

# Testable Upper Bound on $\rho$

N.N. KHURI



1.

PRE LHC

High Energy  $\equiv$  large  $\frac{\sqrt{s}}{m}$

Post LHC

High Energy  $\equiv \ln\left(\frac{s}{m^2}\right) > 10$ .

(Large  $\ln s/m^2$ )

With large  $\ln s$ , old results and bounds become relevant and useful.

Example

N. N. Khuri and T. Kinoshita:

Phys. Rev. B, Vol. 140, 706-720, (1965)

Some inequalities in the above paper are now testable by LHC.



KK Results

Given a forward scattering ampl.,  
 $f(E)$  with

$$f(E) - f(0) = \frac{2E^2}{\pi} \int_{E_0}^{\infty} \frac{dE' \operatorname{Im} f(E')}{E'(E' - E^2)}, \quad (1.)$$

where  $s = 2m^2 + 2mE$  ( $E$  Lab. sys. Energy)

We define  $g(E)$  as,

$$g(E) = \int_0^E \frac{f(E') - f(0)}{(E')^2} dE' \quad (2.)$$

$\operatorname{Im} E \geq 0$

Theorem:  $g(E)$  is univalent for  $\operatorname{Im} E > 0$ ,  
i.e.  $g$  maps upper half plane,  $\operatorname{Im} E > 0$ ,  
in a one to one manner.

From (Eq. 82) in the KK paper we  
have for  $E_2 \gg E_1$

$$\frac{\operatorname{Re} g(E_2)}{\operatorname{Im} g(E_2)} \leq \frac{\pi}{2} \left[ \frac{1}{\ln \frac{E_2}{E_1} - 4 \ln 2} \right] \quad (3.)$$

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Q.1) A particle is moving with a constant velocity of 10 m/s.

Find its displacement after 5 seconds.



∴ Displacement = Area under the graph  
 = Area of rectangle  
 = Velocity × Time  
 = 10 m/s × 5 s  
 = 50 m

(d)

$$s = vt$$

$$= 10 \times 5$$

$$= 50 \text{ m}$$

Let

$$f \equiv \frac{1}{2} [f(p) + f(\bar{p})] \quad (4.)$$

Then with  $m < E_0 < 10m$

$$\left\{ [f(E) - f(0)] - \frac{2E^2}{\pi} \int_{E_0}^{\infty} dE' \frac{\text{Im} f(E')}{E'(E'^2 - E^2)} \right\} \rightarrow 0 \quad (5.)$$

as  $E \rightarrow \infty$ . (Actually as  $O(\frac{1}{E})$ ).

As in KK we set:

$$g(E) \equiv \int_0^E dE' \frac{(f(E') - f(0))}{(E')^2} \quad (6.)$$

;  $\text{Im} E > 0$ .

For  $E \gg m$ ,  $m < E_0 < 10m$ , we define  $\tilde{g}$

$$\tilde{g}(E) \equiv \int_{E_0}^E dE' \frac{f(E')}{(E')^2} \quad (7.)$$

Then,

$$(\tilde{g} - g) \rightarrow 0, \text{ as } E \rightarrow \infty. \quad (8.)$$

$$\tilde{g} - g = O(\frac{1}{E}).$$

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For  $E \gg E_1$ , we have (as is the KK case)

$$\frac{\text{Re } \tilde{g}(E)}{\text{Im } \tilde{g}(E)} \leq \frac{\pi}{2} \left\{ \frac{1}{\left[ \ln \frac{E}{E_1} - 4 \ln 2 \right]} \right\} \quad (9)$$

$$\frac{\text{Re } \tilde{g}(E)}{\text{Im } \tilde{g}(E)} = \frac{\int_{E_1}^E \text{Re } f(E') dE'/E'^2}{\int_{E_1}^E \text{Im } f(E') dE'/E'^2} \quad ; \quad (10)$$

$$\begin{aligned} \text{Re } f &= \rho \text{Im } f & (11) \\ \text{Im } f &= c E \sigma(E) \end{aligned}$$

$$\rightarrow \frac{\text{Re } \tilde{g}}{\text{Im } \tilde{g}} = \frac{\int_{E_1}^E \rho(E') \sigma(E') dE'/E'}{\int_{E_1}^E \frac{\sigma(E')}{E'} dE'} \quad \begin{array}{l} (c \\ \text{cancels out!}) \end{array} \quad (12.)$$

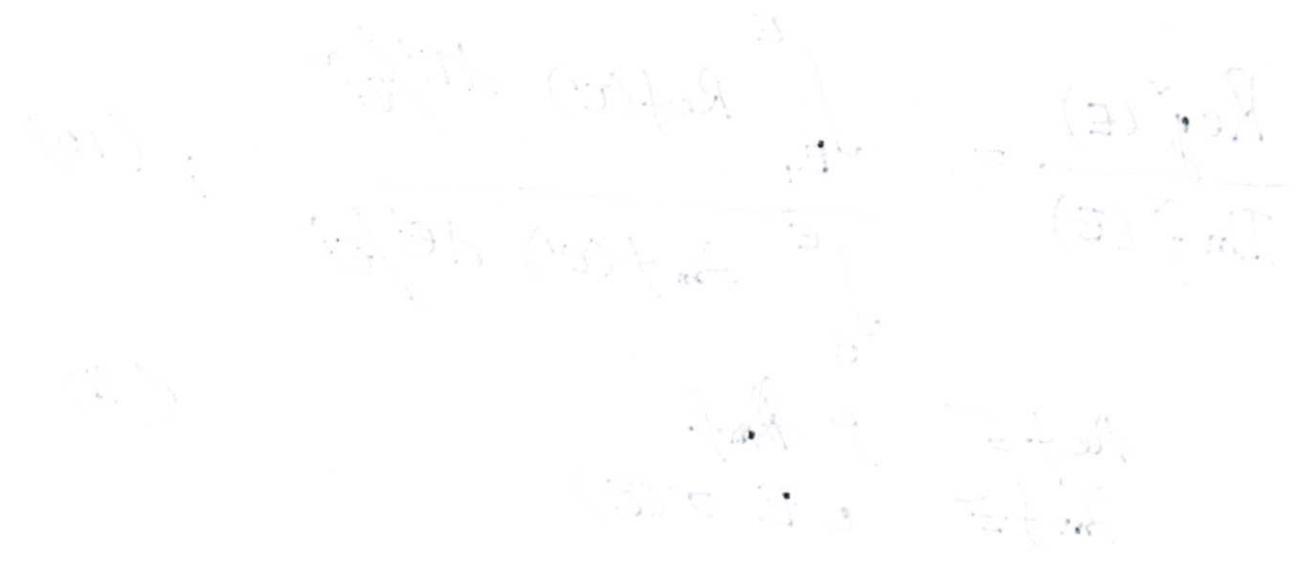
$$\leq \frac{\pi}{2} \left\{ \frac{1}{\left[ \ln \left( \frac{E}{E_1} \right) - 4 \ln 2 \right]} \right\} \quad E_1 \gg m.$$

This bounds  $\rho$  and  $\sigma$ .

(1)

für  $E \in \mathbb{R}$ ,  $\text{det}(A - E \cdot I)$

$$\text{det}(A - E \cdot I) = \begin{vmatrix} 1-E & 1 \\ 1 & 1-E \end{vmatrix} = (1-E)^2 - 1 = E^2 - 2E$$



$$\text{det}(B - E \cdot I) = \begin{vmatrix} 1-E & -1 \\ -1 & 1-E \end{vmatrix} = (1-E)^2 - 1 = E^2 - 2E$$

Das ist die charakteristische Gleichung

With  $\sigma \approx b(\ln E)^2$  ,

( $E, E'$  are  $E/m$  with  $m=1$ )

$$\frac{\text{Re} \tilde{g}}{\text{Im} \tilde{g}} \approx \frac{b \int_{E_1}^E \rho(E') (\ln E')^2 \frac{dE'}{E'}}{b \int_{E_1}^E \frac{dE'}{E'} (\ln E')^2} \quad (13.)$$

$$\frac{\text{Re} \tilde{g}}{\text{Im} \tilde{g}} \approx \frac{3}{(\ln E)^3} \int_{E_1}^E \rho(E') (\ln E')^2 \frac{dE'}{E'} \quad (14.)$$

Hence, from Eq. (12.)

$$\frac{3}{(\ln E)^3} \int_{E_1}^E \rho(E') (\ln E')^2 \frac{dE'}{E'} \leq \frac{\pi}{2} \left[ \frac{1}{\ln \frac{E}{E_1} - 4 \ln 2} \right]$$

(15.)

Finally if  $\rho \approx \frac{\beta}{\ln E/E_1}$   $E \gg E_1$

We get  $\beta < \pi/3$  .

(16.)  
 $s = 2mE$

