TORSIONAL OSCILLATIONS OF STRANGE STARS

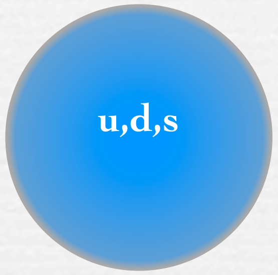
Massimo Mannarelli
INFN-LNGS
massimo@lngs.infn.it

MM, A. Parisi, G. Pagliaroli and L.Pilo, Phys.Rev. D89 (2014) 103014

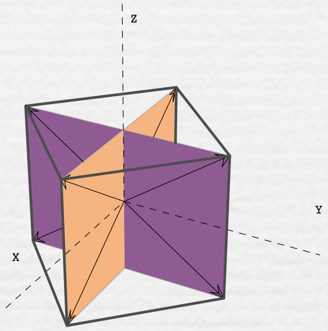
QCD@work 2014

Outline

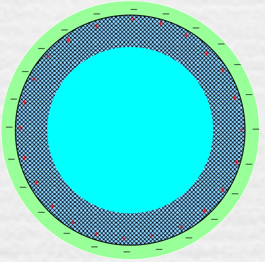
- Strange stars



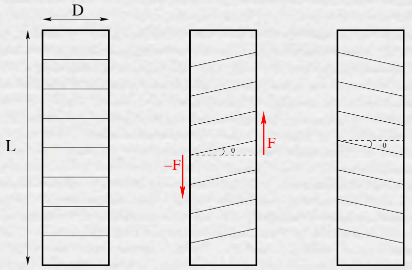
- Color superconductors



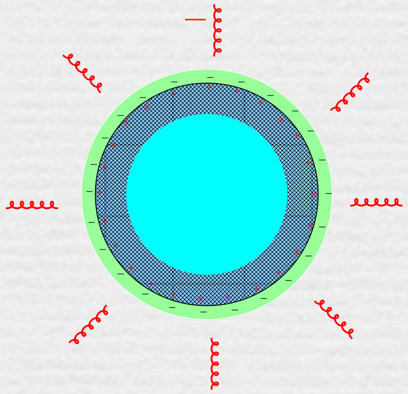
- The strange star surface



- Torsional oscillations of the strange star crust



- Emission of photons



Advertisement

RMP 86, 509 (2014)

REVIEWS OF MODERN PHYSICS, VOLUME 86, APRIL–JUNE 2014

Crystalline color superconductors

Roberto Anglani[*]

Institute of Intelligent Systems for Automation, National Research Council, CNR-ISSIA, Via Amendola 122/D-O, I-70126 Bari, Italy

Roberto Casalbuoni[†]

Department of Physics, University of Florence and INFN Via G. Sansone 1, 50019 Sesto Fiorentino (FI), Italy

Marco Ciminale[‡]

Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR), Viale Trastevere 76/a, 00153 Roma, Italy

Nicola Ippolito[§]

INFN, Sezione di Bari, Via E. Orabona 4, 70126 Bari, Italy

Raoul Gatto[‖]

Departement de Physique Theorique, Universite de Geneve, CH-1211 Geneve 4, Switzerland

Massimo Mannarelli[¶]

INFN, Laboratori Nazionali del Gran Sasso, Via G. Acitelli 22, 67100 Assergi (AQ), Italy

Marco Ruggieri[**]

Department of Physics and Astronomy, University of Catania, Via S. Sofia 64, I-95125 Catania, Italy

(published 30 April 2014)

Inhomogeneous superconductors and inhomogeneous superfluids appear in a variety of contexts including quark matter at extreme densities, fermionic systems of cold atoms, type-II cuprates, and organic superconductors. In the present review the focus is on properties of quark matter at high baryonic density, which may exist in the interior of compact stars. The conditions realized in these stellar objects tend to disfavor standard symmetric BCS pairing and may favor an inhomogeneous color superconducting phase. The properties of inhomogeneous color superconductors are discussed in detail and in particular of crystalline color superconductors. The possible astrophysical signatures associated with the presence of crystalline color superconducting phases within the core of compact stars are also reviewed.

DOI: 10.1103/RevModPhys.86.509 PACS numbers: 12.38.−t, 21.65.Qr, 97.60.Jd, 74.20.−z

CONTENTS

[*]anglani@ba.issia.cnr.it
[†]casalbuoni@fi.infn.it
[‡]marco.ciminale@ba.infn.it
[§]nicola.ippolito@ba.infn.it
[‖]Raoul.Gatto@unige.ch
[¶]massimo@lngs.infn.it
[**]marco.ruggieri@lns.infn.it

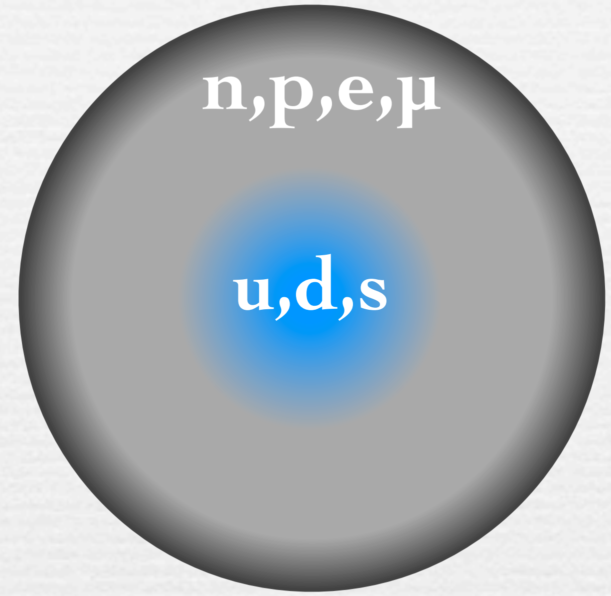
STRANGE STARS

Taxonomy of compact stars

Neutron star

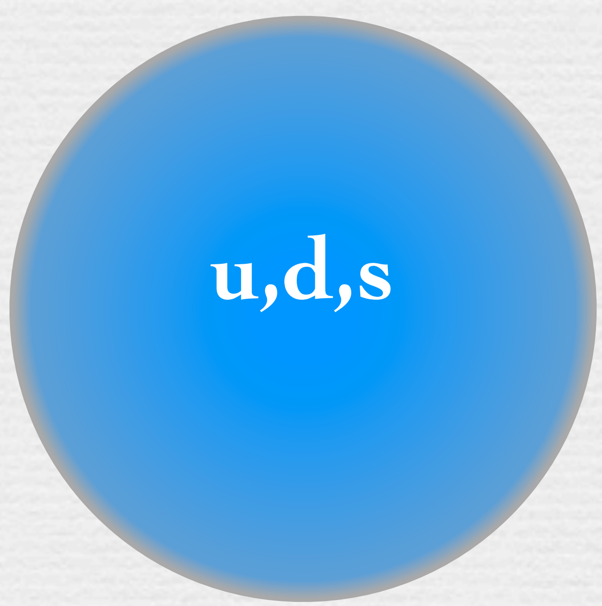
$R \sim 10\ \mathrm{km}\quad M = 1 - 2.5\,M_\odot$

Hybrid star



$R \sim 10\ \mathrm{km}\quad M < 2\,M_\odot$

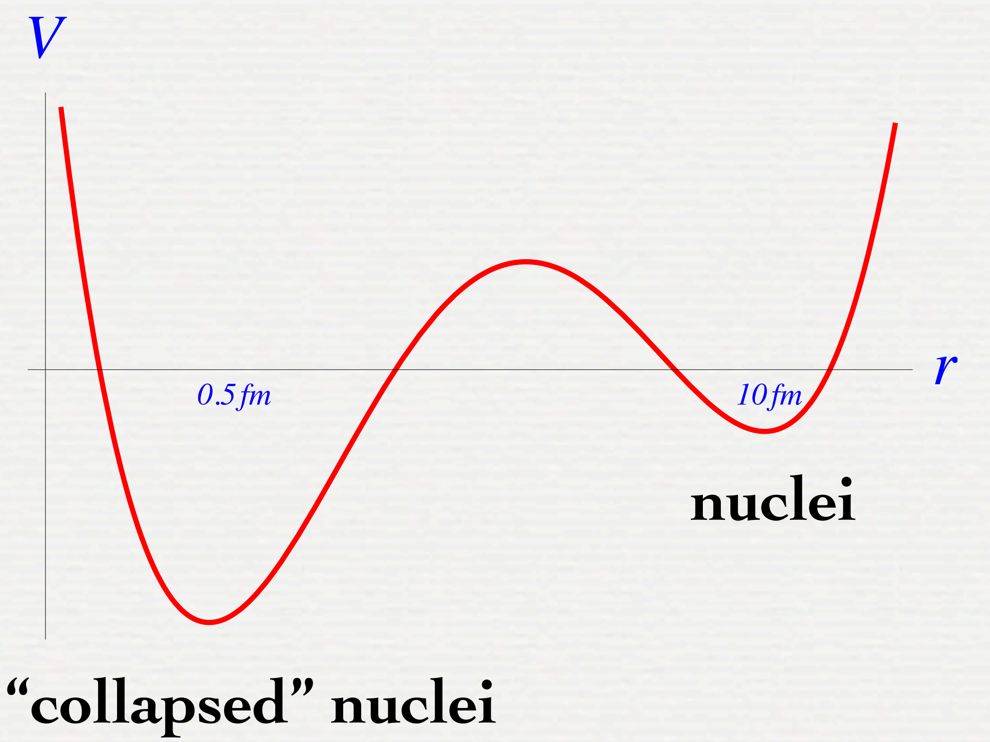
Strange star



$R \sim 0 - 10\ \mathrm{km}\quad M < 2.5\,M_\odot$

Strange stars

pictorial description

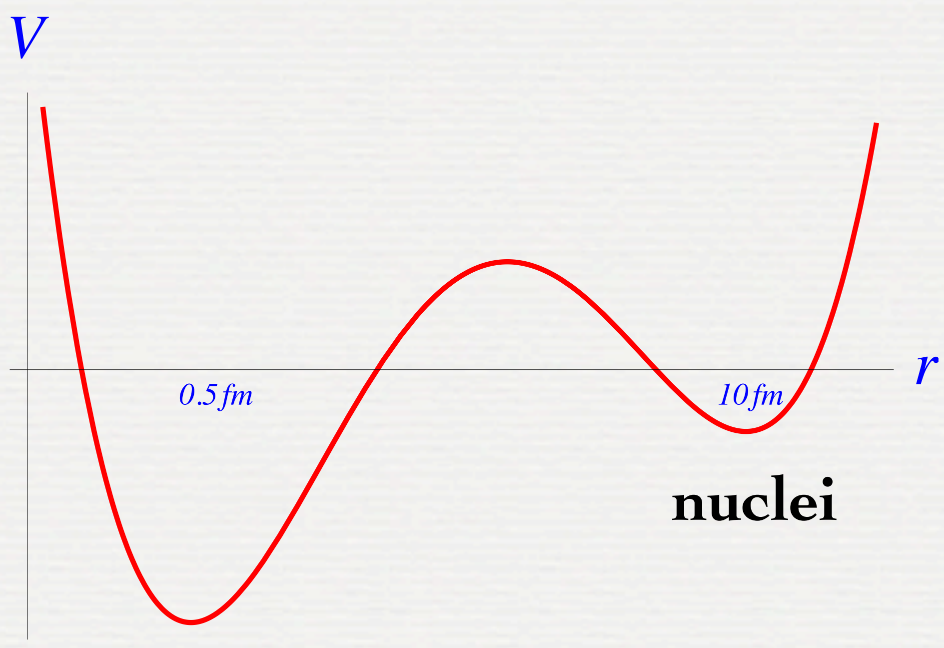


Strange matter hypothesis: uds quark matter more stable than standard hadrons

A.Bodmer Phys. Rev. D4, 1601 (1971)
E.Witten Phys. Rev. D30, 272 (1984)

Strange stars

pictorial description



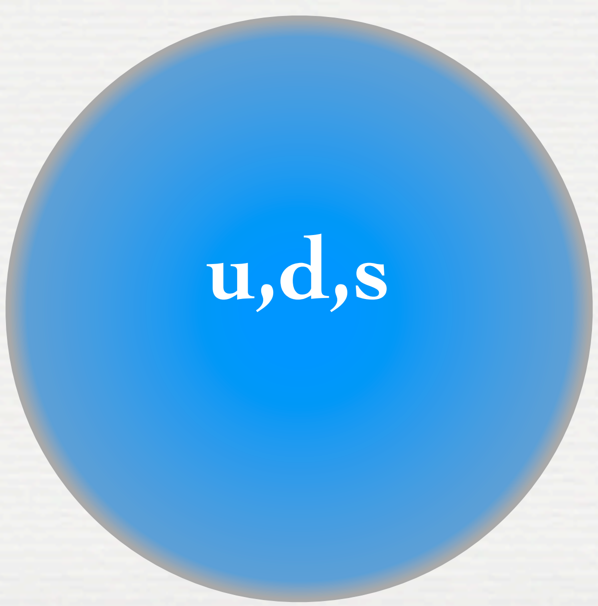
"collapsed" nuclei

Strange matter hypothesis: uds quark matter more stable than standard hadrons

A.Bodmer Phys. Rev. D4, 1601 (1971)
E.Witten Phys. Rev. D30, 272 (1984)

If true, stars entirely made of quark matter exist

C. Alcock, E. Farhi and A. Olinto, Astrophys.J. 310, 261 (1986)



Small clumps of uds matter are bound by the strong interaction. It is like a big hadron.

Gravity plays a role in large objects.

Hydrostatic equilibrium of strange stars

TOV equation $$\frac{\partial p}{\partial r} = -\frac{G(p+\rho)\left(m + 4\pi p\, r^3\right)}{r(r - 2G\, m)}$$

determine of the hydrostatic equilibrium configuration, once the Equation of State P(ϱ) is known

Hydrostatic equilibrium of strange stars

TOV equation

$$\frac{\partial p}{\partial r} = -\frac{G(p+\rho)\left(m + 4\pi p\, r^3\right)}{r(r - 2G\,m)}$$

determine of the hydrostatic equilibrium configuration, once the Equation of State P(ϱ) is known

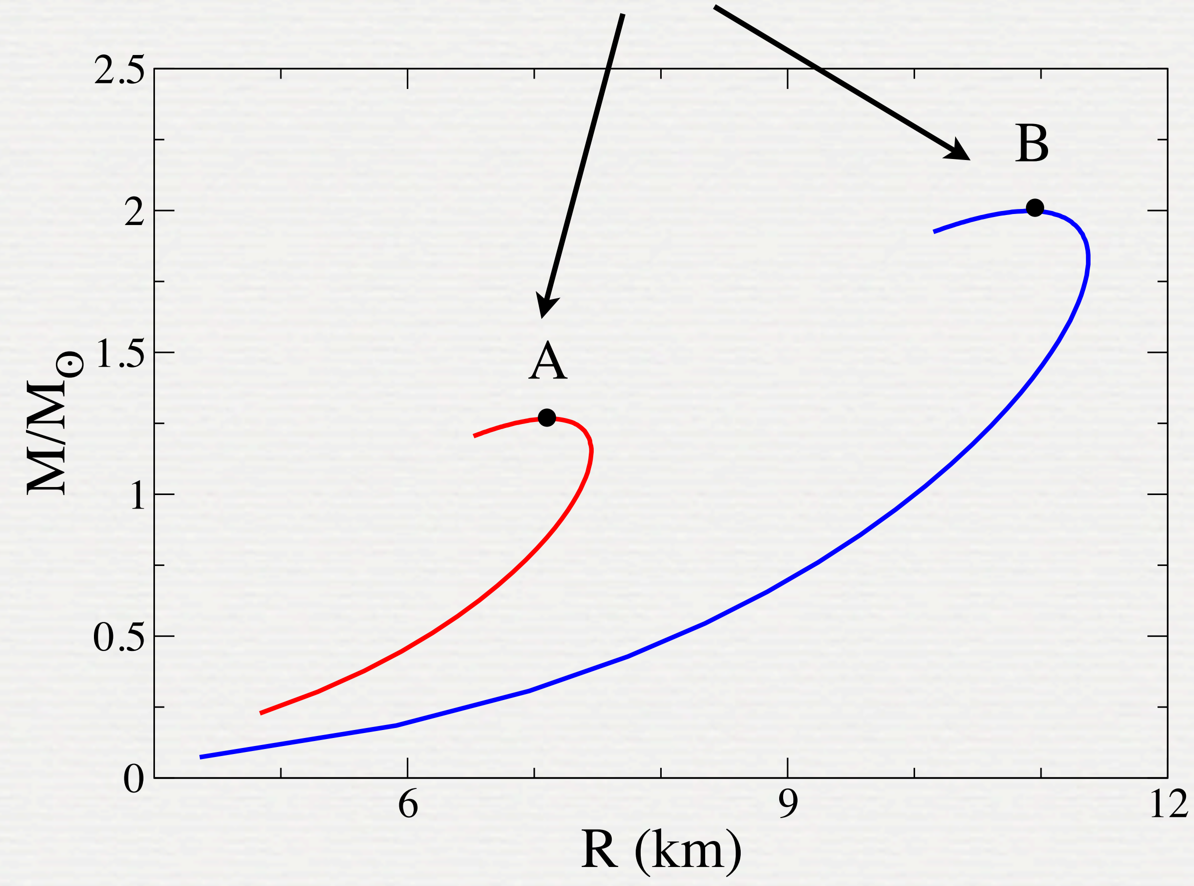
our representative models

We use the free energy

$$\Omega_{\rm QM} = -\frac{3}{4\pi^2} a_4 \mu^4 + \frac{3}{4\pi^2} a_2 \mu^2 + B_{\rm eff}$$

Alford et al. Astrophys.J. 629, 969 (2005)

Varying the parameters several configurations can be created



What quark matter is inside a strange star?

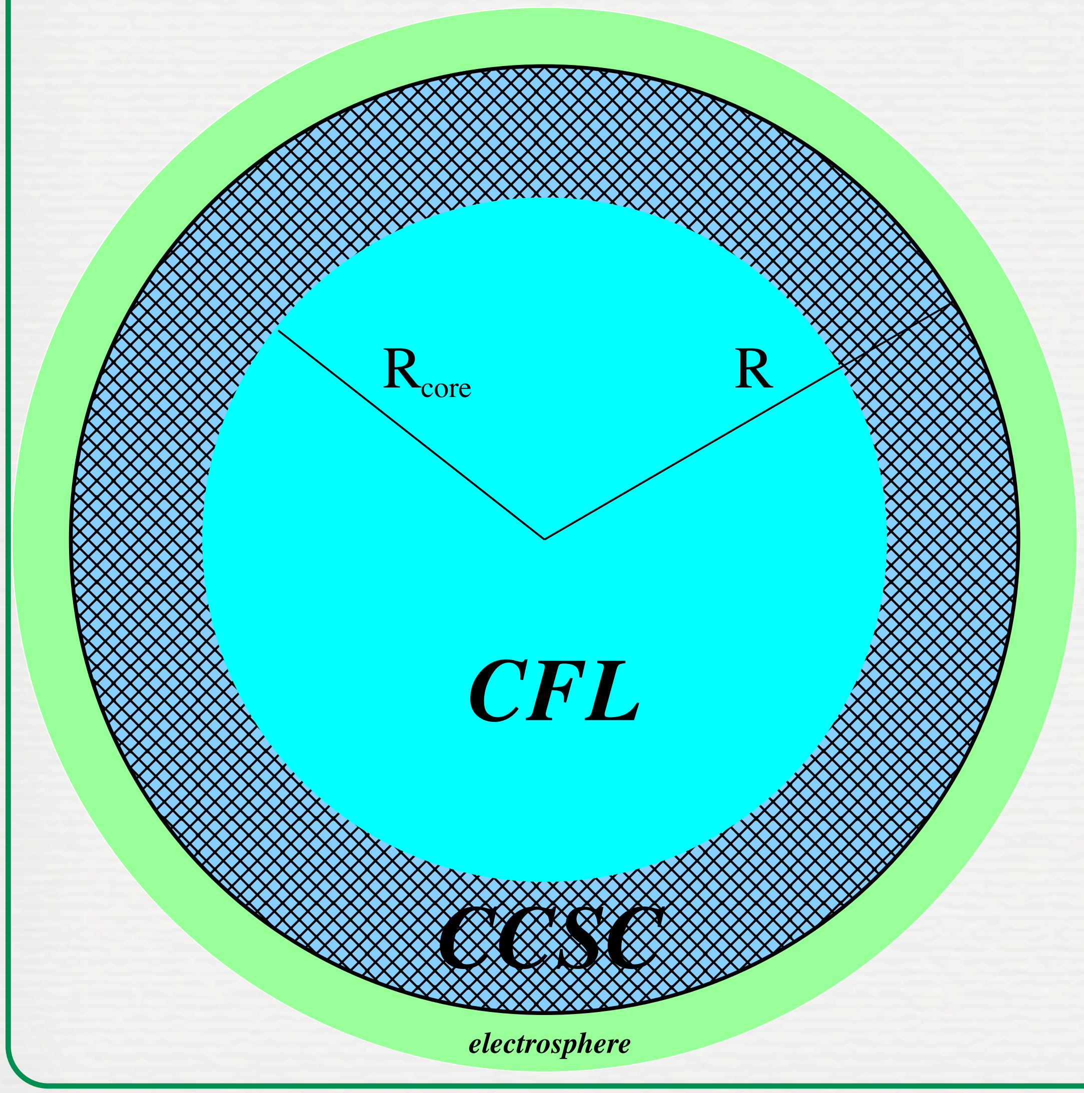
QCD perturbative calculations and lattice QCD simulations are not feasible.

Since the system is cold and there is attractive interaction, we expect that it becomes a **color superconductor**: system with condensation of diquarks.

What quark matter is inside a strange star?

QCD perturbative calculations and lattice QCD simulations are not feasible.

Since the system is cold and there is attractive interaction, we expect that it becomes a **color superconductor**: system with condensation of diquarks.



Strange stars may actually be "color superconducting stars"

Two of the candidate phases:

1) The color flavor locked (CFL) phase at large density

2) The crystalline color superconducting (CCSC) phase at smaller densities.

COLOR SUPERCONDUCTORS

Color Flavor Locking (CFL) phase

Suppose that the strange quark mass is "small".
All quarks should be treated on an equal footing. Pairing of quarks of all flavors and colors

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \Delta_{\rm CFL} \sum_{I=1}^{3} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

Alford, Rajagopal, Wilczek hep-ph/9804403

Color Flavor Locking (CFL) phase

Suppose that the strange quark mass is "small".
All quarks should be treated on an equal footing. Pairing of quarks of all flavors and colors

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \Delta_{\mathrm{CFL}} \sum_{I=1}^{3} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

Alford, Rajagopal, Wilczek hep-ph/9804403

All quarks, contribute coherently to pairing.
Very robust. It is expected to be the ground state of quark matter at very large densities

Color Flavor Locking (CFL) phase

Suppose that the strange quark mass is "small".
All quarks should be treated on an equal footing. Pairing of quarks of all flavors and colors

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \Delta_{\mathrm{CFL}} \sum_{I=1}^{3} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

Alford, Rajagopal, Wilczek hep-ph/9804403

All quarks, contribute coherently to pairing.
Very robust. It is expected to be the ground state of quark matter at very large densities

Breaking pattern

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

BASIC FACTS

1) Cooper pairs have nonzero momentum

2) The crystal reciprocal lattice is given by the direction of these momenta

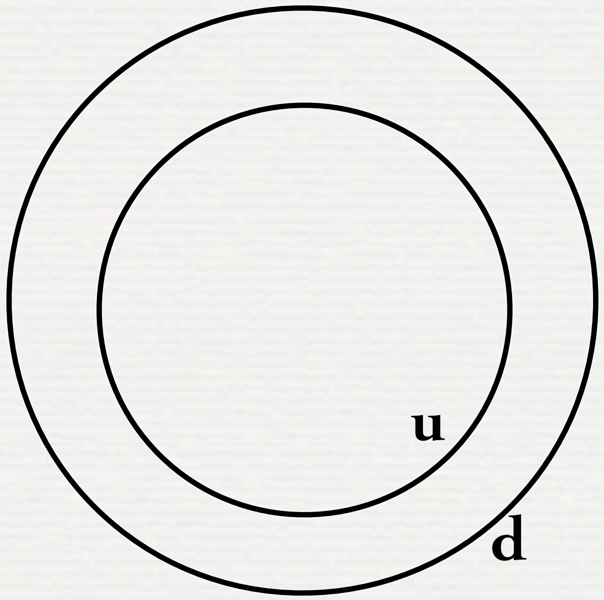
Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

BASIC FACTS

1) Cooper pairs have nonzero momentum

2) The crystal reciprocal lattice is given by the direction of these momenta



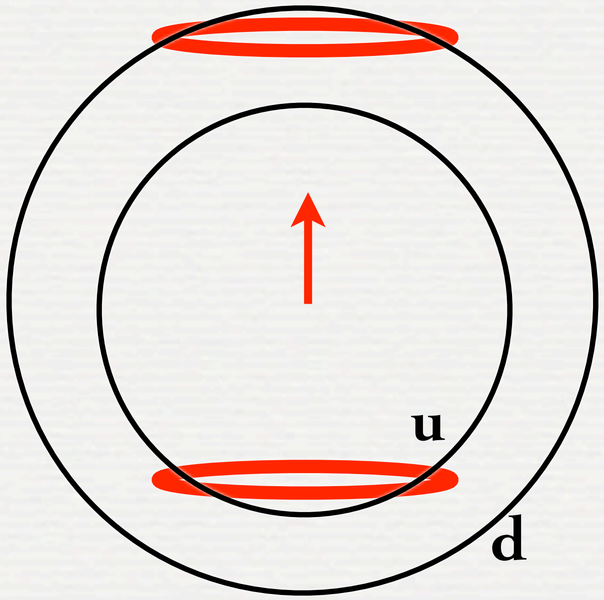
Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

BASIC FACTS

1) Cooper pairs have nonzero momentum

2) The crystal reciprocal lattice is given by the direction of these momenta



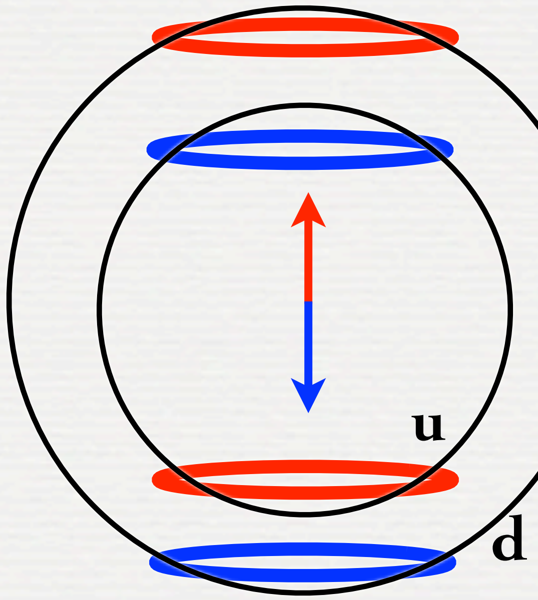
Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

BASIC FACTS

1) Cooper pairs have nonzero momentum

2) The crystal reciprocal lattice is given by the direction of these momenta



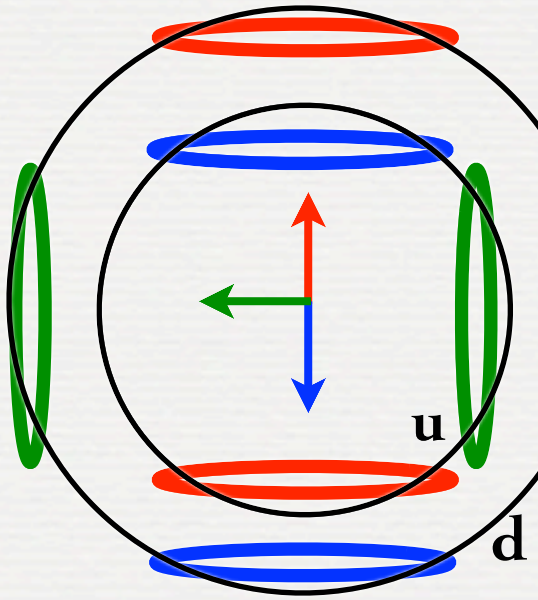
Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

BASIC FACTS

1) Cooper pairs have nonzero momentum

2) The crystal reciprocal lattice is given by the direction of these momenta



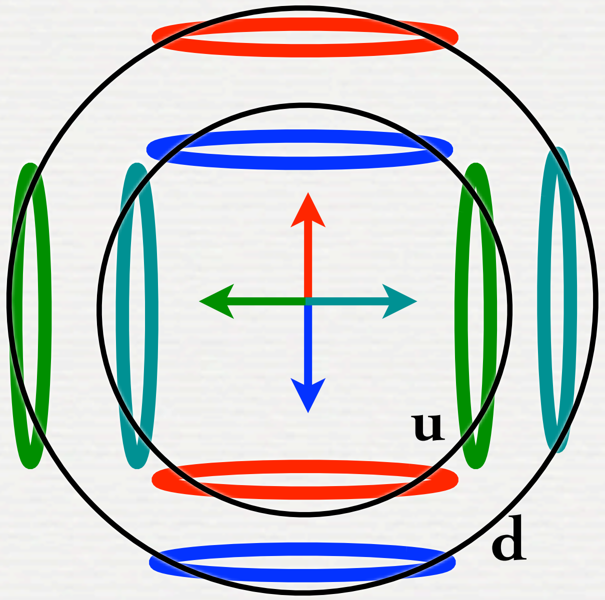
Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

BASIC FACTS

1) Cooper pairs have nonzero momentum

2) The crystal reciprocal lattice is given by the direction of these momenta



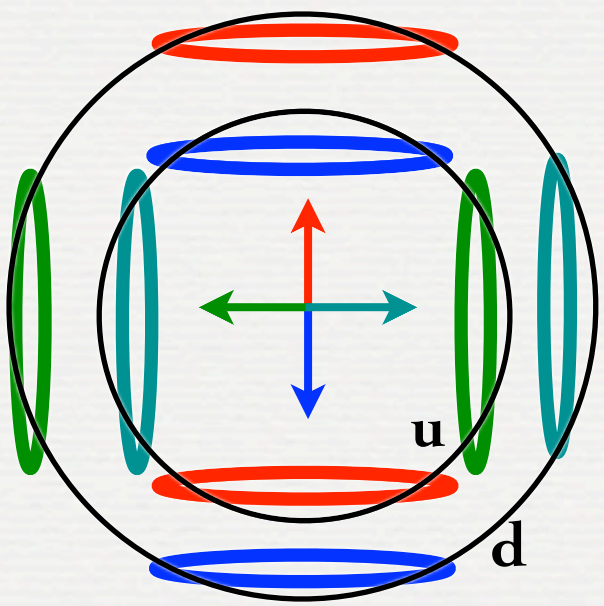
Crystalline Color Superconducting (CCSC) phase

Def: color superconducting phase with a crystalline structure given by the spatial modulation of the condensate

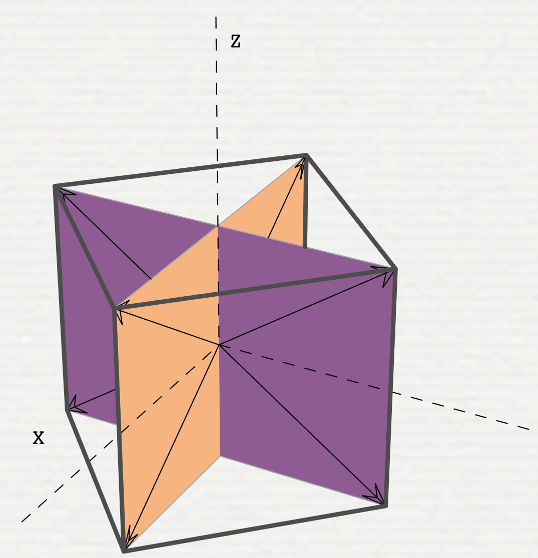
BASIC FACTS

1) Cooper pairs have nonzero momentum

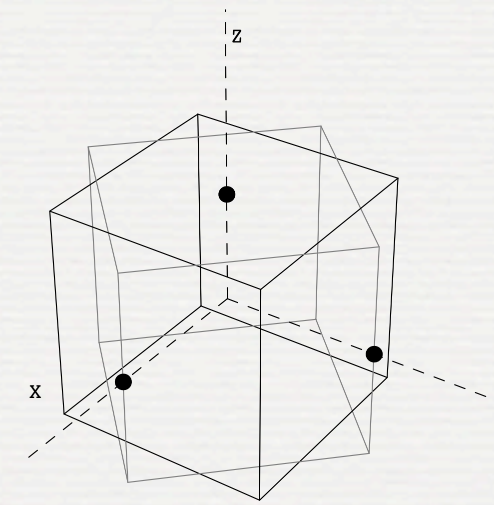
2) The crystal reciprocal lattice is given by the direction of these momenta



CX



2cube45z



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

THE STRANGE STAR SURFACE

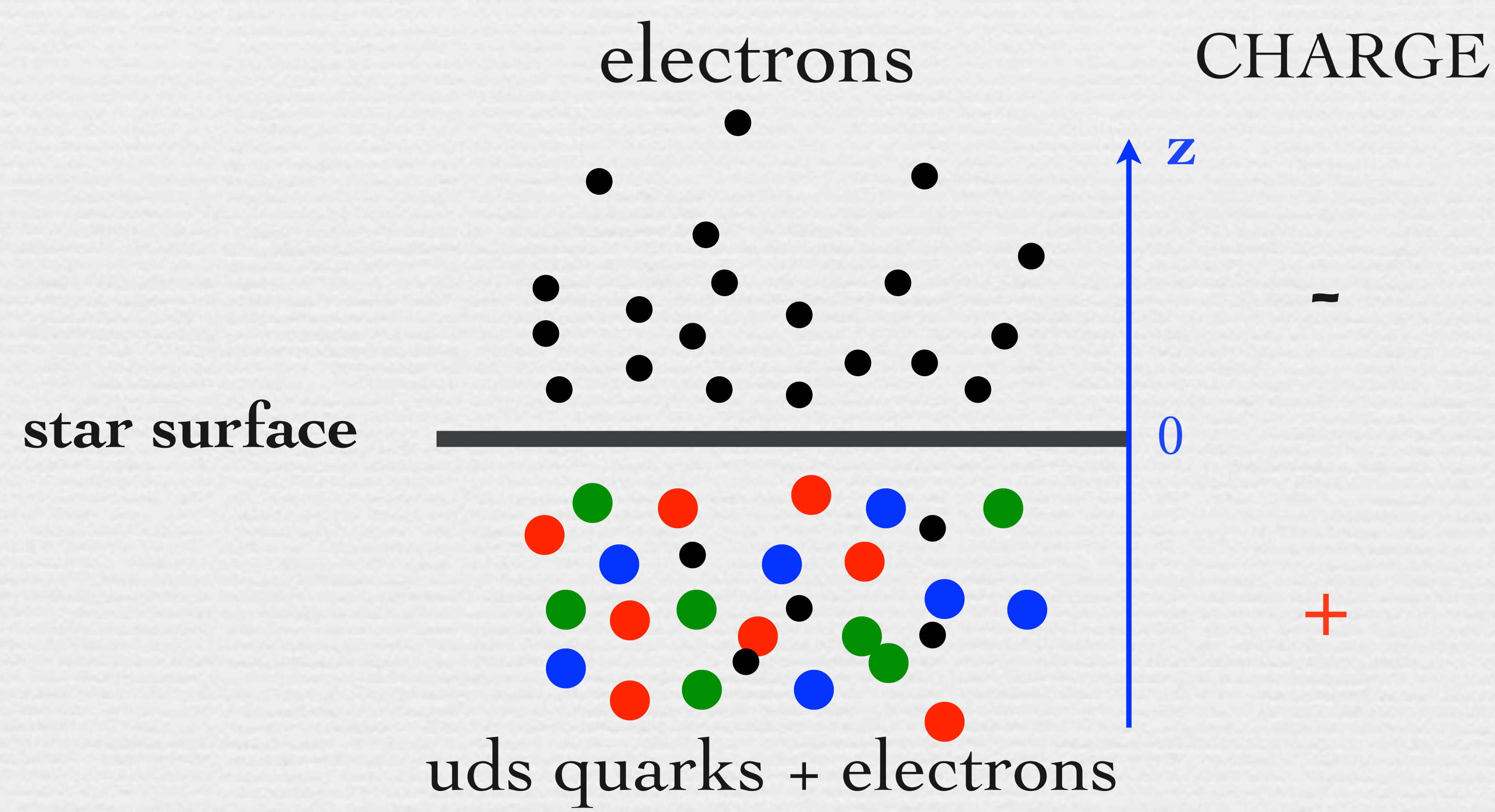
Charge distribution

BASIC OBSERVATION
Quarks are confined inside the strange star by the strong interaction.
Electrons are bound only by the electromagnetic interaction, hovering over the crust

Charge distribution

BASIC OBSERVATION
Quarks are confined inside the strange star by the strong interaction.
Electrons are bound only by the electromagnetic interaction, hovering over the crust

Planar approximation (valid if any length scale << R)



Charge distribution

BASIC OBSERVATION
Quarks are confined inside the strange star by the strong interaction.
Electrons are bound only by the electromagnetic interaction, hovering over the crust

Local density approximation

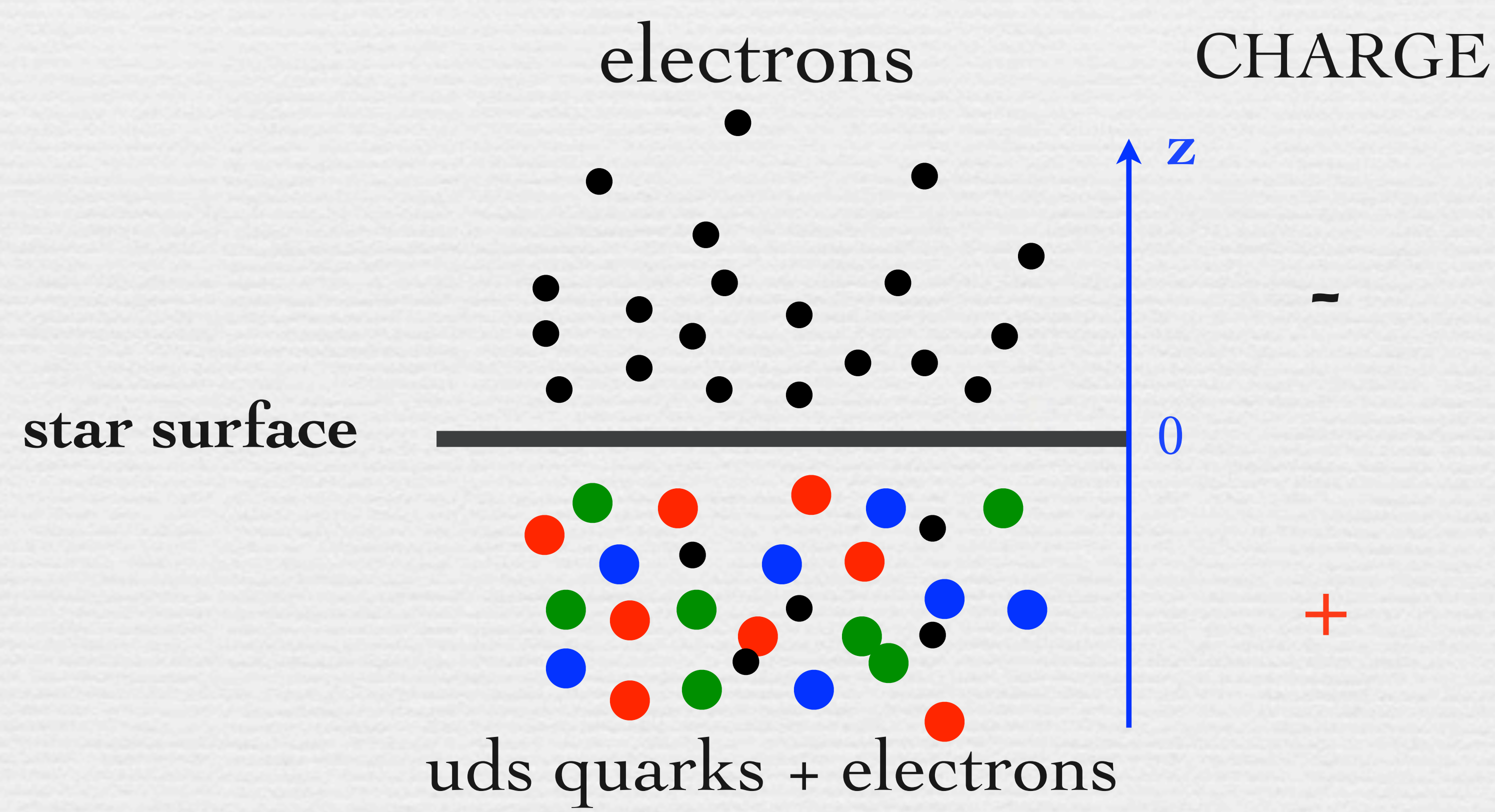
$$\mu_i(z) = \mu_i + eQ_i\phi(z)$$

$$n_i(z) = C_i \frac{k_{F,i}(z)^3}{3\pi^2}$$

Poisson's equation

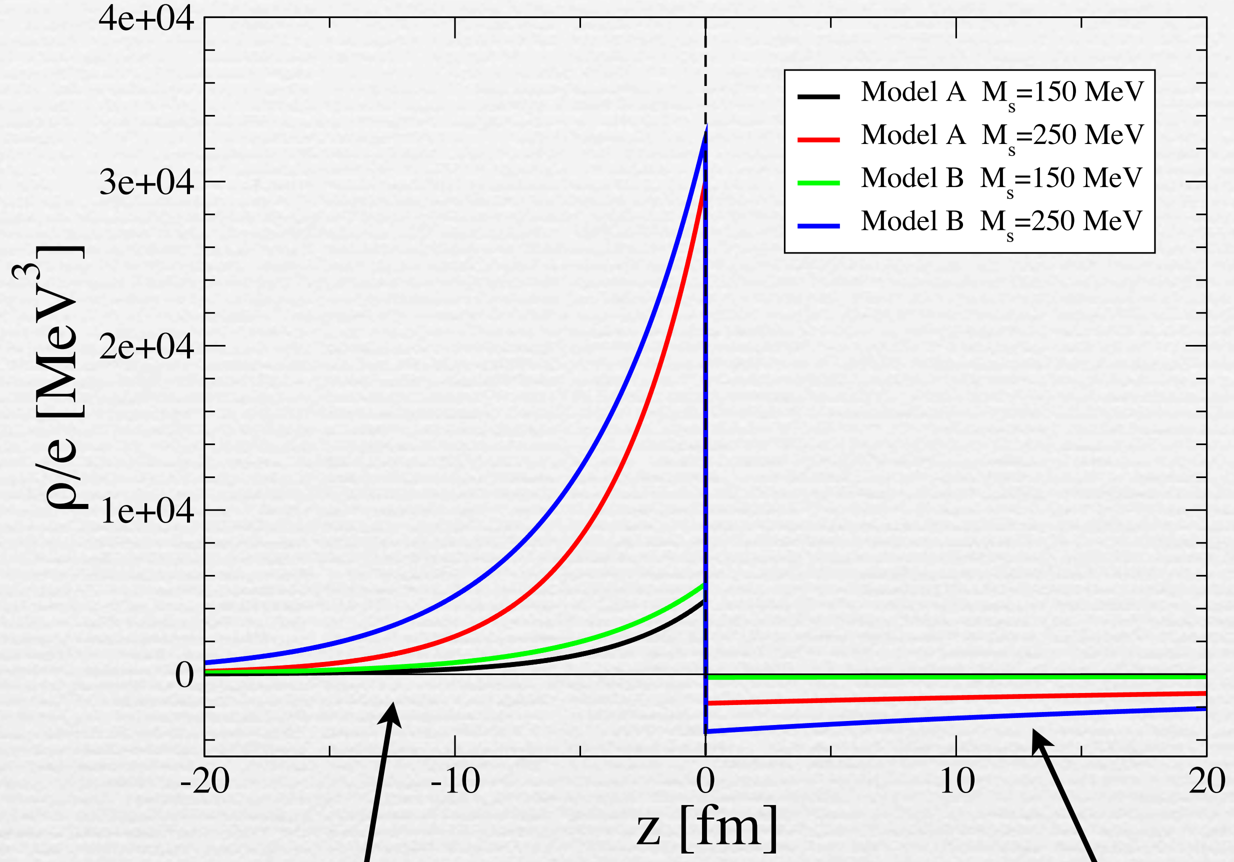
$$\frac{d^2\phi}{dz^2} = e\sum_i Q_i\, n_i(z)$$

Planar approximation (valid if any length scale << R)



eQ_i : charge of the particles i

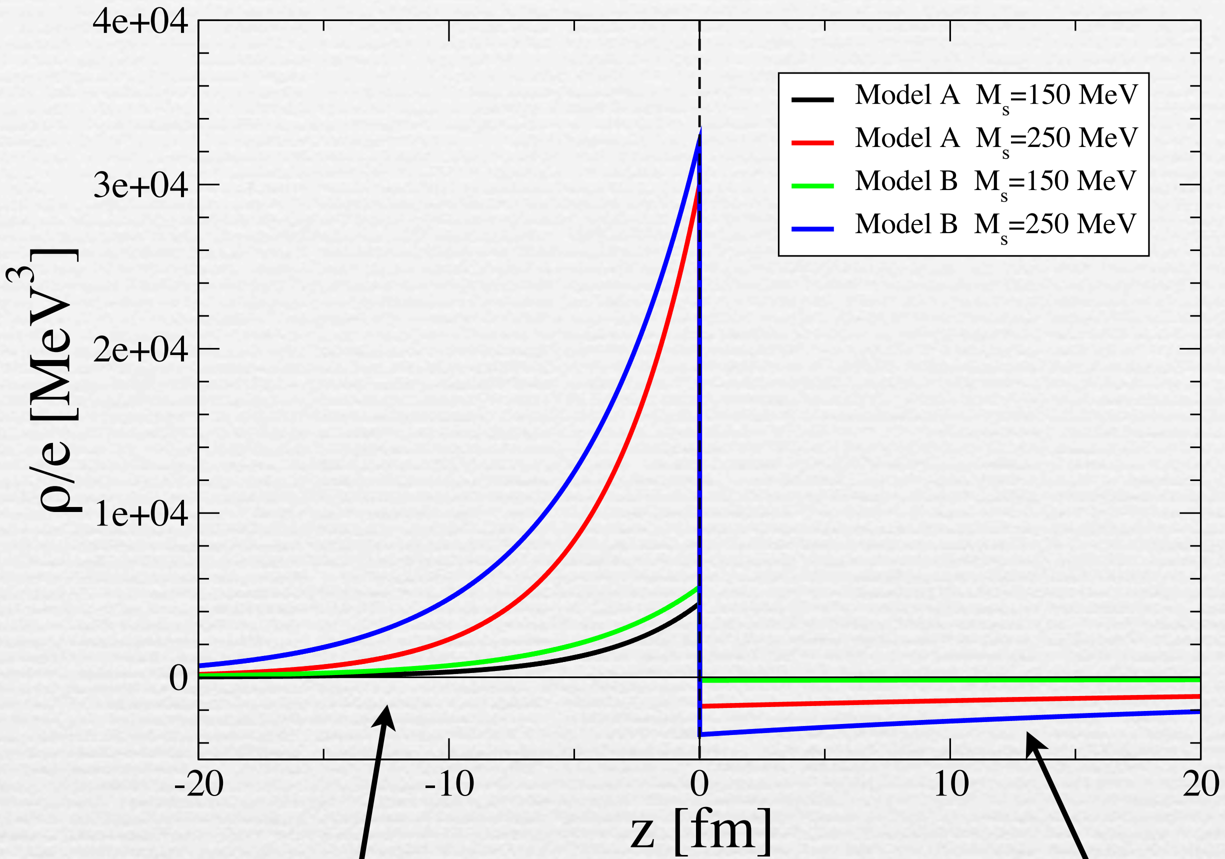
C_i : color degeneracy factor



$$\rho/e \sim e^{z/d_{\text{quark}}} \qquad \rho/e \sim \frac{1}{(1 + z/d_{\text{electron}})^3}$$

Debye length scales

$$d_{\text{quark}} \sim 1 - 10 \text{ fm} \qquad d_{\text{electron}} \sim 10^2 - 10^3 \text{ fm}$$

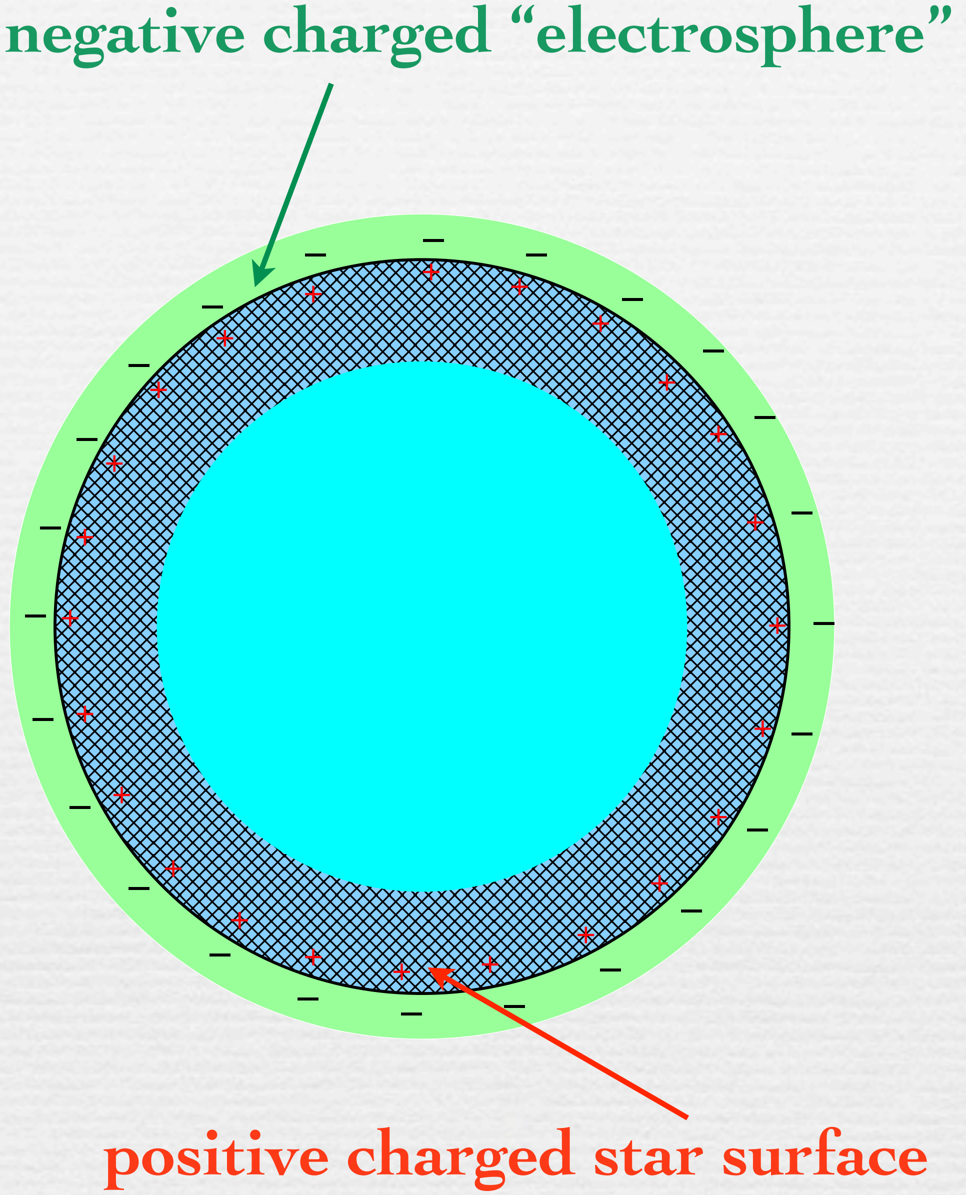


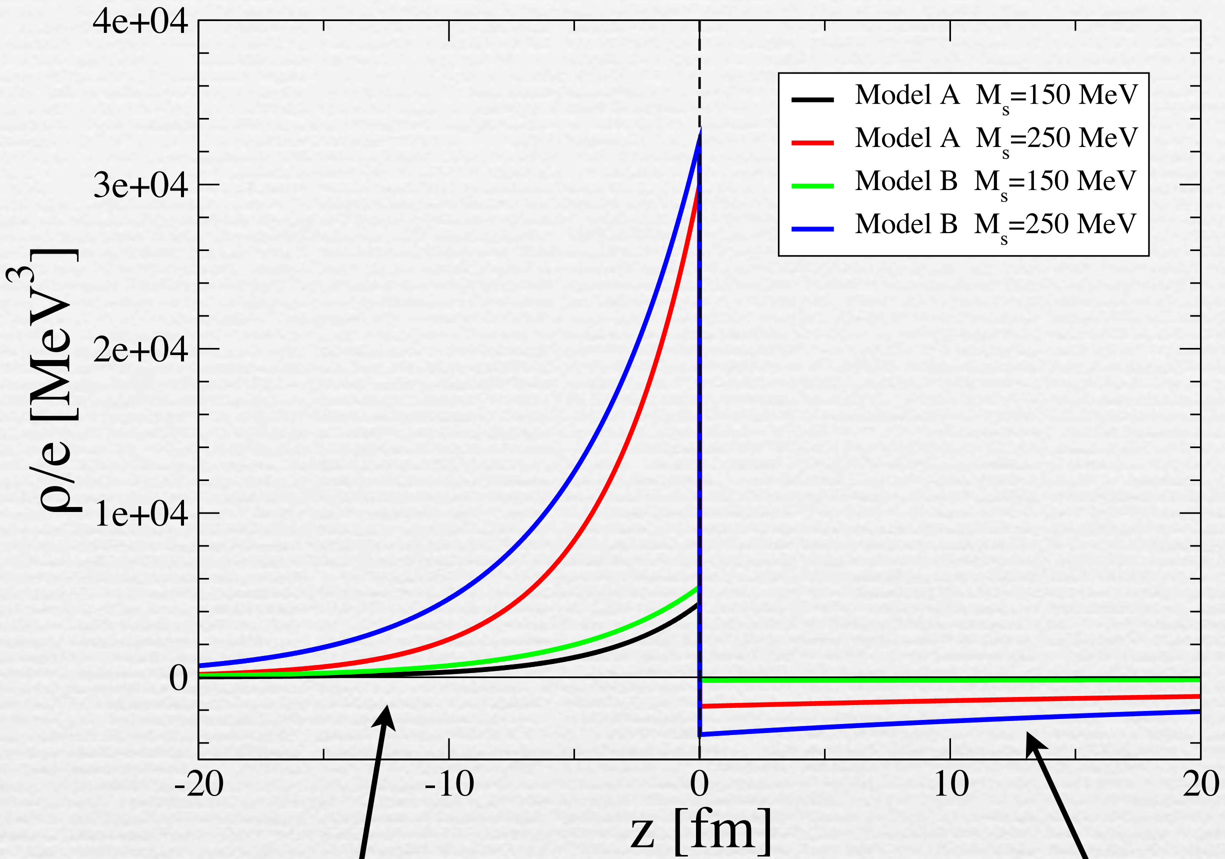
$$\rho/e \sim e^{z/d_{\mathrm{quark}}}$$

$$\rho/e \sim \frac{1}{(1 + z/d_{\mathrm{electron}})^3}$$

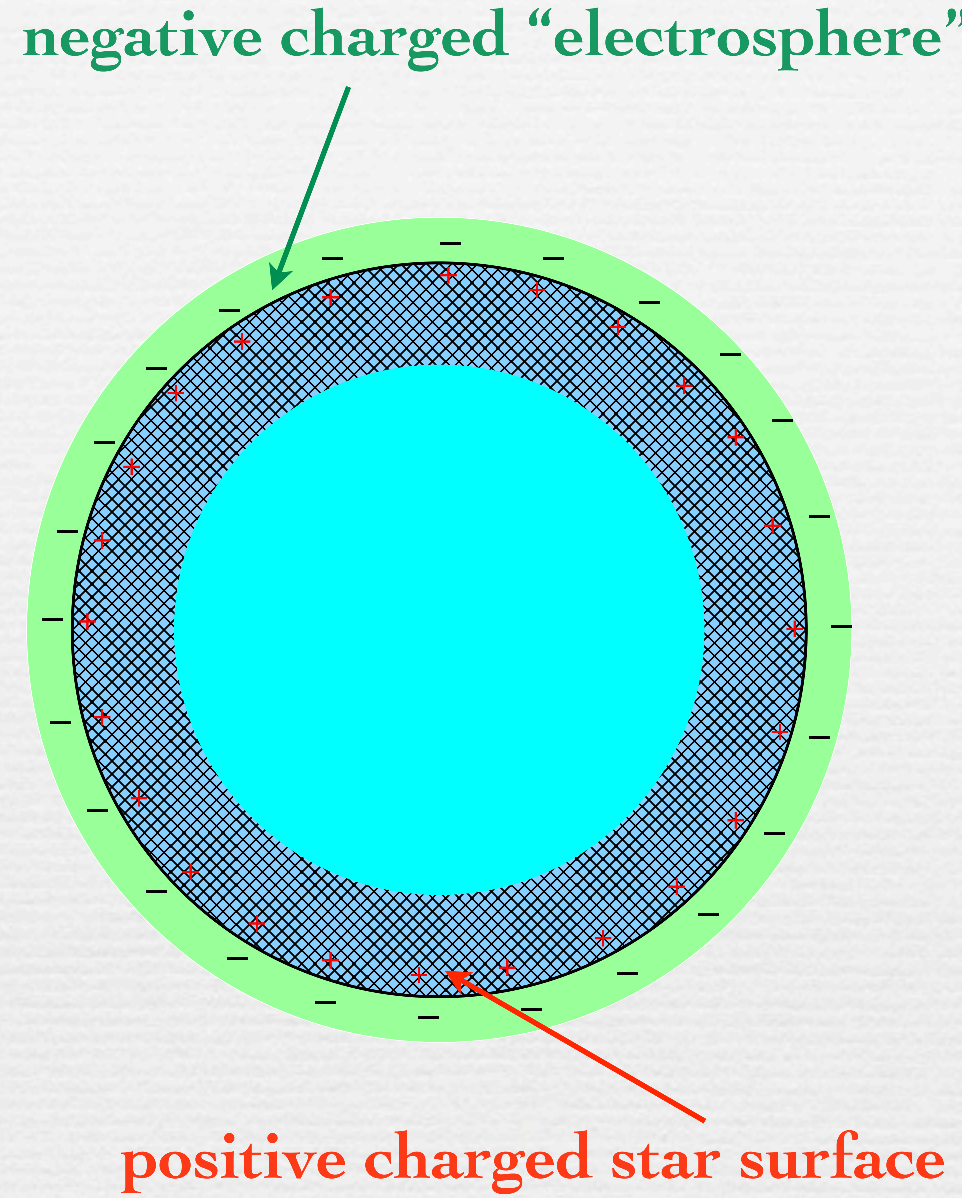
Debye length scales

$$d_{\mathrm{quark}} \sim 1 - 10 \text{ fm} \qquad d_{\mathrm{electron}} \sim 10^2 - 10^3 \text{ fm}$$





$\rho/e \sim e^{z/d_{\rm quark}}$ $\qquad$ $\rho/e \sim \dfrac{1}{(1+z/d_{\rm electron})^3}$



Debye length scales

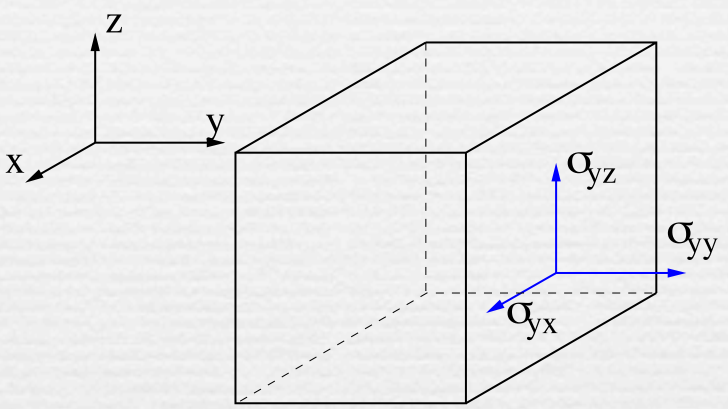
$d_{\rm quark} \sim 1 - 10 \text{ fm}$ $\qquad$ $d_{\rm electron} \sim 10^2 - 10^3 \text{ fm}$

The electric field at the surface is very large $E \sim 10^{17} - 10^{18}\ \text{V/cm}$ and directed outward

TORSIONAL OSCILLATIONS OF THE STRANGE STAR CRUST

Displacement of the crystal structure

Elastic deformation of a stressed crystal (Landau Lifsits, vol. 7)



displacement vector $u_i = x'_i - x_i$

deformation tensor $u_{ij} = \dfrac{u_{i,j} + u_{j,i}}{2}$

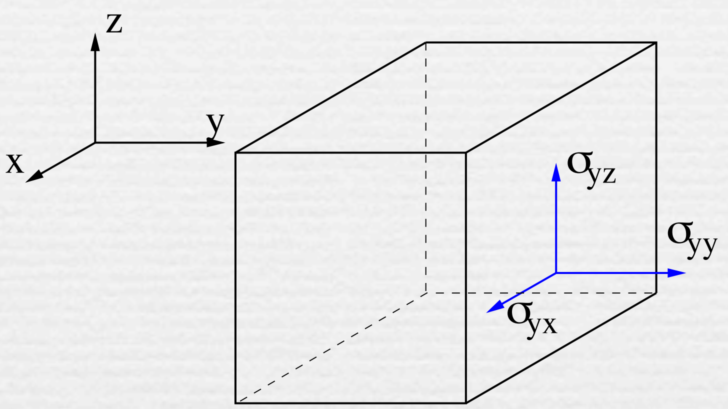
stress tensor $\sigma_{ij} = K u_{kk}\delta_{ij} + 2\nu\left(u_{ij} - \dfrac{1}{3}u_{kk}\delta_{ij}\right)$

compressibility

shear modulus

Displacement of the crystal structure

Elastic deformation of a stressed crystal (Landau Lifsits, vol. 7)



displacement vector $u_i = x'_i - x_i$

deformation tensor $u_{ij} = \dfrac{u_{i,j} + u_{j,i}}{2}$

stress tensor $\sigma_{ij} = K u_{kk}\delta_{ij} + 2\nu\left(u_{ij} - \frac{1}{3}u_{kk}\delta_{ij}\right)$

compressibility shear modulus

$\nu_{\rm CCSC} \sim 2.47\,{\rm MeV/fm^3}$

20 to 1000 times more rigid than the crust of neutron stars

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Aside: gravitational wave emission

Rotating and/or oscillating strange/hybrid stars with a CCSC crust/core, which are somehow deformed to nonaxisymmetric configurations, can efficiently emit gravitational waves

Lin, Phys.Rev. D76 (2007) 081502,
Phys.Rev. D88 (2013)

Haskell et al. Phys.Rev. Lett.99. 231101
(2007)

Knippel et al. Phys.Rev. D79 (2009) 083007

Rupak, Gautam et al. Phys.Rev. C88 (2013) 6,
065801

Torsional oscillations

Crystals can sustain various type of oscillations. We restrict to **torsional oscillations**

$$\boxed{\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \text{ and } u_r = 0}$$

Properties: no volume variation, no radial displacement, the wave is transverse

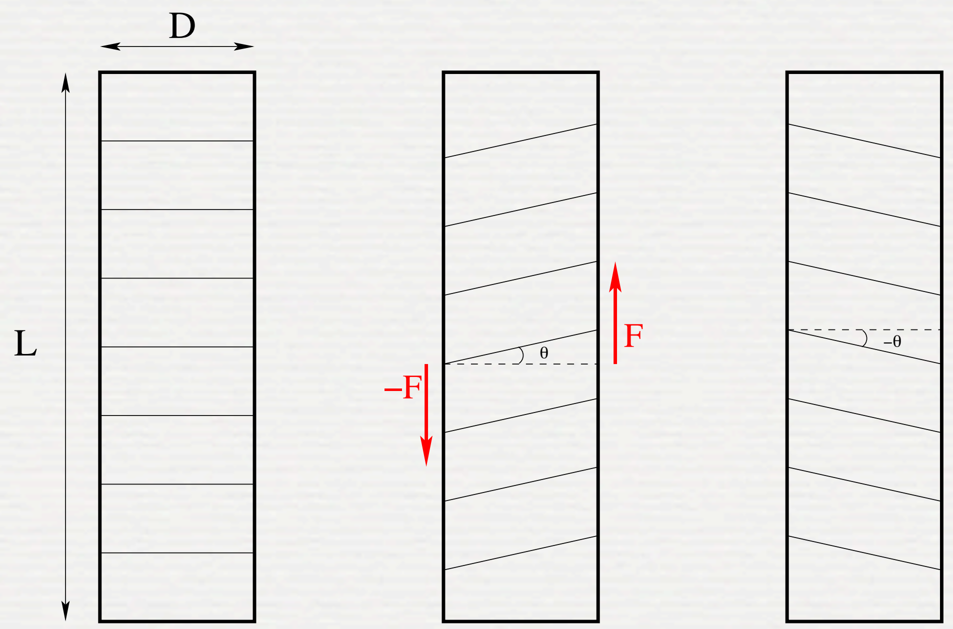
Torsional oscillations

Crystals can sustain various type of oscillations. We restrict to **torsional oscillations**

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \text{ and } u_r = 0$$

Properties: no volume variation, no radial displacement, the wave is transverse

Ex: slab oscillations



$$\omega \sim \frac{c_t}{D}$$

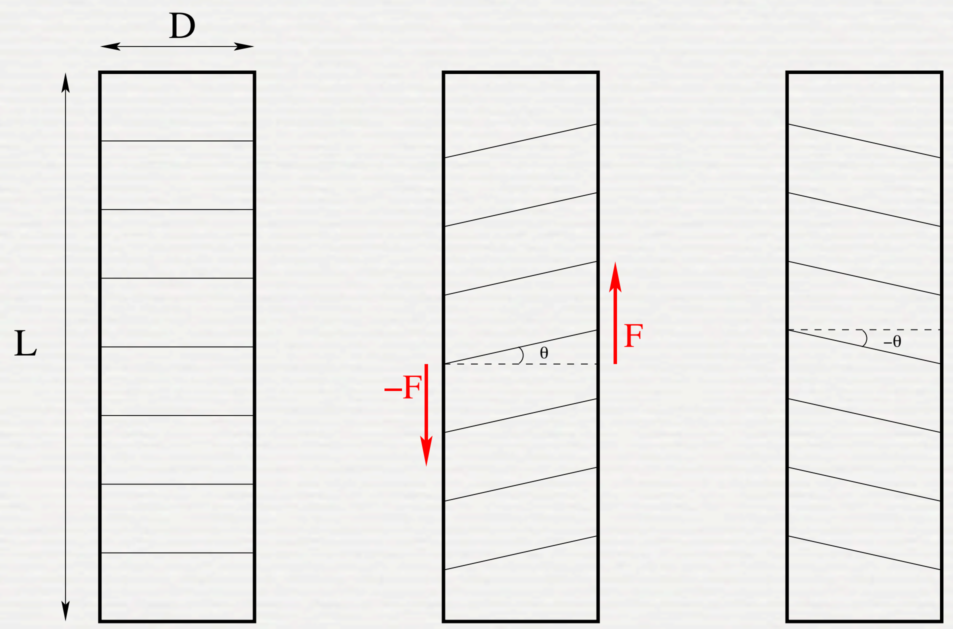
Torsional oscillations

Crystals can sustain various type of oscillations. We restrict to **torsional oscillations**

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \text{ and } u_r = 0$$

Properties: no volume variation, no radial displacement, the wave is transverse

Ex: slab oscillations



$$\omega \sim \frac{c_t}{D}$$

Wave equation

$$\omega_{nl}^2 W_{nl} = c_t^2 \left[-\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dW_{nl}}{dr}\right) + \frac{l(l+1)}{r^2} W_{nl}\right]$$

transverse (shear) velocity: $c_t^2 = \nu/\rho$ **amplitude of the n,l mode:** W_{nl}

Amplitude of the torsional oscillation

Assume that a stellar glitch triggers the torsional oscillation of the l=1, n=1 mode

fraction of the glitch energy going into the torsional oscillation

surface density

$$\alpha E_{\rm glitch} = \frac{\rho_R}{2}\int \omega_{11}^2 |\delta \boldsymbol{u}_{11}|^2 dV$$

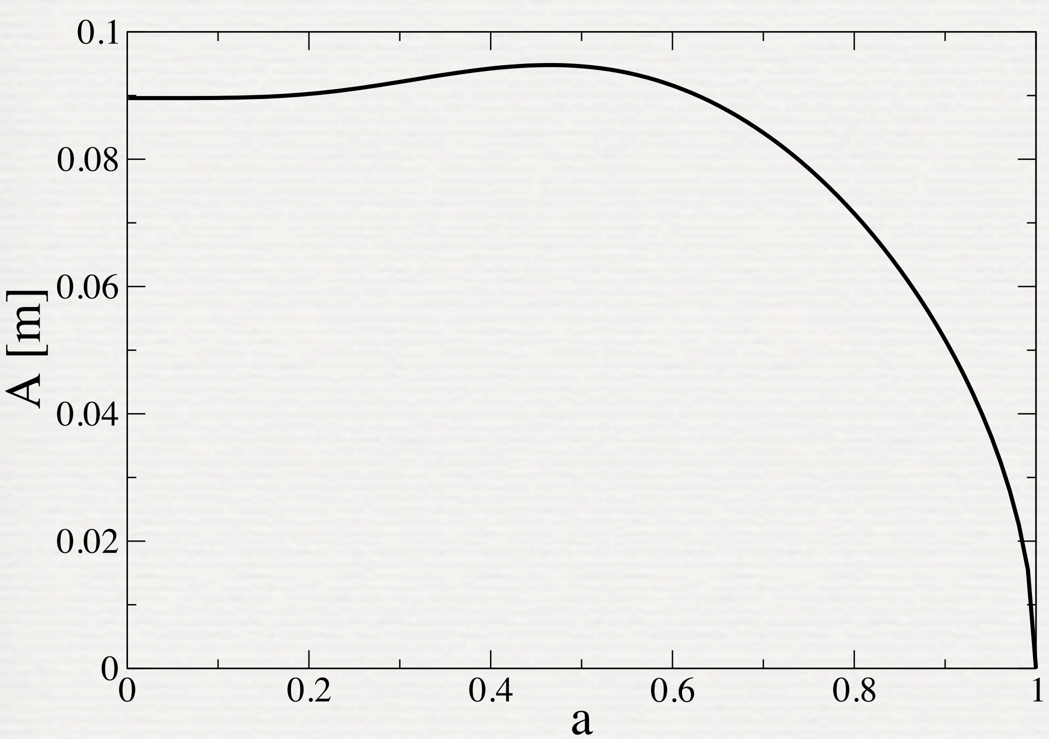
Amplitude of the torsional oscillation

Assume that a stellar glitch triggers the torsional oscillation of the l=1, n=1 mode

fraction of the glitch energy going into the torsional oscillation

surface density

$$\alpha E_{\rm glitch} = \frac{\rho_R}{2}\int \omega_{11}^2 |\delta \boldsymbol{u}_{11}|^2 dV$$

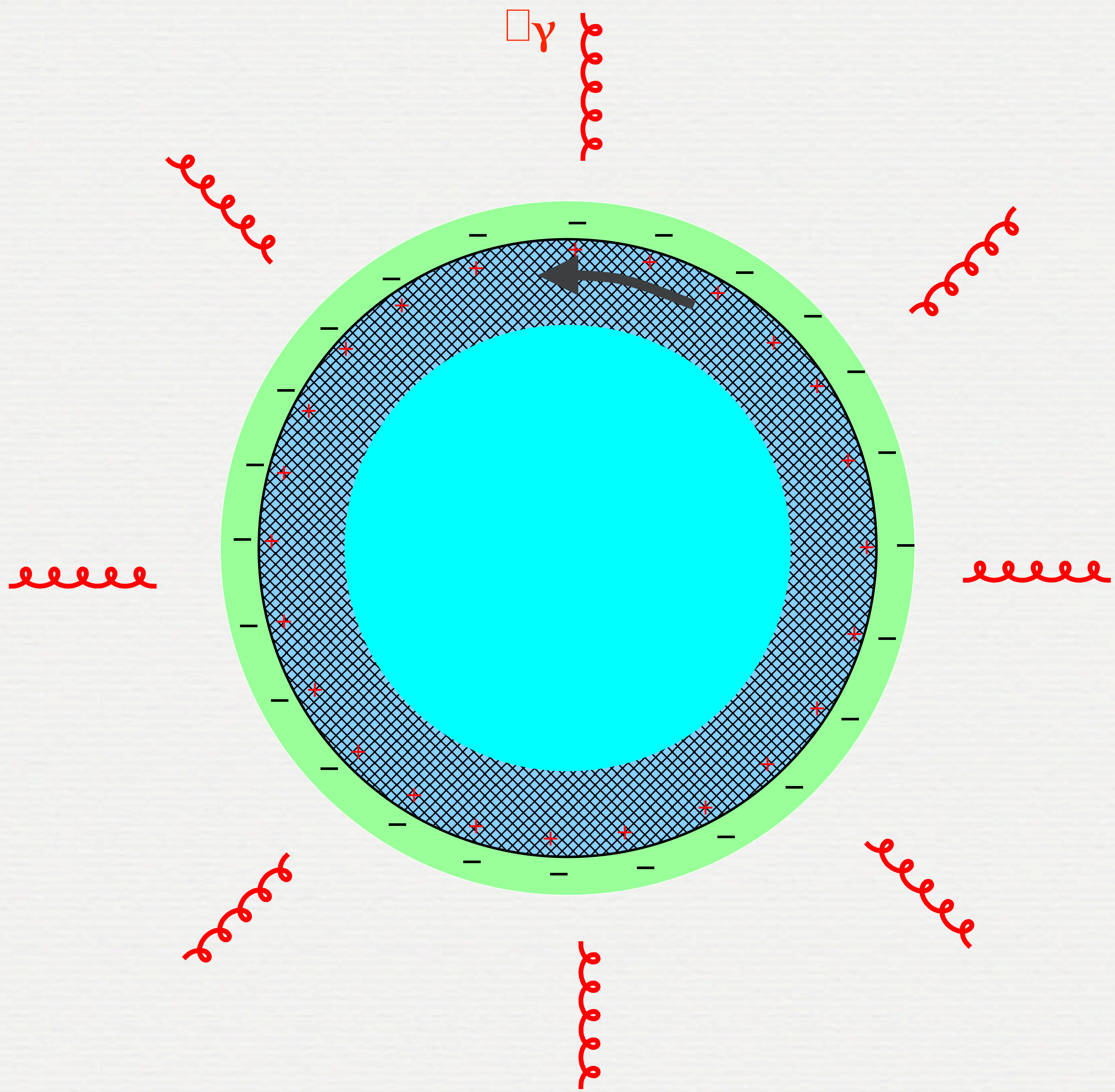


no core **no crust**

$$a = R_{\rm core}/R$$

$$W_{11}(R) = A(a)\left(\frac{\nu}{\nu_0}\right)^{-1/2}\left(\frac{R}{10{\rm km}}\right)^{-1/2}\left(\frac{\alpha E_{\rm glitch}}{E_{\rm glitch}^{\rm Vela}}\right)^{1/2}$$

EMISSION OF PHOTONS



EM emission

We model the system by an oscillating magnetic dipole

Frequency of oscillations

$$\omega_{11} \propto \frac{c_t}{R - R_c}$$

about 10 kHz for a 1 km thick crust
about 1 GHz for a 1 cm thick crust

Estimated emitted power

$$P(a) \simeq 6.4 \times 10^{41} \text{erg/s}$$

(assuming a giant Vela-like glitch as the trigger)

EM emission

We model the system by an oscillating magnetic dipole

Frequency of oscillations

$$\omega_{11} \propto \frac{c_t}{R - R_c}$$

about 10 kHz for a 1 km thick crust
about 1 GHz for a 1 cm thick crust

Estimated emitted power

$$P(a) \simeq 6.4 \times 10^{41} \text{erg/s}$$

(assuming a giant Vela-like glitch as the trigger)

Estimated electrosphere suppression

Many photons will be absorbed by the electrosphere. The Thomson suppression factor is

$$\eta_{\text{Thomson}} \gtrsim 0.1$$

Observables

Rotating Radio Transients (RRTs) are unknown sources of em radiation.
Burst duration: few milliseconds
Frequencies: of the order of GHz

M. A. McLaughlin et al., Nature 439, 817 (2006)
D. Thornton et al., Science 341, 53 (2013)

Observables

Rotating Radio Transients (RRTs) are unknown sources of em radiation.
Burst duration: few milliseconds
Frequencies: of the order of GHz

M. A. McLaughlin et al., Nature 439, 817 (2006)
D. Thornton et al., Science 341, 53 (2013)

CAN OUR STRANGE STAR MODEL EXPLAIN RRTs?

Observables

Rotating Radio Transients (RRTs) are unknown sources of em radiation.
Burst duration: few milliseconds
Frequencies: of the order of GHz

M. A. McLaughlin et al., Nature 439, 817 (2006)
D. Thornton et al., Science 341, 53 (2013)

CAN OUR STRANGE STAR MODEL EXPLAIN RRTs?

If the CCSC crust is thin, say a few centimeters thick, then **GHz frequencies can be attained**. But the **damping time we get is much less than the observed milliseconds**

Observables

Rotating Radio Transients (RRTs) are unknown sources of em radiation.
Burst duration: few milliseconds
Frequencies: of the order of GHz

M. A. McLaughlin et al., Nature 439, 817 (2006)
D. Thornton et al., Science 341, 53 (2013)

CAN OUR STRANGE STAR MODEL EXPLAIN RRTs?

If the CCSC crust is thin, say a few centimeters thick, then **GHz frequencies can be attained**. But the **damping time we get is much less than the observed milliseconds**

However, we are overestimating the emitted power (by orders of magnitude ?) because
1) We use the coherent emission approximation, which conceivably breaks down at GHz frequencies
2) We assume that the electrosphere does not follow the crust oscillations

Observables

Rotating Radio Transients (RRTs) are unknown sources of em radiation.
Burst duration: few milliseconds
Frequencies: of the order of GHz

M. A. McLaughlin et al., Nature 439, 817 (2006)
D. Thornton et al., Science 341, 53 (2013)

CAN OUR STRANGE STAR MODEL EXPLAIN RRTs?

If the CCSC crust is thin, say a few centimeters thick, then **GHz frequencies can be attained**.
But the **damping time we get is much less than the observed milliseconds**

However, we are overestimating the emitted power (by orders of magnitude ?) because
1) We use the coherent emission approximation, which conceivably breaks down at GHz frequencies
2) We assume that the electrosphere does not follow the crust oscillations

More work to be done!!

Summary

- Strange stars stem from the Bodmer-Witten hypothesis
- Quark matter in strange stars should be in a color superconducting phase
- The crystalline color superconducting phase could form a rigid crust
- The surface of strange stars should be positively charged; electrons spill forming a negatively charged electrosphere
- Torsional oscillations of the crust may lead to observable e.m. signals.

BACKUP SLIDES

Fermionic dispersion laws

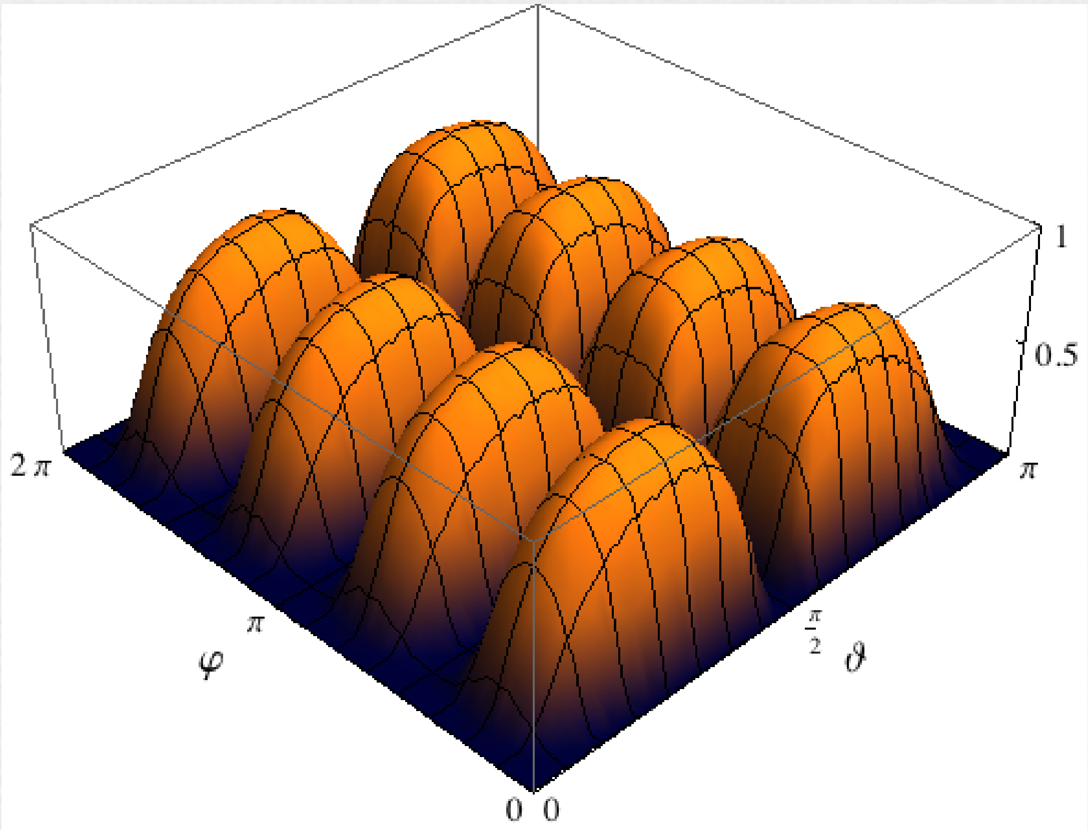
Fermions have an anisotropic gapless dispersion law:

direction dependent velocity

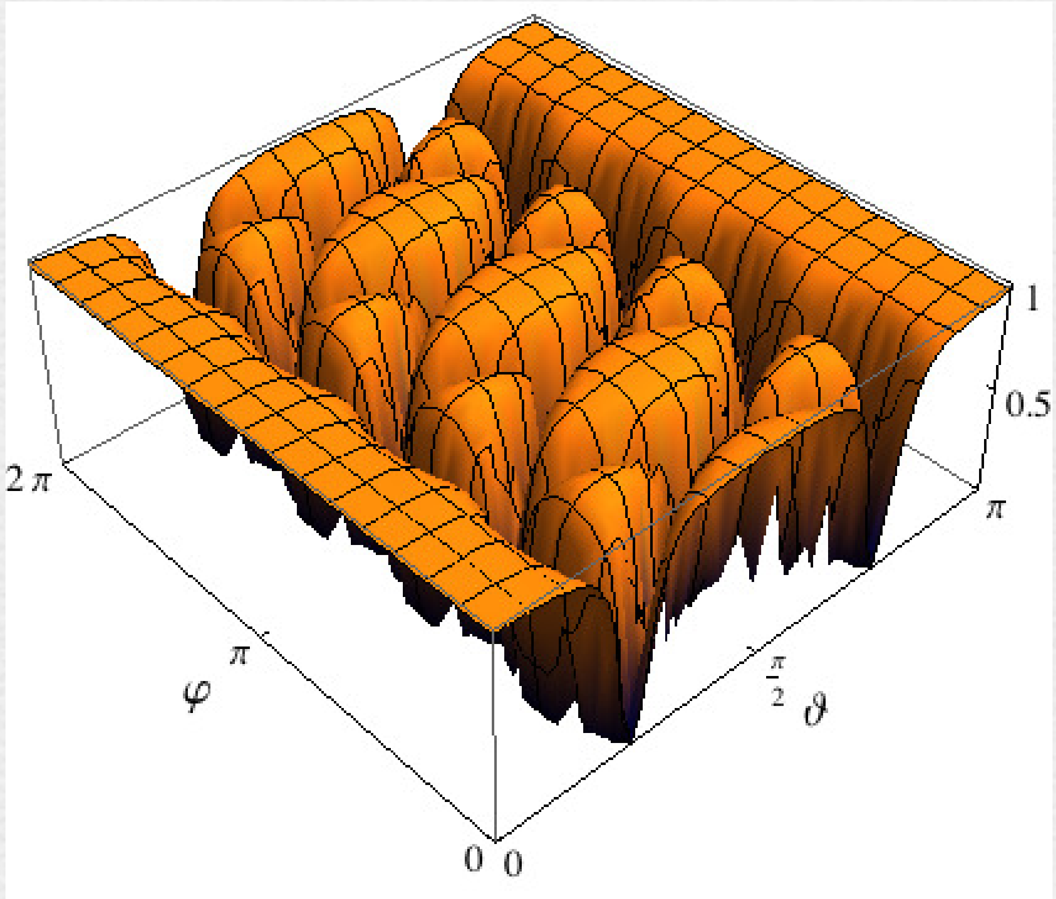
$$E = c(\theta, \phi)\,\xi$$

Velocity of fermions in two different structures

BCC



FCC



MM et al. 1302.4624

LOFF (or FFLO)-phase

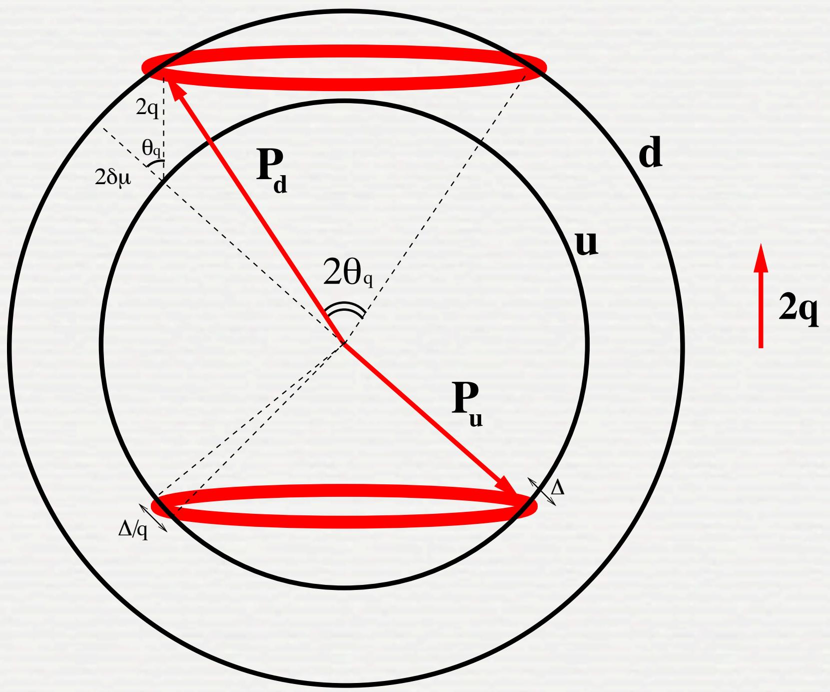
LOFF: Larkin-Ovchinnikov and Fulde-Ferrel (1964)

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ **the LOFF superconducting phase is favored with Cooper pairs of non-zero total momentum**

LOFF (or FFLO)-phase

LOFF: Larkin-Ovchinnikov and Fulde-Ferrel (1964)

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ **the LOFF superconducting phase is favored with Cooper pairs of non-zero total momentum**



- **In momentum space**

$$< \psi(\boldsymbol{p}_u)\psi(\boldsymbol{p}_d) > \sim \Delta\, \delta(\boldsymbol{p}_u + \boldsymbol{p}_d - 2\boldsymbol{q})$$

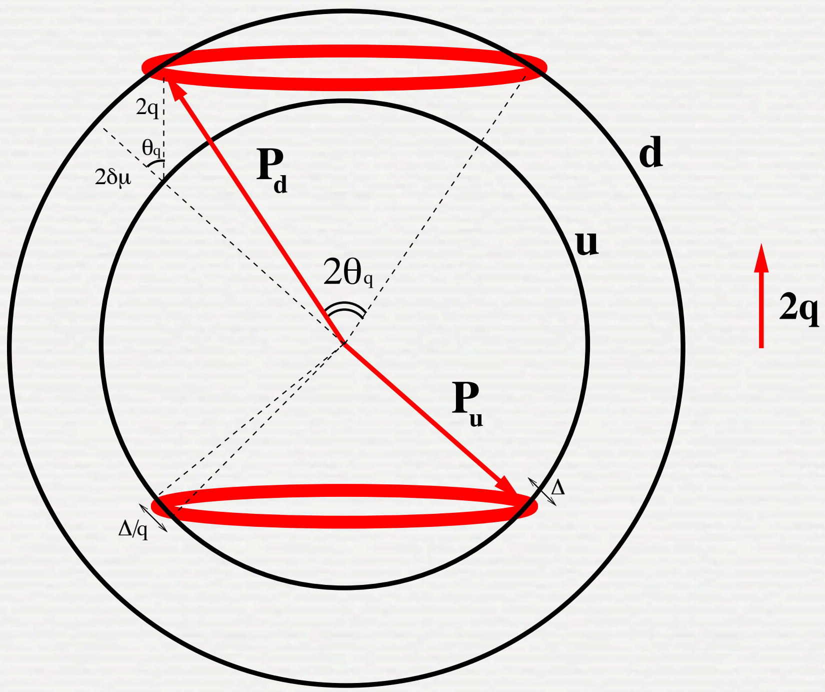
- **In coordinate space**

$$< \psi(\boldsymbol{x})\psi(\boldsymbol{x}) > \sim \Delta\, e^{i2\boldsymbol{q}\cdot\boldsymbol{x}}$$

LOFF (or FFLO)-phase

LOFF: Larkin-Ovchinnikov and Fulde-Ferrel (1964)

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ **the LOFF superconducting phase is favored with Cooper pairs of non-zero total momentum**



- **In momentum space**

$$< \psi(\boldsymbol{p}_u)\psi(\boldsymbol{p}_d) > \sim \Delta\,\delta(\boldsymbol{p}_u + \boldsymbol{p}_d - 2\boldsymbol{q})$$

- **In coordinate space**

$$< \psi(\boldsymbol{x})\psi(\boldsymbol{x}) > \sim \Delta\, e^{i2\boldsymbol{q}\cdot\boldsymbol{x}}$$

Inhomogeneous superconductor, with a spatially modulated condensate in the spin 0 channel

Bulk quark matter in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

$\longrightarrow$ **mismatch of Fermi momenta**

No pairing case

Fermi momenta $k_u^F = \mu_u \quad k_d^F = \mu_d \quad k_s^F = \sqrt{\mu_s^2 - m_s^2}$

Bulk quark matter in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

$\longrightarrow$ **mismatch of Fermi momenta**

No pairing case

Fermi momenta $\quad k_u^F = \mu_u \quad k_d^F = \mu_d \quad k_s^F = \sqrt{\mu_s^2 - m_s^2}$

weak decays

$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

$\longrightarrow$

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

electric neutrality $\longrightarrow$ $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$

Bulk quark matter in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

mismatch of Fermi momenta

No pairing case

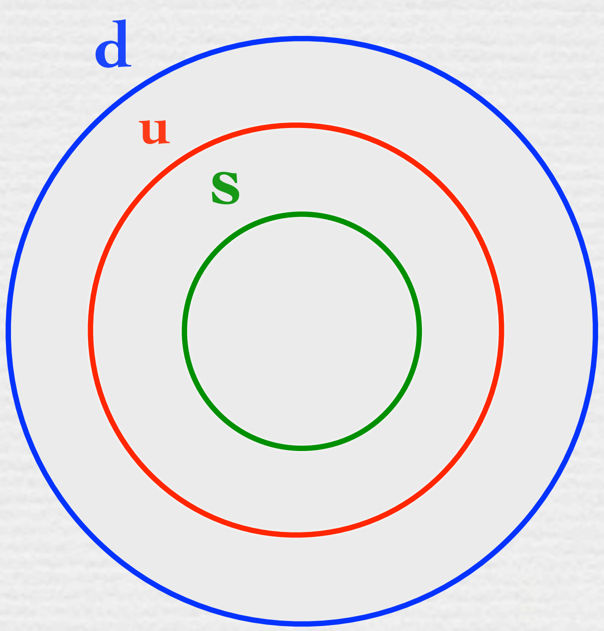
Fermi momenta $k_u^F = \mu_u \quad k_d^F = \mu_d \quad k_s^F = \sqrt{\mu_s^2 - m_s^2}$

weak decays

$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

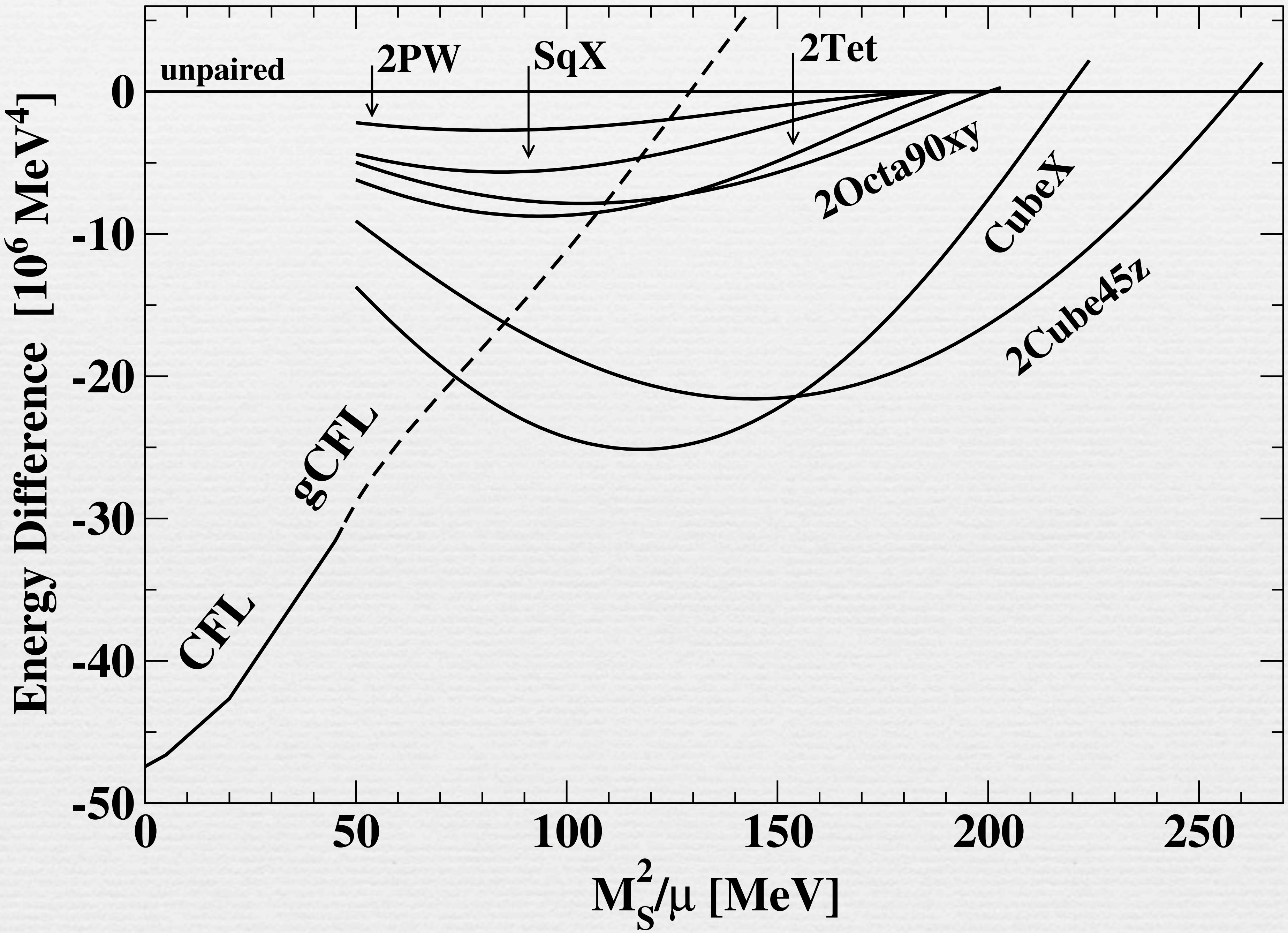
electric neutrality $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$



Fermi spheres of u,d, s quarks

Free energy evaluation

NJL + GL expansion



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019