Anomalous Transport

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EXCELENCIA SEVERO OCHOA

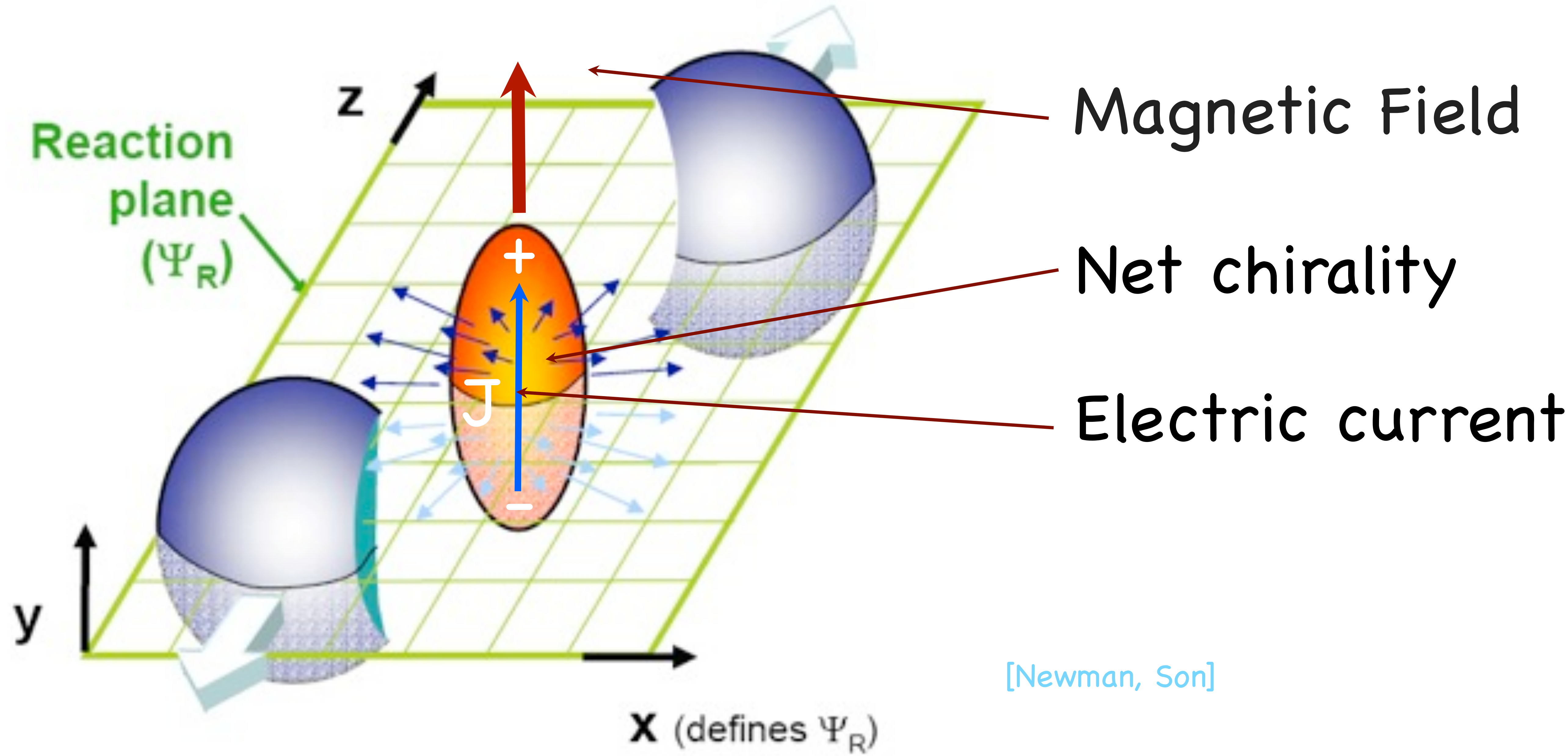
QCD@work, Bari, June 16-19, 2014

Outline

- Motivation
- Kubo formulas
  - Field theory
  - String theory (AdS/CFT)
- Hydrodynamics
- Applications
  - Quark Gluon Plasma
  - Weyl semi-metals

Motivation

[Kharzeev, McLarren, Warringa],
[Fukushima, Kharzeev, Warringa]



[Newman, Son]

parity violating currents: [Vilenkin '80], [Giovannini, Shaposhnikov '98], [Alekseev, Chaianov, Fröhlich] '98] [Newman, Son]

Kubo Formulas

Transport coefficients $\longleftrightarrow$ Correlation functions

Chiral Magnetic Effect : $\vec{J} = \sigma^B \vec{B}$

Kubo formula: $\langle J_i J_j \rangle = i\sigma^B \epsilon_{ijk} p_k$

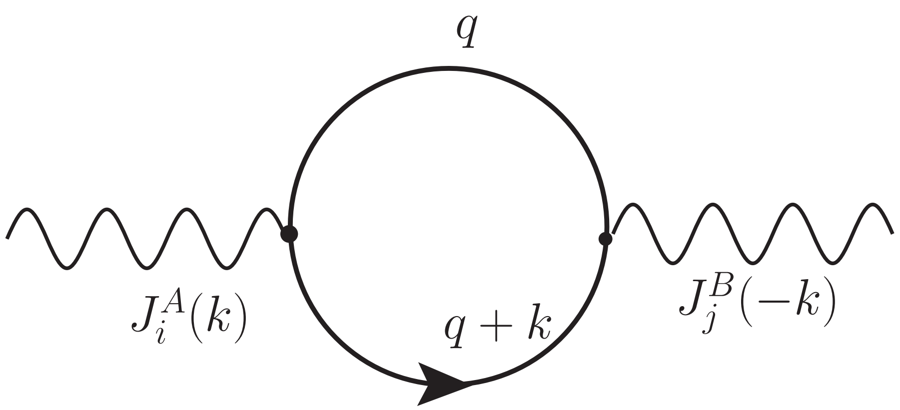
Kinematic regime: $\omega = 0, \lim p_k \to 0$

(general frequency and momentum dependence : see E. Megias' talk)

Kubo Formulas

More general symmetry : $[T^A, T^B] = if^{ABC}T^C$

$$\langle J_i^A J_j^B \rangle = i\sigma_{\mathcal{B}} B^{AB} \epsilon_{ijk} p_k$$

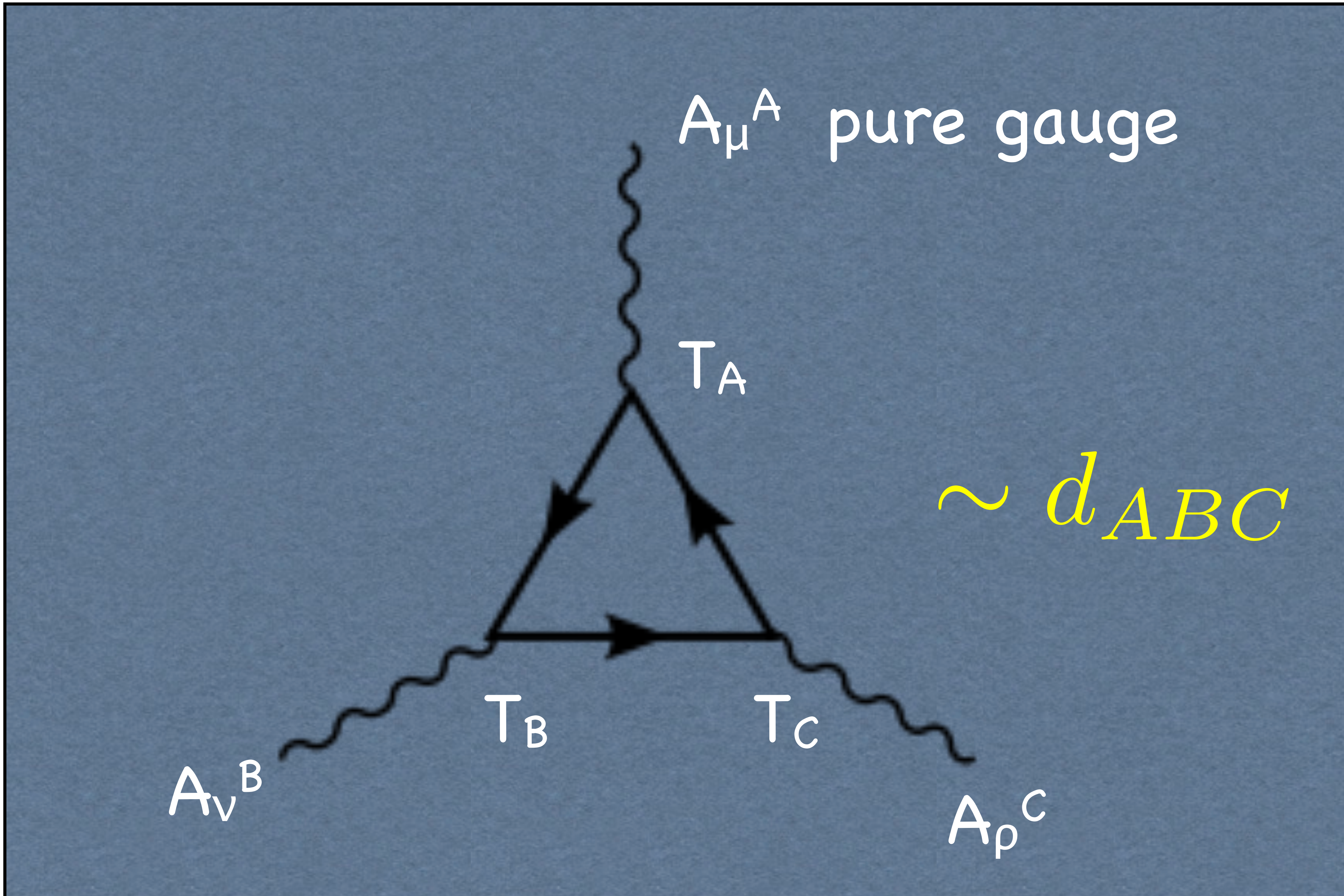


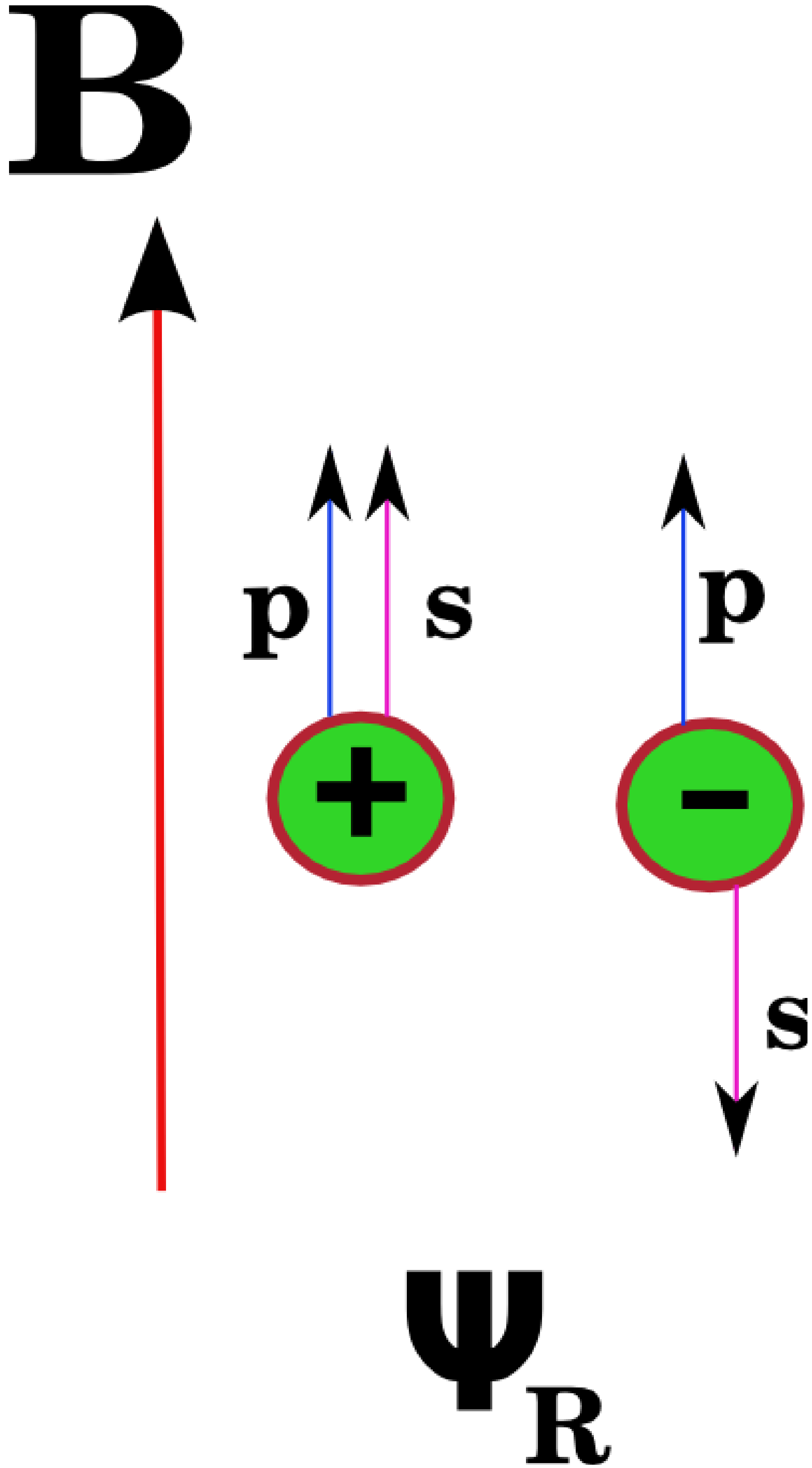
Gas of free chiral fermions at finite temperature and density:

$$\boxed{\sigma_{\mathcal{B}}^{AB} = \frac{d^{ABC}}{4\pi^2}\mu_C}$$

$$d^{ABC} = \mathrm{str}\left(T^A T^B T^C\right)_R - \mathrm{str}\left(T^A T^B T^C\right)_L$$

Triangle Anomalies:




B
p
s
+
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Ψ_R

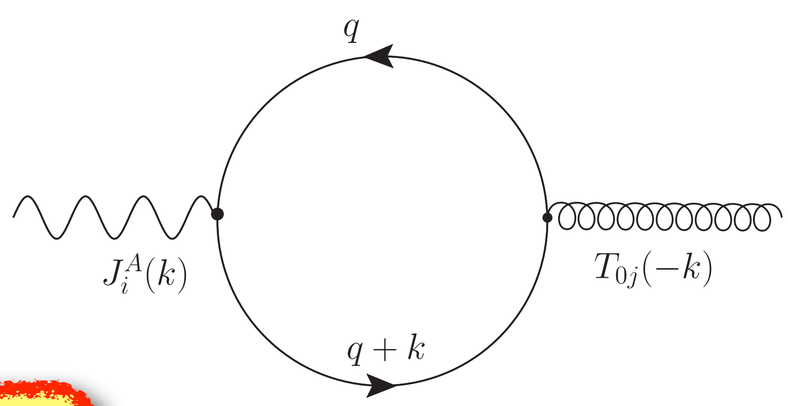
Kubo Formulas

Chemical potential: $\delta E = \mu\, \delta Q$

Energy current: $\delta \vec{J}_\epsilon = \mu\, \delta \vec{J}$

$$\vec{J}_\epsilon = T_{0i} = \sigma^\epsilon_{\mathrm{B}} \vec{B}$$

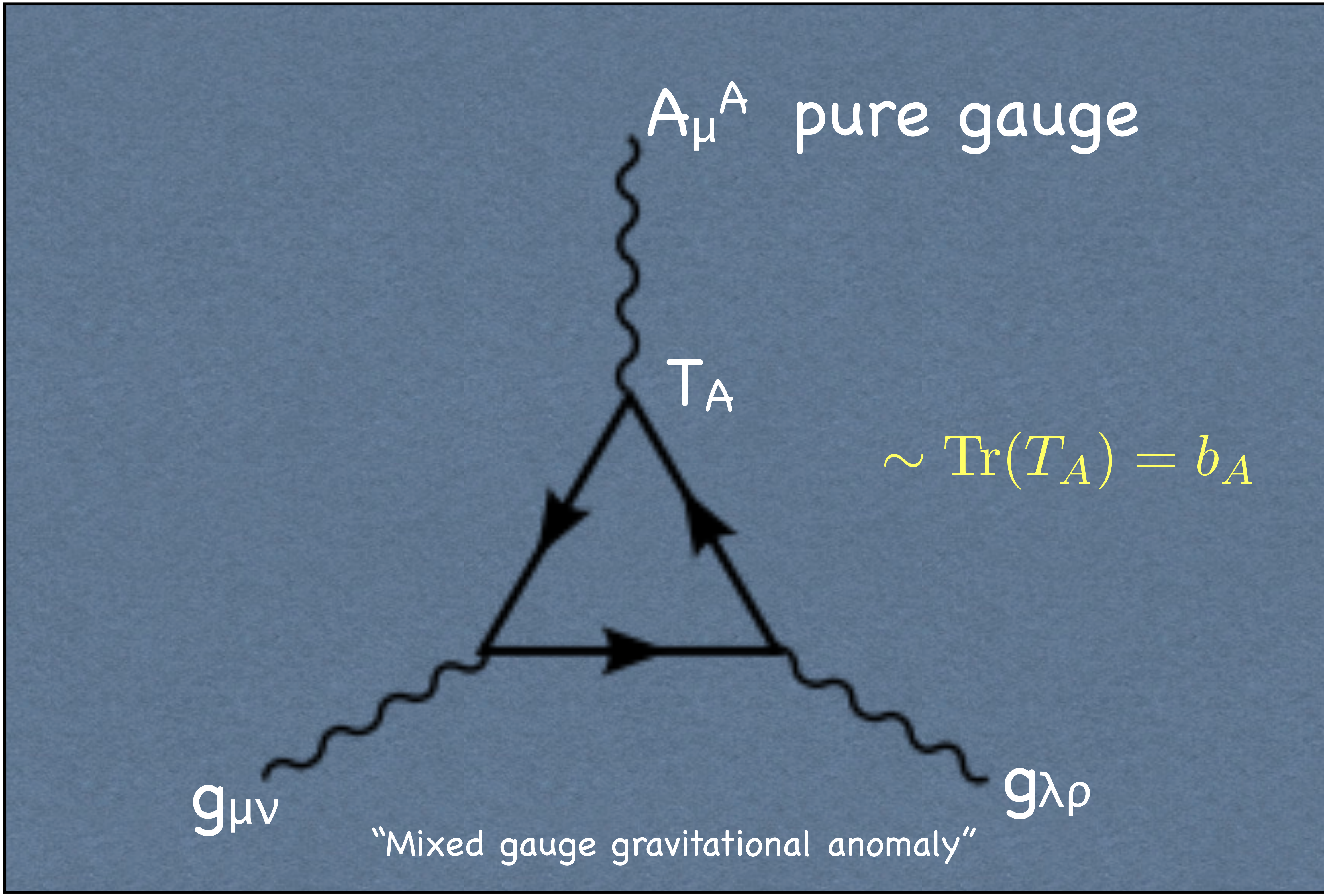
Kubo formula: $\langle T_{0i} J^A_j \rangle = \sigma^{A,\epsilon}_{\mathcal{B}} \epsilon_{ijk} i p_k$



$$\boxed{\sigma^{\epsilon,A}_{\mathcal{B}} = \frac{d^{ABC}}{8\pi^2}\mu_B \mu_C + \frac{b^A}{24}T^2}$$

$$b^A = \mathrm{tr}\left(T^A\right)_R - \mathrm{tr}\left(T^A\right)_L$$

Triangle Anomalies:



Chiral Vortical Effect via Kubo formula

Change order of operators: $\langle J^i T^{0k} \rangle = i p_j \epsilon^{ijk} \sigma^V$

what sort of conductivity?

$$ds^2 = -dt^2 + \vec{A}_g dt d\vec{x} + d\vec{x}^2$$

Rotation via frame dragging (Thirring-Lense effect): ***gravito-magnetic field***

Rotation in fluid: vorticity $\vec{\omega} = \frac{1}{2} \nabla \times \vec{v}$

$$\boxed{\begin{aligned} \sigma_{\mathcal{V}}^{A,\epsilon} &= \sigma_{\mathcal{B}}^{\epsilon,A} \\ \sigma_{\mathcal{V}}^{\epsilon} &= \frac{d^{ABC}}{12\pi^2}\mu_A \mu_B \mu_C + \frac{b_A}{12}\mu_A T^2 \end{aligned}}$$

CME and CVE via Kubo formulae

General symmetry generated by T_a

$$\vec{J}_a = \sigma^{\mathcal{B}}_{ab}\vec{B}_b + 2\sigma^V_a\vec{\omega}$$

$$\vec{J}_\epsilon = \sigma^{\mathcal{B}}_{\epsilon,a}\vec{B}_a + 2\sigma^V_\epsilon\vec{\omega}$$

$$\sigma^{\mathcal{B}}_{ab} = \frac{1}{4\pi^2} d_{abc}\mu_c$$

$$\sigma^V_a = \frac{1}{8\pi^2} d_{abc}\mu_b\mu_c + \frac{T^2}{24} b_a = \sigma^{\mathcal{B}}_\epsilon$$

$$\sigma^V_\epsilon = \frac{1}{12\pi^2} d_{abc}\mu_a\mu_b\mu_c + \frac{T^2}{24} b_a\mu_a$$

Anomaly coefficients:

$$d_{abc} = \mathrm{str}\left(T_a T_b T_c\right)$$

$$b_a = \mathrm{tr}\left(T_a\right)$$

CME and CVE

Interplay of vector and axial symmtries

$$\boxed{\begin{aligned} \vec{J} &= \frac{\mu_5}{2\pi^2}\vec{B} \\ \vec{J}_5 &= \frac{\mu}{2\pi^2}\vec{B} \end{aligned}}$$

Chiral Magnetic Effect (CME)

Chiral Separation Effect (CSE)

$$\boxed{\begin{aligned} \vec{J} &= \frac{\mu\mu_5}{\pi^2}\vec{\omega} \\ \vec{J}_5 &= \left(\frac{\mu^2+\mu_5^2}{2\pi^2}+\frac{T^2}{6}\right)\vec{\omega} \end{aligned}}$$

$$\mu = \frac{\mu_R + \mu_L}{2}$$

$$\mu_5 = \frac{\mu_R - \mu_L}{2}$$

Some comments on CME and CVE

Ohmic transport $\vec{J} = \sigma \vec{E}$

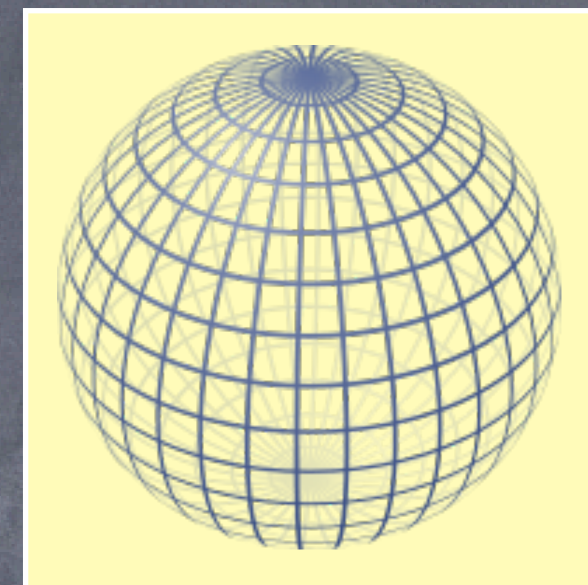
Work done: $\dfrac{dE}{dt} = \vec{J}.\vec{E} = \sigma \vec{E}^2$ $(\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda)$

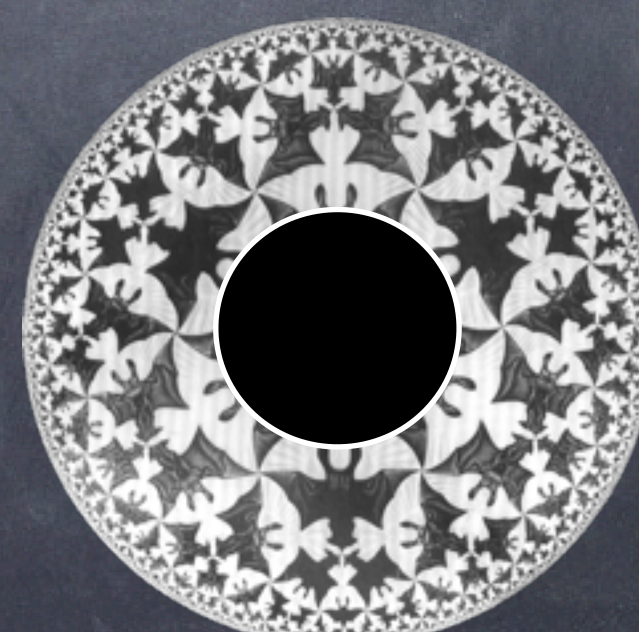
CME (similar for CVE), $\vec{J} = \dfrac{\mu_5}{2\pi^2} \vec{B}$

absence of electric field :

no work done on system dissipationless currents

String Theory as spherical cow of sQGP





String Theory as spherical cow of sQGP

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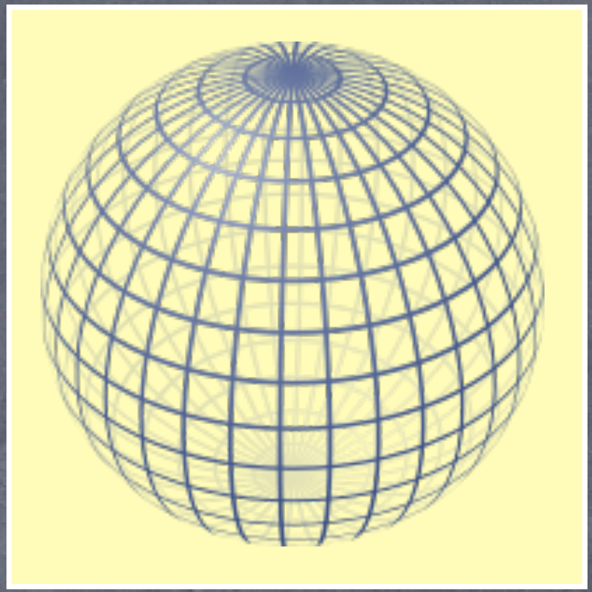

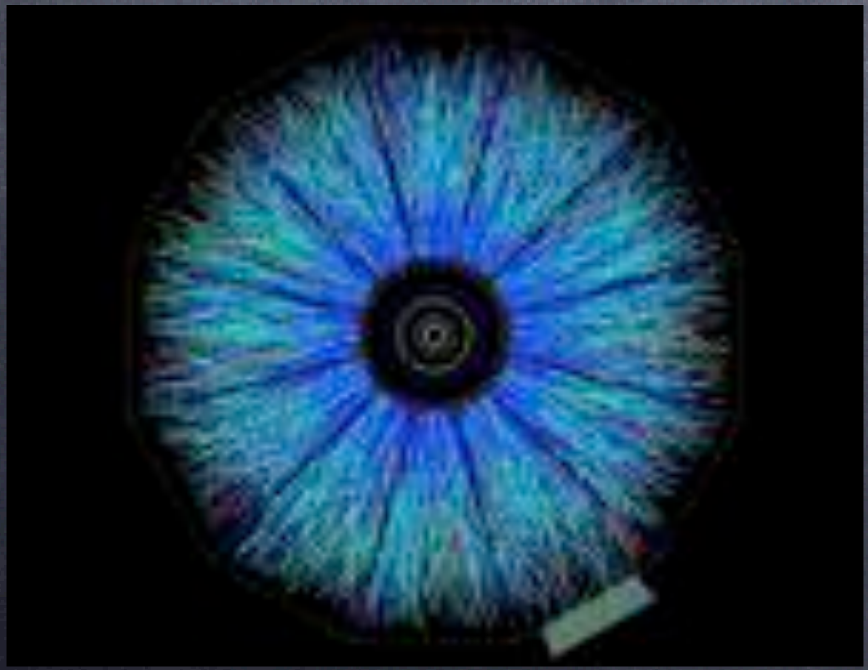

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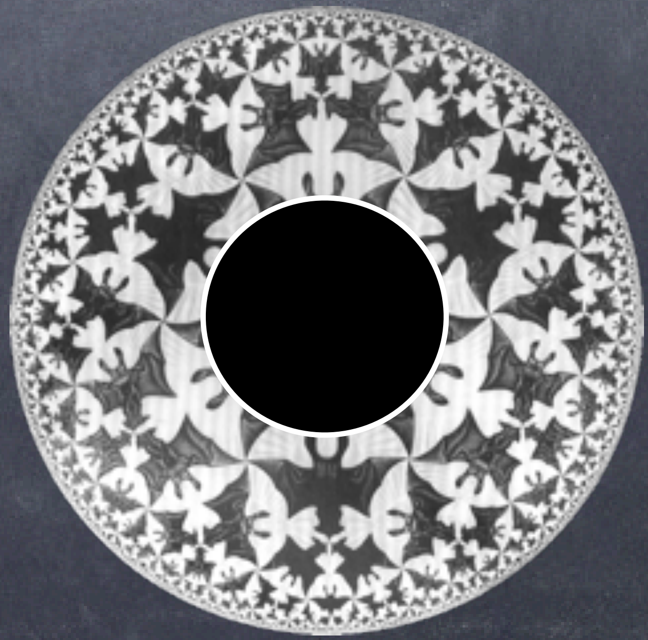


Holography

[Newman], [Banerjee et al.], [Erdmenger et al.] [Yee] [Rebhan, Schmitt, Stricker]
[Khalaydzyan, Kirsch], [Hoyos, Nishioka, OBannon] [Gynther, K.L., Rebhan]

- mixed gauge gravitational Chern Simons term

[E. Megias, K.L., L. Melgar, F. Pena-Benitez]

$$S = S_{ME} + S_{CS} + S_{GH} + S_{CSK}$$

$$S_{EM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right]$$

$$S_{CS} = \frac{1}{16\pi G}\int d^5x\,\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)$$

$$S_{GH} = \frac{1}{8\pi G}\int_\partial d^4x\sqrt{-h}\,K$$

$$S_{CSK} = -\frac{1}{2\pi G}\int_\partial d^4x\sqrt{-h}\,\lambda n_M\epsilon^{MNPQR}A_N K_{PL}D_Q K^L_R$$

Holography

- Holography (String Theory):
  5 dim gravity (Anti de Sitter) dual to strongly coupled quantum field theory
- background: charged AdS black hole

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2 \qquad A_{(0)} = (\beta - \frac{\mu\, r_{\rm H}^2}{r^2})$$

- correlators are

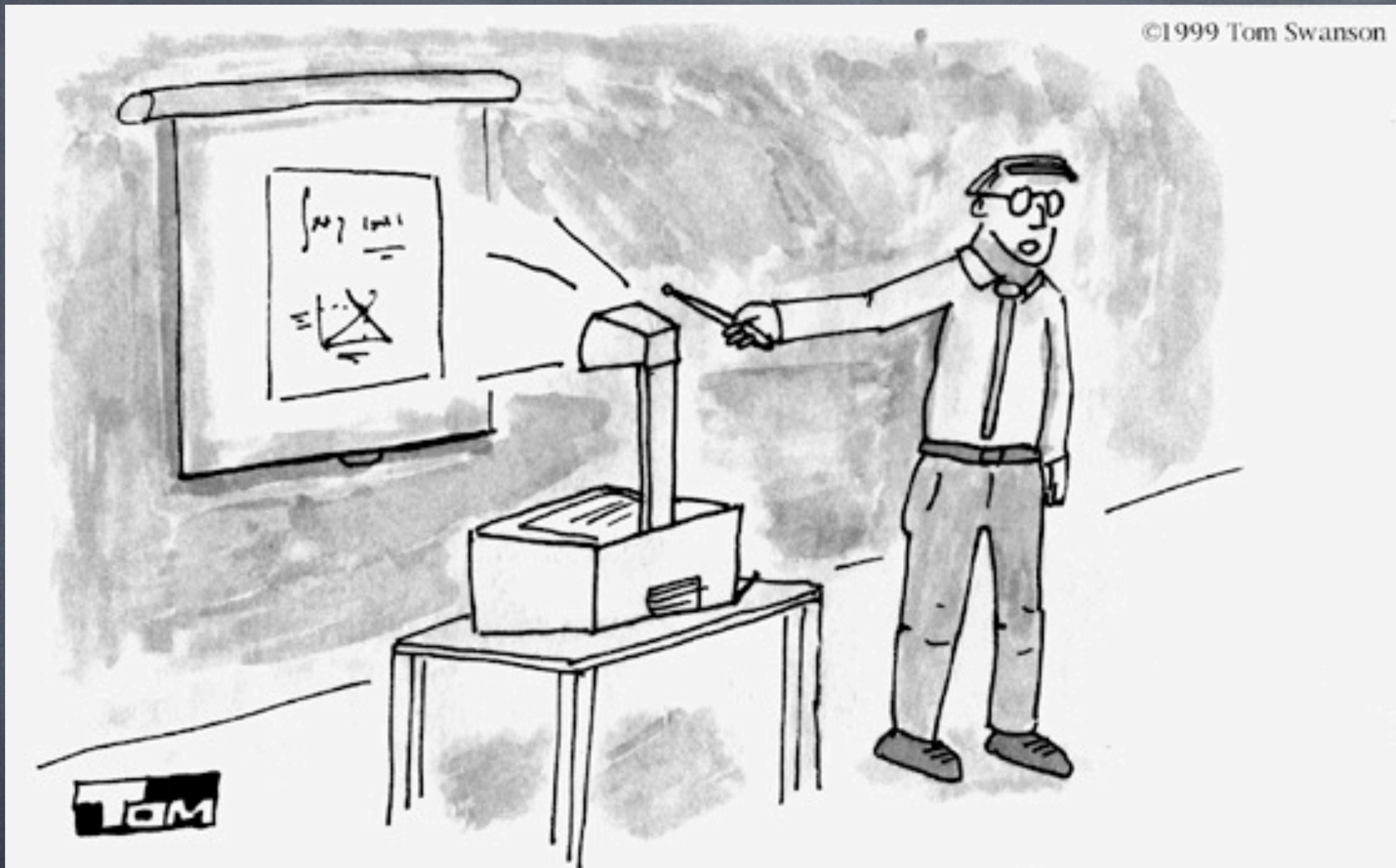
same as weak coupling!
non-renormalization

$$\langle JJ \rangle = -ik_z \left(\frac{\mu}{4\pi^2} - \frac{\beta}{12\pi^2} \right)$$

$$\langle JT \rangle = -ik_z \left(\frac{\mu^2}{8\pi^2} + \frac{T^2}{24} \right)$$

$$\langle TT \rangle = -ik_z \left(\frac{\mu^3}{12\pi^2} + \frac{\mu T^2}{12} \right)$$

Holography



ACTUALLY, THAT ASSUMPTION ISN'T REALLY NECESSARY. WE CAN SEE HERE THAT THE POINT-COW APPROXIMATION WORKS EQUALLY WELL.

CME and CVE in Hydrodynamics

[Son,Surowka],[Neiman,Oz],[Jensen, Loganayagam,Yarom]

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \eta\Sigma^{\mu\nu} - \zeta\Theta + Q^\mu u^\nu + Q^\nu u^\mu$$

$$J^\mu = \rho u^\mu + \sigma_\Omega \left(E^\mu - T\mathcal{P}^{\mu\nu}\nabla_\nu(\frac{\mu}{T}) \right) + \sigma_B B^\mu + 2\sigma_V \omega^\mu$$

$$Q^\mu = \sigma_\epsilon^B B^\mu + 2\sigma_\epsilon^V \omega^\mu$$

$$J_s^\mu = su^\mu - \mu\frac{j_{\mathrm{diss}}}{T}$$

$$\nabla_\mu J_s^\mu \geq 0$$

- Entropy current fixes anomalous transport coeffs up to temperature dependence
- Mismatch in derivative counting
- Non-hydro arguments necessary

$$\boxed{\nabla_\mu J_a^\mu = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{abc}}{32\pi^2}F_{b,\mu\nu}F_{c,\rho\lambda} + \frac{b_a}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)}$$

Holography

- Holography (String Theory):
  5 dim gravity (Anti de Sitter) dual to strongly coupled quantum field theory
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$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2 \qquad A_{(0)} = \left(\beta - \frac{\mu\, r_{\rm H}^2}{r^2}\right)$$

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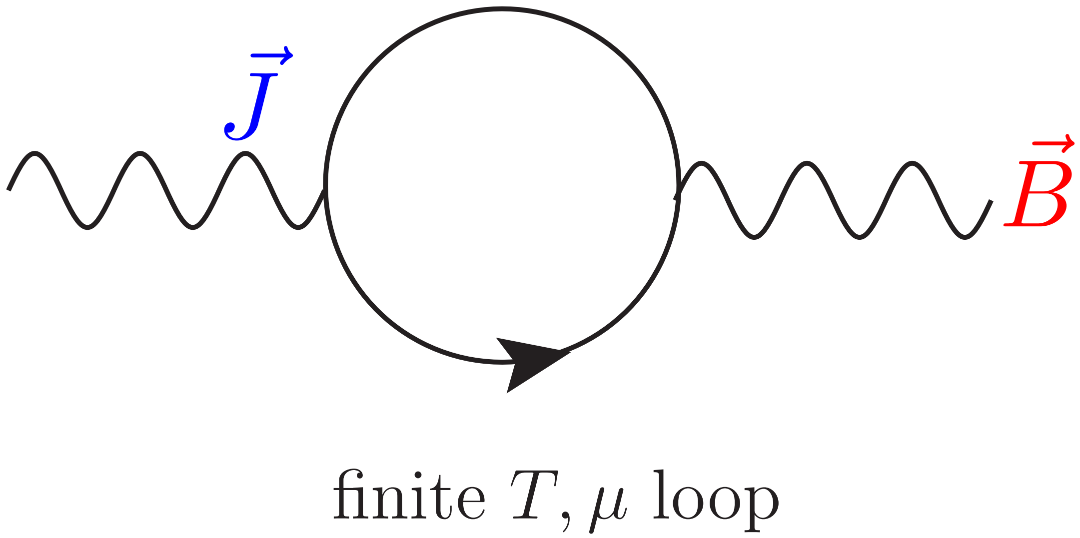
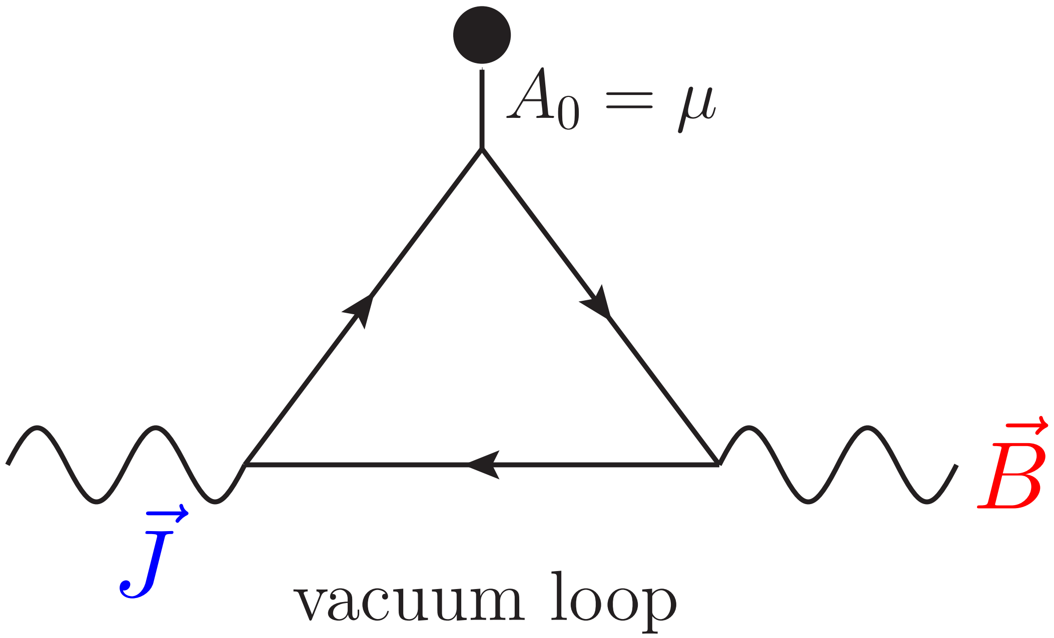
$$\langle TT\rangle = -ik_z\left(\frac{\mu^3}{12\pi^2} + \frac{\mu T^2}{12}\right)$$

what's this?

Consistent vs. Covariant Anomaly





1 chiral fermion:

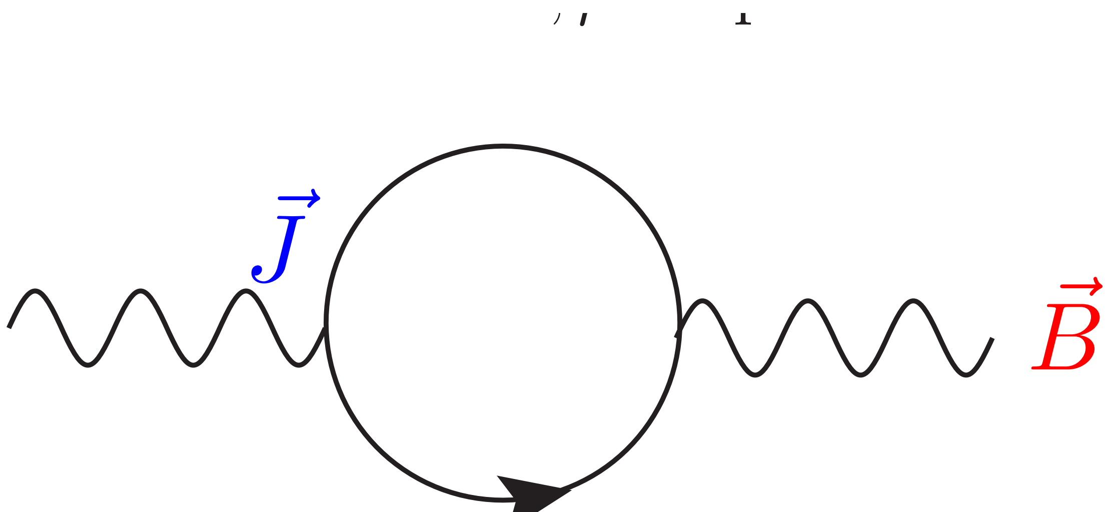
covariant regularization: put all the anomaly into the vertex with current, triangle vanishes

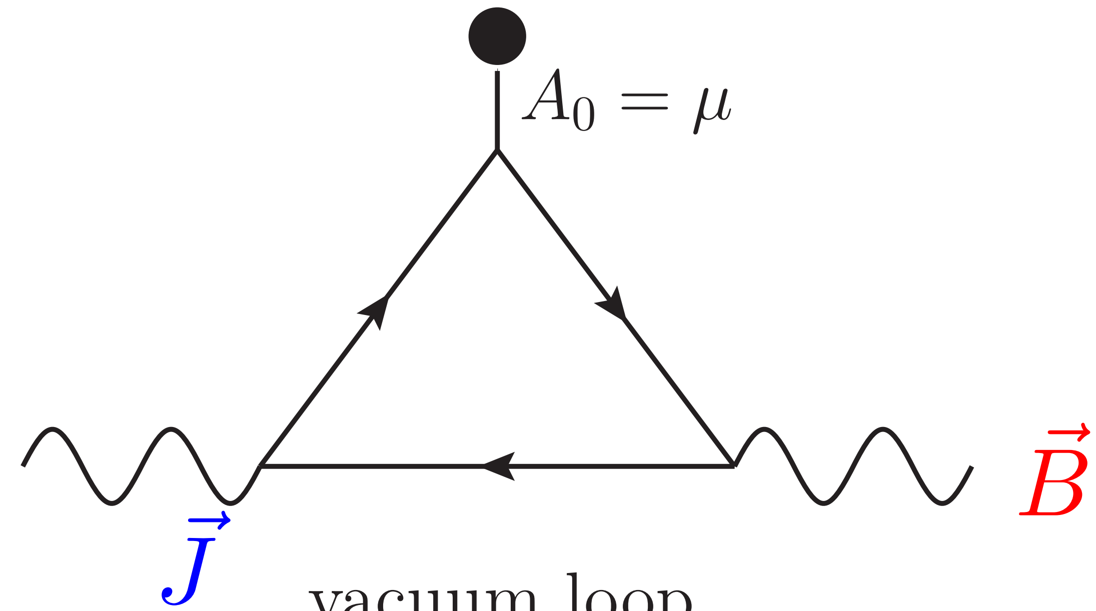
$$\partial_\mu J^\mu = \frac{1}{4\pi^2} \vec{E}\vec{B}$$

$$\vec{J} = \frac{\mu}{4\pi^2} \vec{B}$$

Hydro: covariant current

Consistent vs. Covariant Anomaly





1 chiral fermion:

consistent regularization: distribute the anomaly equally among all three vertices of the triangle,

$$J^\mu = \frac{\delta W_{\text{eff}}[A]}{\delta A_\mu}$$

$$\partial_\mu J^\mu = \frac{1}{12\pi^2}\vec{E}\vec{B}$$

$$\vec{J} = \frac{\mu}{4\pi^2}\vec{B} - \frac{A_0}{12\pi^2}\vec{B}$$

The consistent theory of a Dirac fermion

$$\mathcal{L} = \bar{\Psi}\gamma^\mu(\partial_\mu - iA_\mu - iA^5_\mu\gamma_5)\Psi$$

$$\Gamma'[A_\mu, A^5_\mu] = \Gamma[A_\mu, A^5_\mu] + \int \epsilon^{\mu\nu\rho\lambda} A_\mu A^5_\nu \left(c_1 F_{\rho\lambda} + c_2 F^5_{\rho\lambda}\right)$$

$$J^\mu_{\text{el}} = \frac{\delta\Gamma}{\delta A_\mu} :$$

$$\boxed{\begin{aligned} &\partial_\mu J^\mu_{\text{el}} = 0 \\ &J^\mu_{\text{el}} = J^\mu_V - \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda} A^5_\nu F_{\rho\lambda} \end{aligned}}$$

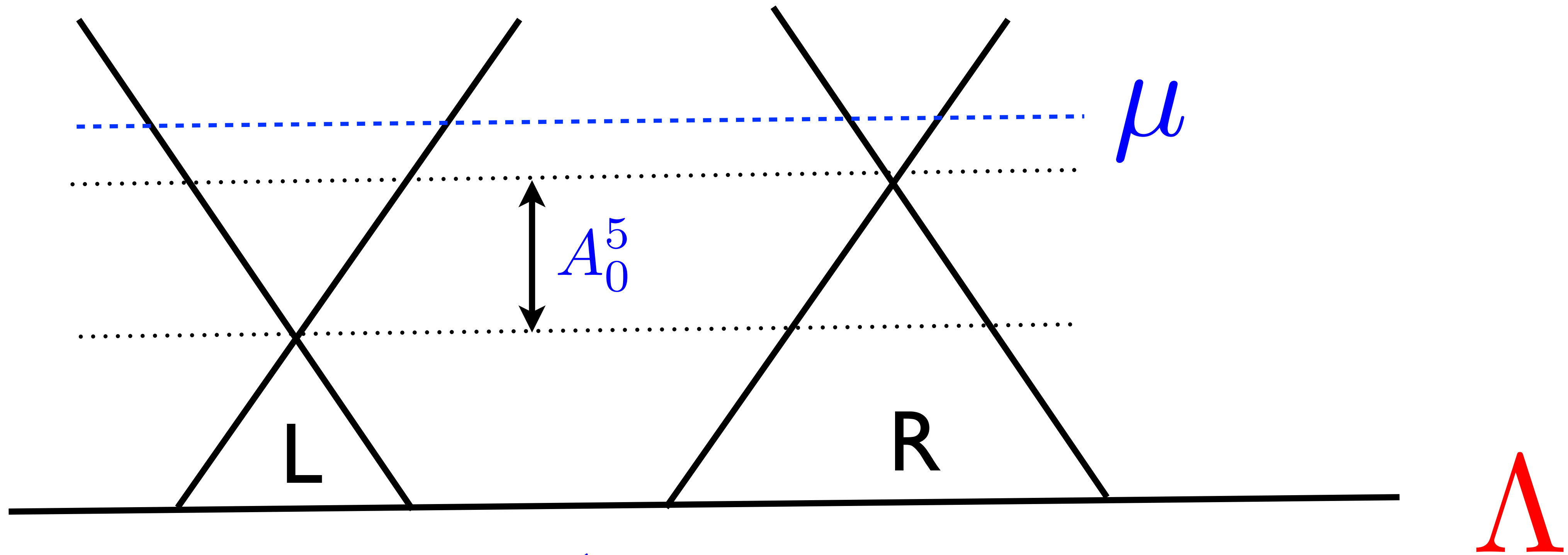
[Bardeen],
[Bardeen,Zumino]

$$\boxed{\vec{J}_{\text{el}} = \frac{\mu_5}{2\pi^2}\vec{B} - \frac{A^5_0}{2\pi^2}\vec{B}}$$

$$\Psi_D \neq \Psi_L \oplus \Psi_R$$

Physical Interpretation of CS current

where does the current come from?



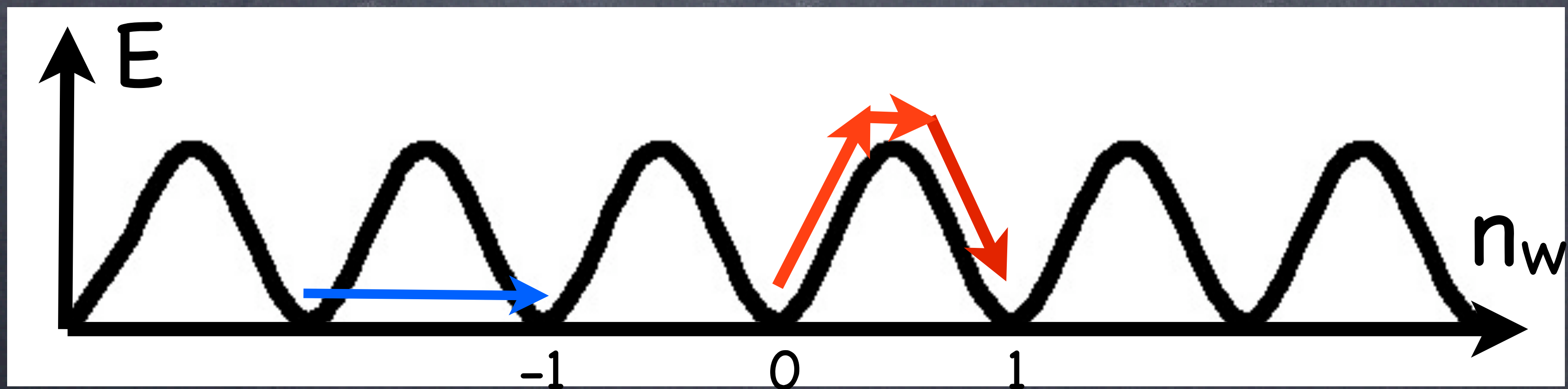
$$J^{\mu}_{\mathrm{CS}} = -\frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_{\nu}F_{\rho\lambda}$$

- CS current stems from states beyond (or at) the cutoff

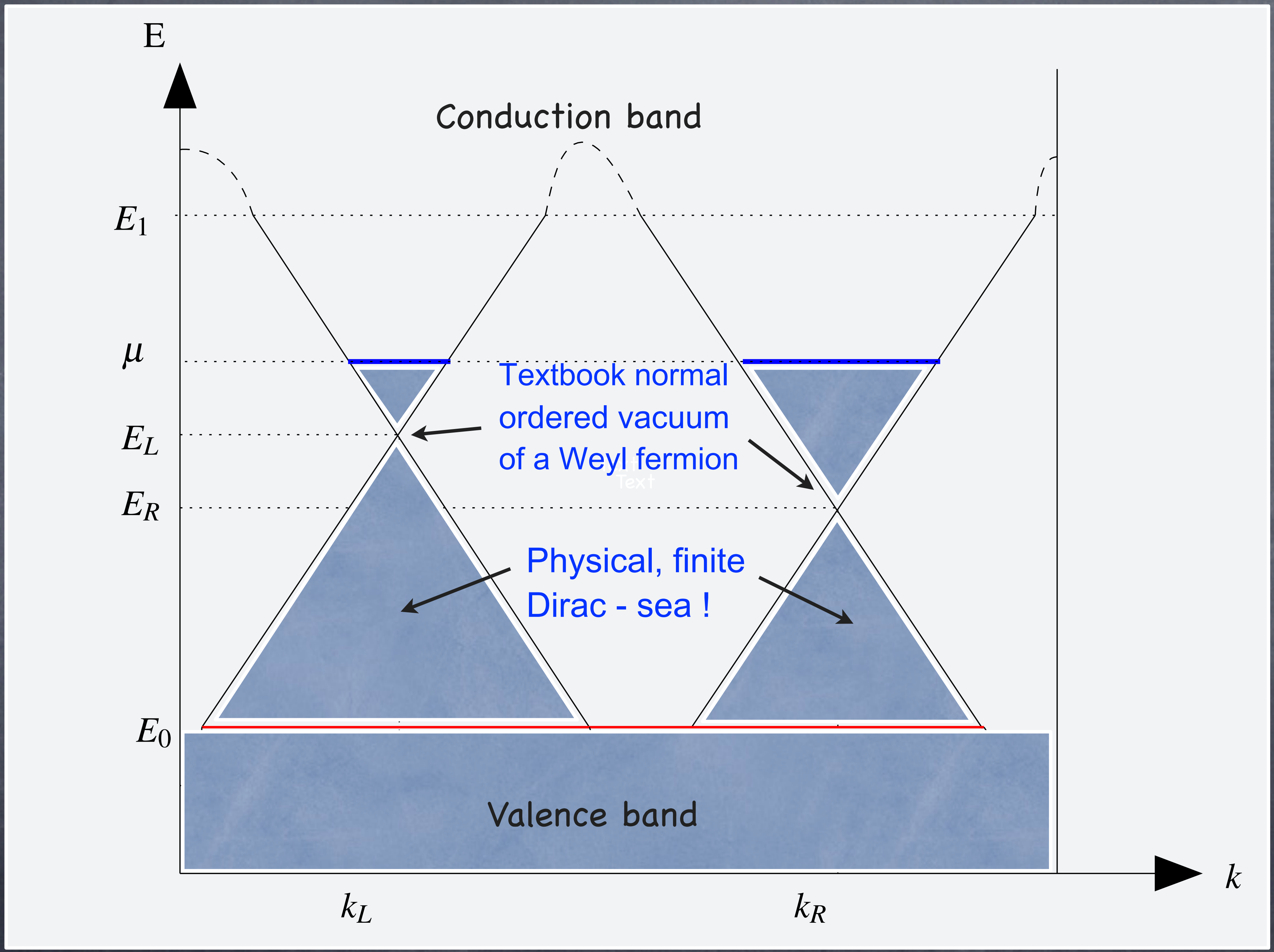
Application: QGP

[Kharzeev, McLarren, Warringa]

- topological charge $Q_w = \frac{g^2}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$
- axial anomaly (QCD) $\partial_\mu j^\mu_5 = 2m_f\langle\bar{\psi}_f i\gamma_5\psi_f\rangle - \frac{N_f g^2}{16\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$
- topologically non trivial gauge field
- effective: axial chemical potential $\mu_5 \leftrightarrow \Delta Q_5 = 2N_f Q_w$
- No axial gauge field at fundamental level in nature $A^5_\mu = 0$



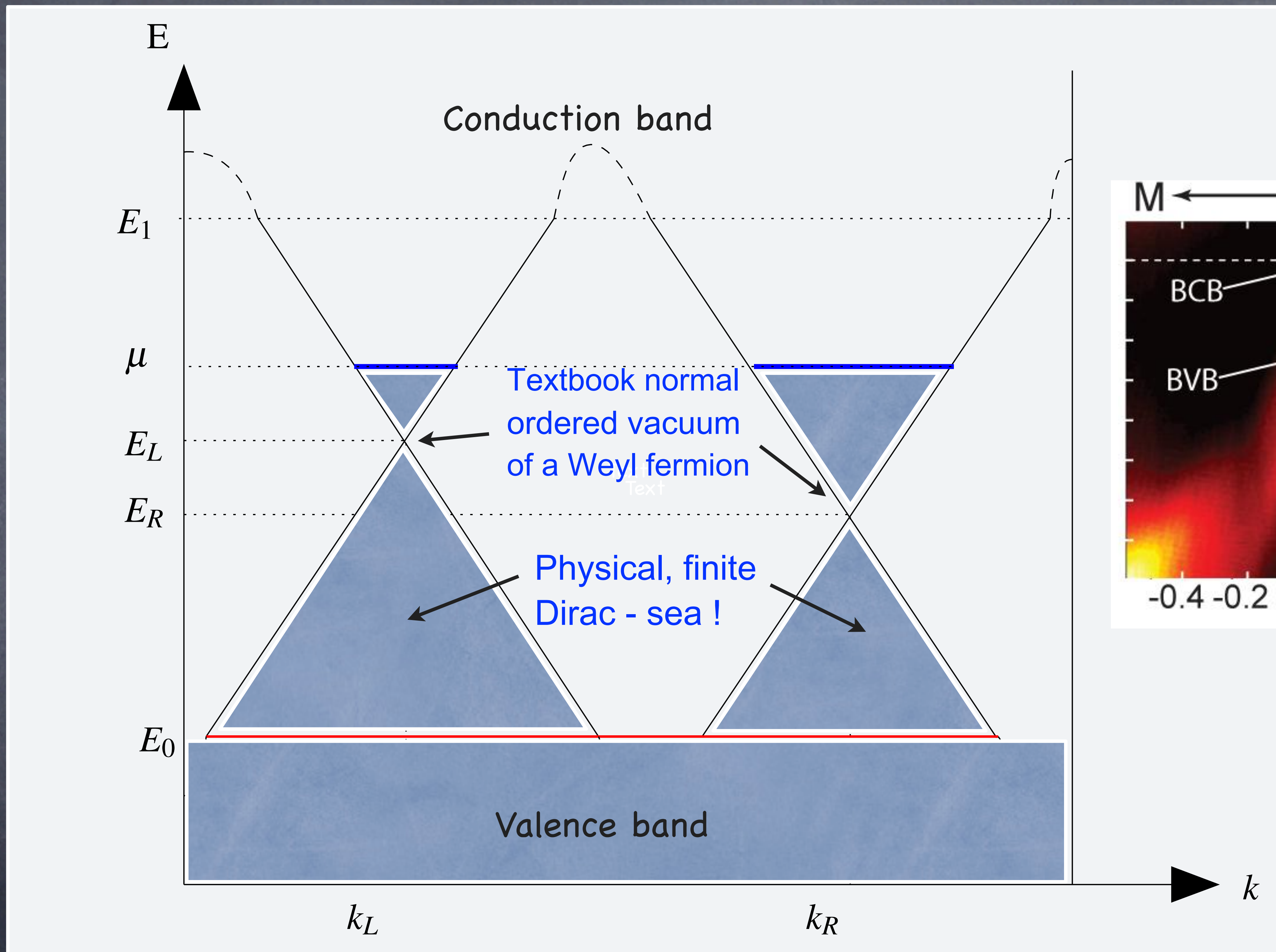
Weyl Semi-metals

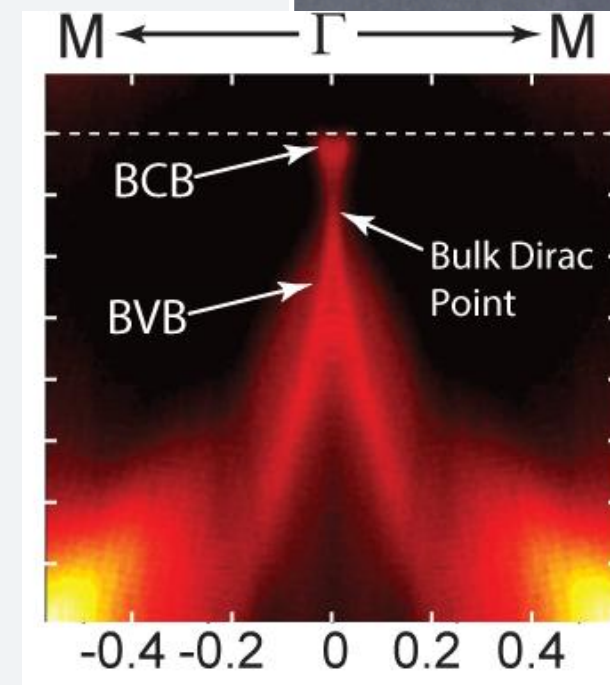


$$A^5_\mu = 1/2(E_R - E_L, \vec{k}_R - \vec{k}_L)$$

[K.L. PRB(!)]

Weyl Semi-metals





[Liu et al. Na_3Bi]

[Neupane et al. Cd_3As_2]

[Borisenko et al. Cd_3As_2]

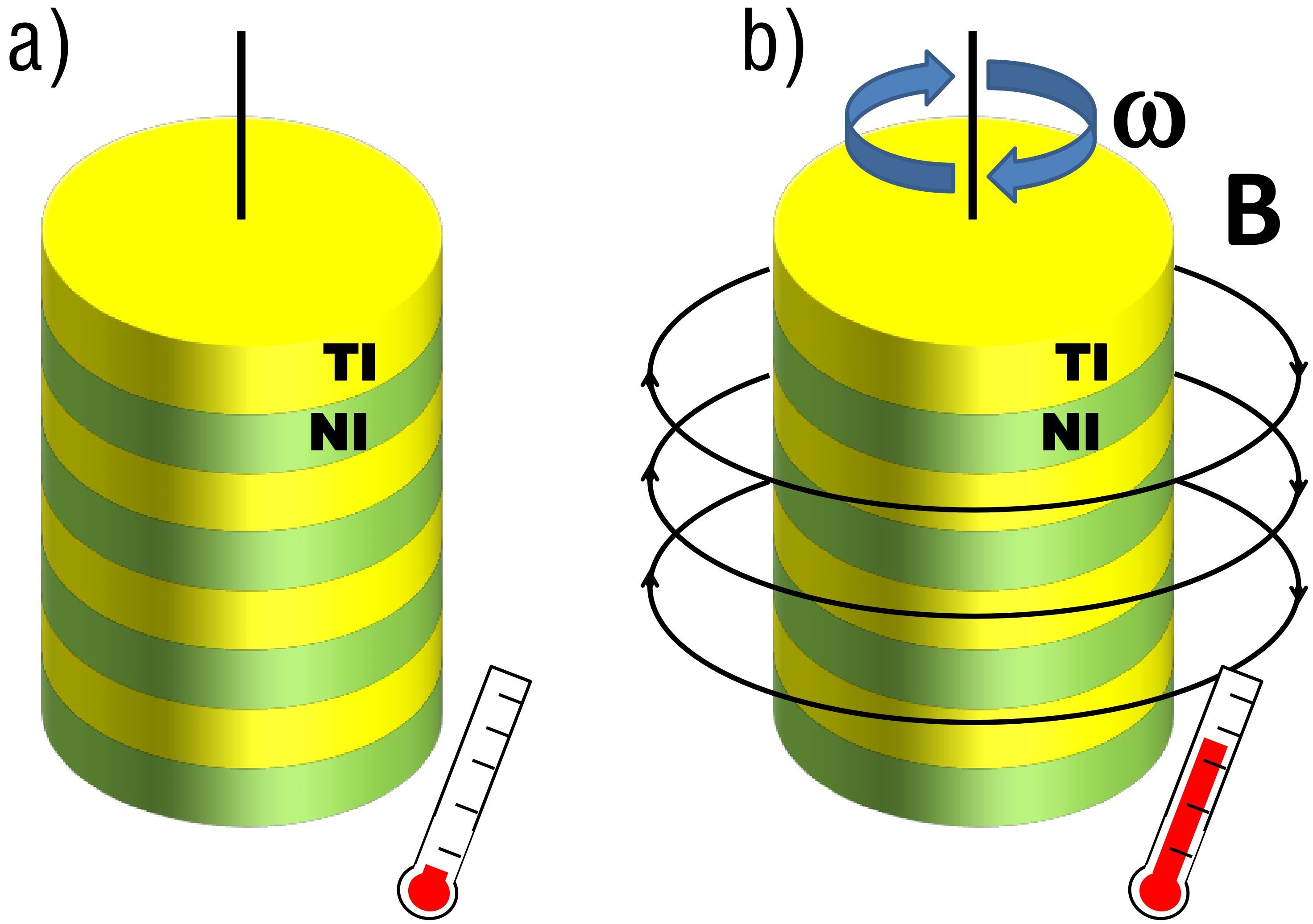
$$A^5_\mu = 1/2(E_R - E_L, \vec{k}_R - \vec{k}_L)$$

[K.L. PRB(!)]

Axial Magnetic Effect in WSM

$$T^{0i} = J_\epsilon^i = \sigma_{AME} B_5^i. \qquad \sigma_{AME} = \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12},$$

$$L_z = \frac{N_f}{6} T^2 b_z \mathcal{V},$$



Gravitational anomaly @ work in the laboratory!

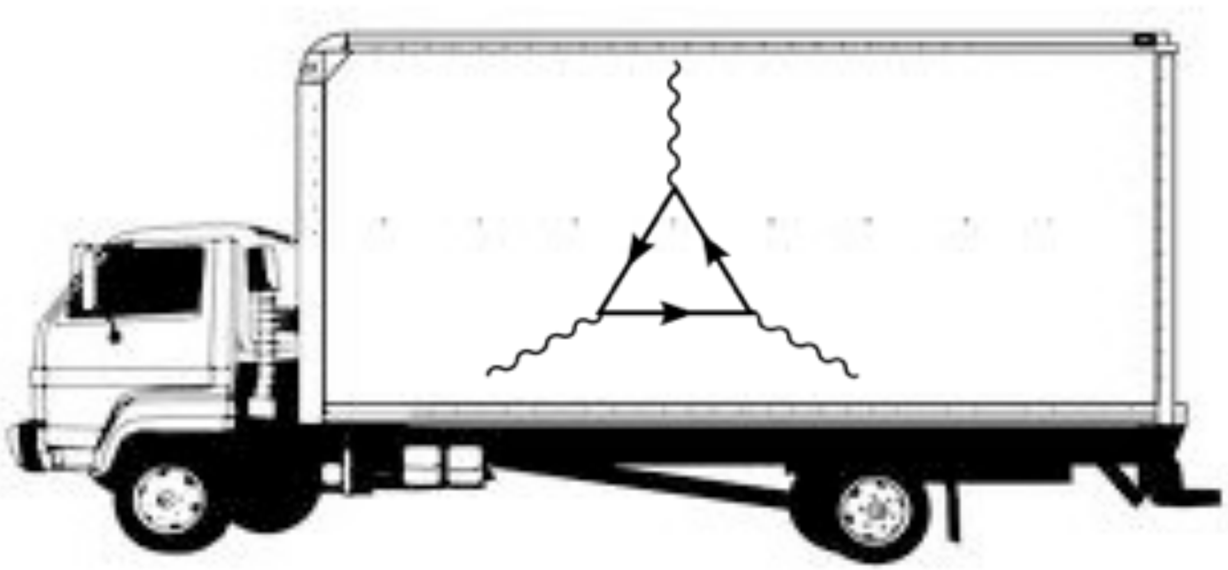
[M. Chernodub, A.Cortijo, A. Grushin, K. L., M.Vozmediano] (PRB)

Summary

- Non-dissipative transport of charge and energy via triangle anomalies!
- Non-renormalization (only for global symmetries)!
- Applications: QGP, WSM (table-top experiments!)

Thank You!

Thank You!



Renormalization via dynamical gauge fields

- Chiral vortical effect in axial current
- No chemical potential necessary
- On Lattice accessible via Axial Magnetic Effect

$$\vec{J}_\epsilon = \sigma_V \vec{B}_5$$

[V.Braguta, M. Chernodub, V.A. Goy, A.V. Molochkov, K.L., M. Polikarpov]