Rev. Lett. **111**, 182001 (2013) [arXiv:1307.8063 [hep-lat]] &
  arXiv:1310.8656 [hep-lat].

- Using non-uniform magnetic field:
  L. Levkova, C. DeTar
  Phys. Rev. Lett. **112**, 012002 (2014) [arXiv:1309.1142 [hep-lat]].

The finite difference method

We are interested in studying the B dependence of F, *i.e.*

$$\Delta F(B,T) \equiv F(B,T) - F(0,T) \qquad a^2 qB = \frac{2\pi b}{L_x L_y} \quad b \in \mathbb{Z}$$

$M(b,T) \equiv \frac{\partial F(B,T)}{\partial b}$ *is not* the magnetization, but we can evaluate it at non quantized values of B (*i.e.* $b \in \mathbb{R}$) in order to get

$$\Delta F(B,T) = \int_0^b M(\tilde{b},T)\mathrm{d}\tilde{b}$$

All the “quantization” artefacts that affect M simplify in the integral to give us the correct answer for the quantized B values!

We work on finite lattices, so everything is analytic and we adopt the previous expression for the $u_i(n)$ also for non quantized B values. These values of B are non physical but are needed only for the purpose of reconstructing ΔF for integer b.

Renormalization prescription

The free energy renormalizes additively and a prescription has to be fixed to perform the continuum limit.

The additive renormalization is temperature independent and can be removed by subtracting the zero temperature value:

$$(\Delta F)_R(B,T) = \Delta F(B,T) - \Delta F(B,T=0)$$

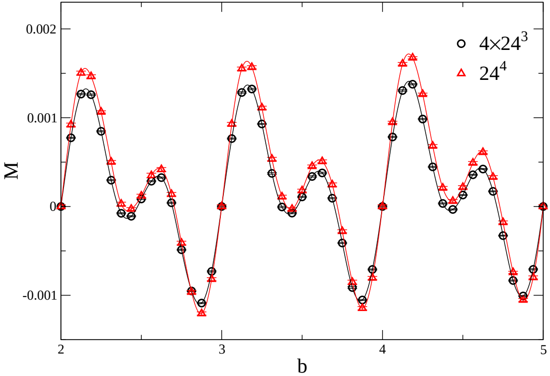
This is motivated by the idea that we want to study the properties of the thermal medium, so the zero temperature value has to be subtracted as a normalization.

Our procedure is thus the following:

1. compute the "magnetization" M for different temperatures and for non quantized B values
2. integrate M to get $\Delta F(B,T)$ for the quantized B values
3. compute the renormalized magnetic free energy $(\Delta F)_R(B,T)$

How M looks like

M computed on 24^4 and 4×24^3 ($T \approx 225$MeV) lattices, $N_f = 2 + 1$, physical masses, $a \approx 0.22$fm. The continuous line is a 3^{rd} order spline interpolation.



The numerical integration of M to get ΔF is performed by means of spline interpolations together with a bootstrap analysis for the error estimation.

Extracting the quadratic term

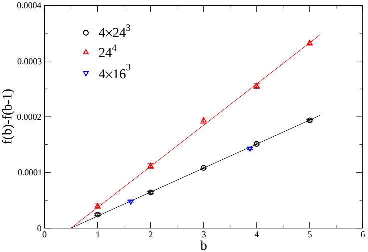
We now need to estimate f_2 defined by $\Delta F(B,T) \approx \frac{1}{2} f_2(T) b^2$ $(B \propto b)$.
In order to minimize the error propagation in the integration we fit

$$\Delta F(B_b, T) - \Delta F(B_{b-1}, T) = \int_{b-1}^{b} M(\tilde{b}, T) \mathrm{d}\tilde{b}$$

with the function

$$\frac{1}{2} f_2(T) \left[b^2 - (b-1)^2\right] = \frac{1}{2} f_2(T)(2b-1)$$

Results for $4 \times 16^3, 4 \times 24^3$ and 24^4 lattices with physical masses and $a \approx 0.22\,\mathrm{fm}$ $(T \approx 225\mathrm{MeV})$.



A note on the susceptibility

In an usual linear medium we have (in SI units)

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \qquad \mathbf{M} = \chi\mathbf{H} \qquad \mathbf{M} = \frac{\tilde{\chi}}{\mu_0}\mathbf{B} \qquad \chi = \frac{\tilde{\chi}}{1-\tilde{\chi}}$$

In our simulations the external field is a background field, so we have to subtract the energy of the magnetic field in vacuum from the free energy:

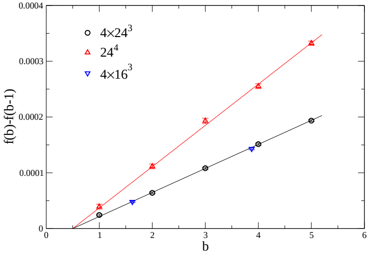
$$\Delta f_R = -\int \mathbf{M}\cdot \mathrm{d}\mathbf{B} = -\frac{\tilde{\chi}}{\mu_0}\int \mathbf{B}\cdot\mathrm{d}\mathbf{B} = -\frac{\tilde{\chi}}{2\mu_0}\mathbf{B}^2$$

B is the total field felt by the medium, but in our simulations the medium has no backreaction, so **B** is just the external field. Once $\tilde{\chi}$ is determined, we can extract the real world behaviour by using

$$\Delta f_R = -\frac{\mu_0\chi(1+\chi)}{2}\mathbf{H}^2$$

Check for systematics

- dependence on the volume



- dependence on the spline interpolation and/or the number of points

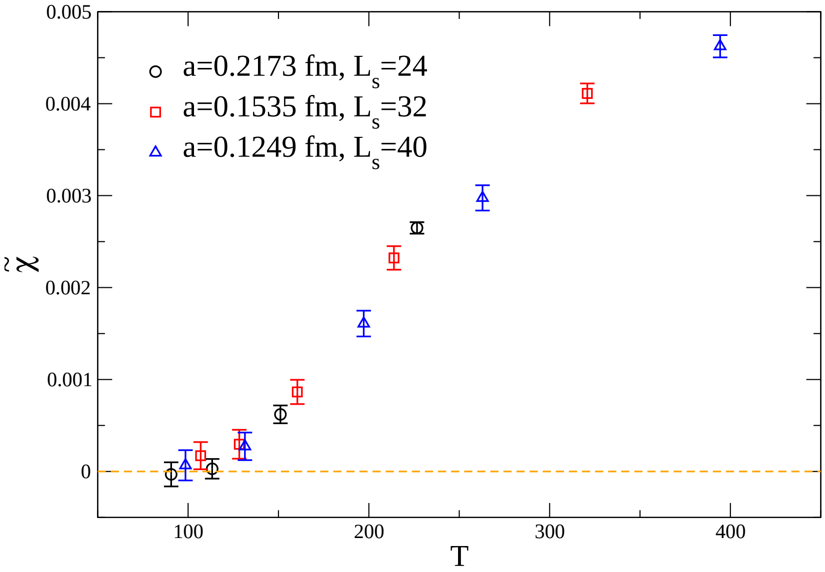
s	16 points	32 points
1	0.001192(32)	0.001187(25)
2	0.001188(35)	0.001186(25)
3	0.001184(35)	0.001188(25)
4	0.001183(34)	0.001188(27)

- dependence on the B field extension out of integers

one string	0.00211(5)
two strings	0.00208(4)

Systematics are always less than statistical errors

The result for $\tilde{\chi}$



Comments & physical interpretation

- The system is paramagnetic in the explored regime
- The QCD medium is linear up to $eB \approx 0.2\mathrm{GeV}^2$
- UV effects are small
- Expectation for the low-T region: Hadron Resonance Model $\tilde{\chi} \approx A \exp(-M/T)$
- Expectation for the high-T region: pQCD, $\tilde{\chi} = A' \log(T/M')$
  Elmfors *et al.*, Phys. Rev. Lett. **71**, 480 (1993)
- Data are well described by a function

$$\tilde{\chi}(T) = \begin{cases} A \exp(-M/T) & T \leq T_0 \\ A' \log(T/M') & T > T_0 \end{cases}$$

with $\mathcal{C}^1$ matching at T_0. The fit gives $M \approx 900\,\mathrm{MeV}$ (lightest hadrons with magnetic moment) and $T_0 \approx 160\,\mathrm{MeV} \approx T_c$.

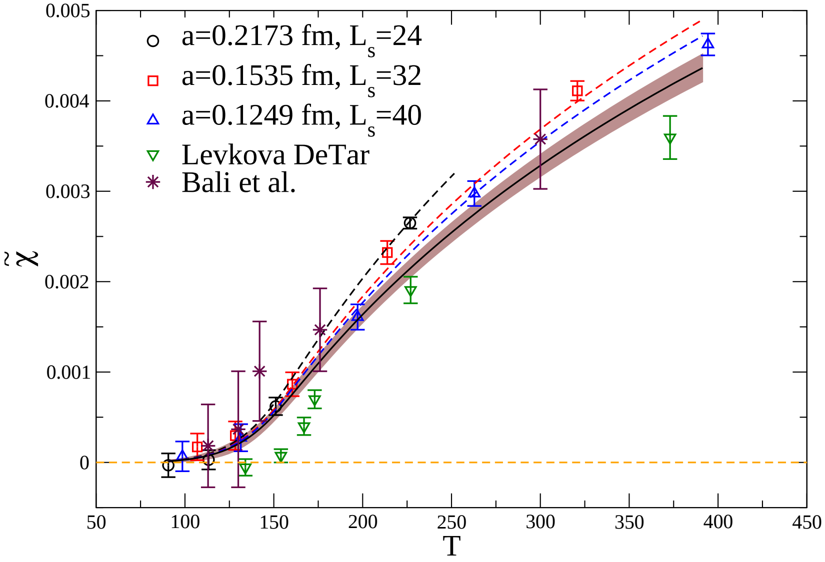
Physical picture (oversimplified but nice)

We have shown that the system is weakly paramagnetic in the confined region and strongly paramagnetic in the deconfined phase.

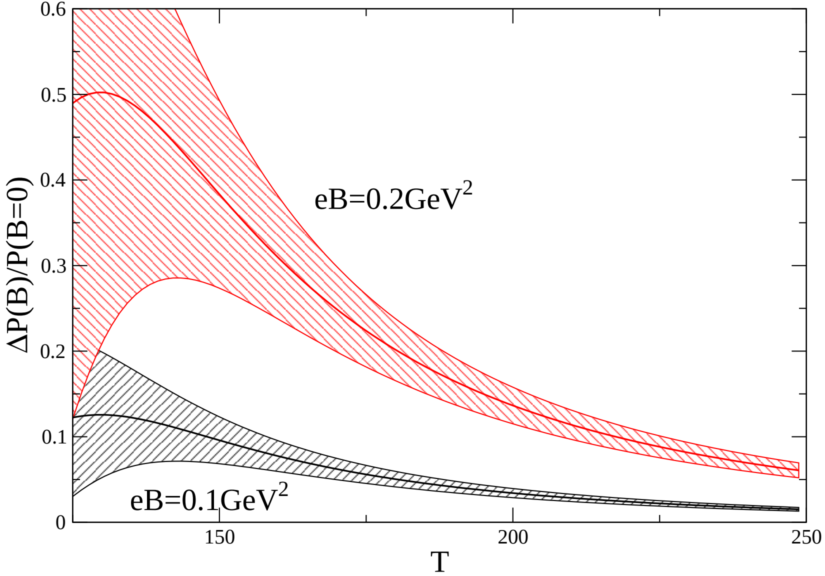
Let's think that the transition is first order. (It isn't!)

- For $T < T_c$ we have $F_c < F_d$ (the confined phase is the stable one) and the transition happens when $F_c = F_d$.
- When a magnetic field is present $F_c(B) \approx F_c(B = 0)$ (since $\chi_c \approx 0$) and $F_d(B) \approx F_d(B = 0) - \frac{1}{2}\chi_d B^2 < F_d(B = 0)$ (since $\chi_d > 0$).
- As a consequence $T_c(B) < T_c(B = 0)$.
- As a consequence $\langle\bar{\psi}\psi\rangle(B, T \approx T_c) < \langle\bar{\psi}\psi\rangle(B = 0, T \approx T_c)$ (*i.e.* inverse magnetic catalisis near the transition).

Continuum limit and comparison with other methods



Magnetic contribution to the pressure



Conclusions

- We introduced an intuitive non-perturbative method to compute the magnetic properties of strongly interacting matter.
- We have shown that the QCD medium is paramagnetic in the explored temperature range.
- The confined phase is weakly paramagnetic, the deconfined phase is strongly paramagnetic.
- The QCD medium is linear up to $B \approx 0.2\,\mathrm{GeV}^2$
- The magnetic contribution to the pressure for $B = 0.1 \div 0.2\,\mathrm{GeV}^2$ can be of order of $10 \div 50\%$ and can play an important role in heavy-ion collision phenomenology.
- For $B > 0.2\,\mathrm{GeV}^2$ nonlinear susceptibilities can play a dominant role (both at zero and finite temperatures). Their study is on the way.