



# Gluon Condensate at finite temperature and density in holographic QCD

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Giovinazzo, 16/06/2014

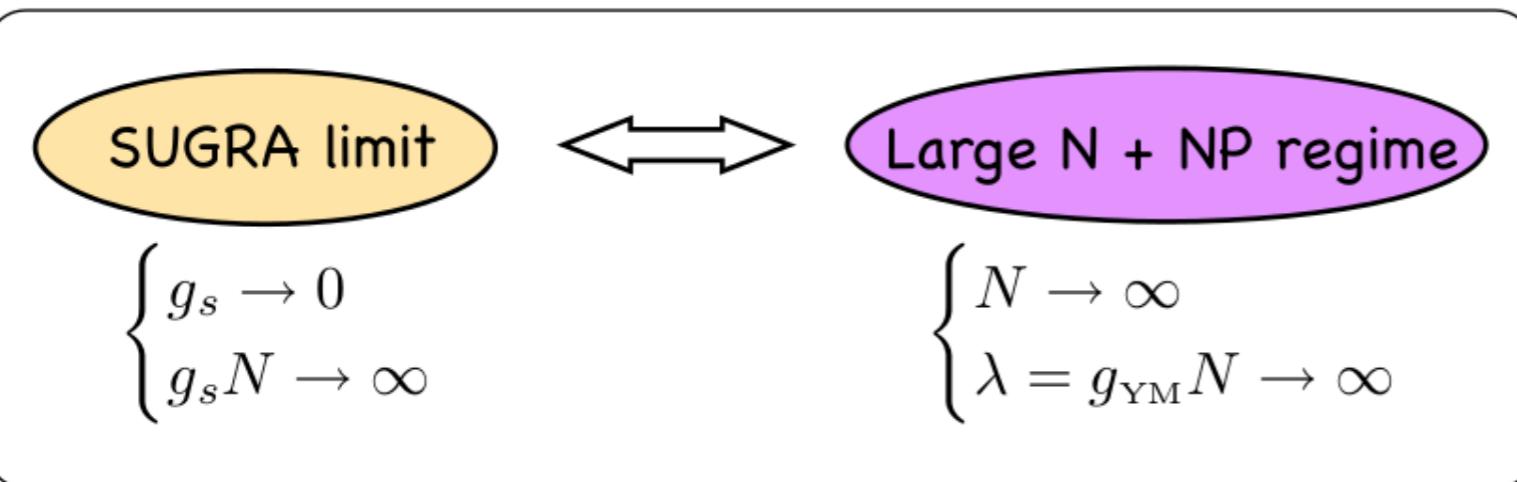
# AdS/CFT correspondence

IIB string theory  
in  $\text{AdS}_5 \times S^5$

$\mathcal{N}=4$  SYM, SU(N)  
in  $\mathcal{M}_4$

$$g_s = g_{\text{YM}}^2$$

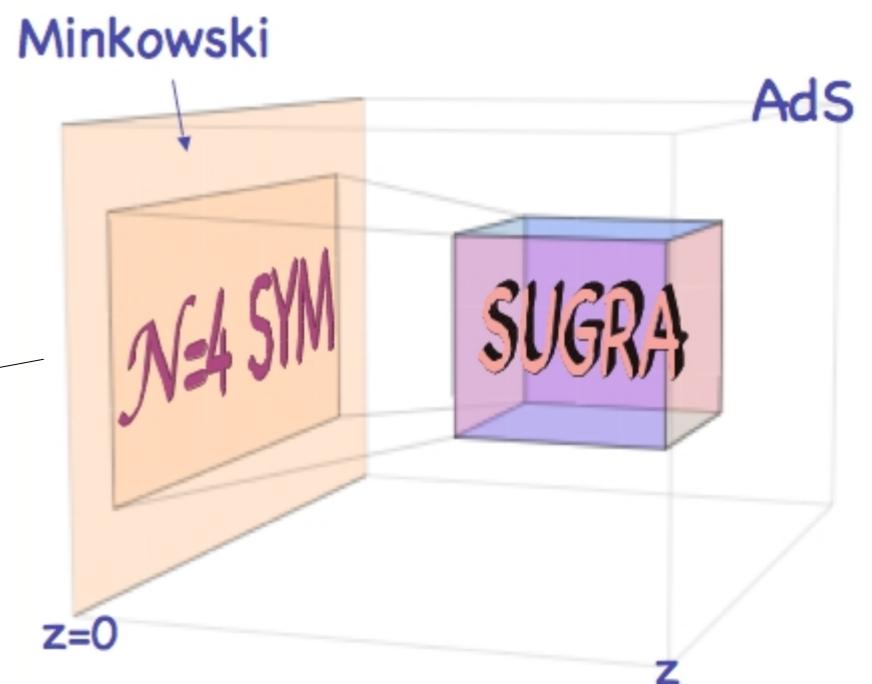
$$R^4 = 4\pi g_s N \alpha'^2$$



J.M. Maldacena  
Adv.Theor.Math.Phys. 2, 231 (1998)

## Holographic description

$$ds^2 = \frac{R^2}{z^2} (-dt^2 + d\bar{x}^2 + dz^2)$$



# AdS/CFT correspondence

How can the two theories be linked?

S.S. Gubser, I.R. Klebanov, A.M. Polyakov  
PLB 428, 105 (1998)

→ dictionary

E. Witten  
Adv.Theor.Math.Phys. 2, 253 (1998)

I. field  $\phi(x, z) \longleftrightarrow$  operator  $\mathcal{O}(x)$

➤ kind of field fixed by operator

Example: 0-form  $\longleftrightarrow$  scalar

➤ mass is related to conformal dimension of operator

$$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$$

➤ boundary value identified with source of operator

$$\phi(x, z) = \int_{\partial AdS_{d+1}} d^d x' K(x - x', z) \phi_0(x')$$

$$K(x - x', z) \xrightarrow{\partial AdS_{d+1}} z^\xi \delta^d(x - x')$$

Bulk-to-boundary propagator

# AdS/CFT correspondence

$$2. \quad Z_S[\phi_0(x)] = \left\langle e^{\int_{\partial AdS_{d+1}} \phi_0(x) \mathcal{O}(x)} \right\rangle_{CFT}$$

↑  
partition function

↑  
generating functional  
of the correlator

Computing two-point correlation function

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle = \frac{1}{Z_{CFT}} \left( -i \frac{\delta}{\delta \phi_0(x_1)} \right) \left( -i \frac{\delta}{\delta \phi_0(x_2)} \right) Z_{CFT} \Big|_{\phi_0=0} \leftrightarrow \frac{\delta^2 S_{OS}}{\delta \phi_0(x_1) \delta \phi_0(x_2)} \Big|_{\phi_0=0}$$

$$Z_{CFT} = Z_S = e^{i S_{OS}}$$

↓  
Semiclassical limit

# Bottom-up: soft wall

QCD is not:

1. supersymmetric
2. conformal (running coupling constant)

Break!

Introduce:

1. independent bosonic and fermionic states
2. mass scale

To break conformal invariance, introduce a mass scale

Soft wall: insert  $e^{c^2 z^2}$  in the action

A. Karch, E. Katz, D.T. Son, M.A. Stephanov  
PRD 74, 015005 (2006)

Regge trajectories:

Vector mesons:

$$m_n^2 = c^2(4n+4)$$

Scalar mesons:

$$m_n^2 = c^2(4n+6)$$

Scalar glueballs:

$$m_n^2 = c^2(4n+8)$$

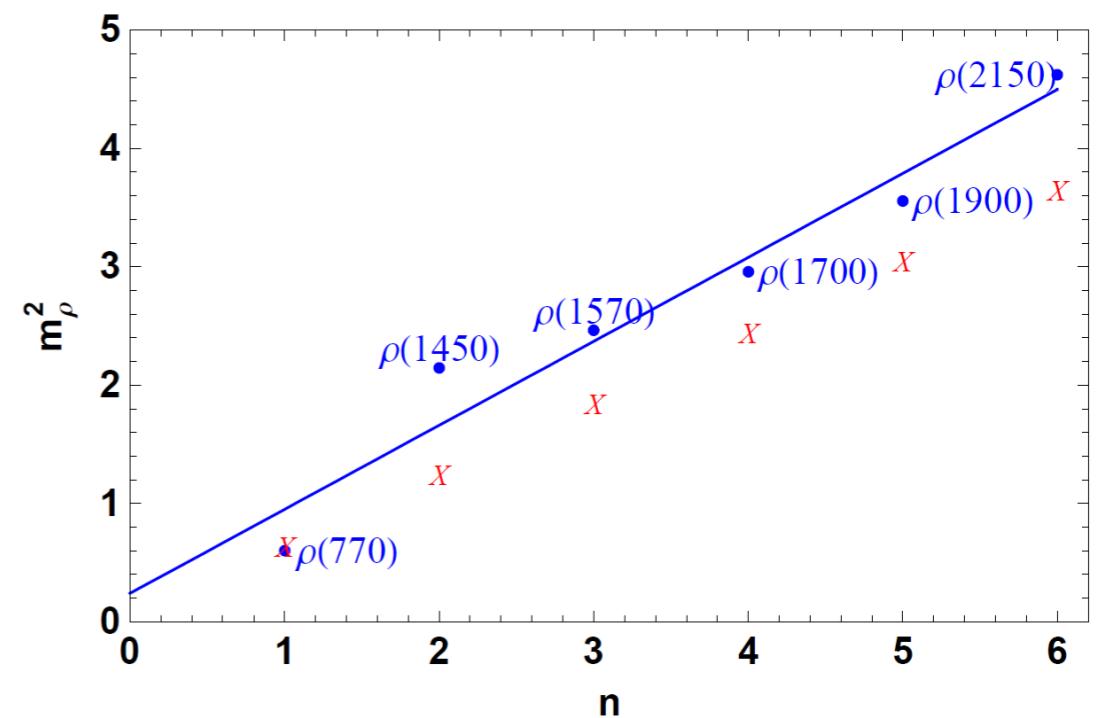
Hybrid mesons:

$$m_n^2 = c^2(4n+8)$$

$$m_\rho = 776 \text{ MeV}$$

$$c = 388 \text{ MeV}$$

mass scale



# Gluon condensate

$$G_2 = \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} | 0 \rangle$$

gluon field strength tensor

- in OPE of two-point correlation function

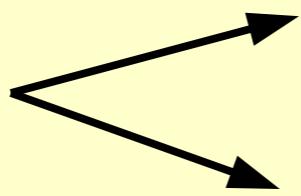
M.A. Shifman, A.I. Vainshtein, V.I. Zakharov,  
NPB 147, 385 (1979)

- $G_2 \simeq 0.012 \text{ GeV}^4$

- related to QCD trace anomaly

$$\Theta_\mu^\mu = \frac{\beta(\lambda)}{\lambda} G_{\mu\nu}^a G^{a,\mu\nu}$$

Two computations:



Finite temperature and density

Small temperature, two components

P. Colangelo, FG, S. Nicotri, F. Zuo  
PRD 88, 115011 (2013)

# Finite temperature and density

AdS + Charged Black Hole  $\longrightarrow$  AdS/RN metric

$$ds^2 = \frac{R^2}{z^2} \left( f(z) dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)} \right) \quad 0 < z < z_h$$

$$f(z) = 1 - \left( \frac{1}{z_h^4} + q^2 z_h^2 \right) z^4 + q^2 z^6 \quad f(z_h) = 0$$

outer horizon of BH

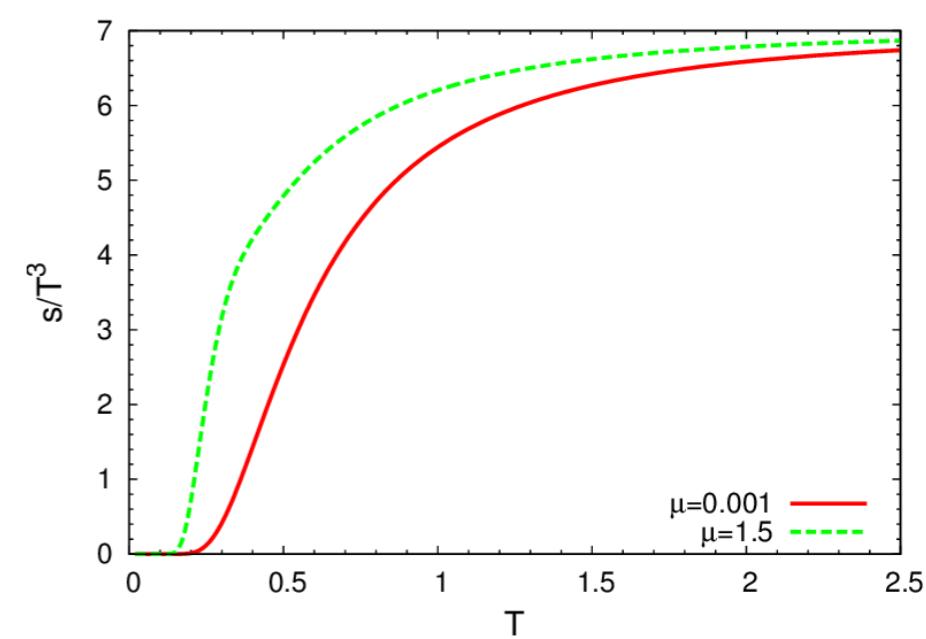
BH charge

Gauge field  $A_0(z)$  dual to  $q^\dagger q = \bar{q}\gamma^0 q$

gauge field from eom:

$$S \propto \int d^5x \sqrt{g} e^{-\phi} F_{MN}F^{MN}$$

$$\longrightarrow A_0(z) = i \left( \mu - \frac{\sqrt{3g_5^2}q}{a_E c^2} \left( 1 - e^{-a_E c^2 z^2} \right) \right)$$



# Finite temperature and density

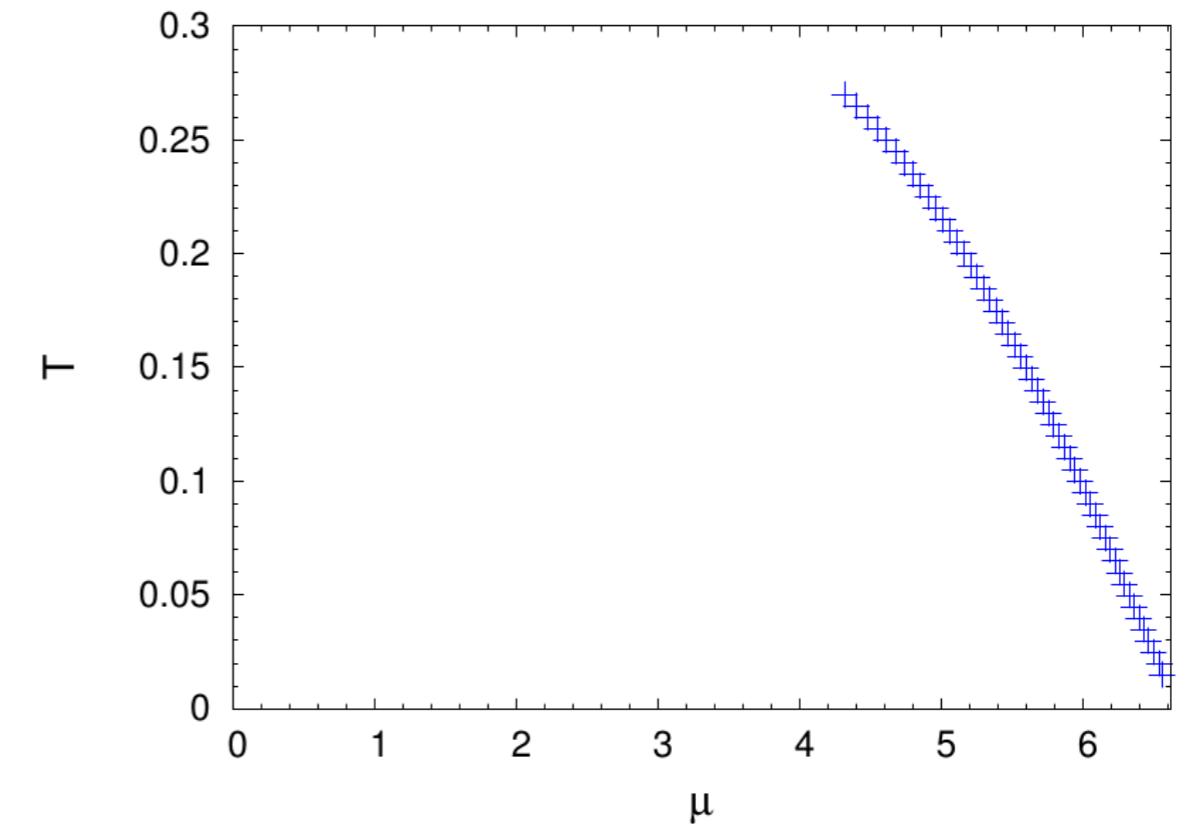
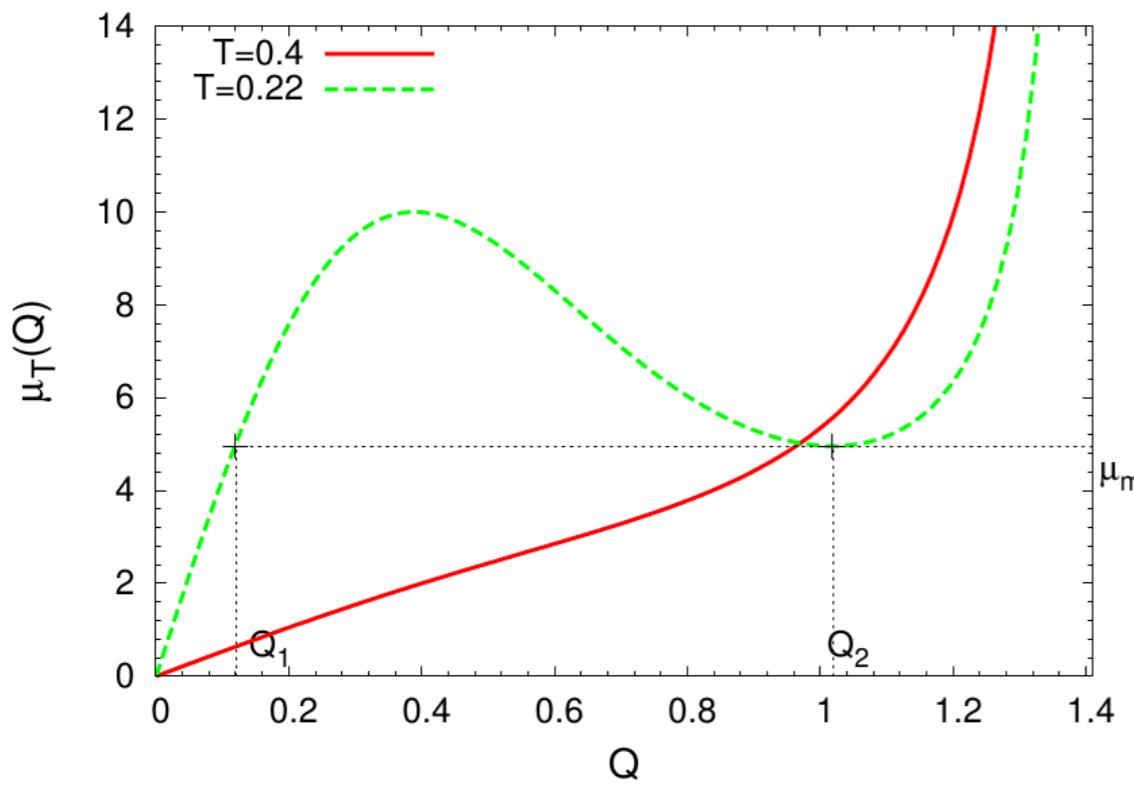
Temperature and chemical potential are related to BH parameters by:

$$T = \frac{1}{4\pi} \left| \frac{df}{dz} \right|_{z=z_h} = \frac{1}{\pi z_h} \left( 1 - \frac{Q^2}{2} \right)$$

$$\mu = \frac{\sqrt{3g_5^2} q}{a_E c^2} \left( 1 - e^{-a_E c^2 z_h^2} \right)$$

from  $A_0(z_h) = 0$

$$\mu_T(Q) = \frac{\sqrt{3g_5^2} Q \pi^3 T^3}{a_E (1 - Q^2/2)^3} \left( 1 - e^{-a_E (1 - Q^2/2)^2 / (\pi T)^2} \right)$$



# Gluon condensate (1)

First computation: from trace anomaly equation

Compute thermodynamic functions:

From AdS/CFT dictionary  $\mathcal{Z} \sim e^{-S}$

$$\mathcal{F} = -\frac{T}{V} \log \mathcal{Z} \quad \longrightarrow \quad \mathcal{F} = -\frac{1}{16\pi G_N} \int_0^{z_h} dz \sqrt{g} e^{a_E c^2 z^2} \left( \mathcal{R} - 2\Lambda - \frac{1}{4g_5^2} F^2 \right)$$

$$p = -\mathcal{F} \quad s = \frac{\partial [T \log \mathcal{Z}]}{\partial T} = \frac{\partial p}{\partial T} \quad \rho = \partial p / \partial \mu \quad \epsilon = Ts - p + \mu \rho$$

From trace anomaly equation:

$$\Delta G_2(T, \mu) = G_2(T, \mu) - G_2(0, 0) = -\epsilon(T, \mu) + 3p(T, \mu)$$

$$\Delta G_2(T, \mu) = 4p(T, \mu) - Ts(T, \mu) - \mu \rho(T, \mu)$$

# Gluon condensate (1)

$$T \rightarrow 0$$

$$p(T, \mu) \rightarrow \frac{1}{8\pi G_N} \frac{1}{2} a_E^2 c^4 (2\gamma_E - 3 + 2 \log(-a_E))$$

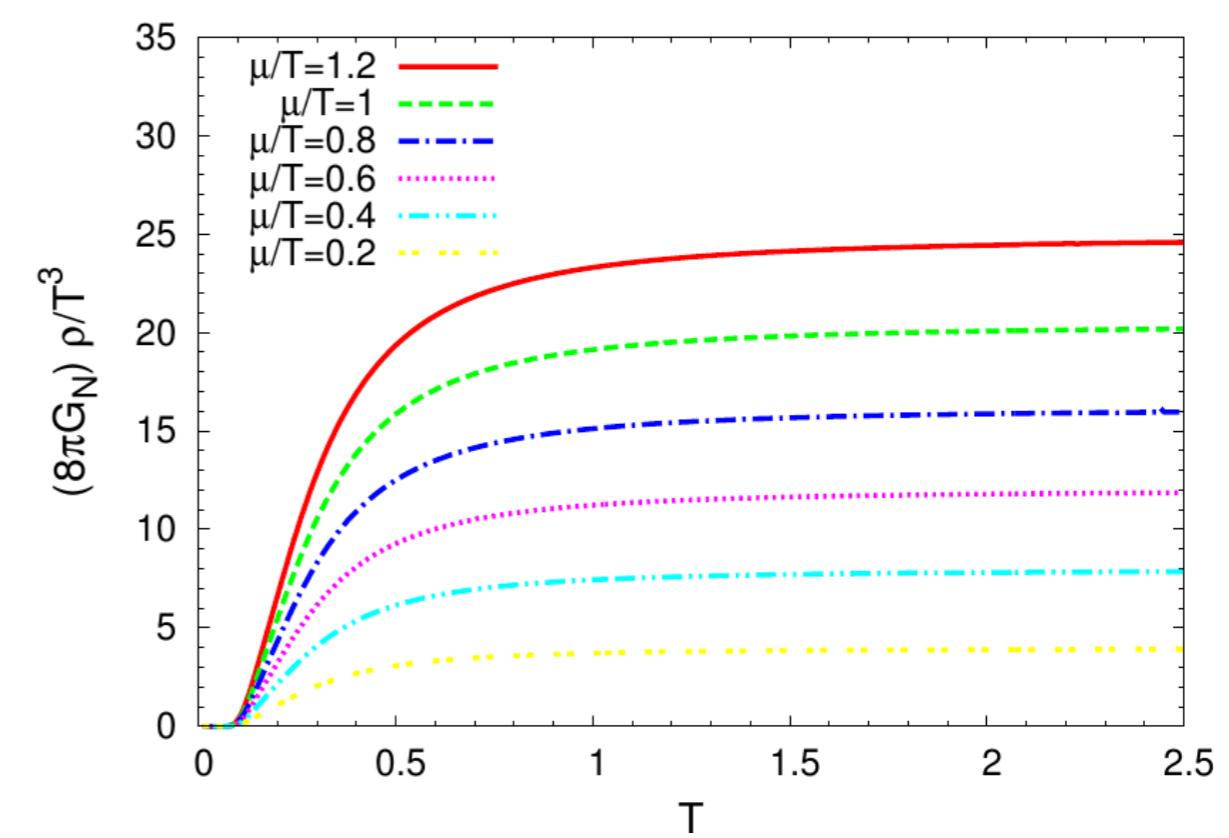
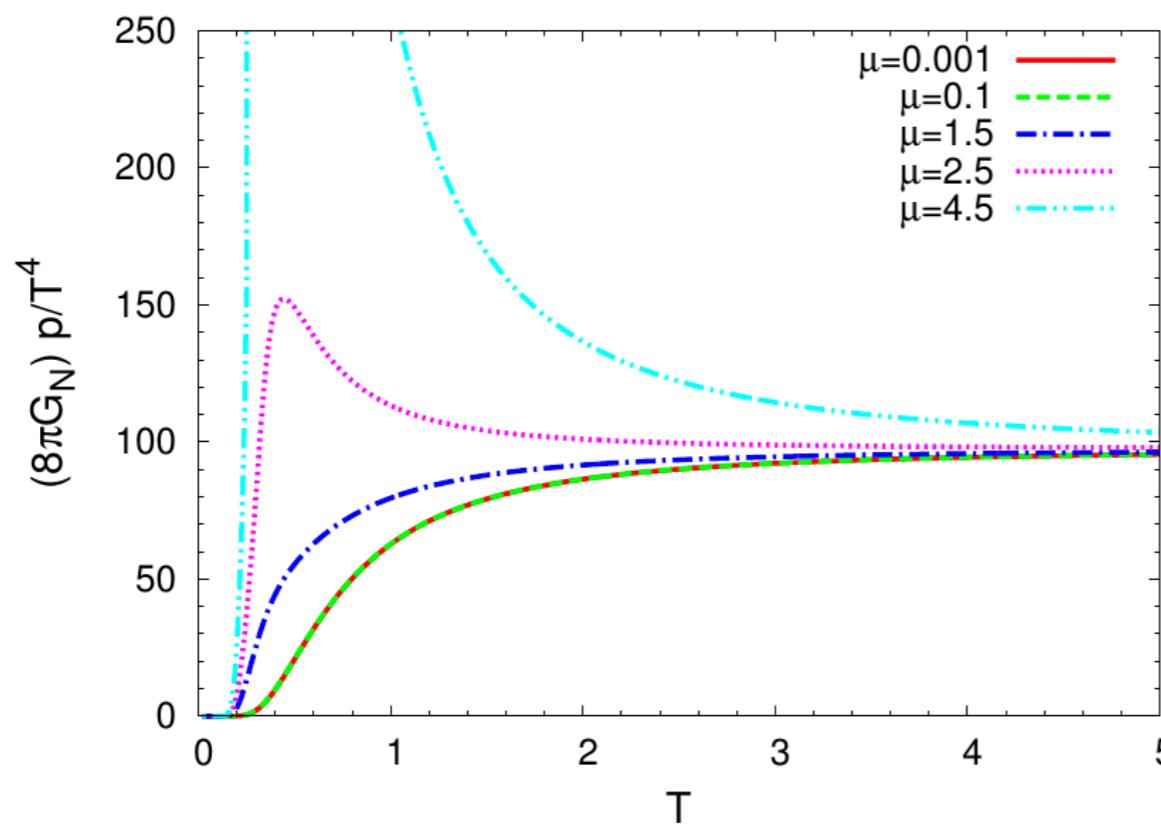


$$a_E = -e^{3/2-\gamma_E} \sim -2.5$$

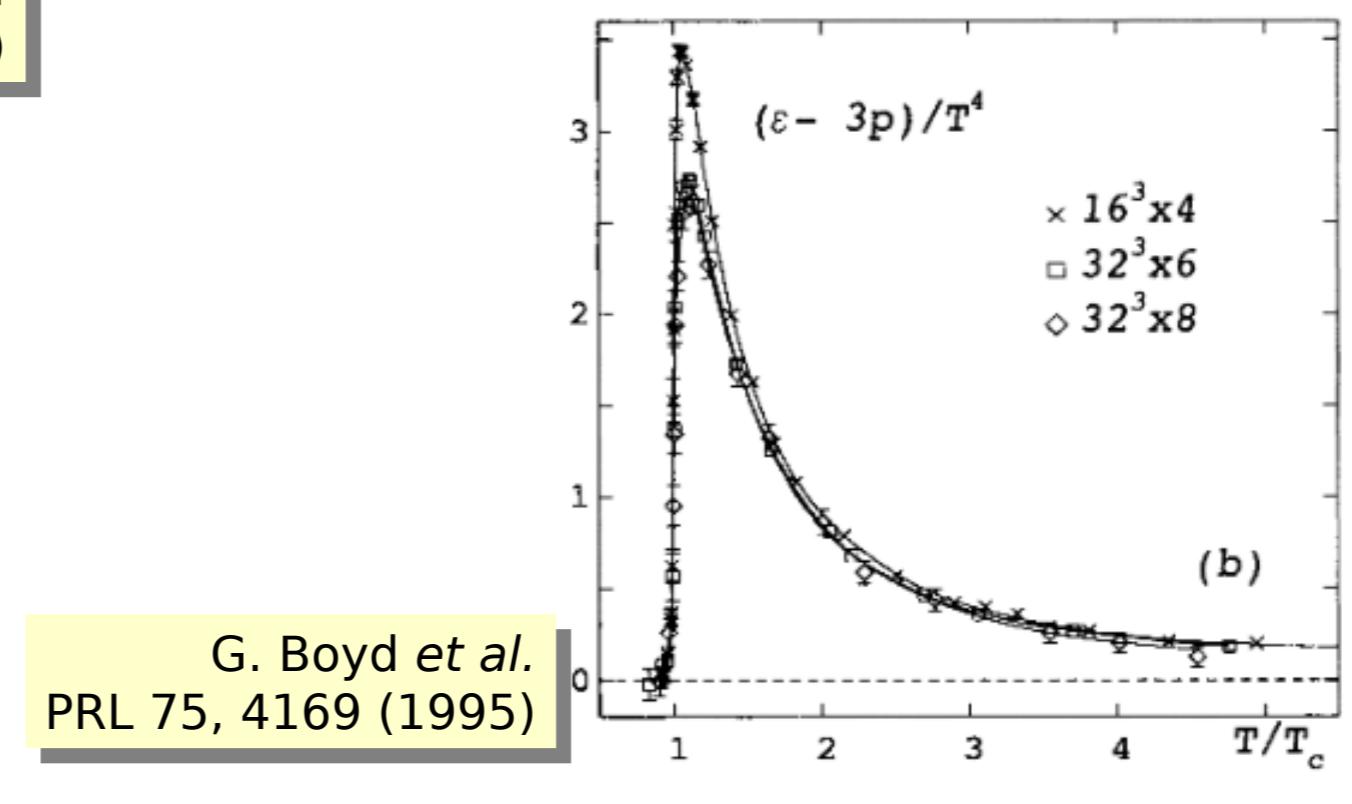
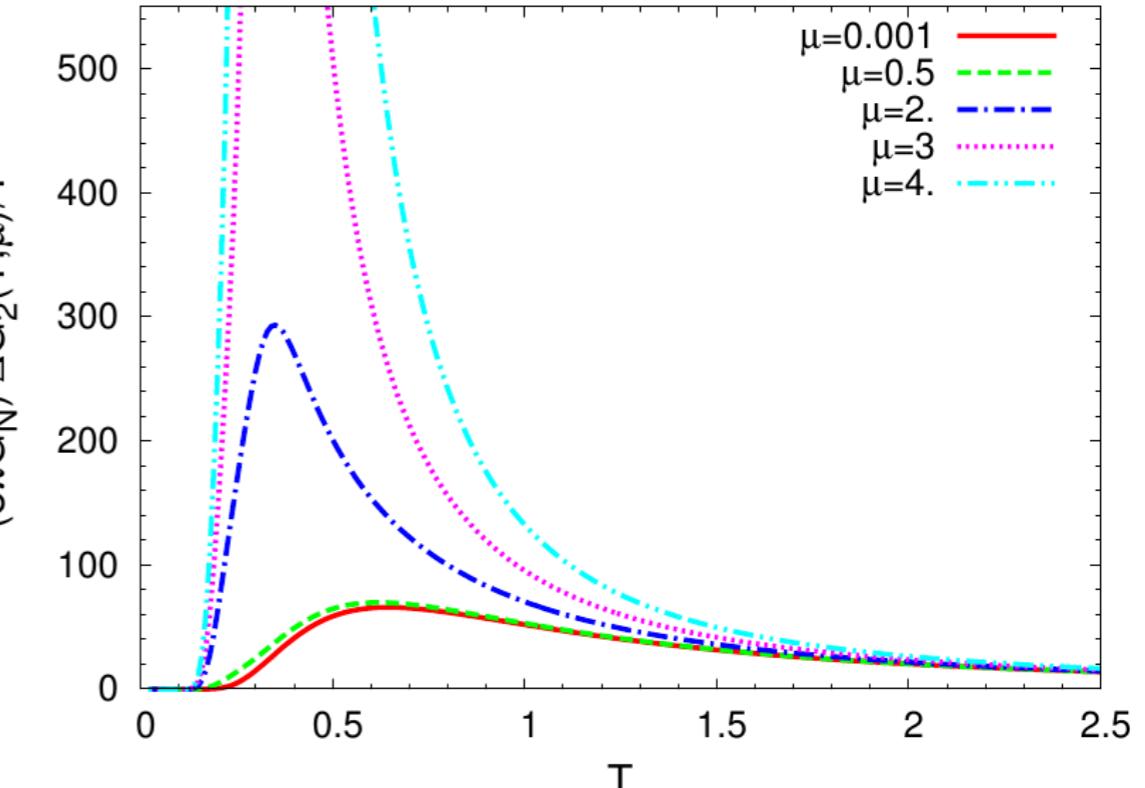
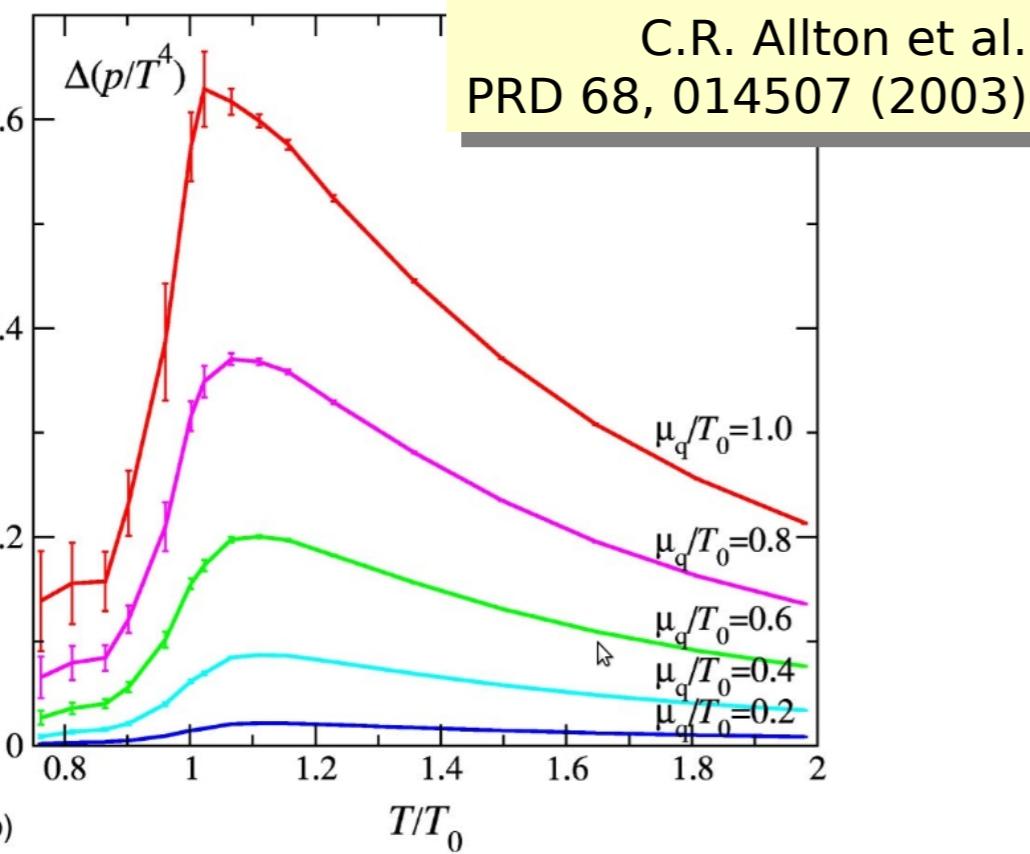
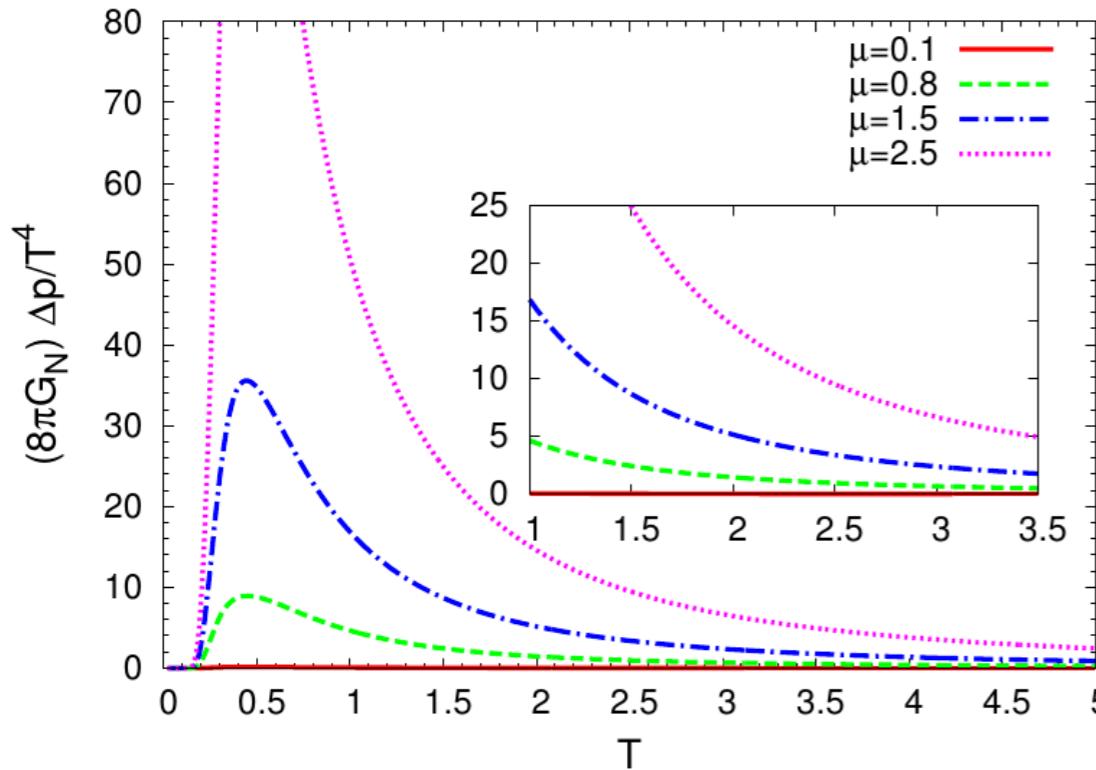
$$c = 0.25 \text{ GeV}$$

$$T \rightarrow \infty$$

$$p \rightarrow \frac{1}{8\pi G_N} (\pi^4 T^4 + \dots)$$



# Gluon condensate (1)



# Gluon condensate (2)

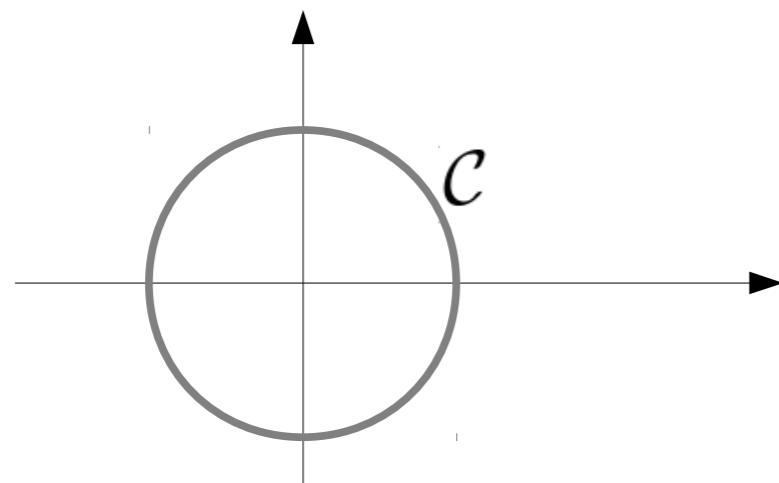
## Second computation: from Wilson loop

related to vacuum expectation value of a small circular Euclidean Wilson loop  $W(C)$

$$\log(\langle W \rangle) = - \sum_n c_n \alpha_s^n - \frac{\pi^2}{36} Z G_2 s^2 + \mathcal{O}(s^3)$$

area

### Holographic description:



O. Andreev, V.I. Zakharov  
Phys. Rev. D 76, 047705 (2007)

$$\langle W(C) \rangle \sim e^{-S_{NG}}$$

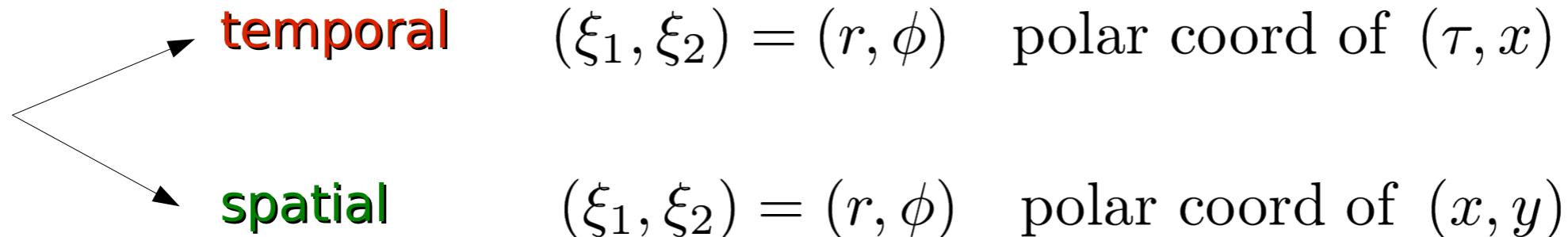
$$S_{NG} = \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{\gamma}$$

minimal area of the worldsheet spanned by a string in the 5d bulk with endpoints attached to  $C$

→  $G_2 = 0.010 \pm 0.0023 \text{ GeV}^4$

# Gluon condensate (2)

Small  $T$ ,  $\mu = 0$



$$ds^2 = \frac{e^{c_S^2 z^2}}{z^2} \left( f(z) d\tau^2 + d\bar{x}^2 + \frac{dz^2}{f(z)} \right) \quad c_S = 0.67 \text{ GeV}$$

$$S_{NG} = S_0 + \lambda S_1 + \lambda^2 S_2 + \mathcal{O}(\lambda^3) \quad \lambda = a^2 c_S^2$$

$$S_0 = -1,$$

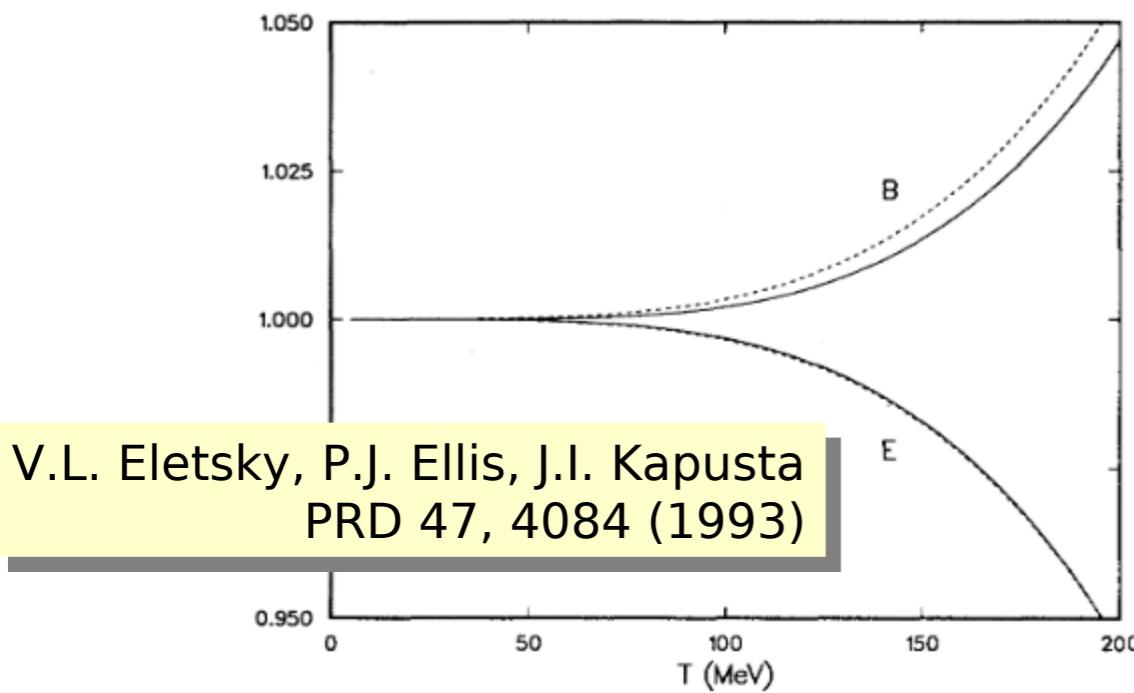
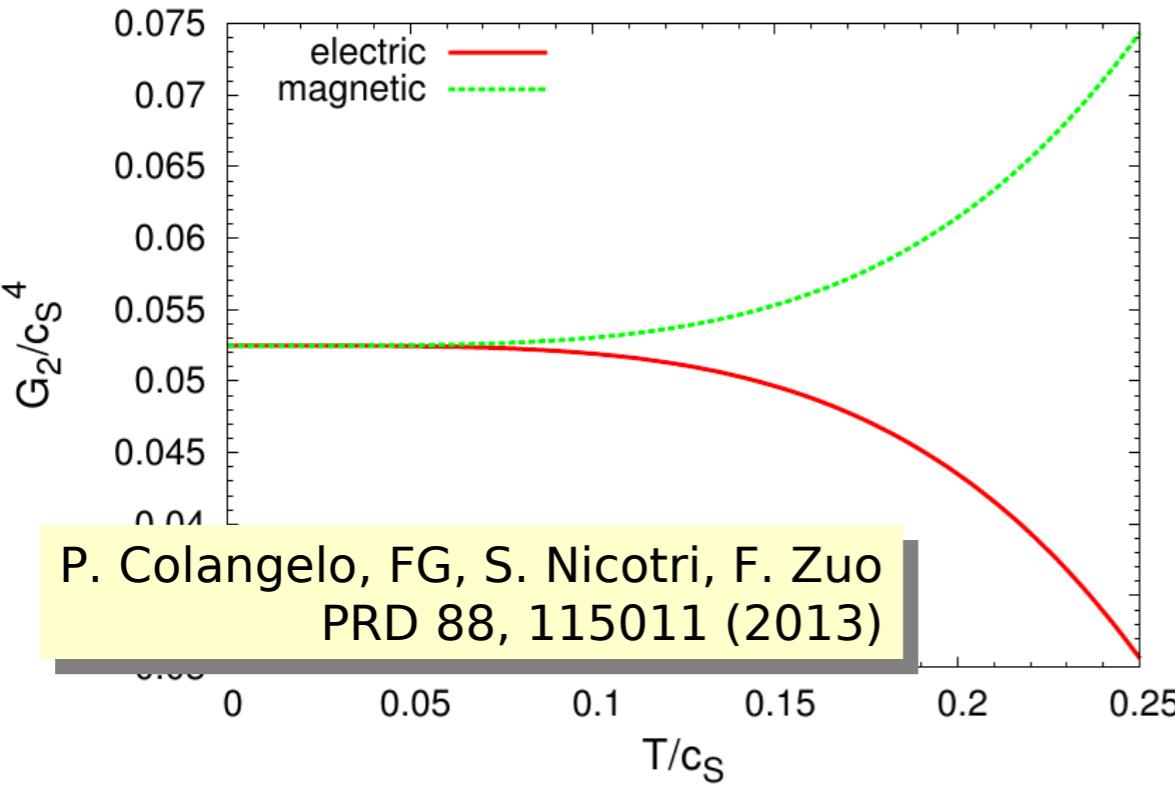
$$S_1 = \frac{5}{3},$$

$$S_2^{\tau/y} = \frac{7}{90} \left( 85 \mp 2\pi^4 \frac{T^4}{c_S^4} - 120 \log 2 \right)$$

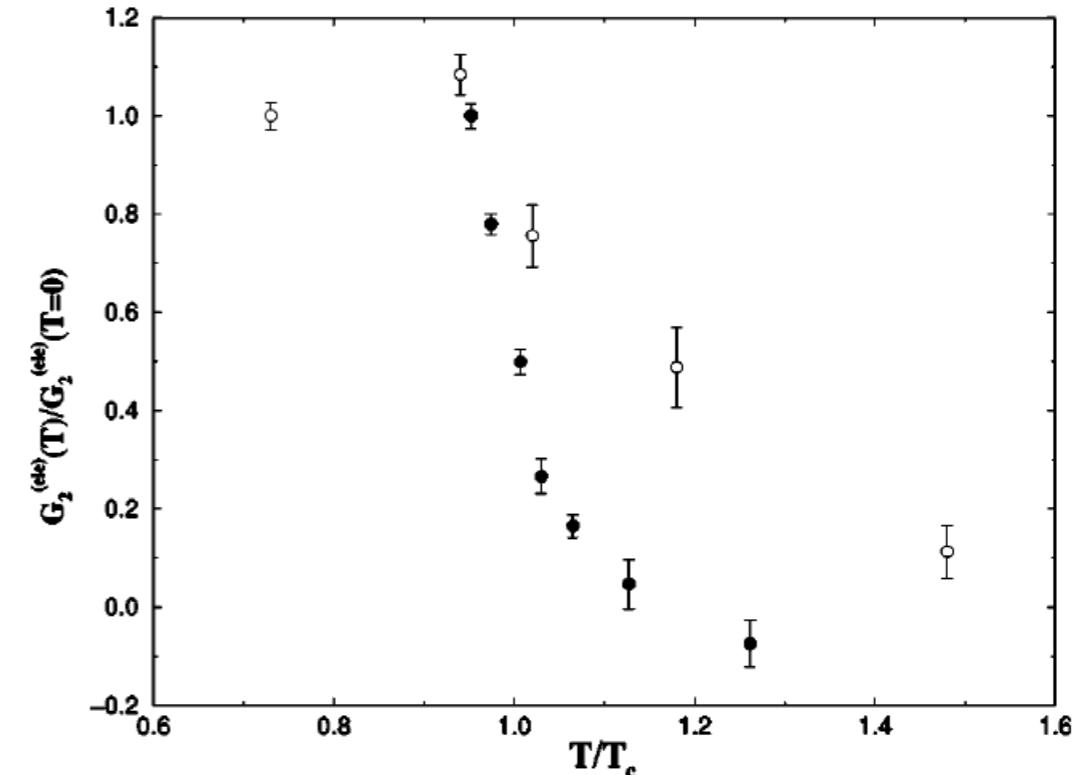
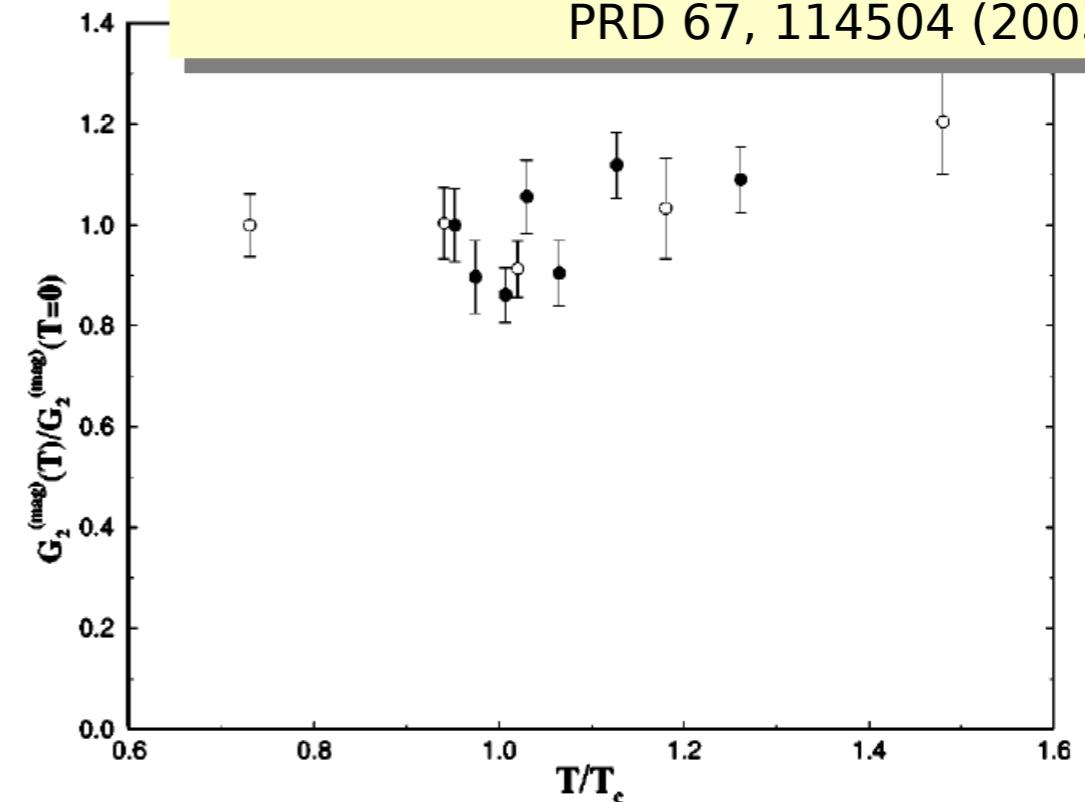
$$G_2^{e/m}(T) = \frac{14c_S^4}{5\pi^4} \left( 85 \mp 2\pi^4 \frac{T^4}{c_S^4} - 120 \log 2 \right)$$

# Gluon condensate (2)

Small  $T, \mu = 0$



M. D'Elia, A. Di Giacomo, E. Meggiolaro  
PRD 67, 114504 (2003)



# Concluding remarks

- Little analytical and numerical effort to compute gluon condensate at finite temperature and density
- Reproduce key features, in spite of the simplicity of the model
- Able to reach finite density
- Model can be improved for quantitative results