

# Callan-Symanzik approach to infrared Yang-Mills theory

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# Quantization of Yang-Mills theory

- ▶  $SU(N)$  Yang-Mills theory in the continuum, Landau gauge
- ▶ Faddeev-Popov determinant  $\Rightarrow$  ghosts (and BRST symmetry)
- ▶ gauge copies [Gribov 1978]  $\Rightarrow$  restriction of the gauge field configurations to the (first) Gribov region  $\Omega$  (properly to the fundamental modular region)
- ▶ Zwanziger's horizon function (*breaks* the BRST symmetry), local formulation  $\Rightarrow$  additional auxiliary fields [Zwanziger 1989]
- ▶ condensates of the auxiliary fields: “refined Gribov-Zwanziger scenario”  $\Rightarrow$  effective mass term for the gluons [Dudal et al. 2008]

# Dyson-Schwinger equations and lattice simulations

- here: formulation *without* (additional) auxiliary fields
- long debate about the IR behavior of gluon propagator and ghost dressing function, Dyson-Schwinger equations yield different types of solutions (scaling and decoupling solutions) [von Smekal, Hauck, Alkofer 1997; Fischer, Alkofer 2002; Aguilar, Natale 2004; Boucaud et al. 2006]
- our conventions for the propagators

$$\langle A_\mu^a(p) A_\nu^b(-q) \rangle = G_A(p^2) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \delta^{ab} (2\pi)^4 \delta(p-q)$$
$$\langle c^a(p) \bar{c}^b(-q) \rangle = G_c(p^2) \delta^{ab} (2\pi)^4 \delta(p-q)$$

and dressing functions

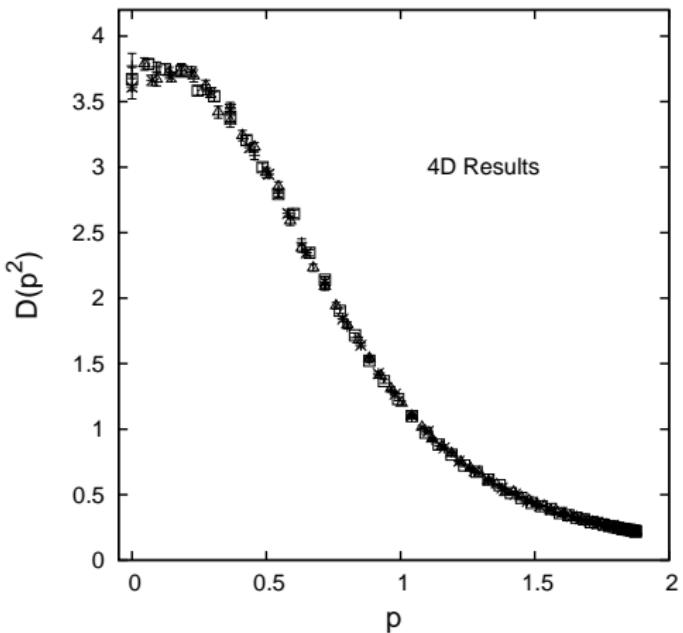
$$G_A(p^2) = \frac{D_A(p^2)}{p^2}$$

$$G_c(p^2) = \frac{D_c(p^2)}{p^2}$$

- lattice simulations** in Landau gauge (gauge field restricted to  $\Omega$ ): decoupling solutions in  $D = 3$  and 4 dimensions [Bogolubsky et al. 2007; Cucchieri, Mendes 2007; Sternbeck et al. 2007], scaling solution in  $D = 2$  dimensions [Maas 2007]

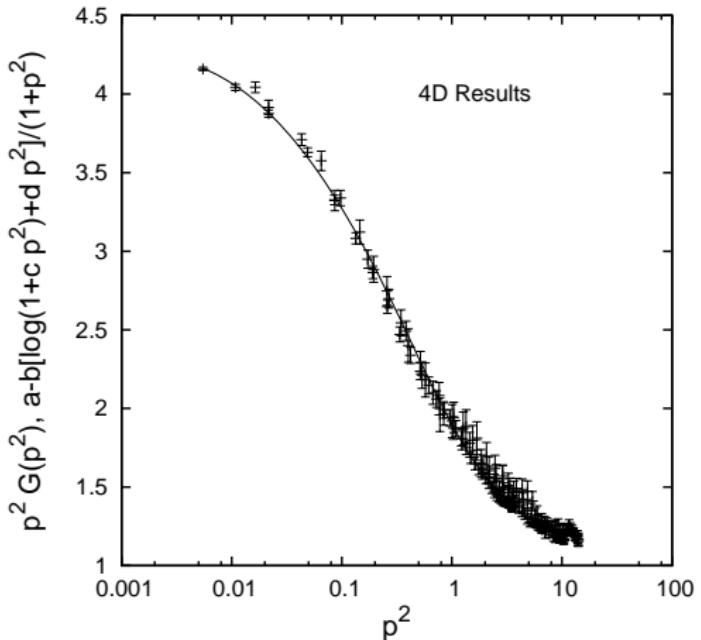
gluon propagator function  $G_A(p^2)$  in  $D = 4$  dimensions

Cucchieri, Mendes 2010



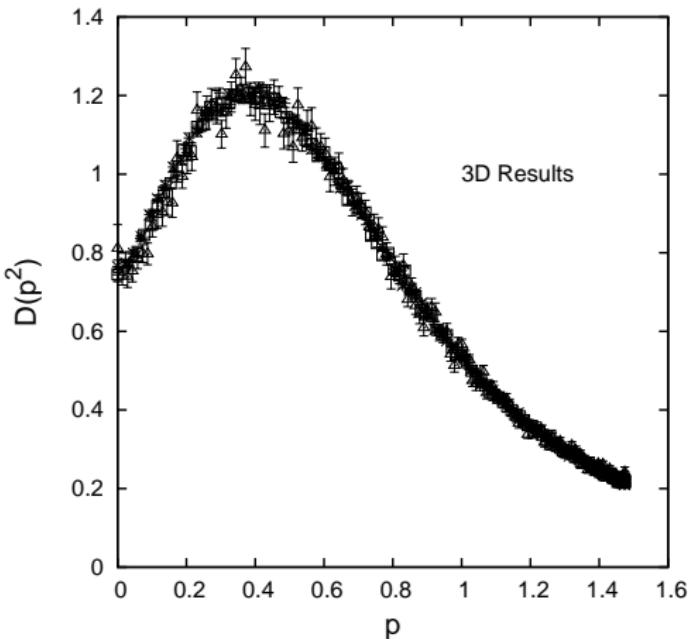
ghost dressing function  $D_c(p^2) = p^2 G_c(p^2)$  in  $D = 4$  dimensions

Cucchieri, Mendes 2010



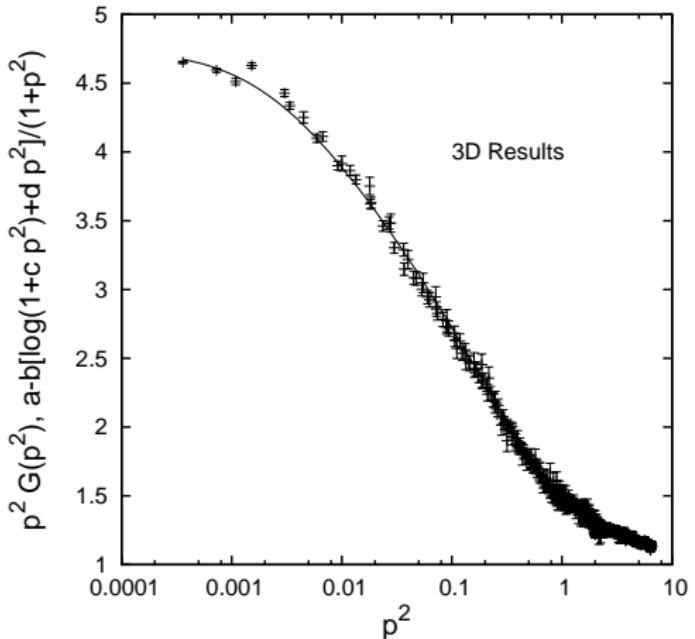
gluon propagator function  $G_A(p^2)$  in  $D = 3$  dimensions

Cucchieri, Mendes 2010



ghost dressing function  $D_c(p^2) = p^2 G_c(p^2)$  in  $D = 3$  dimensions

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## Curci-Ferrari model

- introduce a **mass term** for the gluon field, possible because of the broken BRST symmetry (nevertheless, there is no physical gluonic particle: spectral positivity of the gluon propagator is violated)  $\Rightarrow$  Curci-Ferrari model:

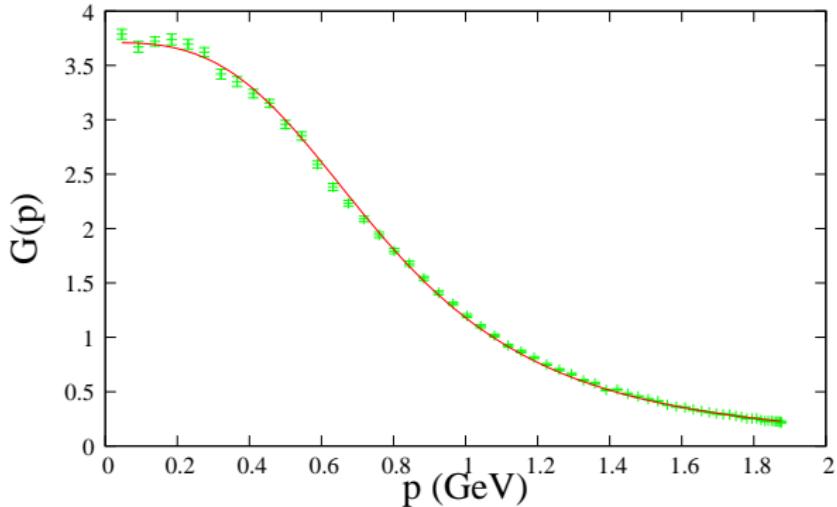
$$S_{CF} = \int d^D x \left( \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} A_\mu^a m^2 A_\mu^a + \partial_\mu \bar{c}^a D_\mu^{ab} c^b + iB^a \partial_\mu A_\mu^a \right)$$

(Nakanishi-Lautrup auxiliary field  $B^a$  to enforce the gauge condition)

- $S_{CF}$  is invariant under a modified BRST transformation (which is *not* nilpotent)
- straightforward **one-loop perturbation theory** qualitatively reproduces the propagators (with decoupling behavior), in the IR regime in  $D = 4$  dimensions even quantitatively [Tissier, Wschebor 2010]

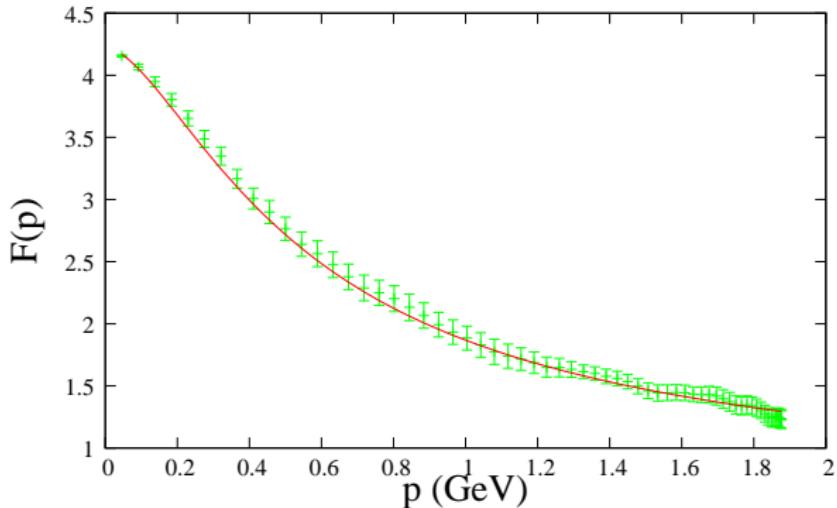
gluon propagator function  $G_A(p^2)$  in  $D = 4$  dimensions

Tissier, Wschebor 2010



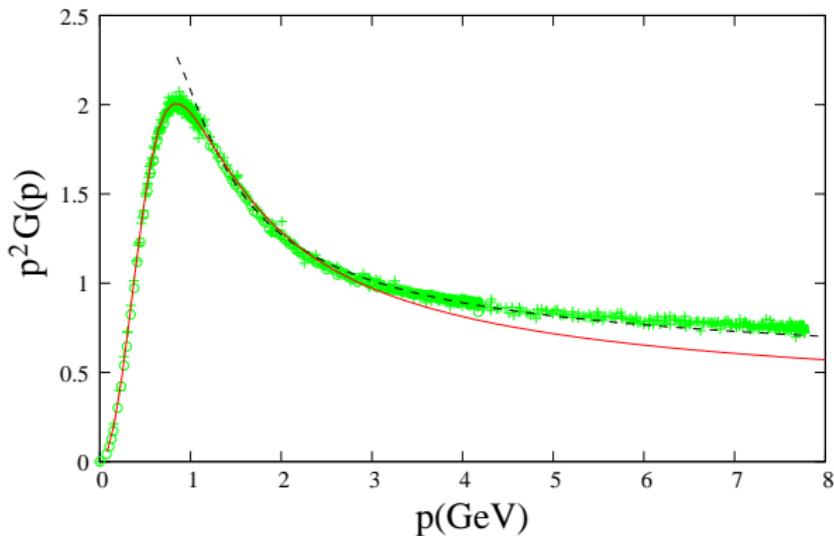
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Tissier, Wschebor 2010



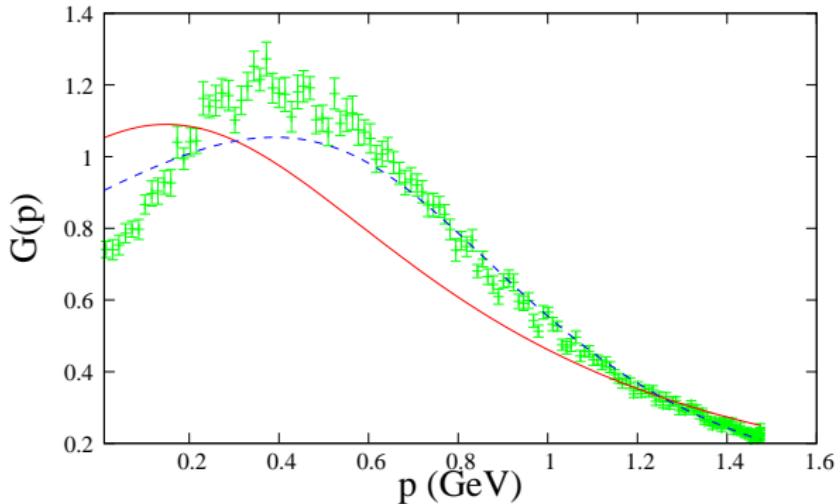
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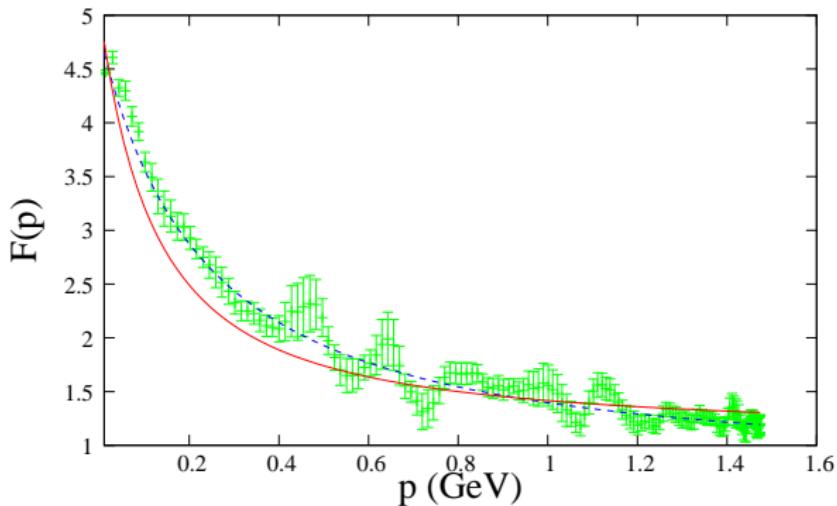
gluon propagator function  $G_A(p^2)$  in  $D = 3$  dimensions

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## Renormalization group improvement

- ▶ solve the Callan-Symanzik equations for the propagators in the renormalization scheme defined by the conditions

$$\Gamma_{AA}^T(p, -q) \Big|_{p^2 = \mu^2} = (\mu^2 + m^2)(2\pi)^D \delta(p - q)$$

$$\Gamma_{AA}^L(p, -q) \Big|_{p^2 = \mu^2} = m^2 (2\pi)^D \delta(p - q)$$

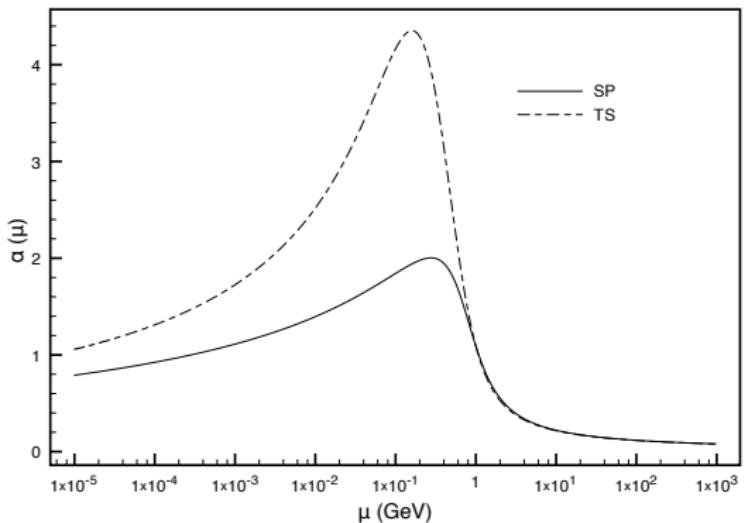
$$\Gamma_{c\bar{c}}(p, -q) \Big|_{p^2 = \mu^2} = \mu^2 (2\pi)^D \delta(p - q)$$

( $\Gamma_{AA}^T$  and  $\Gamma_{AA}^L$  are the transverse and longitudinal parts of the proper gluonic 2-point function)

- ▶ this renormalization scheme maintains the modified BRST symmetry [Tissier, Wschebor 2011: “IR safe” scheme]
- ▶ use the **Taylor scheme** for the definition of the renormalized coupling constant: consider the renormalized ghost-gluon vertex function in the limit of vanishing ghost momentum  $\Rightarrow$  **no** radiative corrections to the vertex function
- ▶ **alternative** scheme: define the renormalized coupling constant from the renormalized ghost-gluon vertex function **at the symmetry point**

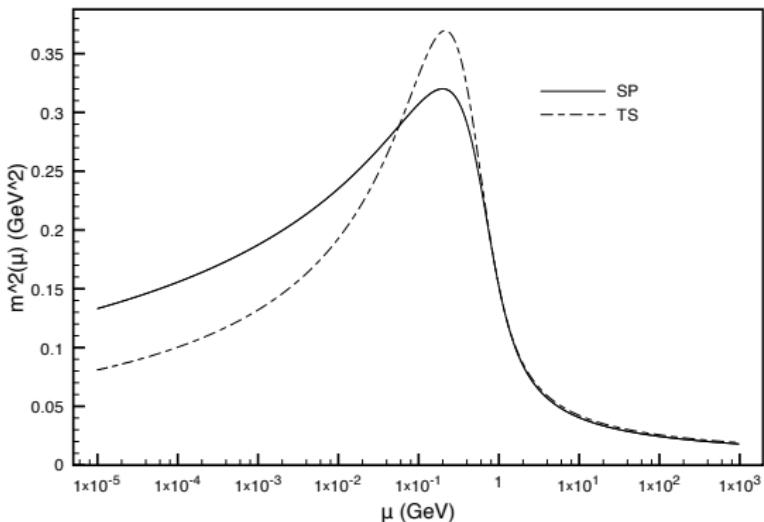
running (strong) fine structure constant  $\alpha(\mu) = g^2(\mu)/4\pi$  in  $D = 4$  dimensions:  
symmetry point (SP) and Taylor scheme (TS)

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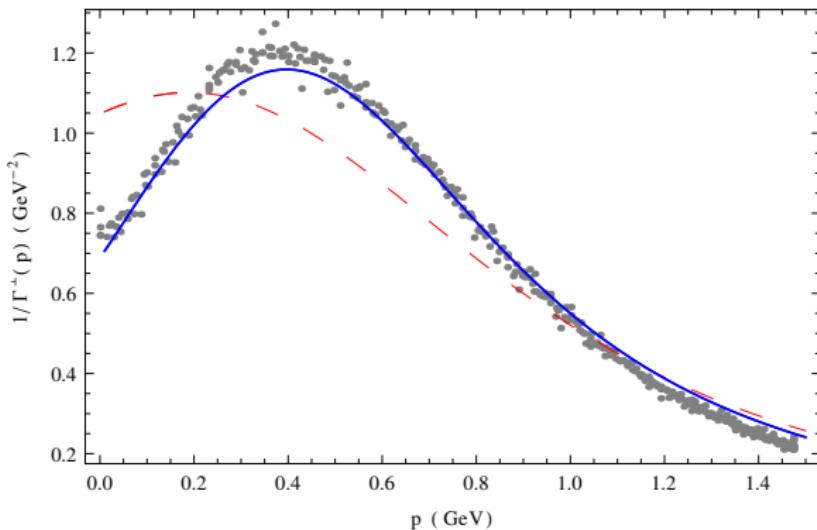
running mass parameter  $m^2(\mu)$  in  $D = 4$  dimensions:  
symmetry point (SP) and Taylor scheme (TS)

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gluon propagator function  $G_A(p^2)$  in  $D = 3$  dimensions:  
solution of the Callan-Symanzik equations in the Taylor scheme (blue line) vs. lattice data

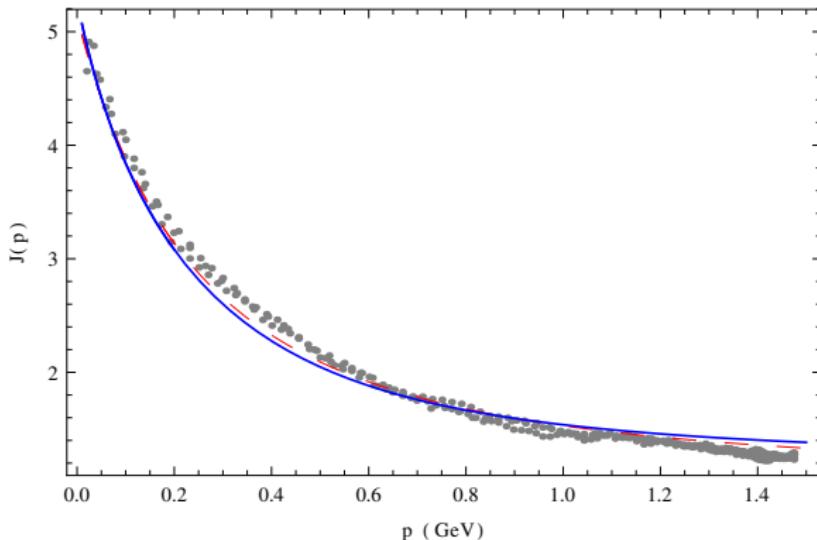
Peláez, Tissier, Wschebor 2013



ghost dressing function  $D_c(p^2) = p^2 G_c(p^2)$  in  $D = 3$  dimensions:

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Peláez, Tissier, Wschebor 2013



## Fitting strategy

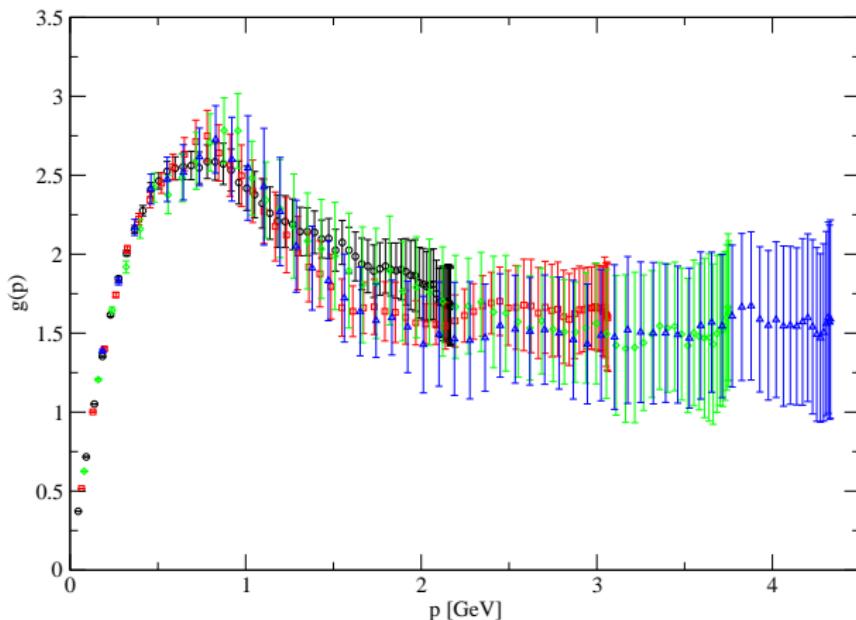
- ▶ (in  $D = 4$  dimensions) adjust the coupling constant in the far UV where the running mass is unimportant
- ▶ use a definition of the running coupling constant in terms of the dressing functions in a (formal) description with zero gluon mass:

$$g_D(p^2) = D_c(p^2) [D_A(p^2)]^{1/2} g_0$$

- ▶ then, adjust the value of the running mass to the IR behavior of the propagators
- ▶ unfortunately, the lattice data in the UV are not sufficiently precise for the determination of the coupling constant; as a compromise, use in addition the gluon propagator function (which is well measured in the UV) to fix the coupling constant

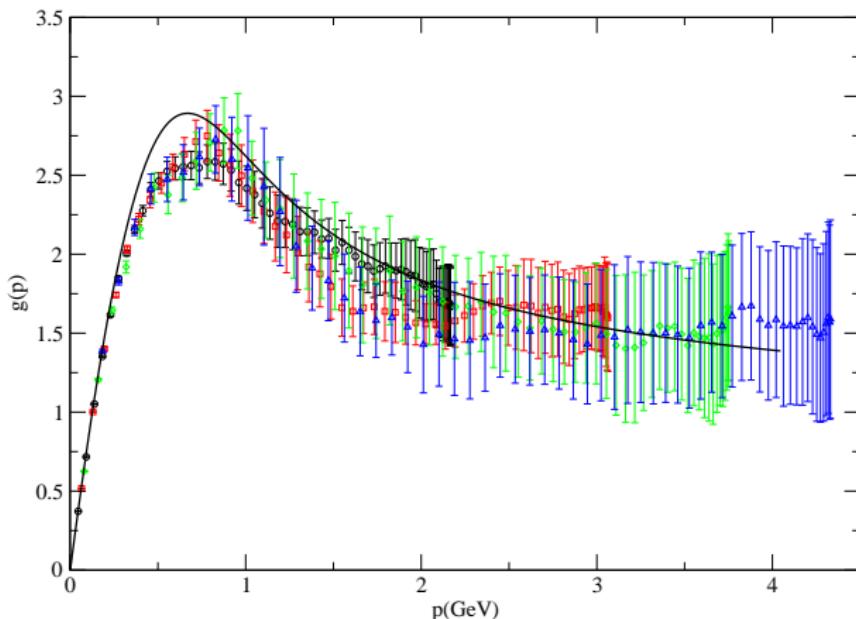
running coupling constant  $g_D(p^2)$  in  $D = 4$  dimensions:  
lattice data from Cucchieri, Mendes 2010

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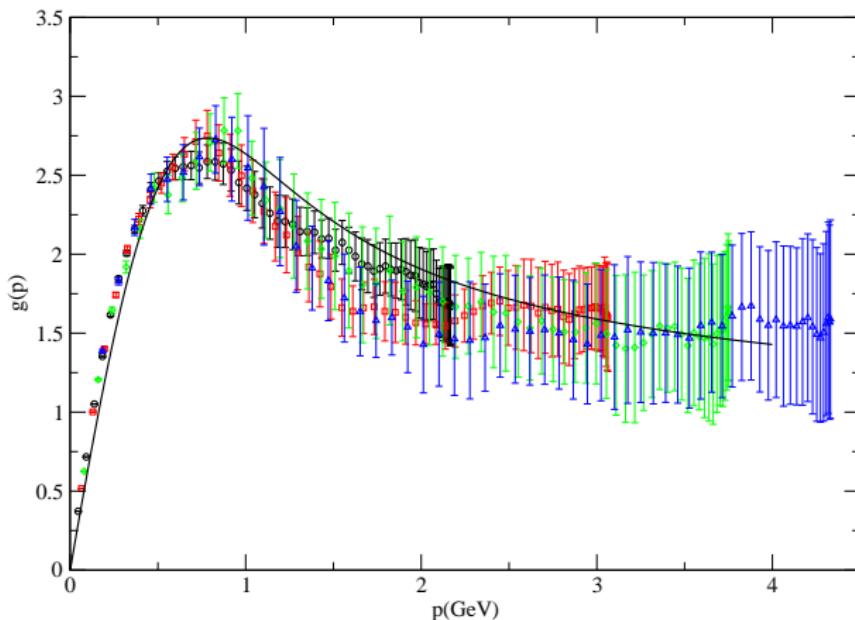
running coupling constant  $g_D(p^2)$  in  $D = 4$  dimensions:  
Taylor scheme

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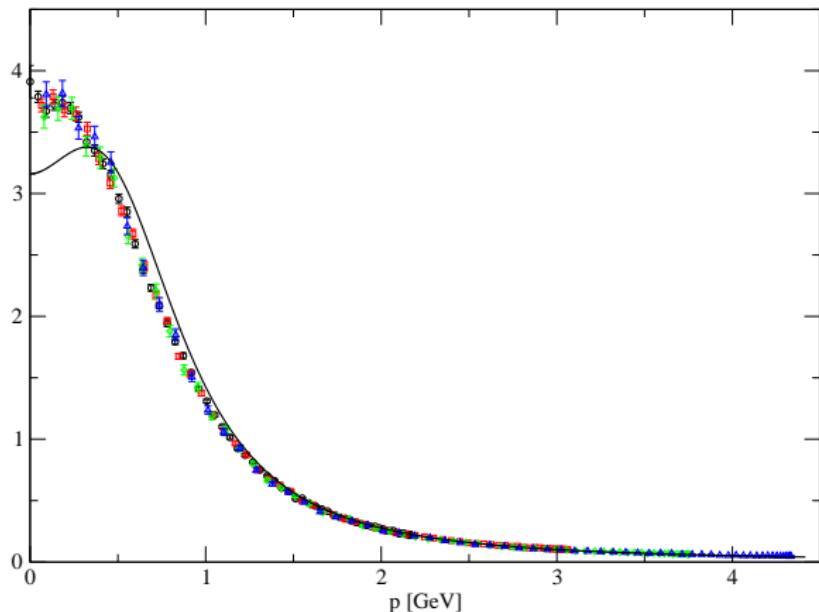
running coupling constant  $g_D(p^2)$  in  $D = 4$  dimensions:  
symmetry point

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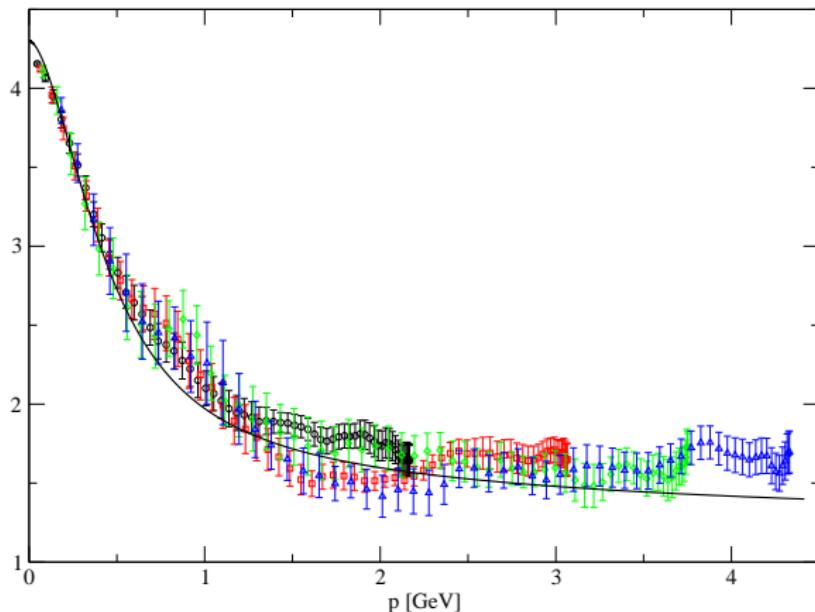
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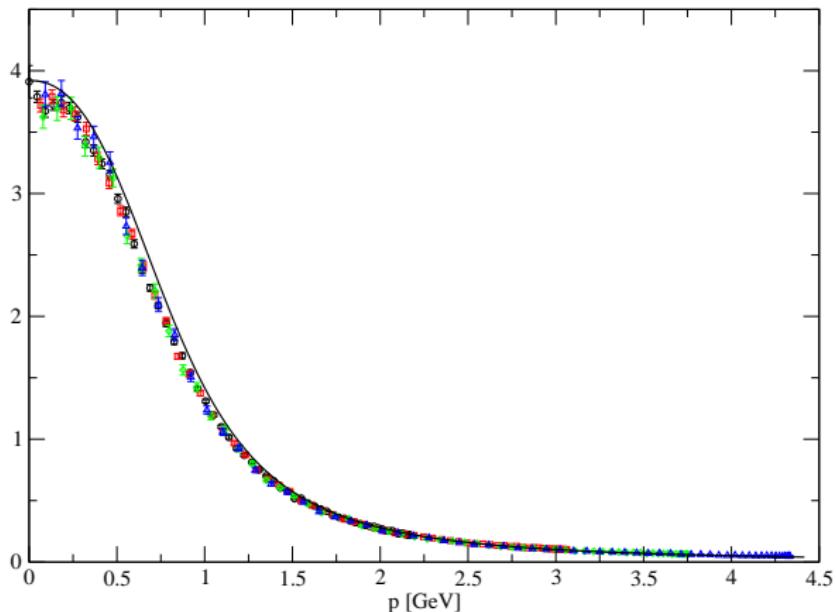
ghost dressing function  $D_c(p^2) = p^2 G_c(p^2)$  in  $D = 4$  dimensions:  
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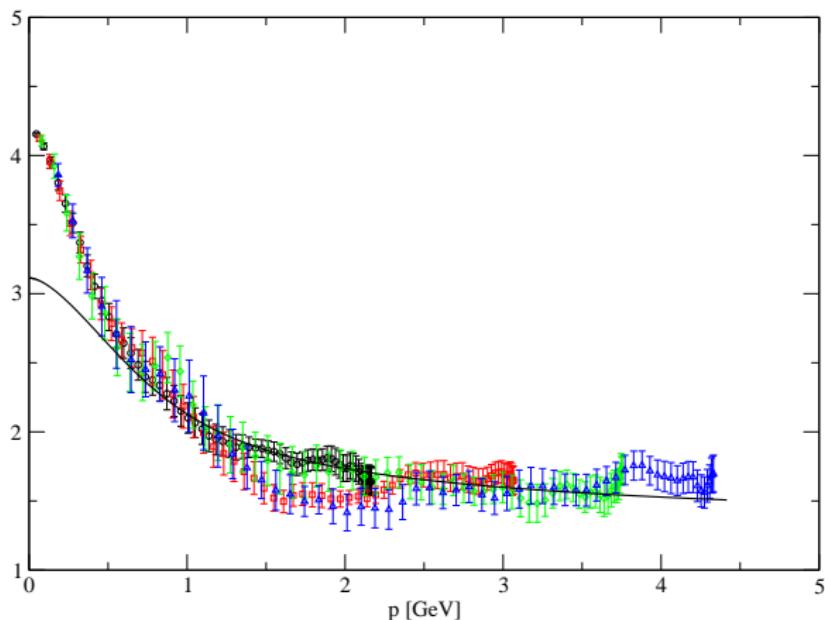
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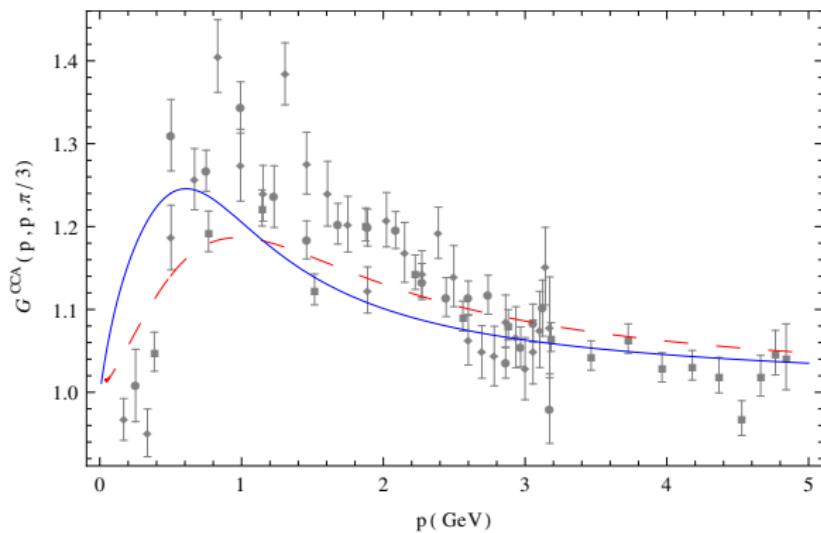
Summary

## Advantages of the Callan-Symanzik approach

- ▶ systematically improvable by going to higher loop orders
- ▶ calculations are analytical, except for the integration of the Callan-Symanzik equations
- ▶ radiative corrections to the vertex functions can be calculated (as compared to the construction of *ansätze* for the structure of the vertex functions necessary in Dyson-Schwinger equations)
- ▶ **further improvement:** treat the ghost-gluon and the 3- and 4-gluon coupling constants as mutually independent because of the break-down of BRST symmetry in the IR; technically, a fine tuning is necessary in order to recover the symmetry in the UV  
[P. Dall'Olio, AW, work in progress]

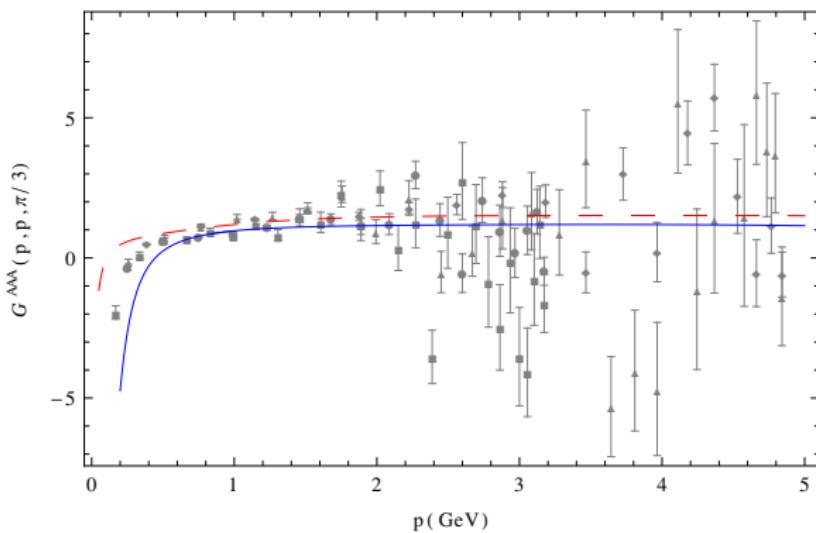
ghost-gluon vertex function in  $D = 3$  dimensions (transverse part, all momenta squared equal): Curci-Ferrari model with Taylor scheme (blue line) vs. lattice data

Peláez, Tissier, Wschebor 2013  
(lattice data from Cucchieri, Maas, Mendes 2008)



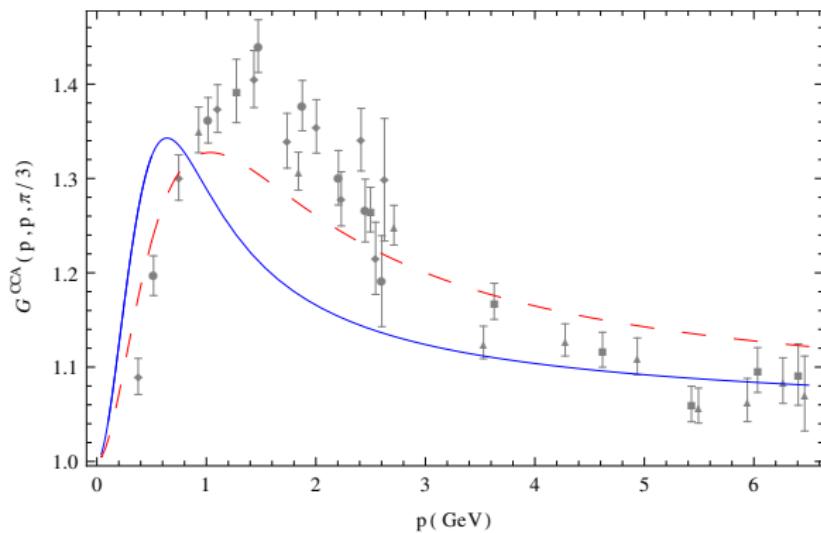
3-gluon vertex function in  $D = 3$  dimensions (completely transverse part, all momenta squared equal): Curci-Ferrari model with Taylor scheme (blue line) vs. lattice data

Peláez, Tissier, Wschebor 2013  
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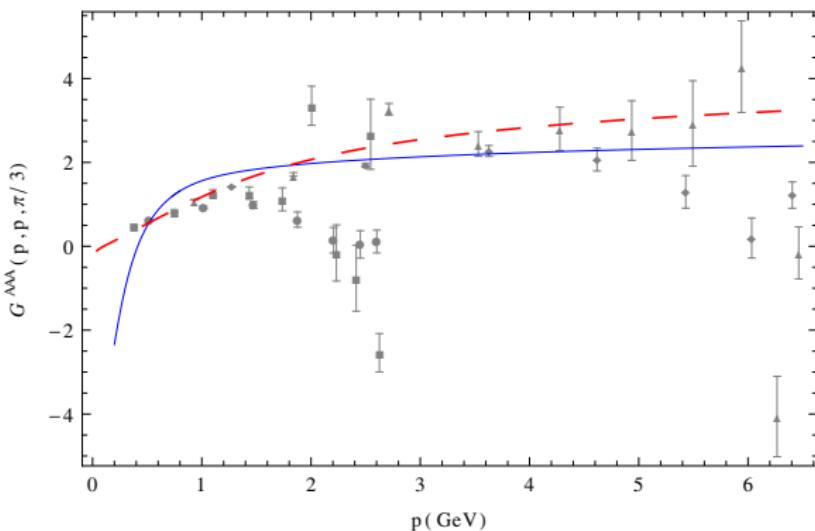
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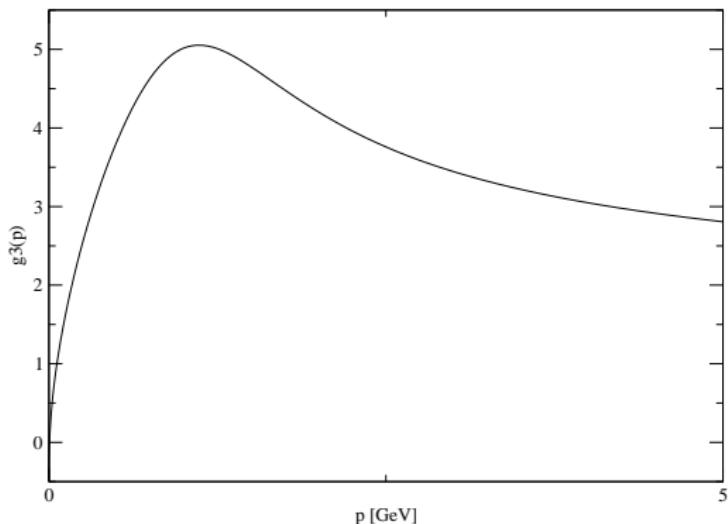
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3-gluon vertex function in  $D = 4$  dimensions (completely transverse part, all momenta squared equal): two independent coupling constants defined at symmetry points

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## Summary: Yang-Mills theory in Landau gauge

- ▶ restricting the integral over the gauge field to the Gribov region  $\Omega$  breaks the BRST symmetry of the theory (after Landau gauge fixing) and generates a mass term for the gluon field
- ▶ straightforward perturbation theory including a gluon mass term reproduces the lattice results for the gluon and ghost propagators in  $D = 3$  and  $4$  dimensions in a qualitative, in parts even quantitative, way: no Landau pole
- ▶ renormalization group improvement: Callan-Symanzik equations in “IR safe” renormalization schemes, strong scheme dependence (Taylor limit vs. symmetry point for the ghost-gluon coupling)
- ▶ in principle, an optimized renormalization scheme can be determined by comparing with the Landau gauge lattice data
- ▶ radiative corrections to the vertex functions can be calculated systematically, important advantage of this approach over Dyson-Schwinger equations
- ▶ BRST symmetry can be completely broken in the IR and recovered in the UV

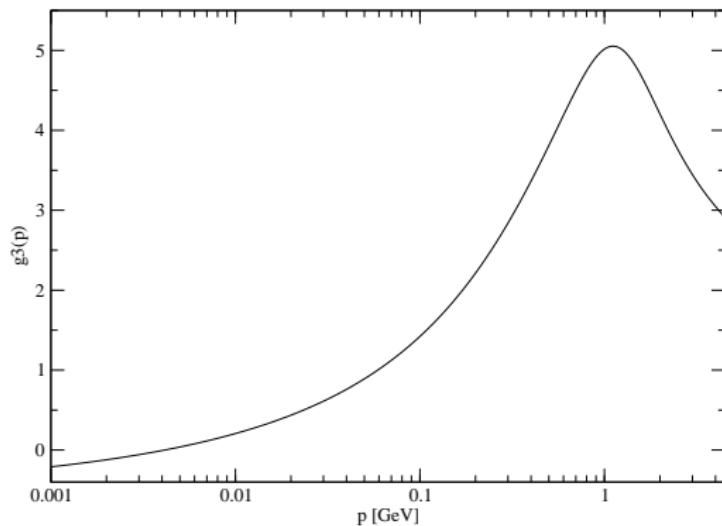
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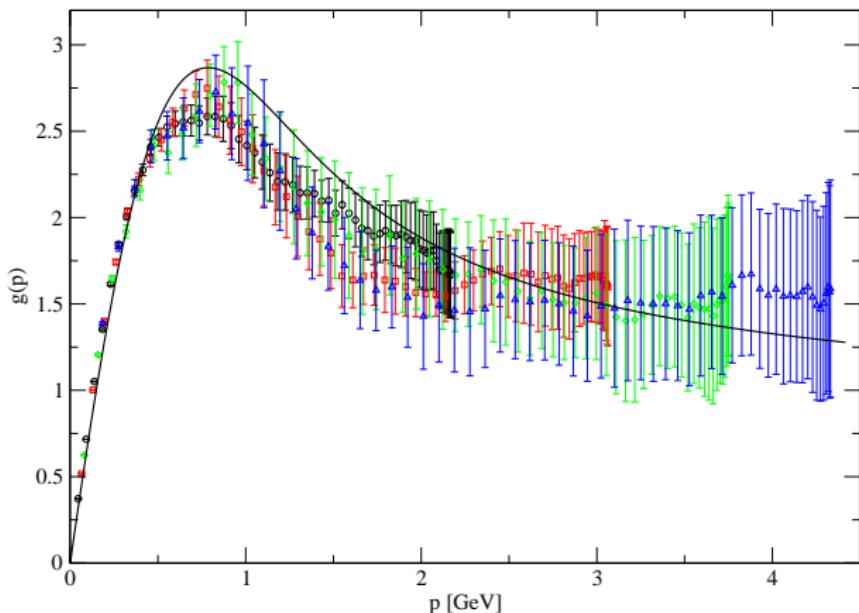
3-gluon vertex function in  $D = 4$  dimensions (completely transverse part, all momenta squared equal): two independent coupling constants defined at symmetry points  
(logarithmic scale)

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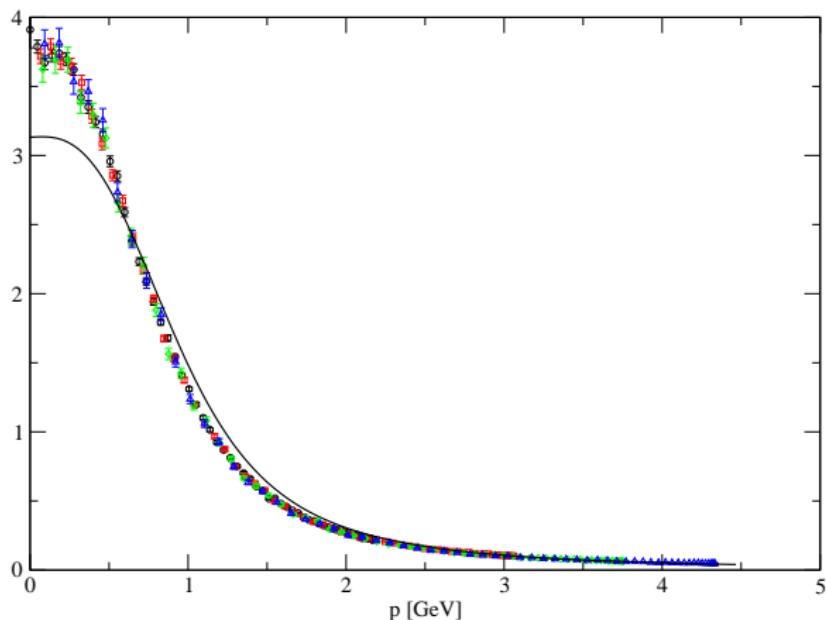
running coupling constant  $g_D(p^2)$  in  $D = 4$  dimensions:  
two independent coupling constants defined at symmetry points

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ghost dressing function  $D_c(p^2) = p^2 G_c(p^2)$  in  $D = 4$  dimensions:  
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