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TORSION GRAVITY WITH SPINORS: AN OVERVIEW

University of Bologna, the 17th of December 2013

Relativity in general requires a connection; connections in general are not symmetric: so the tensor

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The Principle of Equivalence may be used to constrain torsion, but in doing so one may only get torsion to be completely antisymmetric.

If torsion is present beside the metric in metric-compatible connections, then metric and connections are independent; analogously tetrads and spin-connection

$$\omega^{i}{}_{p\alpha} = e^{i}_{\sigma}(\Gamma^{\sigma}_{\rho\alpha} e^{\rho}_{p} + \partial_{\alpha} e^{\sigma}_{p})$$

are independent variables: the torsion and curvature tensor

$$G^{\mu}{}_{\rho\sigma\pi} = \partial_{\sigma}\Gamma^{\mu}_{\rho\pi} - \partial_{\pi}\Gamma^{\mu}_{\rho\sigma} + \Gamma^{\mu}_{\lambda\sigma}\Gamma^{\lambda}_{\rho\pi} - \Gamma^{\mu}_{\lambda\pi}\Gamma^{\lambda}_{\rho\sigma}$$

or equivalently

$$Q^{i}{}_{\alpha\rho} = -\left(\partial_{\alpha} e^{i}_{\rho} - \partial_{\rho} e^{i}_{\alpha} + e^{p}_{\rho}\omega^{i}{}_{p\alpha} - e^{p}_{\alpha}\omega^{i}{}_{p\rho}\right)$$
$$G^{a}{}_{b\sigma\pi} = \partial_{\sigma}\omega^{a}_{b\pi} - \partial_{\pi}\omega^{a}_{b\sigma} + \omega^{a}_{j\sigma}\omega^{j}_{b\pi} - \omega^{a}_{j\pi}\omega^{j}_{b\sigma}$$

are the fundamental variables of the geometrical underlying background.

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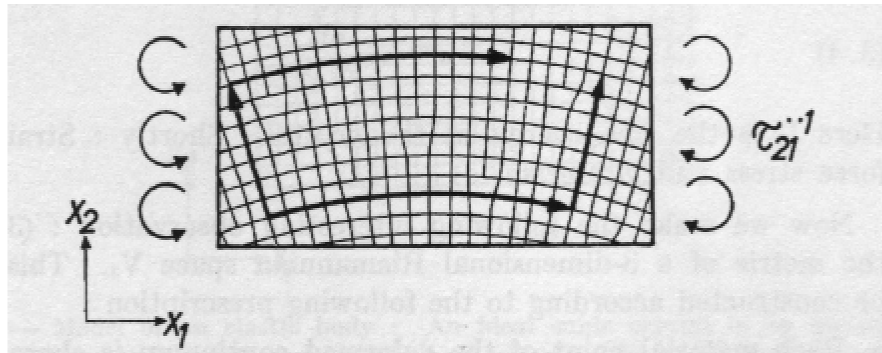
are the fundamental variables of the geometrical underlying background.

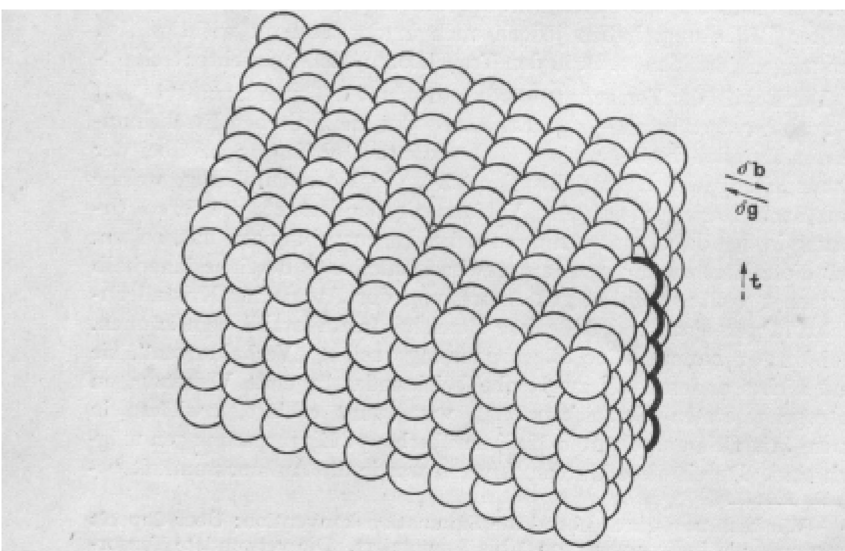
In this geometric background the commutators of covariant derivatives is

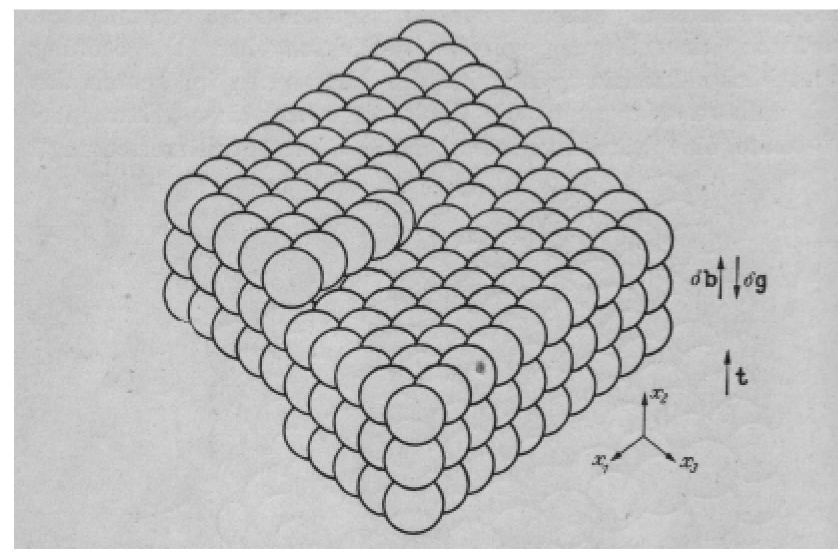
$$[D_{\sigma}, D_{\pi}]V^{\mu} = Q^{\theta}{}_{\sigma\pi} D_{\theta} V^{\mu} + G^{\mu}{}_{\rho\sigma\pi} V^{\rho}$$

so not even for scalars is the commutator of covariant derivatives zero.

The infinitesimal parallelogram does not close, as torsion produces disclination likewise the curvature produces dislocation, when a generic tensor is moved around







So there are many interpretations for torsion and curvature: in particular for the spacetime the Principle of Equivalence provides a natural interpretation for the curvature, but there is no such principle providing a similar interpretation for torsion.

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For the spacetime, torsion is model-dependent, but there is a model that is priviledged with respect to all others.

Field equations coupling
geometrical quantities
to material ones

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The Einstein gravity is based on the assumption that curvature is coupled to energy

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = 8\pi k T_{\mu\nu}$$

obtained from the Lagrangian L=R(g) as known.

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The Sciama-Kibble completion of Einstein gravity maintains in the most general case Einstein's spirit with spin coupled to torsion as curvature is coupled to energy

$$Q^{\rho\mu\nu} = -16\pi k S^{\rho\mu\nu}$$

$$G^\mu{}_\nu - \frac{1}{2}\delta^\mu_\nu G - \lambda \delta^\mu_\nu = 8\pi k T^\mu{}_\nu$$

obtained from the Lagrangian L=G(g,Q) as the simplest generalization.

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obtained from the Lagrangian L=G(g,Q) as the simplest generalization.

The Sciama--Kibble-Einstein gravity for completely antisymmetric torsion is

$$Q^{\rho\mu\nu} = -aS^{\rho\mu\nu}$$
$$\frac{b}{2a}\left(\tfrac{1}{4}\delta^\mu_\nu Q^2 - \tfrac{1}{2}Q^{\mu\alpha\sigma}Q_{\nu\alpha\sigma} + D_\rho Q^{\rho\mu}{}_\nu\right) + \left(G^\mu{}_\nu - \tfrac{1}{2}\delta^\mu_\nu G - \lambda\delta^\mu_\nu\right) = \left(\tfrac{b+a}{2}\right)T^\mu{}_\nu$$

obtained from the Lagrangian L=G(g,Q)+Q^2 as it should be.

The Sciama--Kibble-Einstein SKE gravity in the most general case is given by

$$C\left(Q^{\mu\nu\rho}-Q^{\nu\mu\rho}+2Q^{\rho\mu\nu}\right)+B\left(2Q^{\nu\mu\rho}-2Q^{\mu\nu\rho}\right)+A\left(Q^{\nu}g^{\rho\mu}-Q^{\mu}g^{\rho\nu}\right)+$$
$$+\left(Q^{\rho\mu\nu}+Q^{\mu}g^{\rho\nu}-Q^{\nu}g^{\rho\mu}\right)=-S^{\rho\mu\nu}$$

$$C\left(D_{\mu}Q^{\mu\rho\alpha}-D_{\mu}Q^{\rho\mu\alpha}+Q_{\mu}Q^{\mu\rho\alpha}-Q_{\mu}Q^{\rho\mu\alpha}+Q^{\theta\sigma\alpha}Q_{\sigma\theta}{}^{\rho}-\frac{1}{2}Q^{\theta\sigma\pi}Q_{\pi\sigma\theta}g^{\rho\alpha}\right)+$$
$$+B\left(2D_{\mu}Q^{\alpha\rho\mu}+2Q_{\mu}Q^{\alpha\rho\mu}+2Q^{\theta\sigma\alpha}Q_{\theta\sigma}{}^{\rho}-Q^{\rho\theta\sigma}Q^{\alpha}{}_{\theta\sigma}-\frac{1}{2}Q^{\theta\sigma\pi}Q_{\theta\sigma\pi}g^{\rho\alpha}\right)+$$
$$+A\left(-D^{\alpha}Q^{\rho}+D_{\mu}Q^{\mu}g^{\rho\alpha}+\frac{1}{2}Q_{\mu}Q^{\mu}g^{\rho\alpha}\right)+$$
$$+\left(G^{\rho\alpha}-\frac{1}{2}Gg^{\rho\alpha}\right)=\frac{1}{2}T^{\rho\alpha}$$

from lagrangian $\mathcal{L}_G=G+AQ^{\nu}{}_{\nu\mu}Q_{\rho}{}^{\rho\mu}+BQ_{\rho\mu\nu}Q^{\rho\mu\nu}+CQ_{\rho\mu\nu}Q^{\nu\mu\rho}$ as the most general.

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The Jacobi-Bianchi identities can be worked out into the conservation laws

$$D_{\mu}T^{\mu\rho}+Q_{\mu}T^{\mu\rho}-T_{\mu\sigma}Q^{\sigma\mu\rho}+S_{\beta\mu\sigma}G^{\sigma\mu\beta\rho}=0$$
$$D_{\rho}S^{\rho\mu\nu}+Q_{\rho}S^{\rho\mu\nu}+\frac{1}{2}T^{[\mu\nu]}=0$$

valid in general circumstances because of diffeomorphism and Lorentz invariance.

These are verified for instance by the spin and energy density tensors

$$S^{\rho\mu\nu} = \frac{i\hbar}{4}\overline{\psi}\{\boldsymbol{\gamma}^{\rho}, \boldsymbol{\sigma}^{\mu\nu}\}\psi$$
$$T^{\mu}{}_{\nu} = \frac{i\hbar}{2}\left(\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{D}_{\nu}\psi - \boldsymbol{D}_{\nu}\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi\right)$$

once the Dirac matter field equation

$$i\hbar\boldsymbol{\gamma}^{\mu}\boldsymbol{D}_{\mu}\psi - m\psi = 0$$

is assigned in terms of the mass m of the matter field.

Once decomposed we have that gravitational field equations are

$$R_{\mu\nu}+\lambda g_{\mu\nu}=-4\pi km\overline{\psi}\psi g_{\mu\nu}+$$
$$+2i\pi k\hbar\left(\overline{\psi}\boldsymbol{\gamma}_{\mu}\boldsymbol{\nabla}_{\nu}\psi+\overline{\psi}\boldsymbol{\gamma}_{\nu}\boldsymbol{\nabla}_{\mu}\psi-\boldsymbol{\nabla}_{\nu}\overline{\psi}\boldsymbol{\gamma}_{\mu}\psi-\boldsymbol{\nabla}_{\mu}\overline{\psi}\boldsymbol{\gamma}_{\nu}\psi\right)$$

while the Dirac field equations are given by

$$\equiv i\hbar\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-\tfrac{3a}{16}\hbar^{2}\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi\boldsymbol{\gamma}_{\mu}\psi-m\psi\equiv$$
$$\equiv i\hbar\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-\tfrac{3a}{16}\hbar^{2}\left(\overline{\psi}\psi\mathbb{I}-\overline{\psi}\boldsymbol{\gamma}\psi\boldsymbol{\gamma}\right)\psi-m\psi=0$$

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The non-gravitational non-relativistic limit is given by

$$\frac{\hbar^{2}}{2m}\vec{\nabla}\cdot\vec{\nabla}\phi-\frac{9a^{2}\hbar^{4}}{512m}|\phi^{\dagger}\phi|^{2}\phi-\frac{3a\hbar^{2}}{16}\phi^{\dagger}\phi\phi+(E-m)\phi\approx0$$

with non-linear interactions with the form of Ginzburg-Landau potential.

An application to the case of two-particle interactions.

Take two fermions of which one massive and the other massless single-handed

$$i\gamma^{\mu}\nabla_{\mu}e-\frac{G_{F}}{\sqrt{2}}\overline{e}\gamma_{\mu}e\gamma^{\mu}e-\frac{G_{F}}{\sqrt{2}}\overline{\nu}\gamma_{\mu}\nu\gamma^{\mu}\gamma e-me=0$$

$$i\gamma^{\mu}\nabla_{\mu}\nu-\frac{G_{F}}{\sqrt{2}}\overline{e}\gamma_{\mu}\gamma e\gamma^{\mu}\nu=0$$

as for the electron-neutrino leptonic couple.

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These equations can be Fierz rearranged in the form

$$i\gamma^{\mu}\nabla_{\mu}e+G_F q\tan\theta\frac{\sqrt{2}\cos\theta}{g[2\sin\theta)^2}\left[2(\overline{e}_L\gamma_{\mu}e_L-\overline{\nu}\gamma_{\mu}\nu)-(2\sin\theta)^2\overline{e}\gamma_{\mu}e\right]\gamma^{\mu}e-$$

$$-\frac{gG_F}{2\cos\theta}\frac{\sqrt{2}\cos\theta}{g(2\sin\theta)^2}\left[2(\overline{e}_L\gamma_{\mu}e_L-\overline{\nu}\gamma_{\mu}\nu)-(2\sin\theta)^2\overline{e}\gamma_{\mu}e\right]\gamma^{\mu}e_L+$$

$$+\frac{gG_F}{\sqrt{2}}\frac{\left[1-(2\sin\theta)^2\right]}{g(2\sin\theta)^2}\left(2\overline{\nu}\gamma_{\mu}e_L\right)\gamma^{\mu}\nu-$$

$$-Y\frac{G_F}{Y}\sqrt{2}(\cos\theta)^2\overline{e}ee+G_F\sqrt{2}(\cos\theta)^2\overline{e}\gamma e\gamma e-me=0$$

$$i\gamma^{\mu}\nabla_{\mu}\nu+\frac{gG_F}{2\cos\theta}\frac{\sqrt{2}\cos\theta}{g(2\sin\theta)^2}\left[2(\overline{e}_L\gamma_{\mu}e_L-\overline{\nu}\gamma_{\mu}\nu)-(2\sin\theta)^2\overline{e}\gamma_{\mu}e\right]\gamma^{\mu}\nu+$$

$$+\frac{gG_F}{\sqrt{2}}\frac{\left[1-(2\sin\theta)^2\right]}{g(2\sin\theta)^2}\left(2\overline{e}_L\gamma_{\mu}\nu\right)\gamma^{\mu}e_L=0$$

in very general terms.

These field equations can be written in the form

$$i\gamma^{\mu}\nabla_{\mu}e+G_{F}\sqrt{2}(\cos\theta)^{2}\overline{e}\gamma e\gamma e+q\tan\theta Z_{\mu}\gamma^{\mu}e-$$
$$-\tfrac{g}{2\cos\theta}Z_{\mu}\gamma^{\mu}e_{L}+\tfrac{g}{\sqrt{2}}W^{*}_{\mu}\gamma^{\mu}\nu-YHe-me=0$$
$$i\gamma^{\mu}\nabla_{\mu}\nu+\tfrac{g}{2\cos\theta}Z_{\mu}\gamma^{\mu}\nu+\tfrac{g}{\sqrt{2}}W_{\mu}\gamma^{\mu}e_{L}=0$$

so soon as the following quantities

$$Z_{\mu}=G_{F}\frac{\sqrt{2}\cos\theta}{g(2\sin\theta)^{2}}\left[2\left(\overline{e}_{L}\gamma_{\mu}e_{L}-\overline{\nu}\gamma_{\mu}\nu\right)-(2\sin\theta)^{2}\overline{e}\gamma_{\mu}e\right]$$
$$W_{\mu}=G_{F}\frac{\left[1-(2\sin\theta)^{2}\right]}{g(2\sin\theta)^{2}}\left(2\overline{e}_{L}\gamma_{\mu}\nu\right)$$
$$H=\tfrac{G_{F}}{Y}\sqrt{2}(\cos\theta)^{2}\overline{e}e$$

are introduced: so long as the energy is low enough not to probe the bosons internal compositeness, the weak forces are formally those we would have had if they were to come from the standard U(1)XSU(2) gauge mixing among massless leptons occurring in the GWS Model. L. Fabbri, *Int. J. Theor. Phys.* **50**, 3616 (2011).

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Would a mechanism of *dynamical* symmetry breaking in this case produce a cosmological constant problem as in the standard case?

An application to the case of condensates in cosmology.

How reasonable is to have Dirac field condensates at galactic scales? This is not a new idea and it has been used in varius ways

C. G. Boehmer, T. Harko, *JCAP* **0706**, 025 (2007).

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On the grounds of the gravitational field equations, and for the symmetries and approximations valid for galactic systems, one has the field equations

$$\mathrm{div}\vec{a} \approx -\mathrm{div}\ \mathrm{grad}V \approx -R_{tt} \approx 4\pi k\left[m\overline{\psi}\psi - i\hbar\left(\overline{\psi}\boldsymbol{\gamma}^t\boldsymbol{\nabla}_t\psi - \boldsymbol{\nabla}_t\overline{\psi}\boldsymbol{\gamma}^t\psi\right)\right] \approx$$

$$\approx -4\pi k\left[m\overline{\psi}\psi + \frac{3}{8}a\hbar^2\overline{\psi}\psi\overline{\psi}\psi + i\hbar\left(\vec{\boldsymbol{\nabla}}\overline{\psi}\cdot\vec{\boldsymbol{\gamma}}\psi - \overline{\psi}\vec{\boldsymbol{\gamma}}\cdot\vec{\boldsymbol{\nabla}}\psi\right)\right]$$

or also the non-relativistic limit

$$\mathrm{div}\vec{a} \approx -4\pi k(m\phi^\dagger\phi + \frac{3}{8}a\hbar^2\phi^\dagger\phi\phi^\dagger\phi)$$

in terms of the non-relativistic Schroedinger field equation

$$\frac{\hbar^2}{2m}\vec{\nabla}\cdot\vec{\nabla}\phi - \frac{9a^2\hbar^4}{512m}|\phi^\dagger\phi|^2\phi - \frac{3a\hbar^2}{16}\phi^\dagger\phi\phi + (E-m)\phi \approx 0$$

which now has to be solved.

For high-density condensates, centripetal acceleration in spherical coordinates is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 a\right) \approx \frac{3}{2}\pi k a \hbar^2 u^4$$

while for the condensate we get

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)\right] - \frac{9\hbar^2 a^2}{256}u^5 \approx 0$$

for which a solution is given by

$$u = \sqrt{\frac{8}{3\hbar a r\sin\theta}}$$

which can be substituted to give the tangential velocity as

$$v^2 \approx \frac{32\pi k}{3a}$$

mimicking Dark Matter behaviour. Luca Fabbri, arXiv:1211.3837 [gr-qc].

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The simplest case is given by the action

$$S_{\text{gravity}} = \int \frac{1}{4}(3kW^{\alpha}W_{\alpha}W^{\rho}W_{\rho} + C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu})\sqrt{|g|}dV$$

The conservation laws are the same as above but supplemented by the tracelessness of the energy density of the matter field, as usual.

These are verified for instance by the spin and energy density tensors

$$S^{\rho\mu\nu} = \frac{i\hbar}{4}\overline{\psi}\{\boldsymbol{\gamma}^{\rho}, \boldsymbol{\sigma}^{\mu\nu}\}\psi$$

$$T^{\mu}{}_{\nu} = \frac{i\hbar}{2}\left(\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{D}_{\nu}\psi - \boldsymbol{D}_{\nu}\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi\right)$$

once the Dirac matter field equation

$$i\hbar\boldsymbol{\gamma}^{\mu}\boldsymbol{D}_{\mu}\psi = 0$$

is assigned for massless matter fields.

An application to the case of the standard model.

Suppose we couple it to the scalar field, then it turns out that the scalar couples to the gravitational degrees of freedom according to the action

$$S = \int [\tfrac{3}{4}kW^\alpha W_\alpha W^\mu W_\mu + \tfrac{1}{4}C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu} - \tfrac{1}{4}F_{\alpha\beta}F^{\alpha\beta} +$$
$$+\tfrac{i}{2}(\overline{\psi}\boldsymbol{\gamma}^\rho\boldsymbol{\nabla}_\rho\psi - \boldsymbol{\nabla}_\rho\overline{\psi}\boldsymbol{\gamma}^\rho\psi) - \tfrac{3}{4}\overline{\psi}\boldsymbol{\gamma}_\rho\boldsymbol{\gamma}\psi W^\rho +$$
$$+\nabla_\rho\phi\nabla^\rho\phi + \tfrac{1}{6}R\phi^2 - pW_\alpha W^\alpha\phi^2 - \tfrac{\lambda}{8}\phi^4 - Y\overline{\psi}\psi\phi]|e|dV$$

Field equations for the geometrical background sector are

$$kW^2W_\rho = \frac{1}{4}\overline{\psi}\boldsymbol{\gamma}_\rho\boldsymbol{\gamma}\psi + \frac{2p}{3}\phi^2W_\rho$$

$$\begin{aligned}
&-kW^2(\tfrac{1}{4}g^{\mu\alpha}W^2 - W^\mu W^\alpha) + (\tfrac{1}{4}g^{\mu\alpha}C^2 - C^{\theta\sigma\rho\mu}C_{\theta\sigma\rho}{}^{\alpha}) - \\
&-(C^{\mu\beta\alpha\nu}R_{\beta\nu} + 2\nabla_\beta\nabla_\nu C^{\mu\beta\alpha\nu}) - (\tfrac{1}{4}g^{\mu\alpha}F^2 - F^{\mu\rho}F^{\alpha}{}_{\rho}) = \\
&= \tfrac{i}{4}(\overline{\psi}\boldsymbol{\gamma}^\mu\boldsymbol{\nabla}^\alpha\psi - \boldsymbol{\nabla}^\alpha\overline{\psi}\boldsymbol{\gamma}^\mu\psi + \overline{\psi}\boldsymbol{\gamma}^\alpha\boldsymbol{\nabla}^\mu\psi - \boldsymbol{\nabla}^\mu\overline{\psi}\boldsymbol{\gamma}^\alpha\psi) + \\
&+\tfrac{1}{4}(\tfrac{1}{2}\overline{\psi}\boldsymbol{\gamma}^\alpha\boldsymbol{\gamma}\psi W^\mu + \tfrac{1}{2}\overline{\psi}\boldsymbol{\gamma}^\mu\boldsymbol{\gamma}\psi W^\alpha - g^{\alpha\mu}\overline{\psi}\boldsymbol{\gamma}^\rho\boldsymbol{\gamma}\psi W_\rho) + \\
&+2(\nabla^\alpha\phi\nabla^\mu\phi - \tfrac{1}{2}g^{\alpha\mu}\nabla_\rho\phi\nabla^\rho\phi) + \tfrac{1}{3}(g^{\alpha\mu}\nabla^2\phi^2 - \nabla^\alpha\nabla^\mu\phi^2) + \\
&+\tfrac{1}{3}(R^{\alpha\mu} - \tfrac{1}{2}g^{\alpha\mu}R)\phi^2 + \tfrac{2p}{3}(W^\alpha W^\mu + \tfrac{1}{2}g^{\alpha\mu}W^2)\phi^2 + g^{\alpha\mu}\tfrac{\lambda}{8}\phi^4
\end{aligned}$$

$$\nabla_\rho F^{\rho\mu} = q\overline{\psi}\boldsymbol{\gamma}^\mu\psi$$

Field equations for the material sector are

$$i\gamma^{\mu}\boldsymbol{\nabla}_{\mu}\psi - \frac{3}{4}W_{\sigma}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\gamma}\psi - Y\phi\psi = 0$$

$$\nabla^2\phi - \frac{1}{6}R\phi + pW^2\phi + \frac{\lambda}{4}\phi^2\phi + \frac{Y}{2}\overline{\psi}\psi = 0$$

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The vacuum of the scalar is no longer an absolute constant.

Define the quantity $\Delta \equiv 4\frac{729k\overline{\psi}\boldsymbol{\gamma}_i\psi\overline{\psi}\boldsymbol{\gamma}^i\psi}{512p^3\phi^6}(1+\frac{729k\overline{\psi}\boldsymbol{\gamma}_i\psi\overline{\psi}\boldsymbol{\gamma}^i\psi}{512p^3\phi^6})$

torsion may be inverted as

$$W_\rho = -\frac{9}{8p\phi^2\left[1+u[(1+\Delta)^{\frac{1}{2}}+\Delta^{\frac{1}{2}}]^{\frac{1}{3}}+u^*[(1+\Delta)^{\frac{1}{2}}-\Delta^{\frac{1}{2}}]^{\frac{1}{3}}\right]}\overline{\psi}\boldsymbol{\gamma}_\rho\boldsymbol{\gamma}\psi$$

and substituted everywhere in order to give the geometric field equations

$$\begin{gathered}(\tfrac{1}{4}g_{\mu\alpha}C^2 - C^{\theta\sigma\rho}{}_{\mu}C_{\theta\sigma\rho\alpha} - C_{\mu\beta\alpha\nu}R^{\beta\nu} - 2\nabla^\beta\nabla^\nu C_{\mu\beta\alpha\nu}) - \\ -(\tfrac{1}{4}g_{\mu\alpha}F^2 - F_{\mu\rho}F_\alpha{}^\rho) = \tfrac{i}{4}\left(\overline{\psi}\boldsymbol{\gamma}_{(\mu}\boldsymbol{\nabla}_{\alpha)}\psi - \boldsymbol{\nabla}_{(\alpha}\overline{\psi}\boldsymbol{\gamma}_{\mu)}\psi\right) + \\ +2(\nabla_\alpha\phi\nabla_\mu\phi - \tfrac{1}{2}g_{\alpha\mu}\nabla_\rho\phi\nabla^\rho\phi) + \tfrac{1}{3}(g_{\alpha\mu}\nabla^2\phi^2 - \nabla_\alpha\nabla_\mu\phi^2) + \\ +\tfrac{1}{3}(R_{\alpha\mu} - \tfrac{1}{2}g_{\alpha\mu}R)\phi^2 + \\ +g_{\alpha\mu}[\tfrac{\lambda}{8}+\tfrac{p^2}{27k}[4+u[(1+\Delta)^{\frac{1}{2}}+\Delta^{\frac{1}{2}}]^{\frac{1}{3}}+u^*[(1+\Delta)^{\frac{1}{2}}-\Delta^{\frac{1}{2}}]^{\frac{1}{3}}]\cdot \\ \cdot[2-u[(1+\Delta)^{\frac{1}{2}}+\Delta^{\frac{1}{2}}]^{\frac{1}{3}}-u^*[(1+\Delta)^{\frac{1}{2}}-\Delta^{\frac{1}{2}}]^{\frac{1}{3}}]]\phi^4\end{gathered}$$

$$\nabla_\rho F^{\rho\mu} = q\overline{\psi}\boldsymbol{\gamma}^\mu\psi$$

And also the material field equations

$$i\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-\frac{27}{32p\phi^{2}\left[1+u[(1+\Delta)^{\frac{1}{2}}+\Delta^{\frac{1}{2}}]^{\frac{1}{3}}+u^{*}[(1+\Delta)^{\frac{1}{2}}-\Delta^{\frac{1}{2}}]^{\frac{1}{3}}\right]}\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\boldsymbol{\gamma}^{\rho}\psi-Y\phi\psi=0$$

$$\nabla^{2}\phi-[\tfrac{1}{6}R+\frac{81\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{64p\phi^{4}\left[1+u[(1+\Delta)^{\frac{1}{2}}+\Delta^{\frac{1}{2}}]^{\frac{1}{3}}+u^{*}[(1+\Delta)^{\frac{1}{2}}-\Delta^{\frac{1}{2}}]^{\frac{1}{3}}\right]^{2}}-\frac{\lambda}{4}\phi^{2}]\phi+\frac{Y}{2}\overline{\psi}\psi=0$$

with vacuum $$\left.\left[\frac{\lambda}{2}\phi^{2}-\frac{81\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{32p\phi^{4}\left[1+u[(1+\Delta)^{\frac{1}{2}}+\Delta^{\frac{1}{2}}]^{\frac{1}{3}}+u^{*}[(1+\Delta)^{\frac{1}{2}}-\Delta^{\frac{1}{2}}]^{\frac{1}{3}}\right]^{2}}-\frac{1}{3}R\right]\right|_{\mathrm{v}}=0$$

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The vacuum of the scalar is strictly linked through torsion to the vacuum of the spinor fields: even in absence of gravity, the two vacua are related.

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Focus on non-gravitational limits, we study the high- and low- density cases.

High-density case: $i\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi - \left(\frac{27}{256k\overline{\psi}\boldsymbol{\gamma}_i\psi\overline{\psi}\boldsymbol{\gamma}^i\psi}\right)^{\frac{1}{3}}\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\boldsymbol{\gamma}^{\rho}\psi - Yv\psi \approx 0$

with vacuum $\lambda v^2 \equiv \left(\frac{4p^3\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{k^2}\right)^{\frac{1}{3}}\Bigg|_{\mathrm{v}}$

$$M^2 \equiv \left(\frac{p^3\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{2k^2}\right)^{\frac{1}{3}}\Bigg|_{\mathrm{v}} \quad \text{and} \quad \Lambda \equiv \left(\frac{p^3\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{16k^2\sqrt{\lambda}^3}\right)^{\frac{2}{3}}\Bigg|_{\mathrm{v}}$$

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Low-density case: $i\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi - \frac{9}{32pv^2}\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\boldsymbol{\gamma}^{\rho}\psi - Yv\psi \approx 0$

with vacuum $\lambda v^6 \equiv \frac{9\overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{16p}\Bigg|_{\mathrm{v}}$

$$M^2 \equiv \left(\frac{9\lambda^2\ \overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{128p}\right)^{\frac{1}{3}}\Bigg|_{\mathrm{v}} \quad \text{and} \quad \Lambda \equiv \left(\frac{9\sqrt{\lambda}\ \overline{\psi}\boldsymbol{\gamma}_{\rho}\psi\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi}{1024p}\right)^{\frac{2}{3}}\Bigg|_{\mathrm{v}}$$

High-density case: $i\boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \psi - \left(\frac{27}{256k\overline{\psi}\boldsymbol{\gamma}_i\psi\overline{\psi}\boldsymbol{\gamma}^i\psi}\right)^{\frac{1}{3}} \overline{\psi}\boldsymbol{\gamma}_\rho\psi\boldsymbol{\gamma}^\rho\psi - Yv\psi \approx 0$

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Different values for the generated mass and cosmological constant.

Summary:

1. torsion allows to couple the spin of fermions;
2. non-linearities typical of condensed matter should be taken as fundamental;
3. applications to:
    - I. Leptonic condensation forming massive weak vector mediators,
    - II. WIMPs condensation fitting flat rotation curves,
    - III. Cosmological Constant problem quenched.

"Je t'encourage à continuer, mais si ça s'avérait trop difficile
ne viens pas te plaindre à moi... dans cinquante ans."
F. Englert, La Thuile, Italy, 2013