

Inhomogeneous chiral & pion condensates - from a generalized Ginzburg-Landau approach -

Hiroaki Abuki (Keio U.)

References:

H. Abuki, PRD87 (2013), PLB728 (2014)

18 June 2014 QCD@work2014 Giovinazzo (Bari, Italy)

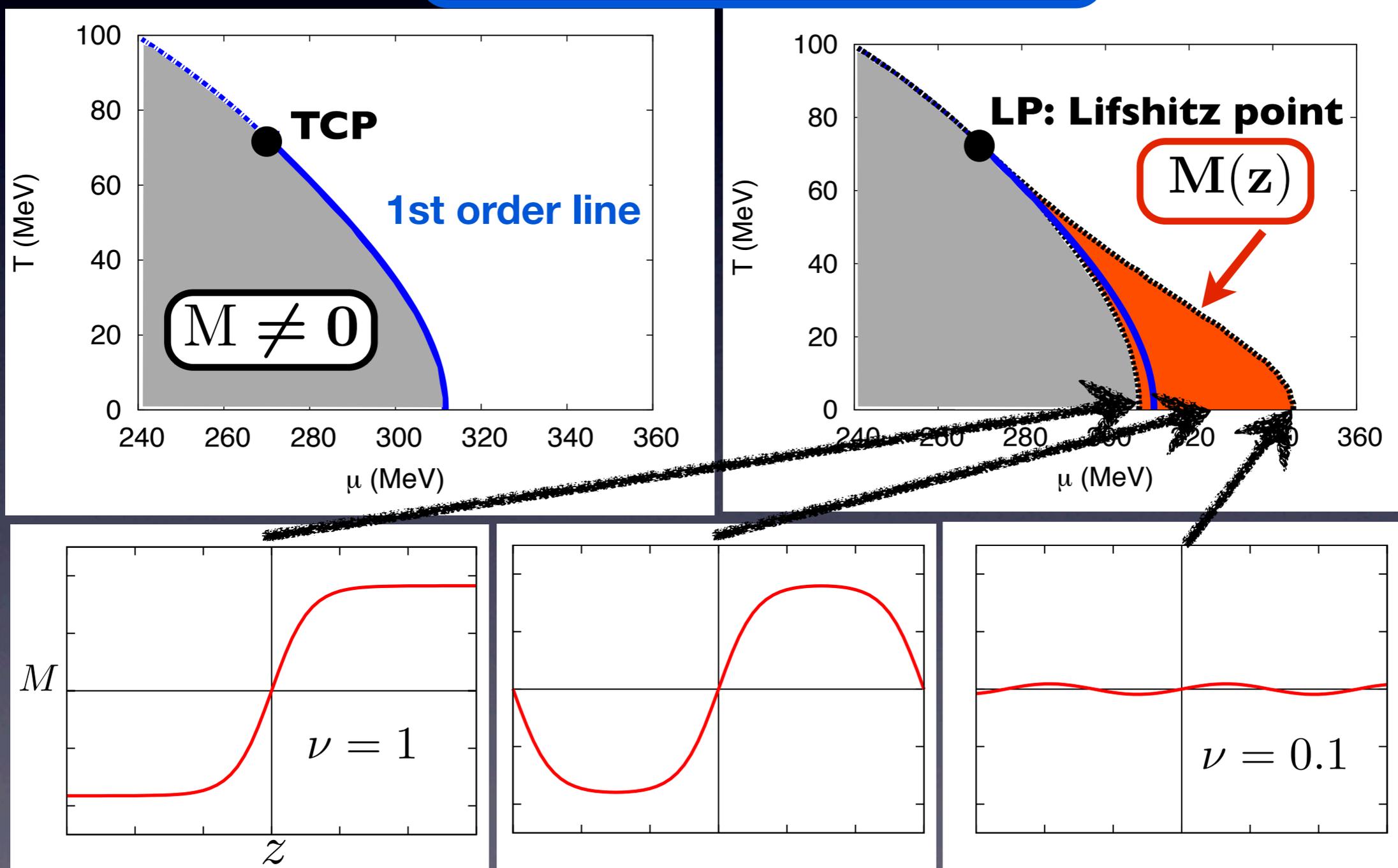
Plan

- What is chiral crystals?
- generalized Ginzburg-Landau (gGL) expansion and the effect of quark mass
- Isospin mismatch: Charged pion crystal
- Summary and Outlook

Solitonic Chiral Crystal: RKC

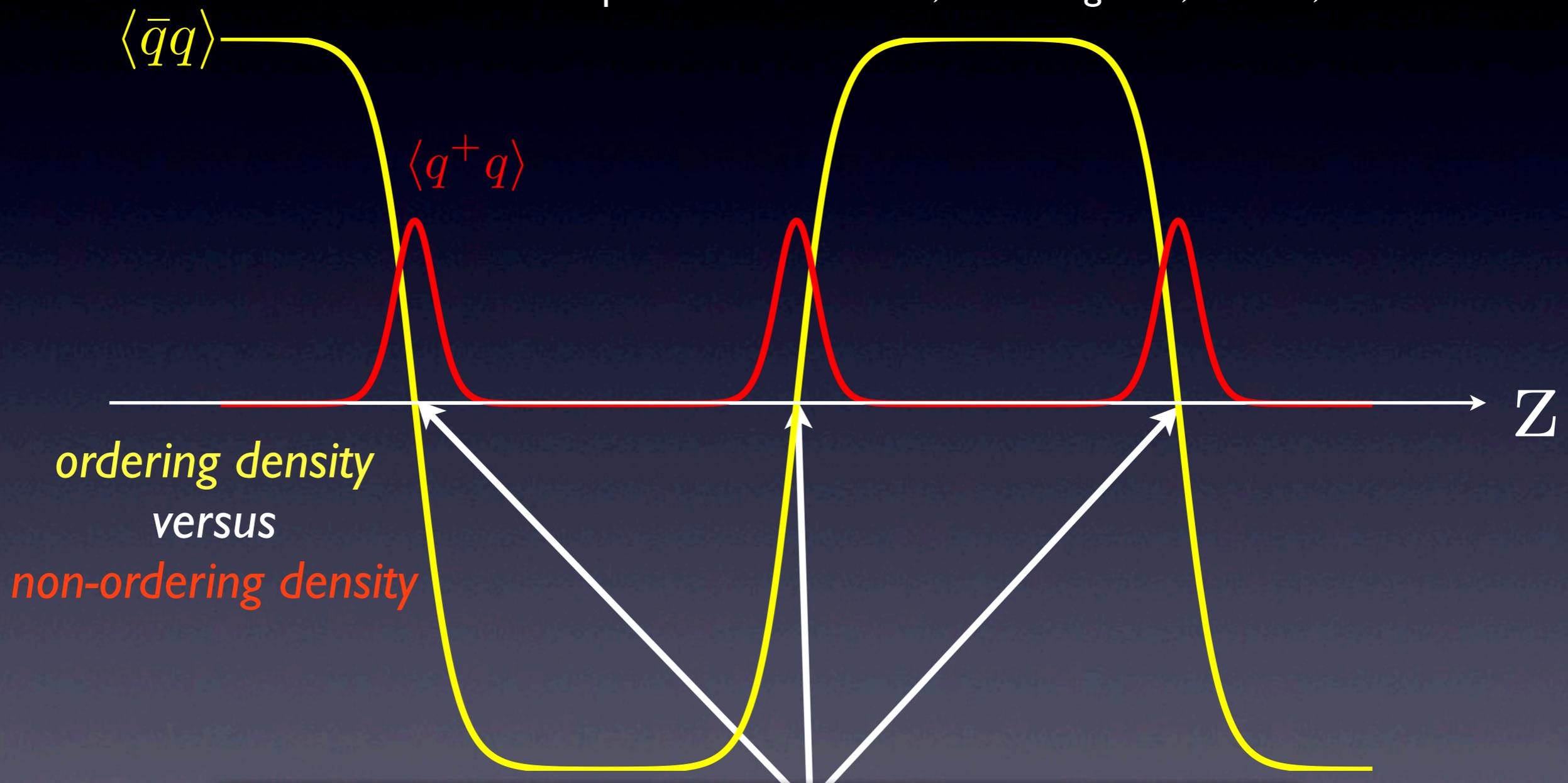
M. Thies, J. Phys. A 2006
D. Nickel, PRL 09, PRD 09

$$M_{sn}(z) = \sqrt{\nu} q \operatorname{sn}(qz, \nu)$$



PS with a microscale spatial order

c.f. for explicit demonstration, see Carignano, Buballa, Nickel 2010



Periodically placed normal phase domains
accommodating for a large q - $qbar$ imbalance

When may crystals develop?

	Chiral Crystal	FFLO	CSC Crystal
<i>order density</i>	$\langle \bar{q}q \rangle$	$\langle \psi_{\uparrow} \psi_{\downarrow} \rangle$	$\langle q_A q_B \rangle$
<i>non-ordering density</i>	$\langle q^{\dagger} q \rangle$ $n_q - n_{\bar{q}}$	$n_{\uparrow} - n_{\downarrow}$	$n_A - n_B$
<i>stress force</i>	μ_q	\hbar $\mu_{\uparrow} - \mu_{\downarrow}$	$\mu_A - \mu_B$

for CSC and CSC crystals, see Casalbuoni, Mannarelli, Ruggieri et al., arXiv:1302.4264
Alford, Schmitt, Rajagopal, Schafer, RMP 80 (2008)

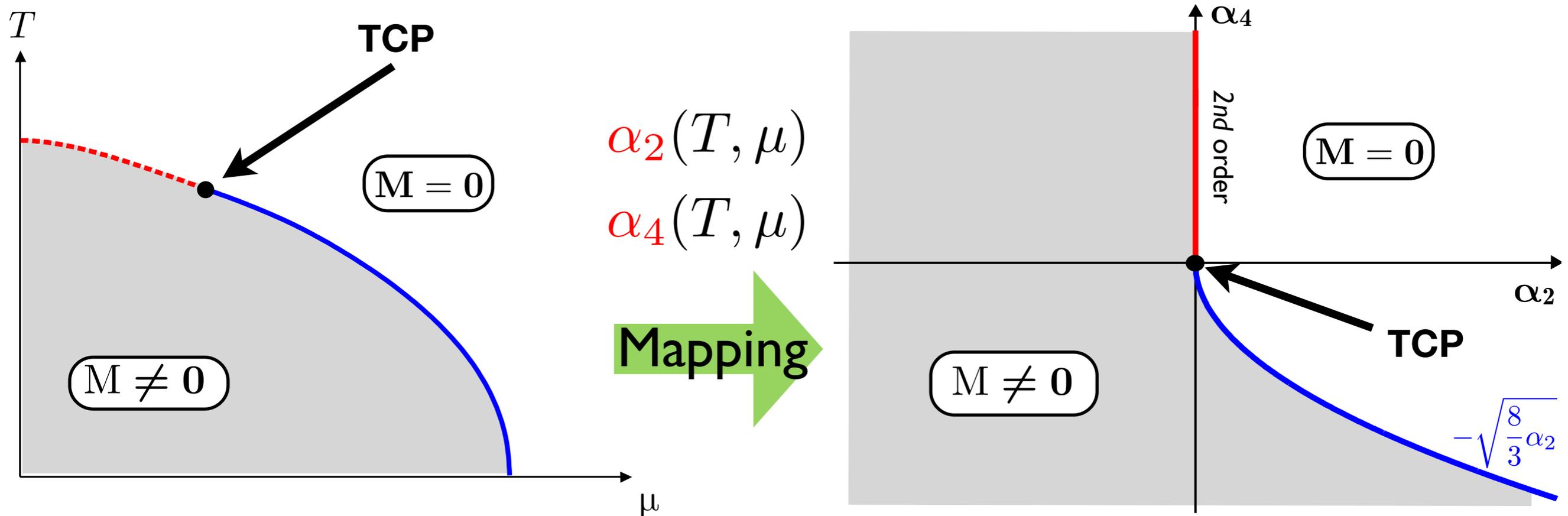
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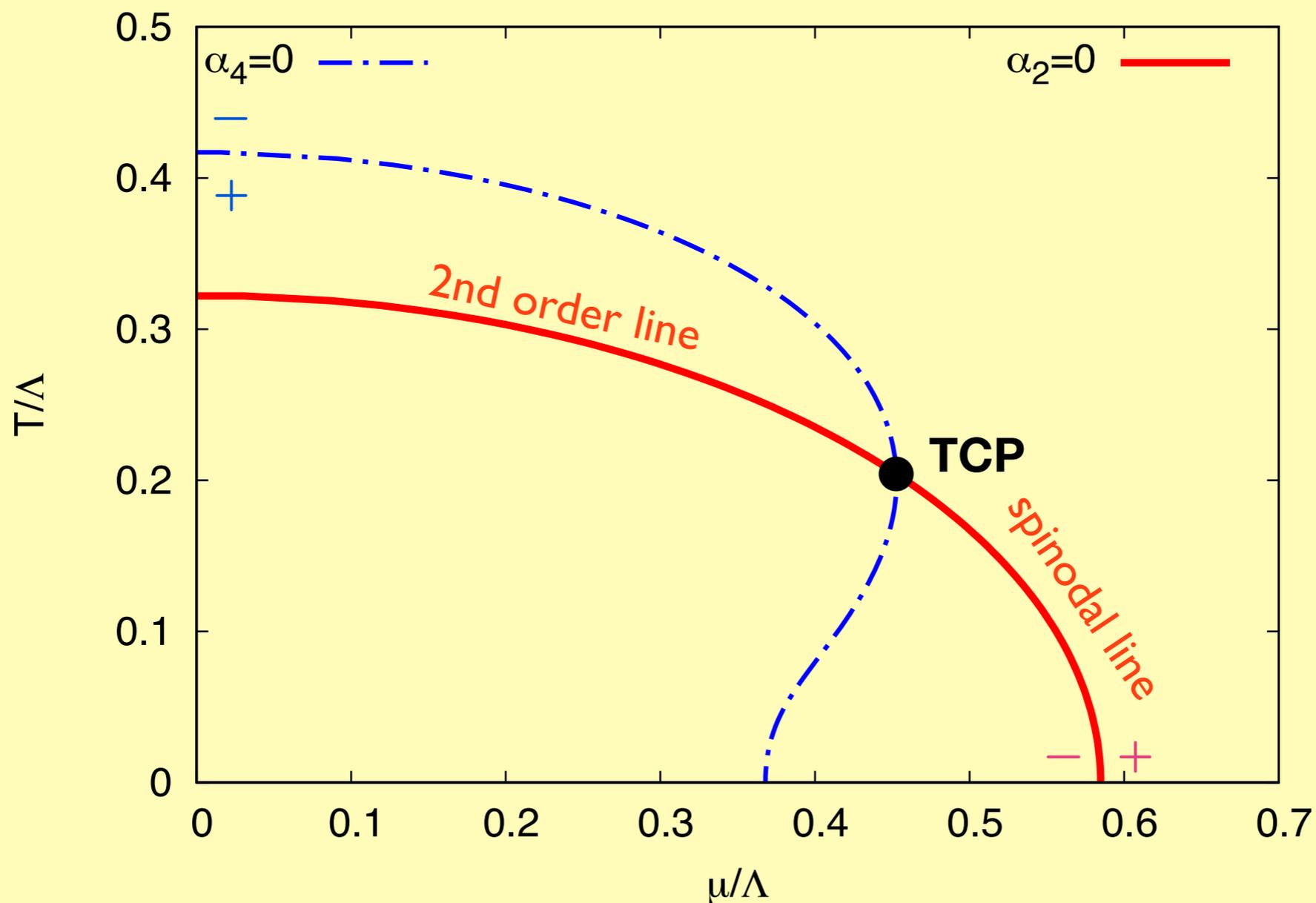
Ginzburg-Landau (GL) approach (homogeneous)

Minimal GL to describe the tricritical point (TCP)

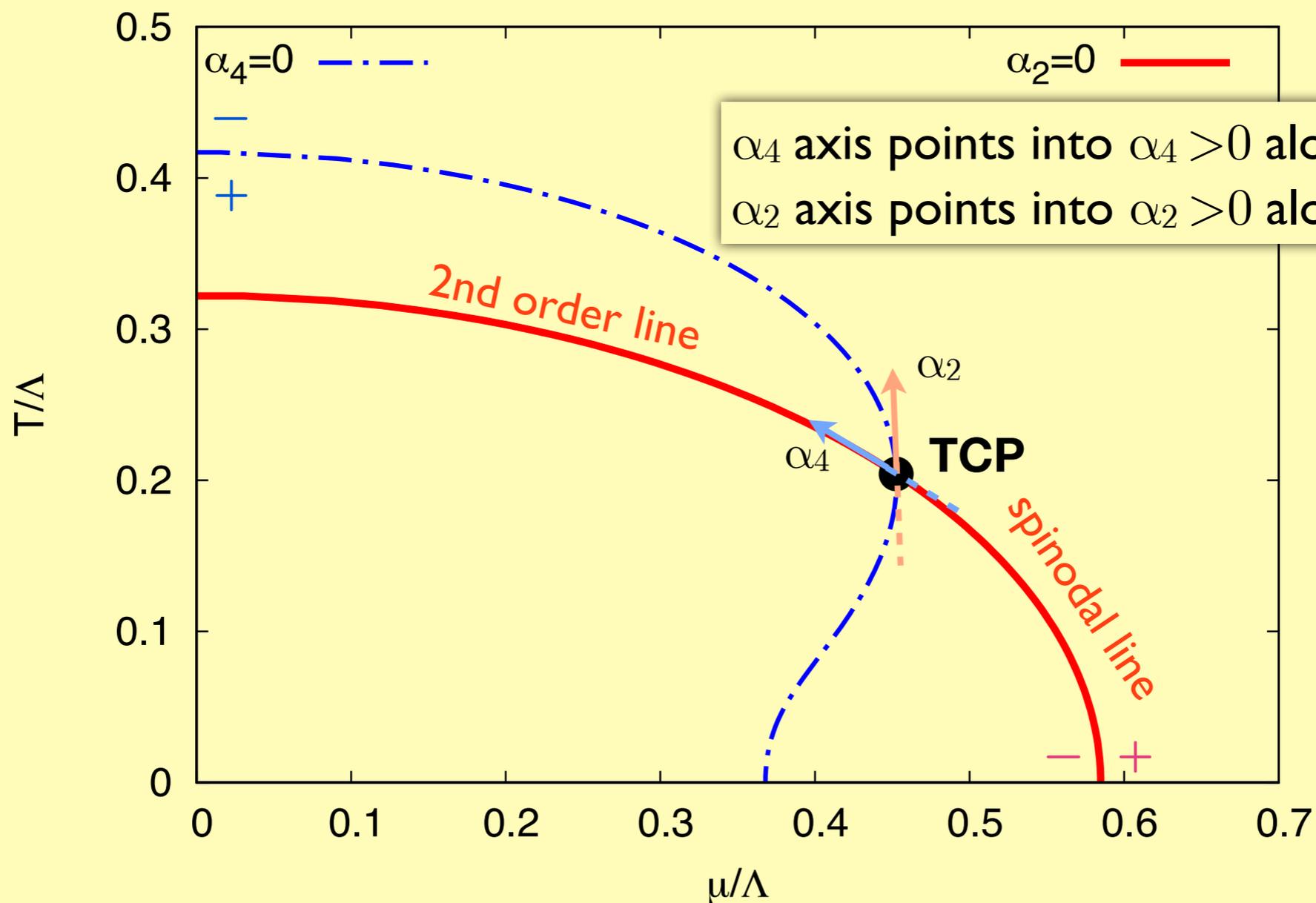
$$\Omega_{\text{GL}} = \frac{\alpha_2}{2} \sigma(\mathbf{x})^2 + \frac{\alpha_4}{4} \sigma(\mathbf{x})^4 + \frac{1}{6} \sigma(\mathbf{x})^6$$



α_2, α_4 in NJL model: How do they map onto (μ, T) ?



α_2, α_4 in NJL model: How do they map onto (μ, T) ?



generalized Ginzburg-Landau (gGL) expansion

D. Nickel, PRL09

mass: *External field* : $O(4) \mapsto O(3)$ isospin

$$\Omega_{\text{GL}} = \boxed{-h\sigma(\mathbf{x})} + \frac{\alpha_2}{2}\sigma(\mathbf{x})^2 \quad \text{gradient terms}$$

$$+ \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 \quad \boxed{+ \frac{\alpha_{4b}}{4}(\nabla\sigma(\mathbf{x}))^2}$$

$$+ \frac{\alpha_6}{6}\sigma(\mathbf{x})^6 \quad \boxed{+ \frac{\alpha_{6b}}{6}\sigma^2(\nabla\sigma)^2 + \frac{\alpha_{6c}}{6}(\Delta\sigma)^2}$$

$\alpha_{4b} = 0$: Lifshitz Point (LP)

$h = \alpha_2 = \alpha_4 = 0$: Tricritical Point (TCP)

$h = \alpha_{4b} = \alpha_2 = \alpha_4 = 0$: Lifshitz TCP (LTCP)

Ansatz for 1D crystal

M. Thies, J. Phys. A 2006
D. Nickel, PRL 09, PRD 09

Saddle point equation (EL eq.):

H. Abuki, PLB (2014)

$$h = \sigma^5 + \alpha_4 \sigma^3 + \alpha_2 \sigma + \frac{1}{6} \sigma^{(4)} - \frac{5}{3} (\sigma^2 \sigma'' + \sigma (\sigma')^2) - \frac{1}{2} \alpha_4 \sigma''$$

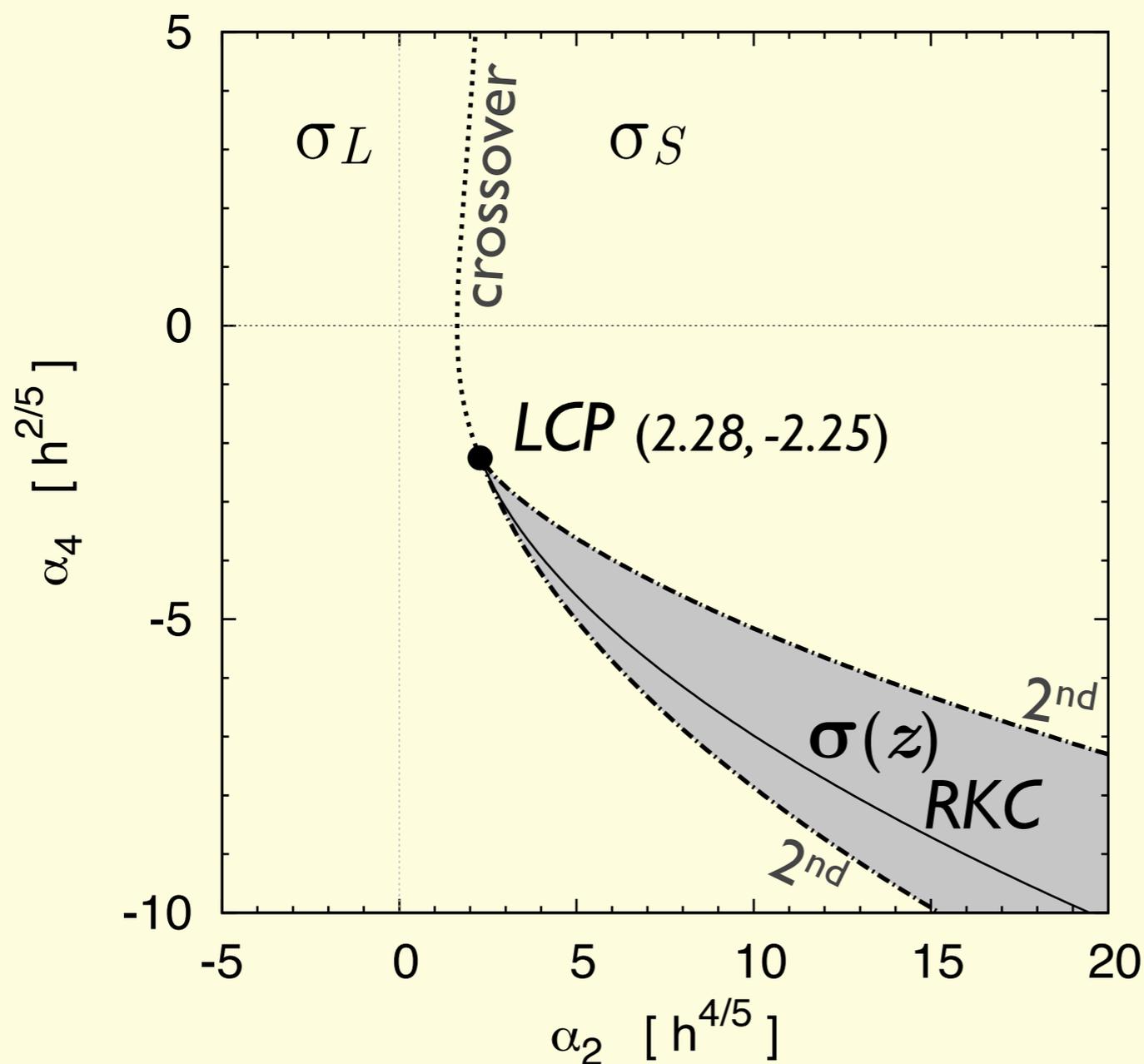
Extended solitonic chiral condensate (RKC)

$$\sigma(z) = k \operatorname{sn}(b, \nu) \left(\nu^2 \operatorname{sn}(kz, \nu) \operatorname{sn}(kz + b, \nu) + \frac{\operatorname{cn}(b, \nu) \operatorname{dn}(b, \nu)}{\operatorname{sn}^2(b, \nu)} \right)$$

In the chiral limit $b_\nu \rightarrow K(\nu)$

$$\sigma(z) = k \nu^2 \frac{\operatorname{sn}(kz, \nu) \operatorname{cn}(kz, \nu)}{\operatorname{dn}(kz, \nu)} \left(= k' \nu' \operatorname{sn}(k' z, \nu') \right)$$

Effect of h @ $\mu_I = 0$

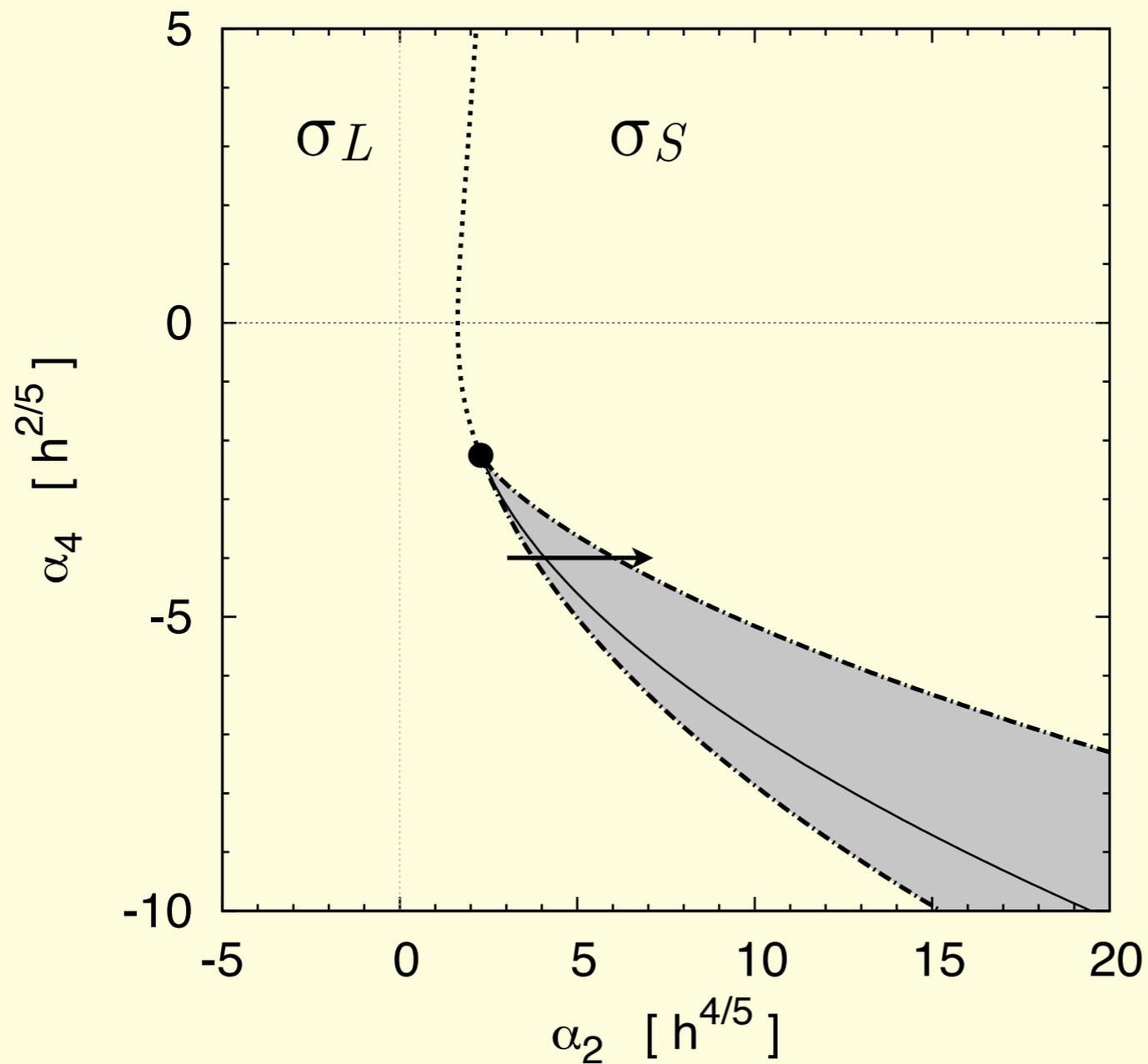


(1) *CP (2.28, -2.25)*
coincides with
Lifshitz point

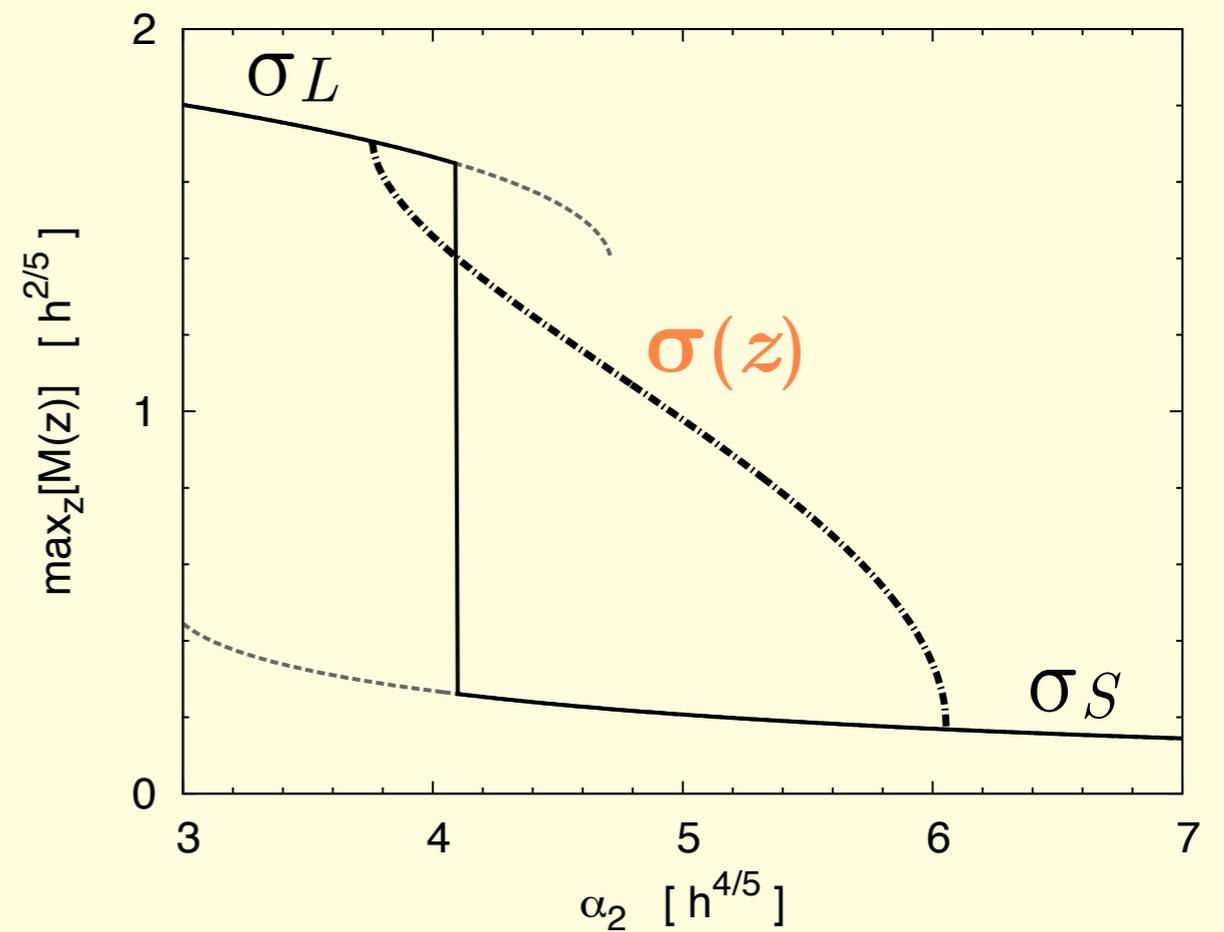
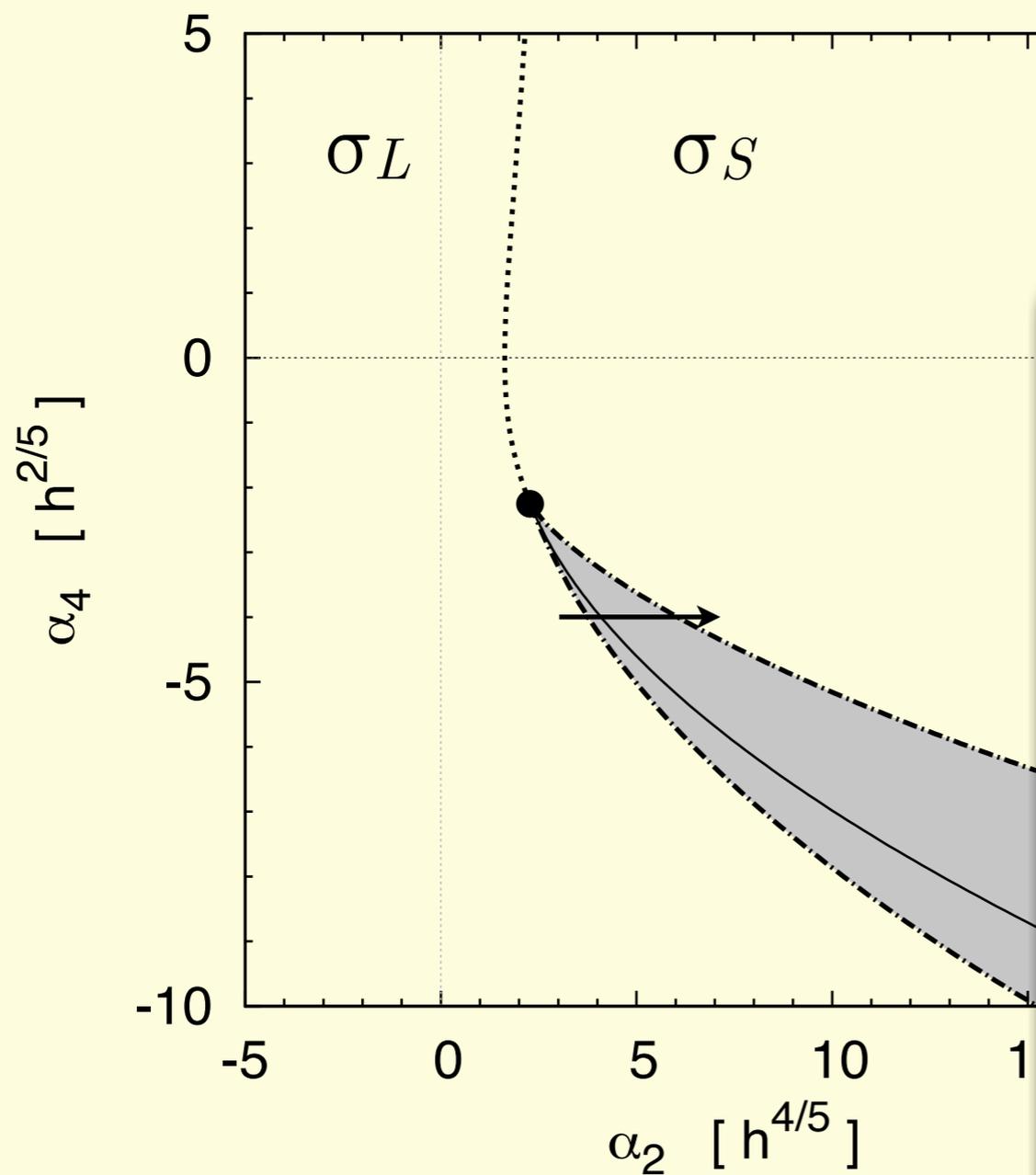
LCP = Lifshitz CP

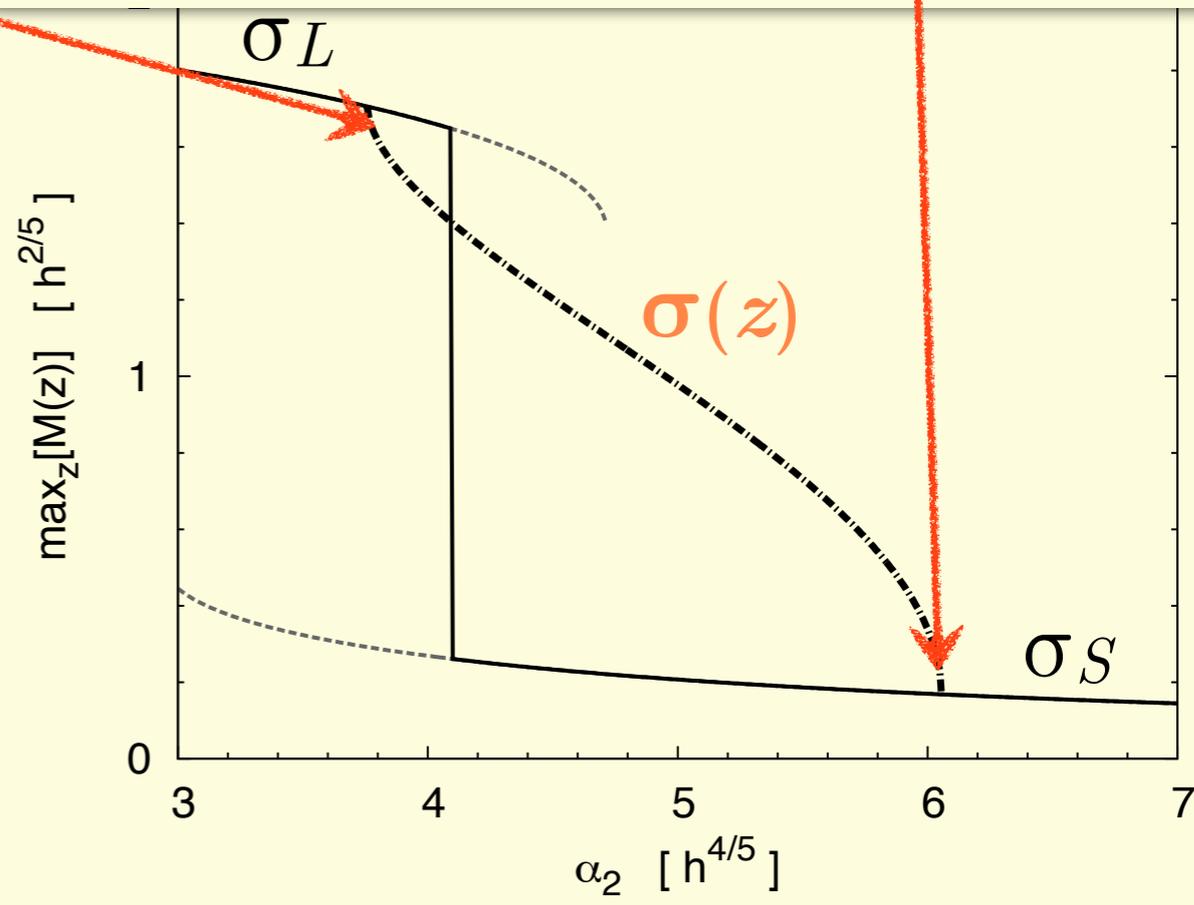
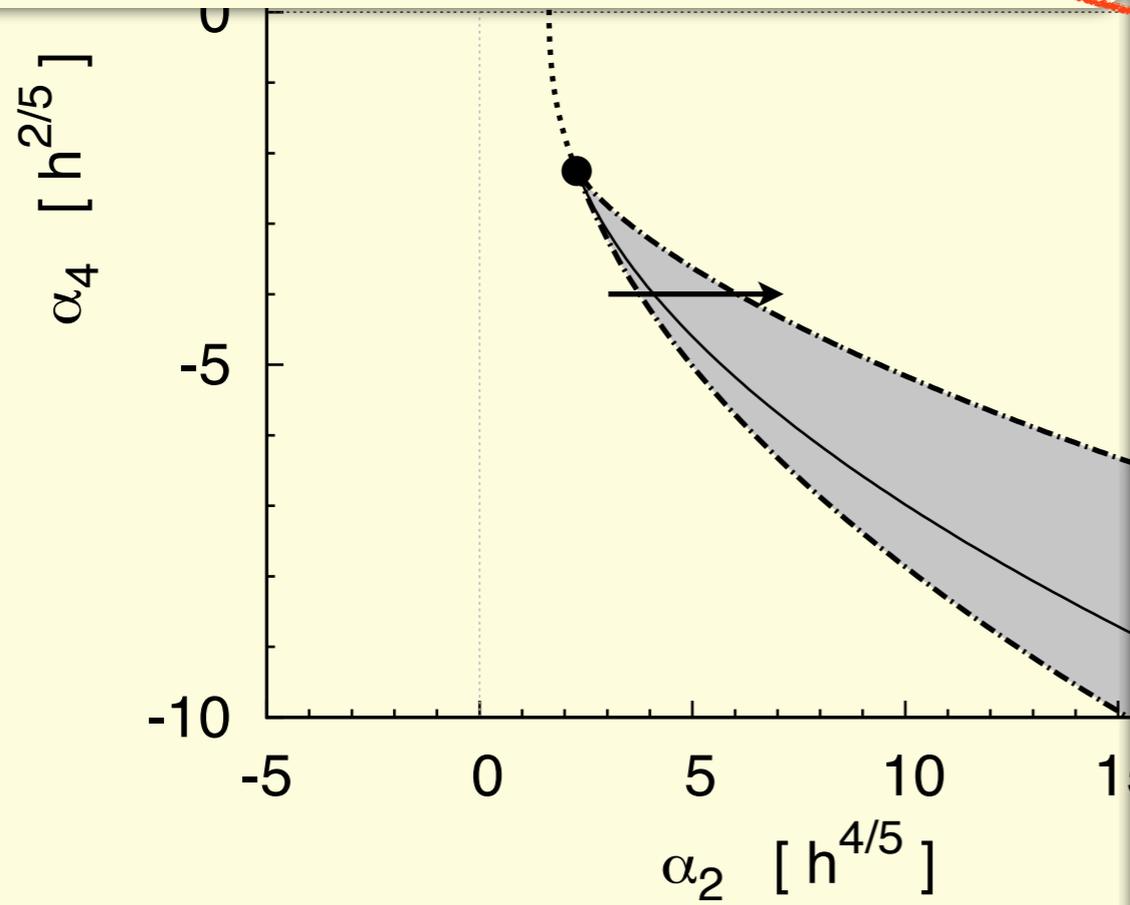
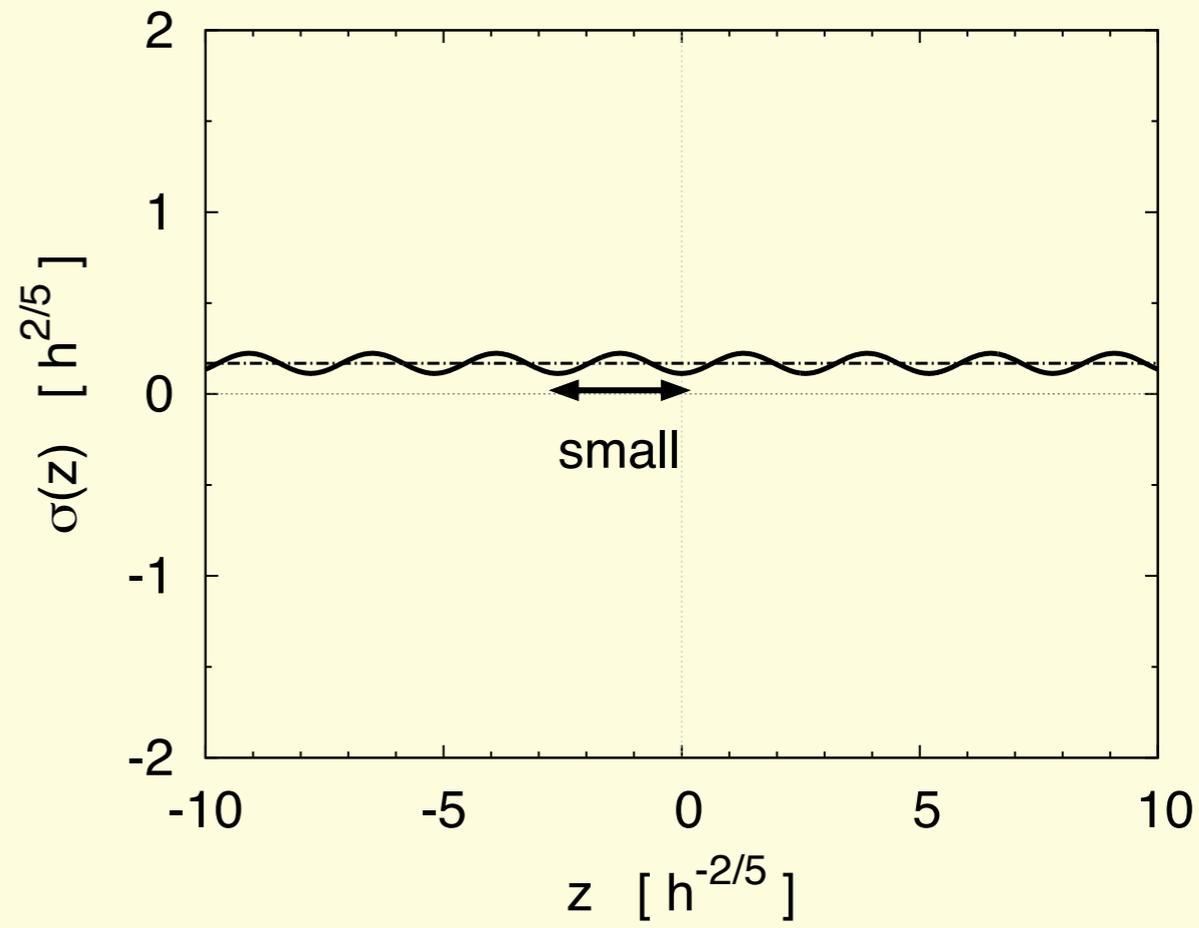
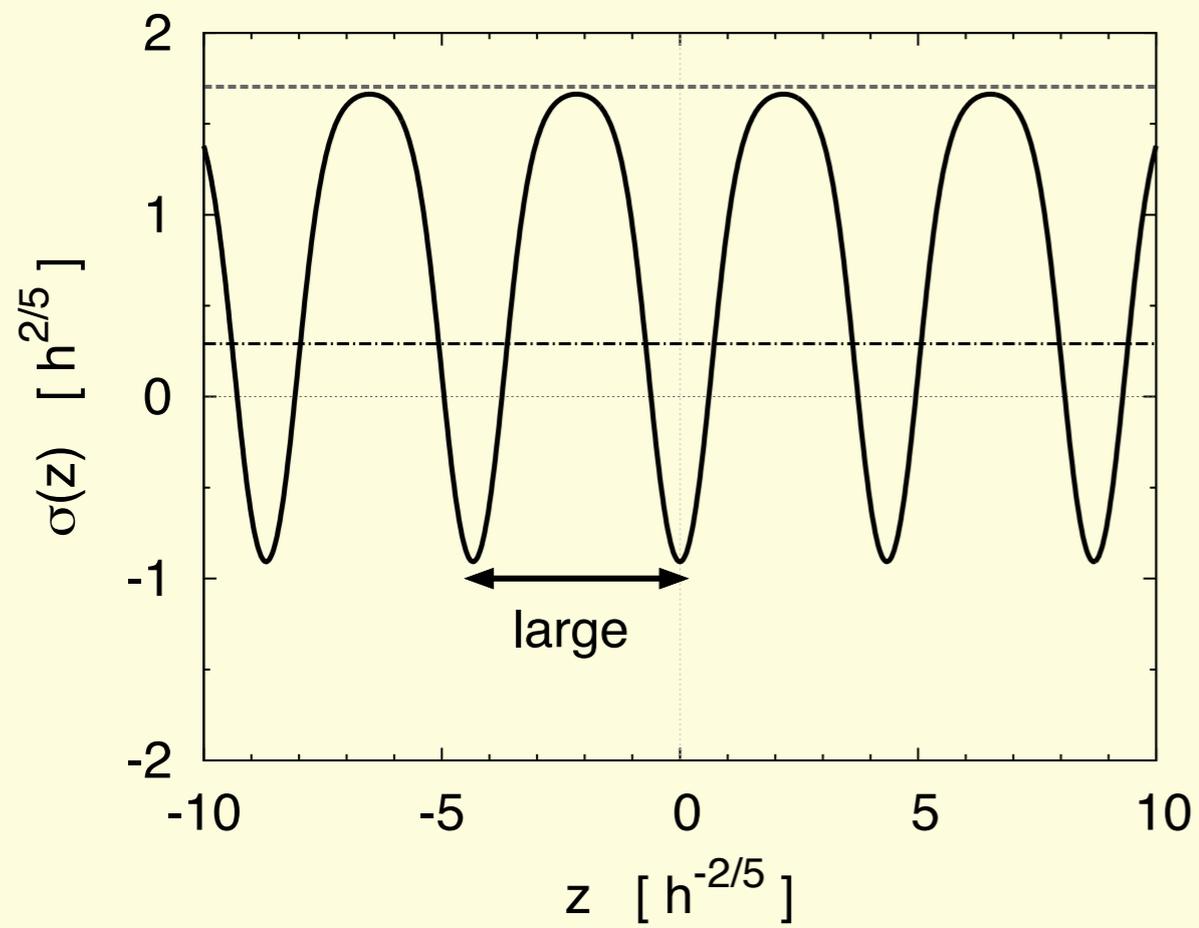
(2) *RKC stabilized:*
Two critical lines
enclosing *RKC* are
both of 2nd order

Profile of RKC phase



Profile of RKC phase



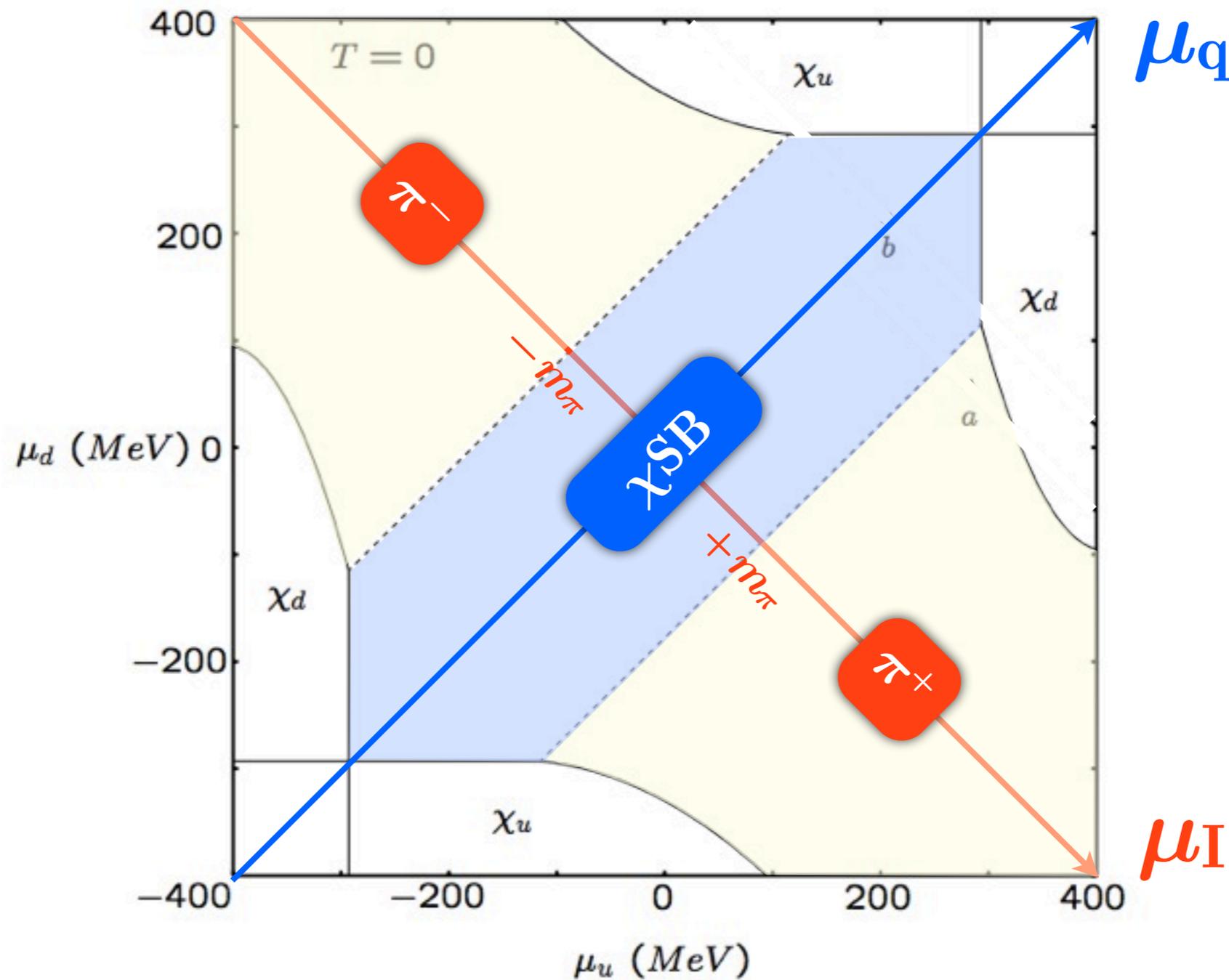


Plan

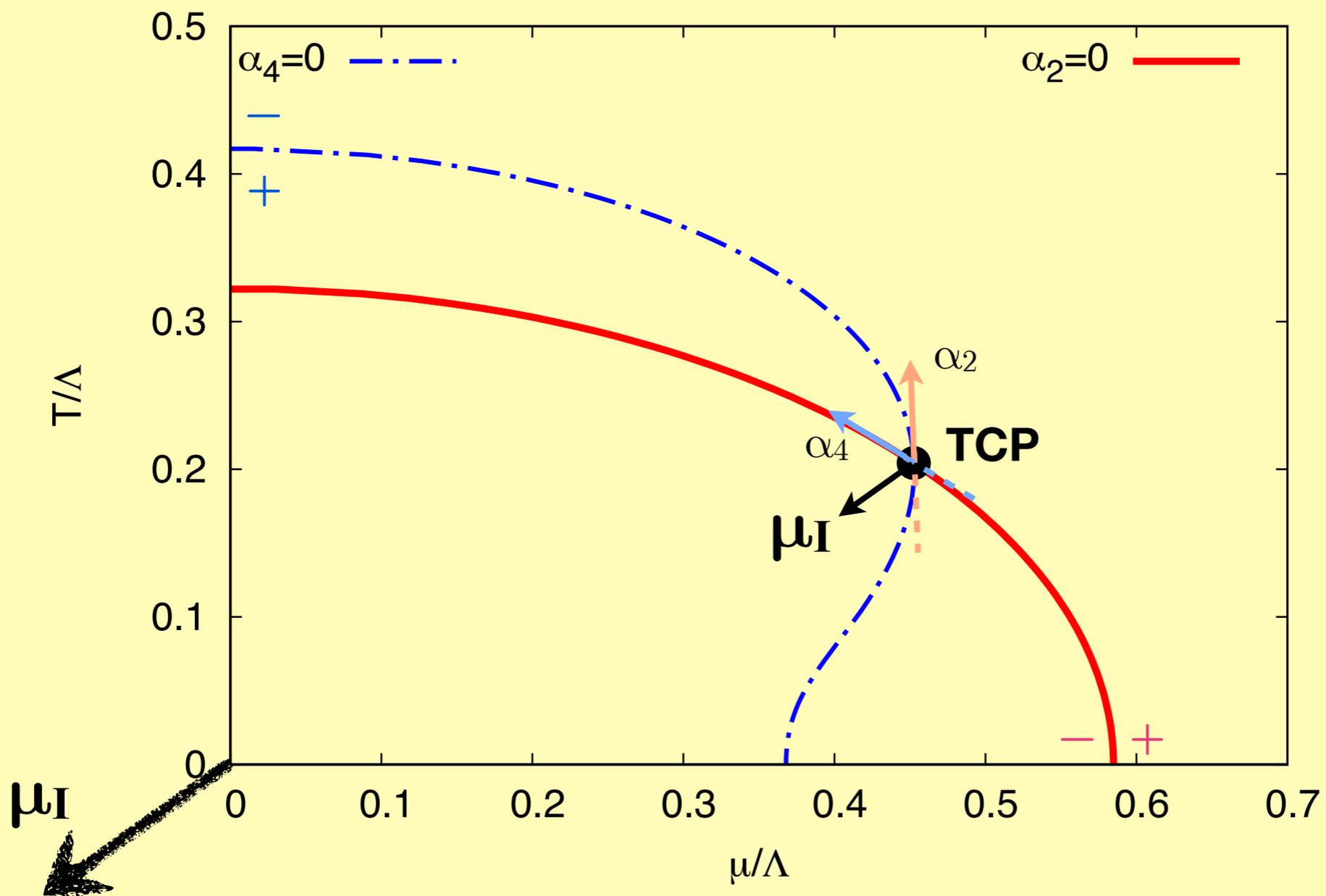
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2-flavor NJL at $T=0$

Barducci, Casalbuoni et al. (2004)



μ_I as a new GL coupling



Extension of gGL

- Rewrite 6th gGL @ $\mu_I=0$, $m_q=0$ in the $O(4)$ invariant form. With 4-vector $\phi=(\pi_1, \pi_2, \pi_3, \sigma)$

$$\begin{aligned} \Omega_{\text{GL}} = & \frac{\alpha_2(\mu_I)}{2} \phi^2 + \frac{\alpha_4(\mu_I)}{4} (\phi^4 + (\nabla\phi)^2) \\ & + \frac{\alpha_6(\mu_I)}{6} \left(\phi^6 + 5\phi^2 (\nabla\phi)^2 + \frac{1}{2} (\Delta\phi)^2 \right) \\ & + \frac{\alpha_{6d}(\mu_I)}{6} (\phi^2 (\nabla\phi)^2 - (\phi \cdot \nabla\phi)^2) \end{aligned}$$

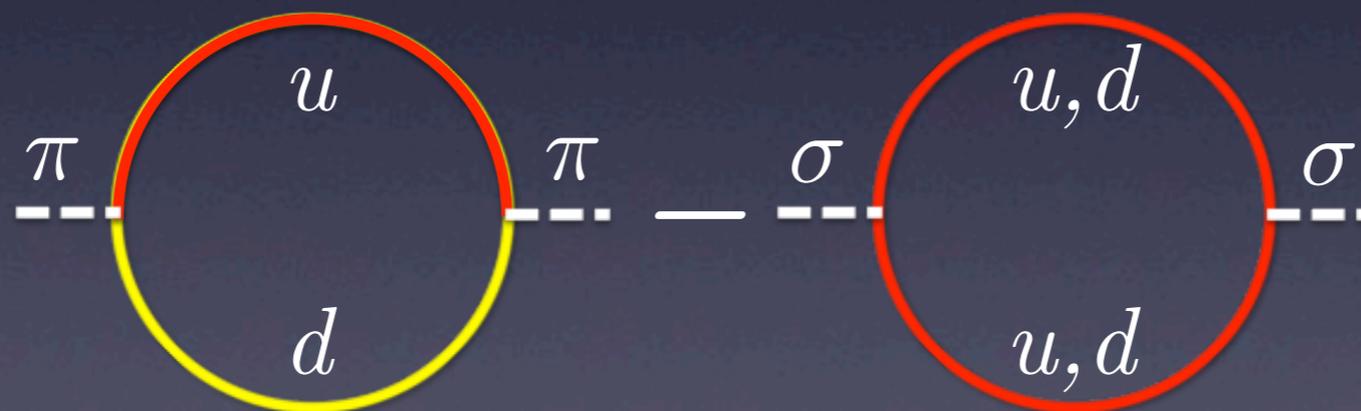
New couplings: $U(1)$ symmetric terms responding to μ_I

$$\delta\Omega = \frac{\beta_2(\mu_I)}{2} \pi_c^2 \quad (\pi_c^2 = \pi_1^2 + \pi_2^2 = 2\pi^+ \pi^-)$$

$$+ \frac{\beta_4(\mu_I)}{4} \pi_c^4 + \frac{\beta_{4b}(\mu_I)}{4} (\phi^2 - \pi_c^2) \pi_c^2 + \frac{\beta_{4c}(\mu_I)}{4} (\nabla \pi_c)^2$$

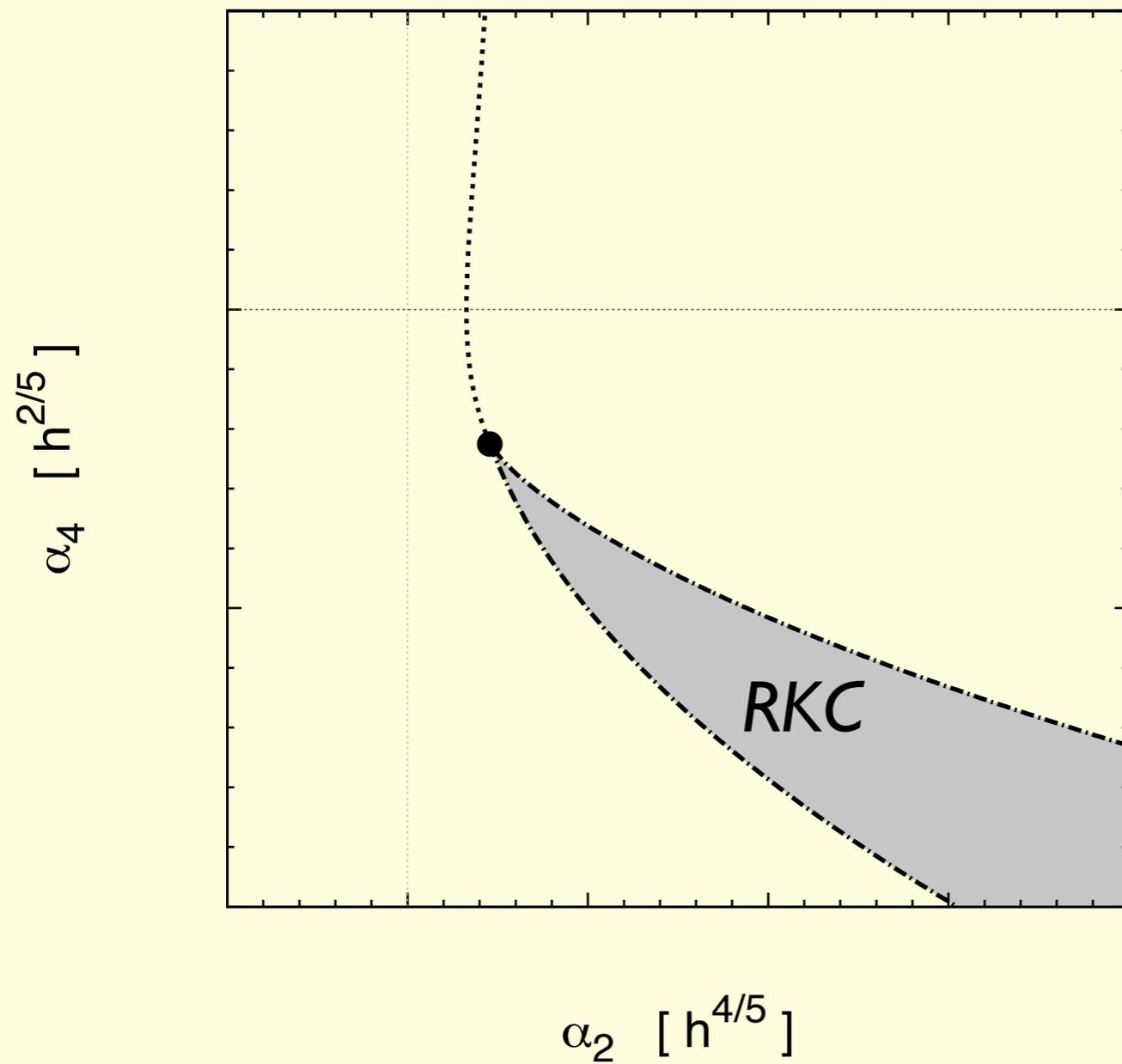
: $SU(2) \mapsto O(2) = U(1)$ rotation about I_3

e.x. Feynman Graphs contributing to β_2

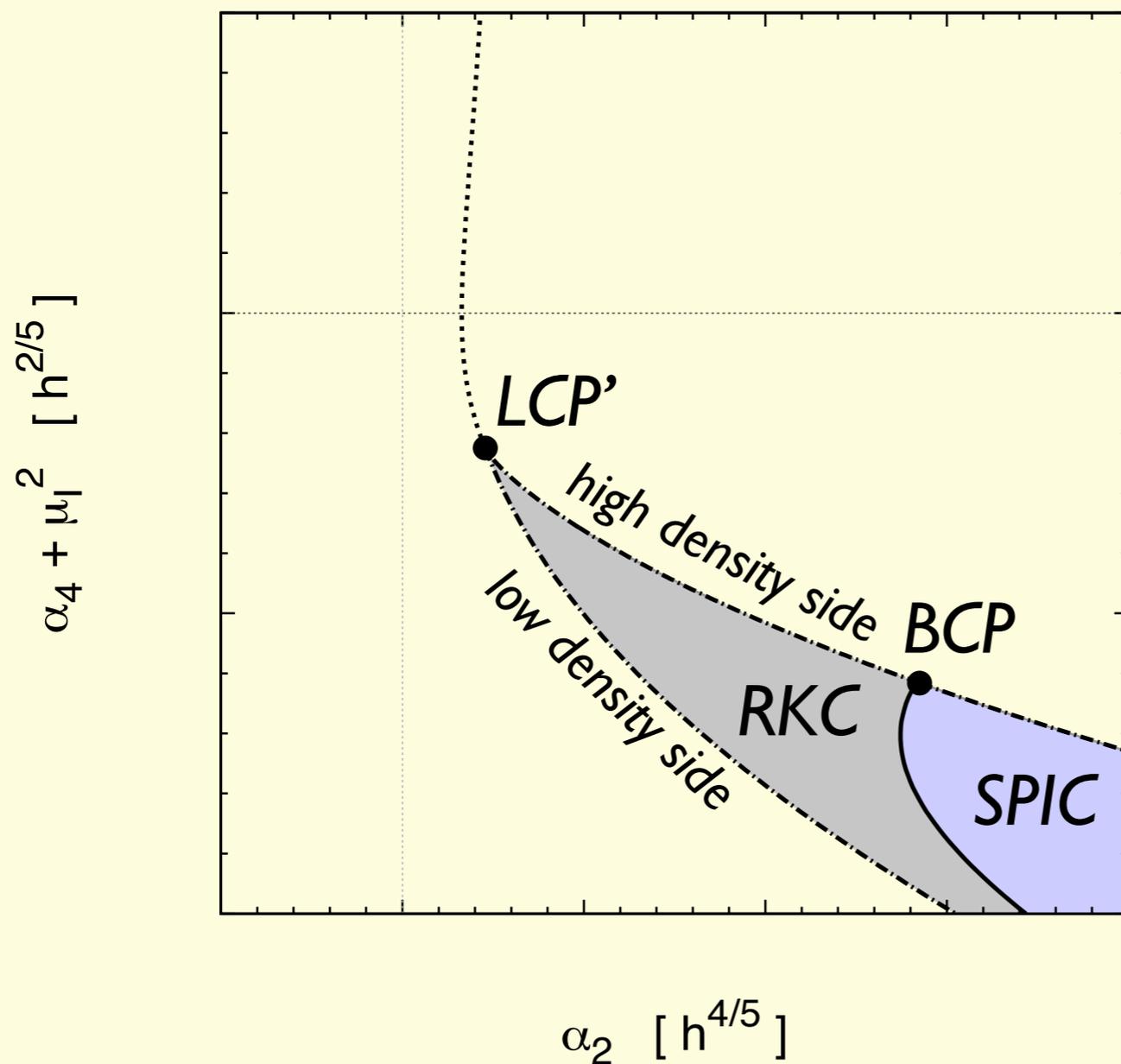


$$\frac{\beta_2(\mu_I)}{2} = 2N_c T \sum_n \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{-(q_u - q_d)^2}{q_u^2 q_d^2} = -\mu_I^2 \frac{\alpha_4}{4} + \mathcal{O}(\mu_I^4)$$

Phases@finite μ_I



Phases@finite μ_I



$$\mu_I^2 = 0.01 h^{2/5}$$

$$(\mu_I \sim 50 \text{ MeV})$$

i. Shift of LCP

$$(2.28, -2.25) \mapsto$$

$$(2.28, -2.25 - \mu_I^2)$$

ii. SPIC replaces a part of RKC

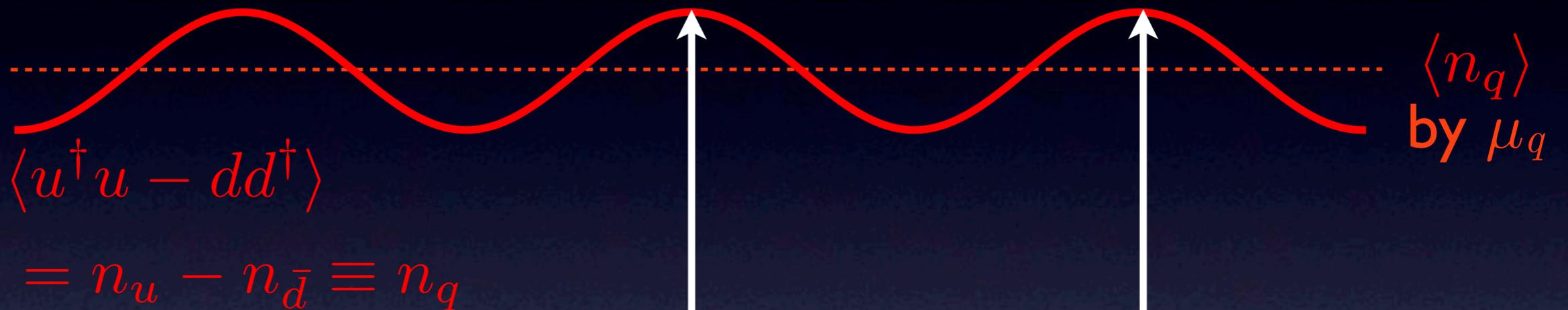
$$\pi(z) = k\nu \text{sn}(kz, \nu)$$

$$\sigma = \text{const.} \neq 0$$

Charged Pion Crystal?

c.f. Pion LOFF superfluidity: He, Jin, Zhuang (2008)

non-ordering density

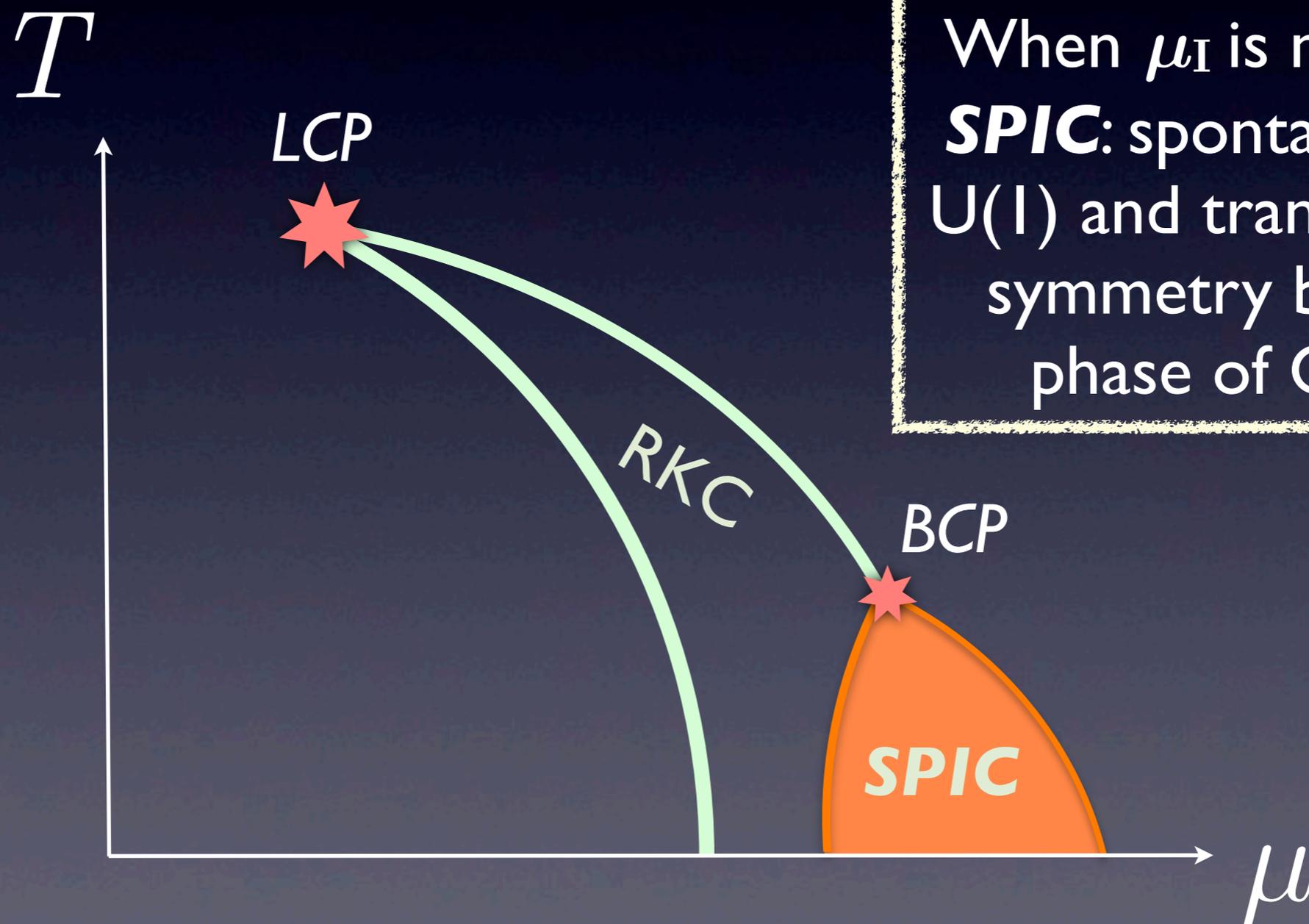


$\langle \bar{d} \gamma_5 u \rangle$

ordering density

z

Implication to QCD



Summary

- Studied the response of TCP against m_q and μ_I within a generalized GL approach.
- Solitonic pion crystal (SPIC) replaces a part of RKC by isospin mismatch.
- In QCD there may be a region of SPIC in high density and low temperature side.

Backups

Outlook

- What about magnetic properties of crystals?

New LP?: Karasawa, Nishiyama, Tatsumi (2013, 2014)

- What is properties of phonon in crystals?

Landau-Pierls instability?: see e.g. Baym, Friman, Grinstein, NPB82

- No shape ansatz for crystals?

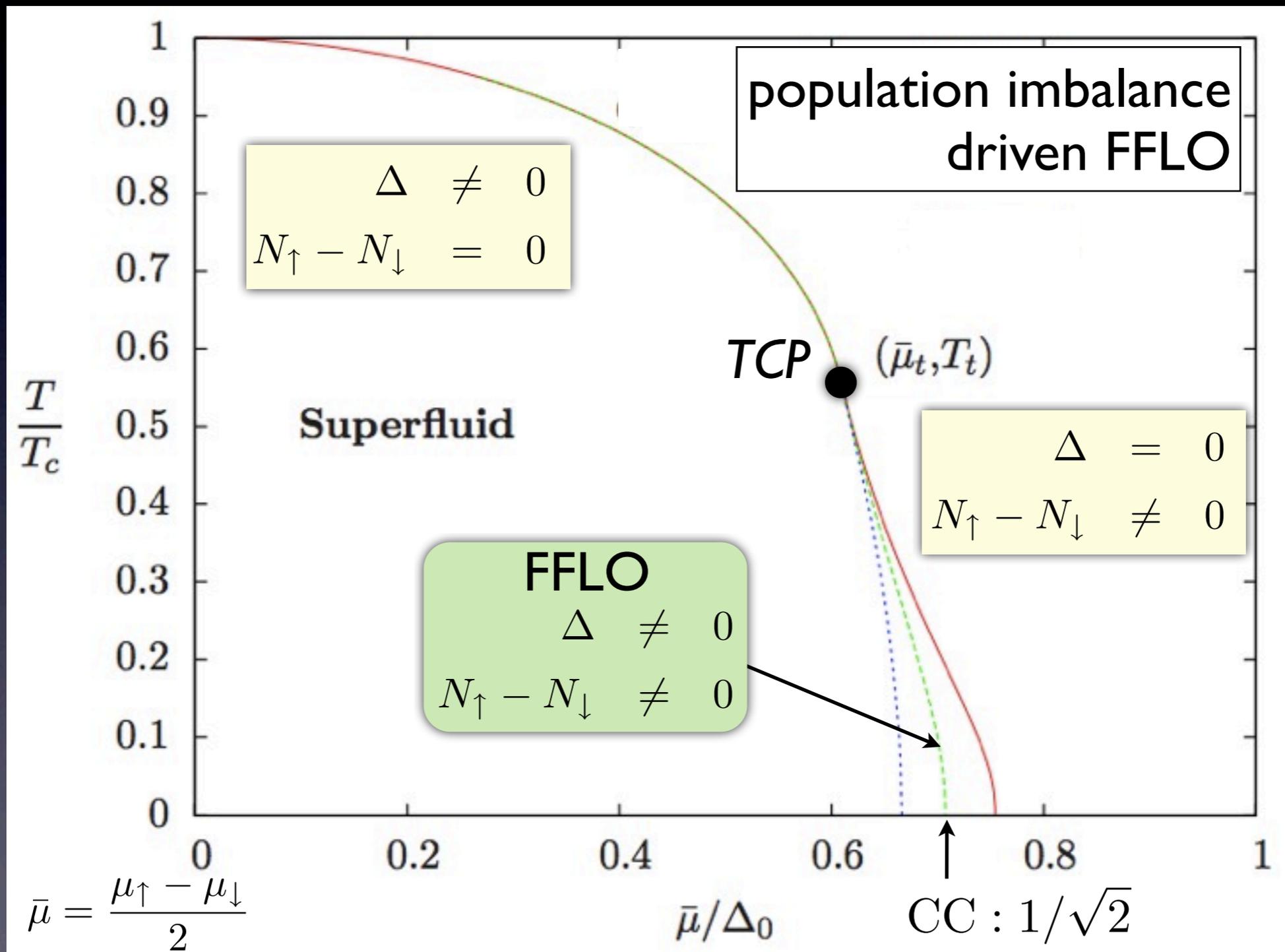
- Phenomenological consequences on compact star physics?

Quark beta decay: Tatsumi, Muto (2014)

- Extension to 3-flavors (including strange quarks):
 μ_S & possible *Kaon condensate*?

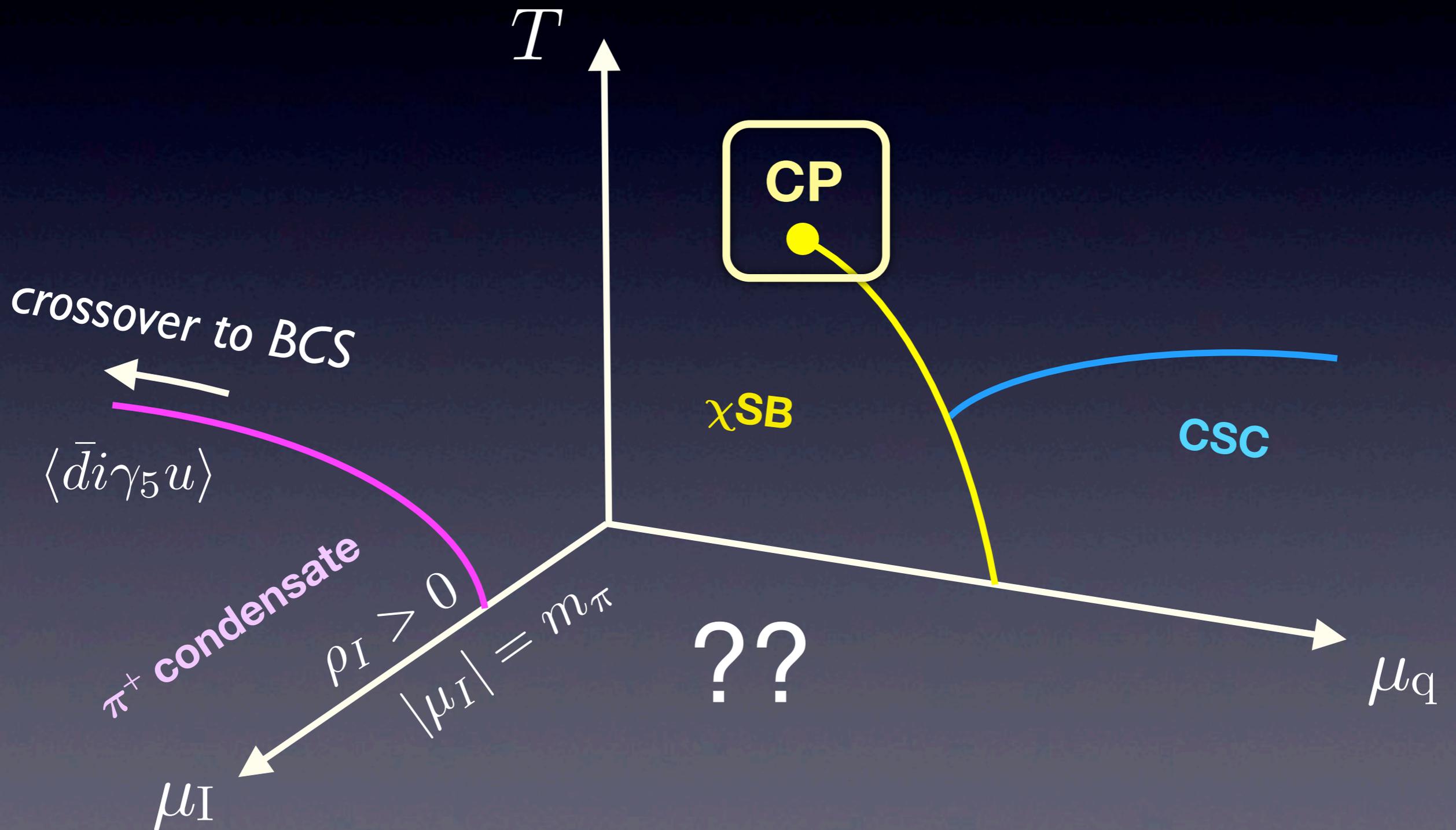
Moreira, et al. (2014)

FFLO @ weak coupling



Focus on CP

Son, Stephanov (2001)



Possible new states

(i) PIC: Pion Condensate

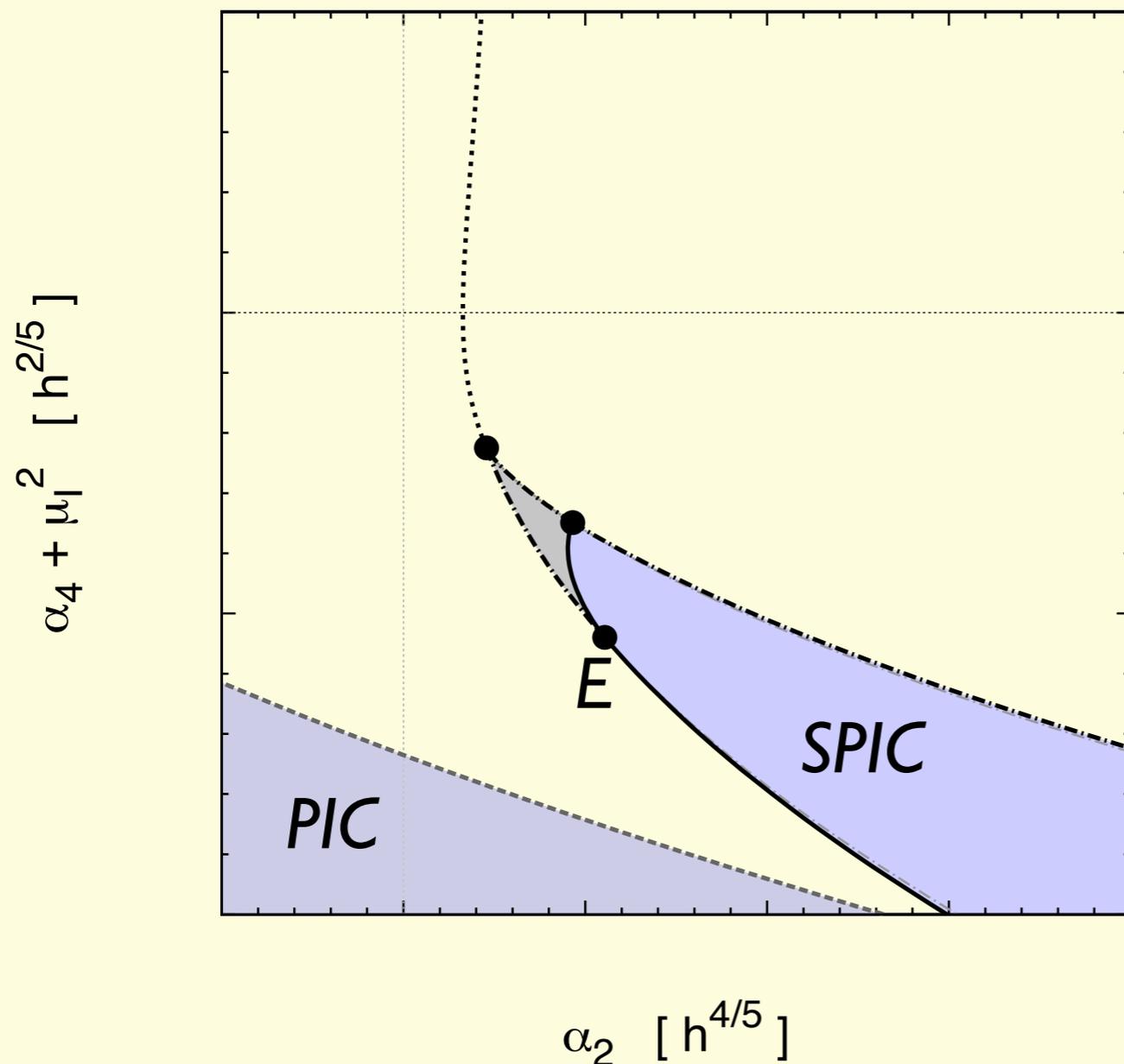
$$\sigma = \text{const.} \neq 0, \quad |\pi_c| = \text{const.} \neq 0$$

(ii) SPIC: Solitonic Pion Crystal (Condensate)

: Pionic analog to solitonic chiral crystal (RKC)

$$\sigma = \text{const.} \neq 0, \quad \pi(z) = k\nu \text{sn}(kz, \nu)$$

Increasing μ_I



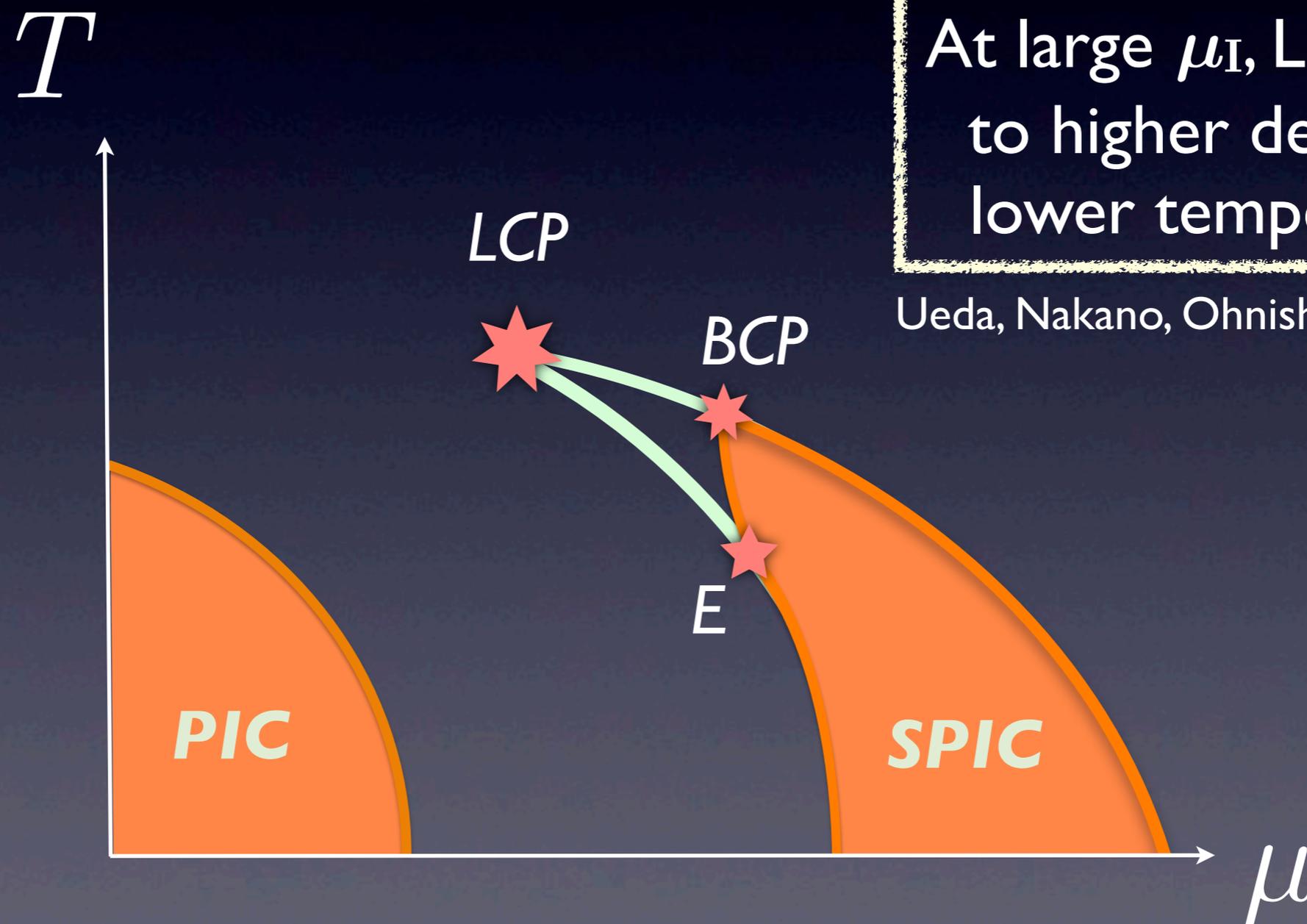
(2) $\mu_I^2 = 0.1 h^{2/5}$
($\mu_I \sim 100$ MeV)

i. SPIC broaden =
CDL shrunk

ii. Point E:
Three phases
meet up

iii. PIC continent:
2nd order

Implication to QCD

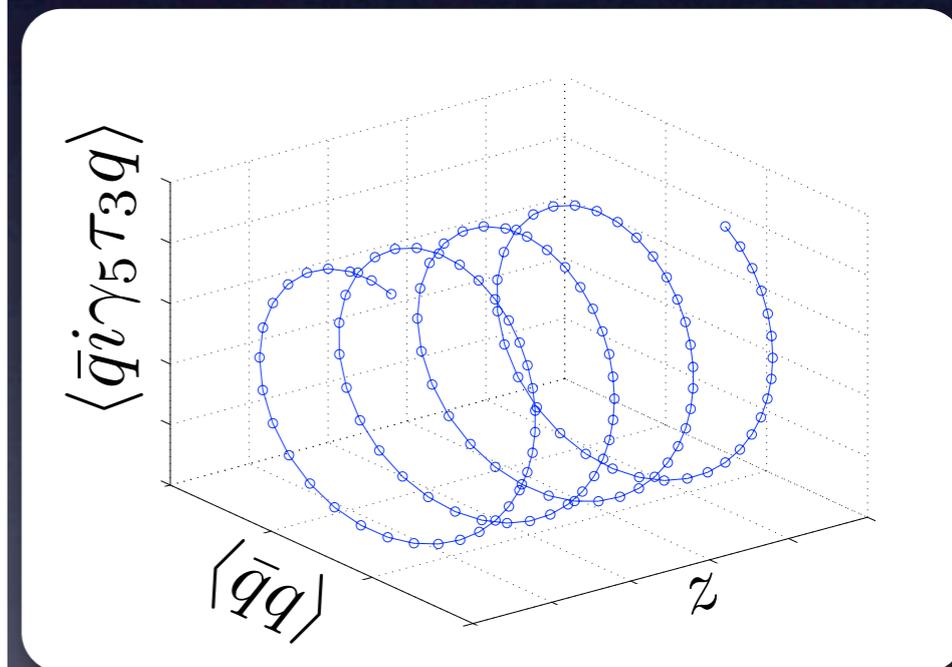
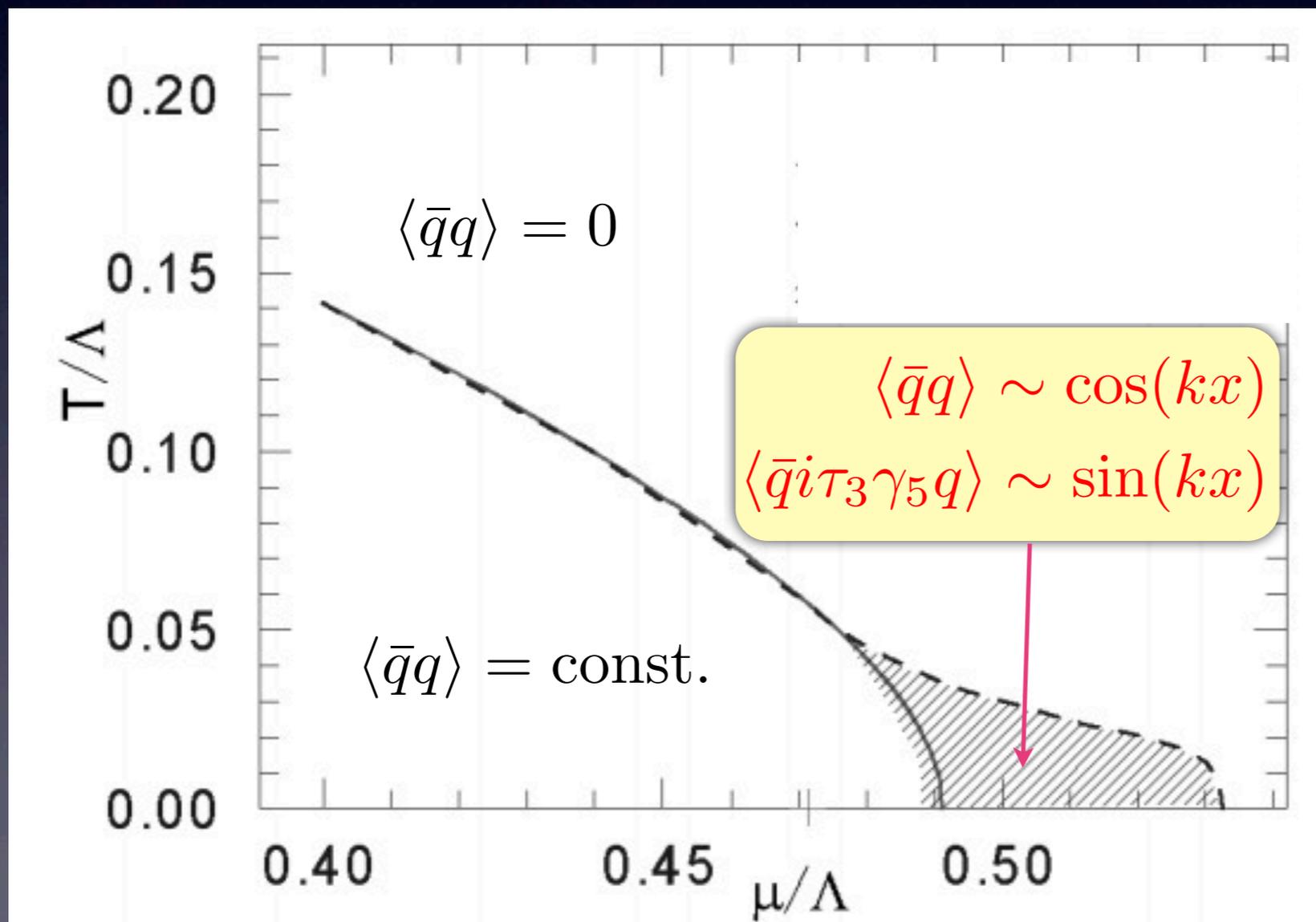


At large μ_I , LCP shifts to higher density & lower temperature

Ueda, Nakano, Ohnishi, Ruggieri (13)
Abuki (13)

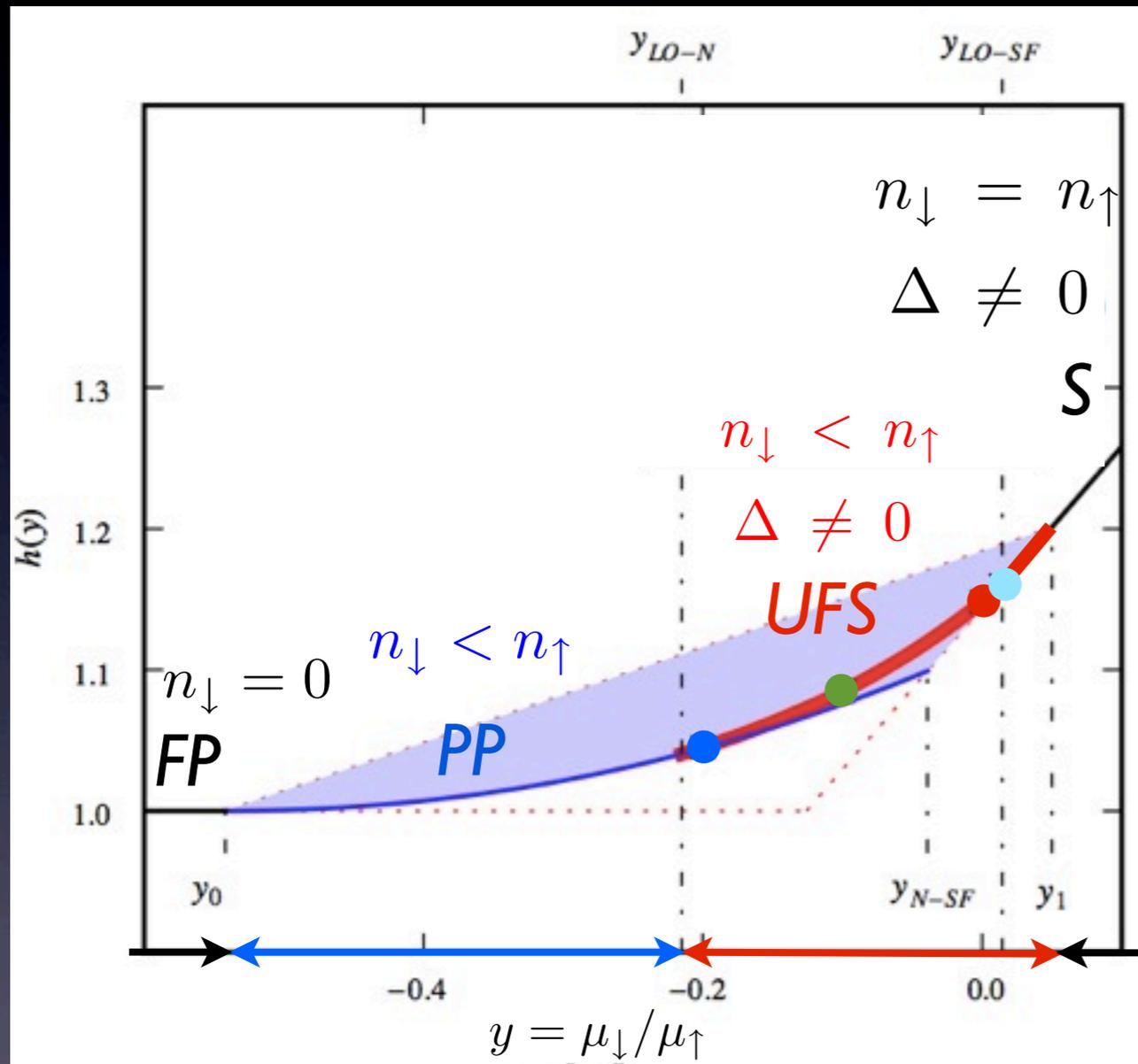
Dual Chiral Density Wave (DCDW)

Seminal work: Nakano, Tatsumi PRD71 (2005) 114006



also known as
“chiral spiral”

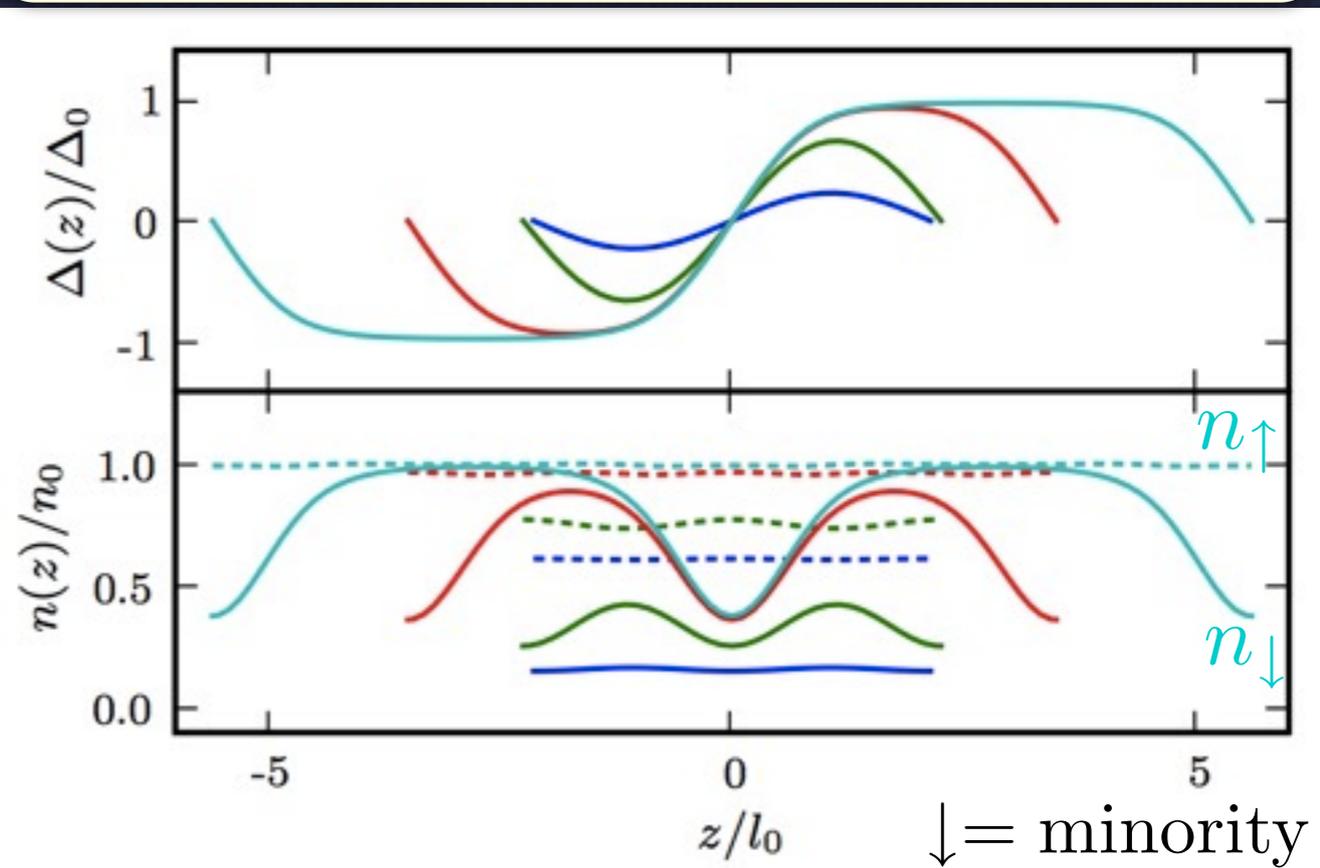
Unitary Fermi Supersolid?



Bulgac & Forbes, RRL (2008)
 Hohenberg-Kohn-Sham DFT solved
 with DVR (discrete variable representation)

$$h(y) = \frac{P(\mu_{\uparrow}, \mu_{\downarrow})}{P_0(\mu_{\uparrow})}$$

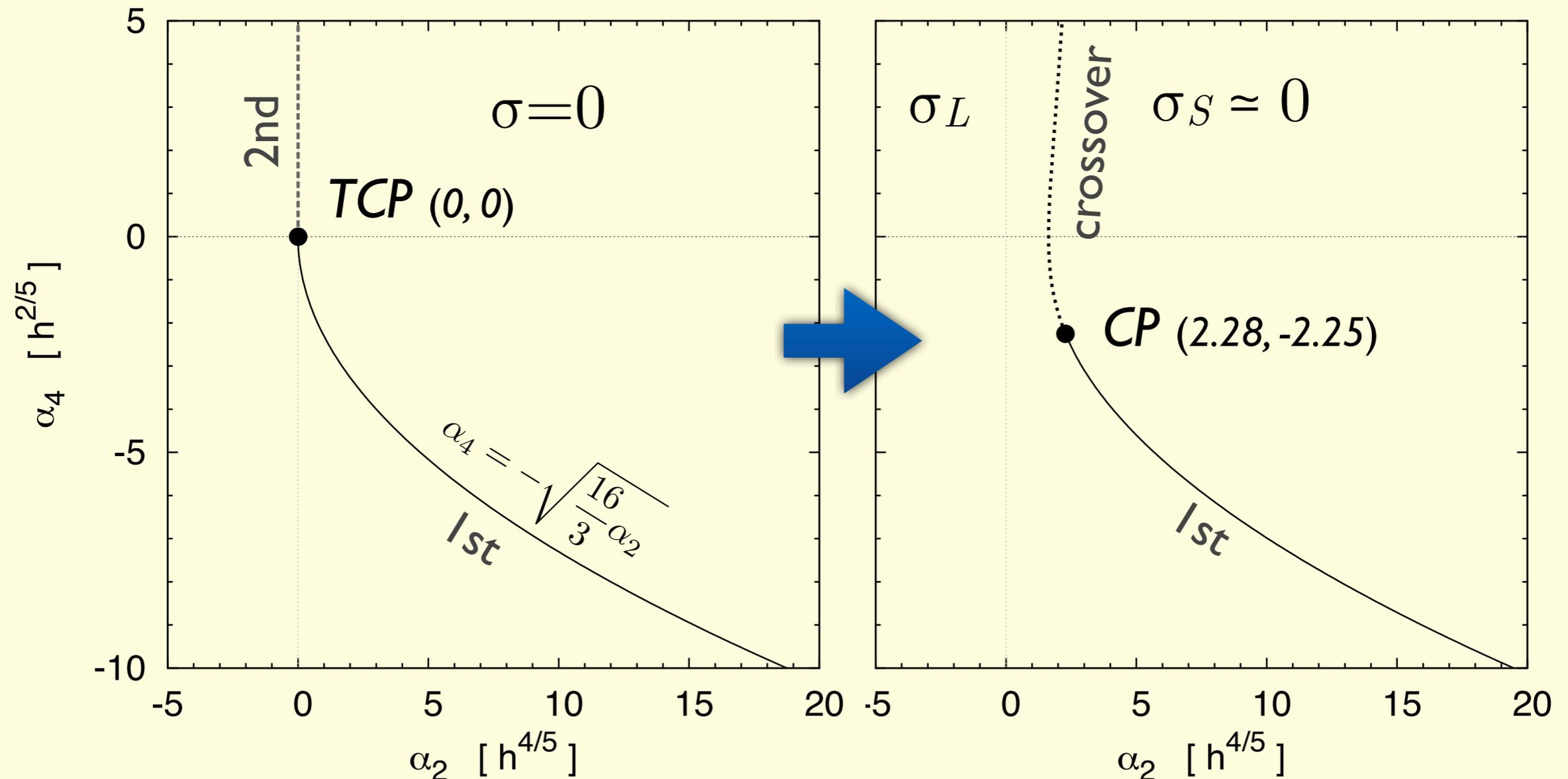
$$h(1) = \frac{2^{2/5}}{\xi^{3/5}} \cong 3.1 \quad (\xi = 0.42(1))$$



see also: Yoshida & Yip (2007)
 MF Bogoliubov-deGennes (B-dG) equation
 solved with an ansatz for pair potential

Effect of current mass: m_q

$\delta\Omega = -h\sigma$: External field : $O(4) \mapsto O(3) = SU(2)$ isospin



reduced gGL potential

$$\Omega_{\text{GL}} = \frac{\alpha_2}{2} M^2 + \frac{\alpha_4}{4} (M^4 + (\nabla M)^2) \quad \text{D. Nickel, PRL09}$$
$$+ \frac{\alpha_6}{6} \left(M^6 + 5M^2 (\nabla M)^2 + \frac{1}{2} (\Delta M)^2 \right)$$

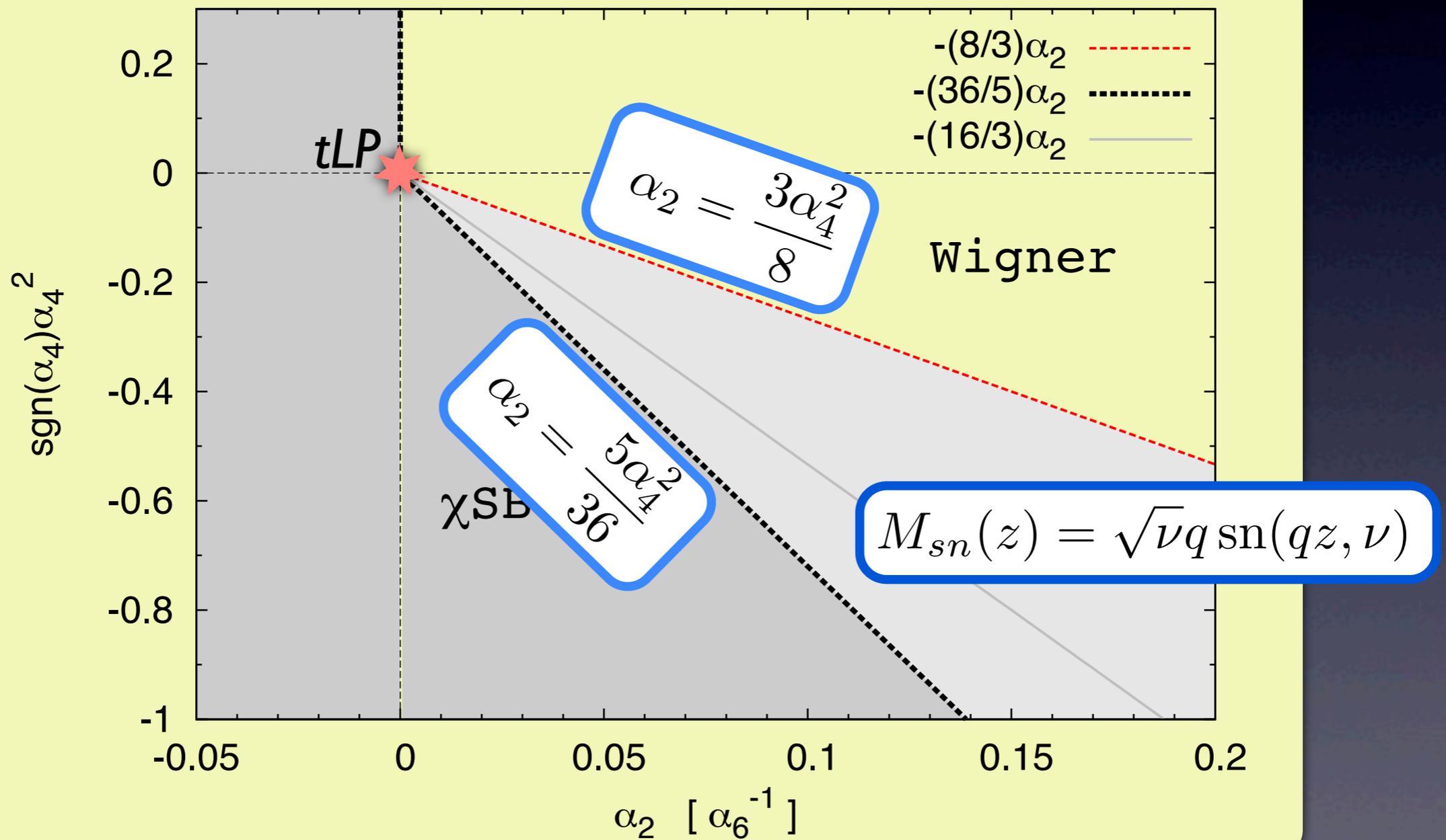
- Three independent GL parameters.

$$[\alpha_6] = \Lambda^{-2}$$

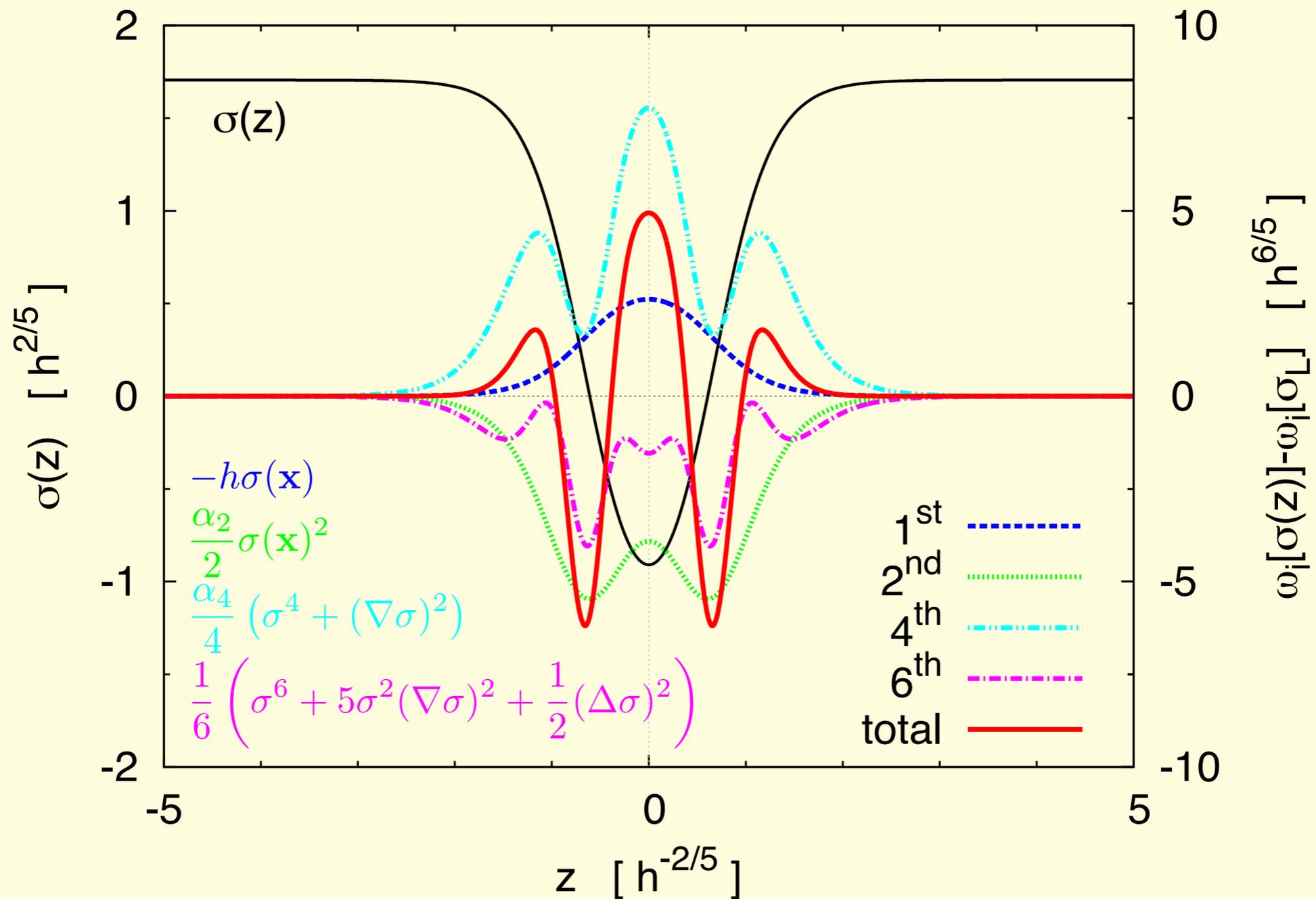
- For thermodynamic stability $\alpha_6 > 0$, so use this to set an energy scale

The vicinity of TCP

D. Nickel, PRL09

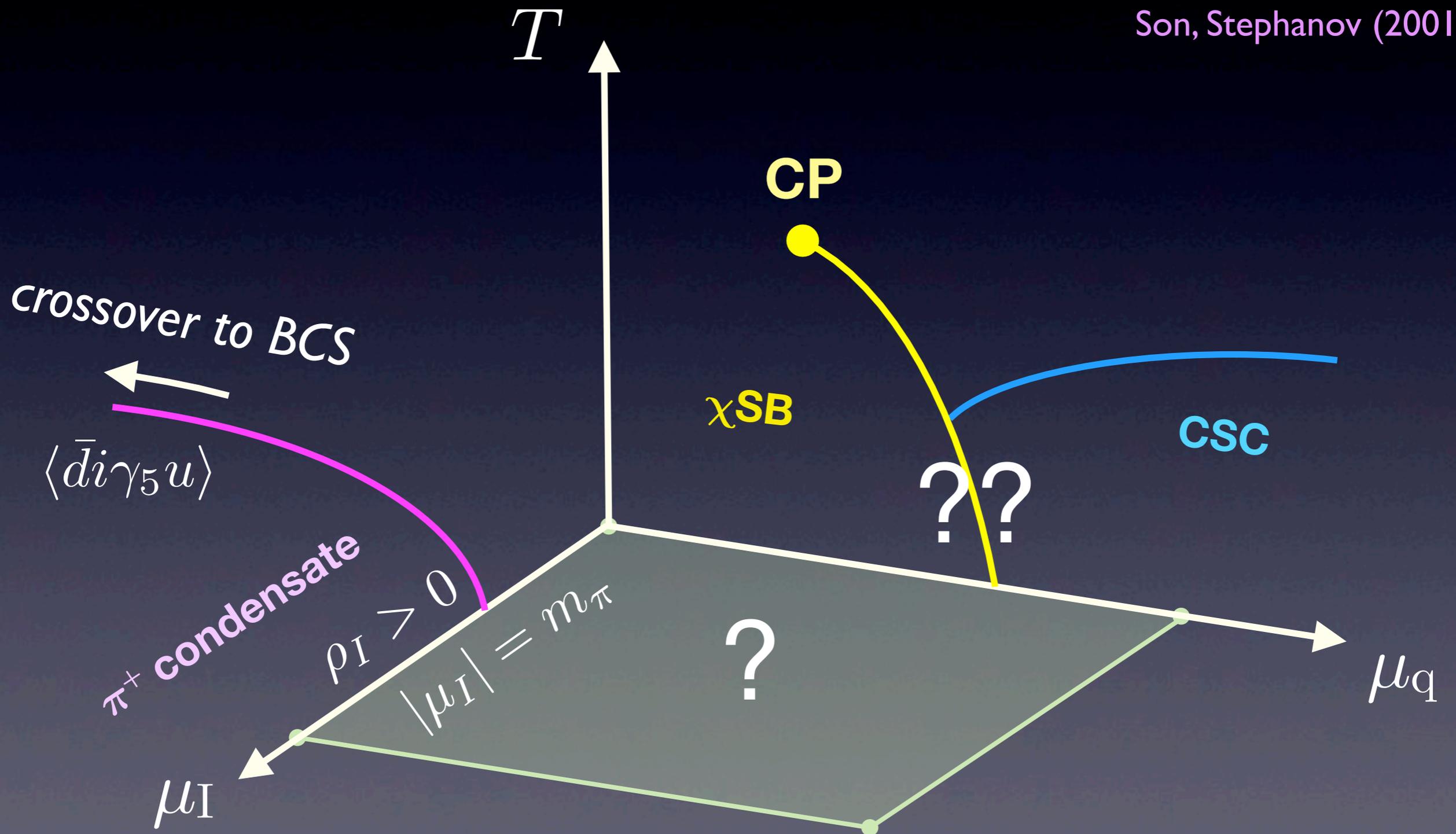


@Single Defect Onset



2-flavor QCD phases

Son, Stephanov (2001)



Final result of gGL

$$\begin{aligned}\Omega_{\text{GL}} = & -h\sigma + \frac{\alpha_2}{2}\sigma^2 + \left(\frac{\alpha_2}{2} - \frac{\mu_1^2\alpha_4}{4}\right)\pi^2 \\ & + \left(\frac{\alpha_4}{4} + \frac{\mu_1^2\alpha_6}{4}\right)\sigma^4 + \frac{\alpha_4}{2}\sigma^2\pi^2 + \left(\frac{\alpha_4}{4} - \frac{\mu_1^2\alpha_6}{12}\right)\pi^4 \\ & + \left(\frac{\alpha_4}{4} + \frac{\mu_1^2\alpha_6}{4}\right)(\nabla\sigma)^2 + \left(\frac{\alpha_4}{4} - \frac{\mu_1^2\alpha_6}{12}\right)(\nabla\pi)^2 \\ & + \frac{\alpha_6}{6}\left(\phi^6 + 8\phi^2(\nabla\phi)^2 - 3(\phi \cdot \nabla\phi)^2 + \frac{1}{2}(\Delta\phi)^2\right)\end{aligned}$$

In other Systems?

