Testing the Muon g-2 Anomaly in the Electron Sector

- A change in perspectives: $a_e \equiv (g-2)/2$ as a probe for new physics
- Two major technological breakthroughs: Penning traps and atomic interferometry
- The a_e systematics budget:
  - ✓ The experimental measurement of a_e
  - ✓ The experimental measurement of α: h/m and the electron mass
- Reaching the a_μ anomaly with a_e in the Naïve Scaling hypothesis
- Conclusions

Based on F. Terranova and G. M. Tino, arXiv:1312.2346

F. Terranova, Univ. Milano-Bicocca and INFN

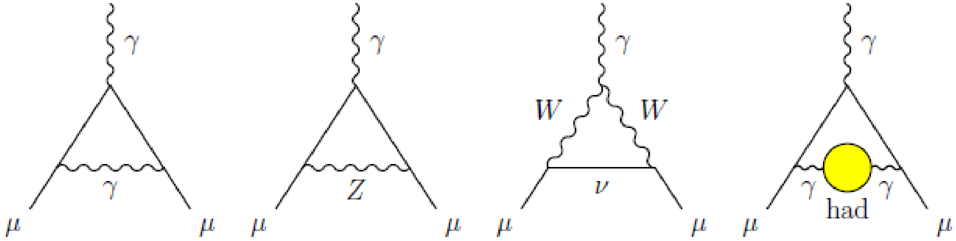
g-2: the standard view

$$\vec{\mu} = g_s \left(\frac{q}{2m} \right) \vec{s}$$

Lepton magnetic moments are exquisite probes of quantum loop effects

$$|\vec{\mu}| = (1 + a_l) \frac{q\hbar}{2m}$$

$$a_l = \frac{g_s - 2}{2}$$



But loop corrections go as $(\mathrm{m}_l/\mathrm{m}_X)^2$ hence…

a_e

Known with very high precision (electrons are stable!). Poorly sensitive to New Physics.
Outstanding test of QED
Best determination of α

a_μ

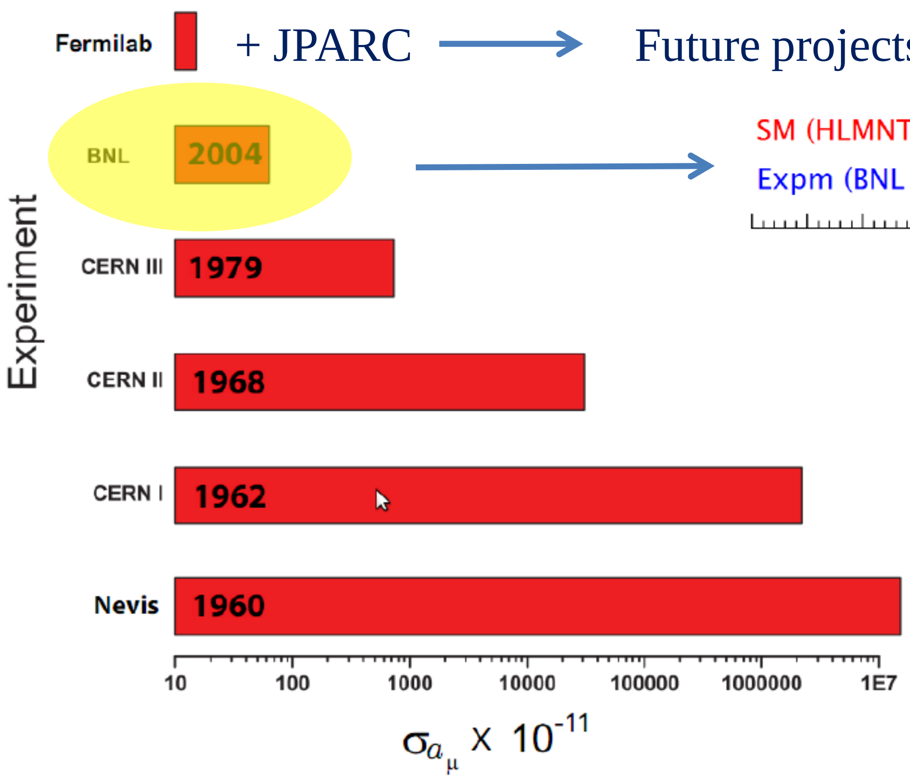
Known with good precision (muons decay but spin precession is visible).
Best place to search for New Physics

a_τ

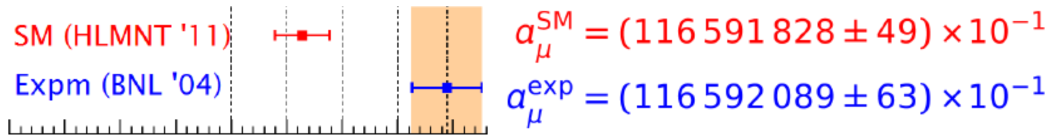
Known with poor precision (too short lifetime).
A priori, very sensitive to New Physics but limited by experimental precision

History confirms this standard view

$a_\mu \equiv (g-2)/2$



+ JPARC → Future projects (~2017)



Tension with SM prediction at the level of 3-3.5σ

Exp. Systematics?
Theory systematics?
Hint of New Physics?

The a_μ "crisis" triggered a vigorous experimental program at Fermilab and JPARC and strengthened the interest for high-precision multi-loop calculations.

In this framework a_e acts as a "standard candle" (used to extract the fine structure constant) while the role of a_τ remains marginal.

Changing the standard approach

The idea has been put forward in 2012 by **G. F. Giudice, M. Passera and P. Paradisi** and, in a nutshell, it sounds like:

In the last ten years, a few major breakthroughs have increased the gap between the experimental precision of a_e and a_μ. Such an increase can compensate for the "natural" suppression of New Physics (NP) effects due to the smallness of m_e.
a_e is ready to become a NP probe provided that:

- There is room for improvement in the experimental determination of a_e
- We can decouple the measurement of a_e from the determination of alpha

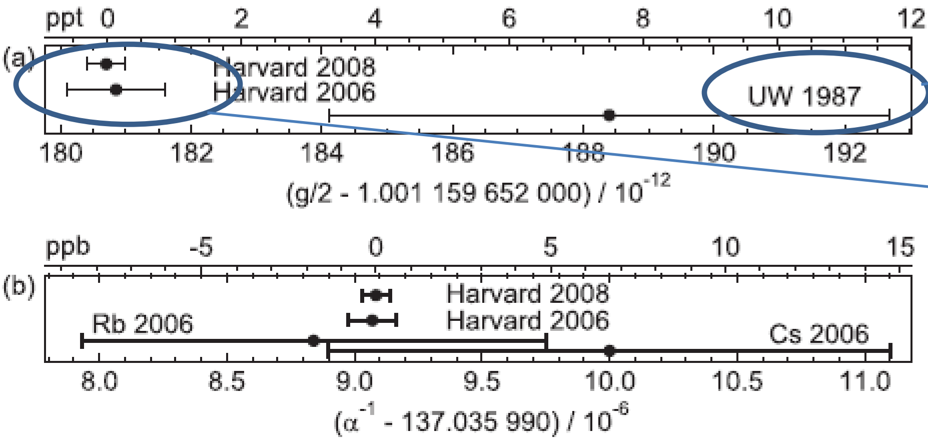
In addition, if the a_μ discrepancy is due to NP, in the vast majority of models, NP effects are expected in a_e, too. The "natural size" of these effects (Naïve Scaling) is of the order of

$$2.9 \times 10^{-9} \times \left(\frac{m_e}{m_\mu}\right)^2 = 6.8 \times 10^{-14} \ (0.06 \text{ ppb})$$

The measurement of a_e can test NP responsible for the a_μ discrepancy provided that the systematics budget is kept below 0.1 ppb (i.e. 10^{-10} relative precision)

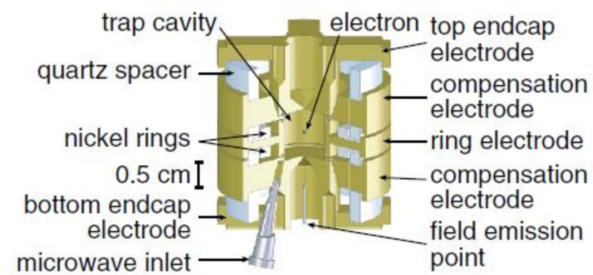
G.F. Giudice, M. Passera and P. Paradisi, JHEP 1211 (2012) 113

Cylindrical Penning traps



Dehmelt et al., single electron hyperbolic Penning trap(**)

Gabrielse et al., cylindrical Penning trap



This measurement corresponds to a 0.24 ppb measurement of a_e, very close already to the a_μ anomaly in the naïve scaling approximation. Further improvements are possible at the level of 0.08 ppb(*)

D.A. Hanneke, Ph.D. Thesis

$\bar{\nu}_c =$	147.5 GHz	149.2 GHz	150.3 GHz	151.3 GHz
$g/2$ - 1.001 159 652 180	-0.15	0.88	0.65	0.41
Uncertainties				
Statistics	0.39	0.17	0.17	0.24
Cavity shift	0.13	0.06	0.07	0.28
Uncorrelated lineshape model	0.56	0.00	0.15	0.30
Correlated lineshape model	0.24	0.24	0.24	0.24
Total uncertainty	0.73	0.30	0.34	0.53

Table 6.6: Corrected g and uncertainties in ppt.

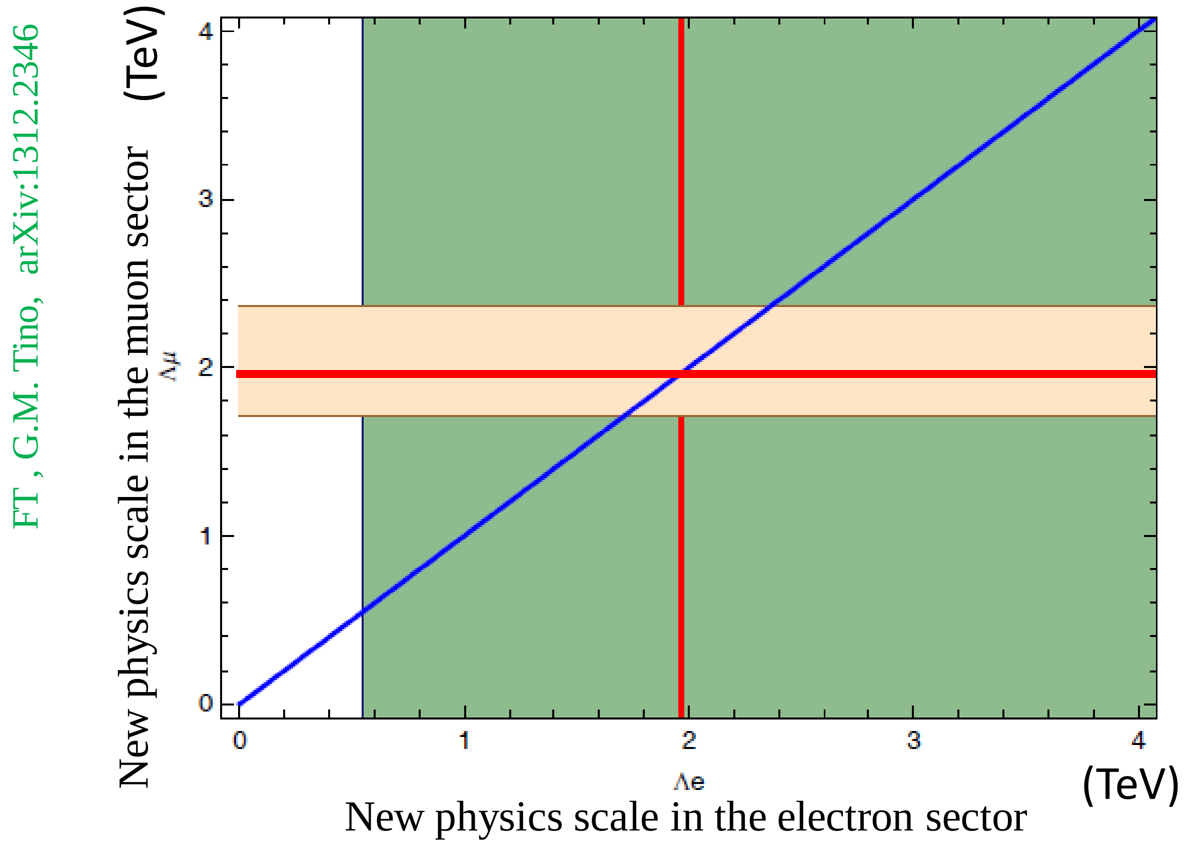
→ Reducible (likely due to B noise)

(*) see FT , G.M. Tino, arXiv:1312.2346 for details

(**) Dehmelt, Paul, Nobel Prize 1989

In spite of this…

The outstanding precision on a_e is not really exploited to constraint NP models



This is due to the fact that $a_e = \alpha/2\pi$ at leading order and, hence, an independent measurement of α is mandatory. The independent measurement of α is the present bottleneck for a test of the muon anomaly in the electron sector.

An independent measurement of α

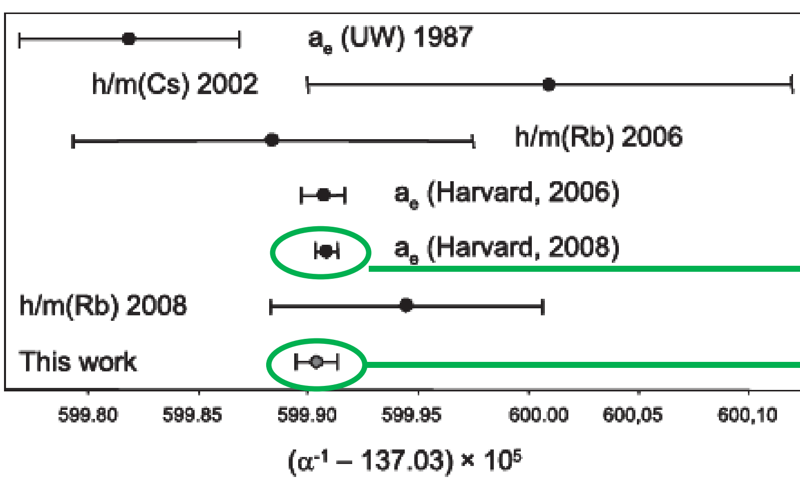
Another major breakthrough: optical comb generators(*) allowed for a 100-fold improved measurement of the Rydberg constant.

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_X}{m_e}\frac{h}{m_X} = 2\frac{R_\infty}{c}\frac{m_X}{m_u}\frac{m_u}{m_e}\frac{h}{m_X}$$

0.007 ppb   exact   <0.1 ppb

The ratio h/m can be measured with outstanding precision using atomic interferometry(**) and, in fact, the world best measurement of α (beyond the one derived from a_e) is based on this technique

Bouchendira et al., PRL 106 (2011) 080801



α from a_e (see above)

Atomic interferometry with Rb

(*) Hall, Hansch, Nobel Prize 2005   (**) Chu, Cohen-Tannoudji, Phillips, Nobel Prize 1999

Can we improve significantly h/m on a timescale comparable with the next round of muon g-2 experiments?

[discussed in depth in FT , G.M. Tino, arXiv:1312.2346]

General principle: transfer a large number of recoils to a population of atoms at rest and measure the velocity distribution. The recoil velocity of an atom when it absorbs a photon of momentum hν is $h\nu/m_X$

Rubidium (^{87}Rb): it leads world measurements (1.24 ppb on h/m, corresponding to 0.6 ppb in α) and, at present, it is not limited by intrinsic systematics. Current precision is limited by laser parameters (Gouy phase, beam alignment etc.). Its mass is known at the 0.17 ppb level.

Cesium (^{133}Cs): workhorse of atomic clocks and largest hyperfine splitting among alkaline atoms. Current error is mostly statistical (1.7 ppb) and there is significant room for improvement down to sub-ppb. Isotope mass known at 0.1 ppb level.

Helium ($^{4}He^{*}$): Not hydrogen-like. Metastable state can be cooled with NIR lasers. Mass known at 0.016 ppb level!

State of the art:

$$\frac{\sigma_\alpha}{\alpha} \simeq \frac{1}{2}[1.24 \text{ ppb} \oplus 0.44 \text{ ppb}] = 0.66 \text{ ppb}$$

h/m_{Rb} m_e/m_{Rb}

A closer look to the mass error budget

Measurements based on Penning traps already bring our knowledge of alkali atom masses (expressed in atomic mass units) in the 0.1 ppb ballpark.

Mount, Redshaw, Myers
PRA 82 (2010) 042513

TABLE III. Final atomic masses (in u) of ^{6}Li, ^{23}Na, ^{39,41}K, ^{85,87}Rb, and ^{133}Cs compared with results of the AME2003 [14] and other recent Penning trap measurements.

Atom	This paper	AME2003	Other recent results
^{6}Li	6.015 122 887 4(16)	6.015 122 795(16)	6.015 122 889(26) [33] 6.015 122 890(40) [34]
^{23}Na	22.989 769 282 8(26)	22.989 769 280 9(29)	
^{39}K	38.963 706 485 6(52)	38.963 706 68(20)	38.963 706 52(17) [35]
^{41}K	40.961 825 257 4(48)	40.961 825 76(21)	
^{85}Rb	84.911 789 739(9)	84.911 789 738(12)	
^{87}Rb	86.909 180 535(10)	86.909 180 527(13)	
^{133}Cs	132.905 451 963(13)	132.905 451 933(24)	

The error is hence dominated by the knowledge of the electron mass in atomic mass units (0.44 ppb, corresponding to 0.22 ppb uncertainty in α)

This considerations bring a remarkable motivation to improve the knowledge of the electron mass by about a factor of 5:

- It allows the exploitation of a_e as a probe of new physics (this talk)
- It allows a measurement of α based on the Rydberg relationship, i.e. independent of QED calculations

The electron mass in a.m.u. : $A_r(m_e)$

Only one direct measurement reported by CODATA: comparison of the cyclotron frequency of an electron and a C^{6+} ion in a Penning trap (Washington, 1995). Relative precision at the 2.1 ppb level (i.e. 1 ppb on alpha). Not appropriate for NP tests on a_e at the 0.1 ppb level!

The best CODATA (and hence PDG) fit comes from the measurement of g-2 in bound-state electrons. I.e. they measure the ratio between the Larmor and cyclotron frequency of C^{5+} and O^{7+} atoms. At leading order (Dirac equation), we have

Free electron

$$g = 2$$

Bound electron in hydrogen-like atom

$$g_b^{Breit} = 2/3(1 + 2\sqrt{1 - Z^2\alpha^2})$$

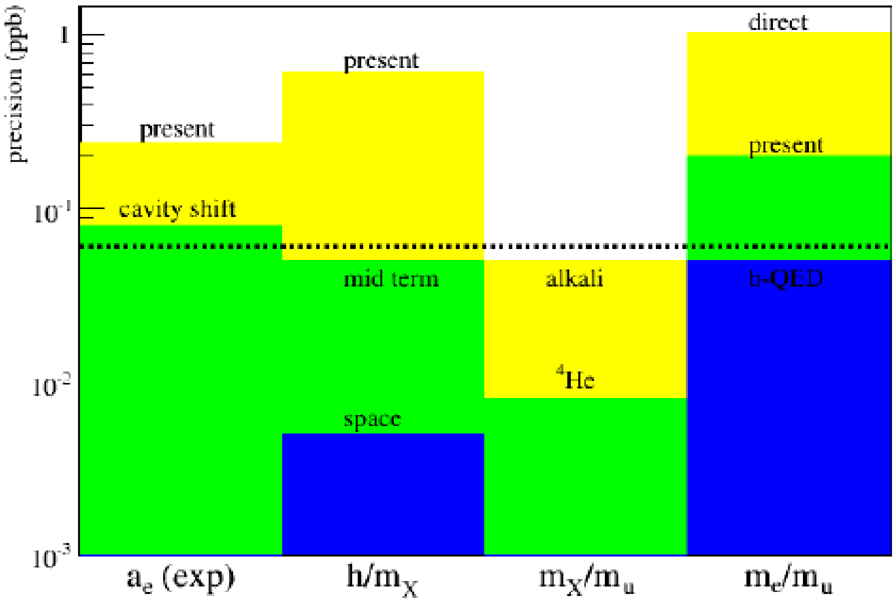
We can measure the "bound counterpart" of a_e $\quad a_e^b \equiv (g_b - g_b^{Breit})/g_b^{Breit}$

$$g_b = 2\frac{\omega_L}{\omega_c}\frac{m_e}{M_{C^{5+}/O^{7+}}} = 2\frac{\omega_L}{\omega_c}A_r(m_e)\frac{m_u}{M_{C^{5+}/O^{7+}}}$$

This measurement is much less precise than a_e (1 ppm versus 0.2 ppb!!), so it is not a probe of NP but can be used to measure $A_r(m_e)$

At GSI (**see yesterday's issue of Nature ☺** - S. Sturm et al., Nature 506 pp.467-470, 27 Feb 2014) they improved $A_r(m_e)$ by a factor of 13

Perspective to improve the precision on a_e at the level of sensitivity required to test the muon anomaly are very bright:



But they deserve at least a note of caution:

- The physics of a_e (as for a_μ !!) is lacking redundancy and cross-checks
  - Only one competitive experimental measurement (Gabrielse et al.)
  - Only one competitive evaluation of QED contributions (Kinoshita et al.)
- a_e and me heavily rely on complex theory calculations (hadronic contribution and bound-state QED)

Conclusions

- The muon anomaly has been with us for more than a decade and new measurements from Fermilab and JPARC will (hopefully) set the issue in about 5 years
- a_e is $(m_e/m_\mu)^2$ less sensitive to new physics but this suppression factor can be overcome by the superior experimental accuracy
- Measurement from cylindrical Penning traps have not saturated their accuracy
- The real bottleneck is the independent determination of α, which can be addressed by atom interferometry
- The knowledge of m_e in a.m.u. is no more a showstopper

We expect to have an independent test of the muon anomaly in the electron sector at about the same timescale of the Fermilab/JPARC projects