Studying Early Universe with Inflationary Gravitational Waves (IGWs)

Takeo Moroi (Tokyo)

Refs:

Jinno, TM and Nakayama, PLB713 ('12) 129

Jinno, TM and Nakayama, PRD86 ('12) 123502

Jinno, TM and Nakayama, JCAP 1401 ('14) 040

Jinno, TM and Takahashi, work in progress

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1. Introduction

Current hot subject in cosmology: B-mode signal in CMB

- BICEP2 announced the discovery of $r \sim 0.2$

  r: tensor-to-scalar ratio

- PLANCK (and other) results will come out soon

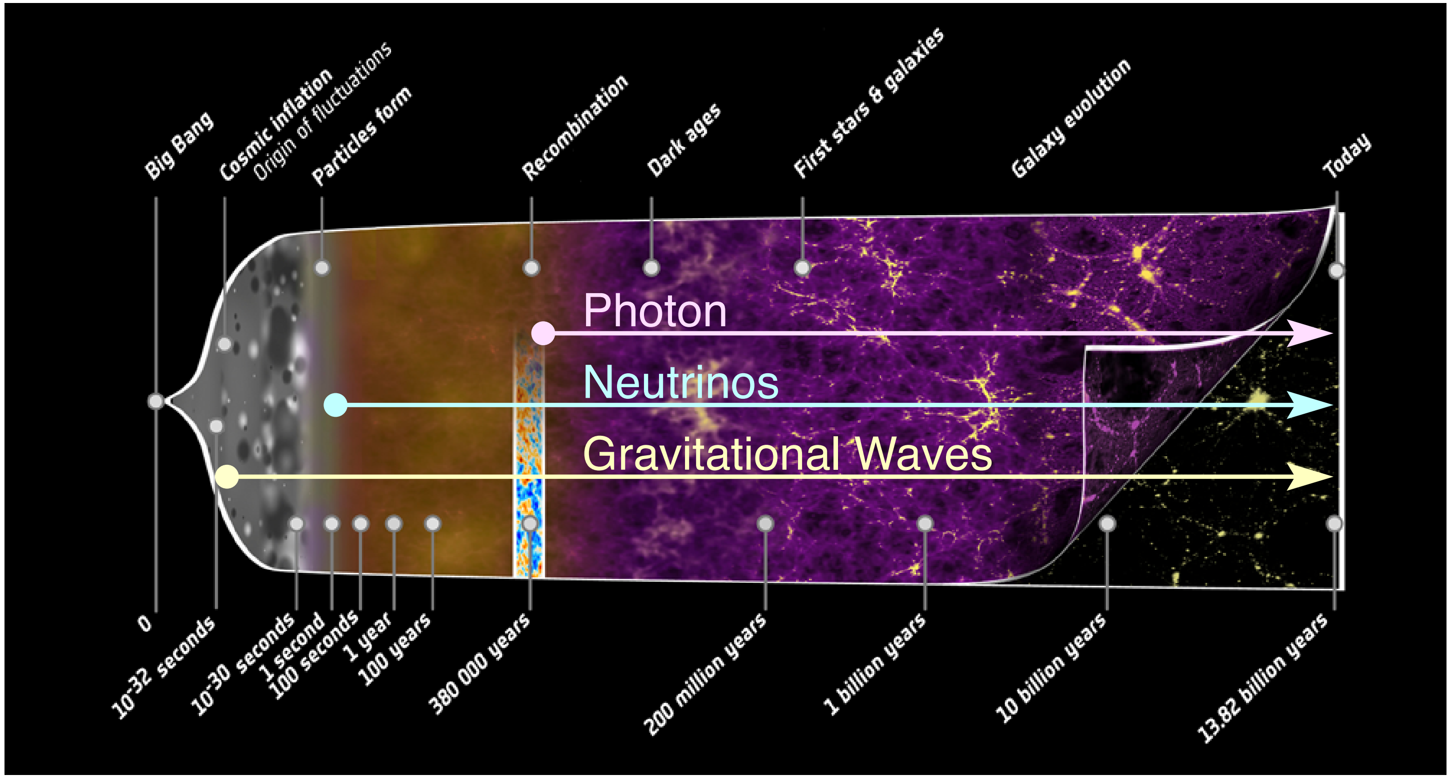
Implications of the discovery of the B-mode signal

- Evidence of inflationary gravitational waves (IGWs)
- Discovery of the B-mode signal provides information about the amplitude of IGW (at the CMB scale)

$$\Omega_{\mathrm{IGW}} \equiv \frac{1}{\rho_{\mathrm{crit}}} \left[\frac{d\rho_{\mathrm{IGW}}}{d\ln k} \right]_{\mathrm{NOW}} \simeq 3 \times 10^{-16} \times \left(\frac{r}{0.1} \right)$$

The history of our universe is imprinted in IGWs

- We may probe the early universe with IGWs



- The IGW spectrum may be measured in (far) future by space-based GW detectors (like BBO / DECIGO)

Today, I will talk about:

- How the IGWs behave
- What kind of information is imprinted in IGWs
- What we may learn with space-based GW detectors

Outline

1. Introduction
2. Gravitational Waves: Basic Properties
3. Determination of Reheating Temperature
4. Case with Cosmic Phase Transition
5. Summary

2. IGWs: Basic Properties

Gravitational wave: Fluctuation of the metric

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + 2h_{ij})dx^i dx^j$$

$$h_{ij}(t,\vec{x}) = \frac{1}{M_{\mathsf{Pl}}} \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^3} \tilde{h}^{(\lambda)}_{\vec{k}}(t)\epsilon^{(\lambda)}_{ij} e^{i\vec{k}\vec{x}}$$

$\epsilon^{(\lambda)}_{ij}$: polarization tensor (transverse & traceless)

Quantum fluctuation generated during inflation

$$\left\langle |\tilde{h}^{(\lambda)}_{\vec{k}}|^2 \right\rangle_{\mathsf{inflation}} \simeq \left(\frac{k^3}{2\pi^2 V}\right)^{-1} \left[\frac{H_{\mathsf{inf}}}{2\pi}\right]^2_{\mathsf{Horizon\text{-}Exit}}$$

$\Rightarrow$ The amplitude of the IGW is proportional to H_{inf}

Wave equation for GWs

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_{\mathsf{Pl}}^2} T_{\mu\nu} \;\;\Rightarrow\;\; \ddot{\tilde{h}}_{\vec{k}}^{(\lambda)} + 3H \dot{\tilde{h}}_{\vec{k}}^{(\lambda)} + \frac{\vec{k}^2}{a^2(t)} \tilde{h}_{\vec{k}}^{(\lambda)} \simeq 0$$

Evolution of IGWs: after inflation

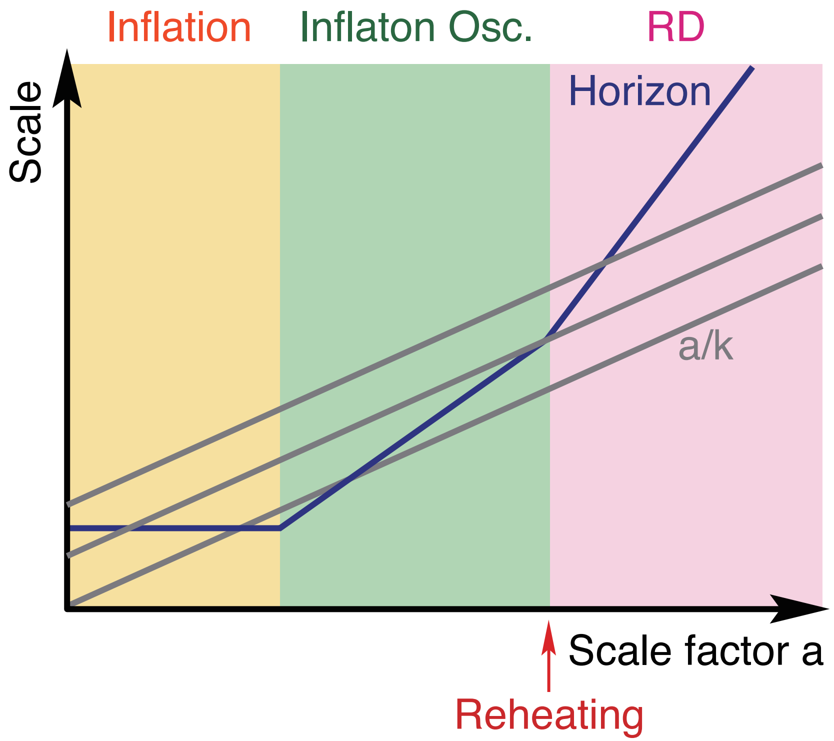
- $k \lesssim aH$: $\tilde{h}_{\vec{k}} \sim \text{const.} \;\;\Rightarrow\;\; \dfrac{d\rho_{\mathsf{GW}}}{d\ln k} \propto a^{-2}$
- $k \gtrsim aH$: $\langle \tilde{h}_{\vec{k}}^2 \rangle_{\mathsf{osc}} \sim a^{-2} \;\;\Rightarrow\;\; \dfrac{d\rho_{\mathsf{GW}}}{d\ln k} \propto a^{-4}$

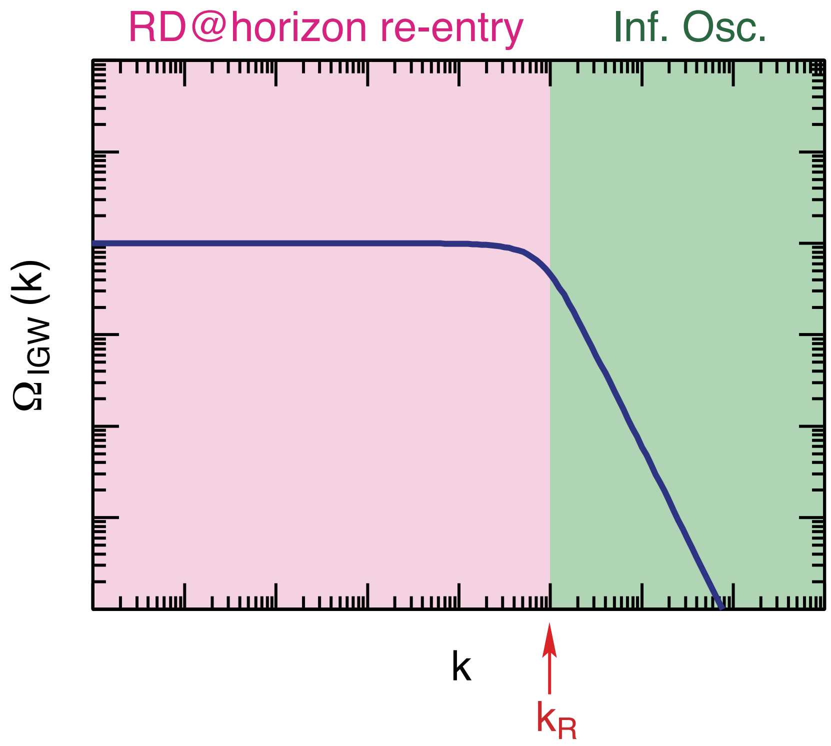
$$\frac{d\rho_{\mathsf{GW}}}{d\ln k} = \frac{k^3}{2\pi^2 V} \sum_{\lambda=+,\times} \left[\frac{1}{2} \left| \dot{\tilde{h}}_{\vec{k}}^{(\lambda)} \right|^2 + \frac{1}{2} \left(\frac{k}{a} \right)^2 \left| \tilde{h}_{\vec{k}}^{(\lambda)} \right|^2 \right]$$

$$\Rightarrow \left[\frac{d\rho_{\mathsf{GW}}}{d\ln k} \right]_{\mathsf{NOW}} \propto \rho_{\mathsf{tot}}(t_{\mathsf{Horizon\text{-}In}}) \left(\frac{a_{\mathsf{Horizon\text{-}In}}}{a_{\mathsf{NOW}}} \right)^4$$

The IGW spectrum depends on the epoch of horizon re-entry

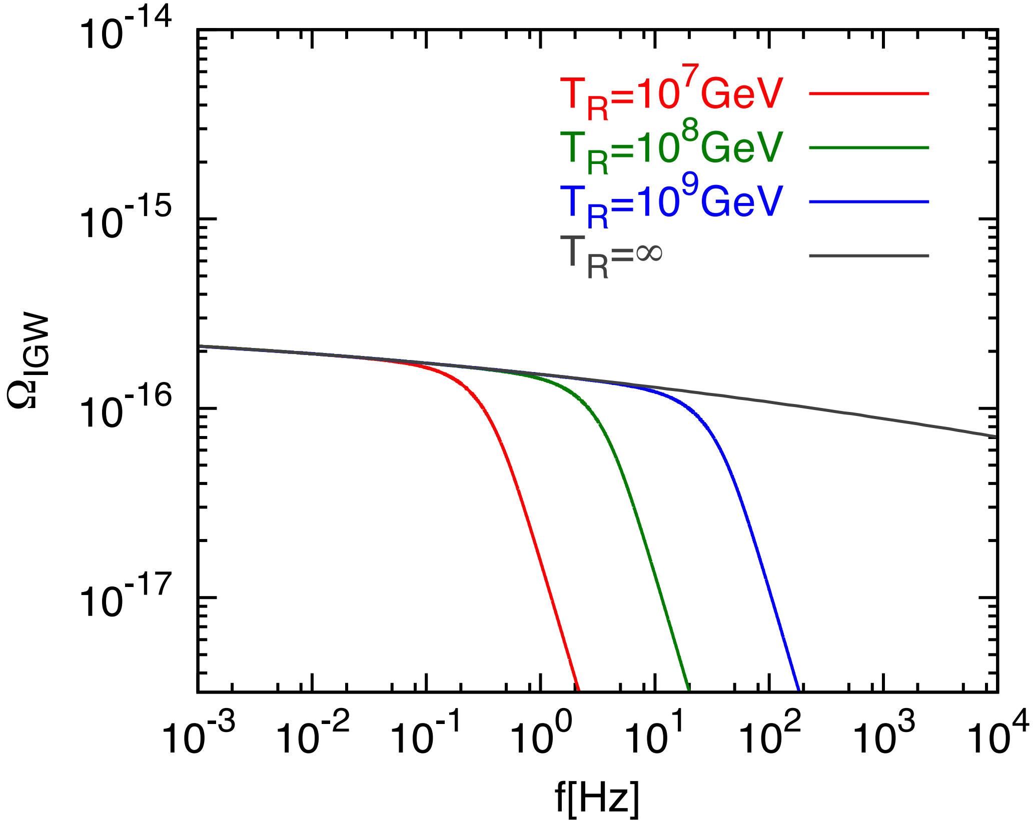
$$\Omega_{\mathsf{IGW}}(k) \equiv \frac{1}{\rho_{\mathsf{crit}}} \left[\frac{d\rho_{\mathsf{IGW}}}{d\ln k}\right]_{\mathsf{NOW}}$$





- $\Omega_{\mathsf{IGW}}(k) \sim$ const, for $k \lesssim k_{\mathsf{R}}$
- $\Omega_{\mathsf{IGW}}(k) \propto k^{-2}$, for $k \gtrsim k_{\mathsf{R}}$

The IGW spectrum ($r_{\mathsf{CMB}} = 0.15$, parabolic chaotic inflation)



$$T_{\mathrm{R}} \equiv \left(\frac{10}{g_* \pi^2}\right)^{1/4} \times M_{\mathsf{Pl}}^{1/2} \Gamma_{\mathsf{inf}}^{1/2}$$

$$f \simeq 2.7 \ \mathsf{Hz} \times \left(\frac{T_{\mathsf{Horizon\text{-}In}}}{10^8 \ \mathsf{GeV}}\right)$$

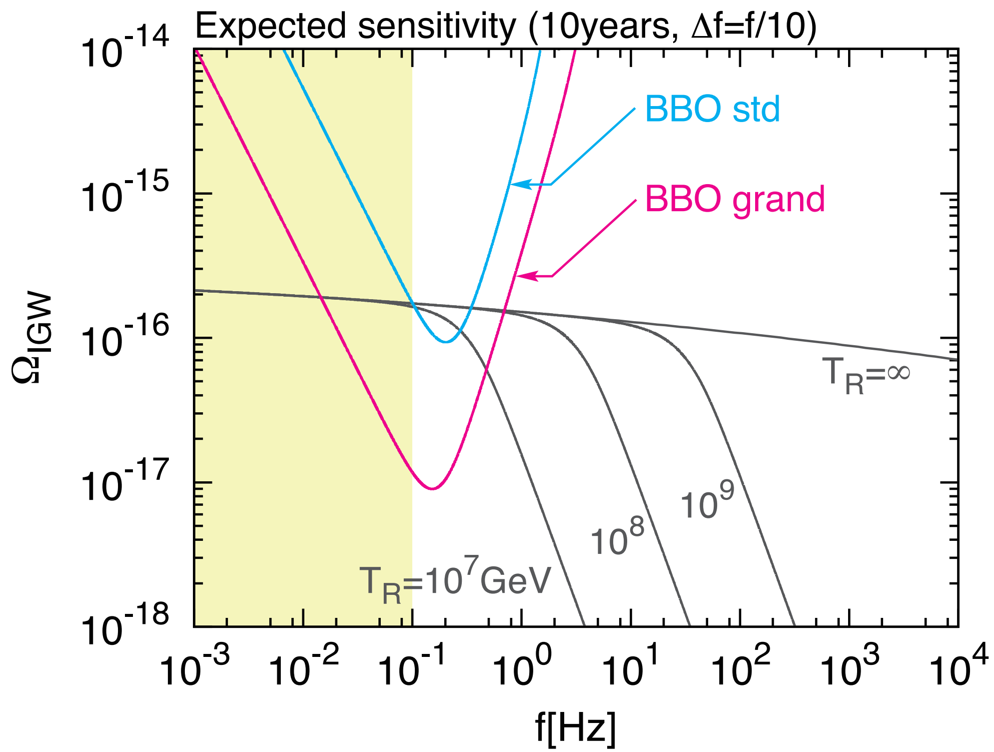
We may determine T_{R}, if the IGW spectrum is measured

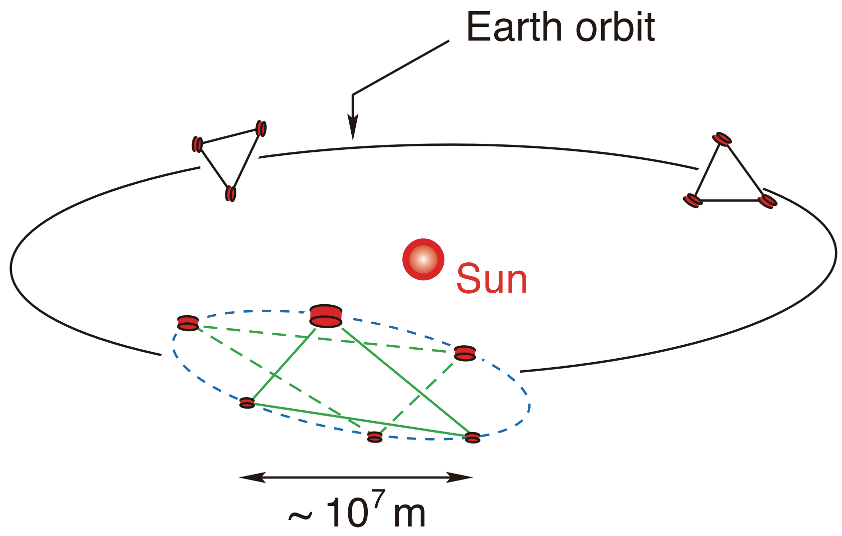
[Nakayama, Saito, Suwa & Yokoyama ('08)]

3. Determination of the Reheating Temperature

[Jinno, TM & Takahashi, work in progress]

With space-based GW detectors, IGWs may be detected

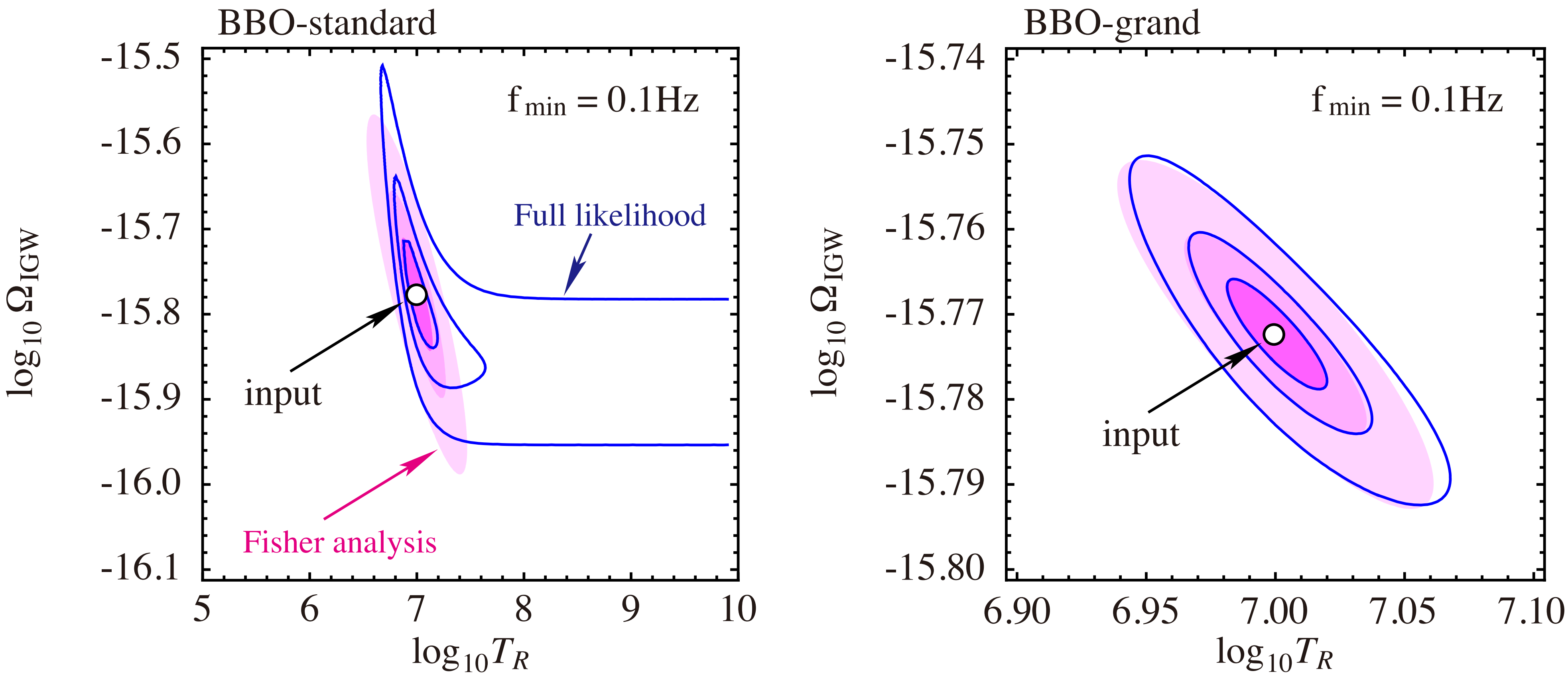




$$\delta\chi^2 \simeq \sum_{f_i} \frac{[\Omega_{\rm obs}(f_i) - \Omega_{\rm IGW}(f_i)]^2}{\Delta\Omega^2(f_i)}$$

$\Delta\Omega^2$ depends on detector parameters (noise, shape, ...)

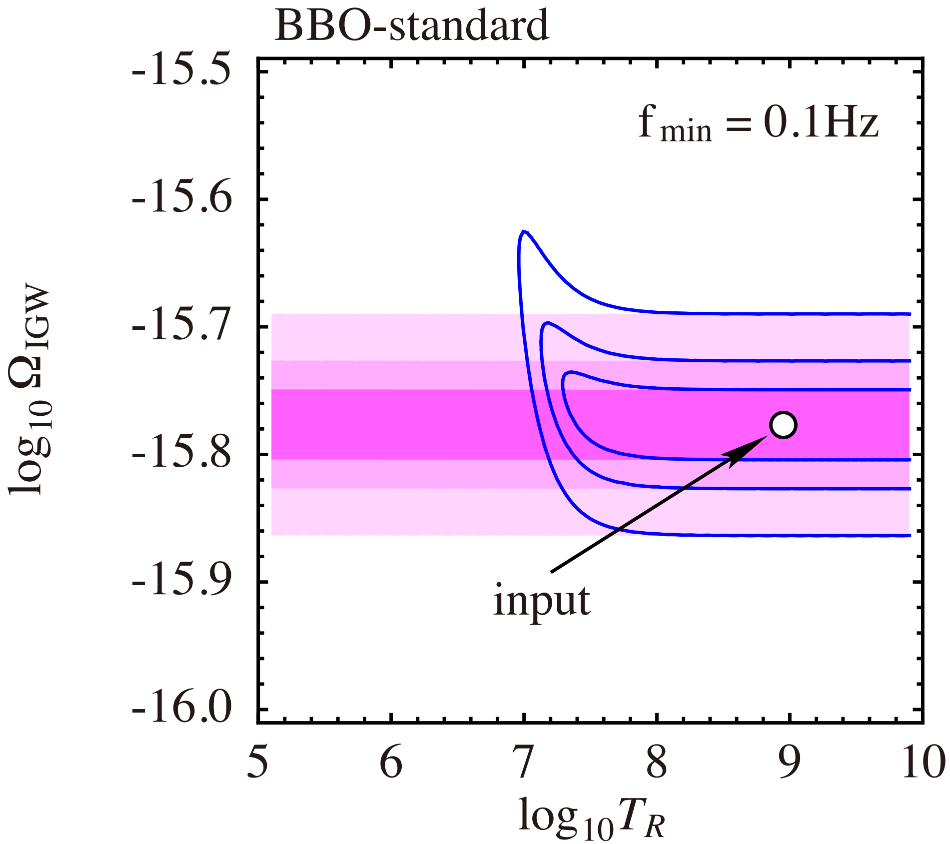
Case with $T_\mathrm{R} = 10^7$ GeV ($t_\mathrm{obs} = 1$, 3, and 10 yr from outside)

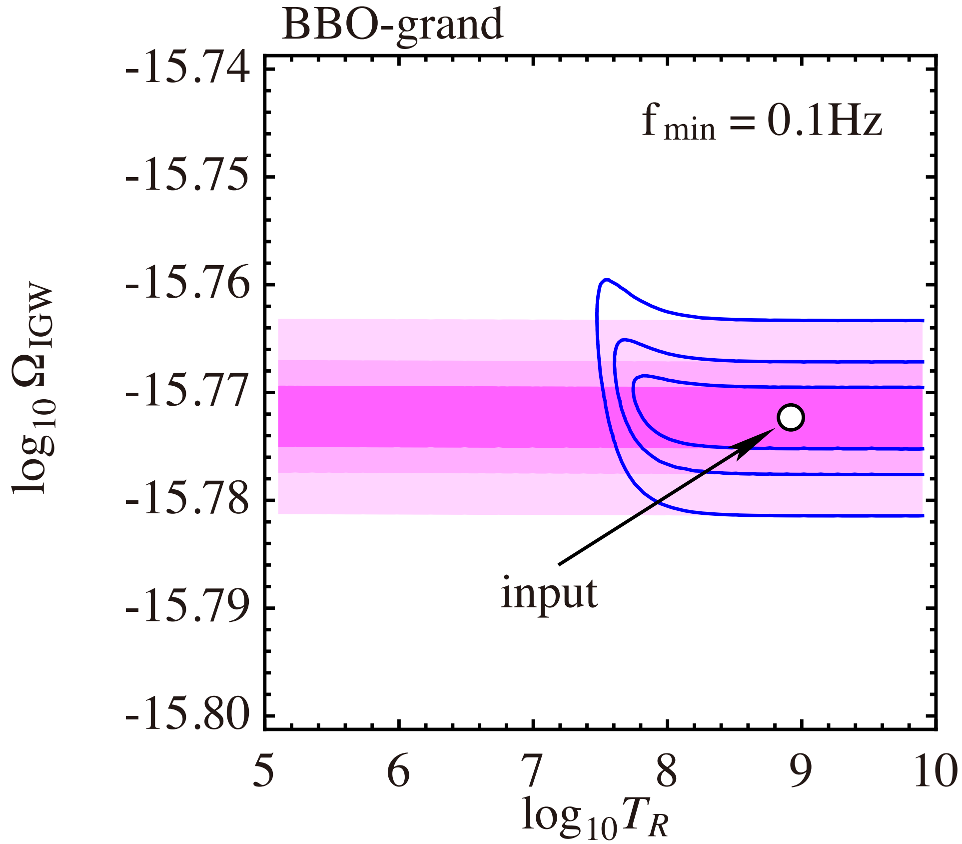


[Jinno, TM & Takahashi (preliminary)]

- Precise determination of Ω_IGW is possible at BBO
- T_R is well constrained if $T_\mathrm{R} \lesssim 10^7 - 10^8$ GeV

Case with $T_{\rm R} = 10^9$ GeV ($t_{\rm obs} = 1$, 3, and 10 yr from outside)





[Jinno, TM & Takahashi (preliminary)]

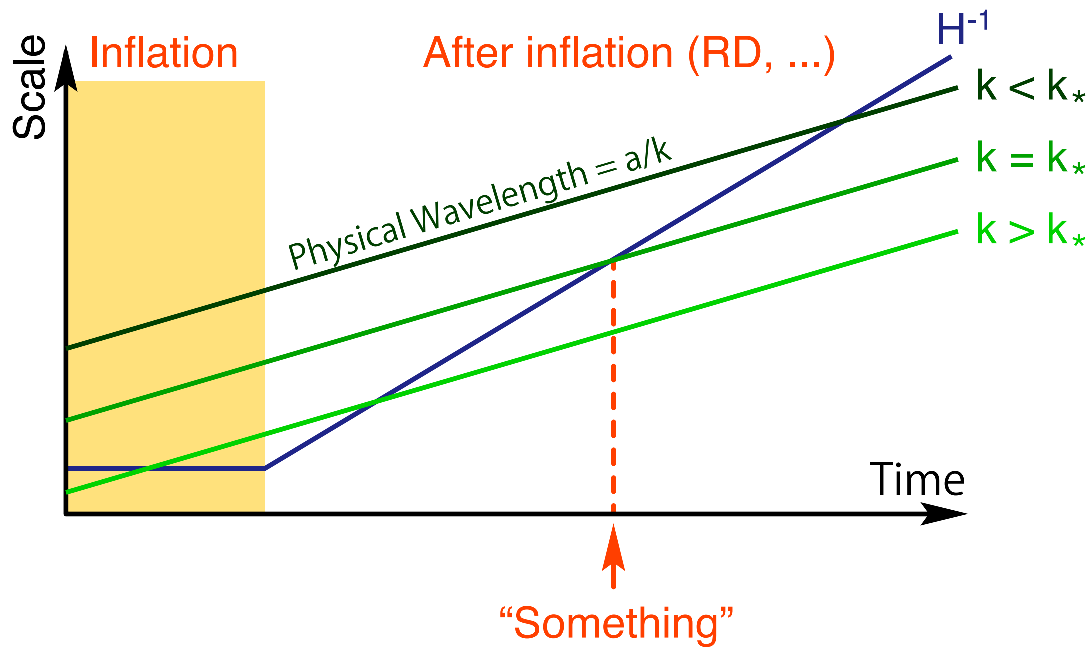
- $T_{\rm R}$ is bounded only from below, if $T_{\rm R} \gtrsim 10^8$ GeV

4. Case with Cosmic Phase Transition

[Jinno, TM & Nakayama]

If something happens after reheating, $\Omega_{\rm GW}(k)$ is deformed

- Cosmic phase transition
- Domination by extra matter
- $\cdots$



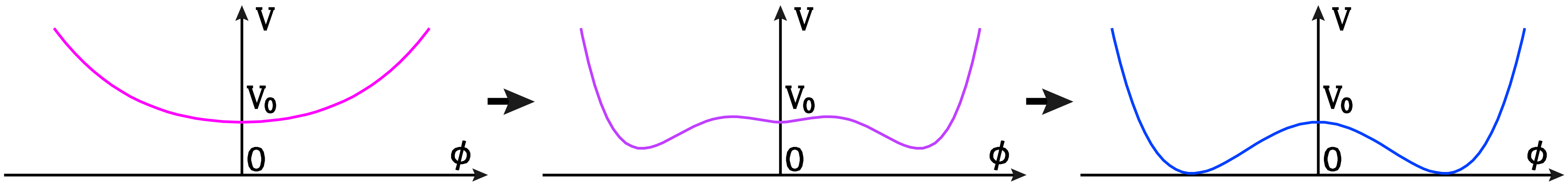
$\Rightarrow \Omega_{\rm GW}(k)$ at $k \sim k_*$ is affected

Case 1: Phase transition (short thermal inflation)

Example: Peccei-Quinn symmetry breaking

Potential with thermal effects:

$$V(\phi) = \frac{g}{24}(\phi^2 - v_\phi^2)^2 + \frac{h}{24}T^2\phi^2$$



$$\langle\phi\rangle \simeq \begin{cases} 0 & : T > T_\mathrm{c} \\ v_\phi & : T < T_\mathrm{c} \end{cases}$$

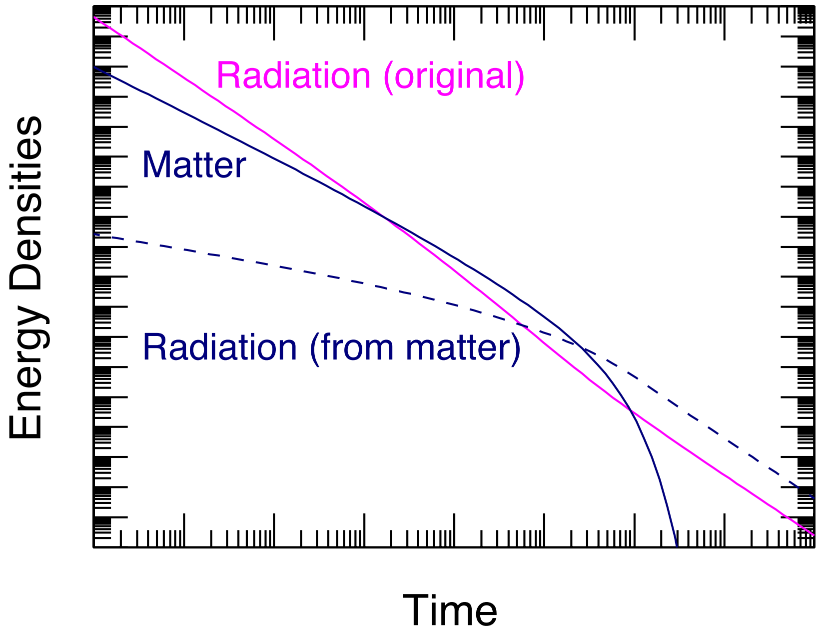
$\Rightarrow$ The universe may be once dominated by the potential energy of ϕ (like thermal inflation)

[Lyth & Stewart]

Case 2: Temporary matter domination

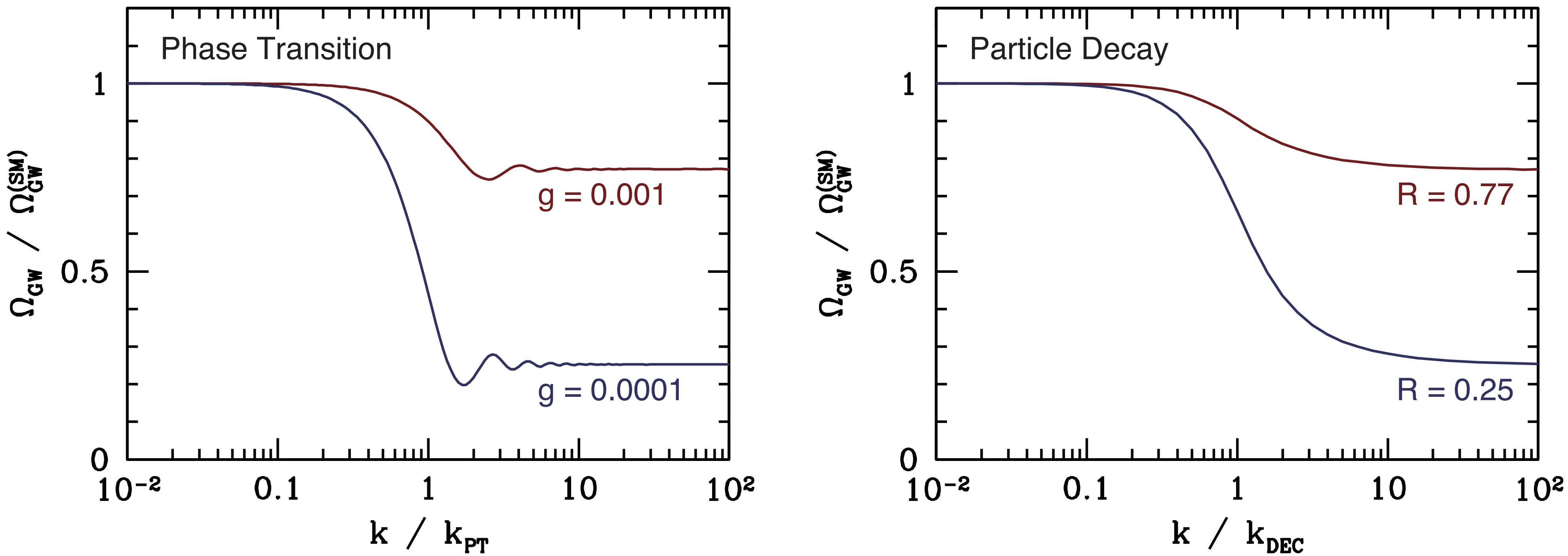
- Scalar condensations (like saxion in SUSY PQ model)
- Other exotic particles

A matter field once dominates the universe, then decays



- $\rho_{\text{rad}} \propto a^{-4}$
- $\rho_{\text{matter}} \propto a^{-3}$

IGW spectrum for two cases:

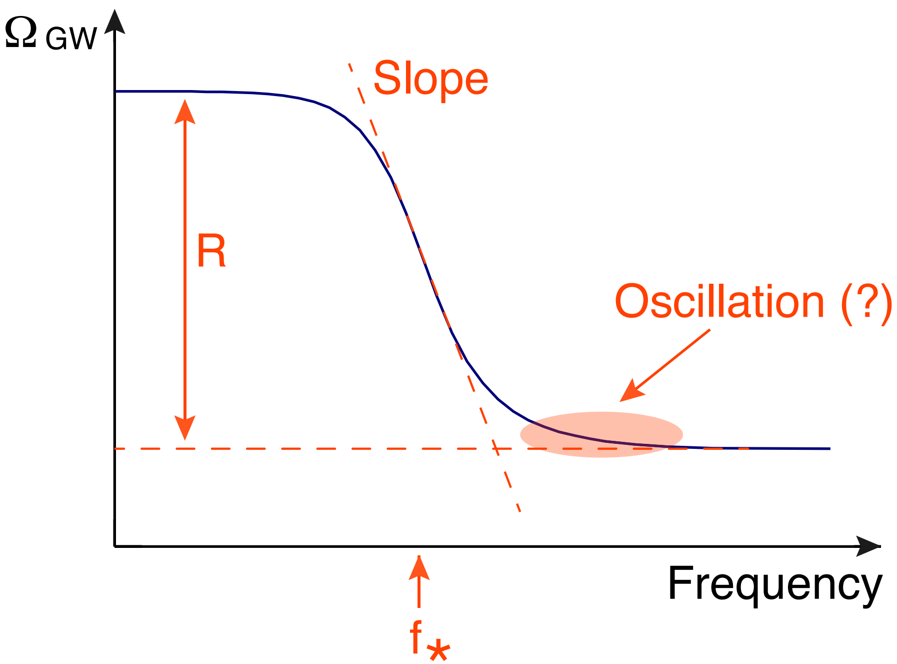


Important parameter:

$$R \equiv \left. \frac{\Omega_{\mathrm{GW}}(k)}{\Omega_{\mathrm{GW}}^{(\mathrm{SM})}(k)} \right|_{k \gg k_{\mathrm{PT}}} = \frac{\rho_{\mathrm{rad}}^{(\mathrm{orig})}}{\rho_{\mathrm{rad}}^{(\mathrm{orig})} + \rho_{\mathrm{rad}}^{(\mathrm{extra})}}$$

Information in the IGW spectrum

- $f_* \Rightarrow$ Temperature of the event
- $R \Rightarrow$ Energy injection
- $d\Omega_{\mathsf{GW}}/d\ln k \Rightarrow$ Time scale of the event



- $f_* \simeq 2.7\ \mathsf{Hz} \times \left(\dfrac{T_*}{10^8\ \mathsf{GeV}} \right)$
- $R = \dfrac{\rho_{\mathsf{rad}}^{(\mathsf{orig})}}{\rho_{\mathsf{rad}}^{(\mathsf{orig})} + \rho_{\mathsf{rad}}^{(\mathsf{extra})}}$

5. Summary

The B-mode signal is a strong evidence of the IGWs

$$\Omega_{\mathsf{IGW}} \equiv \frac{1}{\rho_{\mathsf{crit}}} \left[\frac{d\rho_{\mathsf{IGW}}}{d \ln k} \right]_{\mathsf{NOW}} \simeq 3 \times 10^{-16} \times \left(\frac{r}{0.1} \right)$$

IGW contains rich information about the thermal history

$\Rightarrow$ Direct detection of the IGW is attractive

Possible scenario in the future

- Confirmation of $r > 0$
- Establish the technology with ground-based GW experiments (Advanced LIGO / KAGRA)
- If these are done, the space-based experiment to detect IGW is an interesting possibility

Backups

Case 3: Production of "dark radiation (DR)"

DR: Relativistic particle with large free-streaming length

- Candidates of dark radiation: NG bosons, like axion, $\cdots$

$\Rightarrow$ They can be produced in association with phase transition, for example

$\Rightarrow$ They may decay, or may be diluted afterwards

- Non-vanishing anisotropy inertia shows up

$$\Rightarrow \ddot{\tilde{h}}_{\vec{k}}^{(\lambda)} + 3H\dot{\tilde{h}}_{\vec{k}}^{(\lambda)} + \frac{k^2}{a^2(t)}\tilde{h}_{\vec{k}}^{(\lambda)} = \frac{1}{M_{\text{Pl}}^2} \times \text{(anisotropic inertia)}$$

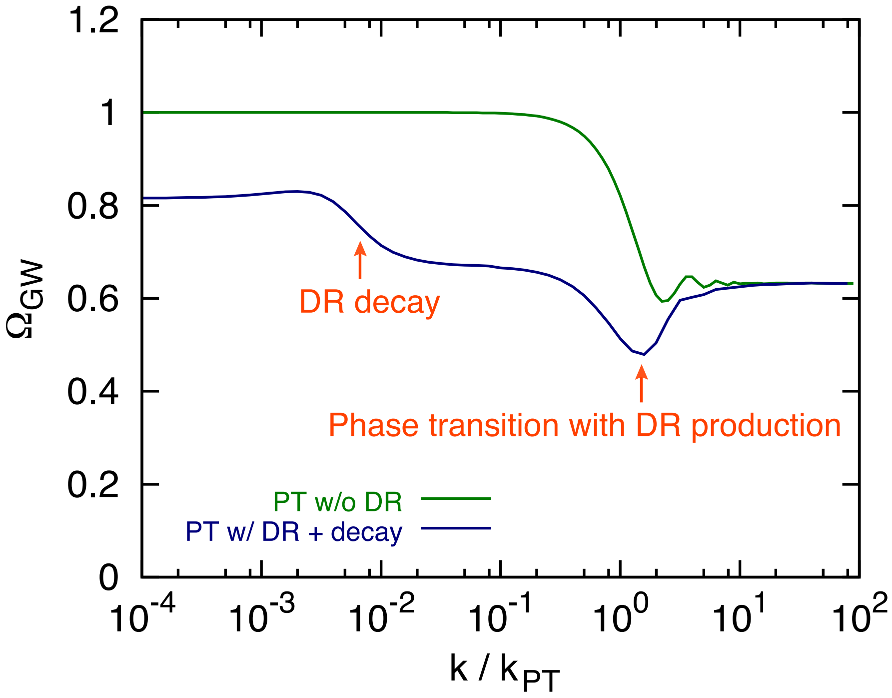
DR changes the shape of the IGW spectrum

[Weinberg]

$\Rightarrow$ Suppression of low frequency mode of the IGW spectrum

Example 1: Phase transition with DR production

1. Phase transition, which produces DR
2. Decay of (some fraction of) “DR”



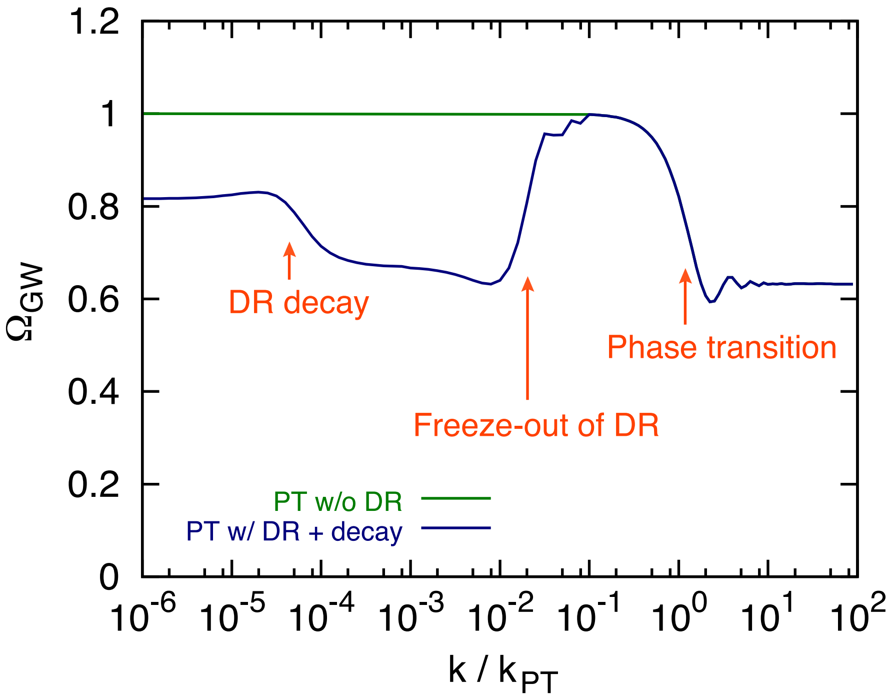
Energy fraction of DR

$33\ \%$ before decay

$13\ \%$ after decay; $\Delta N_{\text{eff}} = 0.5$

Example 2: Freeze-out of DR

1. Phase transition, which produces dark-sector particles
2. Particles in the dark sector freeze-out



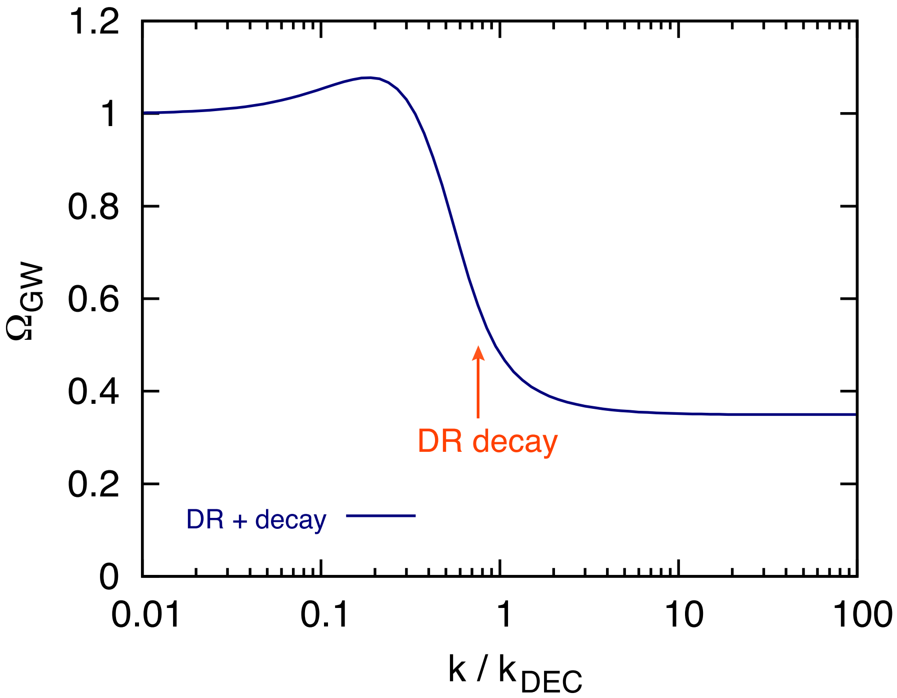
Energy fraction of DR

$33\ \%$ before decay

$13\ \%$ after decay; $\Delta N_{\rm eff} = 0.5$

Example 3: DR domination in the early epoch

1. Universe was once dominated by DR
2. DR decays and reheats the SM sector



Energy fraction of DR

$100\ \%$ before decay

$0\ \%$ after decay