Neutrino mixing: a theoretical overview

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Two different descriptions

- Neutrinos can also be described in terms of mass eigenstates ν_i

$$|\nu_\alpha\rangle = \Sigma_{i=1}^{3} U_{\alpha i} |\nu_i\rangle$$

neutrino matrix

matrix

- Simple time evolutions of the vector $\nu(t) = (\nu_e(t), \nu_\mu(t), \nu_\tau(t))$:

$$i\frac{d}{dt}|\nu(t)\rangle = H|\nu(t)\rangle$$

$$H = \frac{1}{2E_\nu} U\, Diag[0, m_2^2 - m_1^2, m_3^2 - m_1^2] U^+$$

there exist a probability of a change of the neutrino flavour

Transition probabilities

- Flavour changing transitions

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \Sigma_j U_{\beta j} e^{\frac{-i m_j^2 L}{2 E_\nu}} U^*_{\alpha j} \right|^2$$

- In the case of two neutrinos only:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \; \sin^2\left(\frac{\Delta m^2 L}{4 E_\nu}\right)$$

distance source-detector

mixing angle

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

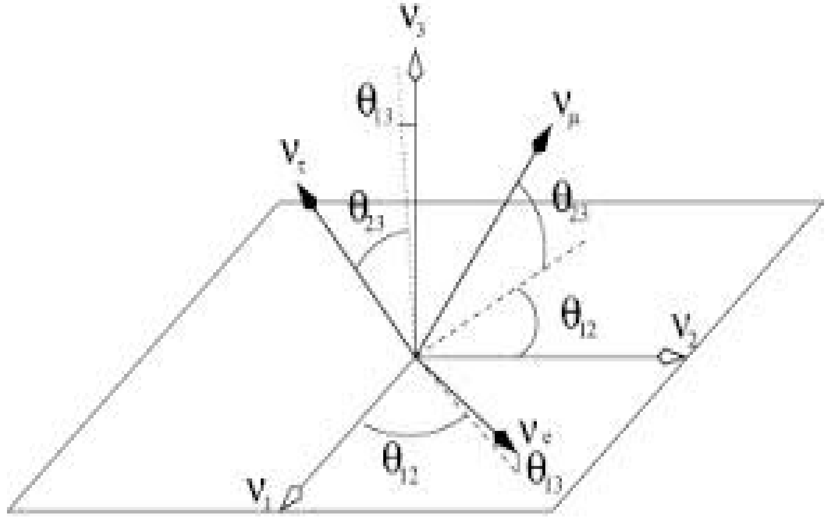
U = unitary matrix

3-flavour transition probabilities

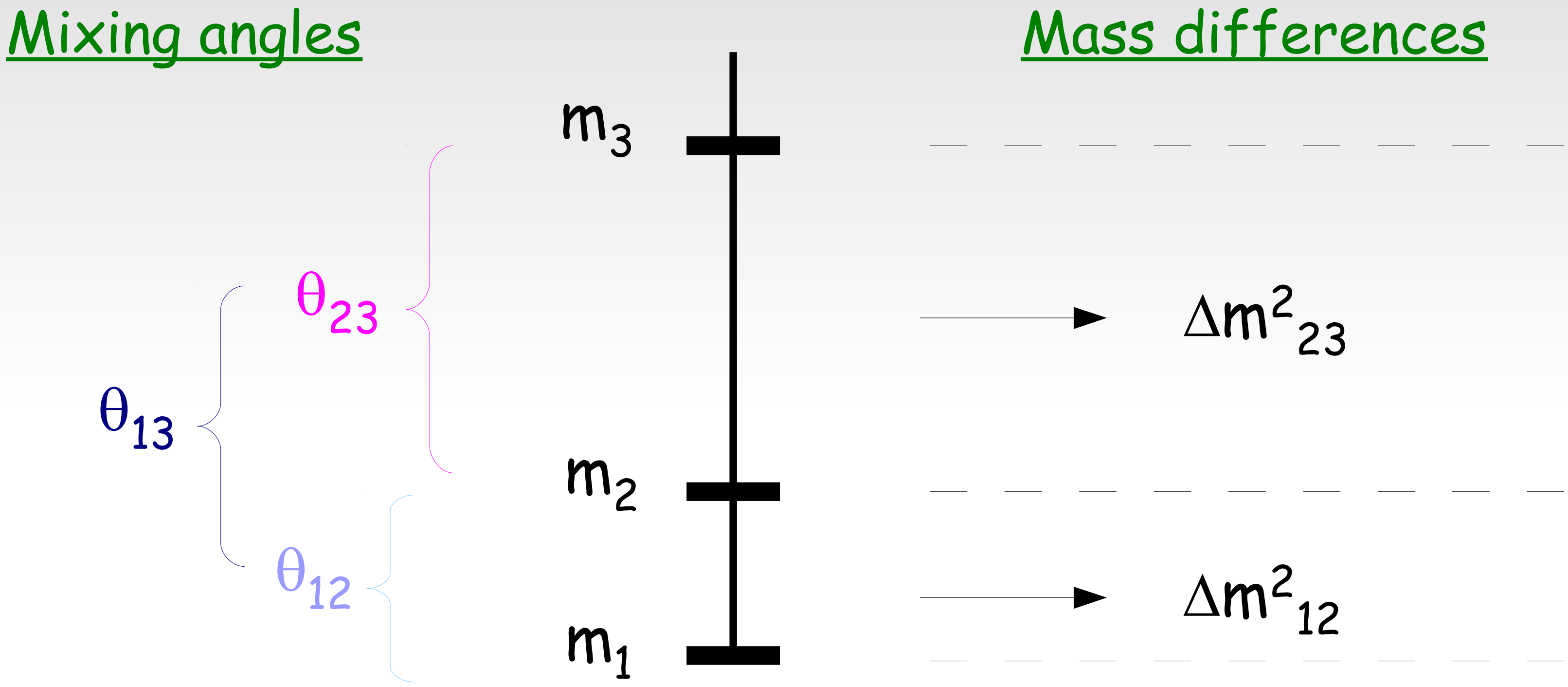
- The neutrino mixing matrix depends on 4 real parameters

$$U_{PMNS} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{atmospheric\ \ mixing} \times \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}}_{reactor\ \ mixing} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{solar\ \ mixing}$$

- A more complicated mismatch between the ν bases



To be determined by oscillation experiments

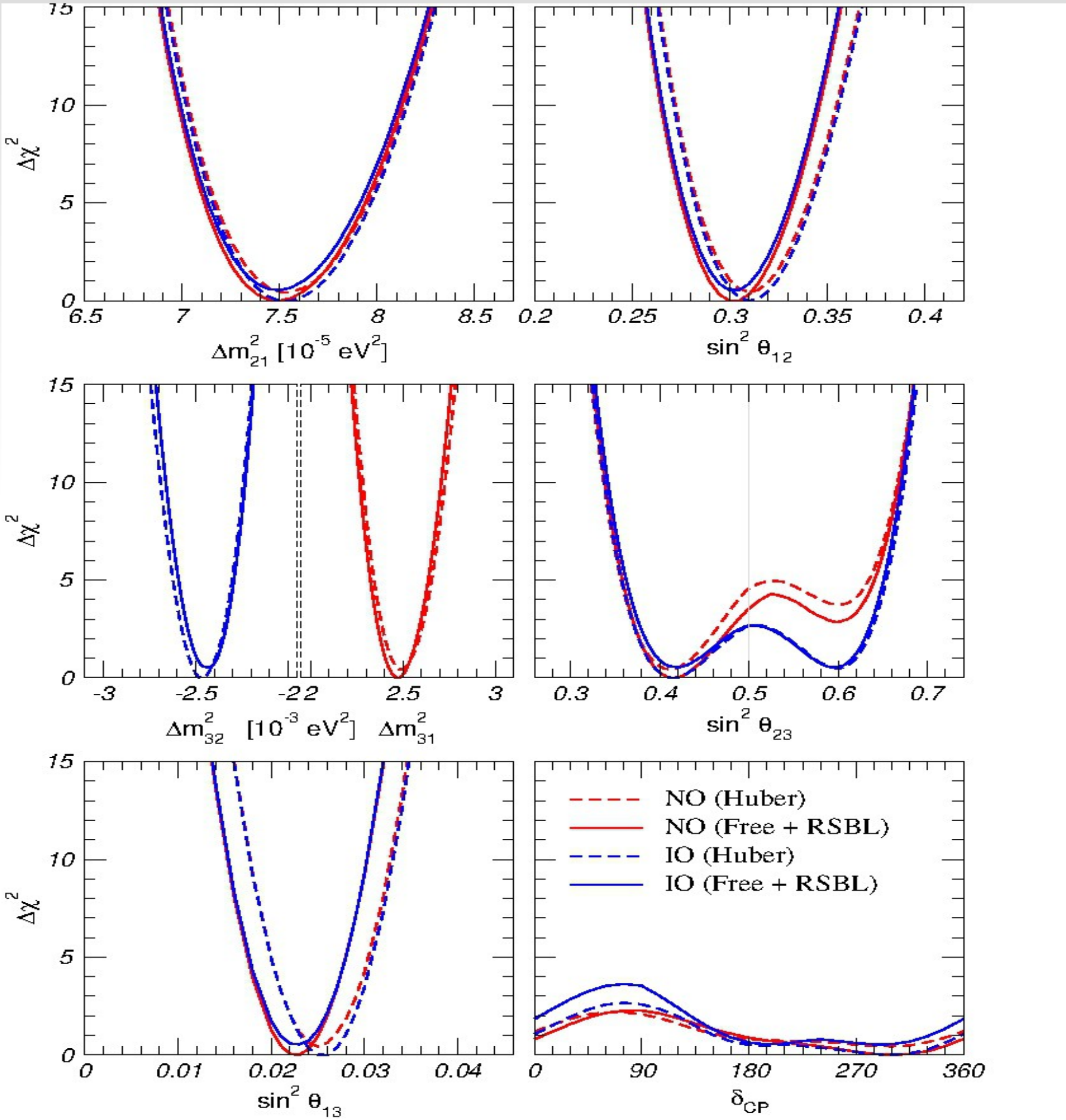


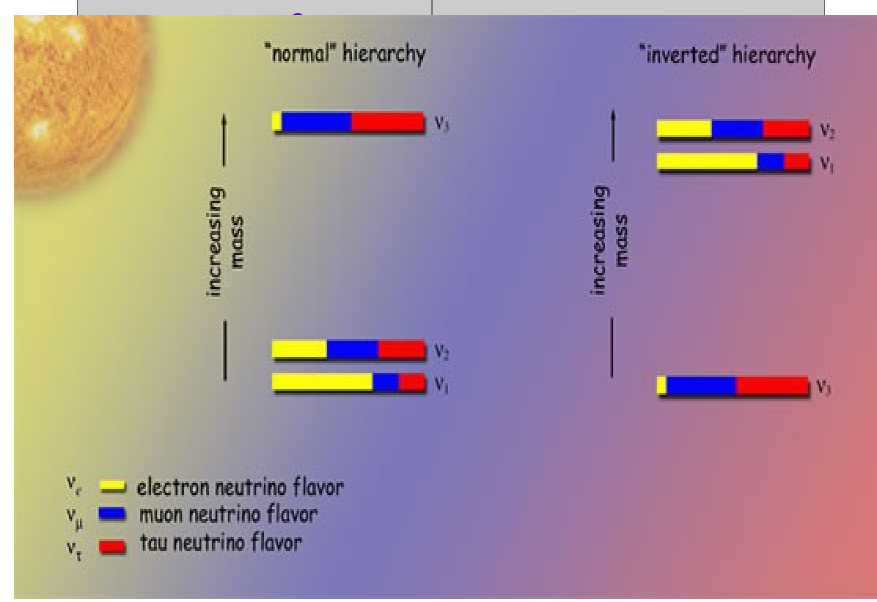
- And: a possible CP phase δ and the absolute order of the mass eigenstates (normal or inverted hierarchy)

Summary of the experimental results

Gonzalez-Garcia et al. JHEP1212,(2012)123

Parameter	Fit results
θ_{12}	$33.36^{+0.81}_{-0.78}$
θ_{13}	$8.66^{+0.44}_{-0.46}$
θ_{23}	$40.0^{+2-1}_{-1.5}$
δ	300^{+66}_{-138}
Δm^2_{23} (10^{-3}	$2.47^{+0.07}_{-0.07}$





What a theorist must explain

hints from experiments

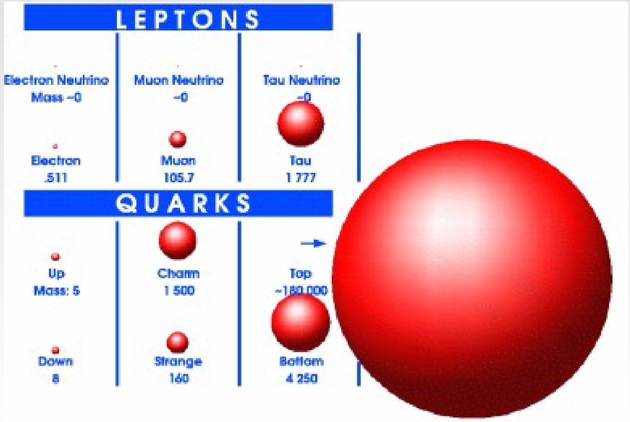
Values of the mixing angles

Values of the mass differences

Smallness of the masses compared to the other fermions

- Some ideas...

About neutrino masses



the assumption of small Yukawa couplings

neutrinos: $Y_\nu \bar{\psi}_L \widetilde{H} \nu_R$ electrons: $Y_e \bar{\psi}_L H e^c$

$$\frac{Y_\nu}{Y_e} \sim 10^{-5}$$

now going beyond these asssumptions...

Neutrino mass terms

we assume the existence of ν_L and ν_R

Smallness of the masses compared to the other fermions

must be conserved: $|\Delta I| = 0$

Weak isospin	ν_L	ν_R	H = (h+,h0)
I	1/2	0	1/2
I_3	1/2	0	(+1/2,-1/2)

	ν	$\bar{\nu}$
Lepton number	1	-1

- Dirac mass term

(same for quarks and leptons)

right-handed neutrinos ν_R must be included in the standard picture; lepton number L is conserved

$$L_D = m_D \bar{\psi}_L \tilde{H} \nu_R$$

- Majorana mass term

if lepton number L is not conserved

$$L_M = m_M \nu_R^T \nu_R$$

The see-saw mechanism

- Total lagrangian

$$L_m = m_D \bar{\psi}_L \tilde{H} \nu_R + m_M \nu_R^T \nu_R$$

Electroweak symmetry breaking → see-saw

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \rightarrow L_m = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}$$

$$m_\nu = -m_D^T m_M^{-1} m_D$$

$m_\nu \sim m_D^2/m_M$

for m_D~100 GeV, m_ν ~ 0.05 eV

↓

$M_M \sim 10^{14}$-10^{15} GeV

Probe into GUT !

About the angles: special patterns of U_{PMNS}

Values of the mixing angles

- observation: one can consider a good starting point the following values of the mixing angle

$$\sin^2\theta_{12} = \frac{1}{3} \qquad \sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0$$

$$U_{PMNS} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Related to θ_{13}

Related to θ_{23}

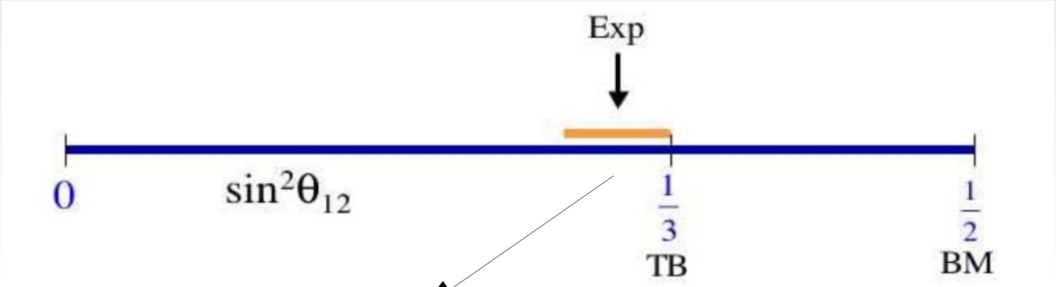
Related to θ_{12}

Tri-Bimaximal Mixing (TBM)

Special patterns of U_{PMNS}

- This cannot be the end of the story: corrections must be included to fit the data

the size of the corrections depends on the solar angle



at most $O(\lambda^2)$, λ being the Cabibbo angle

- As a general result, all mixing angles receive corrections of $O(\lambda^2)$

$$\sin^2\theta_{12}=\frac{1}{3}+O(\lambda_C^2) \quad \sin^2\theta_{23}=\frac{1}{2}+O(\lambda_C^2) \quad \sin^2\theta_{13}=O(\lambda_C^2)$$

good

good

Wrong!

Open field:
the flavour problem
in the lepton sector

Special patterns of U_{PMNS}

a possible solution:

- U_{PMNS} has contributions from the neutrino and the charged leptons

$$\nu_\alpha = U^{\nu}_{\alpha i}\,\nu_i \qquad l_\alpha = U^{l}_{\alpha i}\,l_i$$

matrix elements completely unknown ⇨ mass matrices are (almost) unknown

$$L \sim \bar{l}_\alpha \gamma_\mu \nu_\alpha W^\mu \rightarrow \underbrace{U^{+l}_{\alpha i} U^{\nu}_{\alpha j}}_{U_{PMN}} \bar{l}_i \gamma_\mu \nu_j W^\mu$$

$$|U| = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

in the quarks sector

a remark:

- U_{CKM} has contributions from the up and down quarks

$$u_\alpha = U^u_{\alpha i} u_i \qquad d_\alpha = U^d_{\alpha i} d_i$$

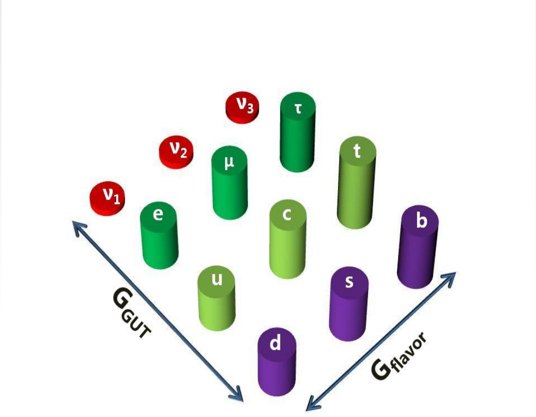
$$L \sim \bar{d}_\alpha \gamma_\mu u_\alpha W^\mu \rightarrow \underbrace{U^{+d}_{\alpha i} U^u_{\alpha j}}_{U_{CKM}} \bar{d}_i \gamma_\mu u_j W^\mu$$

$$|U| = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

$$\begin{bmatrix} 0,97383^{+0,00024}_{-0,00023} & 0,2272^{+0,0010}_{-0,0010} & (3,96^{+0,09}_{-0,09}) \times 10^{-3} \\ 0,2271^{+0,0010}_{-0,0010} & 0,97296^{+0,00024}_{-0,00024} & (42,21^{+0,10}_{-0,80}) \times 10^{-3} \\ (8,14^{+0,32}_{-0,64}) \times 10^{-3} & (41,61^{+0,12}_{-0,78}) \times 10^{-3} & 0,999100^{+0,000034}_{-0,000004} \end{bmatrix}$$

The role of additional symmetries

- The previous patter (and others similar) is easily realized in the context of broken flavor symmetries



Theory invariant under G_F

Less free parameters

Residual symmetry in the neutrino sector G_ν: U_ν

Residual symmetry in the charged lepton sector G_l: U_l

$$U_{PMNS} = U_l^+ \, U_\nu$$

- permutation groups like A_4 and S_4 suitable for TBM
- the breaking of the generators G_ν and G_l usually generate NLO

Non-abelian discrete symmetries

- TB mixing corresponds to m in the basis where charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

x, y, z complex numbers

Important observation:

- m is the most general matrix invariant under

$SmS = m$ and $A_{23} m A_{23} = m$

$$S = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Non-abelian discrete symmetries

- charged lepton masses (a generic diagonal matrix) is invariant under T:

$$T^{+} (ml^{+}\, ml)\, T = (ml^{+}\, ml)$$

$$m_l \sim \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

S, T and A_{23} are all contained in S_4

$S^4 = T^3 = (ST^2)^2 = 1$ define S_4 ⇒ the reference group for TB mixing

The role of A_4

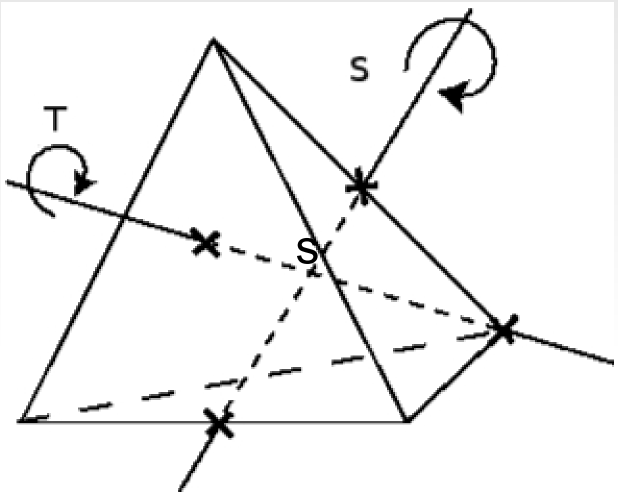
take this as an example

- A_4 is the discrete group of even permutations of 4 objects (4!/2 = 12 elements) generated by S and T

$$S^2=T^3=(ST)^3=1$$

The action of the generators S and T can be assigned as follows:

S: (1234) → (4321) T:(1234) → (2314)



- irreducible representations:

  a triplet and 3 different singlets 3, 1, 1′, 1″ (promising for 3 generations)

- invariance under S and T is automatic while A_{23} is not contained in A_4

  (2-3 symmetry happens in A_4 if 1′ and 1″ symm. breaking flavons are absent or have equal VEV's)

The role of A_4

- we start from a gauge and A_4 invariant lagrangian
- A_4 must be broken by the vacuum expectation values of scalar fields ϕ

↓

At LO TB mixing is exact

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle generically receives corrections of the same order

$$\delta\theta_{ij} \sim \frac{\langle \Phi \rangle}{\Lambda}$$

- special models can be constructed with non-universal $\delta\theta$ (Lin '09)

Typical predictions for A_4 models

$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\,\mathcal{R}e(c^\nu_{23})\right)\xi$$

$$\sin^2\theta_{12} = \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi + \frac{2\sqrt{2}}{3}\mathcal{R}e(c^\nu_{12})\,\xi$$

$$\sin\theta_{13} = \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12} - c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}\,c^\nu_{13} + c^\nu_{23}\right)\right|\xi\,.$$

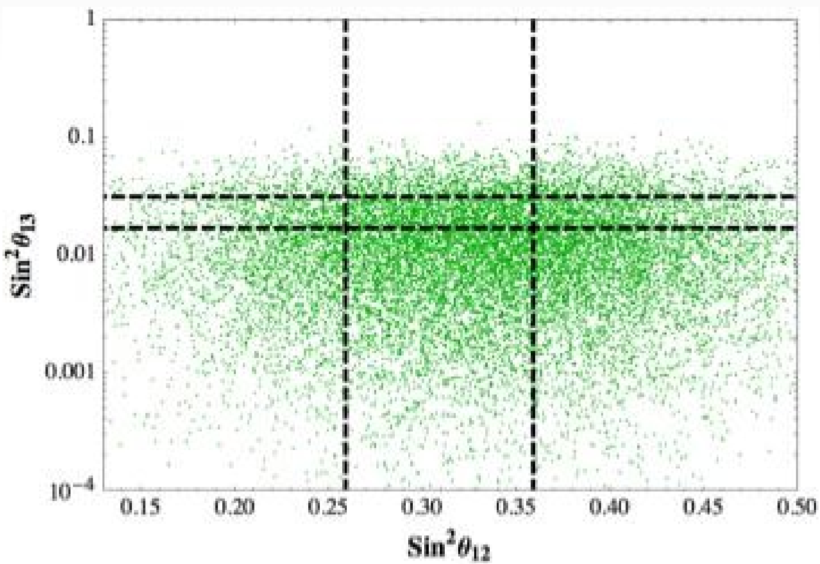
→ $\frac{\langle\Phi\rangle}{\Lambda} \sim O(0.1)$

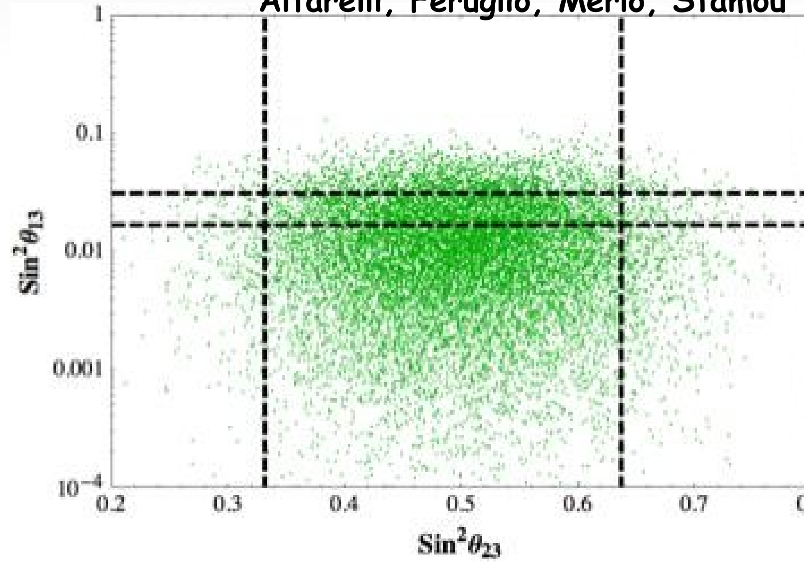
c_{ij} = random complex with abs. value gaussian around 1 with variance 0.5

from charged lepton rotation

from neutrino rotation

Altarelli, Feruglio, Merlo, Stamou '12





main message = TM still a viable LO approximation

No special patterns

- the TBM (BM or whatever) does not play any role
- easy to implement using an abelian U(1) symmetry

Froggatt-Nielsen,NPB147,277 (1979)

- suppose some fields transform as: $\psi \to e^{i q_\psi} \psi$

- a generic mass term transforms as:

$$y \bar{\psi}_L H \psi_R \to e^{i(-q_{\psi_R} + q_{\psi_L} + q_H)} y \bar{\psi}_L H \psi_R$$

if exp=1 the term is invariant, otherwise one has to add some extra scalar field

$$y \bar{\psi}_L H \psi_R \left(\frac{\theta}{\Lambda}\right)^k \to e^{i(-q_{\psi_R} + q_{\psi_L} + q_H + k\, q_\theta)} y \left(\frac{v_\theta}{\Lambda}\right)^k \bar{\psi}_L H \psi_R$$

λ = suppression factor

y = O(1) parameters

An example: SU(5) framework

- Standard Model particles in the 10 and $\bar{5}$ representations

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L , \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_L$$

- SU(5) mass terms:

1 = right-handed neutrino

$$m_{up} \sim 10 \times 10$$
$$m_d \sim m_e \sim 10 \times \bar{5}$$
$$m_{\nu_D} \sim \bar{5} \times 1 \qquad m_M = 1 \times 1$$

An example: SU(5) framework

- Several mass matrix structures for appropriate charge assignments

three entries because of 3 families

- Anarchical models

(A) $10=(3,2,0)$, $\bar{5}=(0,0,0)$, $1=(0,0,0)$

$$m_l = \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}, \quad m_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

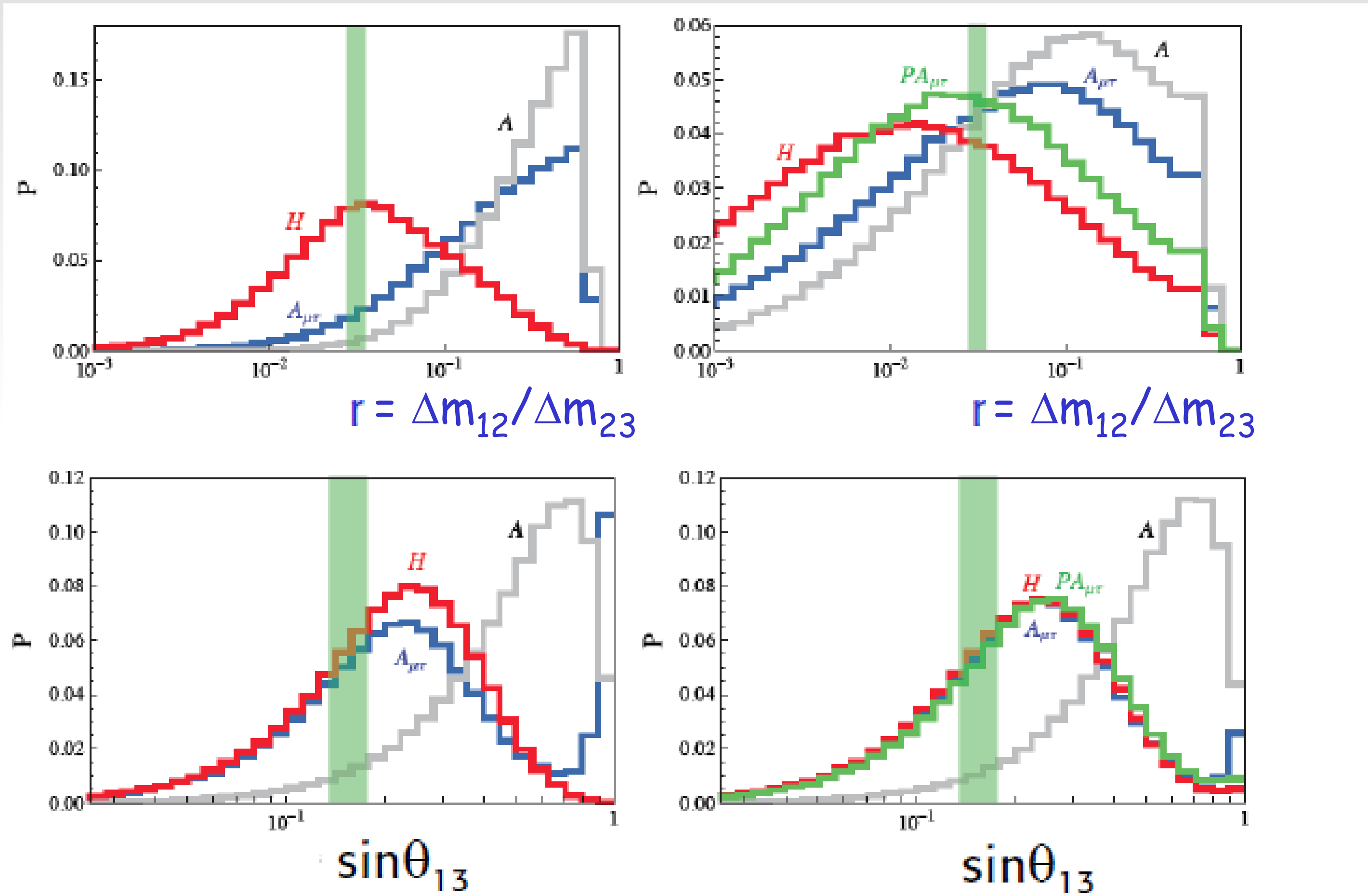
- Hierarchical models

(H) $10=(5,3,0)$, $\bar{5}=(2,1,0)$, $1=(2,1,0)$

$$m_l = \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^3 \\ \lambda^2 & \lambda & 1 \end{pmatrix}, \quad m_\nu = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}$$

Comparing Hierarchy vs Anarchy

no see-saw see-saw



main message = H performs better than A

The future (personal view)

Oscillation sector

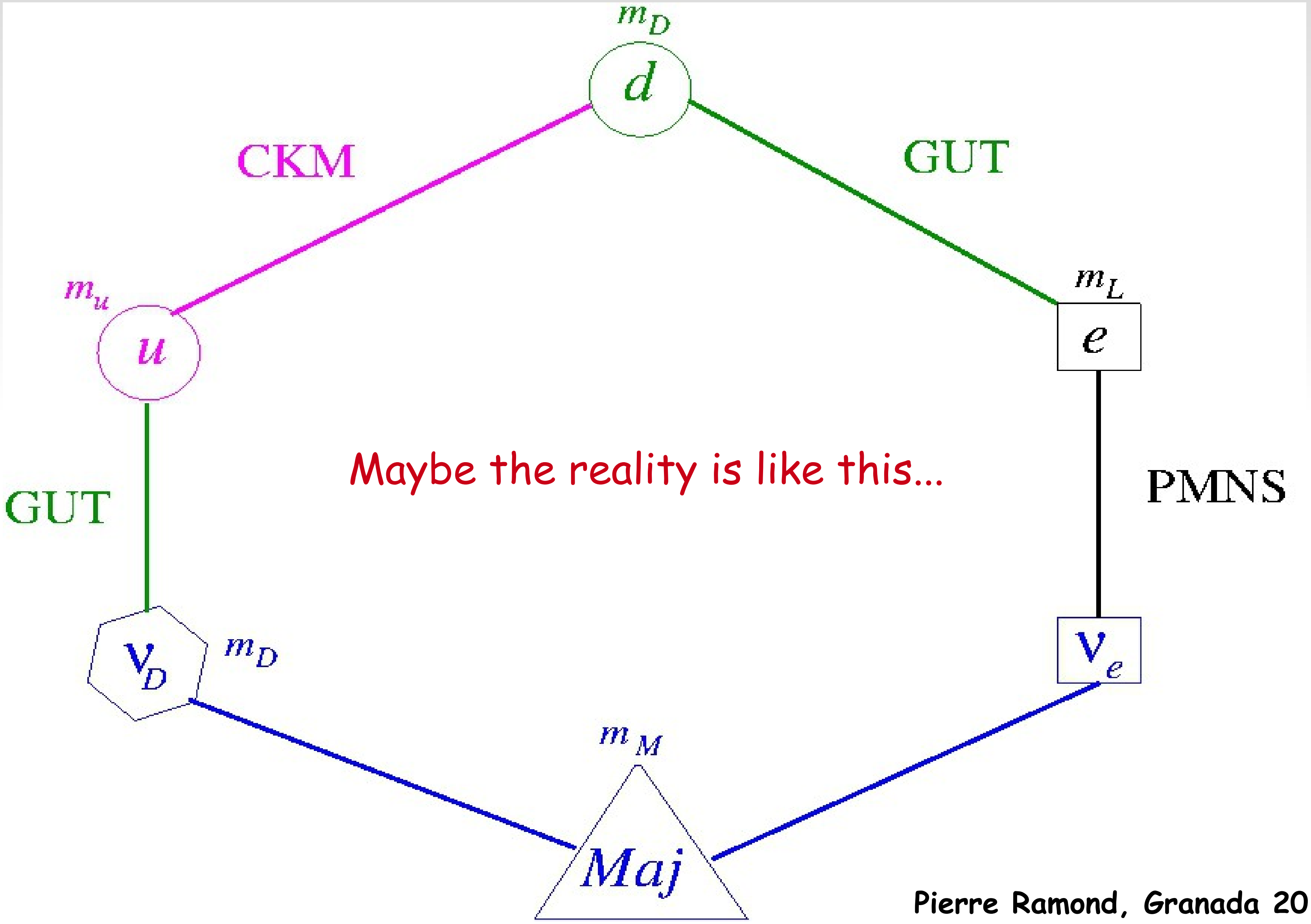
- Better determination of the oscillation parameters and the mass pattern
- Check for new physics effects

Flavor sector

- Interplay of flavor symmetries and realistic GUT theories
- Differences between quarks and leptons

"cosmological" sector

- The problem of high energies neutrinos
- Absolute neutrino masses



Maybe the reality is like this...

Pierre Ramond, Granada 2013

Conclusions

- Neutrino physics is an active field, from both experimental and theoretical point of views
- Many data are now available, which point toward a pattern with two large mixing angles and a smaller θ_{13}
- No clear theoretical explanation is on the market for the understanding of all neutrino properties
- The mystery is a " deformed replica" of what we have in the quark sector →

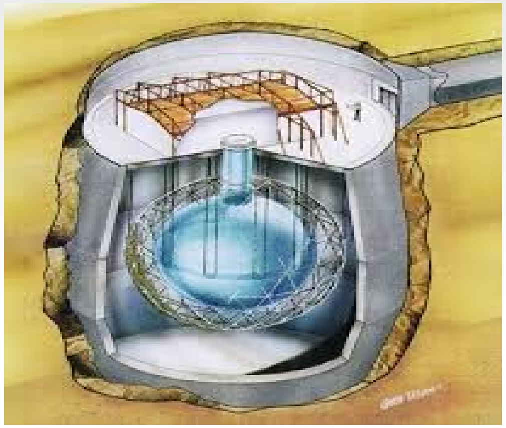
Possible common origin...

backup

Summary of the experimental results

- Superkamiokande:
  $\theta_{23} \sim 45^o$

- SNO
  $\theta_{12} \sim 33^o$



- T2K
  $\theta_{13} \sim 9^o$

$\Delta m^2_{12} \sim 7 \times 10^{-5}\ eV^2$

$\Delta m^2_{23} \sim 2 \times 10^{-3}\ eV^2$

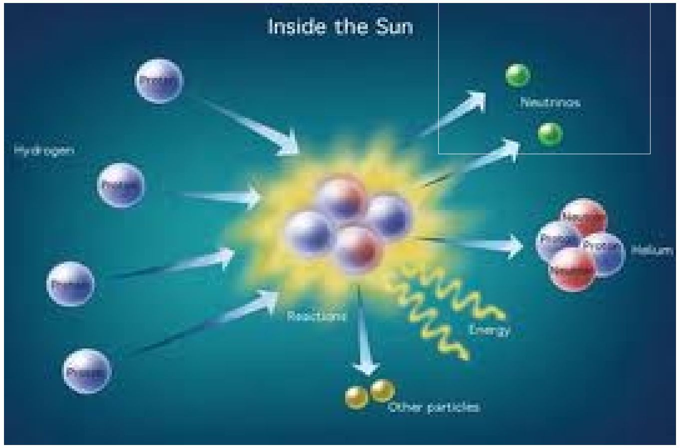
- Hierarchy = ?
- δ = ?

What neutrinos are...

- Electron neutrinos ν_e

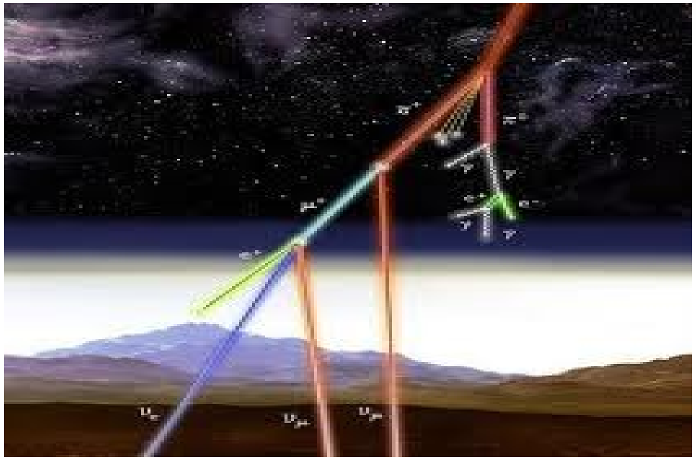
  mainly from the Sun
  (and nuclear plants)

$E_\nu \sim MeV \qquad L \sim 1.5 \cdot 10^5\ Km$



- Atmospheric neutrinos ν_μ

$E_\nu \sim GeV \qquad L \sim 10^3\ Km$

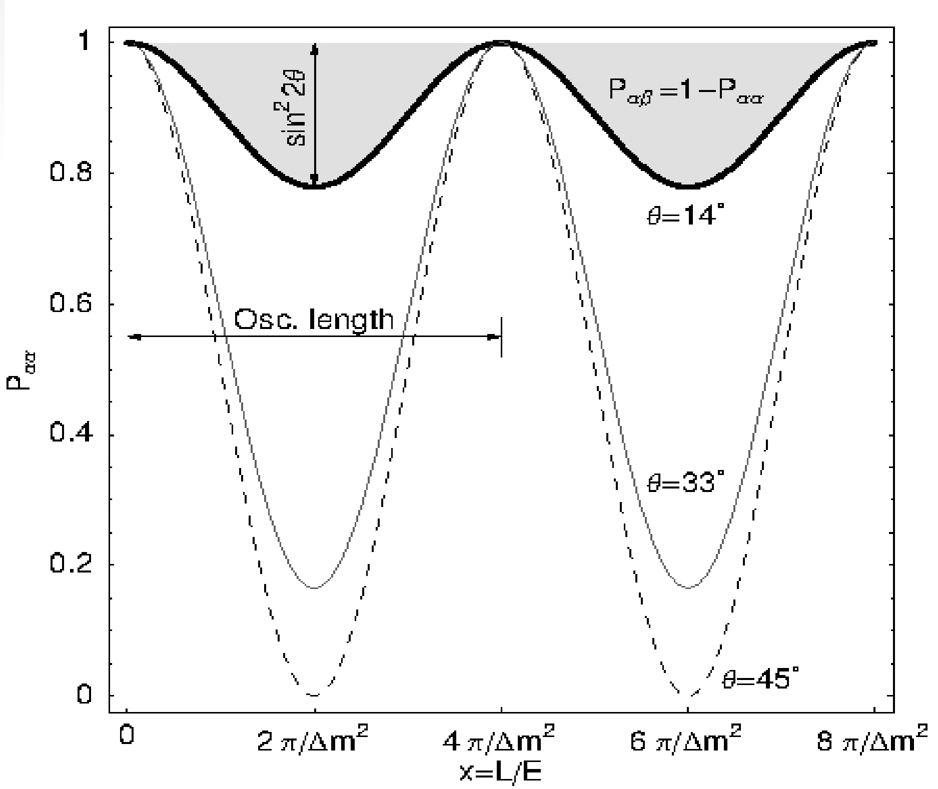


These particles participate to the interactions →
interaction eigenstates ν_α

2-flavour transition probabilities

- In the case of two neutrinos only:

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4 E_\nu}\right)$$



W.Winter, Nucl.Phys.Proc.Suppl.203-204, 45 (2010)

neutrinos from the Sun

$L/E \sim 10^5$ eV^{-2}

sensitivity to $\Delta m^2 \sim 10^{-5}$ eV^2

atmospheric neutrinos

$L/E \sim 10^3$ eV^{-2}

sensitivity to $\Delta m^2 \sim 10^{-3}$ eV^2