Decay constants of heavy mesons from QCD sum rules

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Previously,

• f_D, f_{Ds} f_B, f_{Bs}

Decay constants of heavy pseudoscalar mesons from QCD sum rules, J. Phys. G38, 105002, 2011;

OPE, charm-quark mass, and decay constants of D and Ds mesons from QCD sum rules, Phys. Lett. B701, 82, 2011.

• Combining lattice and QCD sum-rule results for f_B extracted $m_b(m_b)$

Accurate bottom-quark mass from Borel QCD sum rules for fB and fBs, Phys. Rev. D88, 056011, 2013.

• decay constants of charmed vector mesons f_{D^*}, f_{Ds^*}

Decay constants of charmed vector mesons D and Ds* from QCD sum rules*, Phys. Lett. B735, 12, 2014.

Here we present new results for f_{B*} with emphasis on f_{B*}/f_B

Correlation function, OPE, and heavy – quark mass

The basic object is T-product of 2 pseudoscalar currents, $j_5(x) = (m_b + m)\,\bar{q}(x) i\gamma_5 b(x)$,

$$\Pi(p^2) = i \int d^4x\, e^{ipx} \left\langle 0 \left| T\left(j_5(x) j_5^\dagger(0) \right) \right| 0 \right\rangle$$

and its Borel image

$$\Pi(\tau) = f_B^2 M_B^4 e^{-M_B^2 \tau} + \int_{s_{\rm phys}}^{\infty} ds\, e^{-s\tau} \rho_{\rm hadr}(s) = \int_{(m_b+m)^2}^{\infty} ds\, e^{-s\tau} \rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

here $s_{\rm phys} = (M_{B^*} + M_P)^2$, and f_B is the decay constant defined by

$$(m_b + m)\langle 0|\bar{q} i\gamma_5 b|B\rangle = f_B M_B^2.$$

To exclude the excited-state contributions, one adopts the *duality Ansatz*: all contributions of excited states are counterbalanced by the perturbative contribution above an *effective continuum threshold*, $s_{\rm eff}(\tau)$ which differs from the physical continuum threshold.

Applying the duality assumption yields:

$$f_B^2 M_B^4 e^{-M_B^2\tau} = \int_{(m_b+m)^2}^{s_{\rm eff}(\tau)} ds\, e^{-s\tau}\rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu) \equiv \Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

The rhs is the *dual correlator* $\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau))$**.**

Even if the QCD inputs $\rho_{\rm pert}(s,\mu)$ **and** $\Pi_{\rm power}(\tau,\mu)$ **are known, the extraction of the decay constant requires, in addition, a criterion for determining** $s_{\rm eff}(\tau)$**.**

As first step, we need a reasonably convergent OPE for both correlator and dual correlator.

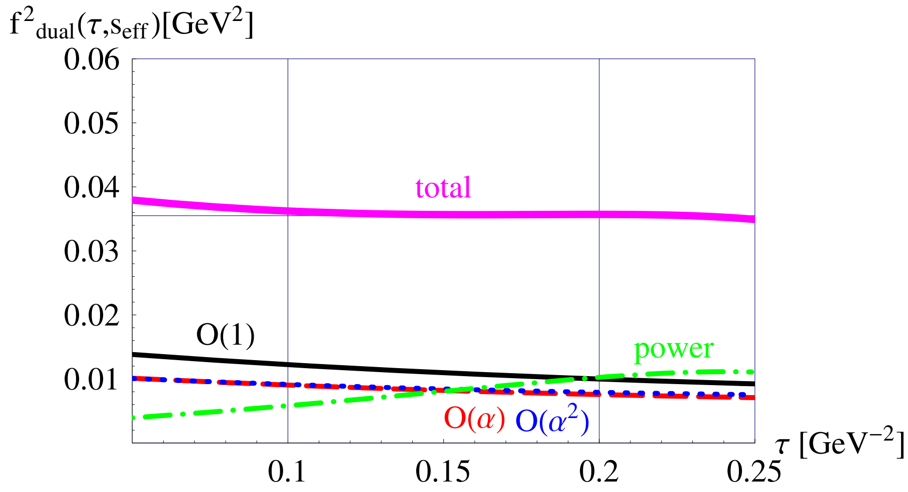
The best-known 3-loop calculations of the perturbative spectral density have been performed in form of an expansion in terms of the $\overline{\rm MS}$ **strong coupling** $\alpha_{\rm s}(\mu)$ **and the pole mass** M_b**:**

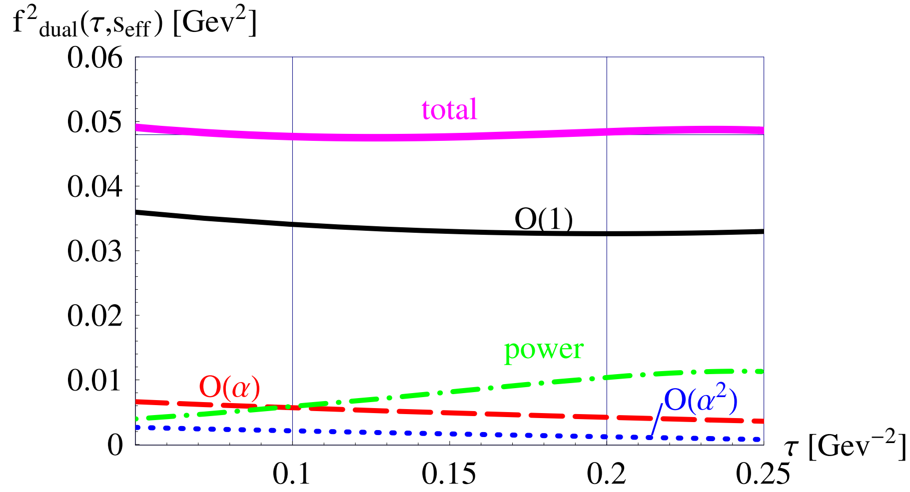
$$\rho_{\rm pert}(s,\mu) = \rho^{(0)}(s, M_b^2) + \frac{\alpha_{\rm s}(\mu)}{\pi}\rho^{(1)}(s, M_b^2) + \left(\frac{\alpha_{\rm s}(\mu)}{\pi}\right)^2 \rho^{(2)}(s, M_b^2, \mu) + \cdots.$$

An alternative option is to reorganize the perturbative expansion in terms of the running $\overline{\rm MS}$ **mass,** $\overline{m}_b(\nu)$**, by substituting** M_b **in the spectral densities** $\rho^{(i)}(s, M_b^2)$ **via its perturbative expansion in terms of the running mass** $\overline{m}_b(\nu)$

$$M_b = \overline{m}_b(\nu)\left(1 + \frac{\alpha_s(\nu)}{\pi} r_1 + \left(\frac{\alpha_s(\nu)}{\pi}\right)^2 r_2 + \dots\right).$$

OPE in terms of b-quark pole (left) and $\overline{\mathrm{MS}}$ mass (right): SR results for f_B; $s_{\rm eff} = 35$ GeV^2.





Lessons:

1. **For dual correlator calculated through the heavy-quark pole mass, perturbative expansion exhibits no sign of convergence; the $O(1)$, $O(\alpha_s)$, and $O(\alpha_s^2)$ terms are of the same magnitude.** *In pole-mass scheme one cannot expect higher orders to give smaller contributions*.

2. **Formulating the perturbative series in terms of the heavy-quark $\overline{\mathrm{MS}}$ mass yields a clear hierarchy of contributions.** *We employ the $\overline{\mathrm{MS}}$-mass OPE in our SR analysis*.

3. **f_B extracted from the pole-mass truncated OPE ($f_B = 188$ MeV) is substantially smaller than that from the $\overline{\mathrm{MS}}$-mass OPE truncated at the same order ($f_B = 220$ MeV). However, both decay constants exhibit stability over a wide range of τ.**

   Borel stability does not guarantee the reliability.

Extraction of the decay constant

According to the standard procedures of QCD sum rules, one executes the following steps:

1. *The Borel window*

The working τ-window is chosen such that the OPE gives an accurate description of the exact correlator (i.e., all higher-order radiative and power corrections are under control) and at the same time the ground state gives a "sizable" contribution to the correlator. Our τ-window for the $B_{(s)}$ mesons is $0.05 \lesssim \tau\,(\mathrm{GeV}^{-2}) \lesssim 0.175$.

2. *The effective continuum threshold*

To find $s_{\rm eff}(\tau)$, we employ a previously developed algorithm which provides a reliable extraction of the ground-state parameters in quantum-mechanics and of the charmed-meson decay constants in QCD. We introduce the *dual invariant mass* $M_{\rm dual}$ and the *dual decay constant* $f_{\rm dual}$

$$M_{\rm dual}^2(\tau) \equiv -\frac{d}{d\tau}\log\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)), \qquad f_{\rm dual}^2(\tau) \equiv M_B^{-4}\, e^{M_B^2\tau}\,\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

The dual mass should reproduce the true ground-state mass M_B; the deviation of $M_{\rm dual}$ from M_B measures the contamination of the dual correlator by excited states. Starting from an Ansatz for $s_{\rm eff}(\tau)$ and requiring a minimum deviation of $M_{\rm dual}$ from M_B in the τ-window generates a variational solution for $s_{\rm eff}(\tau)$. With the latter at our disposal, $f_{\rm dual}(\tau)$ yields the desired decay-constant estimate. We consider polynomials in τ, including also a τ-independent constant:

$$s_{\rm eff}^{(n)}(\tau) = \sum_{j=0}^{n} s_j^{(n)}\tau^j.$$

We obtain $s_j^{(n)}$ by minimizing the squared difference between $M^2_{\rm dual}$ and M^2_B in the τ-window:

$$\chi^2 \equiv \frac{1}{N}\sum_{i=1}^{N}\left[M^2_{\rm dual}(\tau_i) - M^2_B\right]^2 .$$

Uncertainties in the extracted decay constant

The resulting f_B is sensitive to the input values of the OPE parameters — which determines what we call the *OPE-related error* — and to the details of the adopted prescription for fixing the behaviour of the effective continuum threshold $s_{\rm eff}(\tau)$ — the *systematic error*.

OPE – related error

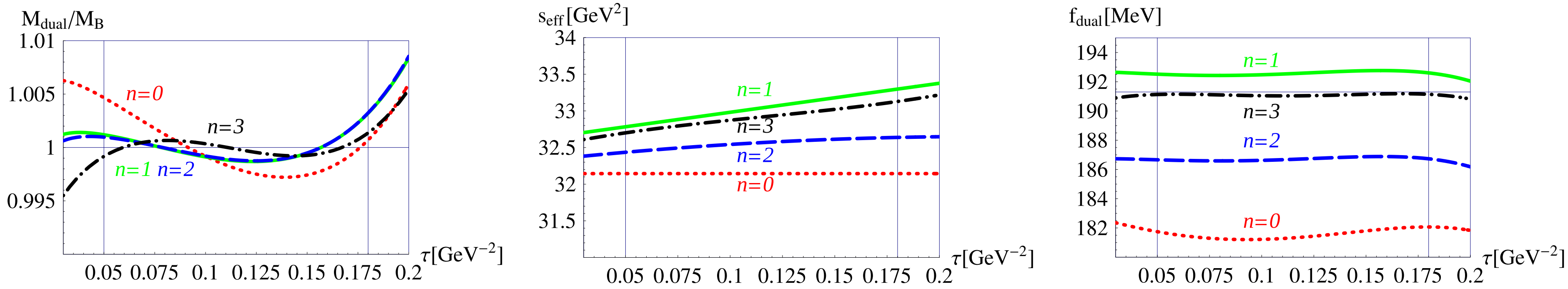
We estimate the size of the OPE-related error by perform a bootstrap analysis, assuming Gaussian distributions for all OPE parameters but the renormalization scales. For the latter, we assume uniform distributions in the range $3 \le \mu, \nu$ (GeV) ≤ 6. The resulting distribution of the decay constant turns out to be close to Gaussian shape. Hence, the quoted OPE-related error is a Gaussian error.

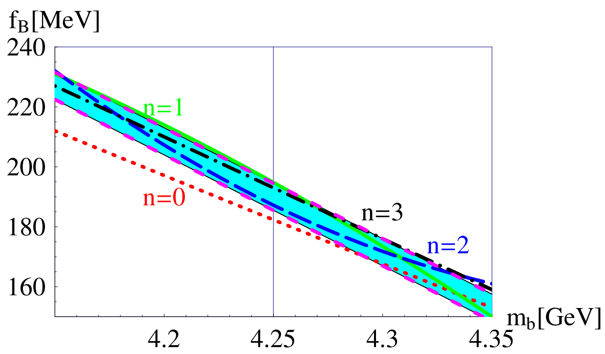
Systematic error

The systematic error, related to the limited intrinsic accuracy of the method of sum rules, is a subtle point. In quantum mechanics, we observed that considering polynomial parameterizations of the effective continuum threshold $s_{\rm eff}(\tau)$, the band of results obtained from linear, quadratic, and cubic Ansätze for $s_{\rm eff}(\tau)$, encompasses the true value of the decay constant. Thus, the half-width of this band may be regarded as a realistic estimate for the systematic uncertainty of the prediction.

Decay constants of B and B^*

For $m_b \equiv \overline{m}_b(\overline{m}_b) = 4.247$ GeV, $\mu = \nu = m_b$, and central values of the other relevant parameters:

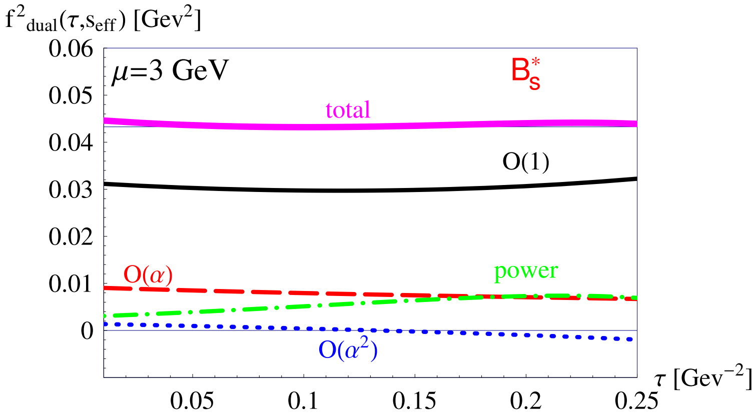


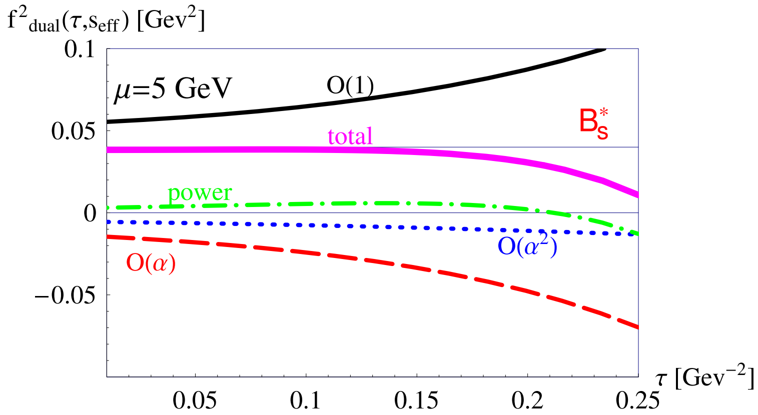


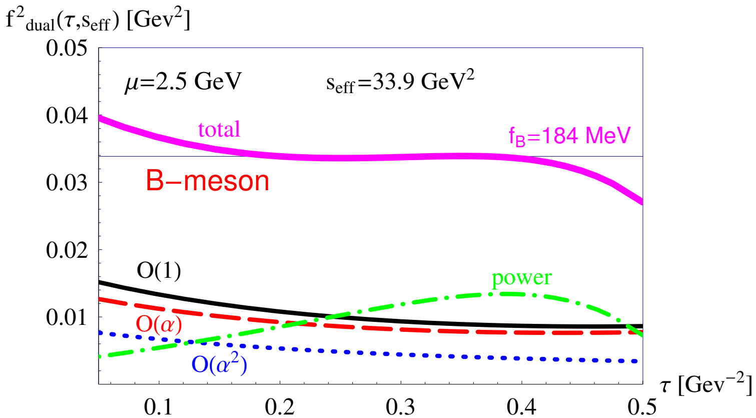
Our results for f_B may be parameterized by (for fixed values of other OPE parameters)

$$f_B^{\rm dual}(m_b, \mu = \nu = m_b, \langle\bar{q}q\rangle) = \left[192.0 - 37\left(\frac{m_b - 4.247\ \textbf{GeV}}{0.1\ \textbf{GeV}}\right) + 4\left(\frac{|\langle\bar{q}q\rangle|^{1/3} - 0.269\ \textbf{GeV}}{0.01\ \textbf{GeV}}\right) \pm 3_{\rm (syst)}\right]\textbf{MeV},$$

Dependence on the scale μ

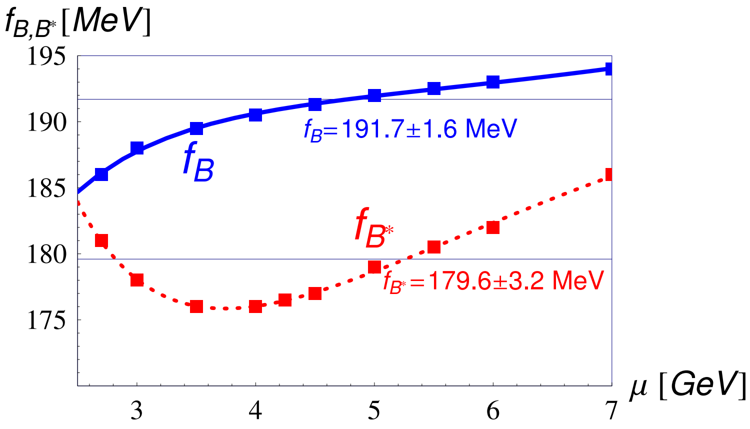


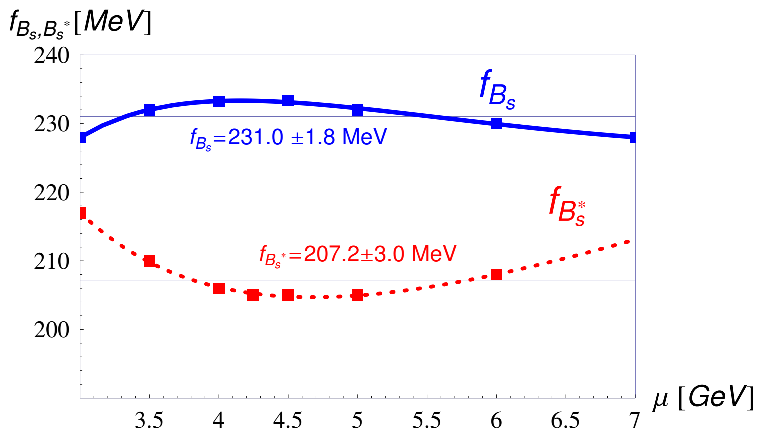


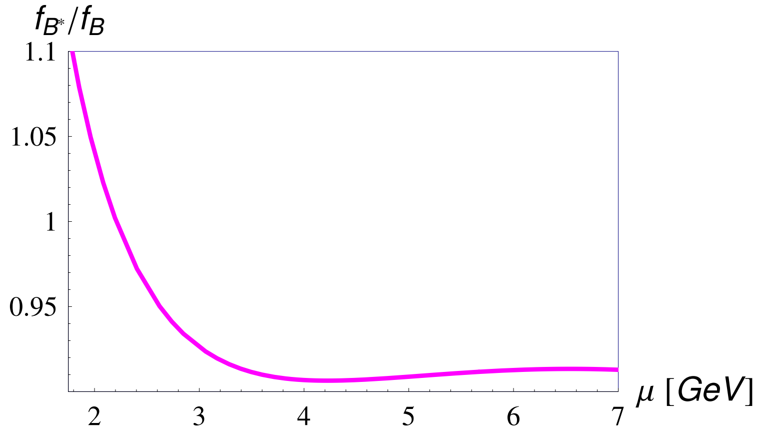


Perturbative hierarchy depends on μ.

Dependence on the scale μ







Summary

• **The extraction of hadronic properties improves by allowing a Borel-parameter dependence for the effective continuum threshold, which increases the accuracy of the duality approximation. Considering suitably optimized polynomial Ansätze for the effective continuum threshold provides an estimate of the intrinsic uncertainty of the method of QCD sum rules.**

• **Result obtained on the basis of pole-mass OPE are not trustable: the pole-mass OPE shows no perturbative hierarchy. Reorganizing the OPE series in terms of the running mass improves the hierarchy.**

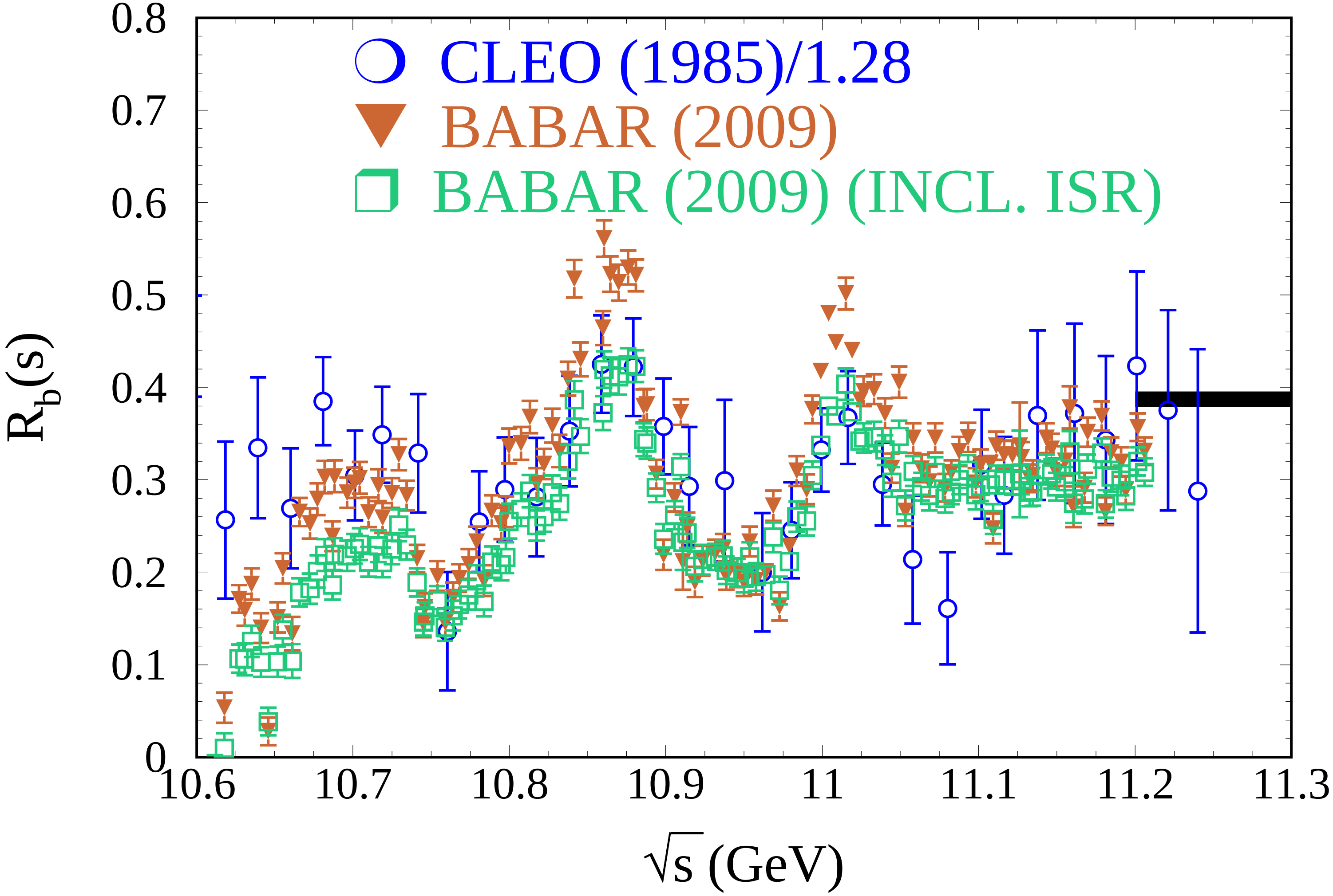
• **For beauty mesons, a strong correlation between** m_b **and the sum-rule result for** f_B **was observed:** $\frac{\delta f_B}{f_B} \approx -8\,\frac{\delta m_b}{m_b}$. **Combining our sum-rule analysis with the latest results for** f_B **and** f_{Bs} **from lattice QCD yields** $m_b = 4.247 \pm 0.027_{(OPE)} \pm 0.018_{(exp)} \pm 0.011_{syst}$ GeV

• **The decay constants** f_B **and** f_{B*} **are sensitive to the precise value of the scale** μ**. Averaging over the range** $3 < \mu[\mathrm{GeV}] < 6$ **yields**

$$f_B = 191.7 \pm 1.6 \text{ MeV} \qquad f_{B^*} = 179.6 \pm 3.2 \text{ MeV}$$

$$f_{Bs} = 231.0 \pm 1.8 \text{ MeV} \qquad f_{Bs^*} = 207.2 \pm 3.0 \text{ MeV}$$

$$f_{B^*}/f_B = 0.927 \pm 0.01 \qquad f_{Bs^*}/f_{Bs} = 0.900 \pm 0.025$$


CLEO (1985)/1.28
BABAR (2009)
BABAR (2009) (INCL. ISR)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$R_b(s)$
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3
$\sqrt{s}$ (GeV)

The OPE parameters:

$m_d(2\ \mathbf{GeV}) = (3.5 \pm 0.5)\ \mathbf{MeV}$, $m_s(2\ \mathbf{GeV}) = (95 \pm 5)\ \mathbf{MeV}$, $\alpha_s(M_Z) = 0.1184 \pm 0.0007$, $\langle\bar{q}q\rangle(2\ \mathbf{GeV}) = -((269 \pm 17)\ \mathbf{MeV})^3$, $\langle\bar{s}s\rangle(2\ \mathbf{GeV})/\langle\bar{q}q\rangle(2\ \mathbf{GeV}) = 0.8 \pm 0.3$, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = (0.024 \pm 0.012)\ \mathbf{GeV}^4$.