Testable Upper Bound on ρ

N. N. KHURI

Pre LHC

High Energy $\equiv$ large $\frac{\sqrt{s}}{m}$

Post LHC

High Energy $\equiv \ln\left(\frac{s}{m^2}\right) > 10$.

(Large $\ln s/m^2$)

With large lns, old results and bounds become relevant and useful.

Example

N. N. Khuri and T. Kinoshita:
Phys. Rev. B, Vol. 140, 706–720, (1965)

Some inequalities in the above paper are now testable by LHC.

KK Results

Given a forward scattering ampl..

$f(E)$ with

$$f(E)-f(0)=\frac{2E^2}{\pi}\int_{E_0}^{\infty} dE' \frac{\operatorname{Im} f(E')}{E'(E'-E^2)}, \quad (1.)$$

where $\boxed{s = 2m^2+2mE}$ (E Lab. sys. Energy)

We define $g(E)$ as,

$$g(E) = \int_0^E \frac{f(E')-f(0)}{(E')^2} dE' \quad (2.)$$

$$\operatorname{Im} E \geq 0$$

Theorem: $g(E)$ is univalent for $\operatorname{Im} E > 0$, i.e. g maps upper half plane, $\operatorname{Im} E > 0$, in a one to one manner.

From (Eq. 82) in the KK paper we have for $E_2 >> E_1$

$$\frac{\operatorname{Re} g(E_2)}{\operatorname{Im} g(E_2)} \leq \frac{\pi}{2}\left[\frac{1}{\ln \frac{E_2}{E_1} - 4\ln 2}\right] \quad (3.)$$

Let

$$f \equiv \frac{1}{2}\left[f(pp) + f(\bar{p}p) \right] \qquad (4.)$$

Then with $m < E_0 < 10\,m$

$$\left\{ \left[f(E) - f(0) \right] - \frac{2E^2}{\pi} \int_{E_0}^{\infty} dE' \frac{\operatorname{Im} f(E')}{E'(E'^2 - E^2)} \right\} \to 0 \qquad (5.)$$

as $E \to \infty$. (Actually as $O(\frac{1}{E})$).

As in KK we set:

$$g(E) \equiv \int_0^E dE' \frac{(f(E') - f(0))}{(E')^2} \qquad (6.)$$

$; \operatorname{Im} E > 0$.

For $E >> m$, $m < E_1 < 10\,m$, we define $\tilde{g}$

as

$$\tilde{g}(E) \equiv \int_{E_1}^{E} dE' \frac{f(E')}{(E')^2}. \qquad (7.)$$

Then, $|\tilde{g} - g| \to 0$, as $E \to \infty$.

$$\tilde{g} - g = O(\tfrac{1}{E}). \qquad (8.)$$

For $E \gg E_1$, we have (as is the KK case)

$$\frac{\mathrm{Re}\,\tilde{g}(E)}{\mathrm{Im}\,\tilde{g}(E)} \le \frac{\pi}{2}\left\{\frac{1}{\left[\ln\frac{E}{E_1} - 4\ln 2\right]}\right\} \quad (9)$$

$$\frac{\mathrm{Re}\,\tilde{g}(E)}{\mathrm{Im}\,\tilde{g}(E)} = \frac{\int_{E_1}^{E} \mathrm{Re}\,f(E')\,dE'/E'^2}{\int_{E_1}^{E} \mathrm{Im}\,f(E')\,dE'/E'^2} \;; \quad (10)$$

$$\mathrm{Re}\,f = \rho\,\mathrm{Im}\,f \qquad (11)$$
$$\mathrm{Im}\,f = c\,E\,\sigma(E)$$

$\rightarrow$

$$\frac{\mathrm{Re}\,\tilde{g}}{\mathrm{Im}\,\tilde{g}} = \frac{\int_{E_1}^{E} \rho(E')\sigma(E')\,dE'/E'}{\int_{E_1}^{E} \frac{\sigma(E')}{E'}\,dE'} \qquad (c \text{ cancels out}) \quad (12.)$$

$$\le \frac{\pi}{2}\left\{\frac{1}{\left[\ln\left(\frac{E}{E_1}\right) - 4\ln 2\right]}\right\} \qquad E_1 \gg m.$$

This bounds ρ and σ.

With $\sigma \cong b(\ln E)^2$,

(E, E' are E/m with m=1)

$$\frac{\mathrm{Re}\,\tilde{g}}{\mathrm{Im}\,\tilde{g}} \cong \frac{b\int_{E_1}^{E} \rho(E')\,(\ln E')^2 \frac{dE'}{E'}}{b\int_{E_1}^{E} \frac{dE'}{E'}\,(\ln E')^2} \qquad (13.)$$

$$\frac{\mathrm{Re}\,\tilde{g}}{\mathrm{Im}\,\tilde{g}} \cong \frac{3}{(\ln E)^3} \int_{E_1}^{E} \rho(E')\,(\ln E')^2 \frac{dE'}{E'} \qquad (14.)$$

Hence, from Eq. (12.)

$$\boxed{\frac{3}{(\ln E)^3} \int_{E_1}^{E} \rho(E')\,(\ln E')^2 \frac{dE'}{E'} \le \frac{\pi}{2}\left[\frac{1}{\ln\frac{E}{E_1} - 4\ln 2}\right]} \qquad (15.)$$

Finally if $\rho \cong \dfrac{\beta}{\ln E/E_1}$ $\qquad E >> E_1$

(16.)

We get $\beta < \pi/3$.

$s = 2mE$