QCD@Work

International Workshop on Quantum Chromodynamics

Sum rules and spectral density flow in QCD and in superconformal theories

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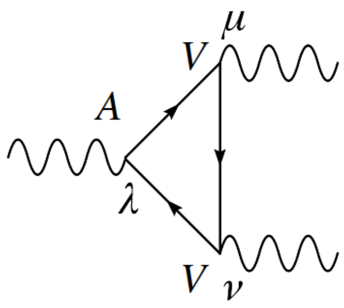
Outline

- ✓ Chiral and conformal anomalies in QCD and its supersymmetric extension
- ✓ Explicit computations in perturbation theory
- ✓ Anomalous spectral densities are characterized by convergent sum rules
- ✓ Evidence of new effective degrees of freedom as a consequence of anomalies

Review: The *AVV* diagram and the pion

The anomalous *AVV* diagram with an axial-vector current (A) and two vector currents (V) is characterized by a massless singularity



$$\Delta^{\lambda\mu\nu} = a_n \frac{k^\lambda}{k^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}$$

Dolgov, Zakharov. Nucl. Phys. B27, 525 (1971)

The axial anomaly pole appears in the AVV diagram

The pole structure clearly describes the pseudoscalar pion π

the anomalous spectral density in the AVV possesses a sum rule

$$\frac{1}{\pi}\int_0^\infty \rho(s,m^2)ds = f$$

Horejsi. Phys. Rev. D32, 1029 (1985)

this is a general feature of anomalies (chiral, conformal, superconformal)

Sum rules

The existence of a sum rule implies a particular behaviour of the spectral density

we consider a sum rule with a mass deformation parameter m to control the spectral density away from the chiral/conformal point

$$\frac{1}{\pi}\int_0^{\infty} \rho(s, m^2)ds = f$$

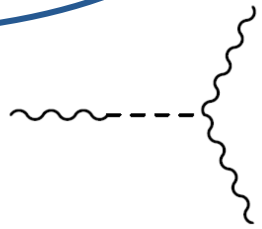
with f ≠ 0 and mass independent

- the spectral density is integrable
- its scale dimension is fixed
- $\rho(s, m^2)$ flows towards a $\delta(s)$ as m goes to zero
- a pole-like behaviour appears also in the UV

this corresponds to a massless state propagating in the theory

anomalies are IR/UV phenomena

$$\lim_{k^2\to\infty} k^2 F(k^2, m^2) = f$$

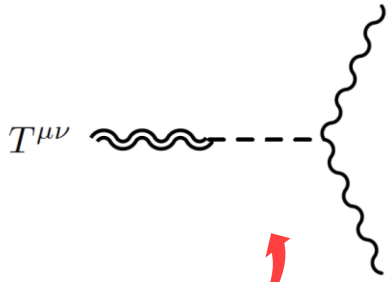


The conformal anomaly case

- Conformally anomalous correlators possess spectral densities with a convergent sum rule

- As the AVV case, it has been shown that the TVV correlator, (T is the energy-momentum tensor EMT) shares a similar behaviour dictated by the conformal anomaly

  this has been proved in QED, QCD and in EW sector of the SM



this can be interpreted as an effective scalar, the ***dilaton***, coupling to the trace of the EMT, in full analogy with the pion

exchange of a conformal anomaly pole

Conformal anomalies and QFT

Quantum field theories examined in the context of conformal anomalies:

- *Abelian gauge field theory (QED)*

  Giannotti, Mottola, Phys.Rev.D79 (2009) 045014,
  Armillis, Corianò, L. D. R., Phys.Rev.D81 (2010) 085001

- *Exact non abelian gauge field theory (QCD)*

  Armillis, Corianò, L. D. R., Phys.Rev.D82 (2010) 064023

- *Electroweak sector of the Standard Model (SM)*

  Corianò, L. D. R., Quintavalle, Serino, Phys.Lett.B700 (2011) 29-38
  Corianò, L. D. R., Serino, Phys.Rev.D83 (2011) 125028

- *N=1 Supersymmetric Yang-Mills*

  Corianò, Constantini, L. D. R., to appear on JHEP

In the following the superconformal anomaly case is considered, in which chiral, conformal and superconformal symmetries are treated in a unified way

Anomalies in *N*=1 Super Yang-Mills

Consider a *N*=1 Super Yang-Mills (for instance SuperQCD)

vector (gauge) supermultiplet $V = \left(A_\mu^a, \lambda^a, D^a\right)$

chiral (matter) supermultiplet $\Phi = \left(\phi_i, \chi_i, F_i\right)$

we can introduce the Ferrara-Zumino (FZ) hypercurrent

$$\mathcal{J}_{A\dot{A}} = \mathrm{Tr}\left[\bar{W}_{\dot{A}} e^V W_A e^{-V}\right] - \frac{1}{3}\bar{\Phi}\left[\overleftarrow{\bar{\nabla}}_{\dot{A}}\, e^V \nabla_A - e^V \bar{D}_{\dot{A}} \nabla_A + \overleftarrow{\bar{\nabla}}_{\dot{A}} \overleftarrow{D}_A\, e^V\right]\Phi$$

$$W_A = 2g W_A^a T^a = -\frac{1}{4}\bar{D}^2 e^{-V} D_A\, e^V$$

which describes a chiral supermultiplet containing the ***EMT***, the ***R-current*** and the ***supersymmetric current***

$$\mathcal{J} = \left(R^\mu, T^{\mu\nu}, S^\mu\right)$$

- *in a classical superconformal theory the FZ hypercurrent is conserved* $\bar{D}^{\dot{A}} \mathcal{J}_{A\dot{A}} = 0$

the chiral superpotential $\mathcal{W}$ must be cubic or vanishing

Anomalies in N=1 Super Yang-Mills

in the quantum mechanical framework, the FZ develops a superconformal anomaly

$$\bar{D}^{\dot{A}} \mathcal{J}_{A\dot{A}} = a_n D_A W^2 \qquad\qquad a_n = -\frac{2}{3} g^2 \frac{3T_G - \sum_f T(R_f)}{16\pi^2}$$

The components of the FZ supermultiplet are

$$\begin{aligned}
R^\mu &= \bar{\lambda}^a \bar{\sigma}^\mu \lambda^a + \frac{1}{3}\left(-\bar{\chi}_i \bar{\sigma}^\mu \chi_i + 2i\phi_i^\dagger \mathcal{D}^\mu_{ij}\phi_j - 2i(\mathcal{D}^\mu_{ij}\phi_j)^\dagger \phi_i\right) , \\
S^\mu_A &= i(\sigma^{\nu\rho}\sigma^\mu \bar{\lambda}^a)_A F^a_{\nu\rho} - \sqrt{2}(\sigma_\nu \bar{\sigma}^\mu \chi_i)_A (\mathcal{D}^\nu_{ij}\phi_j)^\dagger - i\sqrt{2}(\sigma^\mu \bar{\chi}_i)\mathcal{W}_i^\dagger(\phi^\dagger) \\
&- ig(\phi_i^\dagger T^a_{ij}\phi_j)(\sigma^\mu \bar{\lambda}^a)_A + S^\mu_{I\,A} , \\
T^{\mu\nu} &= -F^{a\,\mu\rho} F^{a\,\nu}{}_\rho + \frac{i}{4}\left[\bar{\lambda}^a \bar{\sigma}^\mu (\delta^{ac}\,\overrightarrow{\partial^\nu} - g\, t^{abc} A^{b\,\nu})\lambda^c + \bar{\lambda}^a \bar{\sigma}^\mu(-\delta^{ac}\,\overleftarrow{\partial^\nu} - g\, t^{abc}A^{b\,\nu})\lambda^c + (\mu \leftrightarrow \nu)\right] \\
&+ (\mathcal{D}^\mu_{ij}\phi_j)^\dagger (\mathcal{D}^\nu_{ik}\phi_k) + (\mathcal{D}^\nu_{ij}\phi_j)^\dagger(\mathcal{D}^\mu_{ik}\phi_k) + \frac{i}{4}\left[\bar{\chi}_i \bar{\sigma}^\mu (\delta_{ij}\,\overrightarrow{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j\right. \\
&+ \left.\bar{\chi}_i \bar{\sigma}^\mu(-\delta_{ij}\,\overleftarrow{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j + (\mu \leftrightarrow \nu)\right] - \eta^{\mu\nu}\mathcal{L} + T^{\mu\nu}_I ,
\end{aligned}$$

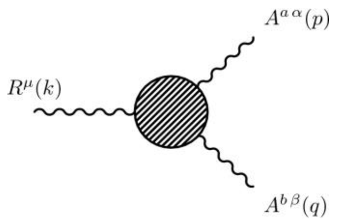
and satisfy the anomaly equations

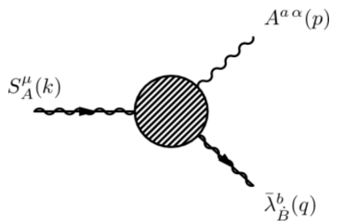
$$\left\{\begin{aligned}
\partial_\mu R^\mu &= -\frac{a_n}{2} F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \\
T^\mu{}_\mu &= \frac{3}{4} a_n F^a_{\mu\nu} F^{a\,\mu\nu} \\
\bar{\sigma}_\mu S^\mu &= 3ia_n \bar{\lambda}^a \bar{\sigma}^{\mu\nu} F^a_{\mu\nu}
\end{aligned}\right.$$

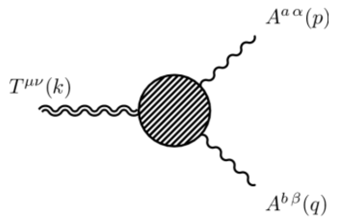
Perturbative computation

We look at the three correlation functions responsible for the appearance of the superconformal anomaly

$$
\begin{aligned}
\delta^{ab}\,\Gamma^{\mu\alpha\beta}_{(R)}(p,q) &\equiv \langle R^\mu(k)\,A^{a\,\alpha}(p)\,A^{b\,\beta}(q)\rangle \qquad \langle RVV\rangle\,,\\
\delta^{ab}\,\Gamma^{\mu\alpha}_{(S)\,A\dot{B}}(p,q) &\equiv \langle S^\mu_A(k)\,A^{a\,\alpha}(p)\,\bar\lambda^b_{\dot B}(q)\rangle \qquad \langle SVF\rangle\,,\\
\delta^{ab}\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) &\equiv \langle T^{\mu\nu}(k)\,A^{a\,\alpha}(p)\,A^{b\,\beta}(q)\rangle \qquad \langle TVV\rangle
\end{aligned}
$$







they are constrained by the vector current conservation

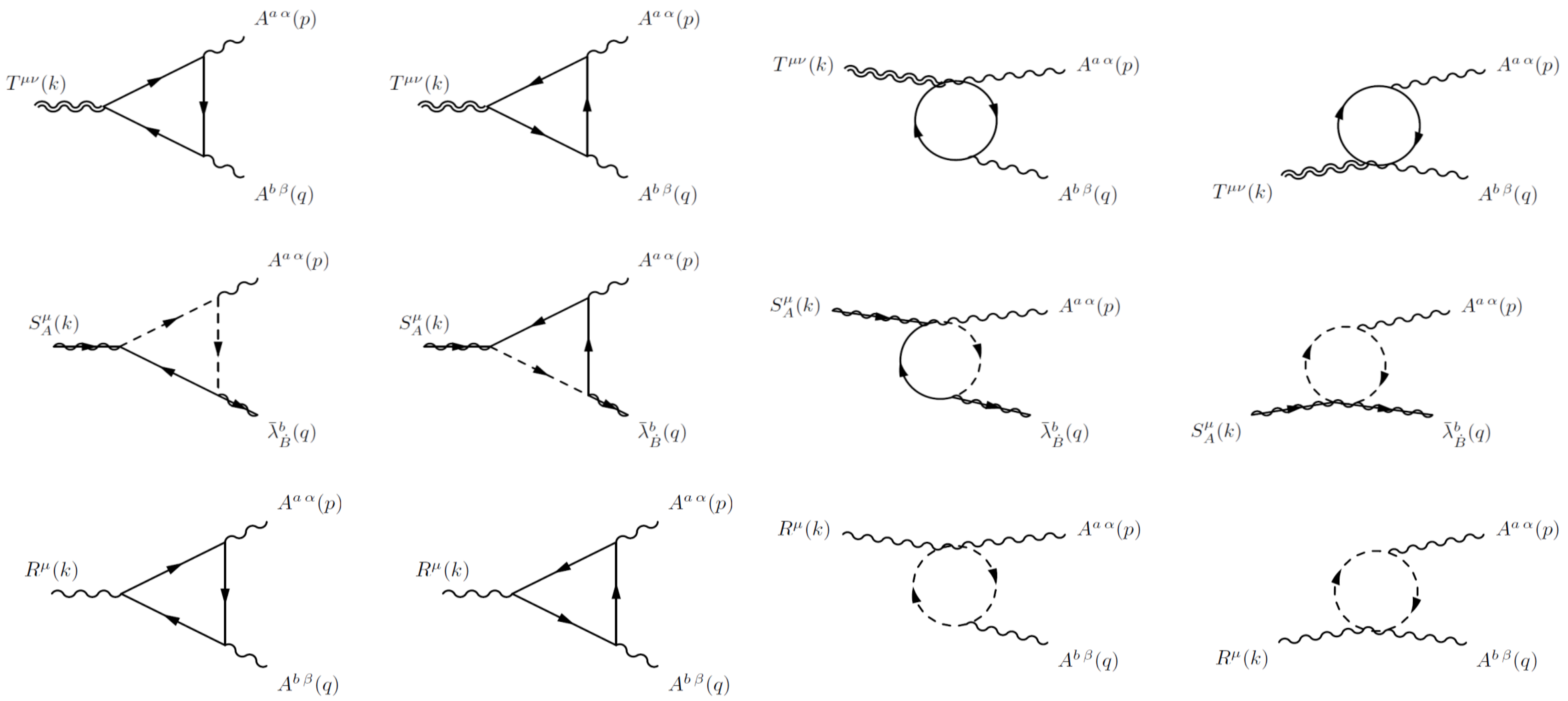
$$
\begin{aligned}
&p_\alpha\,\Gamma^{\mu\alpha\beta}_{(R)}(p,q)=0\,,\quad q_\beta\,\Gamma^{\mu\alpha\beta}_{(R)}(p,q)=0\,,\\
&p_\alpha\,\Gamma^{\mu\alpha}_{(S)}(p,q)=0\,,\\
&p_\alpha\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q)=0\,,\quad q_\beta\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q)=0
\end{aligned}
$$

and by the conservation of the supercurrent and of the EMT

$$
\begin{aligned}
i\,k_\mu\,\Gamma^{\mu\alpha}_{(S)}(p,q) &= -2p_\mu\,\sigma^{\mu\alpha}\hat\Gamma_{(\lambda\bar\lambda)}(q)-i\sigma_\mu\hat\Gamma^{\mu\alpha}_{(AA)}(p)\,,\\
i\,k_\mu\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) &= q_\mu\hat\Gamma^{\alpha\mu}_{(AA)}(p)\eta^{\beta\nu}+p_\mu\hat\Gamma^{\beta\mu}_{(AA)}(q)\eta^{\alpha\nu}-q^\nu\hat\Gamma^{\alpha\beta}_{(AA)}(p)-p^\nu\hat\Gamma^{\alpha\beta}_{(AA)}(q)
\end{aligned}
$$

Perturbative computation

Samples of the one-loop perturbative expansion of the three anomalous correlators



Explicit results

- We show the chiral matter contribution in the on-shell gauge currents kinetimatic region
- A massive chiral supermultiplet is employed to stay away from the conformal point and to study the flow of the spectral densities

$$
\begin{aligned}
\Gamma^{\mu\alpha\beta}_{(R)}(p,q) &= i\frac{g^2\,T(R)}{12\pi^2}\,\Phi_1(k^2,m^2)\,\frac{k^\mu}{k^2}\varepsilon[p,q,\alpha,\beta]\,,\\
\Gamma^{\mu\alpha}_{(S)}(p,q) &= i\frac{g^2 T(R)}{6\pi^2\,k^2}\,\Phi_1(k^2,m^2)\,s_1^{\mu\alpha}+i\frac{g^2 T(R)}{64\pi^2}\,\Phi_2(k^2,m^2)\,s_2^{\mu\alpha}\,,\\
\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) &= \frac{g^2\,T(R)}{24\pi^2\,k^2}\,\Phi_1(k^2,m^2)\,t_{1S}^{\mu\nu\alpha\beta}(p,q)+\frac{g^2\,T(R)}{16\pi^2}\,\Phi_2(k^2,m^2)\,t_{2S}^{\mu\nu\alpha\beta}(p,q)
\end{aligned}
$$

Explicit results

- We show the chiral matter contribution in the on-shell gauge currents kinetimatic region
- A massive chiral supermultiplet is employed to stay away from the conformal point and to study the flow of the spectral densities

superconformal anomaly poles

$$
\begin{aligned}
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\end{aligned}
$$

- The first form factor is the anomaly contribution

this is interpreted as the residue of the pole

$$
\begin{aligned}
\Phi_1(k^2,m^2) &= -1-2\,m^2\,\mathcal{C}_0(k^2,m^2)\,,\\
\Phi_2(k^2,m^2) &= 1-\mathcal{B}_0(0,m^2)+\mathcal{B}_0(k^2,m^2)+2m^2\mathcal{C}_0(k^2,m^2)
\end{aligned}
$$

Anomalous spectral density

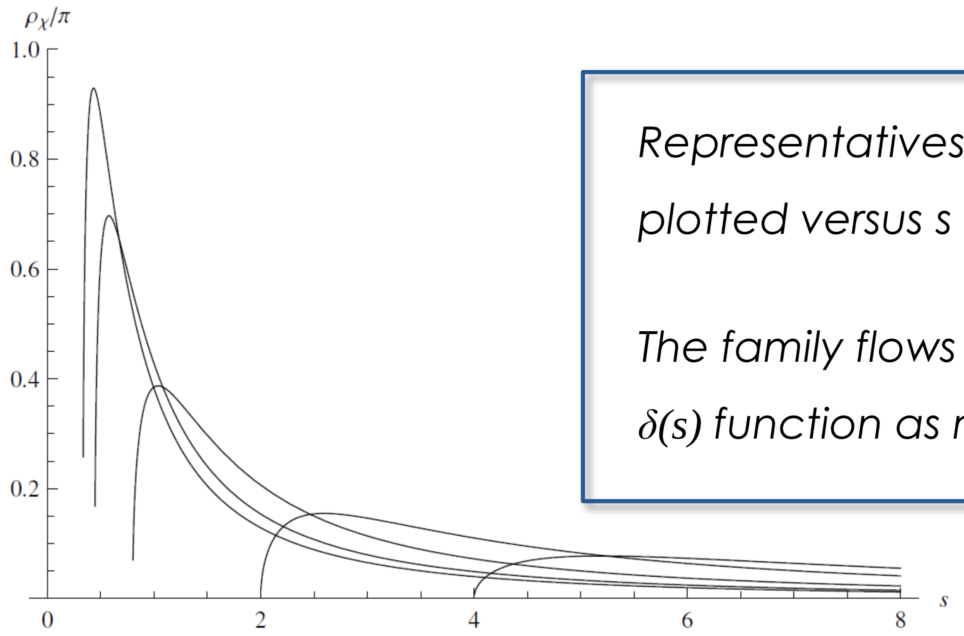
Spectral densities can be computed using cutting rules or exploiting directly the analytic continuations of the two- and three-point scalar integrals

The spectral density of the anomalous form factor

$$\chi(k^2, m^2) \equiv \Phi_1(k^2, m^2)/k^2$$

$$\rho_\chi(s, m^2) = \frac{1}{2i}\mathrm{Disc}\,\chi(s, m^2) = \frac{2\pi m^2}{s^2}\log\left(\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right)\theta(s-4m^2)$$

$$\tau(k^2, m^2) = \sqrt{1-4m^2/k^2}$$



Representatives of the family of spectral densities plotted versus s in units of m^2.

The family flows towards the s=0 region becoming a $\delta(s)$ *function as* m^2 *goes to zero.*

Anomalous spectral density

- The spectral density is integrable and satisfies a convergent sum rule

$$\frac{1}{\pi}\int_{4m^2}^{\infty} ds\,\rho_\chi(s,m^2)=1$$

the anomaly coefficient has been factorized for convenience

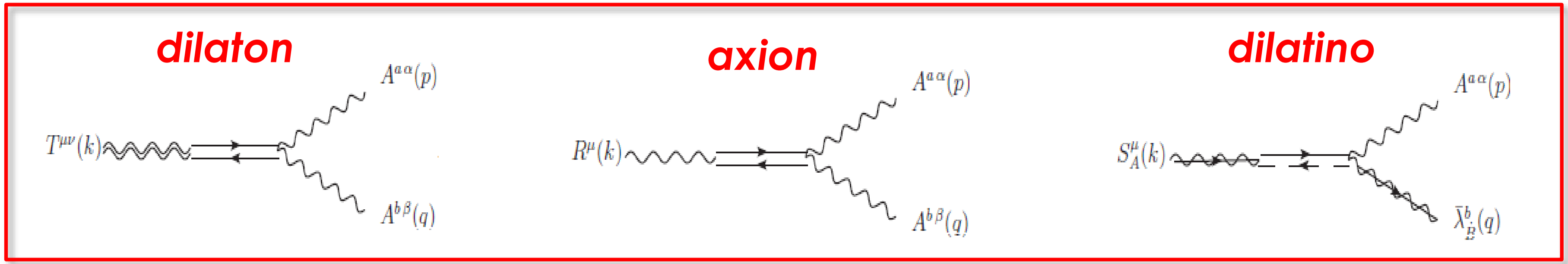
- The discontinuity of the anomalous form factor is characterized by a cut for $k^2 > 4\,m^2$.

  There is any resonant state of the pole at $k^2=0$ in the massive case (*decoupling*)

- The spectral density flows towards a δ(s) as m goes to zero $\lim_{m\to 0}\frac{1}{\pi}\rho_\chi(s,m^2)=\delta(s)$

anomaly accounts for the appearance of massless states in the spectrum,

one for each component of the superconformal hypercurrent

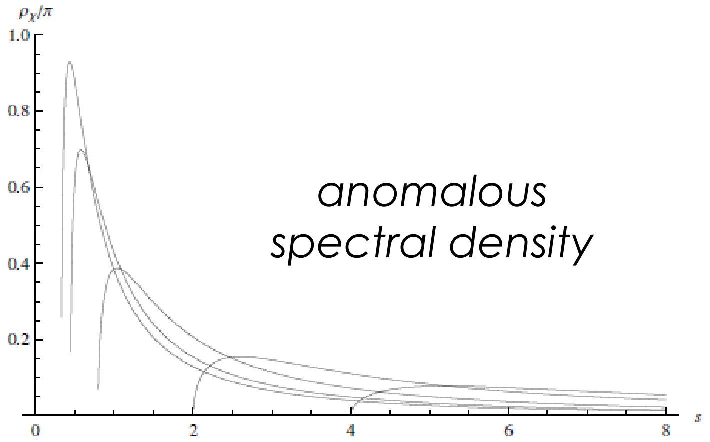


Non-anomalous spectral density

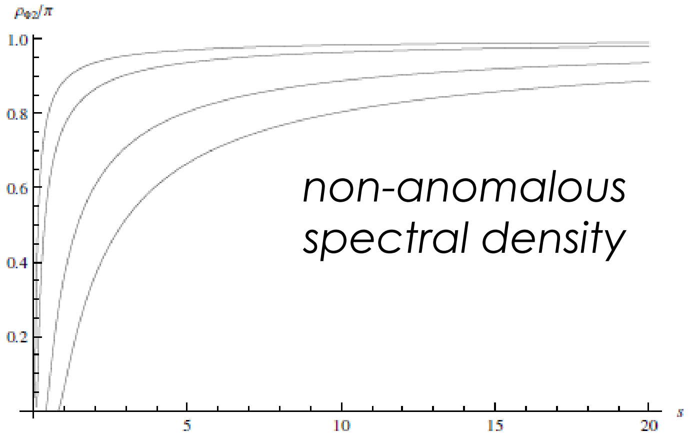
The non-anomalous form factor Φ_2

- needs a subtraction for its integrability (is affected by renormalization)
- does not possess a sum rule
- tends to a uniform distribution as m goes to zero $$\frac{1}{\pi}\lim_{m\to 0}\rho_{\Phi_2}(k^2,m^2)=1$$

there is a sort of ***duality*** between the two spectral densities for m->0



anomalous spectral density



non-anomalous spectral density

Conformal anomaly in QCD

The anomalous correlators (TVV) expands onto ***three*** form factors

$$\Gamma^{\mu\nu\alpha\beta}(p,q) = \Gamma_q^{\mu\nu\alpha\beta}(p,q) + \Gamma_g^{\mu\nu\alpha\beta}(p,q) \qquad \Gamma_{q/g}^{\mu\nu\alpha\beta}(p,q) = \sum_{i=1}^{3} \Phi_{i\,q/g}(k^2,m^2)\,\phi_i^{\mu\nu\alpha\beta}(p,q)$$

The explicit results in quark sector are

$$\Phi_{1\,q}(k^2,m^2) = \frac{g^2}{6\pi^2 k^2}\left\{-\frac{1}{6}+\frac{m^2}{k^2}-m^2\mathcal{C}_0(k^2,m^2)\left[\frac{1}{2}-\frac{2m^2}{k^2}\right]\right\},$$

anomalous

$$\Phi_{2\,q}(k^2,m^2) = -\frac{g^2}{4\pi^2 k^2}\left\{\frac{1}{72}+\frac{m^2}{6k^2}+\frac{m^2}{2k^2}\mathcal{D}(k^2,m^2)+\frac{m^2}{3}\mathcal{C}_0(k^2,m^2)\left[\frac{1}{2}+\frac{m^2}{k^2}\right]\right\},$$

$$\Phi_{3\,q}(k^2,m^2) = \frac{g^2}{4\pi^2}\left\{\frac{11}{72}+\frac{m^2}{2k^2}+m^2\mathcal{C}_0(k^2,m^2)\left[\frac{1}{2}+\frac{m^2}{k^2}\right]+\frac{5\,m^2}{6k^2}\mathcal{D}(k^2,m^2)+\frac{1}{6}\mathcal{B}_0^{\overline{MS}}(k^2,m^2)\right\}$$

non anomalous

$$\rho_{2q}(s,m^2) = -\frac{g^2}{24\pi}\frac{m^2}{s^2}\left[3\sqrt{\tau(s,m^2)}-\left(1+\frac{2m^2}{s}\right)\log\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right]\theta(s-4m^2)$$

This spectral density is not anomalous,
but is integrable and with a convergent sum rule

another massless state
in the spectrum?

Extra pole cancellation in susy theories

Supersymmetric theories force the extra pole to cancel

In a general Yang-Mills theory the contribution to the non-anomalous form factor with the pole-like behaviour is

$$
\begin{aligned}
f_2(k^2) &= \frac{N_f}{2} f_2^{(f)}(k^2) + N_s\, f_2^{(s)}(k^2) + N_A\, f_2^{(A)}(k^2) \\
&= \frac{g^2}{144\pi^2\, k^2}\left[-\frac{N_f}{2} T(R_f) + N_s \frac{T(R_s)}{2} + N_A \frac{T(A)}{2}\right]
\end{aligned}
$$

Bosonic and fermionic contributions have opposite signs

$f_2^{(f)}$ for a Weyl fermion, $f_2^{(s)}$ for a complex scalar, $f_2^{(A)}$ for a gauge vector

- chiral supermultiplet $N_f = 1, N_s = 1, N_A = 0$ $\quad T(R_f) = T(R_s)$
- vector supermultiplet $N_f = 1, N_s = 0, N_A = 1$ $\quad T(R_f) = T(A)$

there is no ambiguity in a susy theory

Superconformal anomalies ⟷ ***Anomaly poles (massless states)***

Conclusions

- ✓ Analysis of chiral, conformal and superconformal anomalies
  and their sum rules in perturbation theory
- ✓ Anomalous spectral densities flow towards massless states
- ✓ Axion-dilaton-dilatino supermultiplet naturally emerges from the quantum breaking of the superconformal symmetry
- ✓ Unambiguous interpretation in a supersymmetric context:
  one-to-one correspondece between anomalies and anomaly poles

Backup slides

The N=1 supersymmetric Lagrangian in the component formalism is

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + i\lambda^a\sigma^\mu\mathcal{D}^{ab}_\mu\bar{\lambda}^b + (\mathcal{D}^\mu_{ij}\phi_j)^\dagger(\mathcal{D}_{ik\,\mu}\phi_k) + i\chi_j\sigma_\mu\mathcal{D}^{\mu\,\dagger}_{ij}\bar{\chi}_i$$
$$-\sqrt{2}g\left(\bar{\lambda}^a\bar{\chi}_i T^a_{ij}\phi_j + \phi^\dagger_i T^a_{ij}\lambda^a\chi_j\right) - V(\phi,\phi^\dagger) - \frac{1}{2}\left(\chi_i\chi_j\mathcal{W}_{ij}(\phi) + h.c.\right)$$

The component expansion of the superfields are

$$\Phi_i = \phi_i + \sqrt{2}\theta\chi_i + \theta^2 F_i$$
$$W^a_A = \lambda^a_A + \theta_A D^a - (\sigma^{\mu\nu}\theta)_A F^a_{\mu\nu} + i\theta^2\,\sigma^\mu_{A\dot{B}}\mathcal{D}_\mu\bar{\lambda}^{a\,\dot{B}},$$
$$V^a = \theta\sigma^\mu\bar{\theta}A^a_\mu + \theta^2\bar{\theta}\bar{\lambda}^a + \bar{\theta}^2\theta\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2\left(D^a + i\partial_\mu A^{a\,\mu}\right)$$

The terms of improvement, necessary only for a scalar field, are

$$S^\mu_{I\,A} = \frac{4\sqrt{2}}{3}i\left[\sigma^{\mu\nu}\partial_\nu(\chi_i\phi^\dagger_i)\right]_A$$
$$T^{\mu\nu}_I = \frac{1}{3}\left(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)\phi^\dagger_i\phi_i$$

These ensure the vanishing of the classical trace of the EMT and of the gamma-trace of the supercurrent for a classical superconformal theory

The two-point functions appearing in the Ward identities are

$$\begin{aligned}
\Gamma^{(AA)}_{\mu\nu}(p) &= -i\delta^{ab}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Sigma^{(AA)}(p^2) \\
\Gamma^{(\lambda\bar{\lambda})}_{A\dot{B}}(p) &= i\delta^{ab}\, p_\mu \sigma^\mu_{A\dot{B}}\, \Sigma^{(\lambda\bar{\lambda})}(p^2)\,,
\end{aligned}$$

with

$$\begin{aligned}
\Sigma^{(AA)}(p^2) &= \frac{g^2}{16\pi^2} p^2 \left\{T(R)\,\mathcal{B}_0(p^2, m^2) - T(A)\,\mathcal{B}_0(p^2, 0)\right\} \\
\Sigma^{(\lambda\bar{\lambda})}(p^2) &= \frac{g^2}{16\pi^2}\left\{T(R)\,\mathcal{B}_0(p^2, m^2) + T(A)\,\mathcal{B}_0(p^2, 0)\right\}\,.
\end{aligned}$$

Two- and three-point scalar integrals

$$\begin{aligned}
\mathcal{B}_0(p_1^2, m_0^2, m_1^2) &= \frac{1}{i\pi^2}\int d^n l \frac{1}{(l^2 - m_0^2)((l+p_1)^2 - m_1^2)}\,, \\
\mathcal{C}_0((p+q)^2, p^2, q^2, m_0^2, m_1^2, m_2^2) &= \frac{1}{i\pi^2}\int d^n l \frac{1}{(l^2 - m_0^2)((l-p)^2 - m_1^2)((l-p-q)^2 - m_2^2)}\,.
\end{aligned}$$

$$\mathcal{B}_0(p_1^2, m^2) \equiv \mathcal{B}_0(p_1^2, m^2, m^2) \qquad \mathcal{C}_0((p+q)^2, m^2) \equiv \mathcal{C}_0((p+q)^2, 0, 0, m^2, m^2, m^2)$$

tensor structures in the susy computation

$$s_1^{\mu\alpha} = \sigma^{\mu\nu} k_\nu \, \sigma^\rho k_\rho \, \bar{\sigma}^{\alpha\beta} p_\beta$$
$$s_2^{\mu\alpha} = 2 p_\beta \, \sigma^{\alpha\beta} \sigma^\mu \, .$$

$$u^{\alpha\beta}(p,q) = \eta^{\alpha\beta} p \cdot q - p^\beta q^\alpha$$

$$\begin{aligned} t_{1S}^{\mu\nu\alpha\beta}(p,q) &\equiv \phi_1^{\mu\nu\alpha\beta}(p,q) = (\eta^{\mu\nu} k^2 - k^\mu k^\nu) u^{\alpha\beta}(p,q) \, , \\ t_{2S}^{\mu\nu\alpha\beta}(p,q) &\equiv \phi_3^{\mu\nu\alpha\beta}(p,q) = (p^\mu q^\nu + p^\nu q^\mu)\eta^{\alpha\beta} + p \cdot q (\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) - \eta^{\mu\nu} u^{\alpha\beta}(p,q) \\ &- (\eta^{\beta\nu} p^\mu + \eta^{\beta\mu} p^\nu) q^\alpha - (\eta^{\alpha\nu} q^\mu + \eta^{\alpha\mu} q^\nu) p^\beta \, , \end{aligned}$$

tensor structures in the qcd computation

$$\begin{aligned} \phi_1^{\mu\nu\alpha\beta}(p,q) &\equiv t_1^{\mu\nu\alpha\beta}(p,q) = (k^2 \eta^{\mu\nu} - k^\mu k^\nu) u^{\alpha\beta}(p,q) \, , \\ \phi_2^{\mu\nu\alpha\beta}(p,q) &\equiv t_3^{\mu\nu\alpha\beta}(p,q) + t_5^{\mu\nu\alpha\beta}(p,q) - 4 t_7^{\mu\nu\alpha\beta}(p,q) = -2 u^{\alpha\beta}(p,q) [k^2 \eta^{\mu\nu} + 2(p^\mu p^\nu + q^\mu q^\nu) \\ &- 4(p^\mu q^\nu + q^\mu p^\nu)] \, , \\ \phi_3^{\mu\nu\alpha\beta}(p,q) &\equiv t_{13}^{\mu\nu\alpha\beta}(p,q) = (p^\mu q^\nu + p^\nu q^\mu)\eta^{\alpha\beta} + p \cdot q (\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) - \eta^{\mu\nu} u^{\alpha\beta}(p,q) \\ &- (\eta^{\beta\nu} p^\mu + \eta^{\beta\mu} p^\nu) q^\alpha - (\eta^{\alpha\nu} q^\mu + \eta^{\alpha\mu} q^\nu) p^\beta , \end{aligned}$$