$X(3872)$ in Heavy Quark Limit of QCD: Its Partners and Isospin Structure

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April 16, 2014
QCD @ WORK, Bari, ITALY

Motivation

- $X(3872)$ is one of the recently discovered and well established states.
- It decays into a final state with $I = 0$ and $I = 1$ with almost equal branching ratio:

$$\frac{B(X(3872) \to J/\psi\rho)}{B(X(3872) \to J/\psi\omega)} \simeq 1 \Rightarrow \frac{A(X(3872) \to J/\psi\rho)}{A(X(3872) \to J/\psi\omega)} \simeq 0.2 \tag{1}$$

- One of the most popular descriptions of $X(3872)$ is as a $D\bar{D}^*$ molecule
- In the molecular picture, isospin violation naturally arises because of the mass deference of charged and neutral $D\bar{D}^*$ components: even if the interaction is isospin conserving, the kinetic energy is isospin violating
- It was shown that in the heavy quark limit, $X(3872)$ should have partners with the same binding energy.

S_h	S_l	J^{PC}
1	1	2^{++} 1^{++} 0^{++}
0	1	1^{+-}
1	0	1^{+-}
0	0	0^{++}

Table : Spectrum of $L = 0$ states of two heavy and two light quarks

In the heavy quark limit, the states which have the same light quark structure are degenerate.

- How does this degeneracy arise
- How to obtain the isospin structure

from a correlation function written in terms of quarks and gluons?

Hidalgo-Duque, J. Nieves, A.O., V. Zamiralov, PLB727 (2013) 432
F.-K. Guo, C. Hidalgo-Duqeu, J. Nieves, A.O., M.P. Valderrama, Eur. Phys. J. C74 (2014) 2885

Correlation Functions and Interpolating Currents

- In frameworks which tries to obtain hadronic properties directly from the quark-gluon description of QCD without using any effective approach, the fundamental object to study is the correlation function

$$\Pi(q) = i \int d^4 x e^{iqx} \langle 0 | \mathcal{T} j(x) j^\dagger(0) | 0 \rangle \tag{2}$$

- This correlation function can be written as a sum over the states that can be created by the operator j as:

$$\Pi(q) = \sum_h \frac{\langle 0 | j | h(q) \rangle \langle h(q) | j^\dagger | 0 \rangle}{q^2 - m_h^2} \tag{3}$$

$$\Pi(q) = \sum_h \frac{\langle 0|j|h(q)\rangle\langle h(q)|j^\dagger|0\rangle}{q^2 - m_h^2} \tag{4}$$

- As long as $\langle 0|j|h(q)\rangle \neq 0$, $\Pi(q)$ will have poles when $q^2 = m_h^2$.
- If one can calculate $\Pi(q)$ exactly, one can find the spectrum and the properties of all hadrons.
- In sum rules, one approximates $\Pi(q)$ in the $q^2 \to -\infty$ limit, in terms of OPE,
- In lattice calculations, $\Pi(q)$ is calculated numerically,

etc.

Interpolating Currents

- In the molecular picture of $X(3872)$, it is formed by two heavy quarks in the $S = 1$ configuration, and two light quarks in the $S = 1$ configuration, i.e. both are in a vector configuration

$$|X(3872)\rangle = |\,[(\bar{c}c)(S = 1)(\bar{q}q)(S = 1)]\,J = 1\rangle \qquad (5)$$

- This hints at an interpolating current which is the product of two vector operators ($C = +1$):

$$j^{q}_{\alpha\beta} = \bar{Q}^a \gamma_\alpha Q^b \bar{q}^b \gamma_\beta q^a \qquad (6)$$

- The color factors are chosen such that they would give colorless D mesons.

- $J^q_{\alpha\beta}$ do not form an irreducible representation of the Lorentz group.
- The irreducible representations can be obtained by the following projection operators.

$$\mathcal{P}^2_{\mu\nu;\bar{\mu}\bar{\nu}} = \frac{1}{2}\left(g_{\mu\bar{\mu}}g_{\nu\bar{\nu}} + g_{\mu\bar{\nu}}g_{\nu\bar{\mu}} - \frac{1}{2}g_{\mu\nu}g_{\bar{\mu}\bar{\nu}}\right) \quad (7)$$

$$\mathcal{P}^1_{\mu\nu;\bar{\mu}\bar{\nu}} = \frac{1}{2}\left(g_{\mu\bar{\mu}}g_{\nu\bar{\nu}} - g_{\mu\bar{\nu}}g_{\nu\bar{\mu}}\right) \quad (8)$$

$$\mathcal{P}^0_{\mu\nu;\bar{\mu}\bar{\nu}} = \frac{1}{4}g_{\mu\nu}g_{\bar{\mu}\bar{\nu}} \quad (9)$$

- The irreducible currents can be obtained as:

$$j^{s}_{\mu\nu} \equiv \mathcal{P}^{s}{}_{\mu\nu\bar{\mu}\bar{\nu}} j^{\bar{\mu}\bar{\nu}} \quad (10)$$

S	# of d.o.f.	States
2	9	$2 \oplus 1 \oplus 0$
1	6	$1 \oplus 1$
0	1	0

By a Fierz transformation, the irreducible representations can also be written in terms of colorless bilinears as:

$$j^q_{\alpha\beta} = \bar{Q}^a \gamma_\alpha Q^b \bar{q}^b \gamma_\beta q^a \tag{11}$$

$$j^0_{\mu\nu} \equiv \mathcal{P}^0{}_{\mu\nu\bar{\mu}\bar{\nu}} j^{\bar{\mu}\bar{\nu}} = \cdots \tag{12}$$

$$\begin{aligned} j^1_{\mu\nu} &\equiv \mathcal{P}^1_{\mu\nu\bar{\mu}\bar{\nu}} j^{\bar{\mu}\bar{\nu}} \\ &= \frac{i}{4}\epsilon_{\mu\nu\alpha\beta}\left[(\bar{Q}\gamma^\beta\gamma_5 q)(\bar{q}\gamma^\alpha Q) + (\bar{Q}\gamma^\beta q)(\bar{q}\gamma^\alpha\gamma_5 Q)\right] \\ &+ \frac{i}{8}\left[(\bar{Q}\sigma_{\mu\nu} q)(\bar{q}Q) + (\bar{Q}q)(\bar{q}\sigma_{\mu\nu}Q)\right] \\ &+ \frac{i}{8}\left[(\bar{Q}\sigma_{\mu\nu}\gamma_5 q)(\bar{q}\gamma_5 Q) + (\bar{Q}\gamma_5 q)(\bar{q}\sigma_{\mu\nu}\gamma_5 Q)\right] \end{aligned} \tag{13}$$

$$j^2_{\mu\nu} \equiv \mathcal{P}^2_{\mu\nu\bar{\mu}\bar{\nu}} j^{\bar{\mu}\bar{\nu}} = \cdots \tag{14}$$

Correlation Function

- Define:

$$\Pi_{\alpha\beta;\gamma\delta} = i\int d^4x e^{iqx}\langle 0|\mathcal{T} j_{\alpha\beta}(x) j^\dagger_{\gamma\delta}(0)|0\rangle \qquad (15)$$

- In the heavy quark limit, $q = 2m_Q v + k$ and
- the spin (γ) of the heavy quark decouples from the theory:

$$\begin{aligned}\Pi_{\alpha\beta;\gamma\delta} &= \mathrm{Tr}\left[\gamma_\alpha \frac{1+\not{v}}{2}\gamma_\gamma\frac{1-\not{v}}{2}\right]\left(\mathcal{R}_1(kv)g_{\beta\delta} + \mathcal{R}_2(kv)v_\beta v_\delta\right) \\ &= 2\left(g_{\alpha\gamma} - v_\alpha v_\gamma\right)\left(\mathcal{R}_1 g_{\beta\delta} + \mathcal{R}_2 v_\beta v_\delta\right) \qquad (16)\end{aligned}$$

- Correlation functions formed of different irreducible representations can be expressed by the SAME functions $\mathcal{R}_1$ and $\mathcal{R}_2$ even though they receive contributions from different particles.

$$\mathcal{R}_1 = \frac{D^2/2}{kv - \Lambda_{2++}} = \frac{A'^2}{kv - \Lambda_{1++}}$$

$$\mathcal{R}_1 = -8\frac{C^2}{kv - \Lambda_{0++}} + \frac{2}{3}\frac{D^2}{kv - \Lambda_{2++}}$$

$$\mathcal{R}_1 - \mathcal{R}_2 = \frac{D^2}{kv - \Lambda_{2++}} + \frac{2F^2}{kv - \Lambda_{1-+}} = \frac{2A^2}{kv - \Lambda_{1-+}} + \frac{2A'^2}{kv - \Lambda_{1++}} \tag{17}$$

Assuming all coefficients are non-zero:

$$\Lambda_{2++} = \Lambda_{0++} = \Lambda_{1++} \tag{18}$$

$$\Lambda_{JPC} = \lim_{m_Q \to \infty} (m_{JPC} - 2m_Q) \tag{19}$$

- An alternative current for $C = -1$ states can be written as:

$$\tilde{j}^q_{\alpha\beta} = \bar{Q}^a \gamma_\alpha \gamma_5 Q^b \bar{q}^b \gamma_\beta q^a \tag{20}$$

- Then, its correlation function can be expressed in terms of the identical functions $\mathcal{R}_1$ and $\mathcal{R}_2$:

$$\Pi_{\alpha\beta;\gamma\delta} = \mathrm{Tr}\left[\gamma_\alpha \gamma_5 \frac{1 + \not{v}}{2} \gamma_\gamma \frac{1 - \not{v}}{2}\right] \left(\mathcal{R}_1 g_{\beta\delta} + \mathcal{R}_2 v_\beta v_\delta\right) \tag{21}$$

- New degeneracies are: $\Lambda_{0^{++}} = \Lambda_{1^{++}} = \Lambda_{2^{++}} = \Lambda_{1^{+-}}$ and $\Lambda_{1^{-+}} = \Lambda_{0^{--}}$

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- By inserting a γ_5 in the light quark sector, we would obtain: $\Lambda_{0^{--}} = \Lambda_{1^{--}} = \Lambda_{2^{--}} = \Lambda_{1^{-+}}$ and $\Lambda_{1^{+-}} = \Lambda_{0^{++}}$

NOTE:

We do not:

- predict the existence (or nonexistence) of any state. If one state exists, its should have degenerate partners.
- make any assumption on the structure of the light quarks. Our results are valid for no light quarks, $I = 0$, $I = 1/2$, $I = 1$, hidden strange mesons (hence also predicting partners for the Z_c)

Some Known Examples:

- 1P states: $\chi_{c0}(0^{++})$, $\chi_{c1}(1^{++})$, $h_c(1^{+-})$, $\chi_{c2}(2^{++})$, $\Delta m = 142\ MeV$
- 1P states: $\chi_{b0}(0^{++})$, $\chi_{b1}(1^{++})$, $h_b(1^{+-})$, $\chi_{b2}(2^{++})$, $\Delta m = 52\ MeV$
- 2P states: $\chi_{b0}(0^{++})$, $\chi_{b1}(1^{++})$, $h_b(1^{+-})(?)$, $\chi_{b2}(2^{++})$, $\Delta m = 37\ MeV$

Isospin Structure of $X(3872)$

- In general, $X(3872)$ should have an isospin $I = 1$ component to explain its decay pattern.
- The physical $X(3872)$ state and its orthogonal state can be written as

$$|X(3872)\rangle = \cos\theta|X(0)\rangle + \sin\theta|X(1)\rangle \quad (22)$$

$$|X_\perp\rangle = -\sin\theta|X(0)\rangle + \cos\theta|X(1)\rangle \quad (23)$$

- And the corresponding currents are:

$$\begin{aligned} j_{X(3872)} &= \cos\theta j^{I=0} + \sin\theta j^{I=1} \\ j_{X_\perp} &= -\sin\theta j^{I=0} + \cos\theta j^{I=1} \end{aligned} \quad (24)$$

where $j^I = 1/\sqrt{2}[j^u - (-1)^I j^d]$

- Since, $X(3872)$ and $X_\perp$ are physical eigenstates, they can not oscillate into each other:

$$i\int d^4x e^{ipx}\langle 0|\mathcal{T} j_{X(3872)}(x) j^\dagger_{X_\perp}(0)|0\rangle + i\int d^4x e^{ipx}\langle 0|\mathcal{T} j_{X_\perp}(x) j^\dagger_{X(3872)}(0)|0\rangle = 0 \qquad (25)$$

- Hence, the mixing angle should be

$$\tan 2\theta = \frac{\Pi^{10}+\Pi^{01}}{\Pi^{00}-\Pi^{11}} = \frac{\Pi^{uu}-\Pi^{dd}}{\Pi^{ud}+\Pi^{du}} = \frac{\Pi^{nn}-\Pi^{cc}}{\Pi^{nc}+\Pi^{cn}} \qquad (26)$$

$$\tan 2\theta = \frac{\Pi^{10} + \Pi^{01}}{\Pi^{00} - \Pi^{11}} = \frac{\Pi^{uu} - \Pi^{dd}}{\Pi^{ud} + \Pi^{du}} \tag{27}$$

Very crude numerical estimate can be obtained:

- In sum rules, Π is expanded in terms of vacuum condensates

$$\Pi = \Lambda^{\delta} \sum_d c_d \frac{\langle O_d \rangle}{\Lambda^d} \tag{28}$$

  In our case, $\delta = 10$.
- $\Pi^{10} + \Pi^{01} = \Pi^{uu} - \Pi^{dd}$ receive contributions only from isospin violating effects. In a QSR framework, the leading contributions are proportional to $(m_u - m_d)\Lambda^9$, and/or $\alpha\Lambda^{10}$. For $\Lambda = m_{X(3872)} - 2m_c = 1.32\ GeV$, those two contributions are of the same order of magnitude.

$$\tan 2\theta = \frac{\Pi^{10} + \Pi^{01}}{\Pi^{00} - \Pi^{11}} = \frac{\Pi^{uu} - \Pi^{dd}}{\Pi^{ud} + \Pi^{du}}$$

- $\Pi^{00} - \Pi^{11} = \Pi^{ud} + \Pi^{du}$ has a leading term $\mathcal{O}(\Lambda^{10})$.
- This difference is responsible for the mass difference between the $I = 0$ state and $I = 1$ state. Assume $\Pi^{00} - \Pi^{11} \simeq \beta\Lambda^{10}$, where β is a measure of the splitting.

$$\tan\theta = \frac{m_u - m_d}{\beta\Lambda} \tag{29}$$

- The splitting of the isospin states is caused by the light quarks which are put in a vector configuration $\Rightarrow$ use $\omega - \rho$ system to estimate its value.

$$\beta \sim \frac{m_\omega - m_\rho}{m_\omega + m_\rho} = 0.0046 \tag{30}$$

- Then the mixing angle is estimated (crudely) as

$$\tan 2\theta = \frac{m_u - m_d}{\beta\Lambda} \simeq 0.56 \longrightarrow \theta \simeq 15^\circ \tag{31}$$

- In the molecular picture of $X(3872)$ can be written as (Gamermann2010)

$$
\begin{aligned}
|X(3872)\rangle &= |\psi_1\rangle|D^0\bar{D}^{*0}\rangle + |\psi_2\rangle|D^+D^{*-}\rangle \\
&= \frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right)|I=0\rangle + \frac{1}{\sqrt{2}}\left(|\psi_1\rangle - |\psi_2\rangle\right)|I=1\rangle \\
\langle\psi_1|\psi_1\rangle &+ \langle\psi_2|\psi_2\rangle = 1
\end{aligned}
$$

- In terms of the wave functions ψ_1 and ψ_2, the mixing angle can be written as

$$\tan^2\theta = \frac{\int d^3r|\psi_1(\vec{r}) - \psi_2(\vec{r})|^2}{\int d^3r|\psi_1(\vec{r}) + \psi_2(\vec{r})|^2} \Rightarrow \theta \sim 39^\circ \qquad (32)$$

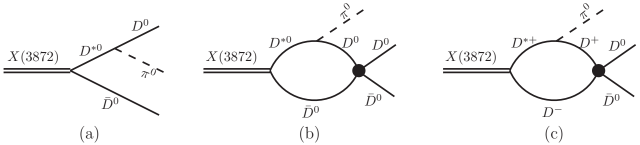
- Note that the $I=1$ strong decay is suppressed by the wave function at the origin; $\psi_1(0) - \psi_2(0)$.

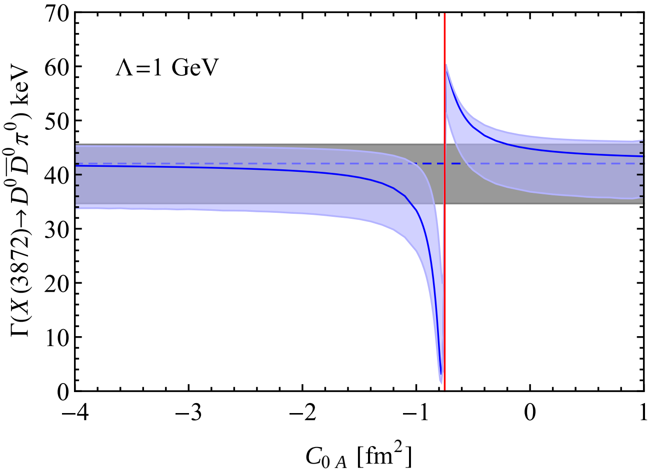
Determination of the Isospin Structure

- The decays of $X(3872)$ into $n\pi$ through $I = 1$ component is suppressed by the wave function at the origin.
- To study the $I = 1$ component, a decay that can proceed at long distances is necessary.
- One such decay is

$$X(3872) \to D\bar{D}^* \to D^0\bar{D}^0\pi$$

- In the model of AlFikky2006, Nieves2012, there are 4 LEC.
- Three are fixed from $m_{X(3872)}$, the branching ratio and $m_{Z_b(10610)}$.
- Fixing the fourth, would fix the isospin structure of $X(3872)$
-




70
60
50
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10
0
−4
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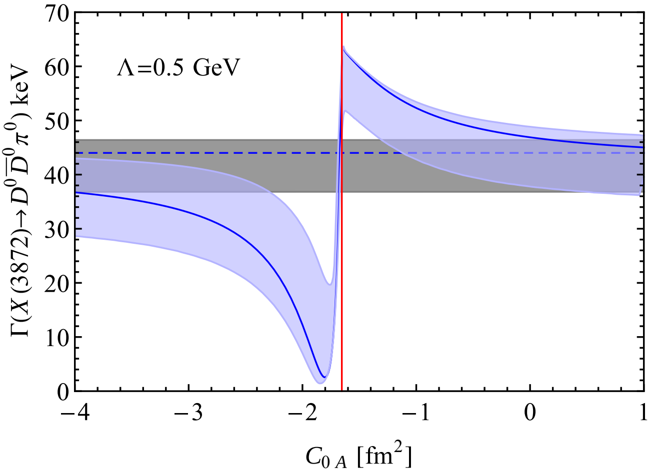
−2
−1
0
1
Λ=1 GeV
Γ(X(3872)→D⁰D̄⁰π⁰) keV
C₀ A [fm²]


Λ=0.5 GeV
$\Gamma(X(3872)\to D^0\overline{D}^0\pi^0)$ keV
$C_{0\,A}$ [fm^2]
70
60
50
40
30
20
10
0
−4
−3
−2
−1
0
1

Summary

- We have proposed interpolating currents that can be used for further studies, using QSR or on the lattice, of the $X(3872)$ meson and its partners
- It is proven that $X(3872)$ should have three other degenerate partners in the heavy quark limit
- It is argued that although the observed isospin violation in the amplitudes of the decays of $X(3872)$ is small, isospin violation in mixing can be very large.
- The decay $X(3872) \to D^0 \bar{D}^0 \pi^0$ can be used to probe the isospin structure
- It can receive large contributions from FSI.