

Fuzzy bags, Polyakov loop and gauge/string duality

Fen Zuo

Huazhong University of Science and Technology

QCD@WORK

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Base on the following works:

Quadratic thermal terms in the deconfined phase from holography, FZ, Yi-Hong Gao, arXiv:1403.2241, to appear in JHEP.

Thermal power terms in the Einstein-dilaton system, FZ, arXiv:1404.4512, to appear in JHEP.

Quark potential at short distance

- 3 + 1 dimension:

$$\begin{aligned} V_{Q\bar{Q}}(R) &\approx -\frac{4}{3} \frac{\alpha_S}{R} + \sigma \cdot R \\ &\simeq -\frac{4}{3} \frac{\alpha_S}{R} \left(1 - \frac{3\sigma}{4\alpha_S} R^2\right). \quad R \rightarrow 0 \end{aligned}$$

- 2 + 1 dimension:
Coulomb potential

$$V_{Q\bar{Q}}(R) \propto g^2 \ln R.$$

Linear potential (?):

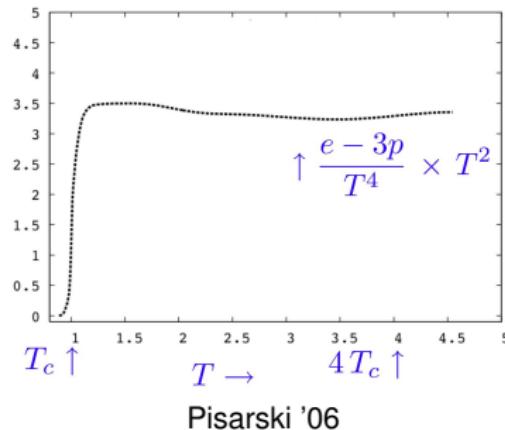
$$V_{Q\bar{Q}}(R) \propto g^4 R.$$

would be roughly as a linear correction at small R .

Fuzzy bags

Trace anomaly

$$\Delta \propto T^2, T_{\max} < T < T_{\text{pert}}$$



- Fuzzy bags (Pisarski '06):

$$p_{\text{QCD}}(T) \approx f_{\text{pert}} T^4 - B_{\text{fuzzy}} T^2 - B_{\text{MIT}} + \dots .$$

- 2 + 1 dimension:

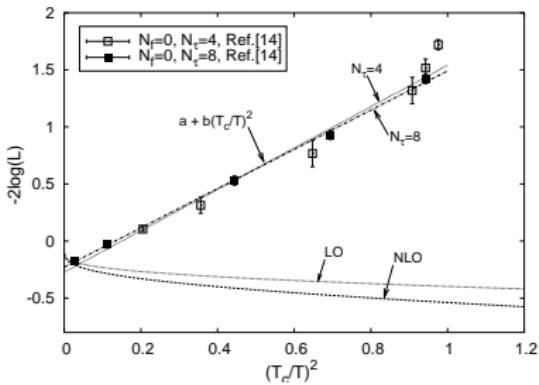
$$\Delta \propto T^2, \quad (\text{Caselle et al, '11})$$

$$p(T) \approx f_{\text{pert}} T^3 - B_{\text{fuzzy}} T^2 + \dots .$$

Polyakov loop

$$\mathcal{L} = \left\langle \text{Tr} \mathcal{P} \exp \left[ig \int_0^{1/T} d\tau A_0(\tau, \vec{x}) \right] \right\rangle.$$

$$-2 \log \mathcal{L} = a + b \left(\frac{T_c}{T} \right)^2$$



Megias, Ruiz Arriola and Salcedo '05

- Strong coupling: AdS-Schwarzschild black hole (infinite mass)

$$ds^2 = -\frac{r^2}{L^2} \left[1 - \left(\frac{r_0}{r} \right)^4 \right] dt^2 + \left\{ \frac{r^2}{L^2} \left[1 - \left(\frac{r_0}{r} \right)^4 \right] \right\}^{-1} dr^2 + \frac{r^2}{L^2} d\mathbf{x}^2.$$

$$\begin{aligned}s_{\lambda=\infty} &= \frac{\pi^2}{2} N_c^2 T^3, & \varepsilon_{\lambda=\infty} &= \frac{3\pi^2}{8} N_c^2 T^4, & p_{\lambda=\infty} &= \frac{\pi^2}{8} N_c^2 T^4 \\c_S^2 &\equiv \frac{dp}{d\varepsilon} = 1/3, & \Delta \equiv \varepsilon - 3p &= 0.\end{aligned}$$

- Strong/Weak coupling ratio (Gubser, Klebanov and Peet '96):

$$\frac{s_{\lambda=\infty}}{s_{\lambda=0}} = \frac{p_{\lambda=\infty}}{p_{\lambda=0}} = \frac{\varepsilon_{\lambda=\infty}}{\varepsilon_{\lambda=0}} = \frac{3}{4}.$$

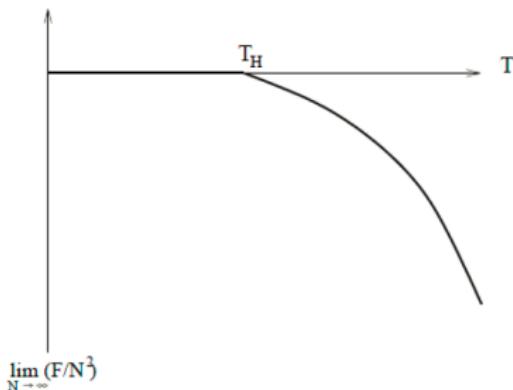
"Confinement" in Finite Volume: Gauss law on a compact manifold.

Hagedorn transition (Sundborg '00):

$$x_H \equiv e^{-\beta_H} \equiv e^{-1/T_H} \approx 0.072.$$

$$T_H \approx 0.38$$

$$\lim_{N \rightarrow \infty} \frac{1}{N^2} F = \begin{cases} 0, & (T < T_H) \\ -\frac{x_H z'(x_H)}{4 T_H} (T - T_H), & (T \gtrsim T_H) \end{cases}$$



(Aharony et al '03)

- Thermal AdS (X_0)

$$ds^2 = - \left(1 + \frac{r^2}{L^2}\right) dt^2 + \left(1 + \frac{r^2}{L^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2.$$

topology: $\mathbb{S}^1 \times \mathbb{R}^4$

AdS-Swarzschild black hole (X_1)

$$ds^2 = - \left(1 - \frac{\omega_4 M}{r^2} + \frac{r^2}{L^2}\right) dt^2 + \left(1 - \frac{\omega_4 M}{r^2} + \frac{r^2}{L^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2.$$

$$r_+ \sim T_H \pm \sqrt{T_H^2 - T_{\min}^2} \quad (+ : \text{big BH}, \quad - : \text{small BH}).$$

topology: $\mathbb{R}^2 \times \mathbb{S}^3$

- Hawking-Page transition (Hawking and Page '83, Witten '98):

$$F_1 - F_0 \sim r_+^2 (L^2 - r_+^2) \quad \begin{cases} > 0, & r_+ < L \ (T < T_c), \\ < 0, & r_+ > L \ (T > T_c). \end{cases}$$

$$T_c = \frac{3}{2\pi L} \sim \frac{0.48}{L} \quad (> T_{\min} = \frac{\sqrt{2}}{\pi L}).$$

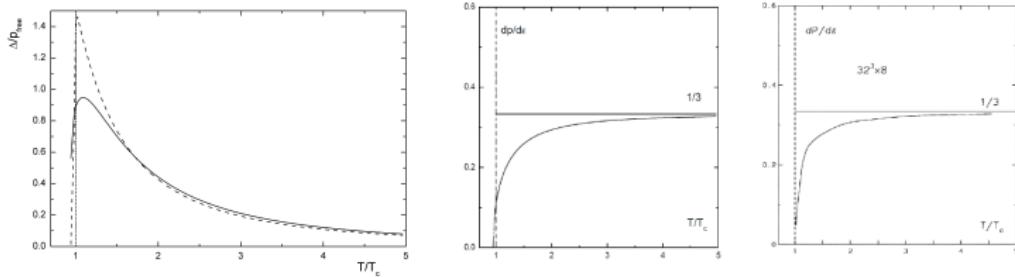
$\mathcal{N} = 4$ SYM on \mathbb{S}^3 : strong coupling (II)

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$$\frac{\Delta}{p_{\lambda=0}} \equiv \frac{(\varepsilon - 3p)}{p_{\lambda=0}} = \frac{T_c^2}{2 T_H^2} \left[1 + \sqrt{1 - \frac{8}{9} \frac{T_c^2}{T_H^2}} \right]^2.$$

$$c_S^2 \equiv \frac{dp}{d\varepsilon} = \frac{1}{3} \sqrt{1 - \frac{8T_c^2}{9T_H^2}}.$$

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dashed line: large-N extrapolation right: lattice data for SU(3) (Boyd et. al '96)
of lattice data (Panero '09)

$\mathcal{N} = 4$ SYM on S^3 : strong coupling (III)

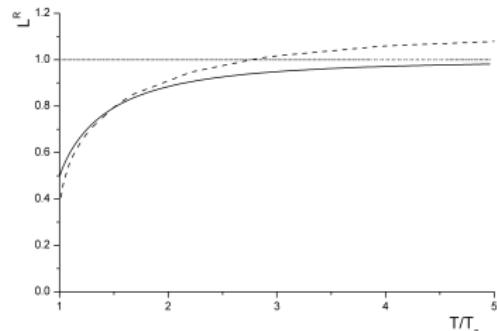
Thermal AdS with topology $S^1 \times \mathbb{R}^4$, no string worldsheet ending on S^1

$$\mathcal{L} = 0.$$

AdS-Swarzschild black hole with topology
 $\mathbb{R}^2 \times S^3$, \mathbb{R}^2 has S^1 as boundary

$$\begin{aligned} -2 \log \mathcal{L}^R &= \frac{\sqrt{\lambda}}{2} \left[1 - \sqrt{1 - \frac{8T_c^2}{9T_H^2}} \right] \\ &\approx \frac{2\sqrt{\lambda}}{9} \frac{T_c^2}{T_H^2} + \mathcal{O}(T_H^{-4}). \end{aligned}$$

$$\mathcal{L}^R(T_c) = \exp \left[-\frac{\sqrt{\lambda}}{6} \right].$$



dashed line: lattice data for SU(3) (Gupta et al, '08)

Perturbative result for $SU(N_C)$:

$$\mathcal{L}^R(T) \sim 1 + \frac{1}{16\pi} \frac{N_C^2 - 1}{N_C} g^2 \frac{m_D}{T} + \dots, \quad m_D \sim g N_C^{1/2} T.$$

Hard wall and soft wall

- Criterion for confinement: area law for the Wilson loop.
- Hard Wall:
“Cut off” the AdS spacetime at some finite radius

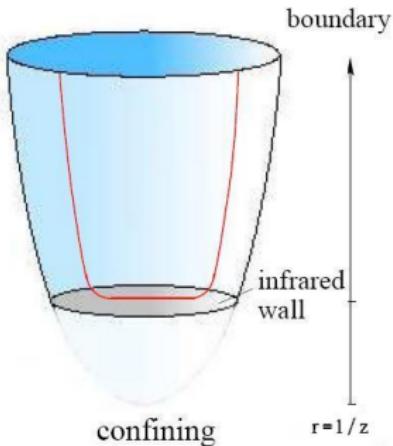
$$m_n^2 \sim n^2.$$

$$p \sim \pi^4 T^4 - 2\Lambda^4.$$

- Soft Wall:
Suppress infrared region by e^{-z^2}

$$m_n^2 \sim n.$$

$$p \sim T^4 \left(1 - \frac{8}{3} \frac{\Lambda^2}{\pi^2 T^2} + \mathcal{O}(T^{-4})\right).$$



Gravity-dilaton system: confinement and phase transition



$$S_5 \sim \int_M d^5x \sqrt{-g} (R - V(\Phi) - \frac{4}{3}(\partial\Phi)^2)$$

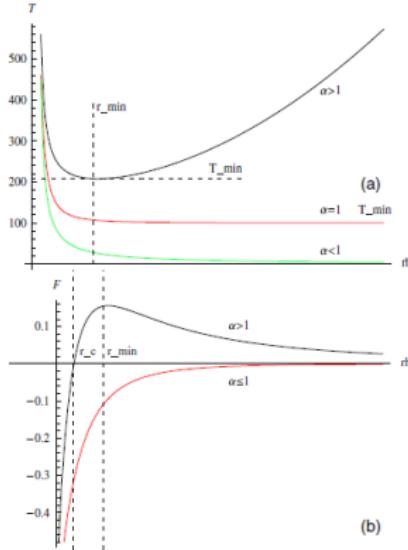
- Infrared asymptotics ($r \rightarrow \infty$)
(Gürsoy et al, '08)

$$\begin{aligned} ds_0^2 &\rightarrow e^{-Cr^\alpha} (dr^2 + dx_4^2), \\ \lambda_0 &\equiv e^\Phi \rightarrow e^{(3C/2)} r^\alpha r^{\frac{3}{4}(\alpha-1)}. \end{aligned}$$

- Confining iff $\alpha \geq 1$
HP transition iff confining.
- Glueball spectrum:

$$m_n^2 \sim n^{2(\alpha-1)/\alpha}.$$

(Gürsoy et al, '08)



Gravity-dilaton system: UV asymptotic solutions

- (Hohler and Stephanov '09, Cherman, Cohen and Nellore '09)

$$ds^2 = \frac{1}{z^2} \left(-f(z)dt^2 + d\bar{x}^2 \right) + e^{2g(z)} \frac{dz^2}{z^2 f(z)}.$$

$$\dot{g} = -\frac{1}{6} \dot{\phi}^2, \quad \dot{y} \equiv z \frac{d}{dz} y.$$

- \bullet ϕ^0 :

$$g(z) = 0, \quad f(z) = 1 - \frac{z^4}{z_H^4}.$$

- ϕ^1 :

$$\phi(z) = \phi_H {}_2F_1 \left(1 - \frac{\Delta_+}{4}, \frac{\Delta_+}{4}, 1, 1 - \frac{z^4}{z_H^4} \right).$$

- ϕ^2 :

Corrections of $g(z)$ and $f(z)$.

Gravity-dilaton system: UV asymptotic expansions

- Thermal quantities:

$$T_H \sim \frac{1}{\pi z_H} (1 + A \phi_H^2),$$
$$s \sim \frac{1}{z_H^3}, \quad \Delta \sim \frac{1}{z_H^4} \phi_H^2,$$

- Transport coefficients:

$$c_s^2 \sim \frac{1}{3} - C \phi_H^2,$$
$$\eta \sim \frac{1}{z_H^3}, \quad \zeta \sim \frac{1}{z_H^3} \phi_H^2.$$

- Quark free energy and Polyakov loop:

$$\frac{dF_Q}{dT_H} \sim -\phi_H.$$

$$\mathcal{L} = e^{-F_Q(T)/T}.$$

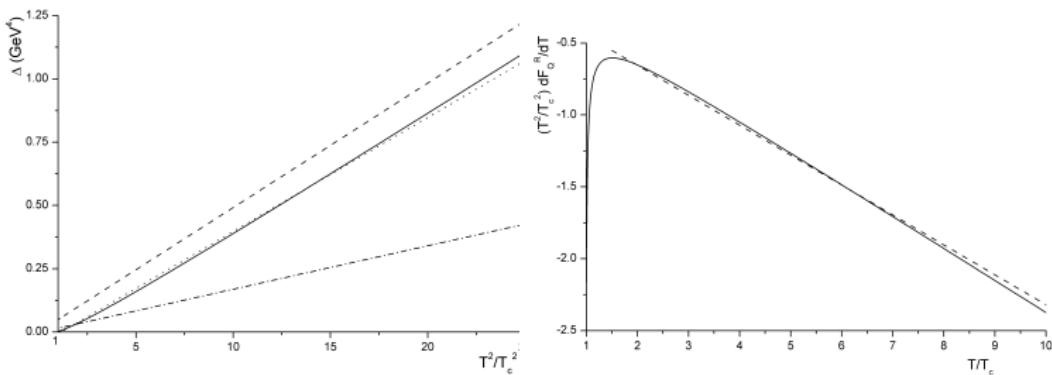
Gravity-dilaton system: Example I

- $\Delta_+ = 3$:

$$\phi_H \sim z_H \sim T_H^{-1}.$$

$$\Delta \sim T_H^2, \quad \frac{dF_Q}{dT} \sim -T_H^{-1}.$$

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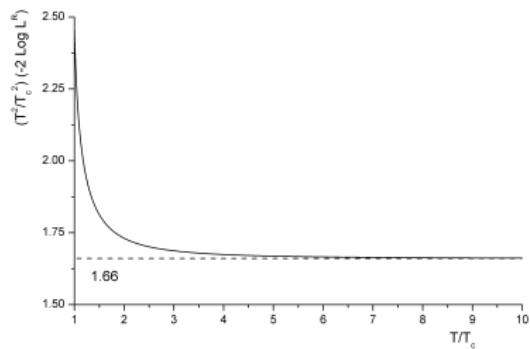
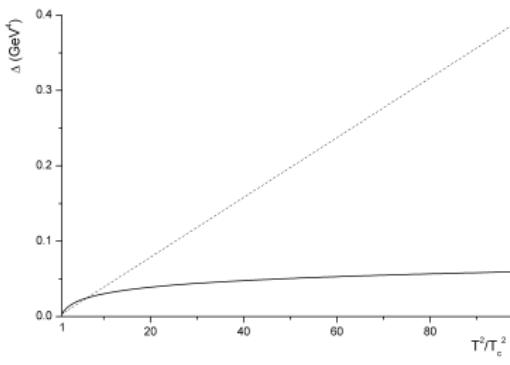
Gravity-dilaton system: Example II

- $\Delta_+ = 2$:

$$\phi_H \sim z_H^2 \sim T_H^{-2}.$$

$$\Delta \sim T_H^0, \quad \frac{dF_Q}{dT} \sim -T_H^{-2}, \quad \log \mathcal{L} \sim -T_H^{-2}.$$

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Summary and discussion

- “Confinement” In Finite Volume: quadratic thermal terms appear in the trace “anomaly” and Polyakov loop, in accordance with lattice data.
- Gravity-dilaton system: can not accommodate both.
- Field theory: no gauge-invariant local operator of dimension 2.

- (Caselle et al, '11):

$$\Delta \propto T^2, \quad \Delta/T^3 \sim T^{-1}.$$

- Hawking-Page Transition in AdS₄-BH:

$$\Delta/p_0 = \frac{T_c^2}{2T_H^2} \left[1 + \sqrt{1 - \frac{3T_c^2}{4T_H^2}} \right] \sim T^{-2}.$$

- Gravity-dilaton:

- (Amos Yarom '10):

$$\Delta/T^3 \sim \phi^2.$$

If $\Delta_+ = 5/2$,

$$\Delta/T^3 \sim T^{-1}.$$

- (Caselle et al, '11)

$$ds_0^2 \sim \frac{e^{-Cr^\alpha}}{r^2} (dr^2 + dx_3^2).$$

$$\alpha \sim 3/2.$$