Towards a model-independent comparison of dark matter direct detection data

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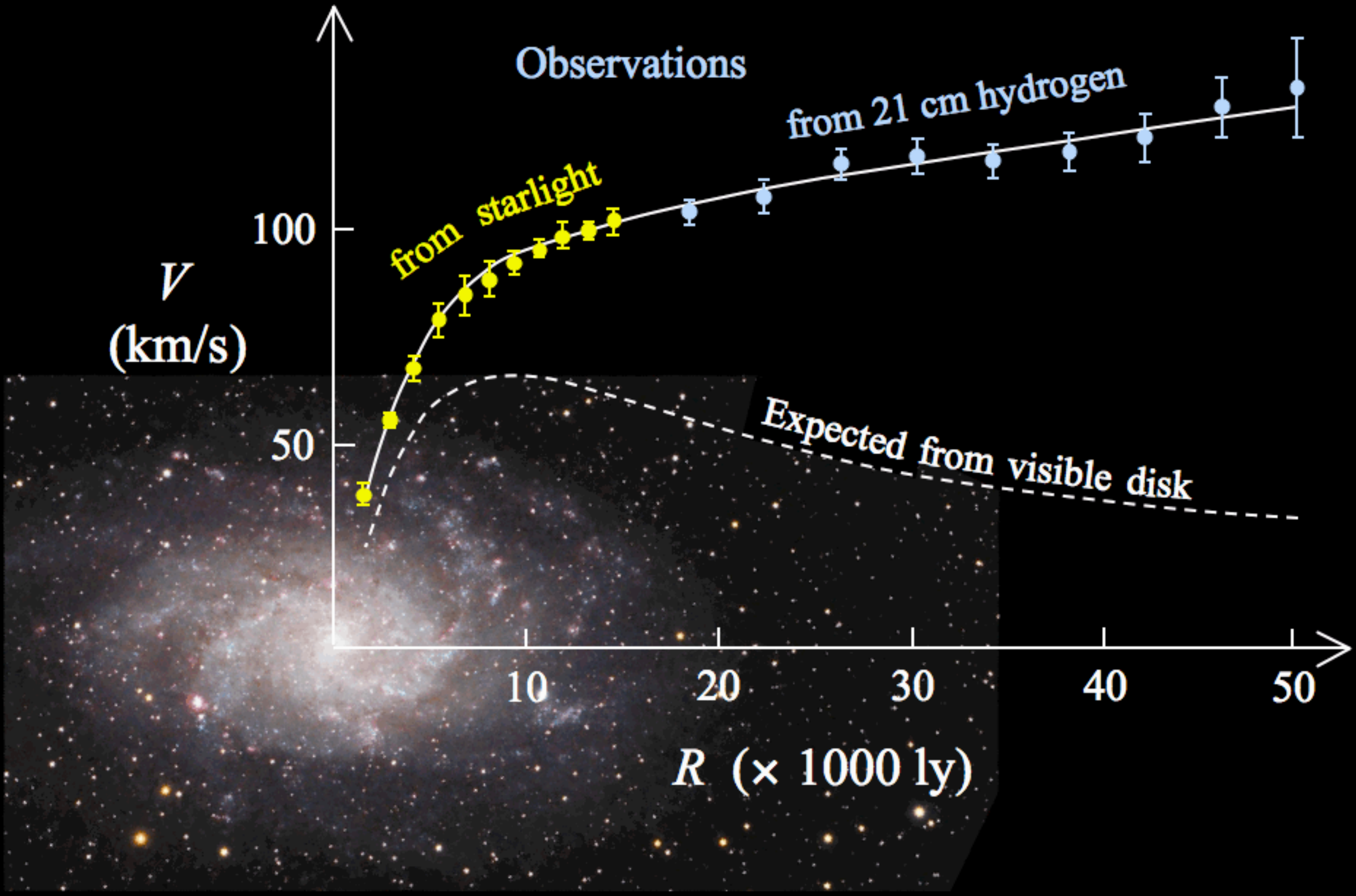
Why do we believe in DM

Because we exist

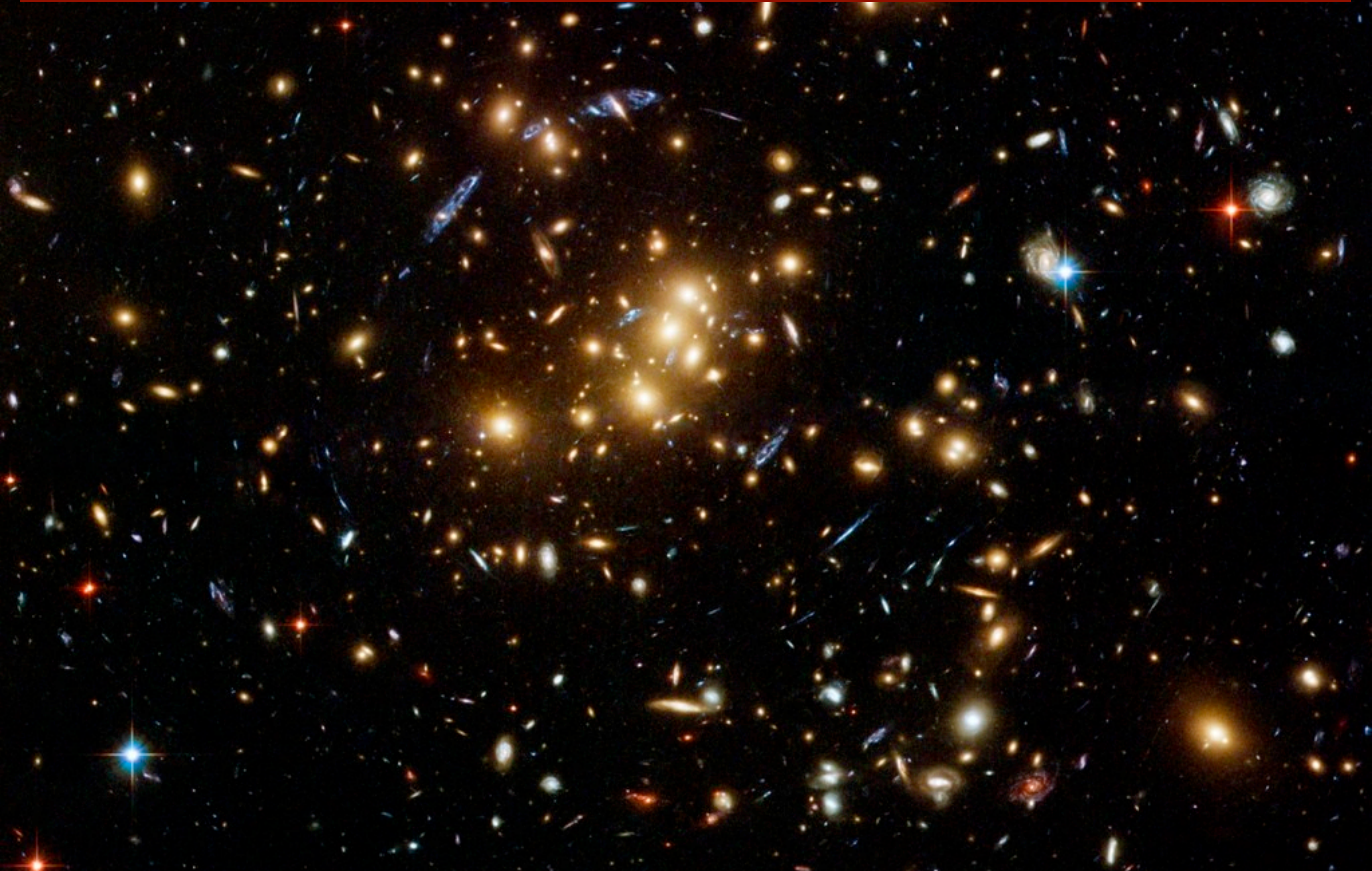
(might sound anthropic, but it's not)

Rotation curves

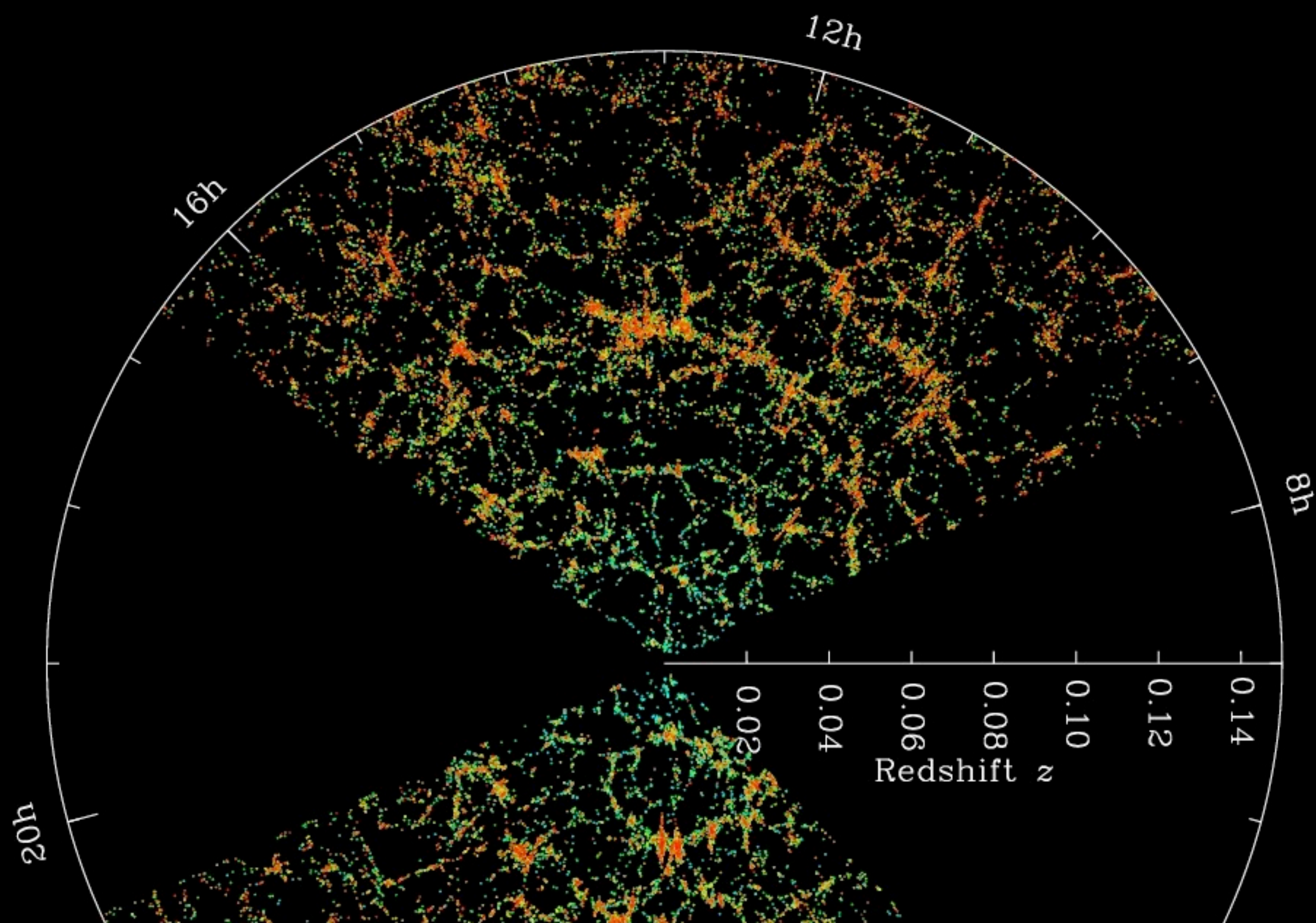


Bullet Cluster

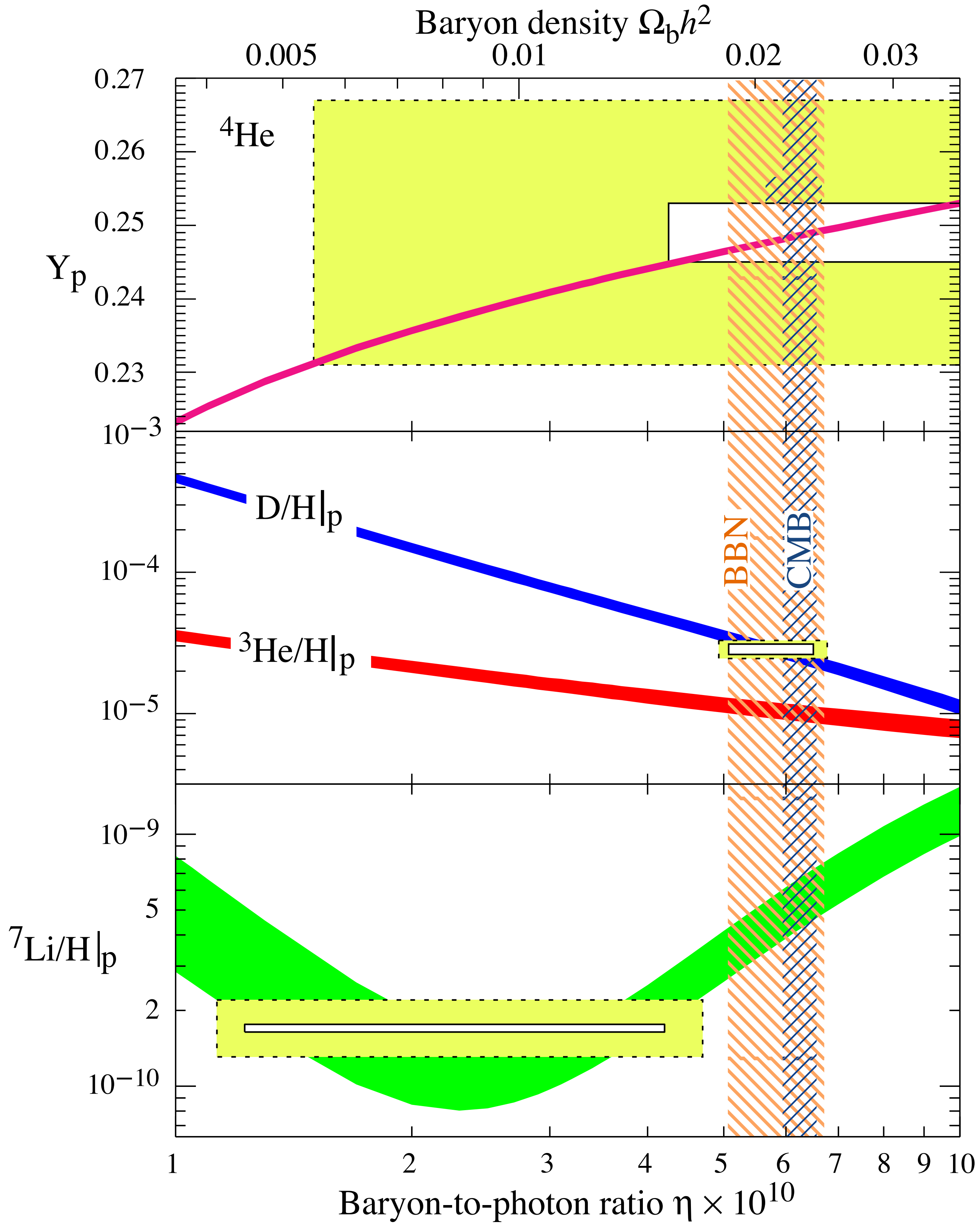
Lensing



Large scale structures

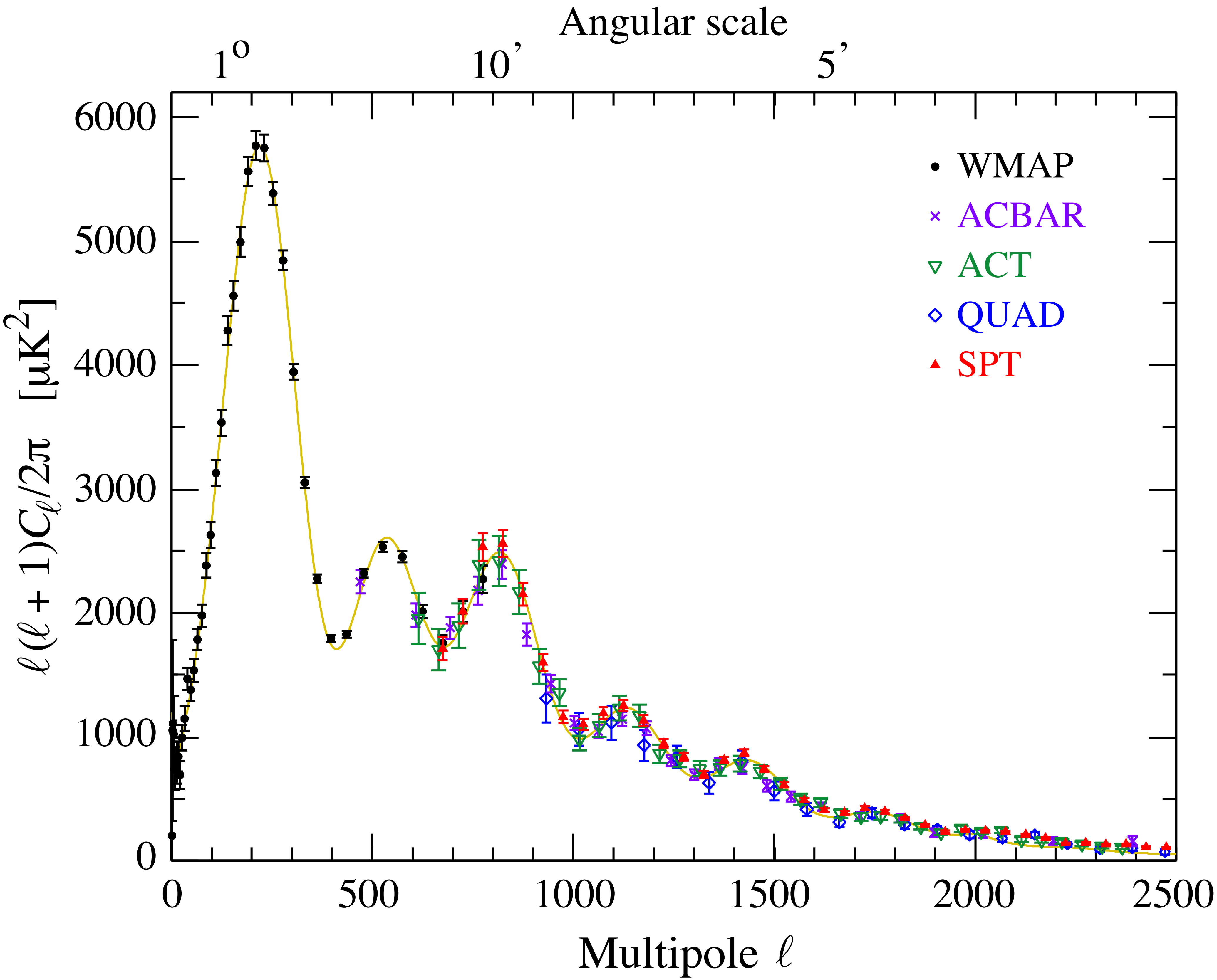


BBN



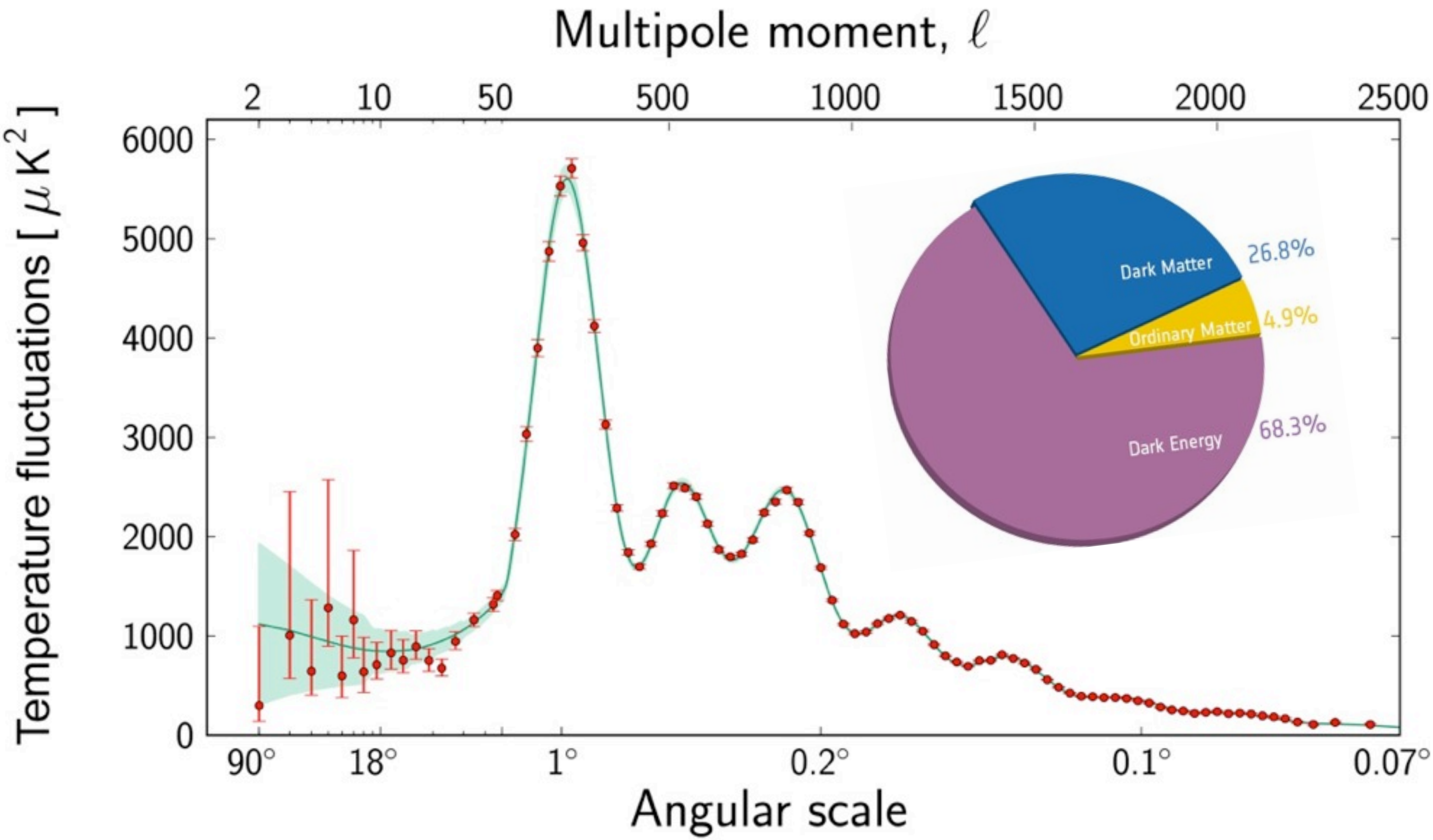
Baryon density $\Omega_b h^2$
0.005
0.01
0.02
0.03
0.27
0.26
0.25
0.24
0.23
^{4}He
Y_p
10^{-3}
10^{-4}
10^{-5}
D/H|p
^{3}He/H|p
BBN
CMB
10^{-9}
5
^{7}Li/H|p
2
10^{-10}
1
2
3
4
5
6
7
8
9
10
Baryon-to-photon ratio $\eta \times 10^{10}$

CMB



Angular scale
1°
10'
5'
6000
5000
4000
3000
2000
1000
0
ℓ(ℓ + 1)Cℓ/2π [μK²]
WMAP
ACBAR
ACT
QUAD
SPT
0
500
1000
1500
2000
2500
Multipole ℓ

CMB as seen by Planck (2013)

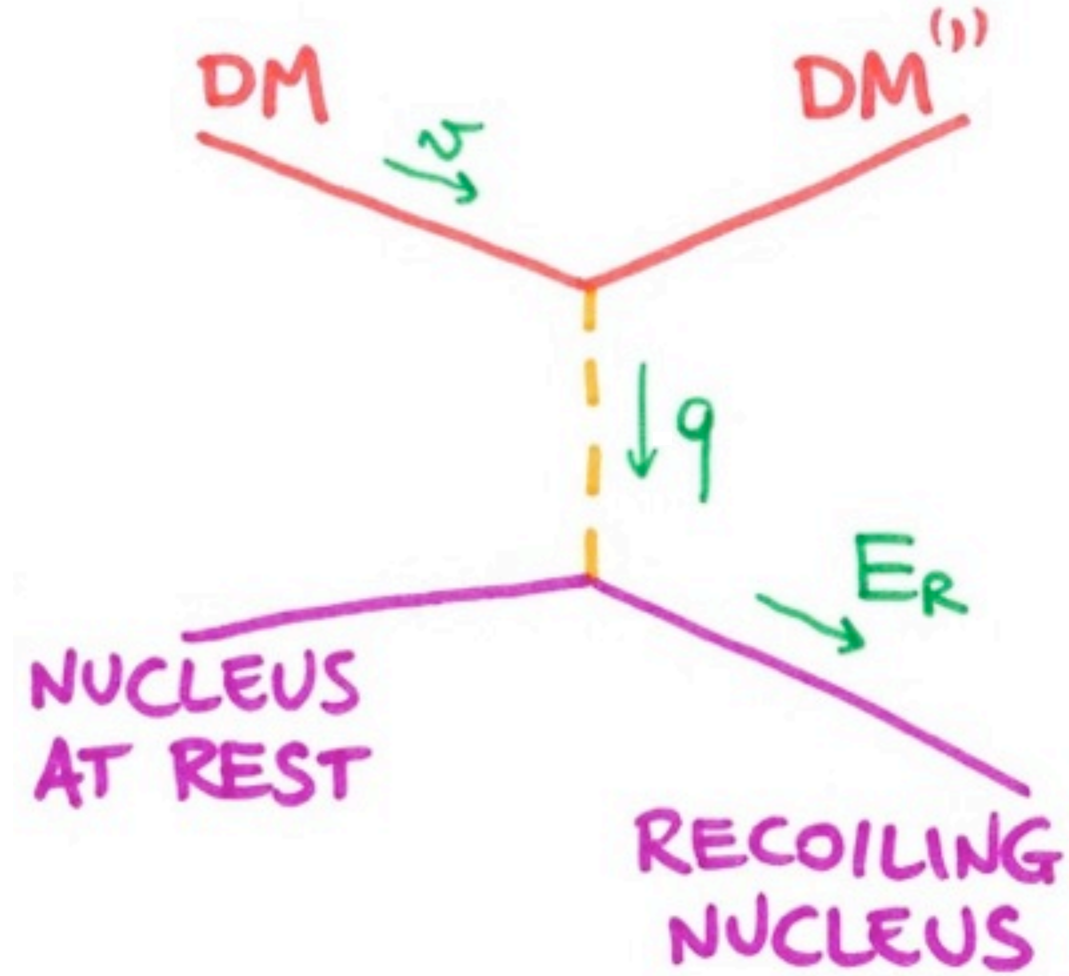


$Assumptions

- Dark Matter (DM) is made of particles
- Actually, only one kind of particle
  (with mass 1 GeV - 10 TeV)
- DM is cold (i.e. non-relativistic)

DM direct detection

Basically explained by



$$\frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}} = n_T \Phi_\mathrm{DM}(t) \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}$$

DM direct detection

The basic ingredient is the recoil rate

$$\frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}} = n_T \Phi_\mathrm{DM}(t) \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}$$

Target density

DM-nucleus cross section

$$\Phi_\mathrm{DM} = \frac{\rho}{m_\mathrm{DM}} v\, f(\vec{v}, t)\, \mathrm{d}^3 v$$

DM flux (has an annual modulation due to Earth's rotation around the sun)

Direct detection rate

$$\frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}} = \frac{\xi_T}{m_T}\frac{\rho}{m_\mathrm{DM}}\int_{v_\mathrm{min}(E_\mathrm{R})}^{v_\mathrm{esc}} \mathrm{d}^3v\, f(\vec{v},t)\, v\, \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}(E_\mathrm{R},\vec{v}\,)$$

ρ DM local density

ξ_T target mass fraction

$f(\vec{v},t)$ DM velocity distribution

v_esc galactic escape velocity

$$R_{[E_1',E_2']}(t) = \sum_T \int_{E_1'}^{E_2'} \mathrm{d}E'\, \epsilon_1(E') \int_0^\infty \mathrm{d}E_\mathrm{R}\, \epsilon_2(E_\mathrm{R}) G_T(E', qE_\mathrm{R}) \frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}}$$

E' detected energy

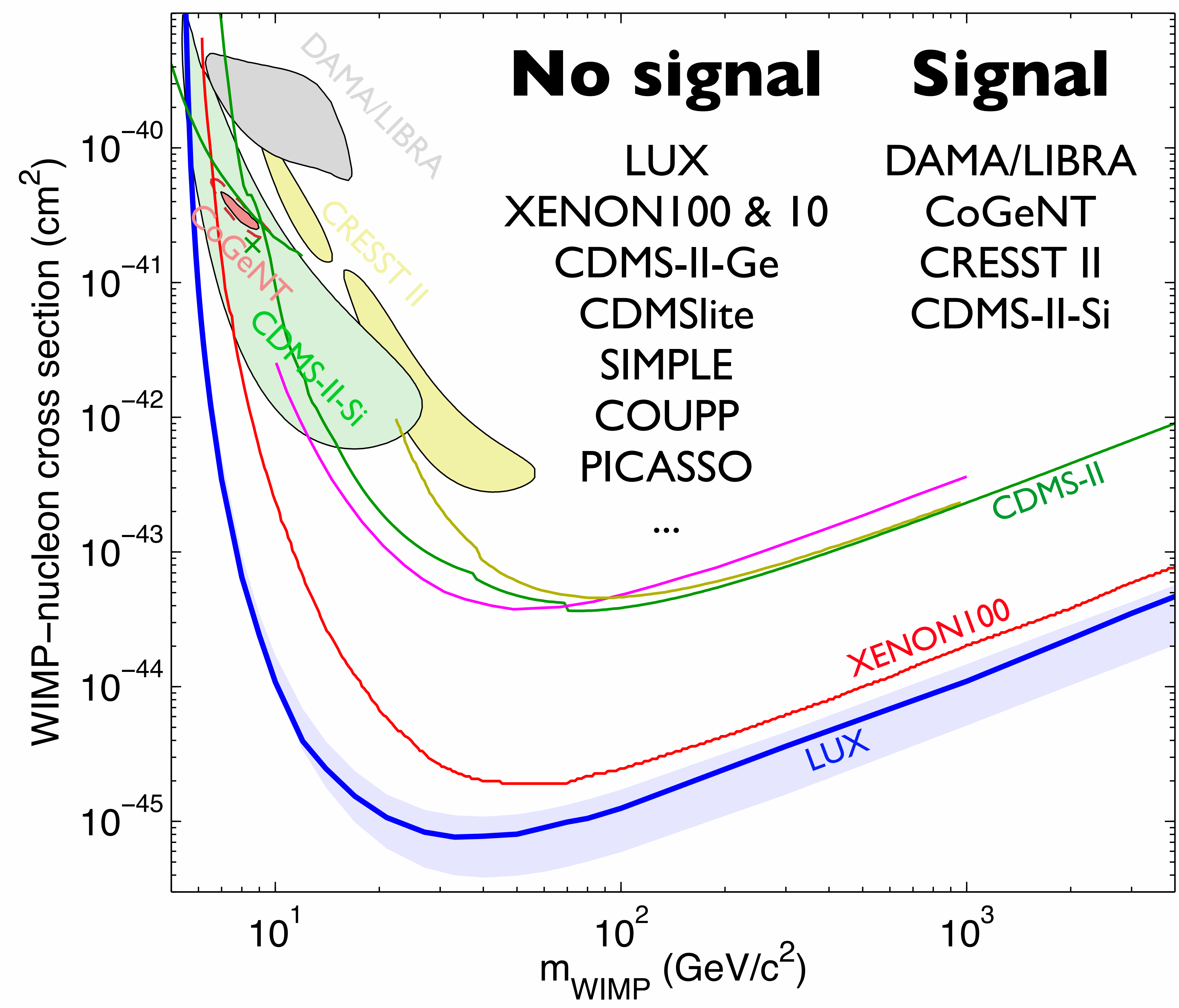
$\epsilon_1,\ \epsilon_2$ efficiencies

$G_T(E', qE_\mathrm{R})$ detector resolution

q quenching factor

Limits and fits

updated Dec 2013



Assumptions

- Spin-independent interaction:
  in QFT language,

$$\mathscr{L} \propto \quad \phi^\dagger \phi \, \bar{N} N \quad \text{or} \quad \bar{\chi}\chi \, \bar{N} N \quad \text{or} \quad \bar{\chi}\gamma^\mu \chi \, \bar{N}\gamma_\mu N$$

(in the NR limit all these interactions look the same)

- Truncated Maxwell-Boltzmann velocity distribution (Standard Halo Model, or SHM):

$$f(\vec{v},t) = f_{\rm G}(\vec{u} = \vec{v} + \vec{v}_{\rm E}(t)) \quad \textbf{with} \quad f_{\rm G}(\vec{u}\,) = \frac{\exp(-u^2/v_0^2)}{(v_0\sqrt{\pi})^3 N_{\rm esc}}\,\theta(v_{\rm esc} - u)$$

Other interactions

One can imagine countless other interaction types

Effective operators

$$\bar{\chi}\, i\gamma^5 \chi\ \bar{N} N \qquad \bar{\chi}\chi\ \bar{N}\, i\gamma^5 N \qquad \phi^*\phi\ \bar{N}\, i\gamma^5 N$$

$$\bar{\chi}\, i\gamma^5 \chi\ \bar{N}\, i\gamma^5 N \qquad \bar{\chi}\gamma^\mu\gamma^5\chi\ \bar{N}\gamma_\mu N \qquad i\left(\phi^* \overleftrightarrow{\partial_\mu} \phi\right)\ \bar{N}\gamma^\mu N$$

$$\bar{\chi}\gamma^\mu\chi\ \bar{N}\gamma_\mu\gamma^5 N \qquad \bar{\chi}\gamma^\mu\gamma^5\chi\ \bar{N}\gamma_\mu\gamma^5 N \qquad i\left(\phi^* \overleftrightarrow{\partial_\mu} \phi\right)\ \bar{N}\gamma^\mu\gamma^5 N$$

$$\bar{\chi}\,\sigma^{\mu\nu}\chi\ \bar{N}\,\sigma_{\mu\nu} N \qquad \bar{\chi}\, i\,\sigma^{\mu\nu}\gamma^5\chi\ \bar{N}\,\sigma_{\mu\nu} N$$

Electromagnetic DM

$$Q_\chi e\ \bar{\chi}\gamma^\mu\chi\, A_\mu \qquad \text{(millicharged DM)}$$

$$\frac{\mu_\chi}{2}\,\bar{\chi}\,\sigma^{\mu\nu}\chi\, F_{\mu\nu} \qquad \text{(anomalous DM magnetic moment)}$$

$$\frac{d_\chi}{2}\, i\,\bar{\chi}\,\sigma^{\mu\nu}\gamma^5\chi\, F_{\mu\nu} \qquad \text{(DM electric dipole moment)}$$

etc.

A unified framework

- Fan, Reece, Wang - *Non-relativistic effective theory of dark matter direct detection* [1008.1591]
- Fitzpatrick, Haxton, Katz, Lubbers, Xu - *The Effective Field Theory of Dark Matter Direct Detection* [1203.3542]+ [1211.2818], [1308.6288]

Set the stage for an interaction-independent analysis of DM signals at direct detection experiments

Note: the DM-nucleus scattering is non-relativistic (NR) because $v \sim q/m \sim 10^{-3}$

Recipe

- Expand $u^s(p) = \begin{pmatrix} \sqrt{p^\mu \sigma_\mu}\, \xi^s \\ \sqrt{p^\mu \bar{\sigma}_\mu}\, \xi^s \end{pmatrix}$ in powers of $\vec{p}$ (NR limit)

- Express $\mathcal{M}_{\text{nucleon}} = \sum_{i=1}^{12} (\mathsf{c}_i^p + \mathsf{c}_i^n)\, \mathcal{O}_i^{\text{NR}}$

$$
\begin{aligned}
\mathcal{O}_1^{\text{NR}} &= \mathbb{1} & \mathcal{O}_2^{\text{NR}} &= (v^\perp)^2 \\
\mathcal{O}_3^{\text{NR}} &= i\, \vec{s}_N \cdot (\vec{q} \times \vec{v}) & \mathcal{O}_4^{\text{NR}} &= \vec{s}_\chi \cdot \vec{s}_N \\
\mathcal{O}_5^{\text{NR}} &= i\, \vec{s}_\chi \cdot (\vec{q} \times \vec{v}) & \mathcal{O}_6^{\text{NR}} &= (\vec{s}_\chi \cdot \vec{q})(\vec{s}_N \cdot \vec{q}) \\
\mathcal{O}_7^{\text{NR}} &= \vec{s}_N \cdot \vec{v}^\perp & \mathcal{O}_8^{\text{NR}} &= \vec{s}_\chi \cdot \vec{v}^\perp \\
\mathcal{O}_9^{\text{NR}} &= i\, \vec{s}_\chi \cdot (\vec{s}_N \times \vec{q}) & \mathcal{O}_{10}^{\text{NR}} &= i\, \vec{s}_N \cdot \vec{q} \\
\mathcal{O}_{11}^{\text{NR}} &= i\, \vec{s}_\chi \cdot \vec{q} & \mathcal{O}_{12}^{\text{NR}} &= \vec{v}^\perp \cdot (\vec{s}_\chi \times \vec{s}_N)
\end{aligned}
$$

$$
\overline{|\mathcal{M}_{\text{nucleus}}|^2} = \frac{m_T^2}{m_N^2} \sum_{i,j=1}^{12} \sum_{N,N'=p,n} \mathsf{c}_i^N \mathsf{c}_j^{N'} F_{i,j}^{(N,N')}
$$

More than form factors

$$\overline{|\mathcal{M}_{\rm nucleus}|^2} = \frac{m_T^2}{m_N^2} \sum_{i,j=1}^{12} \sum_{N,N'=p,n} \mathsf{c}_i^N \mathsf{c}_j^{N'} F_{i,j}^{(N,N')}$$

The "form factors" $F_{i,j}^{(N,N')}$ are tabulated in [1203.3542], [1308.6288]

They contain the nuclear physics associated to each of the NR operators

They form a basis for writing ~any interaction

They provide a parametrization of the differential cross section in terms of the coefficients c_i^N

Factorizing fun

$$\frac{d\sigma_T}{dE_R} = \frac{1}{32\pi}\frac{1}{m_\chi^2 m_T}\frac{1}{v^2}\frac{m_T^2}{m_N^2}\sum_{i,j=1}^{12}\sum_{N,N'=p,n} \mathsf{c}_i^N \mathsf{c}_j^{N'} F_{i,j}^{(N,N')}$$

$$\tilde{\mathfrak{F}}_{i,j}^{(N,N')} \equiv C \sum_T \xi_T \int_{E'_{\mathrm{min}}}^{E'_{\mathrm{max}}} dE'\, \epsilon_1(E') \int_0^\infty dE_R\, \epsilon_2(E_R) G_T(E_R, E') \int_{v_{\mathrm{min}}(E_R)} d^3v\, \frac{1}{v}\, f_E(\vec{v})\, F_{i,j}^{(N,N')}$$

$$R_{\mathrm{tot}} = \sum_{i,j=1}^{12}\sum_{N,N'=p,n} \mathsf{c}_i^N(\lambda, m_{\mathrm{DM}})\, \mathsf{c}_j^{N'}(\lambda, m_{\mathrm{DM}})\, \tilde{\mathfrak{F}}_{i,j}^{(N,N')}(m_{\mathrm{DM}})$$

Contain particle physics

Contain astro+nuclear +experimental physics

Benchmark bound & rescaling

Let us consider the benchmark model

$$\mathcal{M}_{p,\mathrm{B}} = \lambda_\mathrm{B} = \lambda_\mathrm{B}\, \mathcal{O}_1^{\mathrm{NR}} \qquad \Longrightarrow \qquad \mathsf{c}_1^p = \lambda_\mathrm{B}, \text{ all other } \mathsf{c}_i^N = 0$$

e.g. from the QFT Lagrangian $\mathscr{L}_\mathrm{B} = \dfrac{\lambda_\mathrm{B}}{4 m_\mathrm{DM} m_p}\, \bar{\chi}\gamma^\mu\chi\, \bar{p}\gamma_\mu p$

So $R_{\mathrm{tot,B}} = \lambda_\mathrm{B}^2\, \tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})$. Now define $\lambda_\mathrm{B}^{\mathrm{lim}}(m_\mathrm{DM})^2 \equiv \dfrac{R_\mathrm{lim}}{\tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})}$

A bound on another model is given by

$$\sum_{i,j=1}^{12} \sum_{N,N'=p,n} \mathsf{c}_i^N(\lambda, m_\mathrm{DM})\, \mathsf{c}_j^{N'}(\lambda, m_\mathrm{DM})\, \tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\mathrm{DM}) \leqslant R_\mathrm{lim}$$

Benchmark bound & rescaling

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$$\sum_{i,j=1}^{12} \sum_{N,N'=p,n} \mathsf{c}_i^N(\lambda, m_\mathrm{DM})\, \mathsf{c}_j^{N'}(\lambda, m_\mathrm{DM})\, \frac{\tilde{\mathfrak{F}}_{i,j}^{(N,N')}(m_\mathrm{DM})}{\tilde{\mathfrak{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})} \leqslant \frac{R_\mathrm{lim}}{\tilde{\mathfrak{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})}$$

Benchmark bound & rescaling

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$$\mathcal{M}_{p,\mathrm{B}} = \lambda_\mathrm{B} = \lambda_\mathrm{B}\,\mathcal{O}_1^{\mathrm{NR}} \qquad \Longrightarrow \qquad \mathsf{c}_1^p = \lambda_\mathrm{B},\ \text{all other } \mathsf{c}_i^N = 0$$

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So $R_{\mathrm{tot,B}} = \lambda_\mathrm{B}^2\,\tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})$. Now define $\lambda_\mathrm{B}^{\mathrm{lim}}(m_\mathrm{DM})^2 \equiv \dfrac{R_{\mathrm{lim}}}{\tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})}$

A bound on another model is given by

$$\sum_{i,j=1}^{12}\sum_{N,N'=p,n} \mathsf{c}_i^N(\lambda, m_\mathrm{DM})\,\mathsf{c}_j^{N'}(\lambda, m_\mathrm{DM})\,\mathcal{Y}_{i,j}^{(N,N')}(m_\mathrm{DM}) \leqslant \lambda_\mathrm{B}^{\mathrm{lim}}(m_\mathrm{DM})^2$$

where $\mathcal{Y}_{i,j}^{(N,N')}(m_\mathrm{DM}) \equiv \dfrac{\tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\mathrm{DM})}{\tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})}$

Benchmark bound & rescaling

Let us consider the benchmark model

$$\mathcal{M}_{p,\mathrm{B}} = \lambda_\mathrm{B} = \lambda_\mathrm{B}\, \mathcal{O}_1^{\mathrm{NR}} \qquad \Longrightarrow \qquad \mathsf{c}_1^p = \lambda_\mathrm{B}, \text{ all other } \mathsf{c}_i^N = 0$$

e.g. from the QFT Lagrangian $\mathscr{L}_\mathrm{B} = \frac{\lambda_\mathrm{B}}{4 m_\mathrm{DM} m_p} \bar{\chi}\gamma^\mu \chi\, \bar{p}\gamma_\mu p$

So $R_{\mathrm{tot,B}} = \lambda_\mathrm{B}^2\, \tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})$. Now define $\lambda_\mathrm{B}^{\mathrm{lim}}(m_\mathrm{DM})^2 \equiv \frac{R_\mathrm{lim}}{\tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\mathrm{DM})}$

A bound on another model is given by

$$\sum_{i,j=1}^{12} \sum_{N,N'=p,n} \mathsf{c}_i^N(\lambda, m_\mathrm{DM})\, \mathsf{c}_j^{N'}(\lambda, m_\mathrm{DM})\, \mathcal{Y}_{i,j}^{(N,N')}(m_\mathrm{DM}) \leqslant \lambda_\mathrm{B}^{\mathrm{lim}}(m_\mathrm{DM})^2$$

The $\mathcal{Y}_{i,j}^{(N,N')}(m_\mathrm{DM})$ functions and the $\lambda_\mathrm{B}^{\mathrm{lim}}(m_\mathrm{DM})$ for different CL's are available on

http://www.marcocirelli.net/NROpsDD.html

Tools for model-independent bounds in direct dark matter searches

Data and Results from 1307.5955 [hep-ph], JCAP 10 (2013) 019.

If you use the data provided on this site, please cite:
M.Cirelli, E.Del Nobile, P.Panci,
"Tools for model-independent bounds in direct dark matter searches",
arXiv 1307.5955, JCAP 10 (2013) 019.

This is **Release 2.0** (November 2013). *Log of changes at the bottom of this page.*

Test Statistic functions:

The TS.m file provides the tables of TS for the benchmark case (see the paper for the definition), for the five experiments that we consider (XENON100, CDMS-Ge, COUPP, PICASSO, LUX).

Rescaling functions:

The Y.m file provides the rescaling functions $Y_{ij}^{(N,N')}$ and $Y_{ij}^{\mathrm{Ir}(N,N')}$ (see the paper for the definition).

Sample file:

The Sample.nb notebook shows how to load and use the above numerical products, and gives some examples.

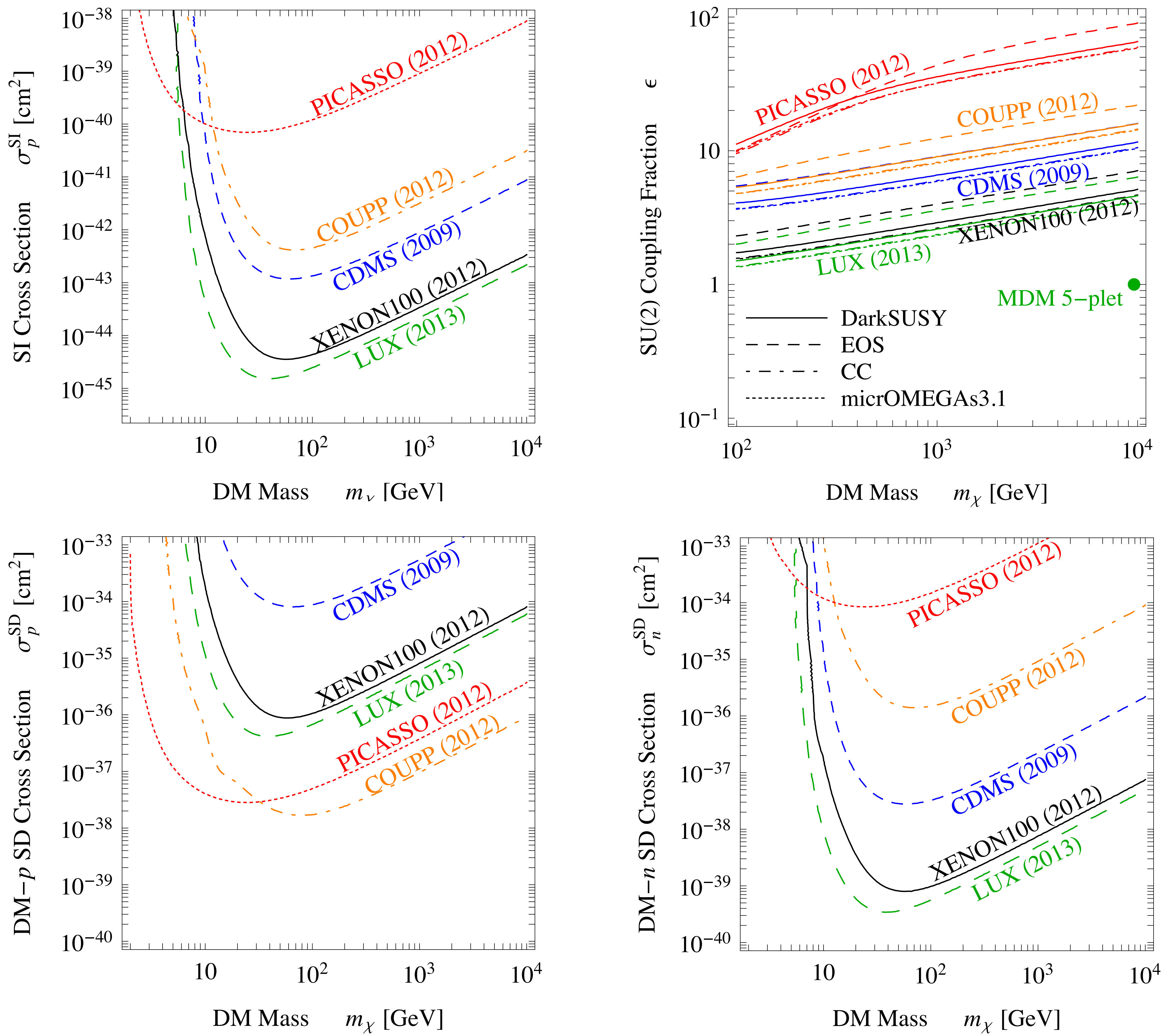
Log of changes and releases:

[23 jul 2013] First Release.

[08 oct 2013] Minor changes in the notations in Sample.nb, to match JCAP version. No new release.

[25 nov 2013] **New Release: 2.0**. Addition of LUX results. This release corresponds to **version 3** of 1307.5955 (with Addendum).

Some results



Conclusions for this part

- The full scattering rate at direct DM search experiments can be factored in two pieces:
  - the coefficients $\mathfrak{c}_i^N$ contain the information on the underlying particle physics model
  - the $\mathcal{Y}_{i,j}^{(N,N')}$ functions contain all the rest, and in principle can be delivered by the experimental collaborations themselves

- With these ingredients, a bound that would otherwise need a PhD student and weeks of coding to perform multiple numerical integrals is computed within minutes

- All the material + a Mathematica sample file is downloadable for instant fun at http://www.marcocirelli.net/NROpsDD.html

Halo-independent stuff

Direct detection rate

$$\frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}} = \frac{\xi_T}{m_T}\frac{\rho}{m_\mathrm{DM}}\int_{v_\mathrm{min}(E_\mathrm{R})}^{v_\mathrm{esc}} \mathrm{d}^3 v\, f(\vec{v},t)\, v\, \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}(E_\mathrm{R},\vec{v}\,)$$

ρ	DM local density	$f(\vec{v},t)$	DM velocity distribution
ξ_T	target mass fraction	v_esc	galactic escape velocity

$$R_{[E_1',E_2']}(t) = \sum_T \int_{E_1'}^{E_2'} \mathrm{d}E'\, \epsilon_1(E') \int_0^\infty \mathrm{d}E_\mathrm{R}\, \epsilon_2(E_\mathrm{R}) G_T(E', qE_\mathrm{R}) \frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}}$$

E'	detected energy	$G_T(E', qE_\mathrm{R})$	detector resolution
$\epsilon_1,\ \epsilon_2$	efficiencies	q	quenching factor

Direct detection rate

$$\frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}} = \frac{\xi_T}{m_T}\frac{\rho}{m_\mathrm{DM}}\int_{v_\mathrm{min}(E_\mathrm{R})}^{v_\mathrm{esc}} \mathrm{d}^3v\, f(\vec{v},t)\, v\, \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}(E_\mathrm{R},\vec{v}\,)$$

ρ DM local density

ξ_T target mass fraction

$f(\vec{v},t)$ DM velocity distribution

v_esc galactic escape velocity

$$R_{[E_1',E_2']}(t) = \sum_T \int_{E_1'}^{E_2'} \mathrm{d}E'\, \epsilon_1(E') \int_0^\infty \mathrm{d}E_\mathrm{R}\, \epsilon_2(E_\mathrm{R}) G_T(E', qE_\mathrm{R}) \frac{\mathrm{d}R_T}{\mathrm{d}E_\mathrm{R}}$$

E' detected energy

$\epsilon_1,\ \epsilon_2$ efficiencies

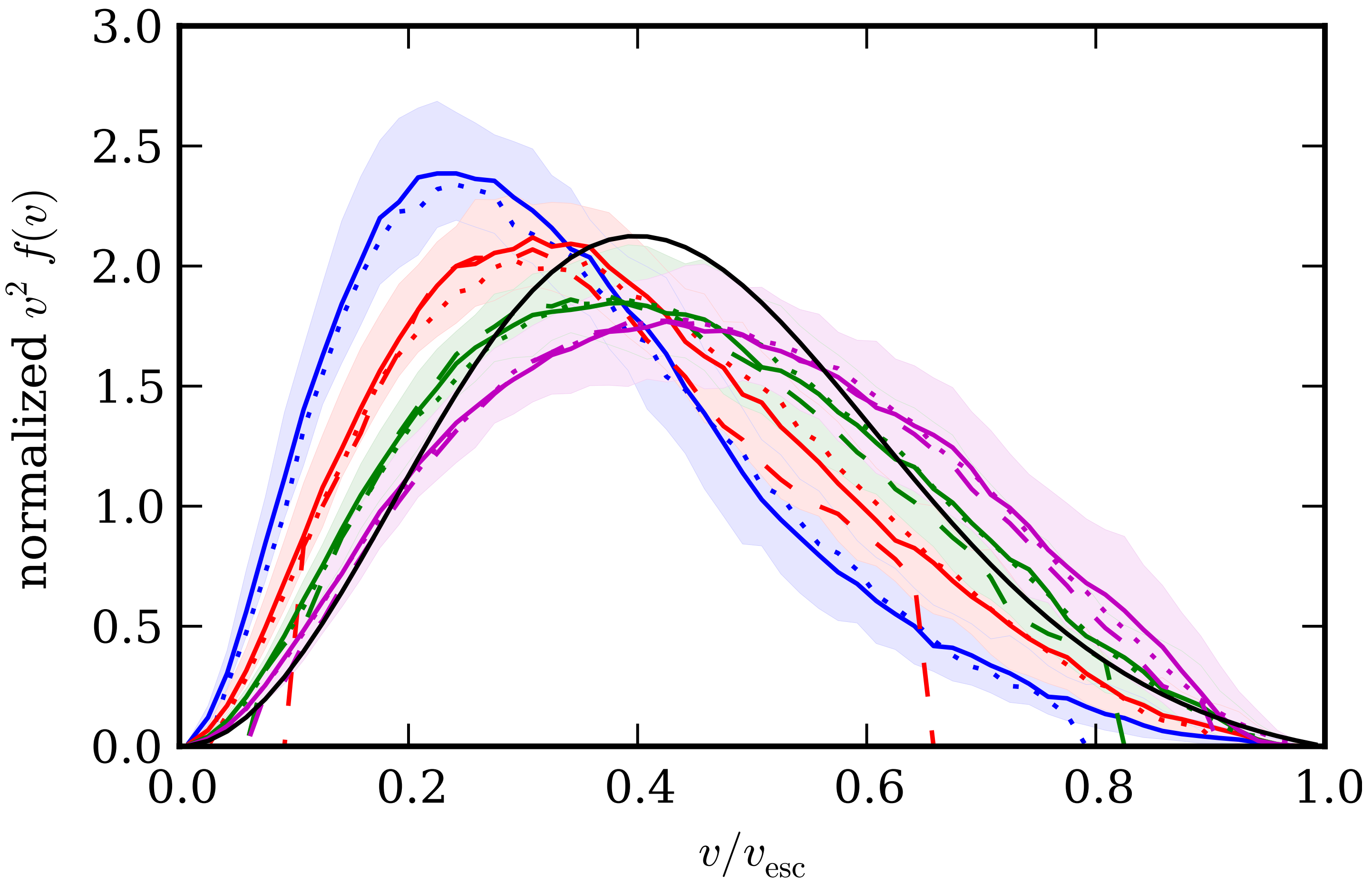
$G_T(E', qE_\mathrm{R})$ detector resolution

q quenching factor

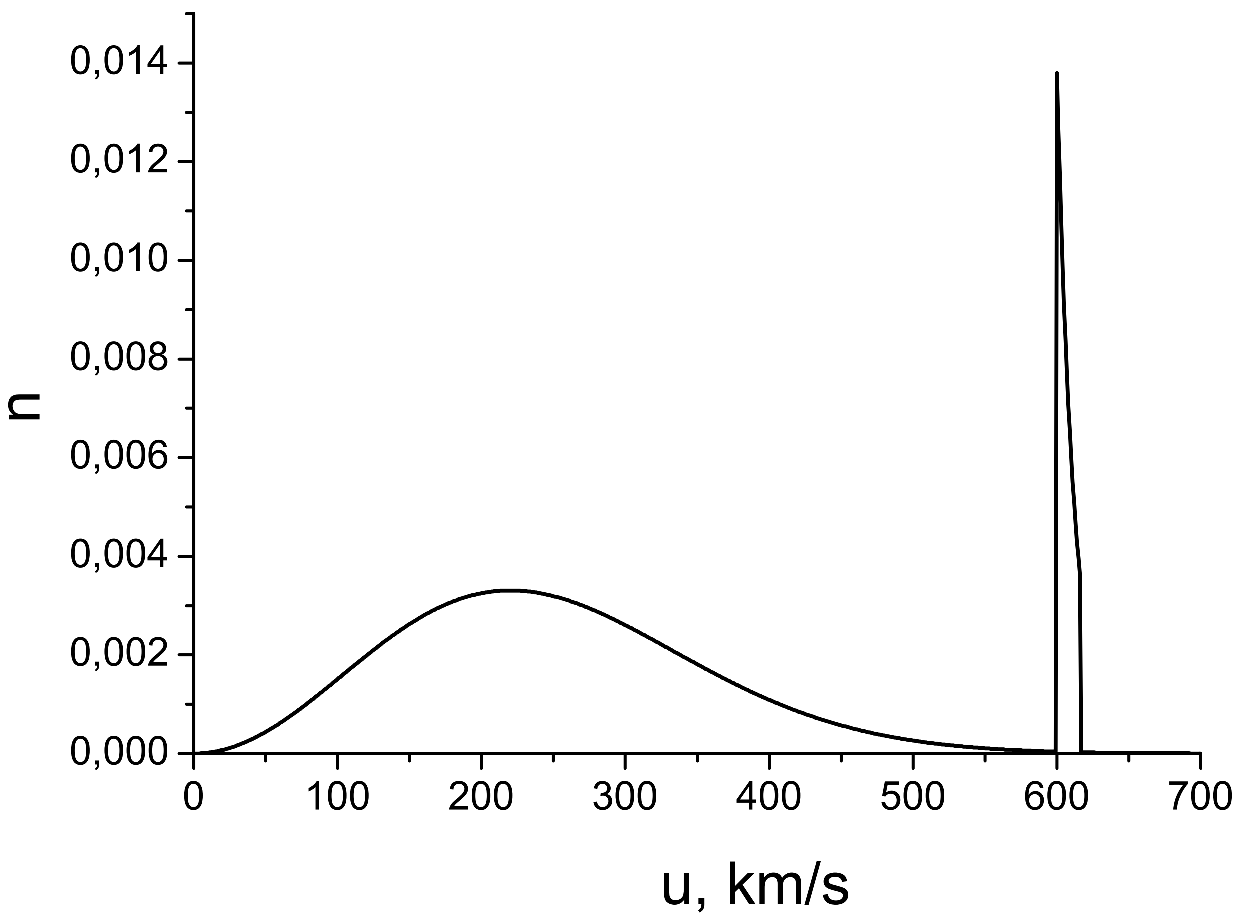
Astrophysical uncertainties

e.g.

Mao, Strigari, Wechsler, Wu, Hahn [1210.2721]



Baushev [1208.0392]



Two approaches

- Try to find alternative halo models, either driven by physical arguments or by fitting simulations or observations
  see e.g. Freese, Lisanti, Savage [1209.3339] and references therein

- Try to factor astrophysics out of your problem as much as you can
  Fox, Liu, Weiner [1011.1915], Frandsen, Kahlhoefer, McCabe, Sarkar, Schmidt-Hoberg [1111.0292][1304.6066], Gondolo, Gelmini [1202.6359] + Del Nobile, Huh [1304.6183][1306.5273][1311.4247], Herrero-Garcia, Schwetz, Zupan [1112.1627][1205.0134] + Bozorgnia [1305.3575]

Direct detection rate

$$R_{[E_1',E_2']}(t) = \frac{\rho}{m_{\mathrm{DM}}} \sum_T \frac{\xi_T}{m_T} \int_{E_1'}^{E_2'} \mathrm{d}E' \, \epsilon_1(E') \\ \times \int_0^\infty \mathrm{d}E_\mathrm{R} \, \epsilon_2(E_\mathrm{R}) G_T(E', qE_\mathrm{R}) \\ \times \int_{v_\mathrm{min}(E_\mathrm{R})}^{v_\mathrm{esc}} \mathrm{d}^3 v \, f(\vec{v}, t) \, v \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}(E_\mathrm{R}, \vec{v})$$

Direct detection rate

$$R_{[E_1',E_2']}(t) = \frac{\rho}{m_{\rm DM}} \sum_T \frac{\xi_T}{m_T} \int_{E_1'}^{E_2'} \mathrm{d}E'\, \epsilon_1(E') \times \int_0^\infty \mathrm{d}E_{\rm R}\, \epsilon_2(E_{\rm R}) G_T(E', qE_{\rm R}) \times \int_{v_{\rm min}(E_{\rm R})}^{v_{\rm esc}} \mathrm{d}^3v\, f(\vec{v},t)\, v \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_{\rm R}}(E_{\rm R},\vec{v})$$

integration by parts

$$R_{[E_1',E_2']}(t) = \frac{\rho}{m_{\rm DM}} \sum_T \frac{\xi_T}{m_T} \int_{E_1'}^{E_2'} \mathrm{d}E'\, \epsilon_1(E') \times \int_0^\infty \mathrm{d}^3v\, f(\vec{v},t)\, v \times \int_{E_{\rm R}^-(v)}^{E_{\rm R}^+(v)} \mathrm{d}E_{\rm R}\, \epsilon_2(E_{\rm R}) G_T(E', qE_{\rm R}) \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_{\rm R}}(E_{\rm R},\vec{v})$$

Algebraic maquillage I

$$R_{[E_1',E_2']}(t) = \int_0^\infty \mathrm{d}^3 v \, \frac{\tilde{f}(\vec{v},t)}{v} \, \mathcal{H}_{[E_1',E_2']}(\vec{v}\,)$$

$$\tilde{f}(\vec{v},t) \equiv \frac{\rho \sigma_{\mathrm{ref}}}{m_{\mathrm{DM}}} \, f(\vec{v},t)$$

$$\mathcal{H}_{[E_1',E_2']}(\vec{v}\,) \equiv \sum_T \frac{\xi_T}{m_T} \int_{E_\mathrm{R}^-(v)}^{E_\mathrm{R}^+(v)} \mathrm{d}E_\mathrm{R} \, \frac{v^2}{\sigma_{\mathrm{ref}}} \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}}(E_\mathrm{R},\vec{v}\,)$$
$$\times \epsilon_2(E_\mathrm{R}) \int_{E_1'}^{E_2'} \mathrm{d}E' \, \epsilon_1(E') G_T(E', qE_\mathrm{R})$$

Algebraic maquillage II

$$R_{[E_1',E_2']}(t) = \int_0^\infty \mathrm{d}v_{\min}\,\tilde{\eta}(v_{\min},t)\,\mathcal{R}_{[E_1',E_2']}(v_{\min})$$

$$\tilde{\eta}(v_{\min},t) \equiv \int_{v_{\min}}^\infty \mathrm{d}^3v\,\frac{\tilde{f}(\vec{v},t)}{v}$$

$$\mathcal{R}_{[E_1',E_2']}(v_{\min}) \equiv \frac{\partial\mathcal{H}_{[E_1',E_2']}(v_{\min})}{\partial v_{\min}}$$

Bounds and fits

$$R_{[E_1',E_2']}(t) = \int_0^\infty \mathrm{d}v_{\min}\, \tilde{\eta}(v_{\min}, t)\, \mathcal{R}_{[E_1',E_2']}(v_{\min})$$

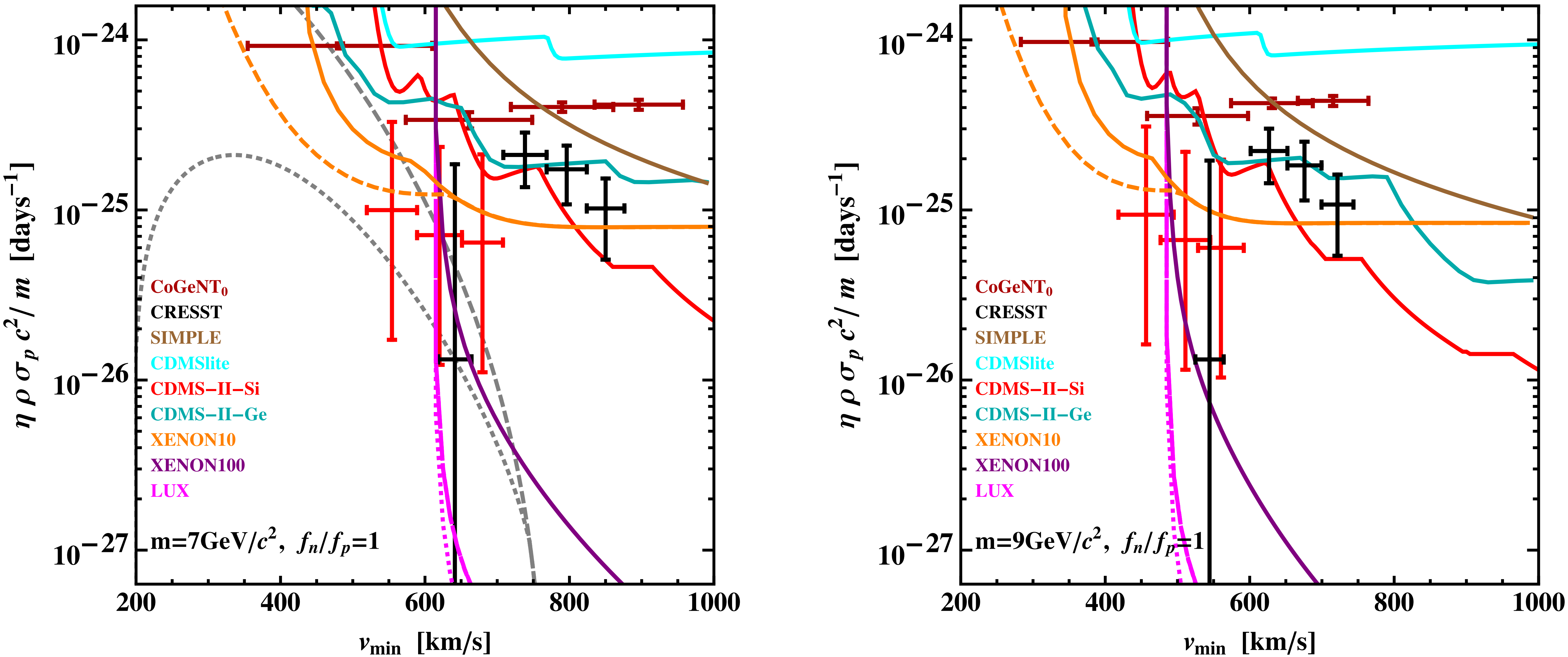
For (conservative) bounds on the unmodulated rate, use

$$\tilde{\eta}_{\mathrm{unmod}}(v_0) \geqslant \tilde{\eta}_0\, \theta(v_0 - v)$$

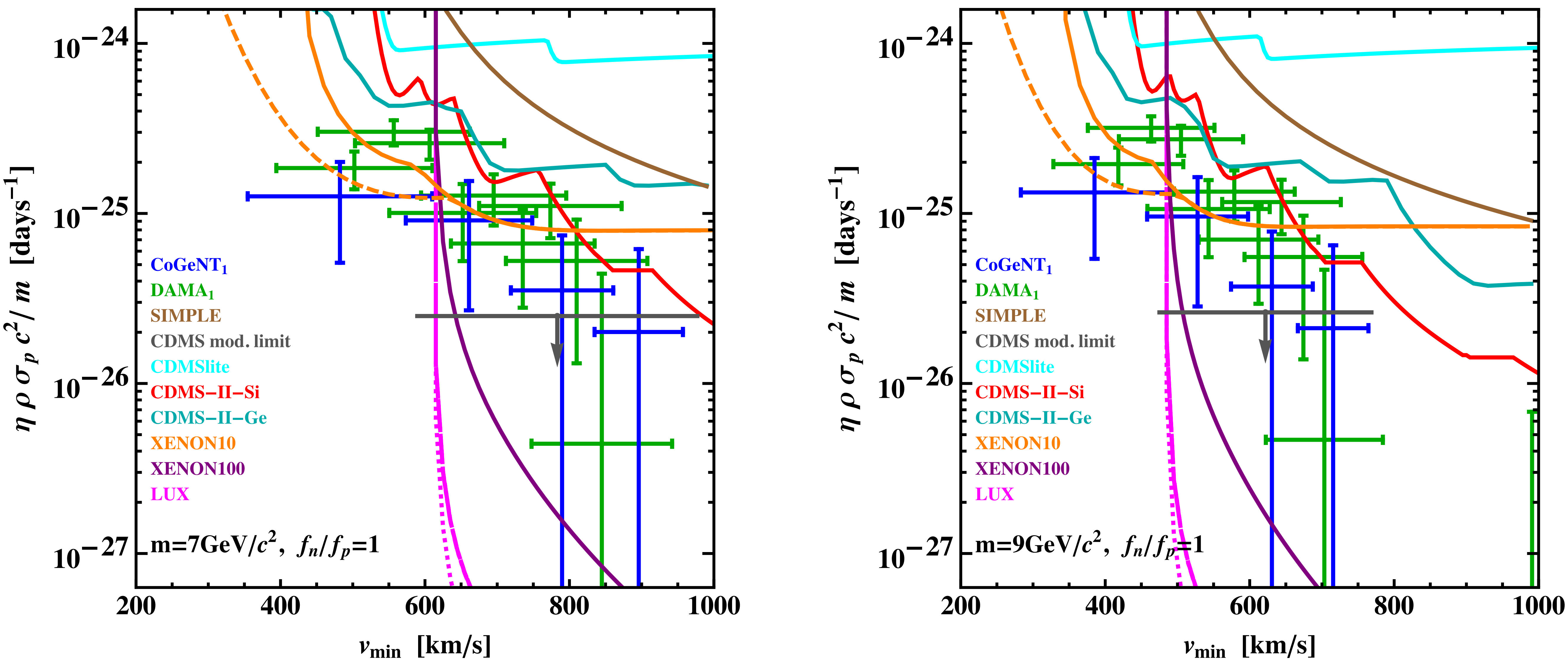
For "fits", use

$$\overline{\tilde{\eta}_{[E_1',E_2']}(v_{\min})} \equiv \frac{R_{[E_1',E_2']}}{\int_0^\infty \mathrm{d}v_{\min}\, \mathcal{R}_{[E_1',E_2']}(v_{\min})}$$

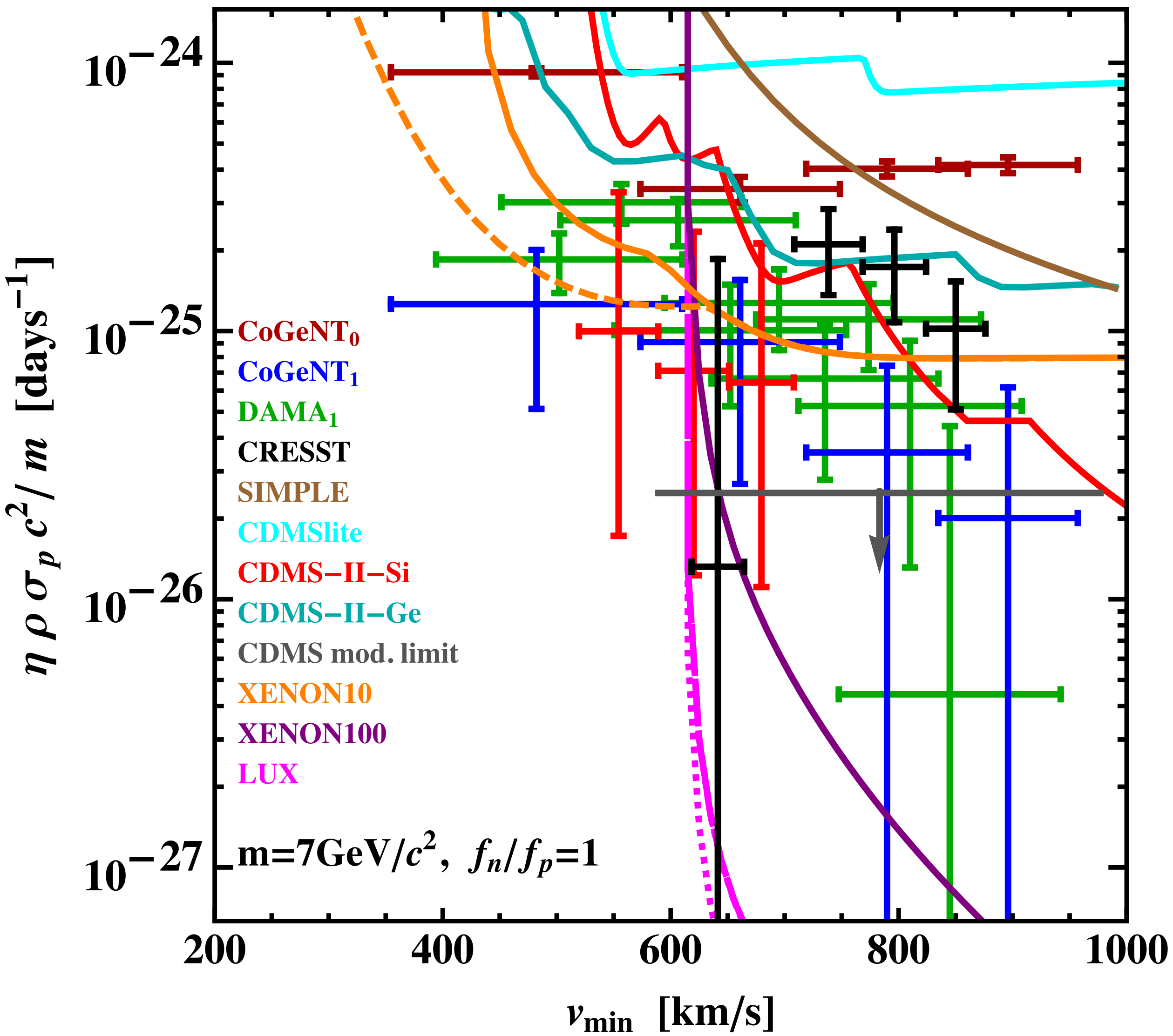
Spin-independent interaction

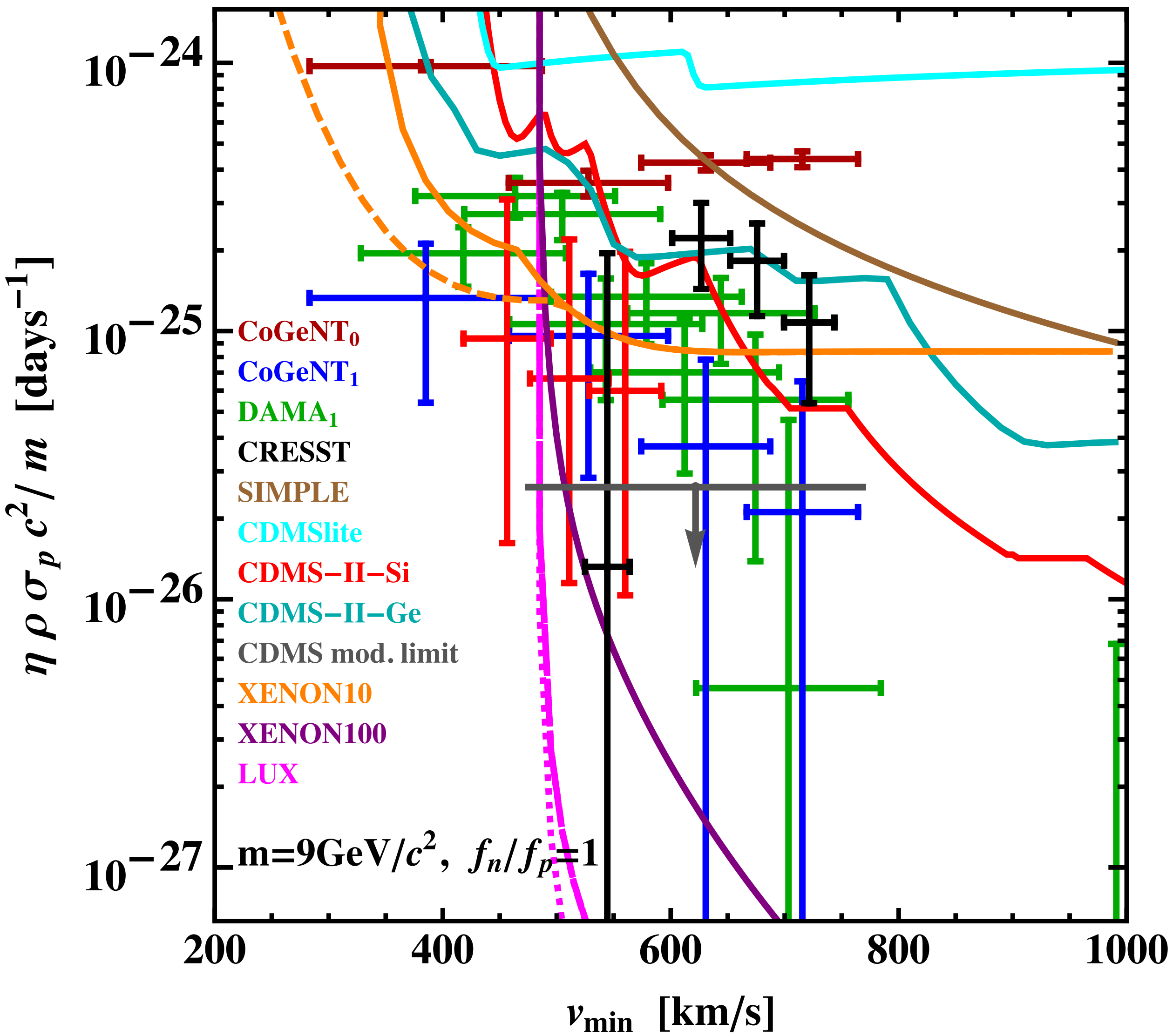


Spin-independent interaction

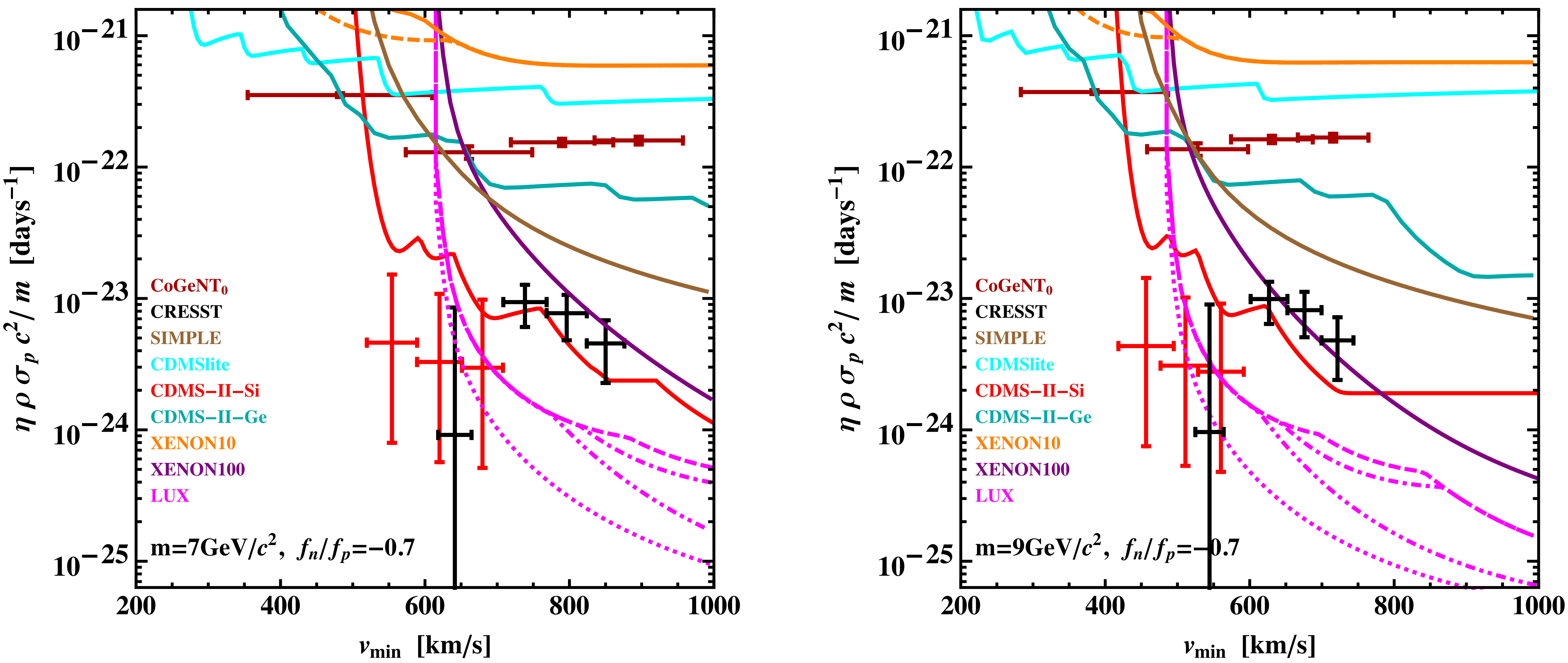


Spin-independent interaction

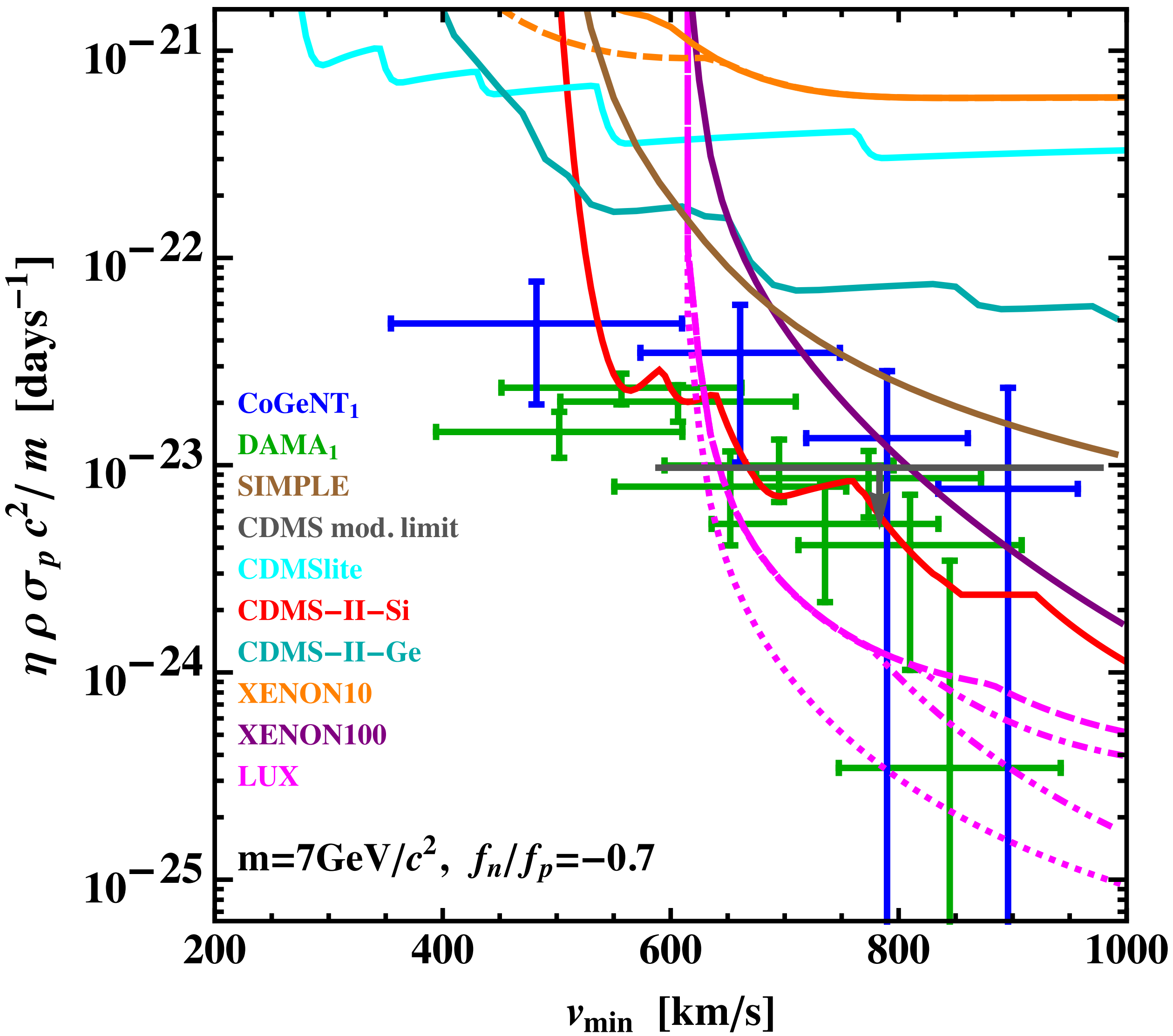


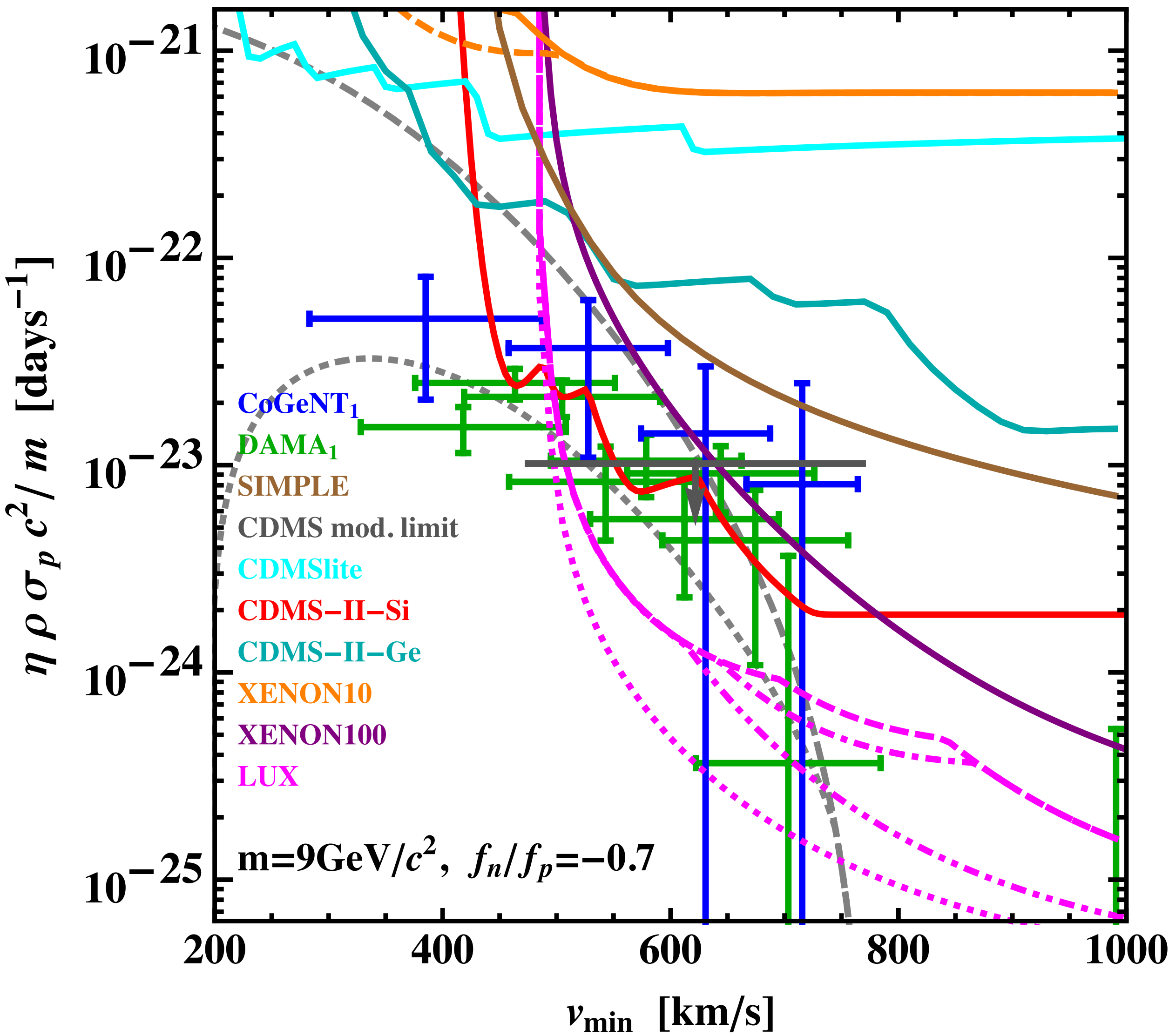


SI isospin violating interaction

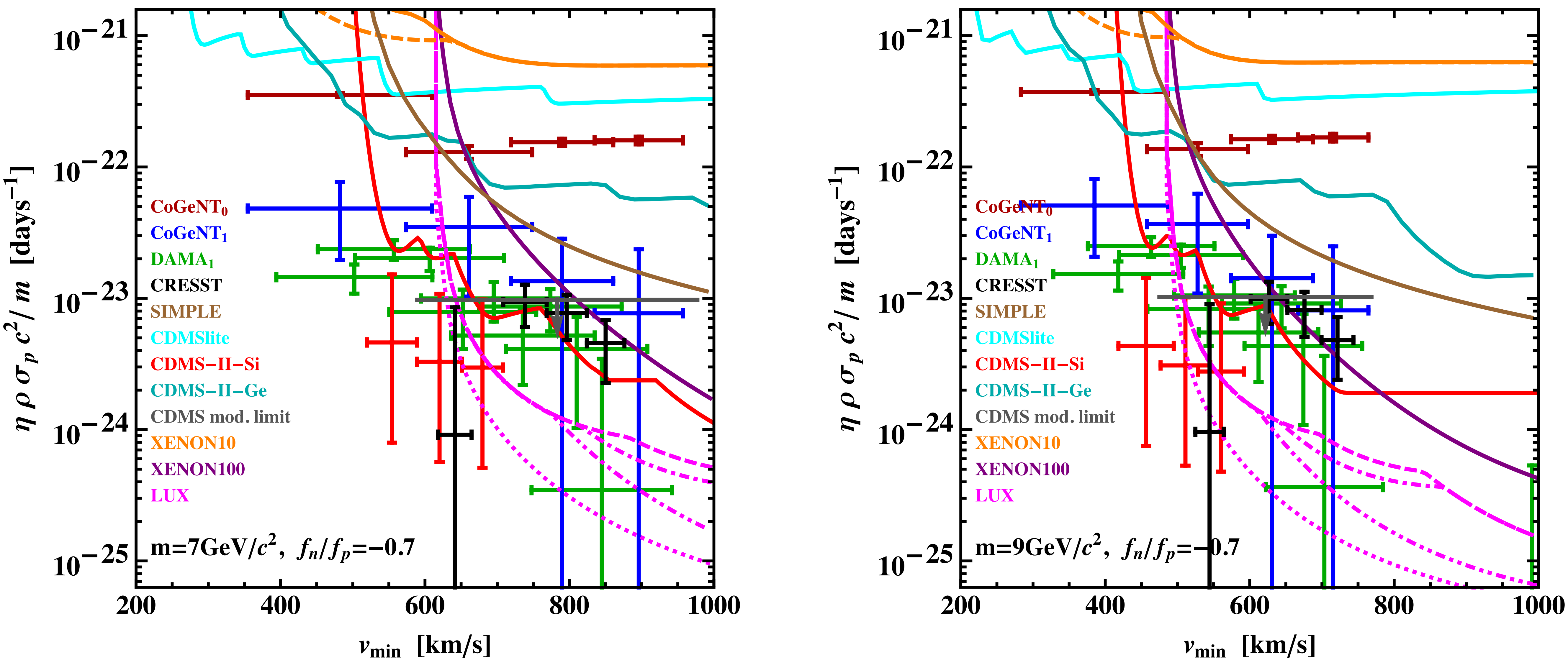


SI isospin violating interaction

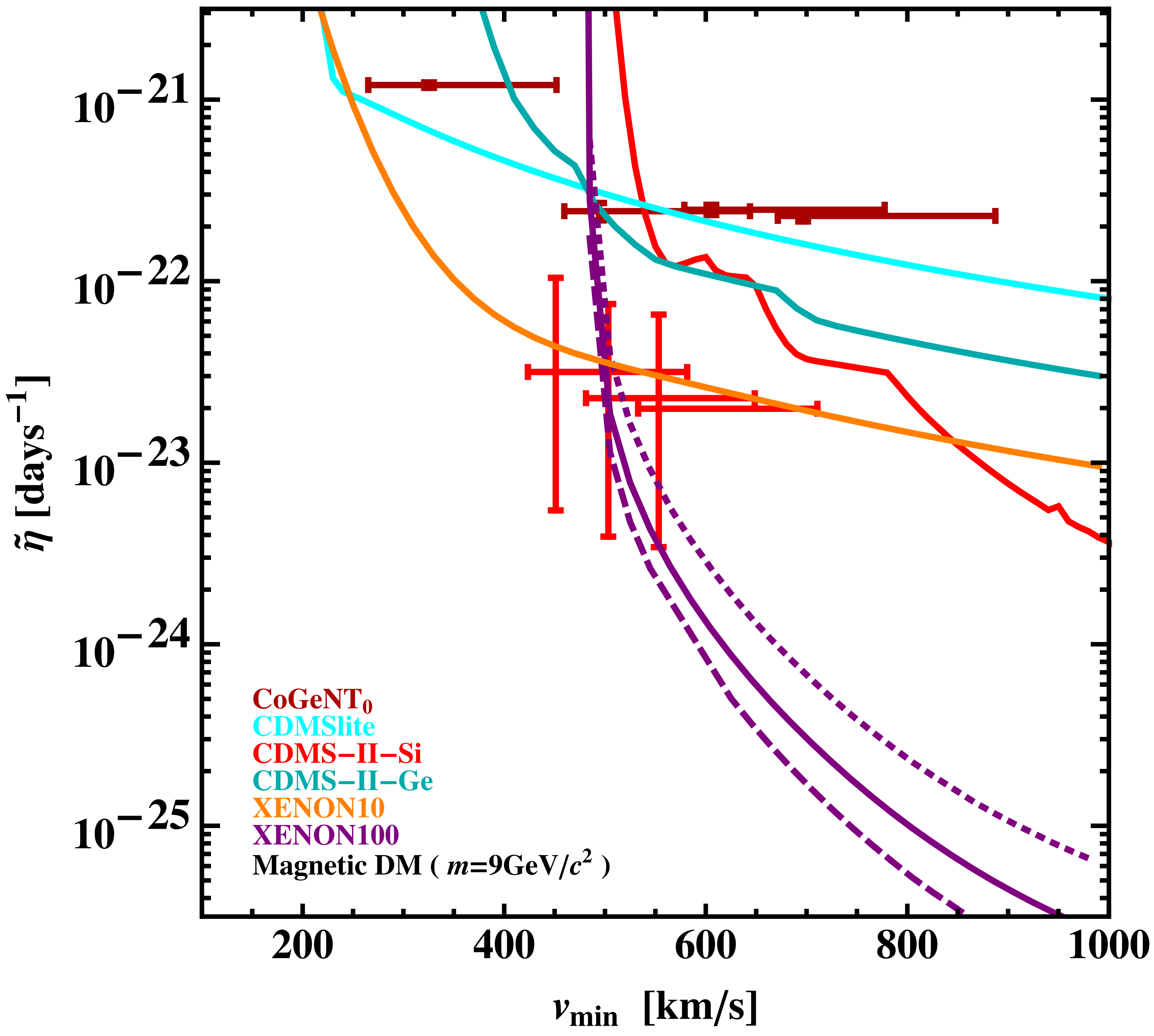




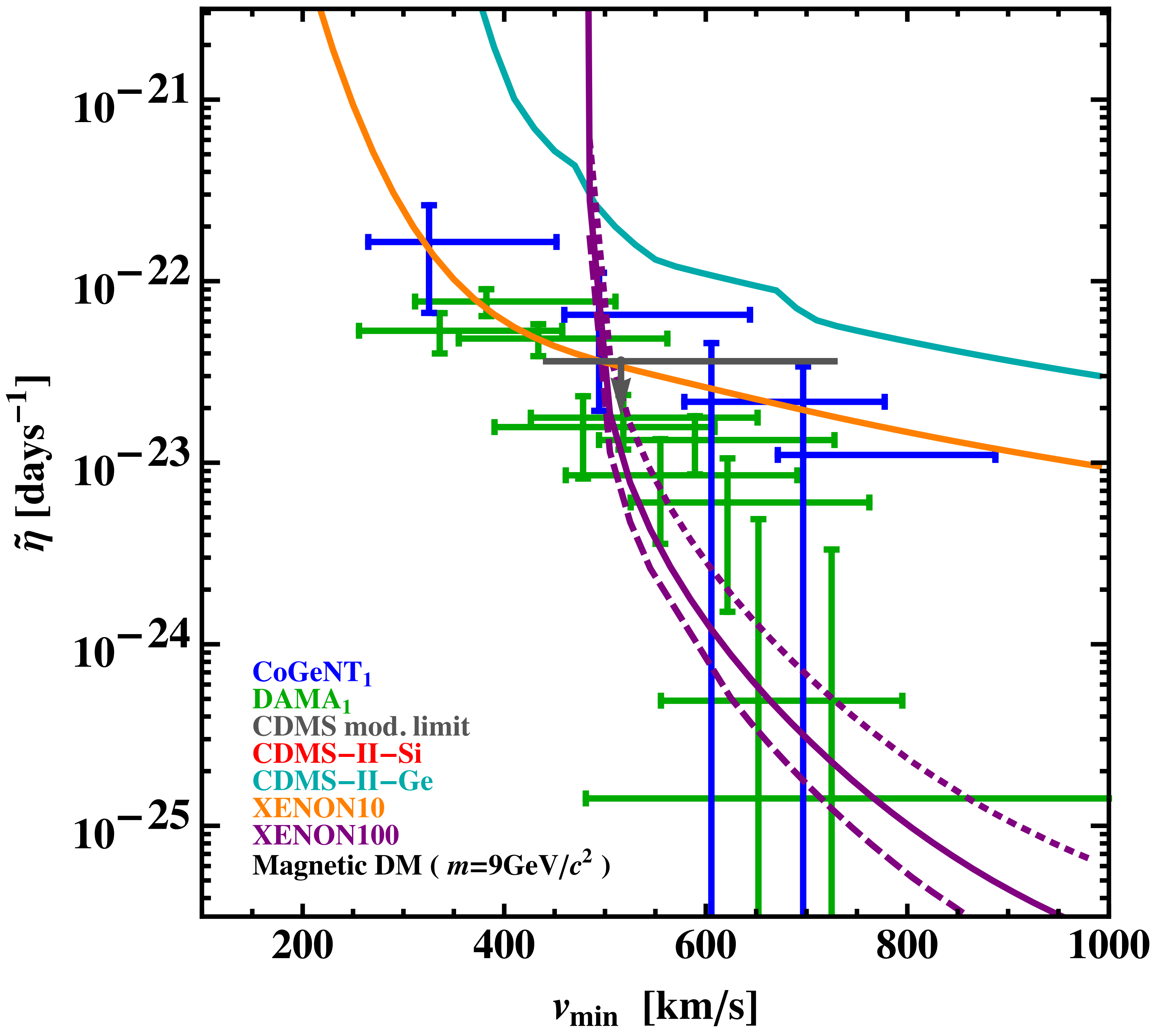
SI isospin violating interaction



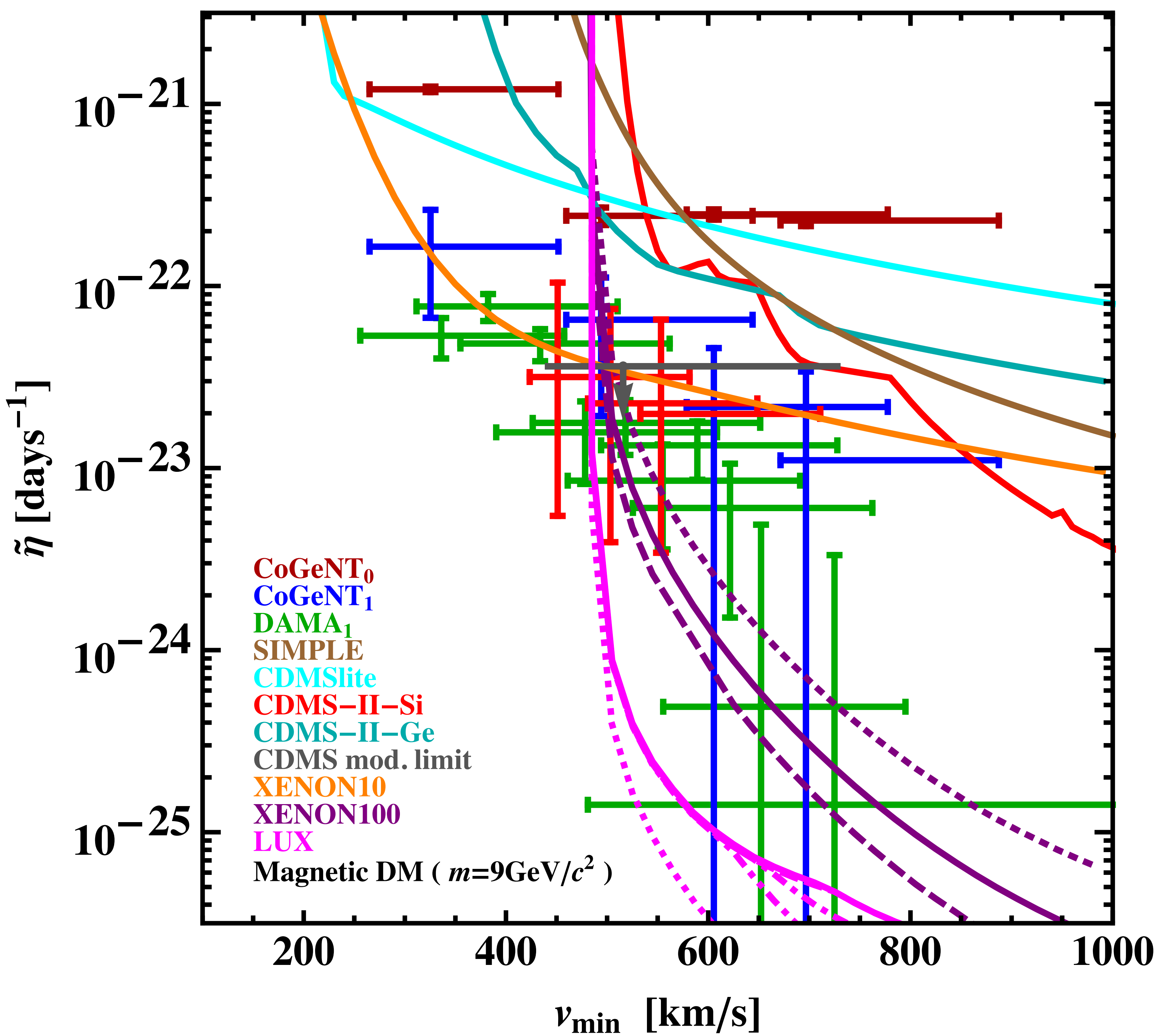
Magnetic moment DM



Magnetic moment DM



Magnetic moment DM



Conclusions

- Promising framework to compare different direct detection experiments in a halo-independent way
- Allows to "compare spectra" of different experiments
- Allows to ~fit the DM velocity distribution
- Quite solid in making (conservative) bounds
- So far it looks like astrophysical uncertainties alone cannot accommodate the discrepancies between different experiments

Drawbacks

(= hopefully future improvements)

- Non straightforward interpretation of the "crosses"
- Crosses lack a precise statistical meaning
- Difficult mapping of the rate onto v_{min}-space for experiments with different nuclei, as DAMA (Na-I) and CRESST (Ca-W-O)
- No information on how compatible unmodulated and modulated signals are