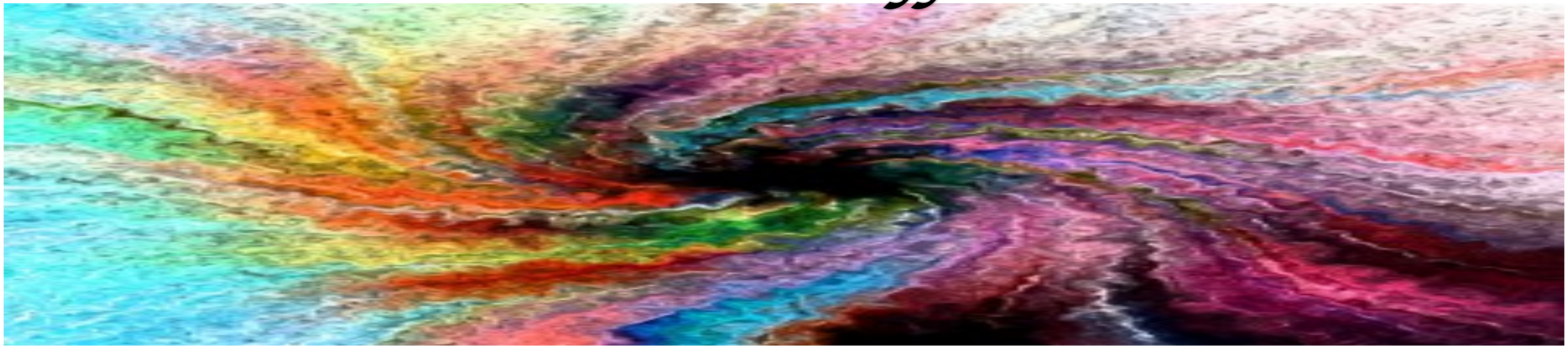
Dark Energy and Dark Matter as Curvature Effects



Salvatore Capozziello

Università di Napoli "Federico II"
and
INFN Sez. di Napoli

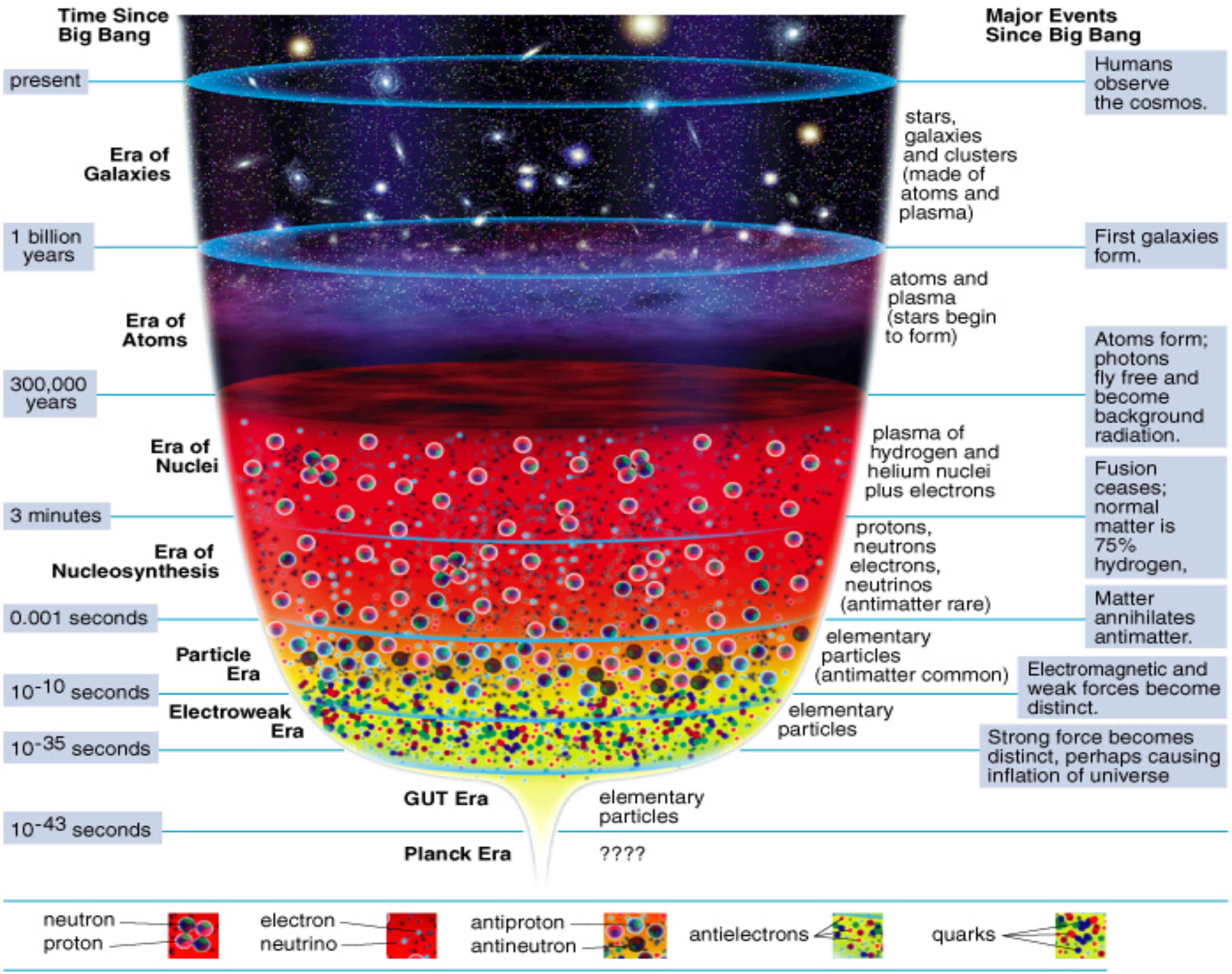
Summary

- Dark Energy and Dark matter problems
- Extending General Relativity
- The weak field limit
- Stellar structures and Jeans instability
- Testing spiral galaxies
- Testing elliptical galaxies
- Modeling clusters of galaxies
- Cosmography
- Conclusions

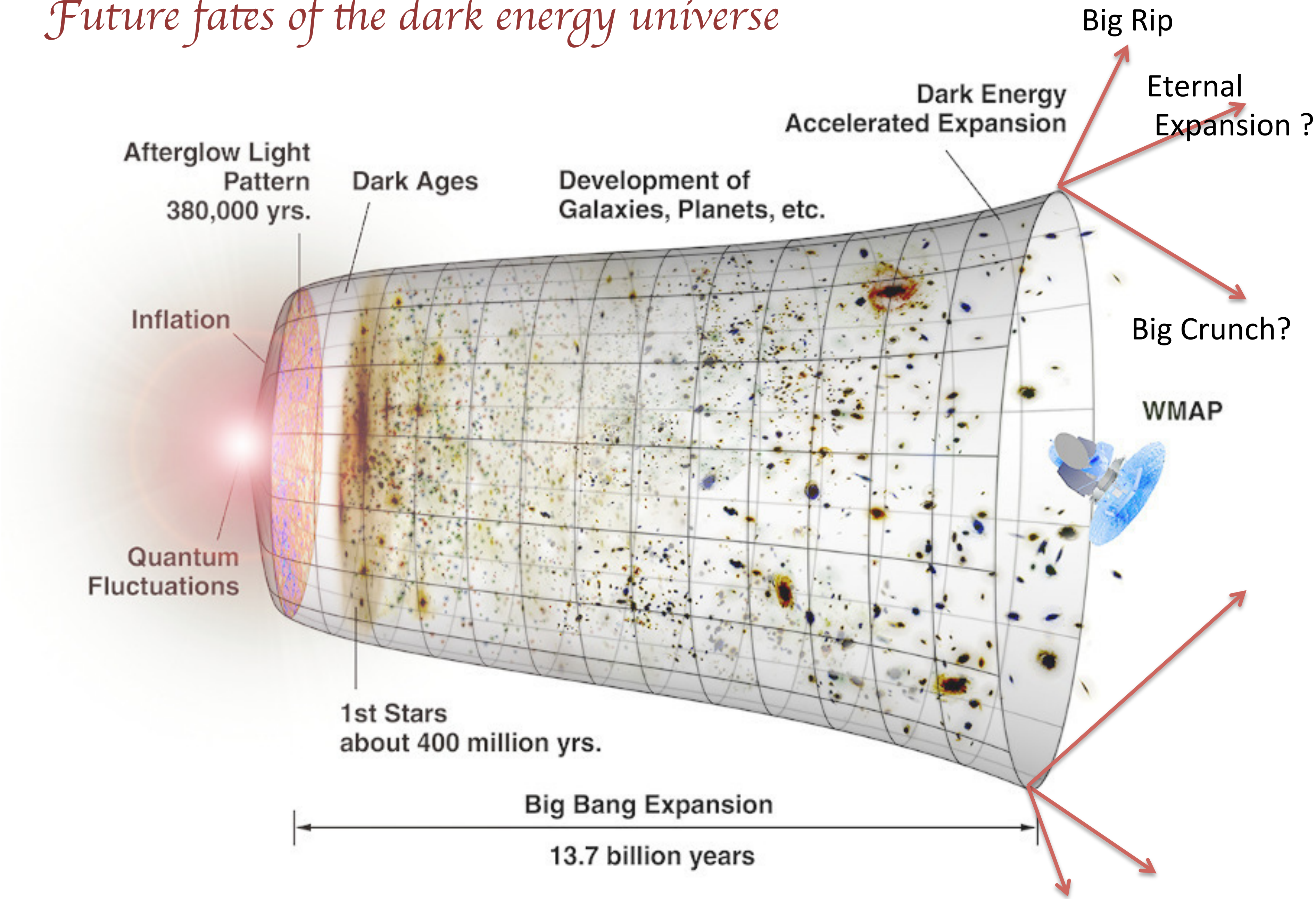
The content of the universe is, up today, absolutely unknown for its largest part. The situation is very "DARK" while the observations are extremely good!

Components of the Universe

The Observed Universe Evolution



Future fates of the dark energy universe



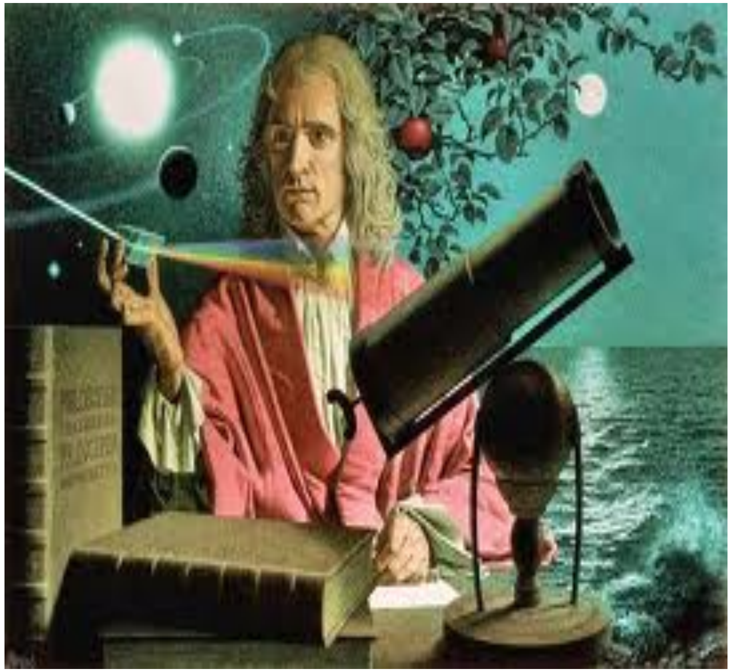
A plethora of theoretical models!!

DARK MATTER

- Neutrinos
- WIMPs
- Wimpzillas, Axions, the "particle forest".....
- MOND
- MACHOS
- Black Holes
- .....

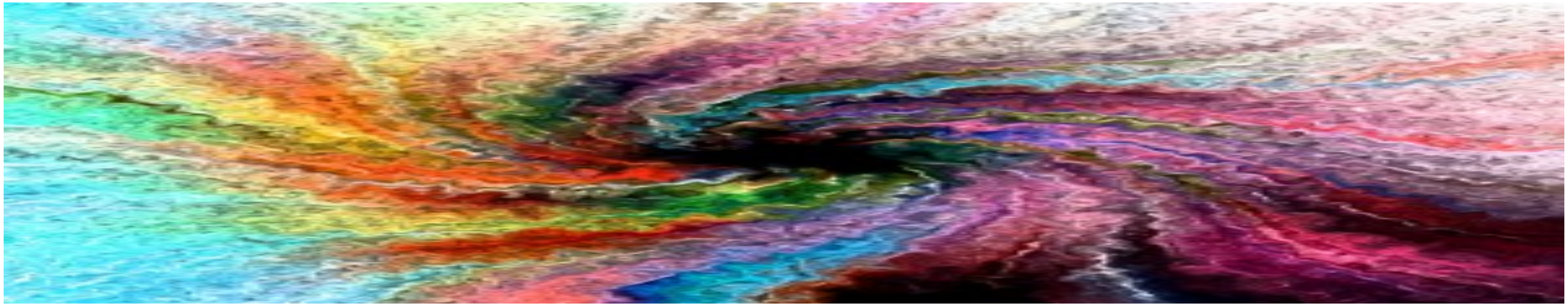
DARK ENERGY

- Cosmological constant
- Scalar field Quintessence
- Phantom fields
- String-Dilaton scalar field
- Braneworlds
- Unified theories
- .....



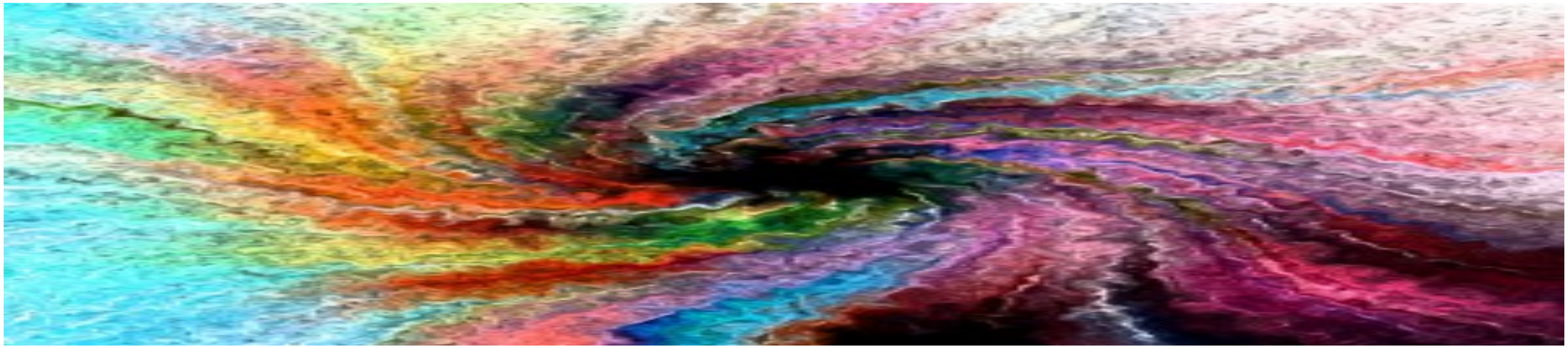
"…there are the ones that invent OCCULT FLUIDS to understand the Laws of Nature. They will come to conclusions, but they now run out into DREAMS and CHIMERAS neglecting the true constitution of things…..
…however there are those that from the simplest observation of Nature, they reproduce New Forces (i.e. New Theories)… "

From the Preface of PRINCIPIA (II Edition) 1687 by Isaac Newton, written by Mr. Roger Cotes



There is a fundamental issue:

Are extragalactic observations and cosmology probing the breakdown of General Relativity at large (IR) scales?



The problem could be reversed

We are able to observe only baryons, radiation, neutrinos and gravity

Dark Energy and Dark Matter as "shortcomings" of GR. Results of flawed physics?

The "correct" theory of gravity could be derived by matching the largest number of observations at ALL SCALES!

Accelerating behaviour (DE) and dynamical phenomena (DM) as CURVATURE EFFECTS

Extending General Relativity

In order to extend General Relativity, we consider two main features:

- the geometry can couple non-minimally to matter and some scalar field;
- higher than second order derivatives of the metric may appear into dynamics

In the first case, we say that we are dealing with scalar-tensor gravity, and in the second case with higher-order theories

A. A. Starobinsky, Phys. Lett. B91, 99 (1980).
S. Capozziello, Int. Jou. Mod. Phys. D 11, 483 (2002) .
A. De Felice, S Tsujikawa, Living Rev.Rel. 13 (2010) 3
S. Capozziello, M. De Laurentis, Phys. Rep. 509, 167 (2011).
S. Nojiri, S.D. Odintsov, Phys. Rep. 505, 59 (2011).

Extending General Relativity

A general class of higher-order-scalar-tensor theories in four dimensions is given by the action

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[F(R, \Box R, \Box^2 R, \ldots, \Box^k R, \phi) - \frac{\epsilon}{2} g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} + \mathcal{L}^{(m)}\right]$$

In the metric approach, the field equations are obtained by varying with respect to $g_{\mu\nu}$

$$\begin{aligned} G^{\mu\nu} = \frac{1}{\mathcal{G}}\Bigg[& \kappa T^{\mu\nu} + \frac{1}{2} g^{\mu\nu}(F - \mathcal{G}R) \\ & + (g^{\mu\lambda}g^{\nu\sigma} - g^{\mu\nu}g^{\lambda\sigma})\mathcal{G}_{;\lambda\sigma} \\ & + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{i}(g^{\mu\nu}g^{\lambda\sigma} + g^{\mu\lambda}g^{\nu\sigma})(\Box^{j-i})_{;\sigma} \\ & \times \left(\Box^{i-j}\frac{\partial F}{\partial \Box^i R}\right)_{;\lambda} - g^{\mu\nu}g^{\lambda\sigma} \\ & \times \left((\Box^{j-1}R)_{;\sigma}\Box^{i-j}\frac{\partial F}{\partial \Box^i R}\right)_{;\lambda}\Bigg], \end{aligned}$$

- $G^{\mu\nu}$ is the Einstein tensor

and

$$\mathcal{G} \equiv \sum_{j=0}^{n} \Box^j \left(\frac{\partial F}{\partial \Box^j R}\right)$$

Extending General Relativity

- The simplest extension of GR is achieved assuming $F = f(R)$, in the action

- The standard Hilbert–Einstein action is recovered for $f(R) = R$

By varying with respect to $g_{\mu\nu}$, we get

$$f'(R)R_{\mu\nu} - \frac{f(R)}{2}g_{\mu\nu} = \nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu}\Box f'(R)$$

and, after some manipulations

$$G_{\mu\nu} = \frac{1}{f'(R)}\left\{\nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu}\Box f'(R) + g_{\mu\nu}\frac{[f(R) - f'(R)R]}{2}\right\}$$

where the gravitational contribution due to higher-order terms can be reinterpreted as a "curvature" stress-energy tensor related to the form of $f(R)$.

Such a tensor disappears for $f(R)=R$

Extending General Relativity

Considering also the standard perfect-fluid matter contribution, we have

$$G_{\alpha\beta} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\alpha\beta}\left[f(R) - Rf'(R)\right] + f'(R)_{;\alpha\beta} - g_{\alpha\beta}\Box f'(R)\right\} + \frac{\kappa T^{(m)}_{\alpha\beta}}{f'(R)} = \underbrace{T^{(\mathrm{curv})}_{\alpha\beta}} + \frac{T^{(m)}_{\alpha\beta}}{f'(R)}$$

In the case of GR, identically vanishes while the standard, minimal coupling is recovered for the matter contribution

is an effective stress-energy tensor constructed by the extra curvature terms

The peculiar behavior of $f(R) = R$ is due to the particular form of the Lagrangian itself which, even though it is a second-order Lagrangian, can be non-covariantly rewritten as the sum of a first-order Lagrangian plus a pure divergence term.

The weak field limit in f(R)-gravity

We assume, analytic Taylor expandable $f(R)$ functions with respect to a certain value $R = R_0$:

$$f(R) = \sum_n \frac{f^n(R_0)}{n!}(R - R_0)^n \simeq f_0 + f_1 R + f_2 R^2 + f_3 R^3 + \ldots .$$

In order to obtain the weak field approximation, one has to insert expansions into field equations and expand the system up to the orders O(0), O(2) e O(4).

If we consider the O(2) - order approximation, the field equations in vacuum, results to be

$$\begin{cases} f_1 r R^{(2)} - 2 f_1 g^{(2)}_{tt,r} + 8 f_2 R^{(2)}_{,r} - f_1 r g^{(2)}_{tt,rr} + 4 f_2 r R^{(2)} = 0\,, \\ f_1 r R^{(2)} - 2 f_1 g^{(2)}_{rr,r} + 8 f_2 R^{(2)}_{,r} - f_1 r g^{(2)}_{tt,rr} = 0\,, \\ 2 f_1 g^{(2)}_{rr} - r \\ \times \left[f_1 r R^{(2)} - f_1 g^{(2)}_{tt,r} - f_1 g^{(2)}_{rr,r} + 4 f_2 R^{(2)}_{,r} + 4 f_2 r R^{(2)}_{,rr} \right] = 0\,, \\ f_1 r R^{(2)} + 6 f_2 \left[2 R^{(2)}_{,r} + r R^{(2)}_{,rr} \right] = 0\,, \\ 2 g^{(2)}_{rr} + r \left[2 g^{(2)}_{tt,r} - r R^{(2)} + 2 g^{(2)}_{rr,r} + r g^{(2)}_{tt,rr} \right] = 0\,. \end{cases} \quad (33)$$

It is evident that the trace equation provides a differential equation with respect to the Ricci scalar which allows to solve exactly the system at O(2) - order

The weak field limit in f(R)-gravity

In order to match at infinity the Minkowskian prescription for the metric, one can discard the Yukawa growing mode in and then we have:

$$\begin{cases} ds^2 = \left[1 - \dfrac{2GM}{f_1 r} - \dfrac{\delta_1(t) e^{-r\sqrt{-\xi}}}{3\xi r}\right] dt^2 \\ -\left[1 + \dfrac{2GM}{f_1 r} - \dfrac{\delta_1(t)(r\sqrt{-\xi}+1) e^{-r\sqrt{-\xi}}}{3\xi r}\right] dr^2 - r^2 d\Omega, \\ R = \dfrac{\delta_1(t) e^{-r\sqrt{-\xi}}}{r}. \end{cases}$$

In particular, since $g_{tt} = 1+2\Phi_{grav} = 1+ g(2)_{tt}$, the gravitational potential of $f(R)$-gravity, analytic in the Ricci scalar R, is

$$\boxed{\Phi_{\rm grav} = -\left(\frac{GM}{f_1 r} + \frac{\delta_1(t) e^{-r\sqrt{-\xi}}}{6\xi r}\right)}$$

This general result means that the standard Newton potential is achieved only in the particular case $f(R) = R$ while it is not so for any analytic $f(R)$ models

The parameters $f_{1,2}$ and the function δ_1 represent the deviations with respect the standard Newton potential

S. Capozziello, M. De Laurentis Ann. Phys. 524, 545 (2012)

The weak field limit in f(R)-gravity

We note that the ξ parameter can be related to an effective mass being

$$m^2 = (3\xi)^{-1} = -\frac{f_1}{3f_2}$$

and can be interpreted also as an effective length $\mathcal{L}$

$$\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1+\delta e^{-\frac{r}{L}}\right)$$

The second term is a modification of the gravity including a scale length.
It gives rise to a further "gravitational length" like the Schwarzschild radius

If $\delta = 0$ the Newtonian potential and the standard gravitational coupling are recovered.

Assuming $1+\delta = f_1$, δ is related to $\delta_1(t)$ through

$$\delta_1 = -\frac{6GM}{L^2}\left(\frac{\delta}{1+\delta}\right)$$

Under this assumption, the scale length $\mathcal{L}$ could naturally arise and reproduce several phenomena that range from Solar System to cosmological scales.

Understanding at which scales the modifications to GeneraL Relativity are working or what is the weight of corrections to gravitational potential is a crucial point that could confirm or rule out these extended approaches to gravitational interaction.

Stellar structures and Jeans instability

It is usually assumed that the dynamics of stellar objects is completely determined by the Newton law of gravity

Considering potential corrections in strong field regimes could be another way to check the viability of Extended Theories of Gravity

In particular, stellar systems are an ideal laboratory to look for signatures of possible modifications of standard law of gravity

Some observed stellar systems are incompatible with the standard models of stellar structure : these are peculiar objects, as star in instability strips, protostars or anomalous neutron stars (the so-called "magnetars" with masses larger than their expected Volkoff mass) that could admit dynamics in agreement with modified gravity and not consistent with standard General Relativity (e.g. PSRJ 1614-2230).

Stellar structures and Jeans instability

Field equations at O (2)-order, that is at the Newtonian level, are

$$R_{tt}^{(2)} - \frac{R^{(2)}}{2} - f''(0) \triangle R^{(2)} = \mathcal{X} T_{tt}^{(0)}$$

$$-3f''(0) \triangle R^{(2)} - R^{(2)} = \mathcal{X} T^{(0)},$$

$f^n(R)=f^n(R^{(2)}+O(4))=f^n(0)+f^{n+1}(0)R^{(2)}+\ldots$

The energy-momentum tensor for a perfect fluid is

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - p g_{\mu\nu}.$$

The pressure contribution is negligible in the field equations of Newtonian approximation

$$\triangle \Phi + \frac{R^{(2)}}{2} + f''(0) \triangle R^{(2)} = -\mathcal{X}\rho$$

modified Poisson equation

$$3f''(0) \triangle R^{(2)} + R^{(2)} = -\mathcal{X}\rho,$$

S. Capozziello, M. De Laurentis Ann. Phys. 524, 545 (2012)

For $f'(R) = 0$ we have the standard Poisson equation $\triangle\Phi = -4\pi G\rho$

From the Bianchi identity we have $T^{\mu\nu}{}_{;\mu} = 0 \rightarrow \frac{\partial p}{\partial x^k} = -\frac{1}{2}(p+\epsilon)\frac{\partial \ln g_{tt}}{\partial x^k}.$

Stellar structures and Jeans instability

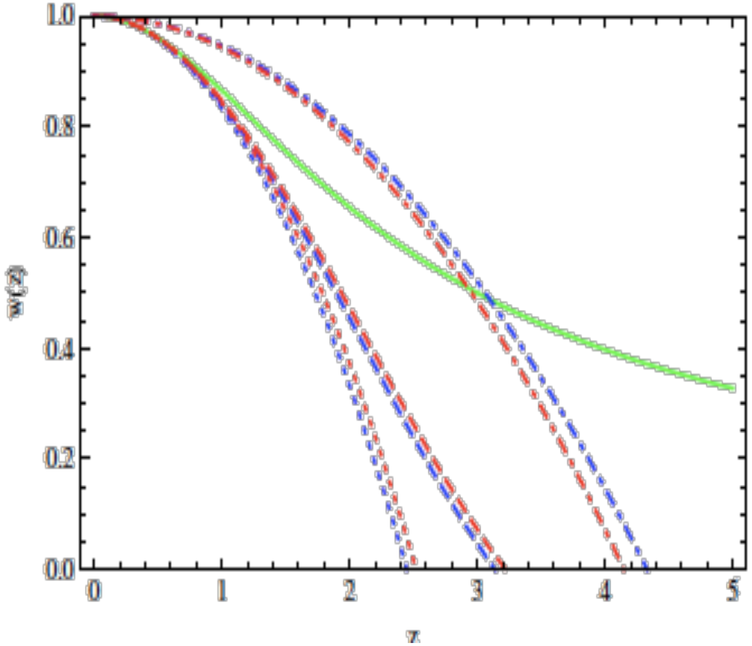
Let us suppose that matter satisfies a polytropic equation $p = K^{\gamma}\rho^{\gamma}$

We obtain an integro-differential equation for the gravitational potential, that is

$$\frac{d^2w(z)}{dz^2}+\frac{2}{z}\frac{dw(z)}{dz}+w(z)^n=\frac{m\xi_0}{8}\frac{1}{z}\int_0^{\xi/\xi_0}dz'\,z'\left\{e^{-m\xi_0|z-z'|}-e^{-m\xi_0|z+z'|}\right\}w(z')^n$$

Lané-Emden equation in f(R)-gravity

We find the radial profiles of the gravitational potential by solving for some values of n (polytropic index)



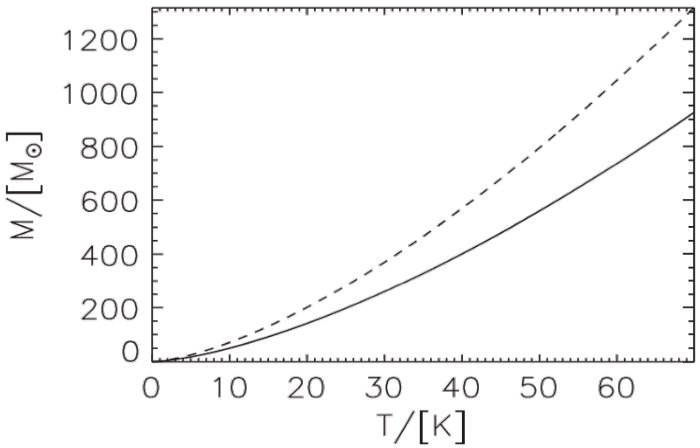
New solutions are physically relevant and could explain exotic systems out of Main Sequence (magnetars, variable stars, very massive neutron stars).

S. Capozziello, M. De Laurentis, A. Stabile, S.D. Odintsov, PRD 83, 064004, (2011)

Stellar structures and Jeans instability

We have also compared the behavior with the temperature of the Jeans mass for various types of interstellar molecular clouds

$$\tilde{M}_J = 6\sqrt{\frac{6}{(3+\sqrt{21})^3}}M_J$$



In our model the limit (in unit of mass) to start the collapse of an interstellar cloud is lower than the classical one advantaging the structure formation.

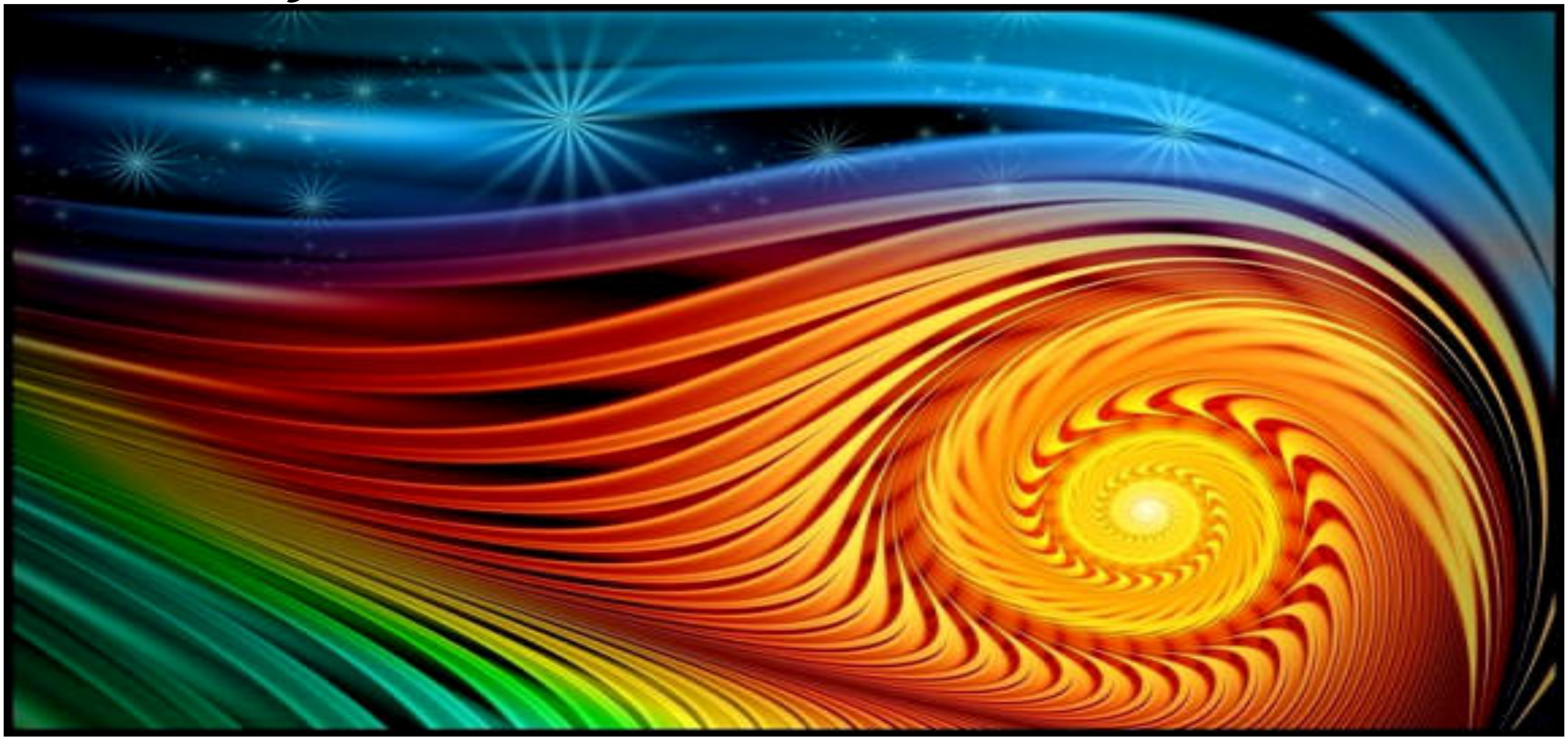
S. Capozziello, M. De Laurentis , I. De Martino, M. Formisano, S.D. Odintsov
Phys.Rev. D85 (2012) 044022

Subject	T (K)	n (10^8 m^{-3})	μ	M_J ($M_\odot$)	$\tilde{M}_J$ ($M_\odot$)
Diffuse hydrogen clouds	50	5.0	1	795.13	559.68
Diffuse molecular clouds	30	50	2	82.63	58.16
Giant molecular clouds	15	1.0	2	206.58	145.41
Bok globules	10	100	2	11.24	7.91

Addressing stellar systems by this approach could be extremely important to test observationally Extended Theories of Gravity. See e.g. Astashenok, Capozziello, Odintsov **JCAP 1312 (2013) 040, PRD 89 (2014) 103509** *where anomalous neutron stars are described by f(R)-gravity.*

Extended Theories of Gravity can also impact on the estimate of DM properties on galactic scales

Modified gravity could be a possible way to solve the cusp/core and similar problems of the DM scenario without asking for huge amounts of DM



Testing spiral galaxies

Yukawa-like corrections are a general feature, in the framework of $f(R)$-gravity

This equation $\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1+\delta e^{-\frac{r}{L}}\right)$ *is the starting point for the computation of the rotation curve of an extended system.*

Considering a general expression derived for a generic potential giving rise to a separable force

$$F_p(\mu, r) = \frac{GM_\odot}{r_s^2} f_\mu(\mu) f_r(\eta)$$

With $\mu = M/M_\odot$, $\eta = r/r_s$ *and* $(M_\odot, r_s)$ *the Solar mass and a characteristic length of the problem*

In our case, $f\mu = 1$ *and:*

$$f_r(\eta) = \left(1 + \frac{\eta}{\eta_L}\right) \frac{\exp(-\eta/\eta_L)}{(1+\delta)\eta^2}$$

With $\eta_L = L/r_s$

Testing spiral galaxies

Using cylindrical coordinates (R, θ , z) and the corresponding dimensionless variables (η , θ , ξ) (with ξ = z/rs), the total force then reads:

$$F(\mathbf{r}) \quad = \frac{G\rho_0 r_s}{1+\delta} \int_0^{\infty} \eta' d\eta' \int_{-\infty}^{\infty} d\zeta' \int_0^{\pi} f_r(\Delta)\tilde{\rho}(\eta', \theta', \zeta') d\theta'$$

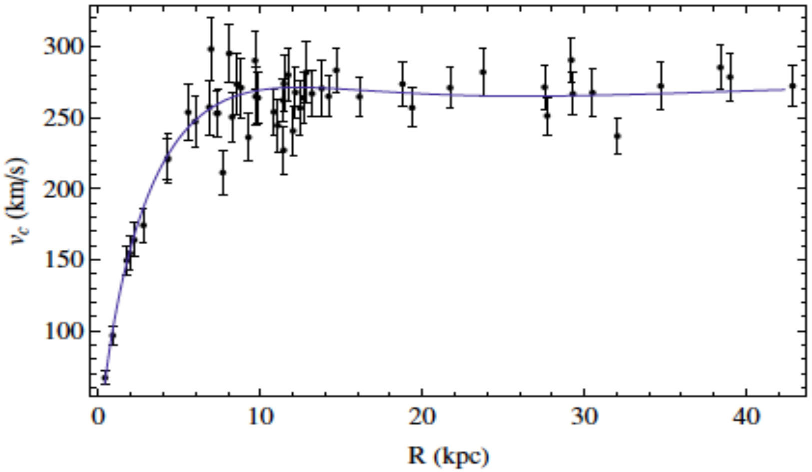
With $\tilde{\rho}$ = ρ / ρ_0, ρ_0 a reference density, we have

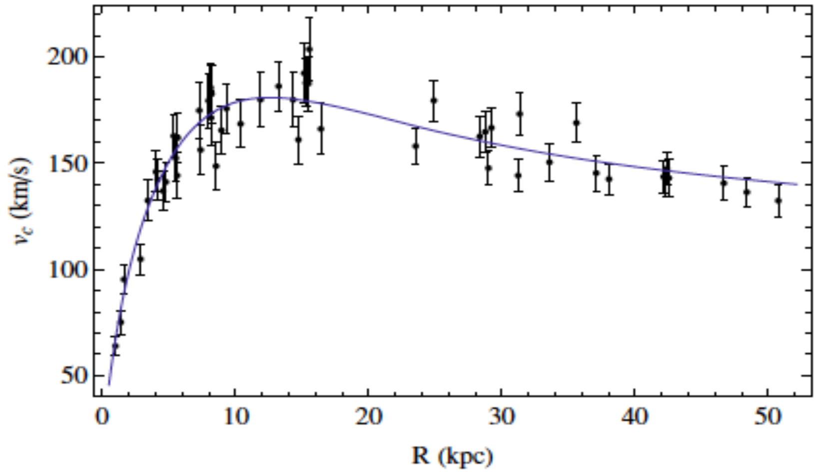
$$\Delta = \left[\eta^2 + \eta'^2 - 2\eta\eta' \cos(\theta - \theta') + (\zeta - \zeta')^2\right]^{1/2}$$

For obtaining axisymmetric systems, one can set $\tilde{\rho} = \tilde{\rho}(\eta, \xi)$.

S. Capozziello, M. De Laurentis Ann. Phys. 524, 545 (2012)

Testing spiral galaxies





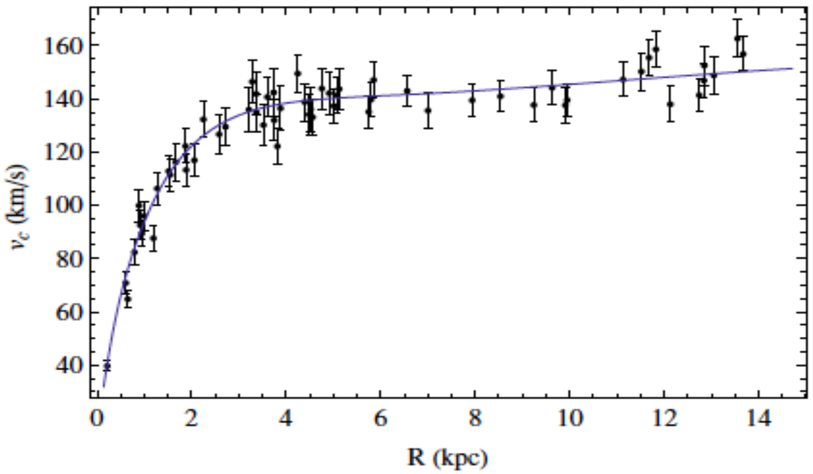


Figure 4 Examples of simulated rotation curves with superimposed theoretical curves. From left to right, model parameters are $(\log M_d, \log M_{\rm vir}, c, f_{DM}, \log \eta_L) =$ (11.15, 12.90, 10.24, 0.47, 0.36),(10.90, 11.76, 14.77, 0.45, −0.92), (10.04, 12.10, 13.76, 0.54, 1.11), while the simulation parameters are set as discussed in the text. Note that, depending on how the model parameters are set, it is possible to get rotation curves which are flat, decreasing or increasing in the outer region.

Testing elliptical galaxies

The modified potential can be tested also for elliptical galaxies checking whether it is able to provide a reasonable match to their kinematics.

Ellipticals are very different with respect to spirals so addressing both classes of objects under the same standard could be a fundamental step versus DM.

One may construct equilibrium models based on the solution of the radial Jeans equation to interpret the kinematics of planetary nebulae

We use the inner long slit data and the extended planetary nebulae kinematics for three galaxies which have published dynamical analyses within DM halo framework (see Napolitano, Capozziello, Capaccioli, Romanowski ApJ 748 (2012) 87).

They are:
NGC 3379 , (DL +09) , NGC 4494 N +09 , NGC 4374 (N + 11).

Testing elliptical galaxies

It is shown the circular velocity of the modified potentiaL as a function of the potential parameters L and δ for NGC 4494 and NGC 4374.

From a theoretical point of view, δ is a free parameter that can assume positive and negative values. Comparing results for spirals and ellipticals, it is clear that the morphology of these two classes of systems strictly depends on the sign and the value of δ .

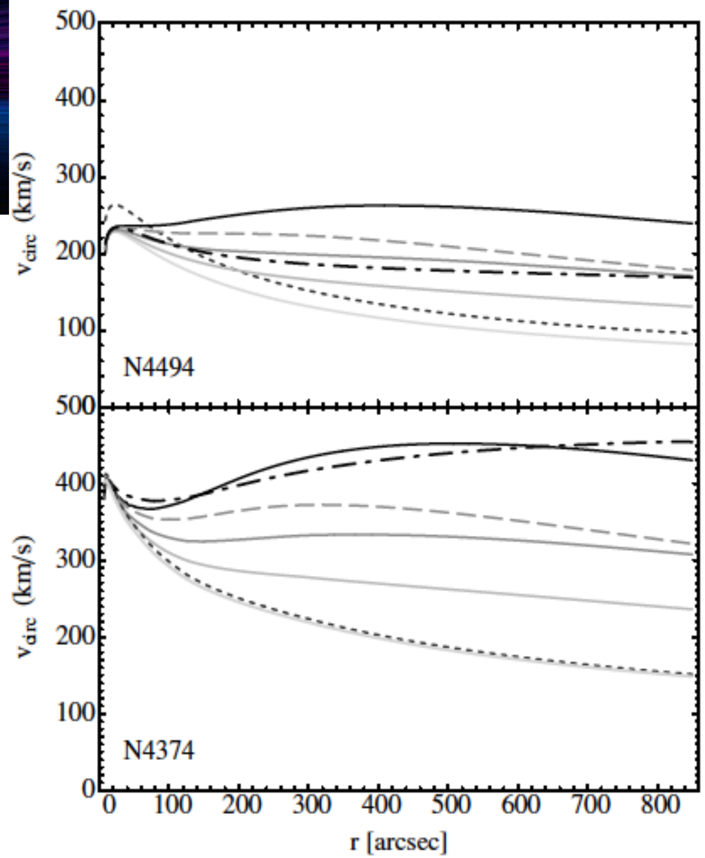


Figure 6 Circular velocity produced by the modified potential for the two galaxies N4494 (top) and N4374 (bottom). In both cases the $M/\mathscr{L}_*$ has been fixed to some fiducial value (as expected from stellar population models and Kroupa 2001 IMF): $M/\mathscr{L}_* = 4.3\Upsilon_{\odot,B}$ for NGC 4494 and $M/\mathscr{L}_* = 5.5\Upsilon_{\odot,V}$ for NGC 4374. The potential parameters adopted are: $L = 250''$ and δ=0, -0.65, -0.8, -0.9 (lighter to darker solid lines) and $L = 180''$ and δ=-0.8 (dashed lines). The dotted line is a case with positive coefficient of the Yukawa-like term and $L = 5000''$ which illustrates that positive δ cannot produce flat circular velocity curves. Finally some reference Navarro-Frenk-White (NFW) models are shown as dot-dashed lines [108].

Testing elliptical galaxies

The overall match of the model curves with data is remarkably good and it is comparable with models obtained with DM modeling (gray lines)

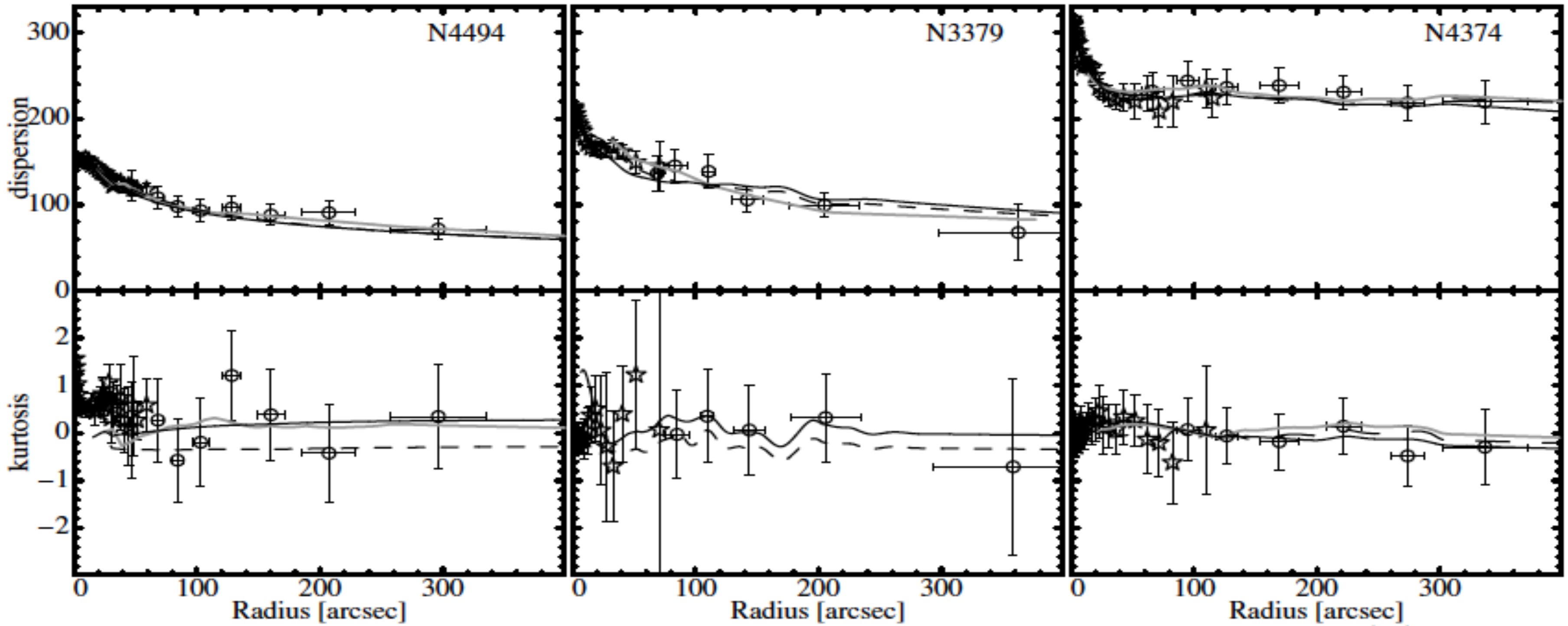


Figure 7 Dispersion in kms (top) and kurtosis (bottom) fit of the galaxy sample for the different $f(R)$ parameter sets: the anisotropic solution (solid lines) is compared with the isotropic case (dashed line – for NGC 4374 and NGC 4494 this is almost indistinguishable from the anisotropic case). From the left, NGC 4494, NGC 3379 and NGC 4374 are shown with DM models as gray lines from N+09, DL+09 (no kurtosis is provided), and N+11 respectively [108].

Modeling clusters of galaxies

A fundamental issue is related to clusters and superclusters of galaxies.

Such structures, essentially, rule the large scale structure, and are the intermediate step between galaxies and cosmology.

As the galaxies, they appear DM dominated but the distribution of DM component seems clustered and organized in a very different way with respect to galaxies. It seems that DM is ruled by the scale and also its fundamental nature could depend on the scale

Our goal is to reconstruct the mass profile of clusters without DM adopting the same strategy as above where DM effects are figured out by corrections to the Newton potential

Modeling clusters of galaxies

Standard Cluster Model: spherical mass distribution in hydrostatic equilibrium

- *Boltzmann equation:*

$$-\frac{d\Phi}{dr} = \frac{kT(r)}{\mu m_p r}\left[\frac{d\ln\rho_{gas}(r)}{d\ln r} + \frac{d\ln T(r)}{d\ln r}\right]$$

- *Newton classical approach:*

$$\begin{cases} \phi(r) = -\dfrac{GM}{r} \\ \rho_{cl,EC}(r) = \rho_{dark} + \rho_{gas}(r) + \rho_{gal}(r) + \rho_{CDgal}(r) \end{cases}$$

- *f(R) approach:*

$$\begin{cases} \phi(r) = -\dfrac{3GM}{4a_1 r}\left(1 + \dfrac{1}{3}e^{-\frac{r}{L}}\right) \\ \rho_{cl,EC}(r) = \rho_{gas}(r) + \rho_{gal}(r) + \rho_{CDgal}(r) \end{cases}$$

- *Rearranging the Boltzmann equation:*

$$\phi_N(r) = -\frac{3GM}{4a_1 r}$$

$$\phi_C(r) = -\frac{GM}{4a_1}\frac{e^{-\frac{r}{L}}}{r}$$

$$\begin{cases} M_{bar,th}(r) = \dfrac{4a_1}{3}\left[-\dfrac{kT(r)}{\mu m_p G}r\left(\dfrac{d\ln\rho_{gas}(r)}{d\ln r} + \dfrac{d\ln T(r)}{d\ln r}\right)\right] - \dfrac{4a_1}{3G}r^2\dfrac{d\Phi_C}{dr}(r) \\ M_{bar,obs}(r) = M_{gas}(r) + M_{gal}(r) + M_{CDgal}(r) \end{cases}$$

Modeling clusters of galaxies

Fitting mass Profile with data:

- Sample: 12 clusters from Chandra (Vikhlinin 2005, 2006)
- Temperature profile from spectroscopy
- Gas density: modified beta-model

$$n_p n_e = n_0^2 \cdot \frac{(r/r_c)^{-\alpha}}{(1+r^2/r_c^2)^{3\beta-\alpha/2}} \cdot \frac{1}{(1+r^\gamma/r_s^\gamma)^{\epsilon/\gamma}} + \frac{n_{02}^2}{(1+r^2/r_{c2}^2)^{3\beta_2}}$$

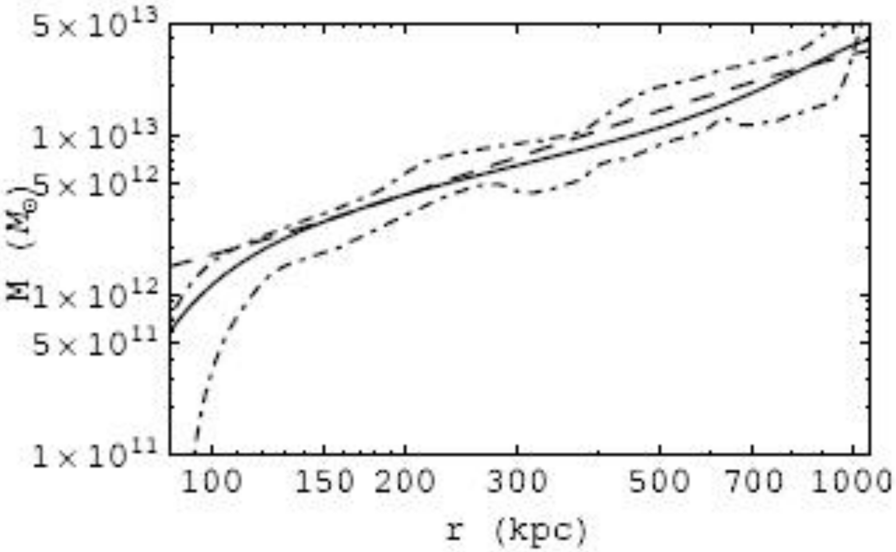
- Galaxy density:

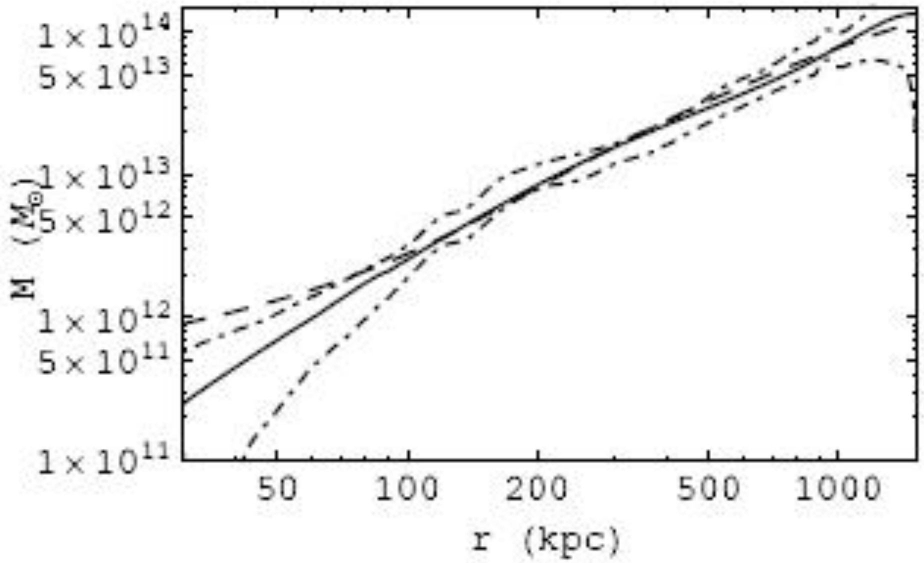
$$\rho_{gal}(r) = \begin{cases} \rho_{gal,1} \cdot \left[1+\left(\frac{r}{R_c}\right)^2\right]^{-\frac{3}{2}} & r < R_c \\ \rho_{gal,2} \cdot \left[1+\left(\frac{r}{R_c}\right)^2\right]^{-\frac{2.6}{2}} & r > R_c \end{cases} \qquad \rho_{CDgal} = \frac{\rho_{0,J}}{\left(\frac{r}{r_c}\right)^2\left(1+\frac{r}{r_c}\right)^2}$$

Table 1. Column 1: Cluster name. Column2: Richness. Column 2: cluster total mass. Column 3: gas mass. Column 4: galaxy mass. Column 5: cD-galaxy mass. All mass values are estimated at $r = r_{max}$. Column 6: ratio of total galaxy mass to gas mass. Column 7: minimum radius. Column 8: maximum radius.

name	R	$M_{cl,N}$ ($M_\odot$)	M_{gas} ($M_\odot$)	M_{gal} ($M_\odot$)	M_{cDgal} ($M_\odot$)	$\frac{gal}{gas}$	r_{min} (kpc)	r_{max} (kpc)
A133	0	$4.35874 \cdot 10^{14}$	$2.73866 \cdot 10^{13}$	$5.20269 \cdot 10^{12}$	$1.10568 \cdot 10^{12}$	0.23	86	1060
A262	0	$4.45081 \cdot 10^{13}$	$2.76659 \cdot 10^{12}$	$1.71305 \cdot 10^{11}$	$5.16382 \cdot 10^{12}$	0.25	61	316
A383	2	$2.79785 \cdot 10^{14}$	$2.82467 \cdot 10^{13}$	$5.88048 \cdot 10^{12}$	$1.09217 \cdot 10^{12}$	0.25	52	751
A478	2	$8.51832 \cdot 10^{14}$	$1.05583 \cdot 10^{14}$	$2.15567 \cdot 10^{13}$	$1.67513 \cdot 10^{12}$	0.22	59	1580
A907	1	$4.87657 \cdot 10^{14}$	$6.38070 \cdot 10^{13}$	$1.34129 \cdot 10^{13}$	$1.66533 \cdot 10^{12}$	0.24	563	1226
A1413	3	$1.09598 \cdot 10^{15}$	$9.32466 \cdot 10^{13}$	$2.30728 \cdot 10^{13}$	$1.67345 \cdot 10^{12}$	0.26	57	1506
A1795	2	$1.24313 \cdot 10^{14}$	$1.00530 \cdot 10^{13}$	$4.23211 \cdot 10^{12}$	$1.93957 \cdot 10^{12}$	0.11	79	1151
A1991	1	$1.24313 \cdot 10^{14}$	$1.00530 \cdot 10^{13}$	$1.24608 \cdot 10^{12}$	$1.08241 \cdot 10^{12}$	0.23	55	618
A2029	2	$8.92392 \cdot 10^{14}$	$1.24129 \cdot 10^{14}$	$3.21543 \cdot 10^{13}$	$1.11921 \cdot 10^{12}$	0.27	62	1771
A2390	1	$2.09710 \cdot 10^{15}$	$2.15726 \cdot 10^{14}$	$4.91580 \cdot 10^{13}$	$1.12141 \cdot 10^{12}$	0.23	83	1984
MKW4	-	$4.69503 \cdot 10^{13}$	$2.83207 \cdot 10^{12}$	$1.71153 \cdot 10^{11}$	$5.29855 \cdot 10^{11}$	0.25	60	434
RXJ1159	-	$8.97997 \cdot 10^{13}$	$4.33256 \cdot 10^{12}$	$7.34414 \cdot 10^{11}$	$5.38799 \cdot 10^{11}$	0.29	64	568

Modeling clusters of galaxies





- Differences between theoretical and observed fit less than 5%
- Typical scale in [100; 150] kpc range where is a turning-point:
  - Break in the hydrostatic equilibrium
  - Limits in the expansion series of f(R): $R - R_0 << \frac{a_1}{a_2}$ in the range [19;200] kpc

  Proper gravitational scale (as for galaxies, see Capozziello et al MNRAS 2007)

- Similar issues in Metric-Skew-Tensor-Gravity (Brownstein, 2006): we have better and more detailed approach

Modeling clusters of galaxies

Results

name	a_1	$[a_1 - 1\sigma,\ a_1 + 1\sigma]$	a_2 (kpc^2)	$[a_2 - 1\sigma,\ a_2 + 1\sigma]$ (kpc^2)	L (kpc)	$[L - 1\sigma,\ L + 1\sigma]$ (kpc)
A133	0.085	[0.078, 0.091]	$-4.98 \cdot 10^3$	$[-2.38 \cdot 10^4,\ -1.38 \cdot 10^3]$	591.78	[323.34, 1259.50]
A262	0.065	[0.061, 0.071]	-10.63	$[-57.65,\ -3.17]$	31.40	[17.28, 71.10]
A383	0.099	[0.093, 0.108]	$-9.01 \cdot 10^2$	$[-4.10 \cdot 10^3,\ -3.14 \cdot 10^2]$	234.13	[142.10, 478.06]
A478	0.117	[0.114, 0.122]	$-4.61 \cdot 10^3$	$[-1.01 \cdot 10^4,\ -2.51 \cdot 10^3]$	484.83	[363.29, 707.73]
A907	0.129	[0.125, 0.136]	$-5.77 \cdot 10^3$	$[-1.54 \cdot 10^4,\ -2.83 \cdot 10^3]$	517.30	[368.84, 825.00]
A1413	0.115	[0.110, 0.119]	$-9.45 \cdot 10^4$	$[-4.26 \cdot 10^5,\ -3.46 \cdot 10^4]$	2224.57	[1365.40, 4681.21]
A1795	0.093	[0.084, 0.103]	$-1.54 \cdot 10^3$	$[-1.01 \cdot 10^4,\ -2.49 \cdot 10^2]$	315.44	[133.31, 769.17]
A1991	0.074	[0.072, 0.081]	-50.69	$[-3.42 \cdot 10^2,\ -13]$	64.00	[32.63, 159.40]
A2029	0.129	[0.123, 0.134]	$-2.10 \cdot 10^4$	$[-7.95 \cdot 10^4,\ -8.44 \cdot 10^3]$	988.85	[637.71, 1890.07]
A2390	0.149	[0.146, 0.152]	$-1.40 \cdot 10^6$	$[-5.71 \cdot 10^6,\ -4.46 \cdot 10^5]$	7490.80	[4245.74, 15715.60]
MKW4	0.054	[0.049, 0.060]	-23.63	$[-1.15 \cdot 10^2,\ -8.13]$	51.31	[30.44, 110.68]
RXJ1159	0.048	[0.047, 0.052]	-18.33	$[-1.35 \cdot 10^2,\ -4.18]$	47.72	[22.86, 125.96]

Modeling clusters of galaxies

name	a_1	$[a_1 - 1\sigma,\ a_1 + 1\sigma]$	a_2 (kpc^2)	$[a_2 - 1\sigma,\ a_2 + 1\sigma]$ (kpc^2)	L (kpc)	$[L - 1\sigma,\ L + 1\sigma]$ (kpc)
A133	0.085	[0.078, 0.091]	$-4.98 \cdot 10^3$	$[-2.38 \cdot 10^4,\ -1.38 \cdot 10^3]$	591.78	[323.34, 1259.50]
A262	0.065	[0.061, 0.071]	-10.63	$[-57.65,\ -3.17]$	31.40	[17.28, 71.10]
A383	0.099	[0.093, 0.108]	$-9.01 \cdot 10^2$	$[-4.10 \cdot 10^3,\ -3.14 \cdot 10^2]$	234.13	[142.10, 478.06]
A478	0.117	[0.114, 0.122]	$-4.61 \cdot 10^3$	$[-1.01 \cdot 10^4,\ -2.51 \cdot 10^3]$	484.83	[363.29, 707.73]
A907	0.129	[0.125, 0.136]	$-5.77 \cdot 10^3$	$[-1.54 \cdot 10^4,\ -2.83 \cdot 10^3]$	517.30	[368.84, 825.00]
A1413	0.115	[0.110, 0.119]	$-9.45 \cdot 10^4$	$[-4.26 \cdot 10^5,\ -3.46 \cdot 10^4]$	2224.57	[1365.40, 4681.21]
A1795	0.093	[0.084, 0.103]	$-1.54 \cdot 10^3$	$[-1.01 \cdot 10^4,\ -2.49 \cdot 10^2]$	315.44	[133.31, 769.17]
A1991	0.074	[0.072, 0.081]	-50.69	$[-3.42 \cdot 10^2,\ -13]$	64.00	[32.63, 159.40]
A2029	0.129	[0.123, 0.134]	$-2.10 \cdot 10^4$	$[-7.95 \cdot 10^4,\ -8.44 \cdot 10^3]$	988.85	[637.71, 1890.07]
A2390	0.149	[0.146, 0.152]	$-1.40 \cdot 10^6$	$[-5.71 \cdot 10^6,\ -4.46 \cdot 10^5]$	7490.80	[4245.74, 15715.60]
MKW4	0.054	[0.049, 0.060]	-23.63	$[-1.15 \cdot 10^2,\ -8.13]$	51.31	[30.44, 110.68]
RXJ1159	0.048	[0.047, 0.052]	-18.33	$[-1.35 \cdot 10^2,\ -4.18]$	47.72	[22.86, 125.96]

Modeling clusters of galaxies

- Gravitational length: $$L \equiv L(a_1, a_2) = \left(-\frac{6a_2}{a_1}\right)^{1/2}$$

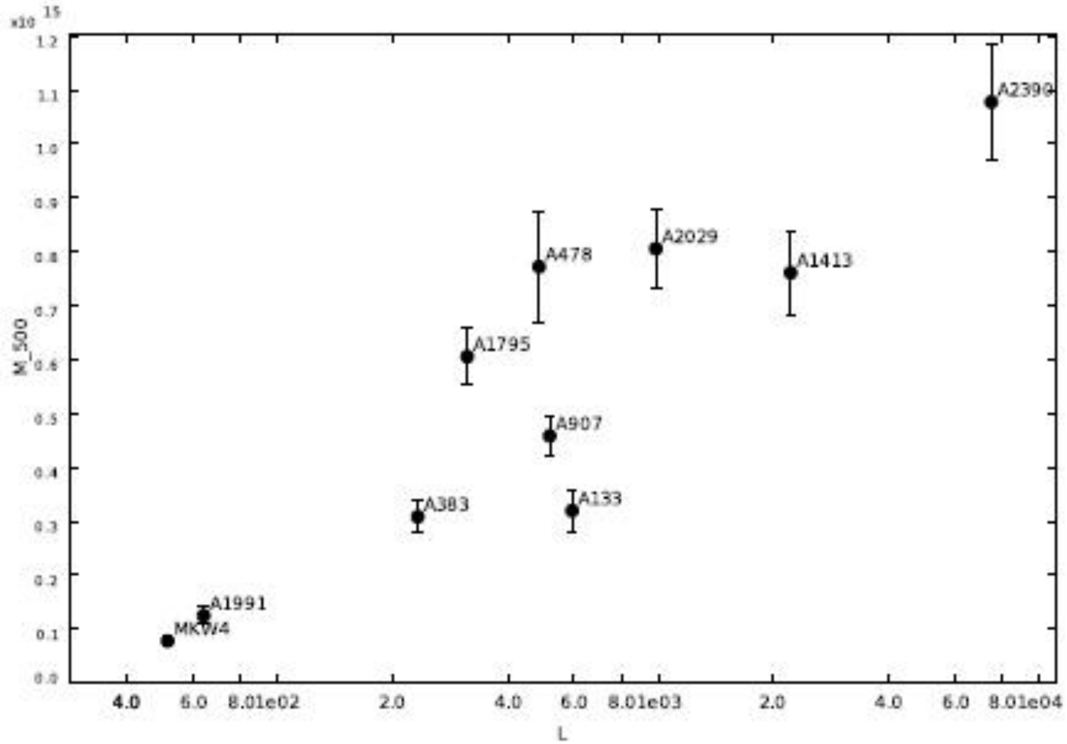
Strong characterization of Gravitational potential

- Mean length:

$$< L >_\rho \quad = \quad 318 \text{ kpc} \qquad < a_2 >_\rho = -3.40 \cdot 10^4$$
$$< L >_M \quad = \quad 2738 \text{ kpc} \qquad < a_2 >_M = -4.15 \cdot 10^5$$

- Strongly related to virial mass (the same for gas mass):

- Strongly related to average temperature:



Cosmography

GR based models vs f(R) gravity

Agreement with Data...

How can we discriminate?

- No a priori dynamical model = Model Independent Approach;
- Robertson – Walker metric;
- Expansion series of the scale factor with respect to cosmic time:

$$\frac{a(t)}{a(t_0)} = 1 + H_0(t-t_0) - \frac{q_0}{2}H_0^2(t-t_0)^2 + \frac{j_0}{3!}H_0^3(t-t_0)^3 + \frac{s_0}{4!}H_0^4(t-t_0)^4 + \frac{l_0}{5!}H_0^5(t-t_0)^5 + O[(t-t_0)^6$$

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2} \qquad j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3} \qquad s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4} \qquad l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$$

Deceleration Jerk Snap Lerk

Expansion up to fifth order :

$$\begin{cases} \text{error on } d_L(z) \text{ less than } 10\% \text{ up to } z = 1 \\ \text{error on } \mu(z) \text{ less than } 3\% \text{ up to } z = 2 \end{cases}$$

Cosmography with f(R)- gravity

- Definition: $H(t) = \frac{1}{a}\frac{da}{dt}, \; q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2}, \; j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3}, \; s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4}, \; l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$

- Derivatives of H(t):

$$\dot{H} = -H^2(1+q)$$

$$\ddot{H} = H^3(j+3q+2)$$

$$d^3H/dt^3 = H^4\left[s-4j-3q(q+4)-6\right]$$

$$d^4H/dt^4 = H^5\left[l-5s+10(q+2)j+30(q+2)q+24\right]$$

- Derivatives of scalar curvature:

$$R_0 = -6H_0^2(1-q_0)$$

$$\dot{R}_0 = -6H_0^3(j_0-q_0-2)$$

$$R = -6(\dot{H}+2H^2)$$

$$\ddot{R}_0 = -6H_0^4\left(s_0+q_0^2+8q_0+6\right)$$

$$d^3R_0/dt^3 = -6H_0^5\left[l_0-s_0+2(q_0+4)j_0-6(3q_0+8)q_0-24\right]$$

Cosmography with f(R)-gravity

- 1st Friedmann eq. :

$$H_0^2 = \frac{H_0^2 \Omega_M}{f'(R_0)} + \frac{f(R_0) - R_0 f'(R_0) - 6H_0 \dot{R}_0 f''(R_0)}{6f'(R_0)},$$

- 2nd Friedmann eq. :

$$-\dot{H}_0 = \frac{3H_0^2 \Omega_M}{2f'(R_0)} + \frac{\dot{R}_0^2 f'''(R_0) + (\ddot{R}_0 - H_0 \dot{R}_0) f''(R_0)}{2f'(R_0)}$$

- Derivative of 2nd Friedmann eq. :

$$\begin{aligned} \ddot{H} &= \frac{\dot{R}^2 f'''(R) + \left(\ddot{R} - H\dot{R}\right) f''(R) + 3H_0^2 \Omega_M a^{-3}}{2\left[\dot{R} f''(R)\right]^{-1} [f'(R)]^2} - \frac{\dot{R}^3 f^{(iv)}(R) + \left(3\dot{R}\ddot{R} - H\dot{R}^2\right) f'''(R)}{2f'(R)} \\ &- \frac{\left(d^3R/dt^3 - H\ddot{R} + \dot{H}\dot{R}\right) f''(R) - 9H_0^2 \Omega_M H a^{-3}}{2f'(R)} \end{aligned}$$

- Constraint from gravitational constant:

$$H^2 = \frac{8\pi G}{3f'(R)} [\rho_m + \rho_{\rm curv} f'(R)]$$

$$G_{\rm eff}(z=0) = G \to f'(R_0) = 1.$$

Cosmography with f(R)-gravity

- Final solutions:

$$\frac{f(R_0)}{6H_0^2} = -\frac{\mathcal{P}_0(q_0, j_0, s_0, l_0)\Omega_M + \mathcal{Q}_0(q_0, j_0, s_0, l_0)}{\mathcal{R}(q_0, j_0, s_0, l_0)}$$

$$f'(R_0) = 1$$

$$\frac{f''(R_0)}{(6H_0^2)^{-1}} = -\frac{\mathcal{P}_2(q_0, j_0, s_0)\Omega_M + \mathcal{Q}_2(q_0, j_0, s_0)}{\mathcal{R}(q_0, j_0, s_0, l_0)}$$

$$\frac{f'''(R_0)}{(6H_0^2)^{-2}} = -\frac{\mathcal{P}_3(q_0, j_0, s_0, l_0)\Omega_M + \mathcal{Q}_3(q_0, j_0, s_0, l_0)}{(j_0 - q_0 - 2)\mathcal{R}(q_0, j_0, s_0, l_0)}$$

- Taylor expansion f(R) in series of R up to third order (higher not necessary)

- Linear equations in f(R) and derivatives

- Ω_M is model dependent:

$$\Omega_M = 0.041$$

$$\Omega_M = 0.250.$$

f(R) derivatives and CPL models

"Precision cosmology" $\nRightarrow$ Values of cosmographic parameters?

Cosmographic parameters $\Rightarrow$ Dark energy parameters = equivalent f(R)

CPL approach:
(Chevallier, Polarski, Linder)

$$w = w_0 + w_a(1-a) = w_0 + w_a z(1+z)^{-1}$$

Cosmographic parameters:

$$\left\{\begin{aligned}
q_0 &= \frac{1}{2} + \frac{3}{2}(1-\Omega_M)w_0 \\
j_0 &= 1 + \frac{3}{2}(1-\Omega_M)\left[3w_0(1+w_0)+w_a\right] \\
s_0 &= -\frac{7}{2} - \frac{33}{4}(1-\Omega_M)w_a - \frac{9}{4}(1-\Omega_M)\left[9+(7-\Omega_M)w_a\right]w_0 + \\
&\quad - \frac{9}{4}(1-\Omega_M)(16-3\Omega_M)w_0^2 - \frac{27}{4}(1-\Omega_M)(3-\Omega_M)w_0^3 \\
l_0 &= \frac{35}{2} + \frac{1-\Omega_M}{4}\left[213+(7-\Omega_M)w_a\right]w_a + \frac{(1-\Omega_M)}{4}\left[489+9(82-21\Omega_M)w_a\right]w_0 + \\
&\quad + \frac{9}{2}(1-\Omega_M)\left[67-21\Omega_M+\frac{3}{2}(23-11\Omega_M)w_a\right]w_0^2 + \frac{27}{4}(1-\Omega_M)(47-24\Omega_M)w_0^3 + \\
&\quad + \frac{81}{2}(1-\Omega_M)(3-2\Omega_M)w_0^4
\end{aligned}\right.$$

CPL Cosmography and f(R): the ΛCDM Model

ΛCDM model: $(w_0, w_a) = (-1, 0)$

$$q_0 = \frac{1}{2} - \frac{3}{2}\Omega_M; \quad j_0 = 1; \quad s_0 = 1 - \frac{9}{2}\Omega_M; \quad l_0 = 1 + 3\Omega_M + \frac{27}{2}\Omega_M^2$$

$$f(R_0) = R_0 + 2\Lambda, \qquad f''(R_0) = f'''(R_0) = 0,$$

ΛCDM fits well many data ⇒ cosmographic values strictly depend on Ω_M

$$q_0 = q_0^\Lambda \times (1 + \varepsilon_q), \qquad j_0 = j_0^\Lambda \times (1 + \varepsilon_j),$$
$$s_0 = s_0^\Lambda \times (1 + \varepsilon_s), \qquad l_0 = l_0^\Lambda \times (1 + \varepsilon_l),$$

$$\eta_{20} = f''(R_0)/f(R_0) \times H_0^4$$

$$\eta_{30} = f'''(R_0)/f(R_0) \times H_0^6$$

$$\left\{ \begin{array}{l} \eta_{20} = \dfrac{64 - 6\Omega_M(9\Omega_M + 8)}{[3(9\Omega_M + 74)\Omega_M - 556]\,\Omega_M^2 + 16} \times \dfrac{\varepsilon}{27} \\[2ex] \eta_{30} = \dfrac{6\,[(81\Omega_M - 110)\Omega_M + 40]\,\Omega_M + 16}{[3(9\Omega_M + 74)\Omega_M - 556]\,\Omega_M^2 + 16} \times \dfrac{\varepsilon}{243\Omega_M^2} \\[2ex] \begin{cases} \eta_{20} \simeq 0.15 \times \varepsilon & \text{for} \quad \Omega_M = 0.041 \\ \eta_{20} \simeq -0.12 \times \varepsilon & \text{for} \quad \Omega_M = 0.250 \end{cases} \\[2ex] \begin{cases} \eta_{30} \simeq 4 \times \varepsilon & \text{for} \quad \Omega_M = 0.041 \\ \eta_{30} \simeq -0.18 \times \varepsilon & \text{for} \quad \Omega_M = 0.250 \end{cases} \end{array} \right.$$

Constraining f(R) models by Cosmography

- Procedure:
  1. Estimate (q(0), j(0), s(0), l(0)) observationally
  2. Compute f(Ro), f '(Ro), f " (Ro), f '''(Ro)
  3. Solve for f(R) parameters from derivatives
  4. Constraint f(R) models

- e.g. Double Power-Law: $f(R) = R(1 + \alpha R^n + \beta R^{-m})$

$$
\begin{cases}
f(R_0) = R_0(1 + \alpha R_0^n + \beta R_0^{-m}) \\
f'(R_0) = 1 + \alpha(n+1)R_0^n - \beta(m-1)R_0^{-m} \\
f''(R_0) = \alpha n(n+1)R_0^{n-1} + \beta m(m-1)R_0^{-(1+m)} \\
f'''(R_0) = \alpha n(n+1)(n-1)R_0^{n-2} \\
\qquad\qquad - \beta m(m+1)(m-1)R_0^{-(2+m)}.
\end{cases}
$$

$$
\begin{cases}
\alpha = \frac{1-m}{n+m}\left(1 - \frac{\phi_0}{R_0}\right)R_0^{-n} \\
\beta = -\frac{1+n}{n+m}\left(1 - \frac{\phi_0}{R_0}\right)R_0^{m},
\end{cases}
$$

$$
\begin{cases}
\alpha = \frac{\phi_2 R_0^{1-n}[1+m+(\phi_3/\phi_2)R_0]}{n(n+1)(n+m)} \\
\beta = \frac{\phi_2 R_0^{1+n}[1-n+(\phi_3/\phi_2)R_0]}{m(1-m)(n+m)}.
\end{cases}
$$

$$
\begin{cases}
\frac{n(n+1)(1-m)(1-\phi_0/R_0)}{\phi_2 R_0[1+m+(\phi_3/\phi_2)R_0]} = 1 \\
\frac{m(n+1)(m-1)(1-\phi_0/R_0)}{\phi_2 R_0[1-n+(\phi_3/\phi_2)R_0]} = 1.
\end{cases}
$$

$$
\begin{cases}
m = -[1 - n + (\phi_3/\phi_2)R_0] \\
n = \frac{1}{2}\left[1 + \frac{\phi_3}{\phi_2}R_0 \pm \frac{\sqrt{\mathcal{N}(\phi_0, \phi_2, \phi_3)}}{\phi_2 R_0(1 + \phi_0/R_0)}\right]
\end{cases}
$$

Constraining f(R) models by Cosmography

- Cosmographic parameters from SNeIa:
  What we have to expect from data

$$q_0 = -0.90 \pm 0.65, \qquad j_0 = 2.7 \pm 6.7,$$
$$s_0 = 36.5 \pm 52.9, \qquad l_0 = 142.7 \pm 320.$$

- Fisher information matrix method:

$$F_{ij} = \left\langle \frac{\partial^2 L}{\partial \theta_i \partial \theta_j} \right\rangle$$

- FM ingredients :

$$\begin{cases} \chi^2(H_0, \mathbf{p}) = \sum_{n=1}^{\mathcal{N}_{SNeIa}} \left[\frac{\mu_{obs}(z_i) - \mu_{th}(z_n, H_0, \mathbf{p})}{\sigma_i(z_i)} \right]^2 \\ d_L(z) = \mathcal{D}_L^0 z + \mathcal{D}_L^1 \; z^2 + \mathcal{D}_L^2 \; z^3 + \mathcal{D}_L^3 \; z^4 + \mathcal{D}_L^4 \; z^5 \\ \sigma(z) = \sqrt{\sigma_{\rm sys}^2 + \left(\frac{z}{z_{\rm max}}\right)^2 \sigma_m^2} \end{cases}$$

- Estimating error on g:

$$\sigma_g^2 = \left| \frac{\partial g}{\partial \Omega_M} \right|^2 \sigma_M^2 + \sum_{i=1}^{i=4} \left| \frac{\partial g}{\partial p_i} \right|^2 \sigma_{p_i}^2 + \sum_{i \neq j} 2 \frac{\partial g}{\partial p_i} \frac{\partial g}{\partial p_j} C_{ij}$$

- Survey: Davis (2007)

$\sigma_M/\Omega_M = 10\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 2000$; $\sigma_m = 0.33$

$z_{max} = 1.7$

$\sigma_1 = 0.38$

$\sigma_2 = 5.4$

$\sigma_3 = 28.1$

$\sigma_4 = 74.0$

$\sigma_{20} = 0.04$

$\sigma_{30} = 0.04$

- Snap like survey:

$\sigma_M/\Omega_M = 1\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 2000$; $\sigma_m = 0.02$

$z_{max} = 1.7$

$\sigma_1 = 0.08$

$\sigma_2 = 1.0$

$\sigma_3 = 4.8$

$\sigma_4 = 13.7$

$\sigma_{20} = 0.007$

$\sigma_{30} = 0.008$

- Ideal PanSTARRS survey:

$\sigma_M/\Omega_M = 0.1\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 60000$; $\sigma_m = 0.02$

$z_{max} = 1.7$

$\sigma_1 = 0.02$

$\sigma_2 = 0.2$

$\sigma_3 = 0.9$

$\sigma_4 = 2.7$

$\sigma_{20} = 0.0015$

$\sigma_{30} = 0.0016$

Conclusions (DE)

- *Extended Gravity seems a viable approach to describe the Dark Side of the Universe. It is based on a straightforward generalization of Einstein Gravity and does not account for exotic fluids.*
- *Following Starobinsky, R can be considered a "geometric" scalar field!).*
- *Comfortable results are obtained by matching the theory with data (SNeIa, Radio-galaxies, Age of the Universe, CMBR).*
- *Transient dust-like Friedman solutions evolving in de Sitter- like expansion (DE) at late times are particularly interesting (debated issue).*
- *Generic quintessential and DE models can be easily "mimicked" by f(R) through an inverse scattering procedure. Cosmography.*
- *A comprehensive cosmological model from early to late epochs should be achieved by f(R). LSS issues have to be carefully addressed.*

Conclusions (DM)

- Rotation curves of galaxies can be naturally reproduced, without huge amounts of DM, thanks to the corrections to the Newton potential, which come out in the low energy limit.
- The baryonic Tully- Fisher relation has a natural explanation in the framework of f(R) theories.
- Effective haloes of elliptical galaxies are reproduced by the same mechanism..
- Good evidences also for galaxy clusters

Furthermore…..

- Orbital period for PSR 1913 + 16 and other binary systems in agreement with f(R)-gravity (probe for massive GWs?).
- Exotic stellar structures could be compatible with f(R)..
- Search for EXPERIMENTUM CRUCIS

Perspectives:

DE & DM as curvature effects

- Matching other DE models
- Jordan Frame and Einstein Frame
- Systematic studies of rotation curves for other galaxies
- Galaxy cluster dynamics (virial theorem, SZE, etc.)
- Luminosity profiles of galaxies in f(R).
- Faber-Jackson & Tully-Fisher, Bullet Cluster

Weak Fields, GW, Further results

- Systematic studies of PPN formalism
- Relativistic Experimental Tests in f(R)
- Gravitational waves and lensing
- Birkhoff 's Theorem in f (R)-gravity
- f(R) with torsion

WORK in PROGRESS! (suggestions are welcome!)