# Thermodynamical and transport properties of sQGP from hQCD

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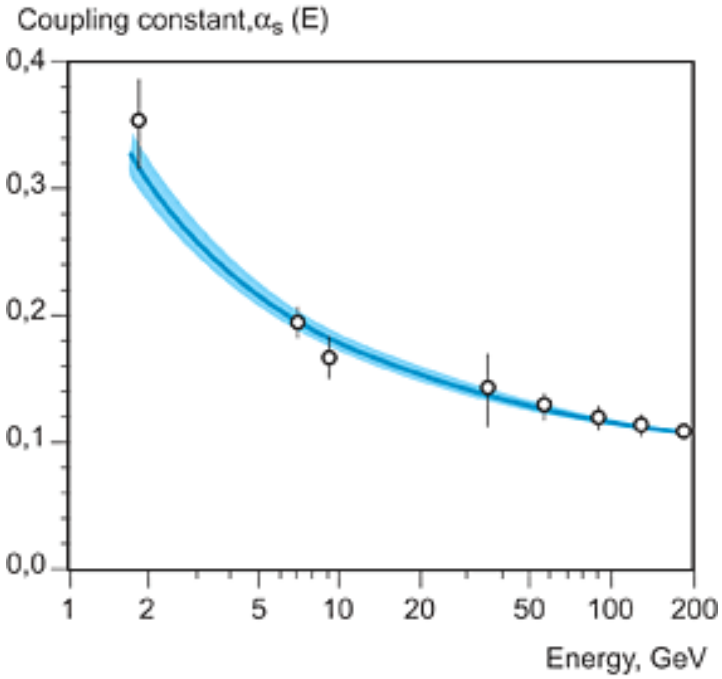
I. Motivation
II. The dynamical hQCD model
III. Phase transition
IV. Transport properties
V. Conclusion and discussion

D.N.Li, S.He, M.H. in preparation
D.N. Li, J.F. Liao, M.H., PRD in press, arXiv:1401.2035
D.N. Li, M.H., JHEP2013, arXiv:1303.6929
D.N, Li, S. He, M. H., Q. S. Yan, JHEP2011, arXiv:1103.5389

# I. Motivation

# QCD




UV (Weak coupling):

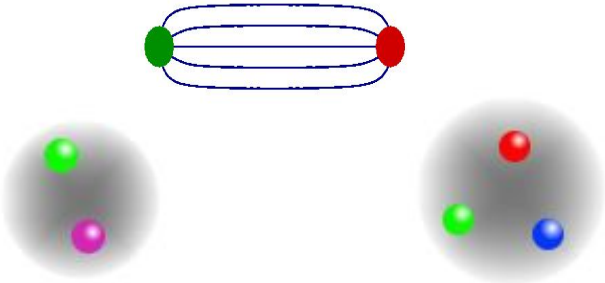
Asymptotic freedom

Asymptotically conformal

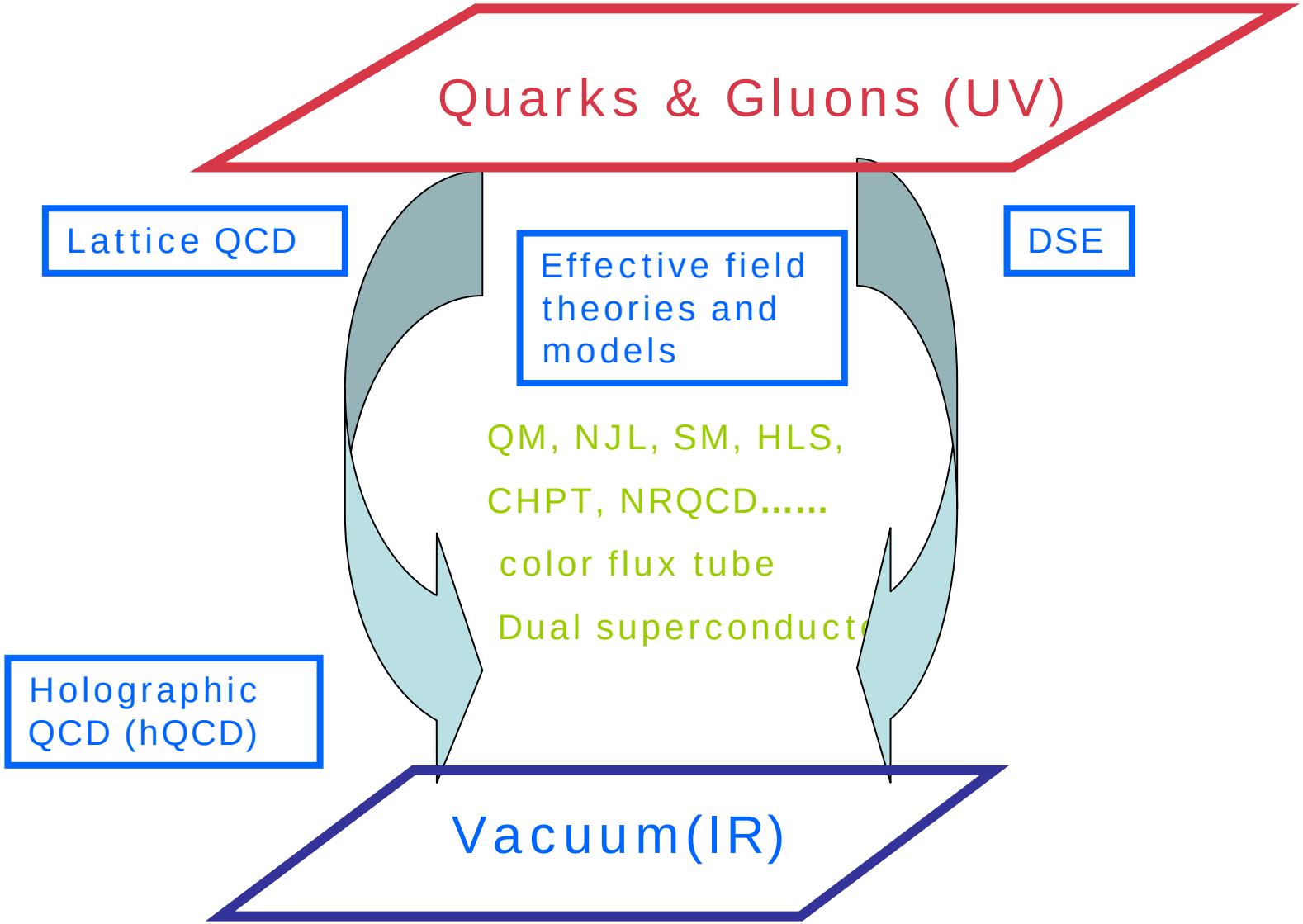
IR (Strong coupling):

Chiral symmetry breaking

& Confinement

# Strong QCD


Quarks & Gluons (UV)
Lattice QCD
Effective field theories and models
DSE
QM, NJL, SM, HLS,
CHPT, NRQCD.....
color flux tube
Dual superconducto
Holographic QCD (hQCD)
Vacuum(IR)

# Holographic Duality: Gravity/QFT

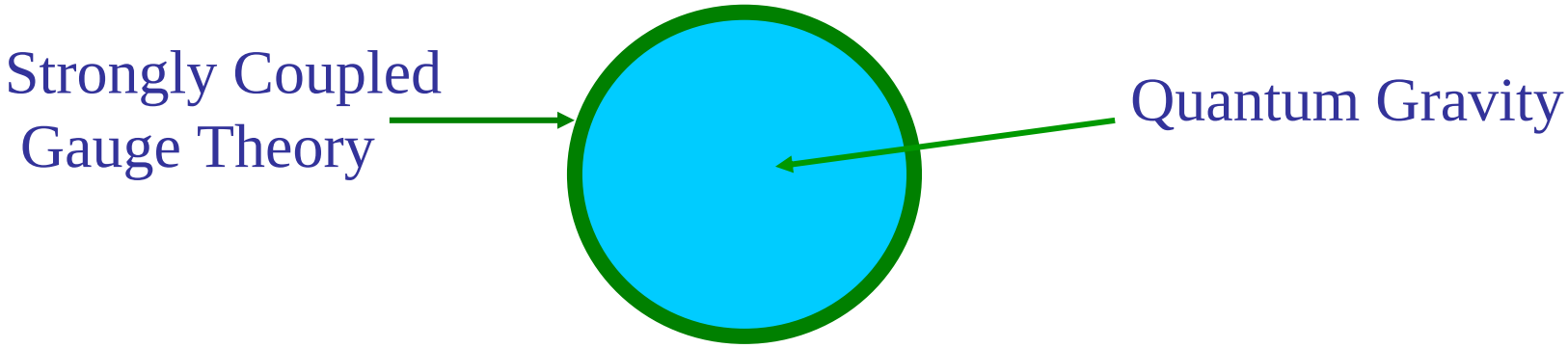
## AdS/CFT : Original discovery of duality

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

**Supersymmetry and conformality are required for AdS/CFT.**

**In general, supersymmetry and conformality are not necessary**

## General Gravity/QFT:



# Holographic QCD or gravity dual of QCD

**holographic QCD (5D)**

**String theorists' business: whether it can be deduced from 10D string theory ?**

**Build the connection between QCD dynamics and geometry**

**3rd step: gravity dual**
**systematic framework**

**2nd step: deformed $AdS_5$**
**intelligent guess**

**1st step: just $AdS_5$**
**naïve try**

Real QCD world:

Rich experimental data and lattice data

# A systematic framework: Graviton-dilaton system

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R_s + 4\partial_M\Phi\partial^M\Phi - V_G^s(\Phi)\right)$$

N=4 Super YM

conformal

$AdS_5$

$$ds^2 = \frac{L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

$$V_E(\phi) = -\frac{12}{L^2}$$

QCD

nonconformal

deformed $AdS_5$

$$ds^2 = \frac{h(z)L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

**Dilaton field breaks conformal symmetry**

**Input: QCD dynamics at IR**

**Solve: Metric structure, dilaton potential**

The goal is to describe

Hadron spectra
chiral symmetry breaking
& linear confinement

Phase transitions
equation of state

Transport properties

in the same systematic framework

# II. The dynamical hQCD model

# Holography & Emergent critical phenomena:

**When system is strongly coupled, new weakly-coupled degrees of freedom dynamically emerge.**

**The emergent fields live in a dynamical spacetime with an extra spatial dimension.**

**The extra dimension plays the role of energy scale in QFT, with motion along the extra dimension representing a change of scale, or renormalization group (RG) flow.**

**arXiv:1205.5180** Allan Adams,[1] Lincoln D. Carr,[2,3] Thomas Schäfer,[4] Peter Steinberg[5] and John E. Thomas[4]

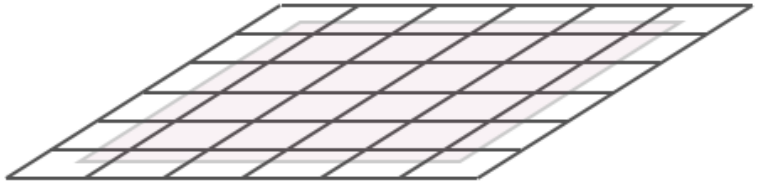
# Holographic Duality & RG flow

## Coarse graining spins on a lattice: Kadanoff and Wilson

$$H = \sum_{x,i} J_i(x)\mathcal{O}^i(x)$$

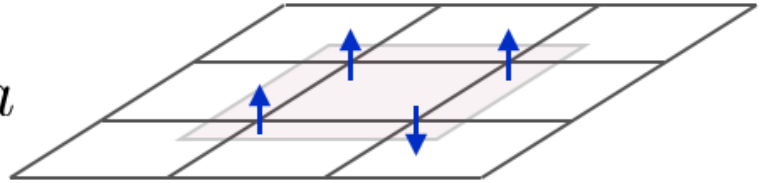
**J(x): coupling constant or source for the operator**

$a$



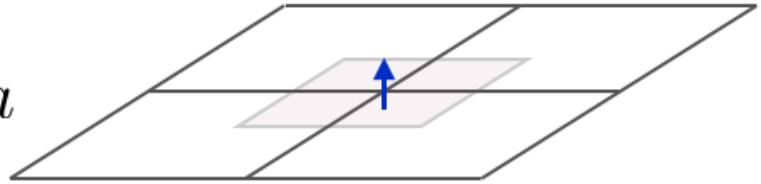
$$H = \sum_i J_i(x,a)\,\mathcal{O}^i(x)$$

$2a$



$$H = \sum_i J_i(x,2a)\,\mathcal{O}^i(x)$$

$4a$



$$H = \sum_i J_i(x,4a)\,\mathcal{O}^i(x)$$

$$u\frac{\partial}{\partial u}J_i(x,u) = \beta_i(J_j(x,u),u)$$

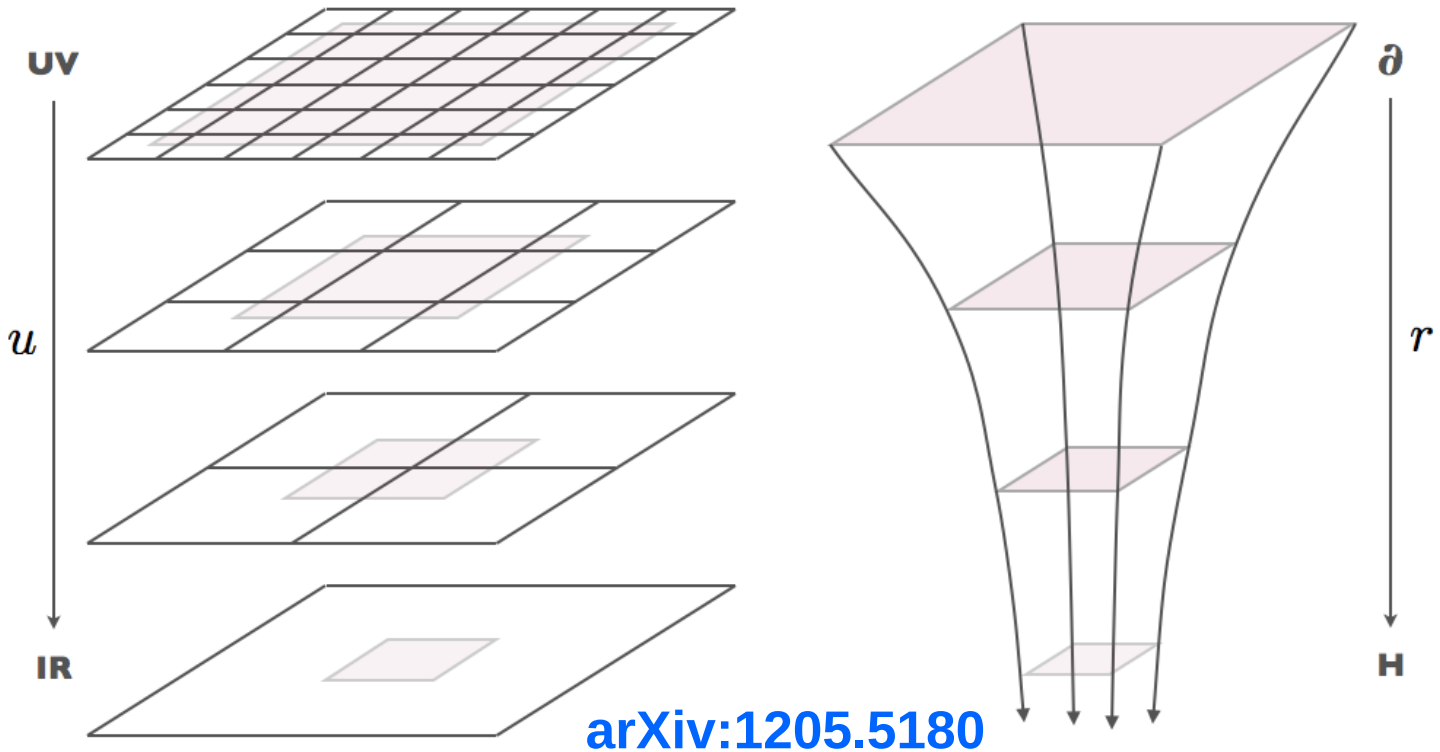
**arXiv:1205.5180**

# Holographic Duality & RG flow

**QFT on lattice equivalent to GR problem from Gravity**

**RG scale -> an extra spatial dimension**
**Coupling constant -> dynamical filed**

$$J_i|_{UV} = \Phi_i|_{\partial}$$




**arXiv:1205.5180**

# Holographic Duality: Dictionary

| Boundary QFT | Bulk Gravity |
|---|---|
| Local operator $\mathcal{O}_i(x)$ | Bulk field $\Phi_i(x,r)$ |

$$\Delta(d-\Delta) = m^2 L^2$$

| Strongly coupled | Semi-classical |
|---|---|

$$\boxed{Z_{\mathrm{QFT}}[J_i] \;=\; Z_{\mathrm{QG}}[\Phi[J_i]]}$$

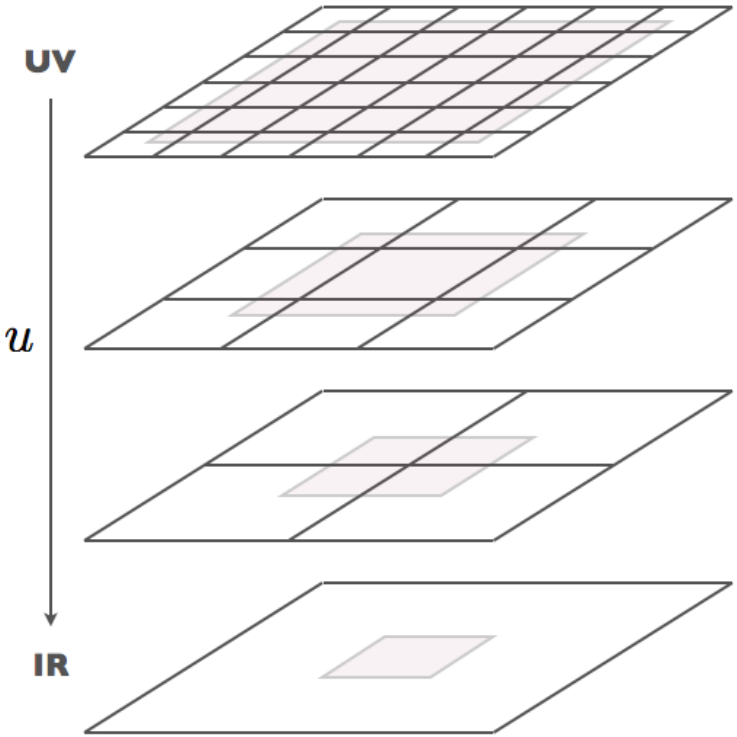
$$Z_{\mathrm{QFT}}[J] \;\simeq\; e^{-I_{\mathrm{GR}}[\Phi[J]]}$$

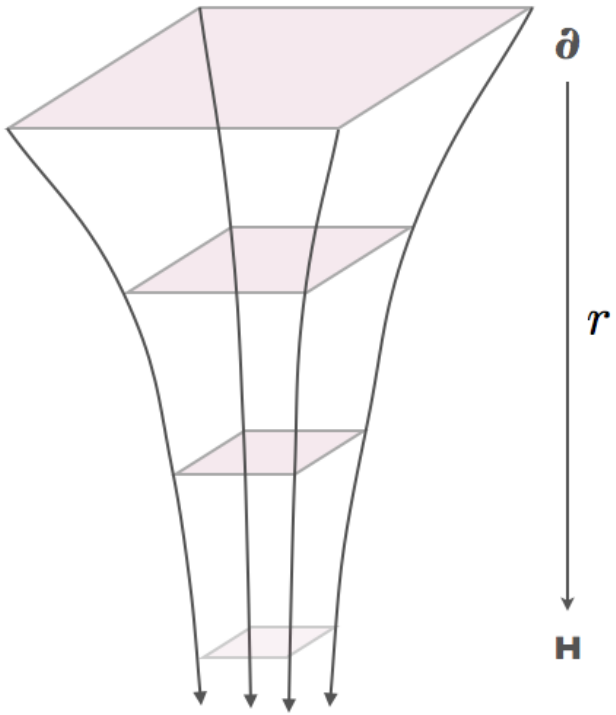
$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n I_{\mathrm{GR}}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\bigg|_{J_i=0}$$

# Dynamical hQCD & RG

$$J_i|_{UV} = \Phi_i|_{\partial}$$

A $AdS_5$




QCD Dynamics at IR

deformed $AdS_5$

# Pure gluon system:

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

$$\mathscr{L}_G = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x),$$

Gluon condensate at IR: $\mathrm{Tr}\langle G^2 \rangle$

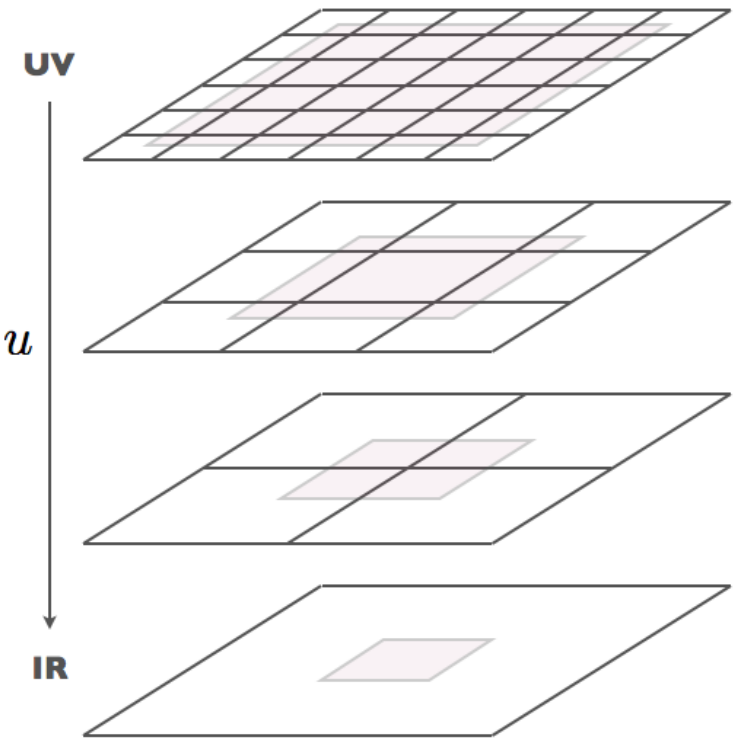
## 5D action: graviton-dilaton

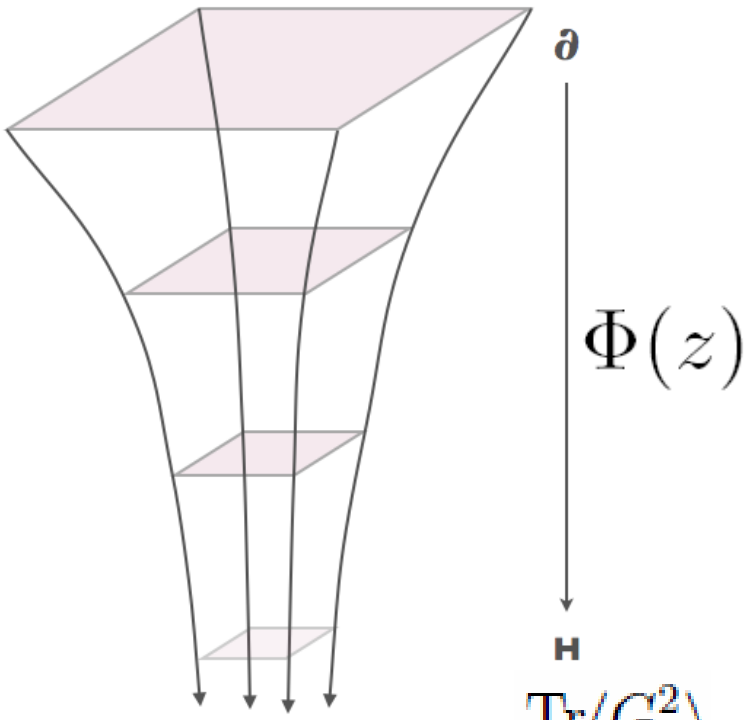
$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} \left( R_s + 4\partial_M \Phi \partial^M \Phi - V_G^s(\Phi) \right)$$

$\mathrm{Tr}\langle G^2 \rangle$ dual to $\Phi(z)$

# Graviton-dilaton system

$$J_i|_{UV} \quad = \quad \Phi_i|_{\partial}$$




A $AdS_5$

$\mathrm{Tr}\langle G^2\rangle$

deformed $AdS_5$

$$g^s_{MN} = b_s^2(z)(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \quad b_s(z) \equiv e^{A_s(z)}$$

## 5D action for scalar glueball:

$$S_{\mathscr{G}} = \int d^5x \sqrt{g_s} \frac{1}{2} e^{-\Phi} \left[ \partial_M \mathscr{G} \partial^M \mathscr{G} + M^2_{\mathscr{G},5} \mathscr{G}^2 \right]$$

scalar glueball: $\mathscr{G}$ dual to $tr(G_{\mu\nu}G^{\mu\nu})$ $M^2_{\mathscr{G},5} = 0$

$$-\mathscr{G}_n'' + V_{\mathscr{G}} \mathscr{G}_n = m^2_{\mathscr{G},n} \mathscr{G}_n,$$

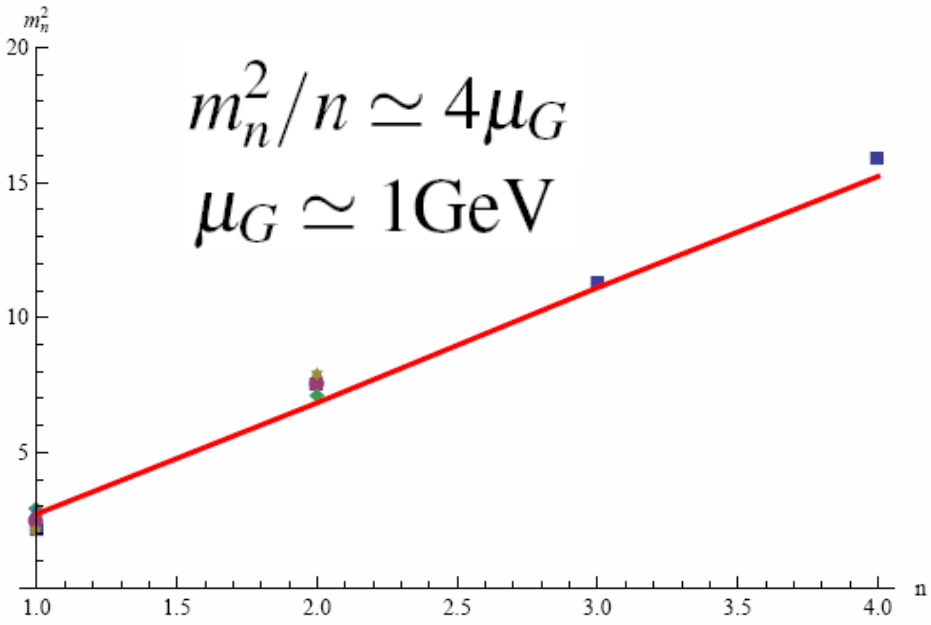
$$V_{\mathscr{G}} = \frac{3A_s'' - \Phi''}{2} + \frac{(3A_s' - \Phi')^2}{4}$$

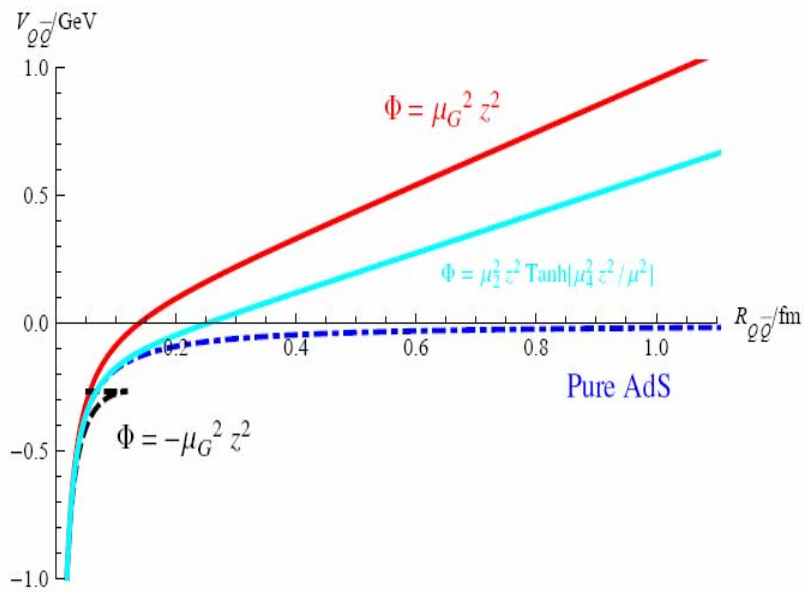
# Dilaton field: quartic at UV and quadratic at IR

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$$

$$\Phi(z) \overset{z\to 0}{\to} \mu_{G^2}^4 z^4 \qquad \Phi(z) \overset{z\to\infty}{\to} \mu_G^2 z^2$$




Linear confinement: linear Regge and linear potential

# Glueball spectra:

| n($0^{++}$) | Lat1 | Lat2 | Lat3 | Lat4 | Lat5 |
|---|---|---|---|---|---|
| | $N_c = 3$ | $N_c = 3$ | $N_c \to \infty$ | $N_c = 3$ | $N_c = 3$ |
| 1 | 1475(30)(65) | 1580(11) | 1480(07) | 1730(50)(80) | 1710(50)(80) |
| 2 | 2755(70)(120) | 2750(35) | 2830(22) | 2670(180)(130) | |
| 3 | 3370(100)(150) | | | | |
| 4 | 3990(210)(180) | | | | |

hep-lat/0508002 [hep-lat/0103027]. [hep-lat/9901004] [hep-lat/0510074]

# Light flavor system: Graviton-dilaton-scalar

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Action for pure gluon system: Graviton-dilaton coupling

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Action for light hadrons: KKSS model

$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2))$$

Total action:

$$S = S_G + \frac{N_f}{N_c}S_{KKSS}$$

Background with gluon condensate $\Phi(z)$

and quark-antiquark condensate $< X >= \frac{\chi(z)}{2}$

$$S_{vac} = S_{G,vac} + \frac{N_f}{N_c} S_{KKSS,vac}$$

$$dS_s^2 = B_s^2(-dt^2 + d\vec{x}^2 + dz^2) \qquad B_s^2 \equiv e^{2A_s} \equiv L^2 b_s^2$$

$$-A_s'' + A_s'^2 + \frac{2}{3}\Phi'' - \frac{4}{3}A_s'\Phi' - \frac{\lambda}{6}e^{\Phi}\chi'^2 = 0,$$

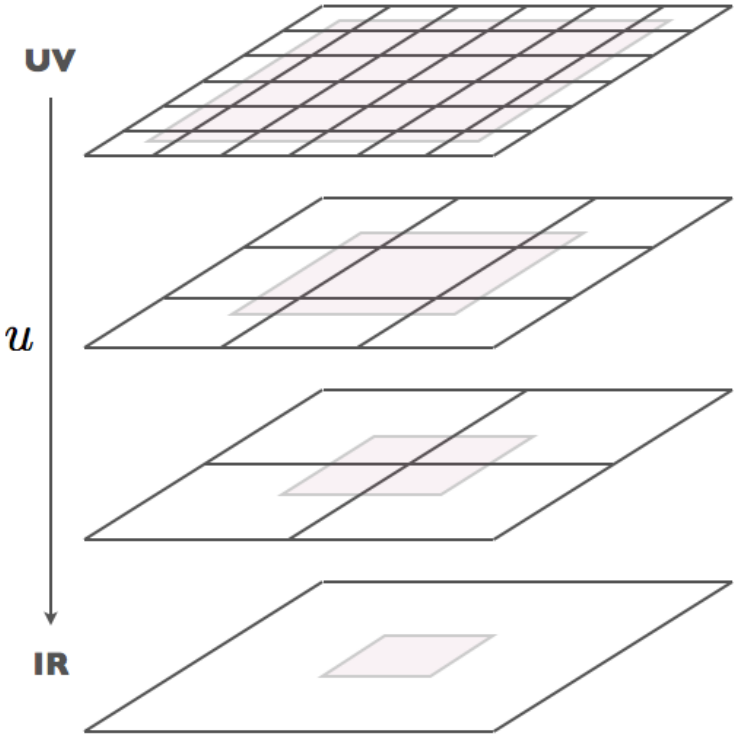
$$\Phi'' + (3A_s' - 2\Phi')\Phi' - \frac{3\lambda}{16}e^{\Phi}\chi'^2 - \frac{3}{8}e^{2A_s - \frac{4}{3}\Phi}\partial_\Phi\left(V_G(\Phi) + \lambda e^{\frac{7}{3}\Phi}V_C(\chi,\Phi)\right) = 0,$$

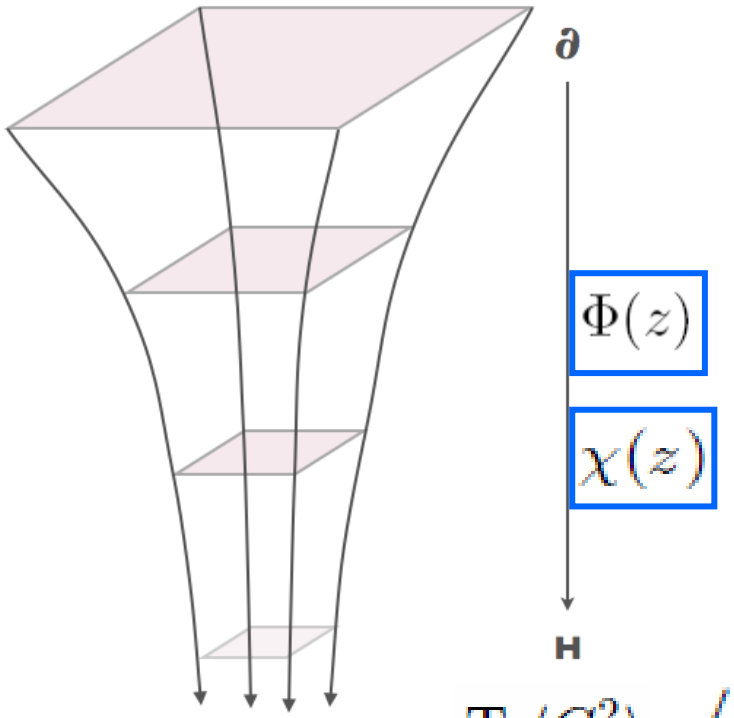
$$\chi'' + (3A_s' - \Phi')\chi' - e^{2A_s}V_{C,\chi}(\chi,\Phi) = 0.$$

# Graviton-dilaton-scalar system

$$J_i|_{UV} \quad = \quad \Phi_i|_{\partial}$$

A $AdS_5$




$\mathrm{Tr}\langle G^2 \rangle \quad \langle \bar{q} q \rangle$

deformed $AdS_5$

$$Dilaton\ in\ Mod\ I: \quad \Phi(z) = \mu_G^2 z^2$$

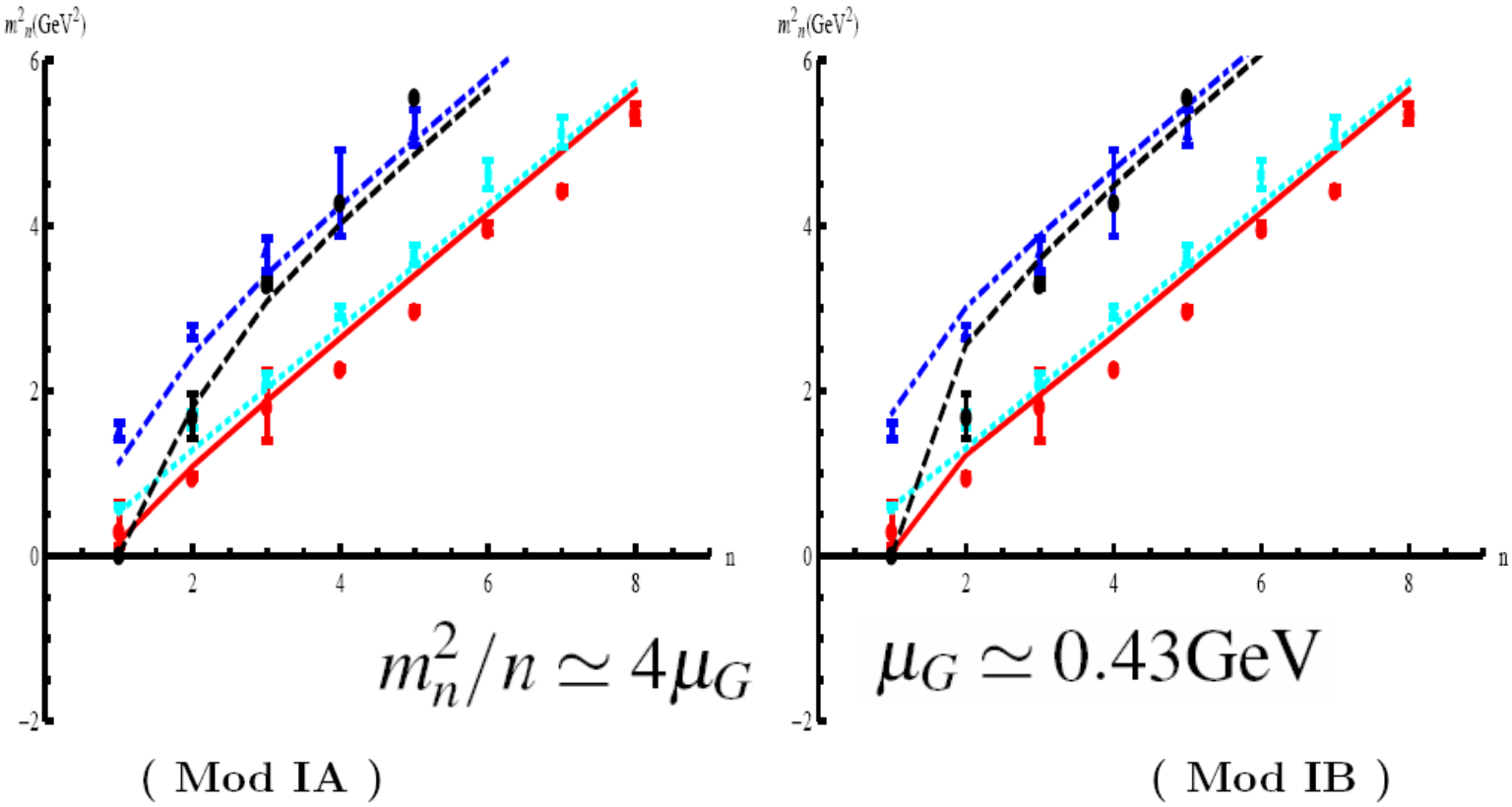
$$Dilaton\ in\ Mod\ II: \quad \Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$$

| | Mod IA | Mod IB | Mod IIA | Mod IIB |
|---|---|---|---|---|
| $G_5/L^3$ | 0.75 | 0.75 | 0.75 | 0.75 |
| $m_q$ (MeV) | 5.8 | 5.0 | 8.4 | 6.2 |
| $\sigma^{1/3}$ $(MeV)$ | 180 | 240 | 165 | 226 |
| $\mu_G$ | 0.43 | 0.43 | 0.43 | 0.43 |
| $\mu_{G^2}$ | - | - | 0.43 | 0.43 |

**Table 7**. Two sets of parameters.

# Produced hadron spectra compared with data

D.N. Li, M.H., JHEP2013, arXiv:1303.6929




Ground states: chiral symmetry breaking
Excitation states: linear confinemnt

# III. HQCD for

# Phase transitions

# Color electric deconfinement phase transition

5D graviton action:

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^E} \left( R - \frac{4}{3} \partial_\mu \phi \partial^\mu \phi - V_E(\phi) \right)$$

$$ds_S^2 = \frac{L^2 e^{2A_s}}{z^2} \left( -f(z) dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i \right),$$

**Metric structure, blackhole, Dilaton field and Dilaton potential should be solved self-consistently from the Einstein equations.**

**Experiences in constructing holographic QCD model tells us that: a quadratic correction in the deformed warp factor is responsible for the linear confinement.**

$$A_s(z) = ck^2 z^2$$

$$\phi(z) = \phi_0 + \phi_1 \int_0^z \frac{e^{2A_s(x)}}{x^2}\, dx + \frac{3A_s(z)}{2} + \frac{3}{2}\int_0^z \frac{e^{2A_s(x)} \int_0^x y^2 e^{-2A_s(y)} A_s'(y)^2 dy}{x^2}\, dx,$$

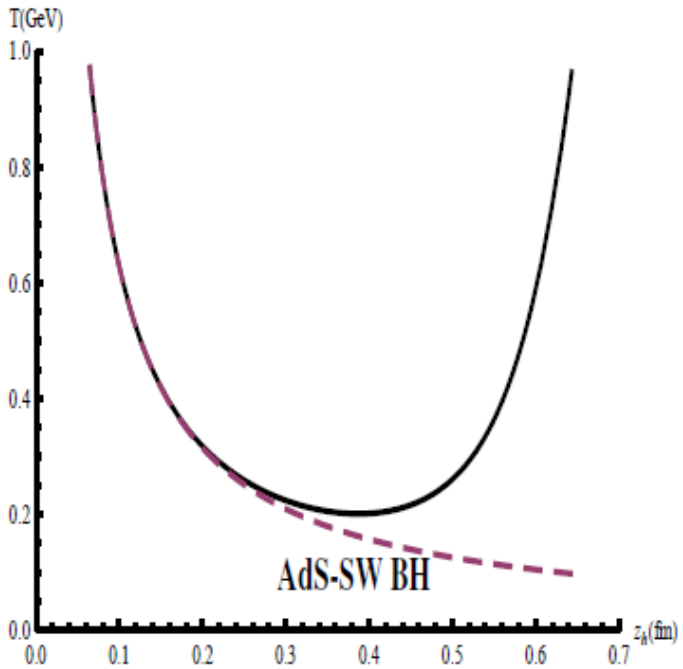
$$f(z) = f_0 + f_1 \left( \int_0^z x^3 e^{2\phi(x) - 3A_s(x)}\, dx \right),$$

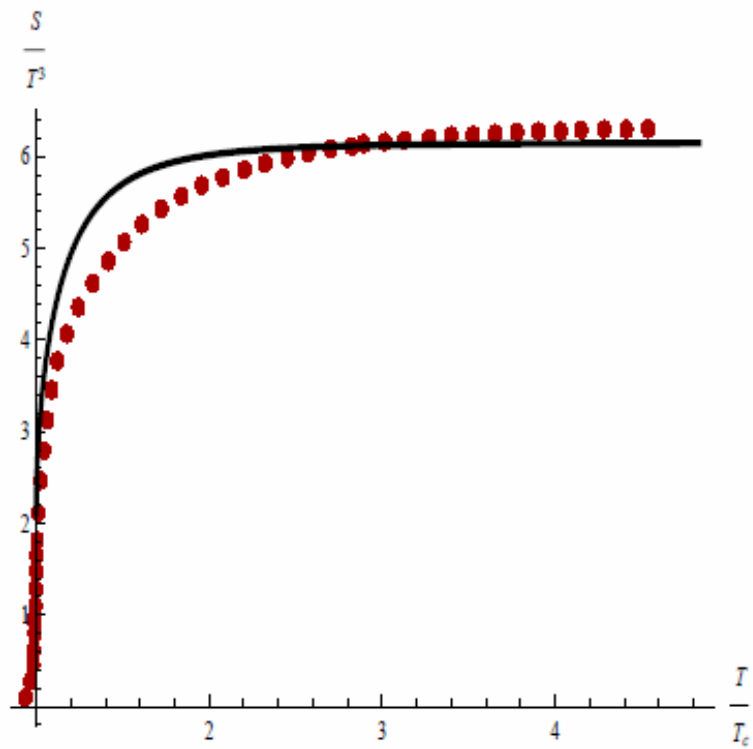
$$V_E(\phi) = \frac{e^{\frac{4\phi(z)}{3} - 2A_s(z)}}{L^2} \left( z^2 f''(z) - 4f(z) \left( 3z^2 A_s''(z) - 2z^2 \phi''(z) + z^2 \phi'(z)^2 + 3 \right) \right).$$

**D.N, Li, S. He, M. H., Q. S. Yan, arXiv:1103.5389, JHEP2011**

$$T = \frac{|f'(z_h)|}{4\pi}.$$

$$s = \frac{A_{area}}{4G_5V_3} = \frac{L^3}{4G_5}\left(\frac{e^{A_s-\frac{2}{3}\phi}}{z}\right)^3.$$



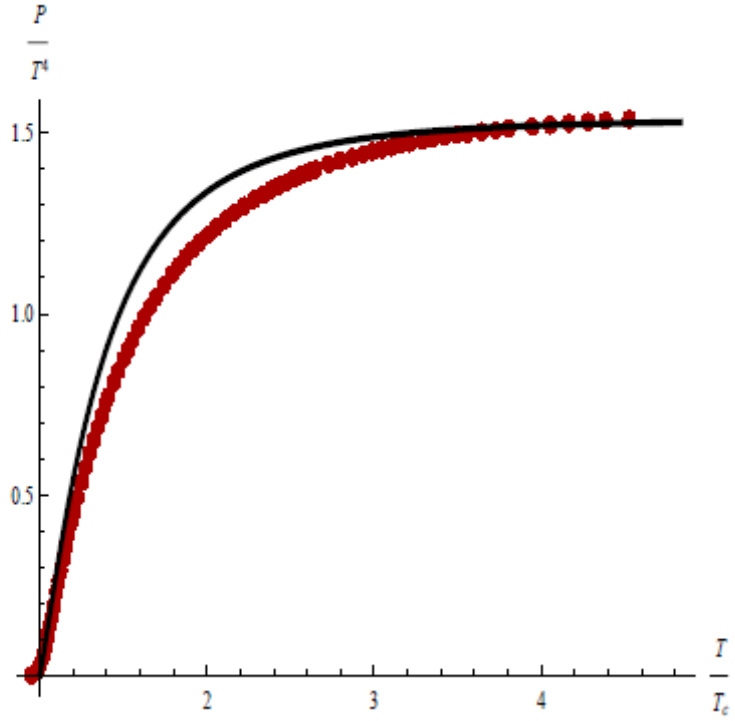



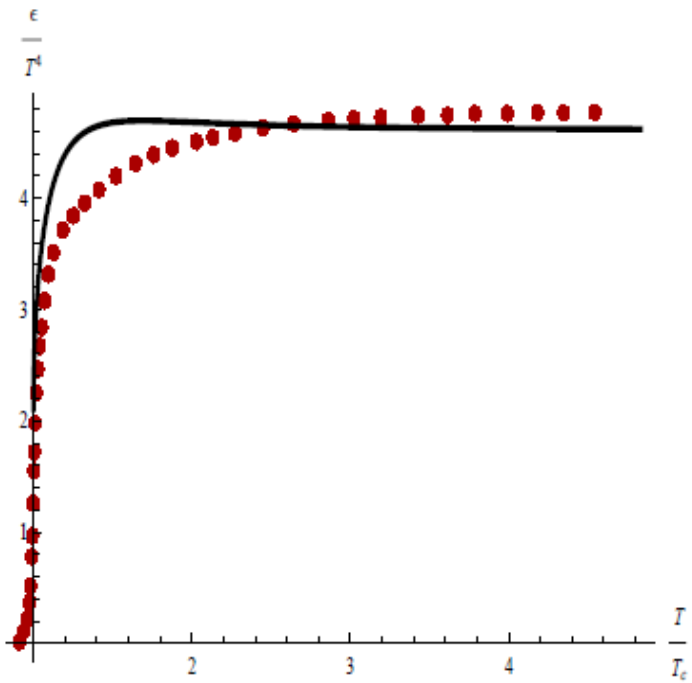

$$T_c = 201\text{MeV}$$

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

$$\frac{dp(T)}{dT} = s(T).$$

$$\epsilon = -p + sT.$$

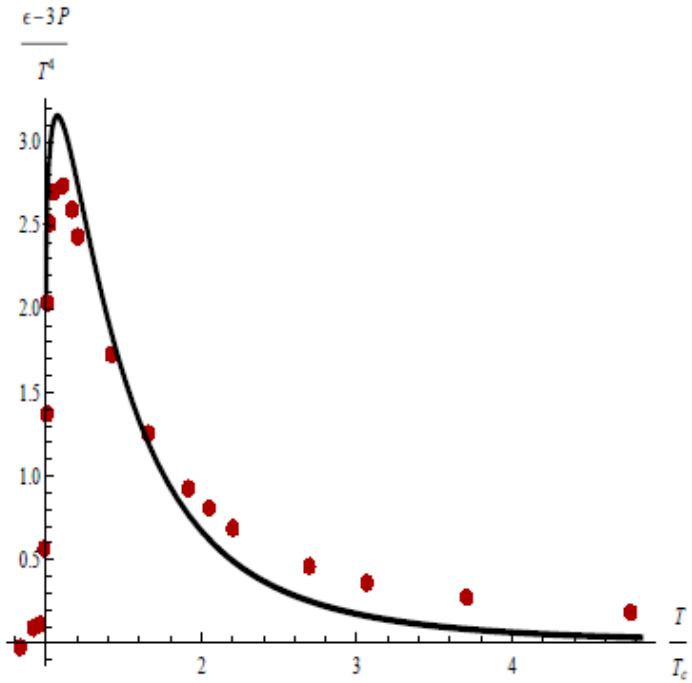


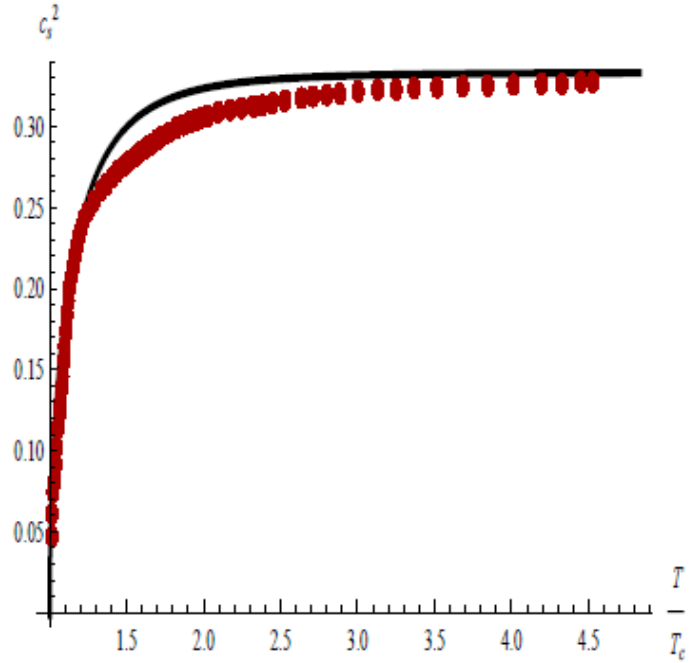




D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

# Trace anomaly

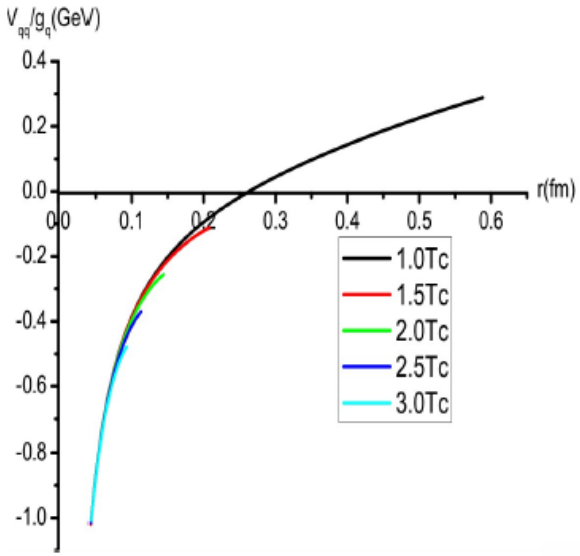
$$c_s^2 = \frac{d\log T}{d\log s} = \frac{s}{T ds/dT},$$



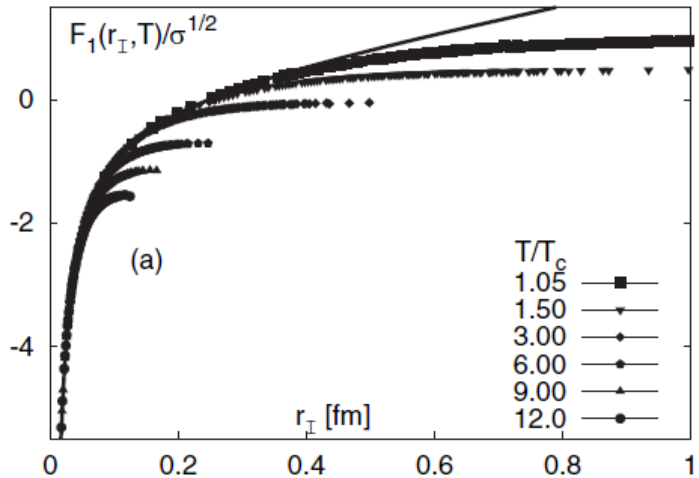




D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

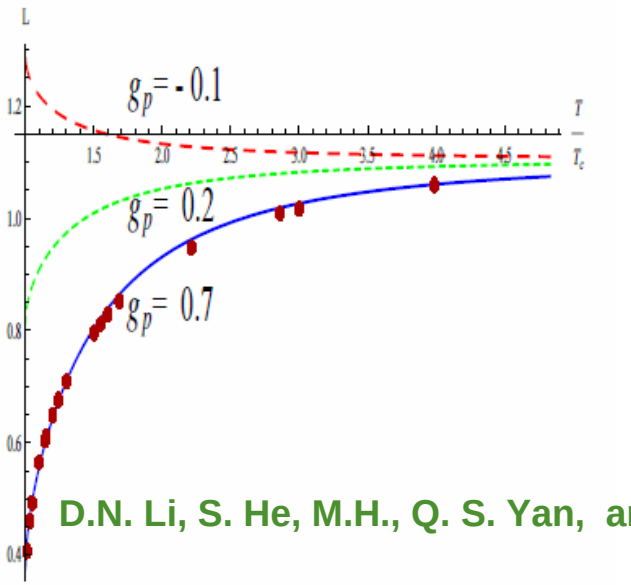
## Electric screening




## Heavy quark potential

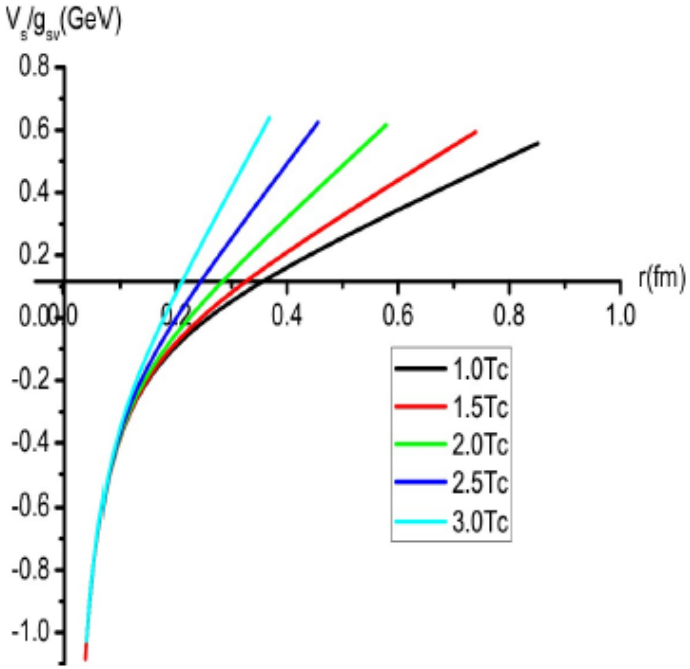


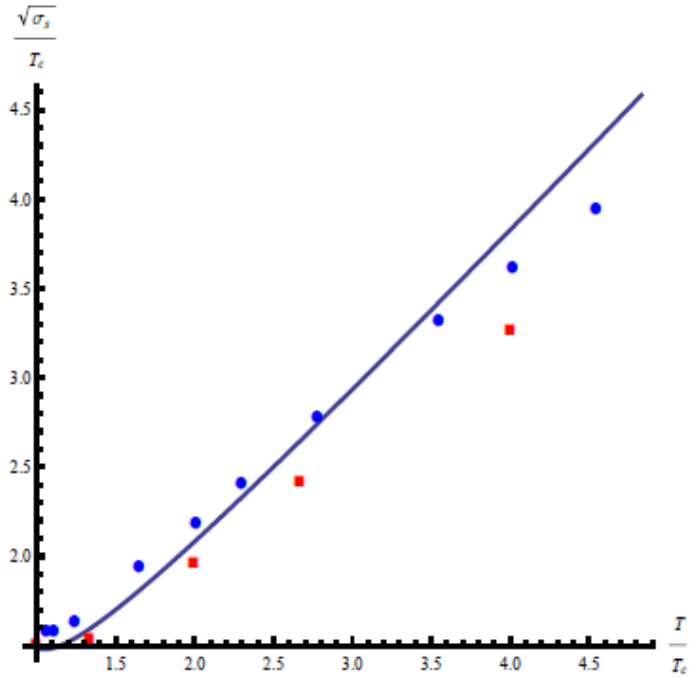

## Polyakov loop: color electric deconfinement




D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

# Magnetic screening and magnetic confinement

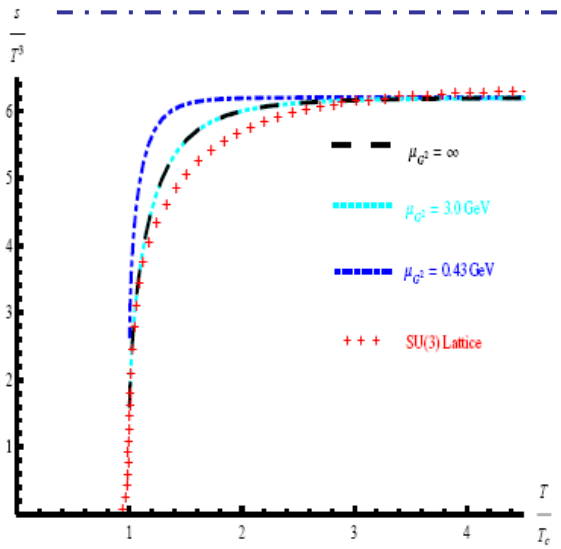



spatial Wilson loop  spatial string tension

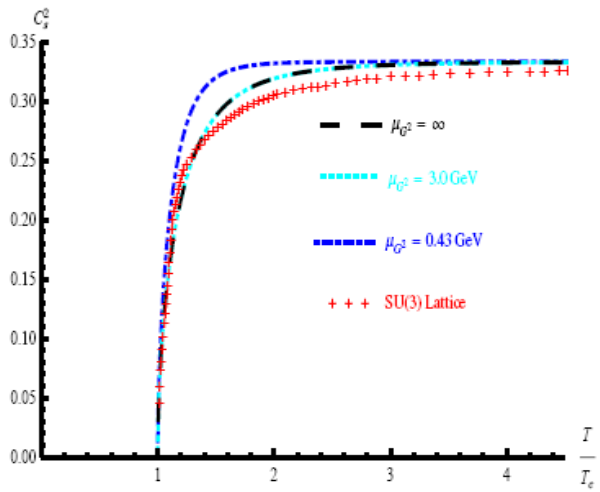
D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

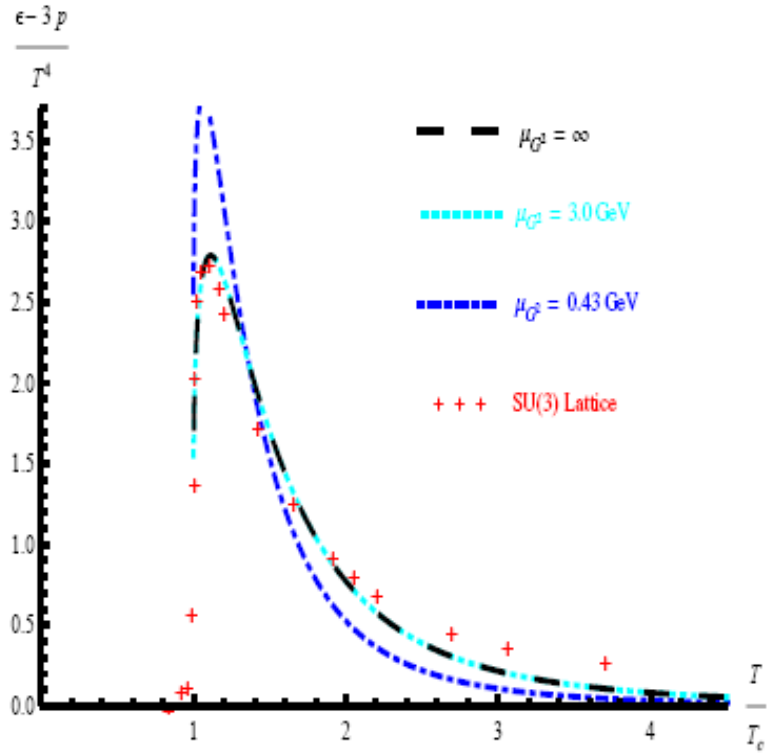
# EOS from dynamical hQCD




Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035

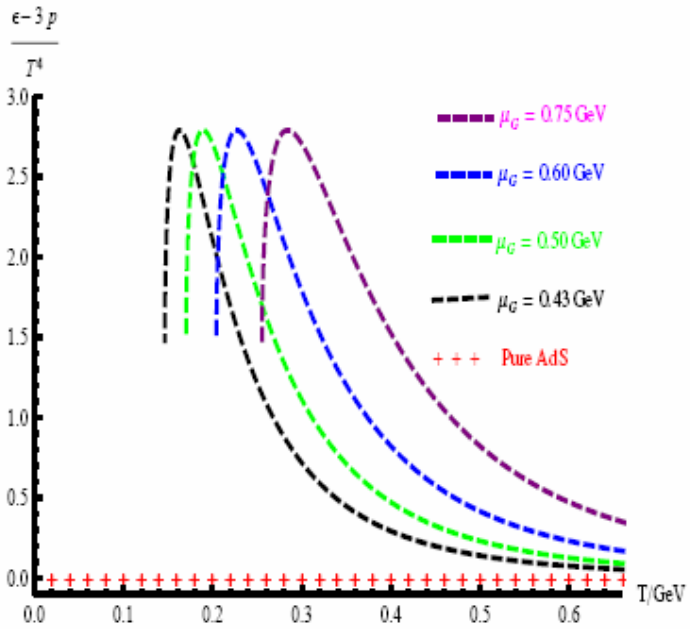
$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$$

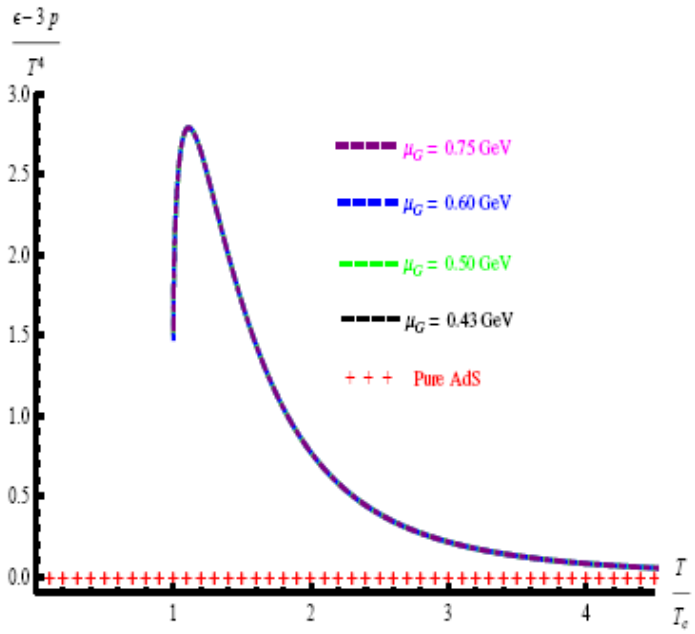


# EOS from dynamical hQCD
# Non-conformal around Tc

Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035



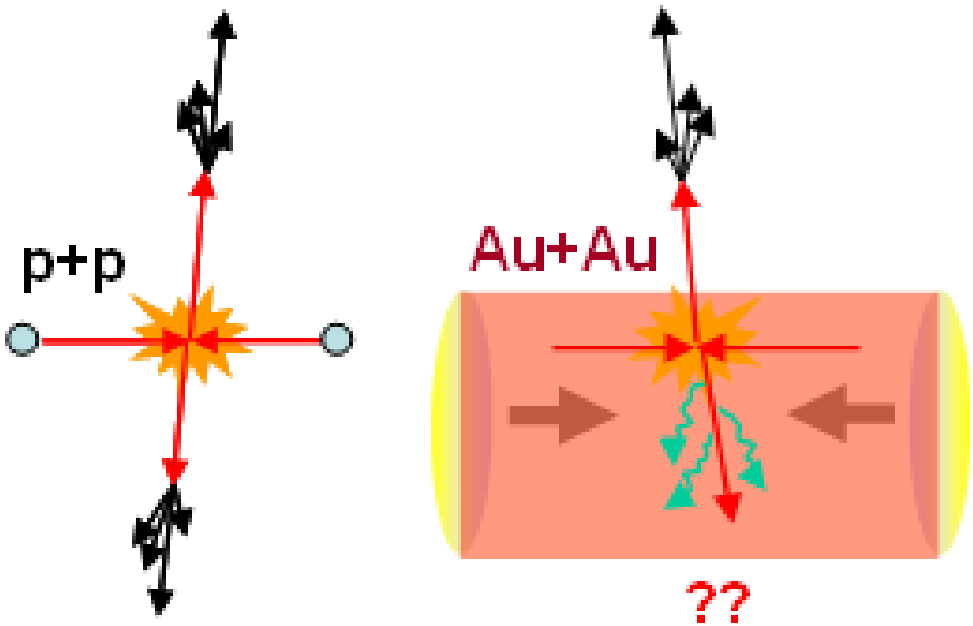




$$\mu_G = 0.43, 0.5, 0.6, 0.75\text{GeV}$$
$$T_c = 146, 170, 204, 255\text{MeV}$$

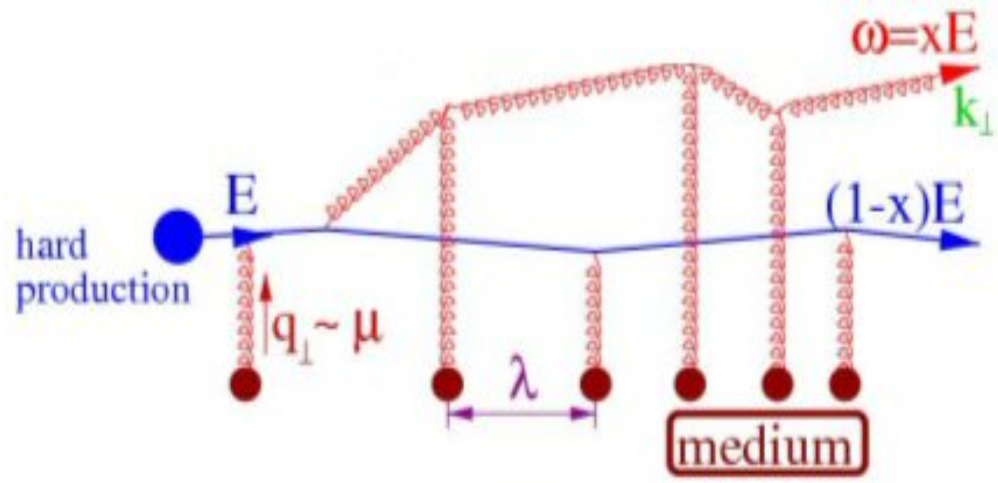
# IV. Transport properties

**Jet quenching parameter,**
**shear viscosity and bulk viscosity**

# What do we know about jet quenching parameter?



# Parton energy loss in QGP




The dominant effect of the medium on a high energy parton is medium-induced Bremsstrahlung.

$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

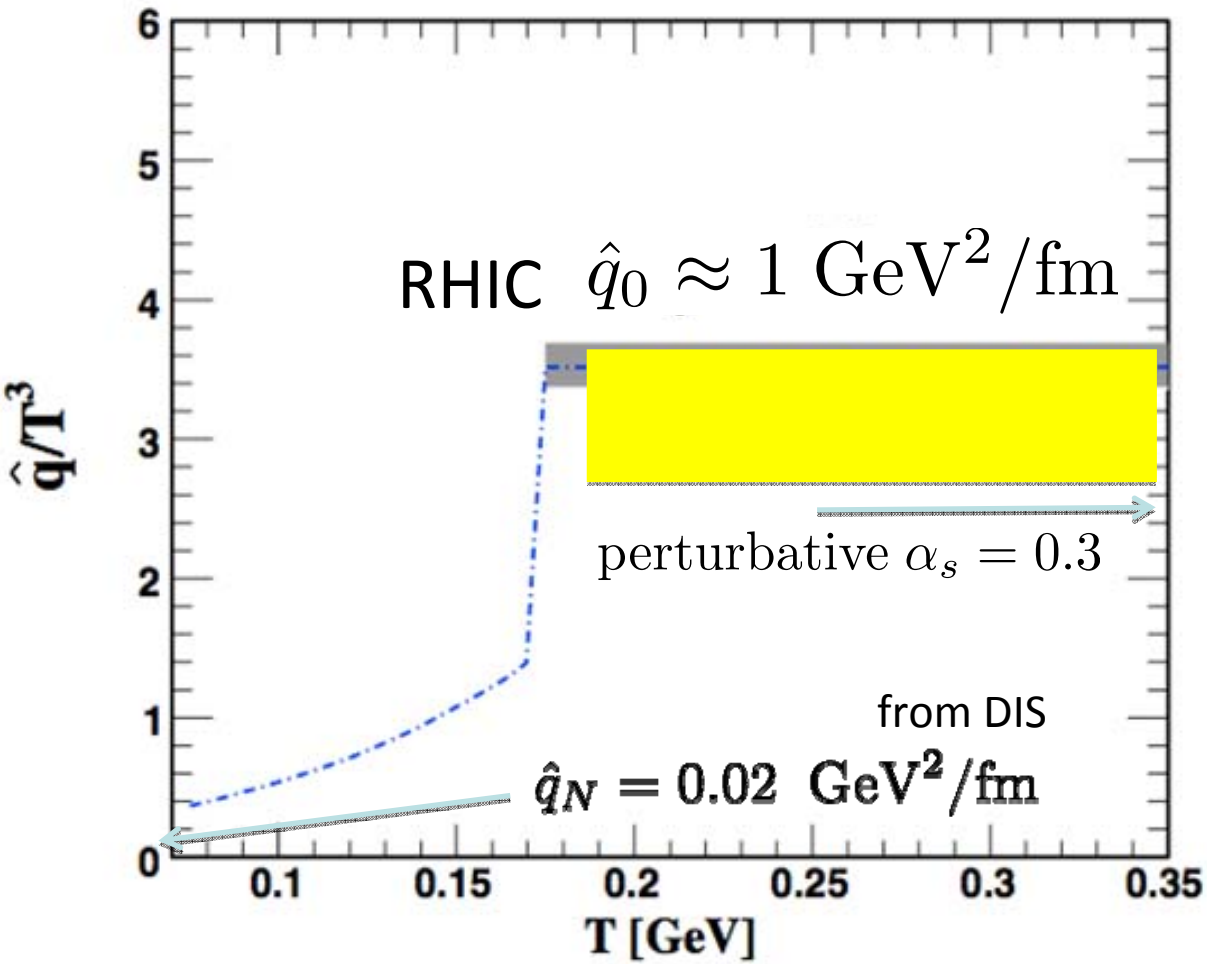
Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

$\hat{q}$ : reflects the ability of the medium to "quench" jets.

$$\hat{q} = \frac{\langle k_T^2 \rangle}{L} \approx \frac{\mu^2}{\lambda}$$

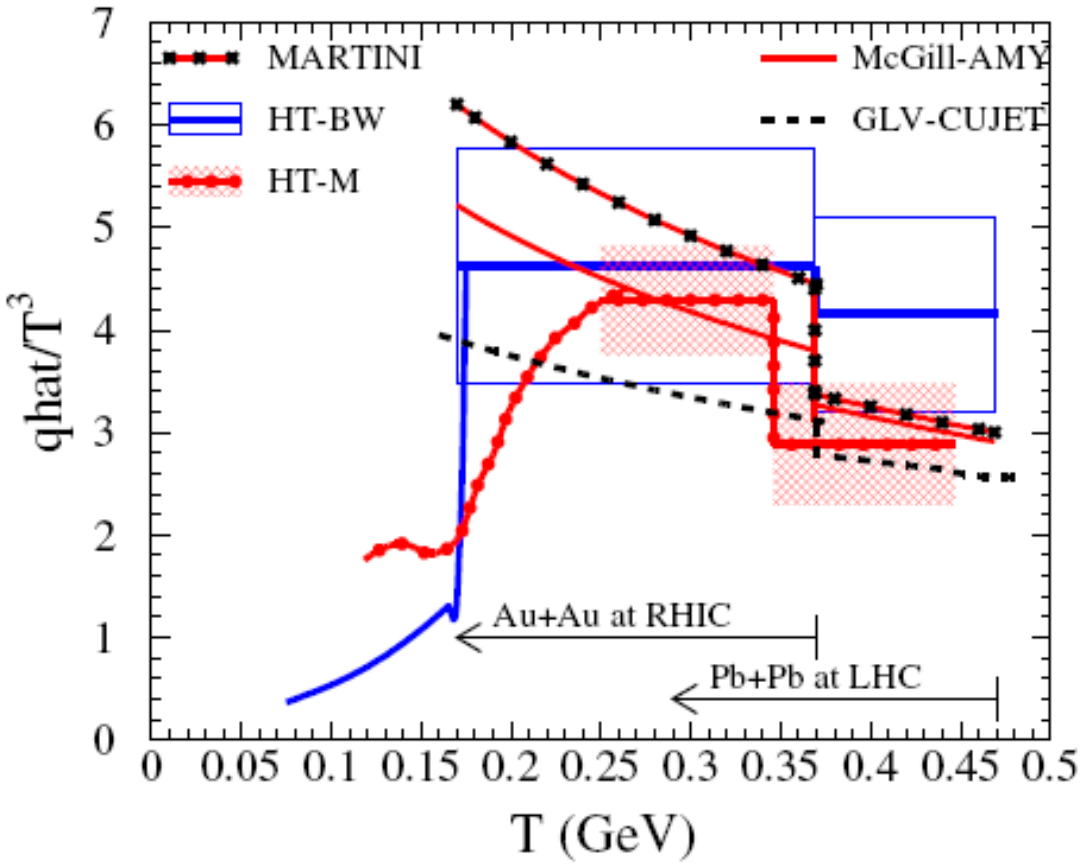
$\mu$ : Debye mass $\lambda$ : mean free path

## Fundamental question: What's the property of $\hat{q}$ ?




Chen, Greiner, Wang, XNW, Xu, PRC 81 (2010) 064908

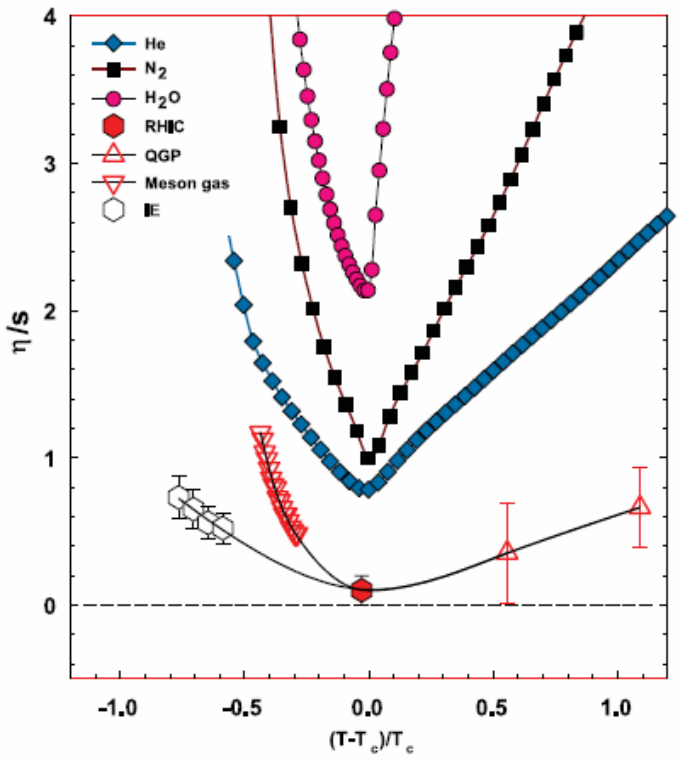
## Assumptions:
## Temperature dependence of jet quenching parameter
## [Jet Collaboration] arXiv:1312.5003



# Can jet quenching characterizing phase transition?

$$\frac{\eta_A}{s} = \frac{8\pi^2}{63}\frac{T^3}{\hat{q}}$$

Majumder,Muller,Wang, PRL 2007

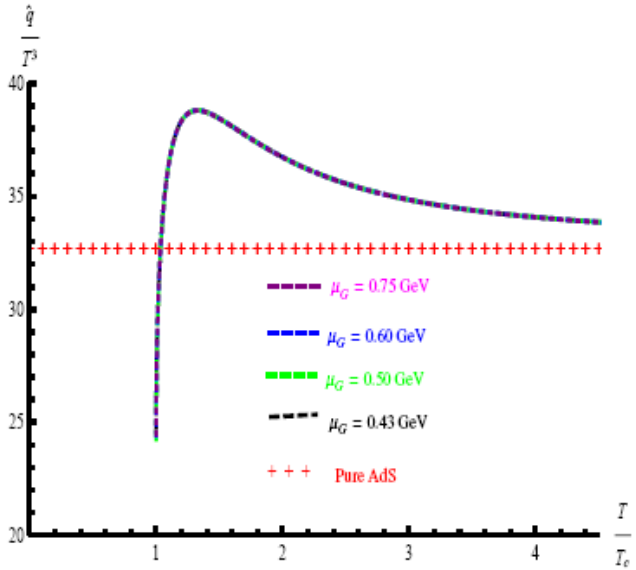



Lacey et al., PRL 98:092301,2007

Naively extend to general case:

$$\eta/s \sim T^3/\hat{q}$$

One can expect a peak of $\hat{q}/T^3$ around phase transition !?



# How can we calculate jet quenching parameter?

$$\hat{q}_R = \frac{4\pi C_R \alpha_s}{N_c^2 - 1} \int dy^- \left\langle F^{ai+}(0) F_i^{a+}(y^-) \right\rangle e^{i\xi p^+ y^-}$$

1, pQCD: cannot go to phase transition region;

2, LQCD: waiting for temperature dependent result

Majumder, arXiv:1202.5295, Panero et.al., arXiv:1307.5850

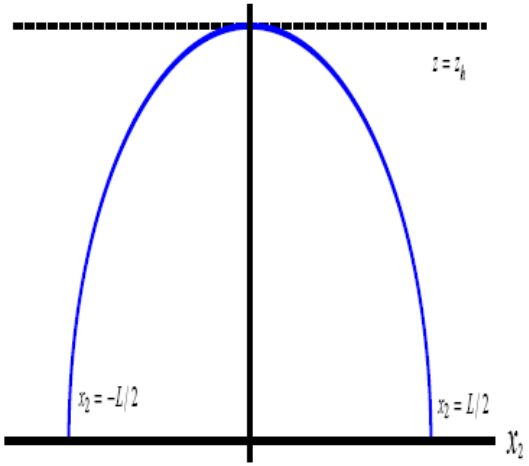
3, Effective Models: how ???

4, AdS/CFT: conformal, constant value

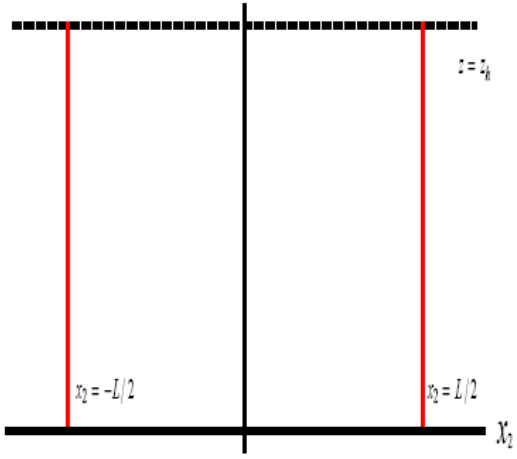
5, hQCD model: this work

$$W^{Adj}[\mathscr{C}] \approx exp(-\frac{1}{4\sqrt{2}}\hat{q}L^{-}L^{2})$$

Liu et.al. PRL2006




$(\tau = x^{-}, \sigma = x_2)$




$(\tau = x^{-}, \sigma = z)$

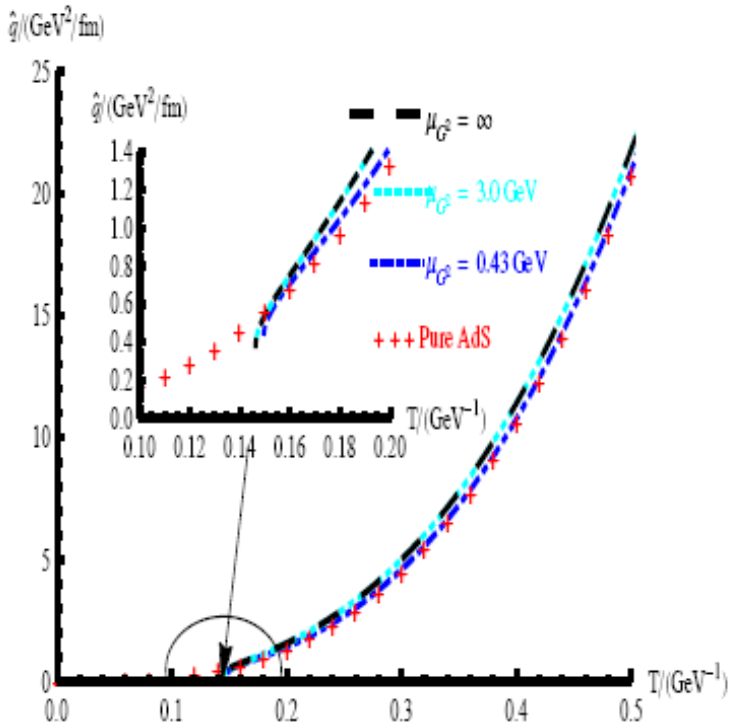
$$W^{Adj} = \exp(2i(S_1 - S_2))$$

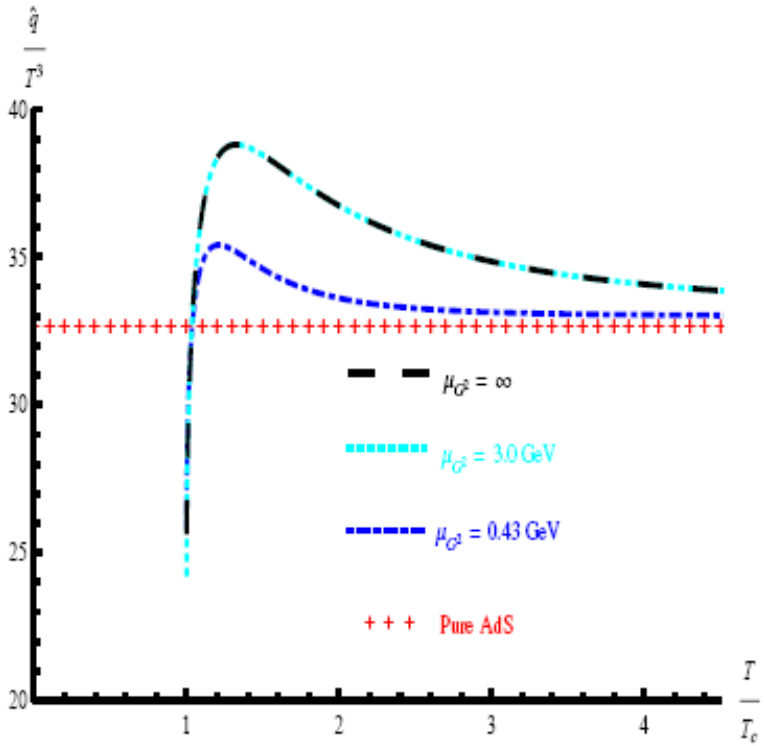
$$S_1 = \frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{g_{\tau\tau}g_{zz}z'(\sigma) + g_{\tau\tau}g_{22}}, \qquad S_2 = \frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{g_{\tau\tau}g_{zz}}.$$

$$\hat{q} = \frac{\sqrt{2}\sqrt{\lambda}}{\pi\int_0^{z_h} dz\sqrt{g_{zz}/g_{22}}},$$

# Jet quenching from dynamical hQCD

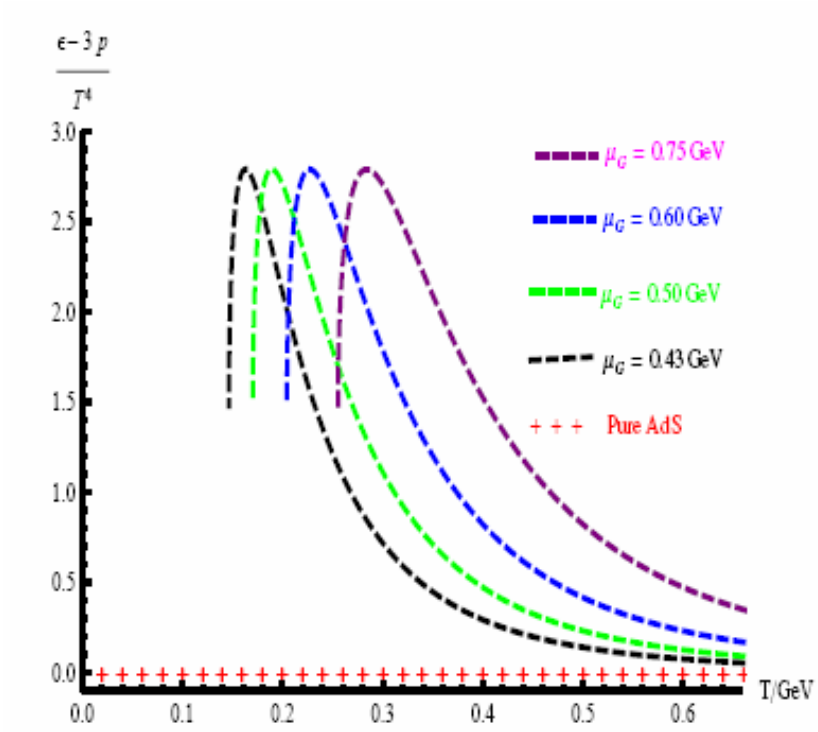
Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035

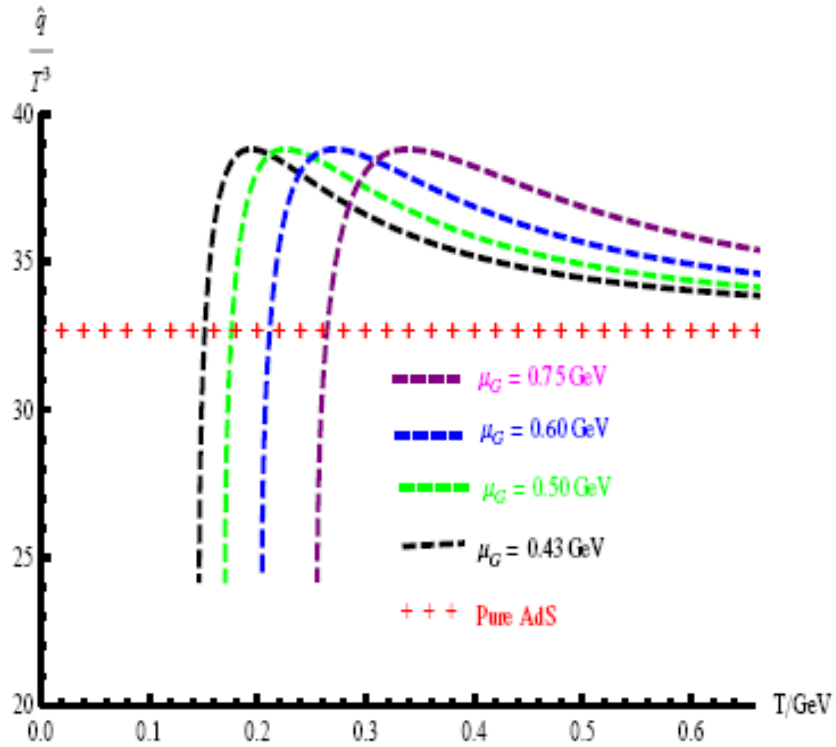


# Jet quenching from dynamical hQCD

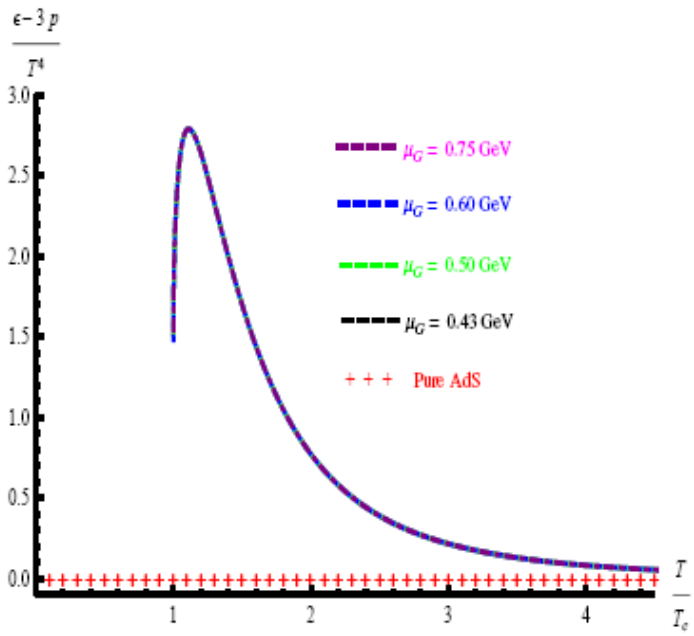
Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035

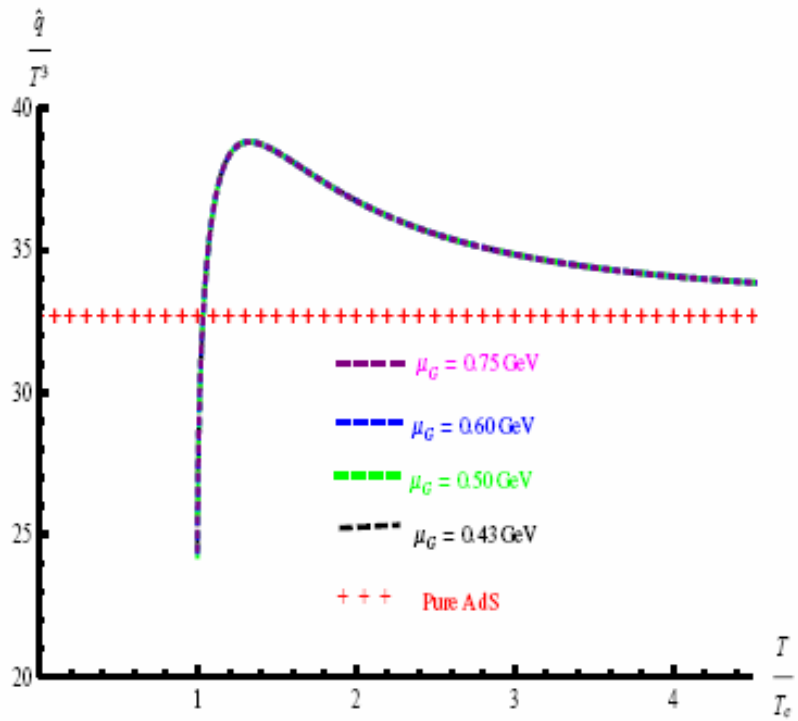


# Jet quenching characterizing phase transition!

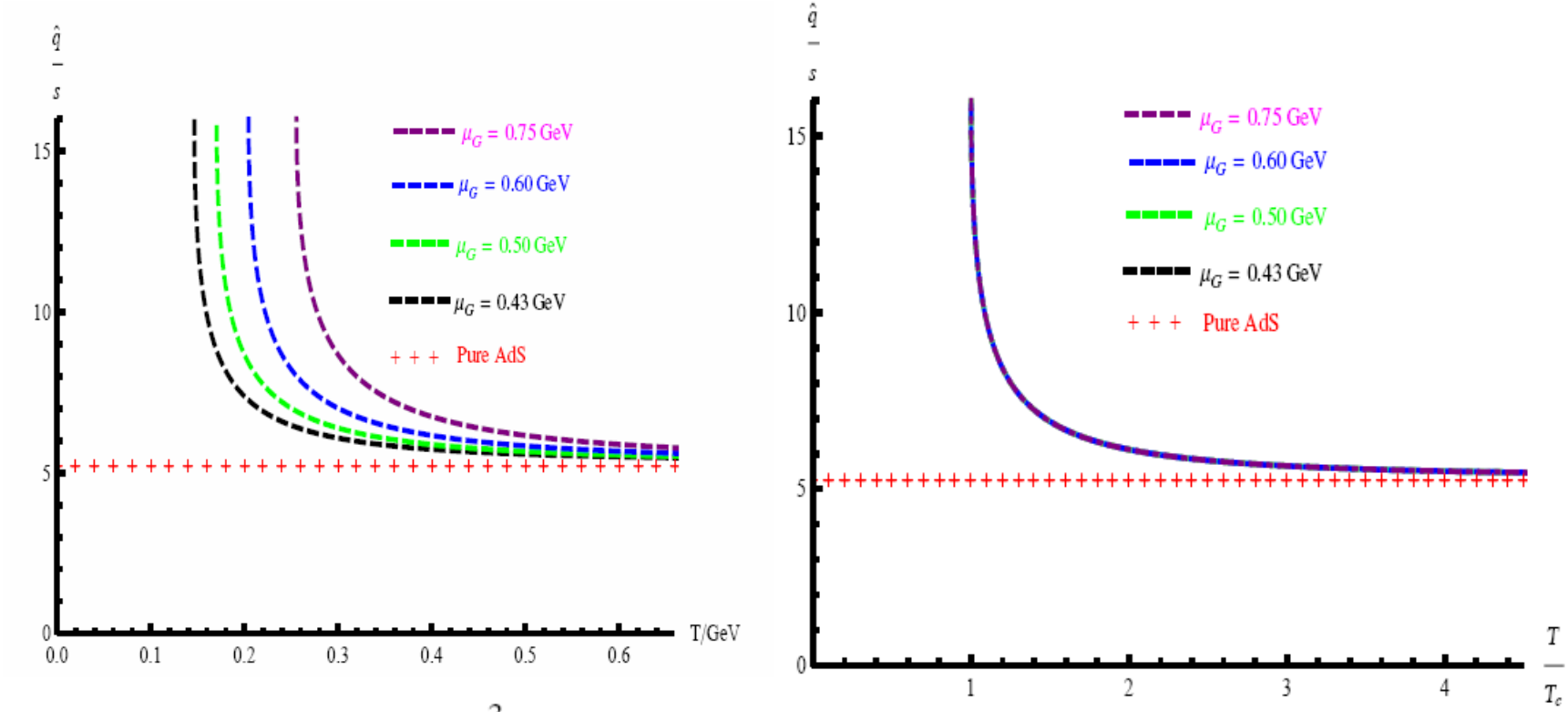
Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035



# Jet quenching characterizing phase transition!

Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035




$$s_{AdS_5} = \frac{1}{4G_5}\frac{1}{z_h^3} = \frac{\pi^3}{4G_5}T^3 \simeq 7.75\frac{1}{G_5}T^3$$

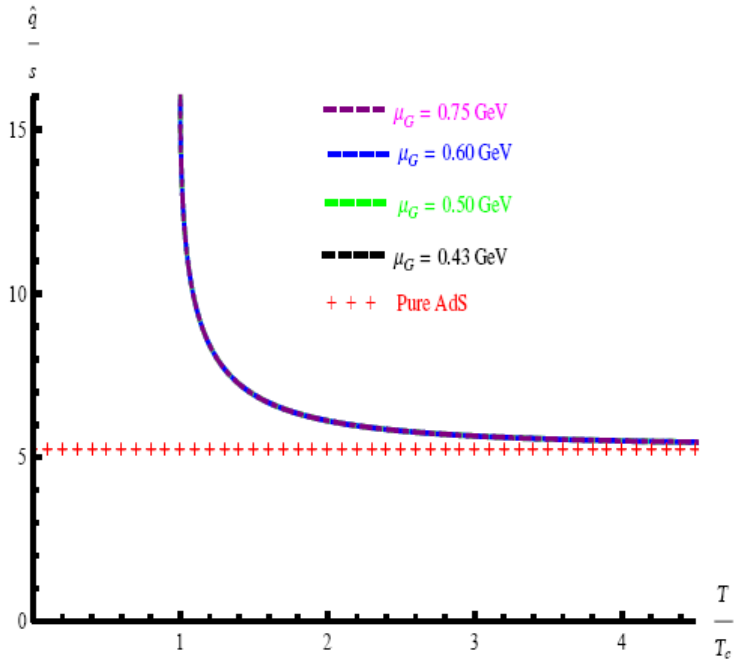
$$\hat{q}_{AdS_5} = \frac{\pi^{3/2}\sqrt{\lambda}\Gamma[3/4]}{\Gamma[5/4]}T^3 \simeq 7.53\sqrt{\lambda}T^3$$

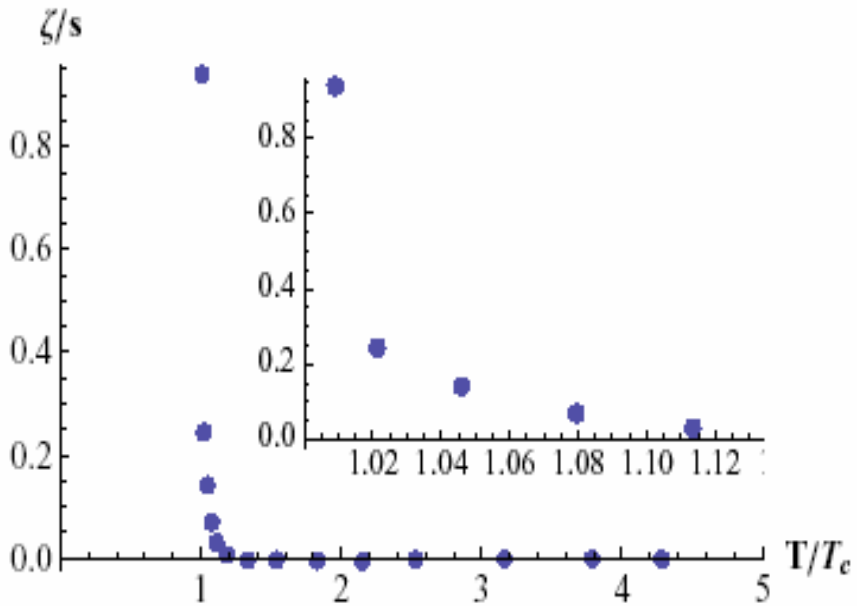
$$\hat{q}_{AdS_5}/s_{AdS_5} = 0.97G_5\sqrt{\lambda}$$

$$\lambda = 6\pi$$

# Jet quenching characterizing phase transition!

Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035



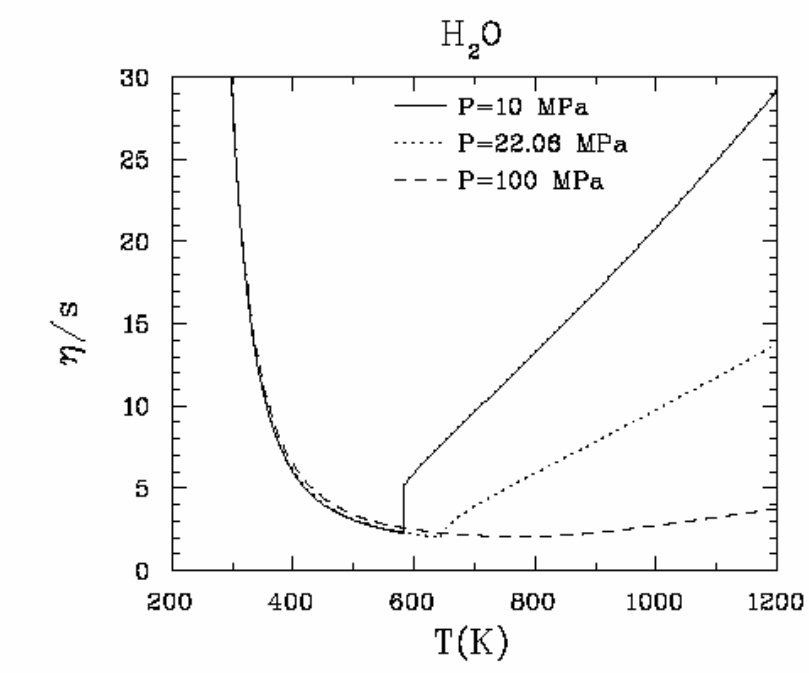



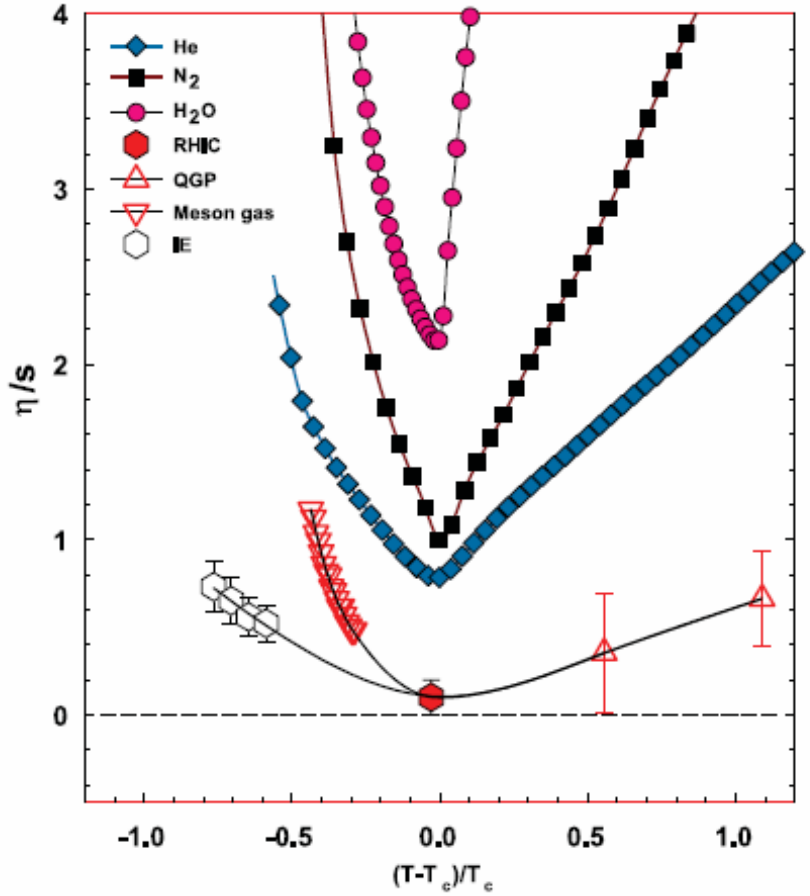

$\hat{q}/s$ similar to $\zeta/s$

# shear viscosity and bulk viscosity

# Shear viscosity over entropy density: LQCD + Model

## minimum near phase transition

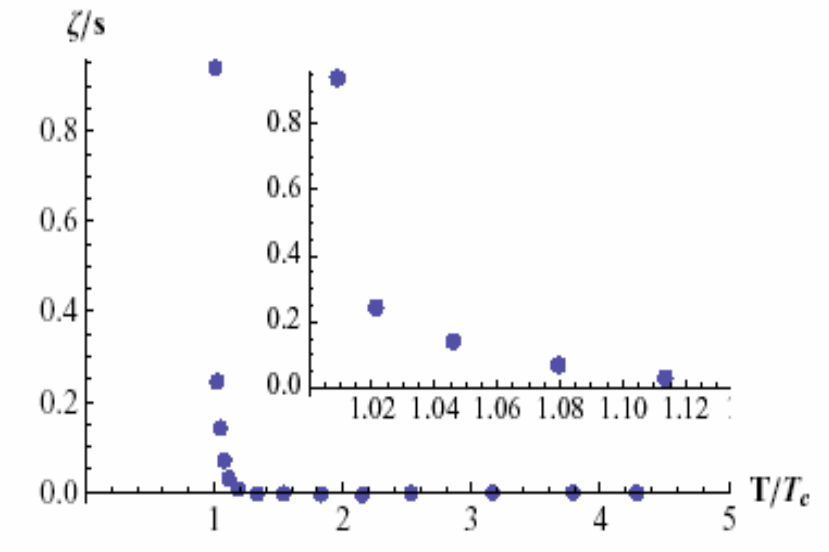


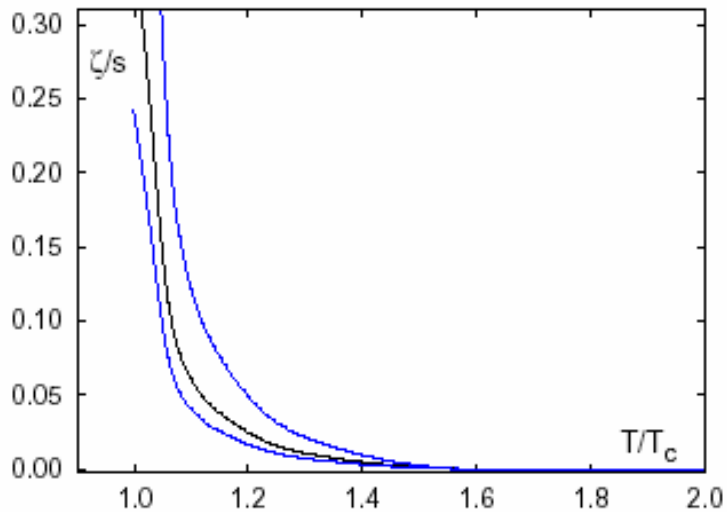

**Csernai et.al. Phys.Rev.Lett.97:152303,2006**

**Lacey et al., PRL 98:092301,2007**

# Bulk viscosity over entropy density: LQCD
## sharply rising near phase transition




Pure gluodynamics



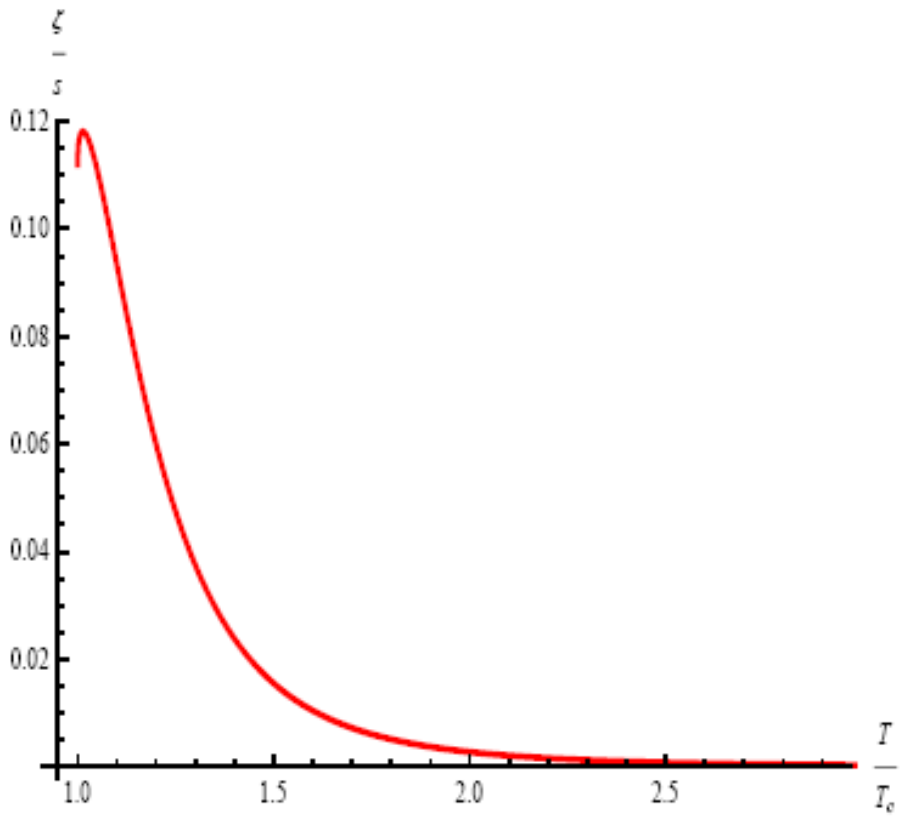

2-flavor case

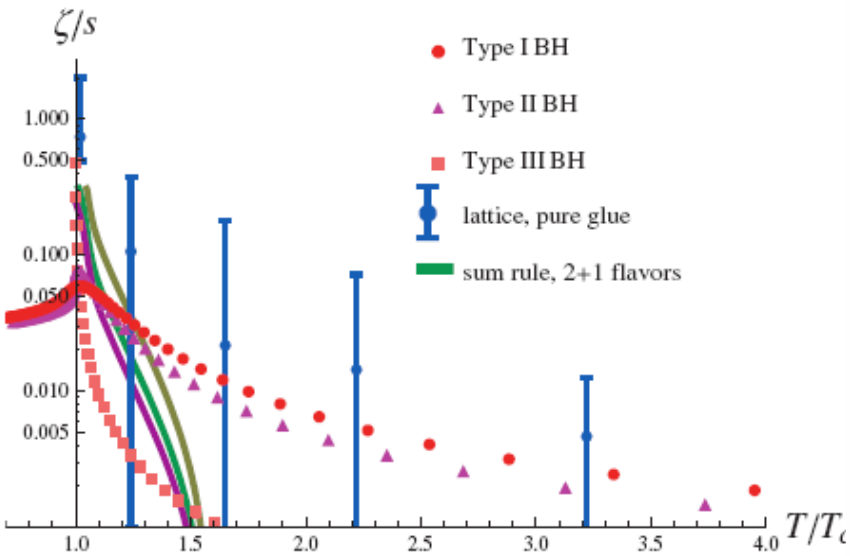
$$\zeta = \frac{1}{9\,\omega_0}\left\{T^5 \frac{\partial}{\partial T}\frac{(\epsilon_T - 3p_T)}{T^4} + 16|\epsilon_v|\right\}$$

Dmitri Kharzeev, Kirill Tuchin arXiv:0705.4280 [hep-ph],
F.Karsch, Dmitri Kharzeev, Kirill Tuchin arXiv:0711.0914 [hep-ph],
Harvey Meyer arXiv:0710.3717 [hep-ph],

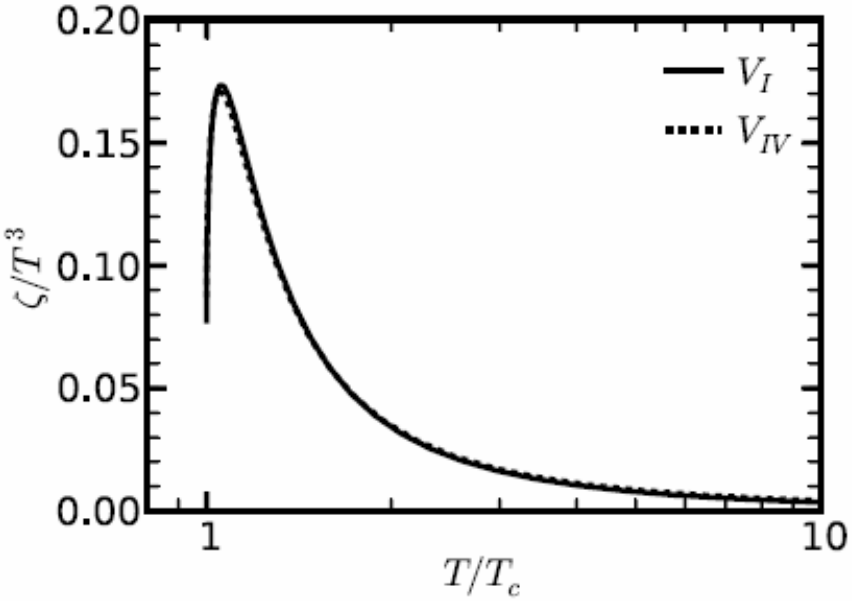
# Bulk viscosity from dynamical hQCD

Danning Li, Song He, M.H. work in progress





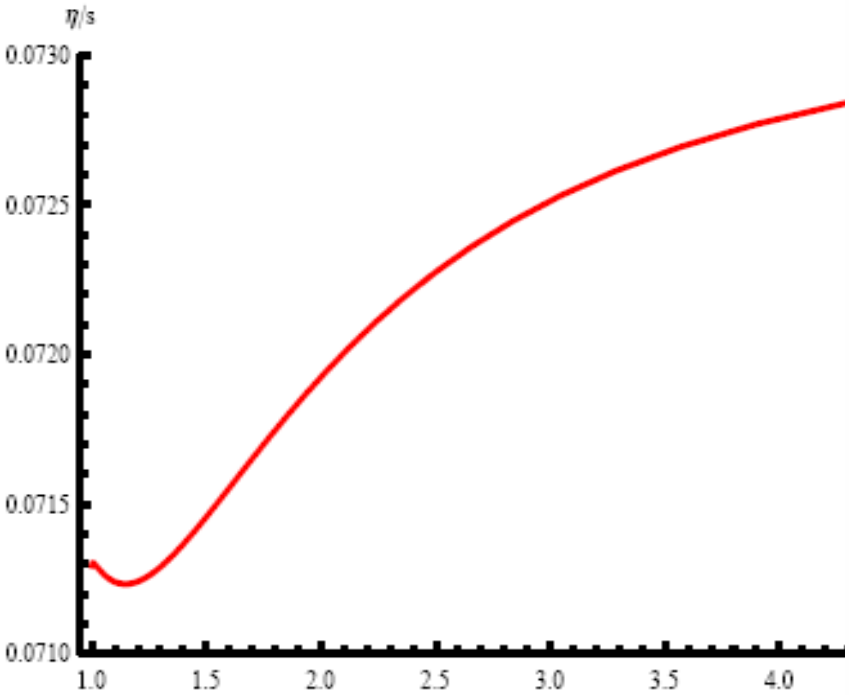

S.Gubser, et.al PRL101(2008)

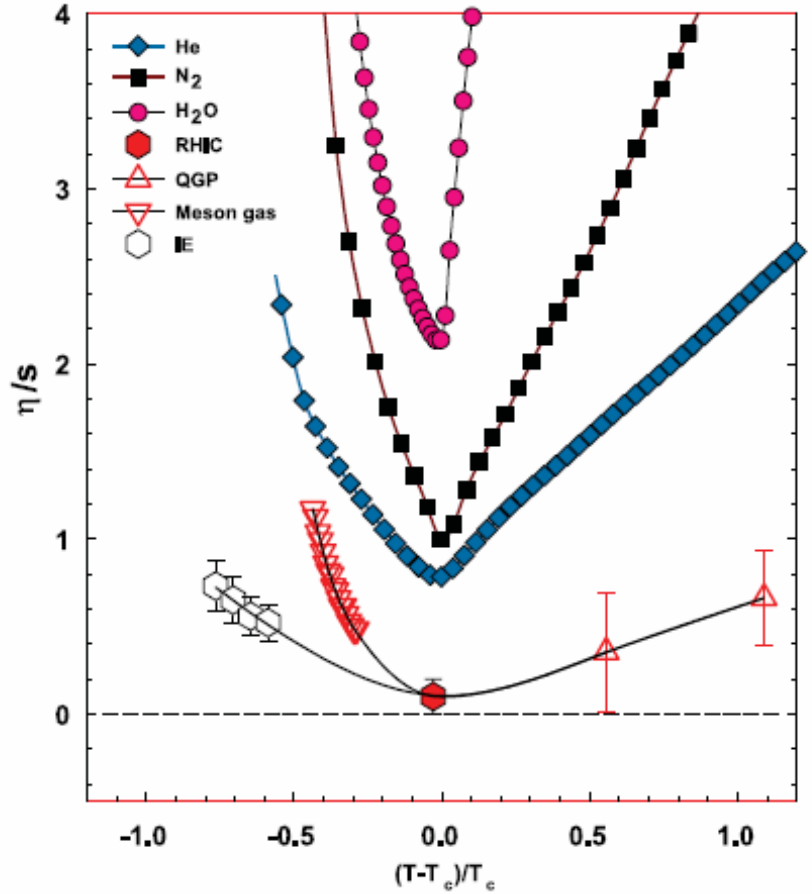



Yaresko, Kampfer, arXiv:1306.0214

# Shear viscosity from dynamical hQCD

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{4}{3}(\nabla\phi)^2 + V(\phi) + \ell^2\beta e^{\sqrt{\frac{2}{3}}\gamma\phi} R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}\right]$$




Danning Li, Song He, M.H. work in progress

**Lacey et al., PRL 98:092301,2007**

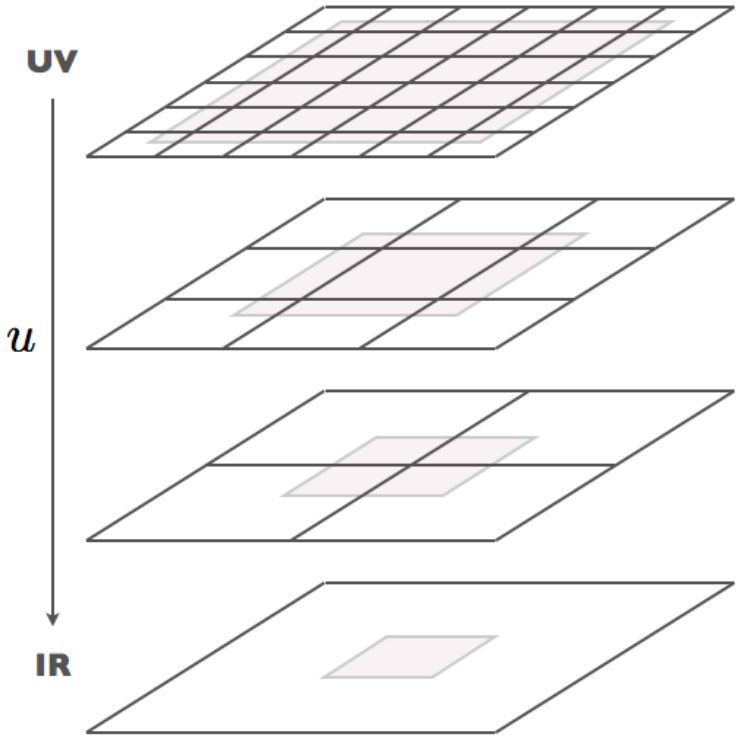
# V. Conclusion and discussion

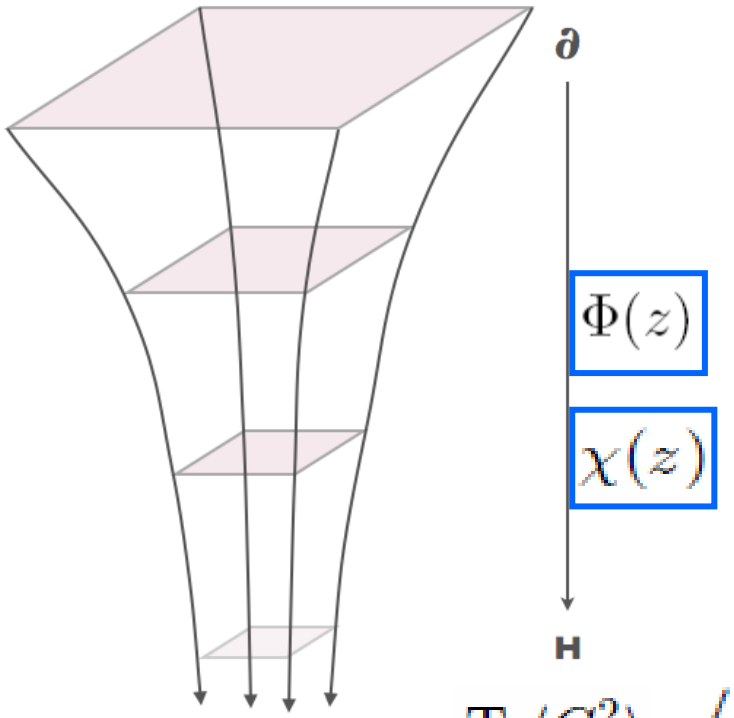
The ambitious goal is to build a standard hQCD model, which can describe hadron spectra, EOS as well as transport properties.

# Graviton-dilaton-scalar system

$$J_i|_{UV} \quad = \quad \Phi_i|_{\partial}$$

A $AdS_5$

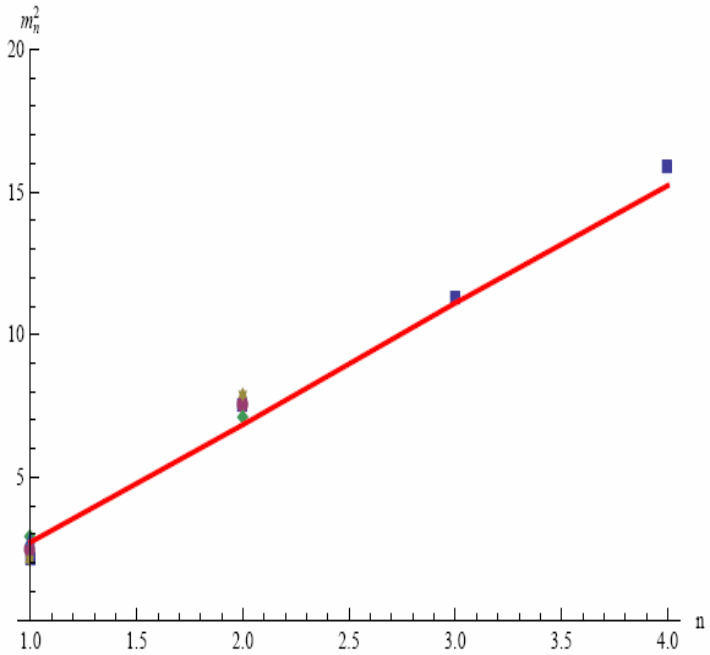


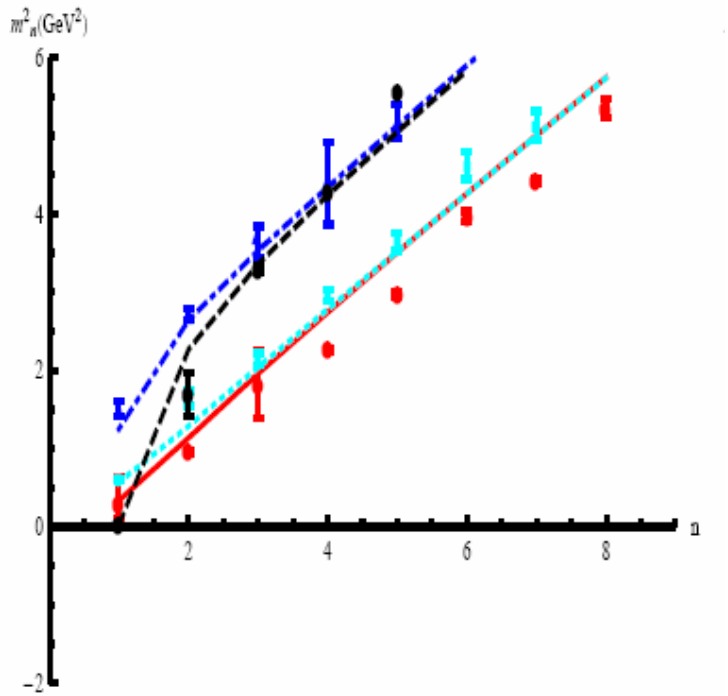



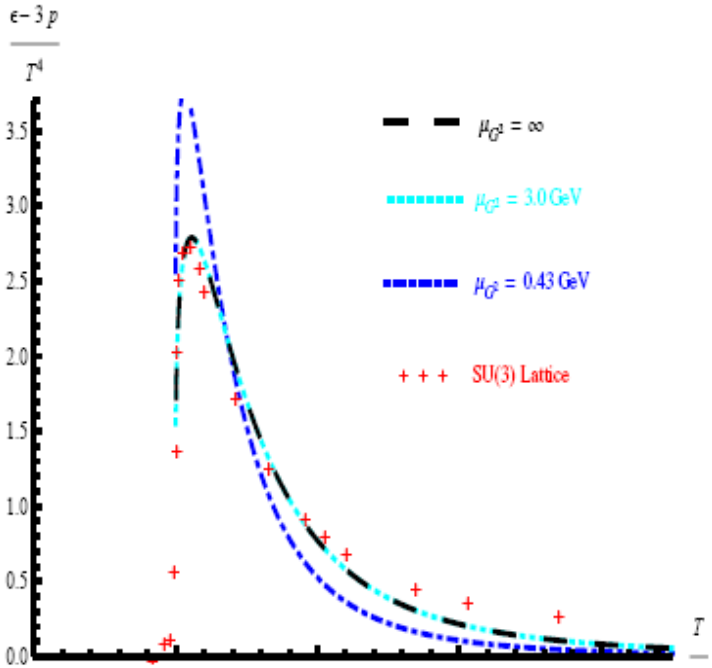

$\mathrm{Tr}\langle G^2 \rangle \quad \langle \bar{q} q \rangle$

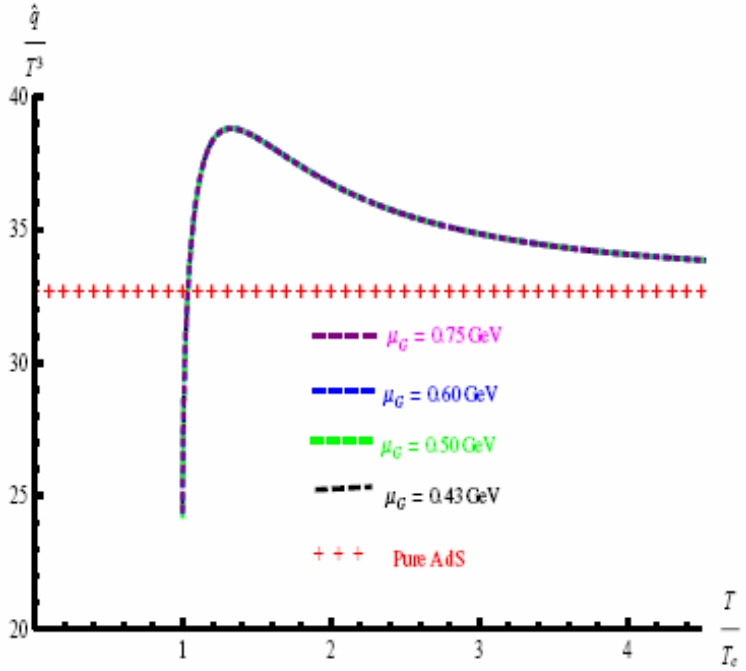
deformed $AdS_5$

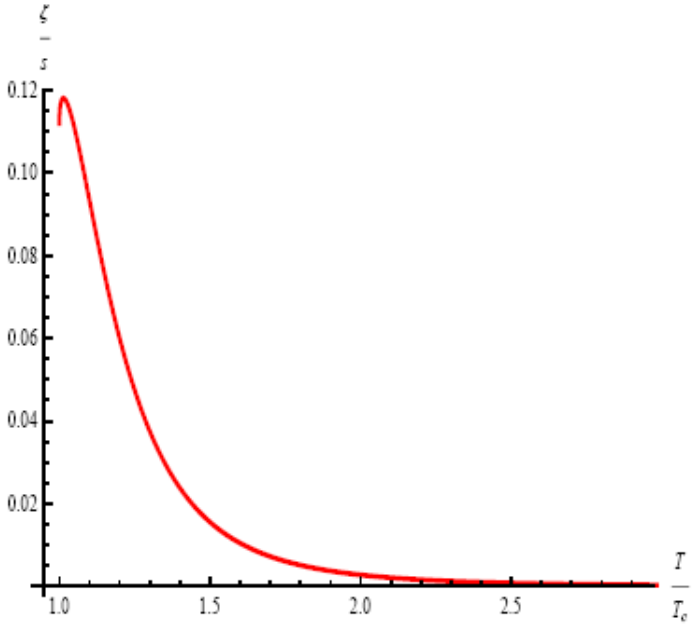
# Glueball and meson spectra

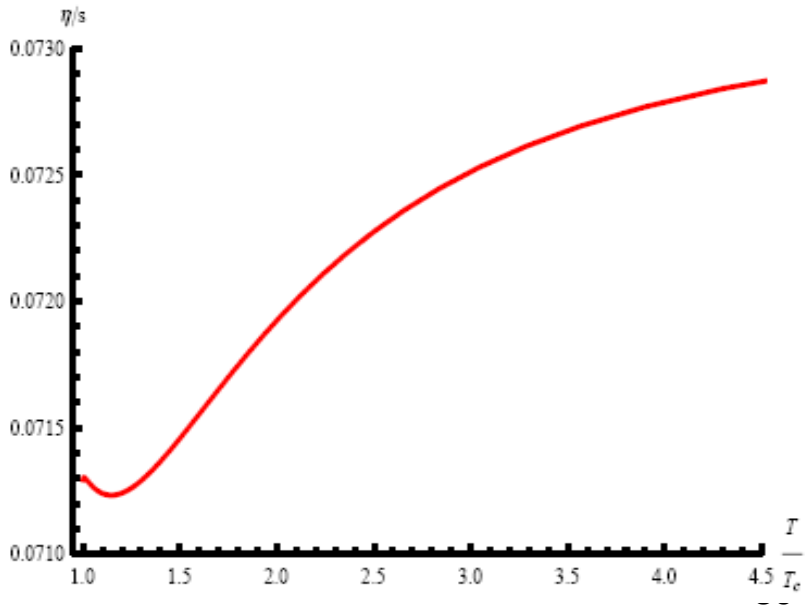



$\frac{\epsilon - 3p}{T^4}$
$\mu_{G^2} = \infty$
$\mu_{G^2} = 3.0$ GeV
$\mu_{G^2} = 0.43$ GeV
SU(3) Lattice
$\frac{T}{T_c}$


$\frac{\hat{q}}{T^3}$
$\mu_G = 0.75$ GeV
$\mu_G = 0.60$ GeV
$\mu_G = 0.50$ GeV
$\mu_G = 0.43$ GeV
Pure AdS
$\frac{T}{T_c}$


$\frac{\zeta}{s}$
$\frac{T}{T_c}$


$\eta/s$
$\frac{T}{T_c}$

# Thanks for your attention!