$\Delta I = 1/2$ Rule 2014

Andrzej J. Buras
(Technical University Munich, TUM-IAS)

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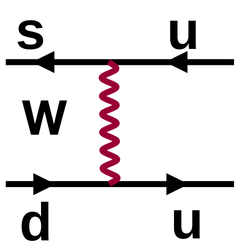
$\Delta I = 1/2$ Rule for $K \to \pi\pi$

Gell-Mann + Pais (1955), Gell-Mann + Rosenfeld (1957)

$$\mathrm{Re}\,A_0 = 27.04 \cdot 10^{-8}\,\mathrm{GeV}$$
$$\mathrm{Re}\,A_2 = 1.21 \cdot 10^{-8}\,\mathrm{GeV}$$

$$R = \frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 22.35$$

But:



$$Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}; \quad \langle Q_2 \rangle_0, \quad \langle Q_2 \rangle_2 \quad N \to \infty$$

(factorization)

$$\mathrm{Re}\,A_0 = 3.59 \cdot 10^{-8}\,\mathrm{GeV}$$
$$\mathrm{Re}\,A_2 = 2.54 \cdot 10^{-8}\,\mathrm{GeV}$$

$$R = \sqrt{2}$$

Puzzle !

Missing 15.8 in R

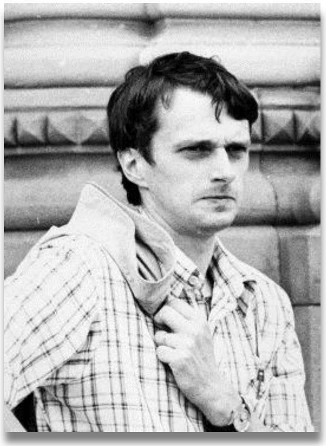
Search for dynamics: enhancing $\mathrm{Re}\,A_0$ by 7.5
suppressing $\mathrm{Re}\,A_2$ by 2.1

Dual QCD Large N Team

1985

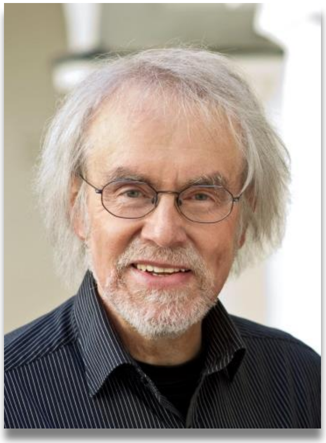
W. Bardeen

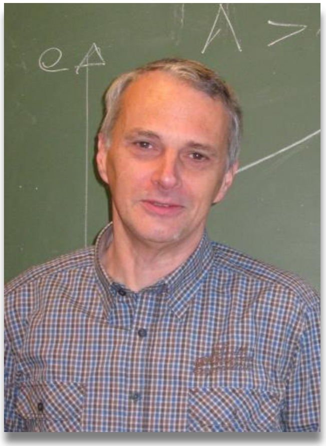
AJB



J.-M. Gérard

2014





Dominant Dynamics behind $\Delta I = 1/2$ Rule

Bardeen, AJB, Gérard (1986), (2014) 1401.1385

Step 1 : **Renormalization Group Evolution** (long, slow)

(Q_1, Q_2)

$$\boxed{M_W \to \mu = 0 \;(1\,\text{GeV})}$$

Altarelli + Maiani (1974)
Gaillard + Lee (1974)

within quark-gluon picture of QCD

Step 2 : **Continuation of RG Evolution** (short, fast)

(Q_1, Q_2)

$$\boxed{\mu = 0 \;(1\,\text{GeV}) \to \mu \approx 0}$$

Meson evolution
BBG (1986, 2014)

$\begin{pmatrix}\text{'t Hooft} \\ \text{Witten}\end{pmatrix}$

within the dual representation of QCD as a theory of weakly interacting mesons for Large N

at $\mu \approx 0$ factorization of hadronic matrix elements

Step 3 : **Inclusion of QCD Penguins**

Shifman, Vainshtein
Zakharov (1977)

$\langle Q_6 \rangle_0$ calculated within Large N (BBG, 1986)

Step 1 : **Quark - Gluon Evolution ("Octet Enhancement")**

- Enhances $\mathrm{Re}\,A_0$
- Suppresses $\mathrm{Re}\,A_2$

Altarelli + Maiani (1974)
Gaillard + Lee (1974)

The result depends on μ and renormalization scheme.

For $\boxed{\mu = 0.8\ \mathrm{GeV}}$ (Wilson Coefficients) $\boxed{\mu = 0 \quad \text{for hadronic matrix elements}}$

$$\boxed{\mathrm{NDR} - \overline{\mathrm{MS}}\ :\ R_{cc} \simeq 3} \qquad \boxed{\overline{\mathrm{MOM}}\ :\ R_{cc} \approx 4.4}$$

(BBG) (2014)

$\overline{\mathrm{MOM}}$:

$$\mathrm{Re}\,A_0 = 7.1 \cdot 10^{-8}\,\mathrm{GeV} \qquad (\mathrm{Exp}: 27.0 \cdot 10^{-8}\,\mathrm{GeV})$$

$$\mathrm{Re}\,A_2 = 1.6 \cdot 10^{-8}\,\mathrm{GeV} \qquad (\mathrm{Exp}: 1.2 \cdot 10^{-8}\,\mathrm{GeV})$$

$$\left.\begin{array}{r}\text{Further enhancement of } \mathrm{Re}\,A_0 \\ \text{suppression of } \mathrm{Re}\,A_2\end{array}\right\} \text{needed} \ +$$

Removal of scale and renormalization scheme dependence.

Step 2 : **Meson Evolution to $\mu \approx 0$** (1986) + 1401.1385

BBG

Starting with factorizable hadronic matrix elements $\langle Q_2 \rangle_{0,2}, \langle Q_1 \rangle_{0,2}$, at $\mu = 0$ allows to calculate these matrix elements at $\mu \approx 0$ (1GeV)

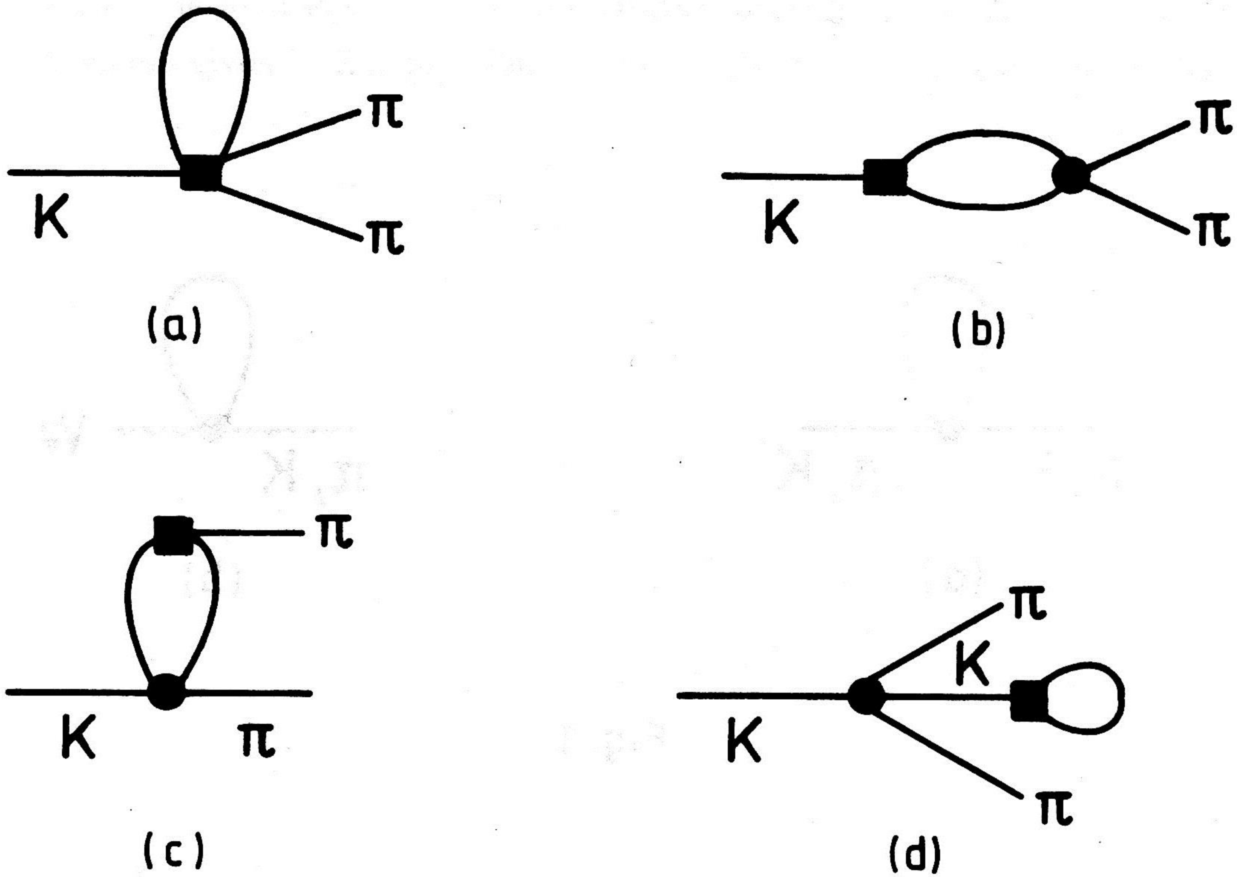
These matrix elements are scale and scheme dependent and cancel approximately these dependences in WC.

2014 Significant improvements over 1986 calculations through

- Inclusion of vector meson contributions in addition to pseudoscalars
- Matching performed at NLO in QCD in a MOM scheme suitable for Meson Evolution

$\boxed{\mu = M}$ Physical cut-off of the truncated Meson Theory

Meson Evolution



Structure of Meson Evolution

$$C_1(M^2) > 0$$
$$C_2(M^2) > 0$$

$$Q_1(M^2) = Q_1(0) - C_1(M^2)Q_2(0)$$
$$Q_2(M^2) = Q_2(0) - \underbrace{C_1(M^2)Q_1(0)}_{Q_1 - Q_2 \text{ mixing}} + \underbrace{C_2(M^2)\left[Q_2(0) - Q_1(0)\right]}_{Q_2 - Q_6 \text{ mixing}}$$

$$\hat{\gamma}^M = \begin{array}{c} \\ \end{array}\begin{array}{cccc} Q_1 & Q_2 & Q_4 & Q_6 \end{array}$$

$$\hat{\gamma}^M = \begin{bmatrix} 0 & \gamma^M_{12} & 0 & 0 \\ \gamma^M_{21} & 0 & \gamma^M_{24} & \gamma^M_{26} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma^M_{66} \end{bmatrix}$$

Anomalous Dimension Matrix

$$\gamma^M_{12} = \gamma^M_{21} = 2M^2 \left[\frac{\partial C_1(M^2)}{\partial M^2}\right] > 0$$

$$\gamma^M_{24} = \gamma^M_{26} = 2M^2 \frac{\Lambda_X^2}{r^2 - \Lambda_X^2}\left[\frac{\partial C_2(M^2)}{\partial M^2}\right] > 0$$

Precisely the structure of $\hat{\gamma}^{QG}$

$$\hat{\gamma}^{QG} = \frac{\alpha_s N}{2\pi}\begin{bmatrix} 0 & 3/N & 0 & 0 \\ 3/N & 0 & 1/3N & 1/3N \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\frac{\gamma^M_{12}}{\gamma^M_{26}} = 8.7 \qquad \frac{\gamma^{QG}_{12}}{\gamma^{QG}_{26}} = 9$$
$$\gamma^M_{66} = \gamma^{QG}_{66}$$

Fast Meson Evolution for $\mu < 1$ GeV

$0 < \mu < 0.5$ GeV Pseudoscalar Dominance

$$\frac{1}{f_\pi^2} \approx \frac{1}{N}$$

$$C_1^P(M^2) = \frac{1}{16\pi^2 f_\pi^2}\left[2\ln(2)M^2 - \frac{m_K^2}{4}\ln\left(1+\frac{M^2}{m^2}\right)\right]$$

$$m \approx 0.3\ \text{GeV}$$

$$C_2^P(M^2) = \frac{1}{16\pi^2 f_\pi^2}\left[\ln(2)M^2 + m_K^2\ \ln\left(1+\frac{M^2}{m^2}\right)\right]$$

$0.5\ \text{GeV} < \mu < 1$ GeV Contribution of Vector Mesons

M^2 dependence significantly softened in both C_1 and C_2

$M = \mu$ [GeV]	0.6	0.7	0.8	0.9	1.0	Exp
$10^8\,\text{Re}\,A_2$[GeV]	1.11	1.11	1.07	1.00	0.91	1.21
$(10^8\ \text{Re}\,A_0)_{CC}$[GeV]	13.9	13.4	13.3	13.4	13.6	27.0

↑
No QCD Penguins

$\underbrace{13.4 \quad 13.6}$
Inclusion of heavier resonances would help

QCD Penguins

Step 3

$\begin{pmatrix} \mathrm{Re}A_0 \\ \varepsilon'/\varepsilon \end{pmatrix}$ $$\boxed{\langle Q_6(\mu)\rangle_0 = -4\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi) B_6^{(1/2)}}$$ **Dominant QCD Penguin**

$(\varepsilon'/\varepsilon)$ $$\boxed{\langle Q_8(\mu)\rangle_2 = -1.74\left[\frac{B_8^{(3/4)}}{B_6^{(1/2)}}\right]^2 \langle Q_6(\mu)\rangle_0}$$ **Dominant Electroweak Penguin**

Large N-Limit $$\boxed{B_6^{1/2} = B_8^{3/4} = 1}$$

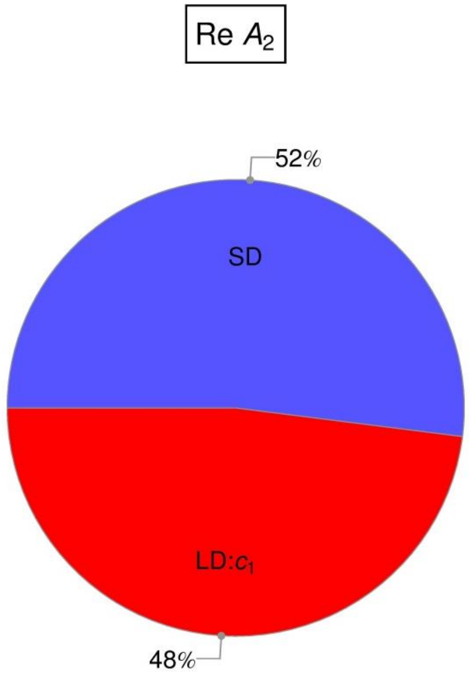
Very weak μ-dependence of $B_6^{(1/2)}$ and $B_8^{(3/4)}$: $$\boxed{\gamma_{66} \approx \gamma_{88} \approx 2\gamma_m}$$

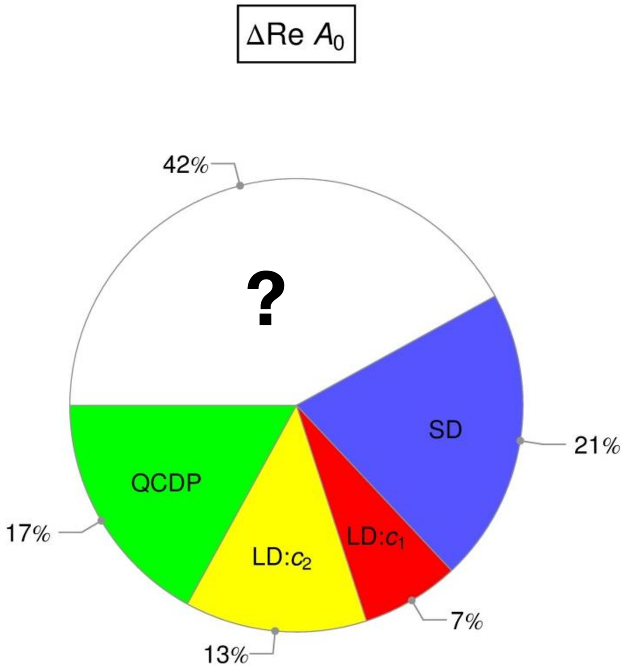
BBG 86 : **Incomplete GIM for $\mu > m_c$ allows to enhance Q_6 contribution to Re A_0 at $\boxed{\mu \approx 0.8\ \mathrm{GeV}}$ by a factor of 2.**

Contribution of QCD Penguin to Re A_0 : **~ 15% of Exp. Value** $\leftrightarrow$ $\boxed{3 \cdot \mathrm{Re}A_2}$

BBG (2014)

Budgets for Re A_2 and ΔRe A_0





$$\mathrm{Re}\,A_0 \approx 17.0\cdot 10^{-8}\,\mathrm{GeV} \qquad (\mathrm{Exp}: 27.0\cdot 10^{-8}\,\mathrm{GeV})$$

$$\mathrm{Re}\,A_2 \approx 1.07\cdot 10^{-8}\,\mathrm{GeV} \qquad (\mathrm{Exp}: 1.21\cdot 10^{-8}\,\mathrm{GeV})$$

$$R \approx 16.0 \qquad (\mathrm{Exp}: 22.4)$$

Comments on Lattice QCD Results

1

Very important progress in last five years: inclusion of dynamical fermions

→ Precise values for weak decay constants (F_π, F_K, F_{B_s}, F_{B_d} and B_i parameters for $\Delta F=2$

2

Two important results:

Relevant for ε'/ε	$B_8^{3/2}(3\ \mathrm{GeV}) = 0.65 \pm 0.05$ (RBC - UKQCD)
Relevant for ε_K	$\hat{B}_K = 0.766 \pm 0.010$ (Lattice Average 2013)

25 years effort

(BBG 1986) : $\boxed{\hat{B}_K = 0.66 \pm 0.07} \rightarrow \boxed{\hat{B}_K = 0.73 \pm 0.02}$ (BBG 2014)

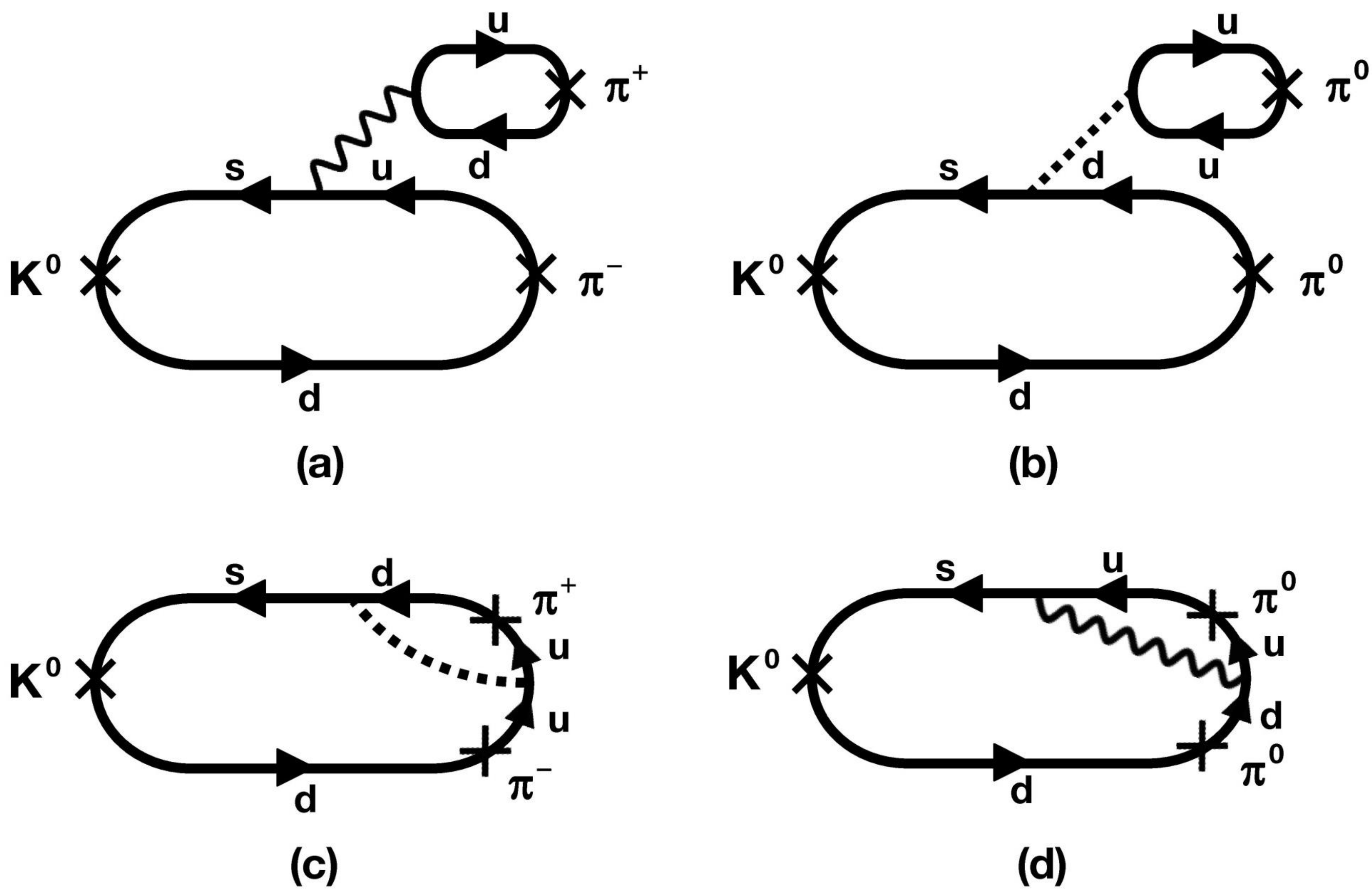
Why is $\hat{B}_K$ so close to 3/4 ?

Large N-Limit : $\boxed{\hat{B}_K = \frac{3}{4}}$ Scheme and scale independent

Answer in Dual QCD Approach : Cancellation between pseudoscalar and vector meson loop contributions at 1/N level.

Answer in Lattice QCD : ?

Contractions



$\Delta I = 1/2$ Rule and Lattice QCD (RBC-UKQCD)

$$\mathrm{Re}\,A_2 = \left(1.13 \pm 0.21\right)\cdot 10^{-8}\,\mathrm{GeV}$$

(Exp: 1.21)

$R \approx 11$

$(\mathrm{Exp}: 22.4)$

$\mathrm{Re}\,A_0$ not yet for physical kinematics

$B_6^{(1/2)}$ unknown but QCD-Penguins small at $\mu \approx 2-3\,\mathrm{GeV}$

The suppression of $\mathrm{Re}\,A_2$ and enhancement of $\mathrm{Re}\,A_0$ originate in:

$$2 \approx -0.7\ 1$$

2 , 1 **contractions**

But can this result be explained physically within Lattice QCD?

Explanation from Large N approach : (1986, 2014)

$$\left(\mu \simeq 0.8\,\mathrm{GeV}\right)\quad 1 = \frac{X_F}{\sqrt{2}} \qquad 2 = -C_1\frac{X_F}{\sqrt{2}} \qquad X_F = \sqrt{2}F_\pi\left(m_K^2 - m_\pi^2\right) \quad C_1 \approx 0.33$$

$$\Rightarrow\quad \mathrm{Re}\,A_2 \simeq 1.07\cdot 10^{-8}\,\mathrm{GeV}$$

Personal View on the Importance of Lattice QCD for $K \to \pi\pi$

1.

As long as Lattice calculations of hadronic matrix elements are performed at $\mu \simeq 2-3$ GeV, understanding of the dynamics (physics) behind $\Delta I = 1/2$ rule will not be possible within Lattice QCD

All the physics happening for $\mu < 2$ GeV : QCD penguin effects and meson evolution for $\mu < 1$ GeV hidden in two black boxes : Re A_2 and Re A_0 or numerical values of 1 and 2.

2.

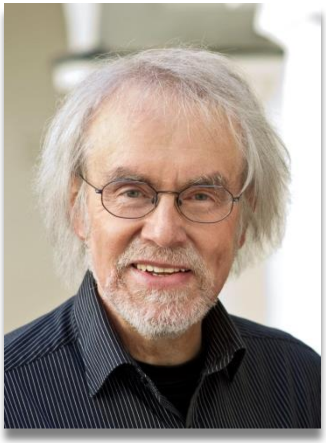
But Lattice QCD is the only existing non-perturbative framework which can tell us one day with high accuracy whether there is any room for New Physics contributions in Re A_0, Re A_2, and ε'/ε.

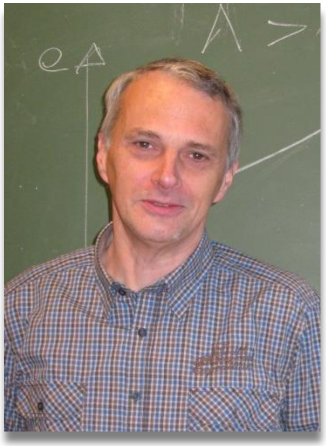
2014 Question in the context of the $\Delta I = 1/2$ Rule

Dual QCD approach and Lattice QCD reproduce well Re A_2 within the Standard Model but seem both to fail at present in obtaining fully Re A_0.

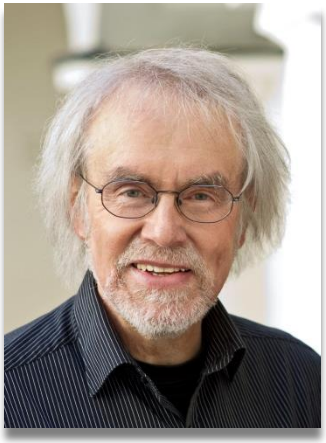
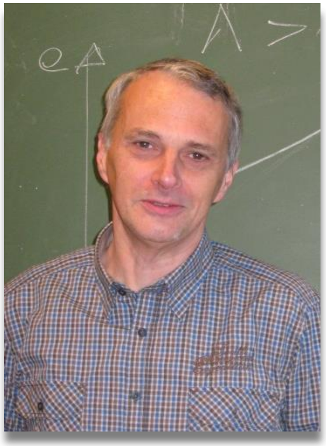
Could the missing piece of 30% (Dual QCD) in Re A_0 be due to some special kind of New Physics, still consistent with other constraints ?

Dual QCD Large N Team

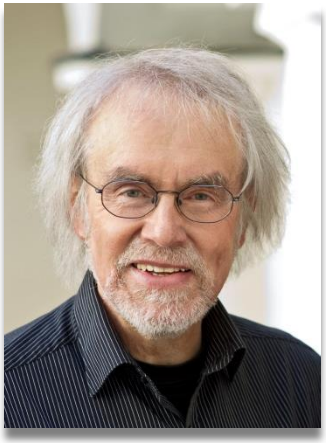




Dual QCD Large N Team

New Physics in $\Delta I = 1/2$ Rule Team

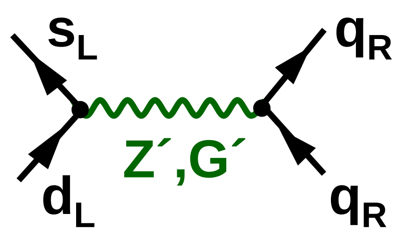

Constraints from $\boxed{\Delta M_K, \varepsilon'/\varepsilon,\ K^+ \to \pi^+ \nu\bar{\nu},\ K_L \to \pi^0 \nu\bar{\nu}}$

Basic Idea

Enhance QCD Penguin

$$Q_6 = \left(\bar{s}_\alpha d_\beta\right)\sum_q\left(\bar{q}_\beta q_\alpha\right)_{V+A}$$

through



Flavour Universal

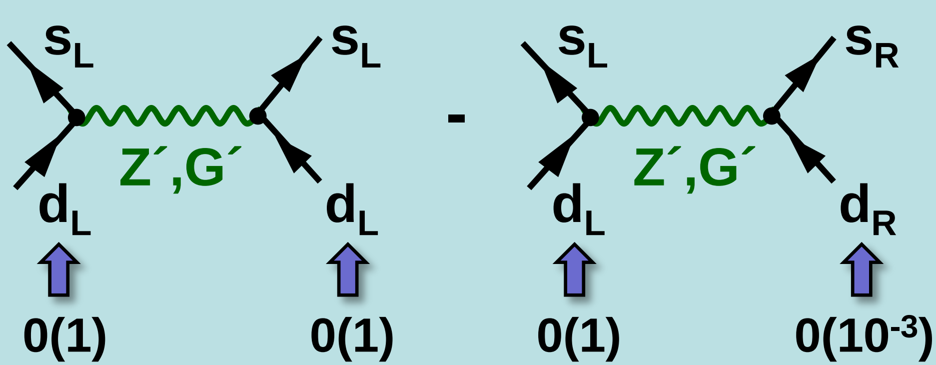
Need 0(1) Couplings

$$g_R^{q\bar{q}}\left(Z'\right),\ g_L^{\bar{s}d}\left(Z'\right)$$
$$g_R^{q\bar{q}}\left(G'\right),\ g_L^{\bar{s}d}\left(Z'\right)$$

Challenge

How to enhance $\mathrm{Re}A_0$ while being consistent with ΔM_K?

$\Delta M_K \sim$



LR operators being enhanced through RG and chiral structure allow to keep (with some fine-tuning) ΔM_K under control.

Z´with very special Properties can do it $R \simeq 18 \pm 2$

AJB, De Fazio, Girrbach 1404.3824

Enhancement of Q_6 through tree-level Z´ exchange :

Renormalization Group $\left(M_{Z'} \to \mu \simeq 0(1\text{GeV})\right)$
$+$ Chiral Enhancement of $\langle Q_6 \rangle_0$
allow for 20% Effect in Re A_0
for $M_{Z'} \approx 3$ TeV

$\text{Re}\ g_L^{sd}(Z') \approx 3-4$

$\text{Re}\ g_R^{qq}(Z') \approx 1$ (flavour universal to high degree to keep Re A_2 unchanged)

$\text{Re}\ g_R^{sd}(Z') \simeq 10^{-3}$ (to satisfy ΔM_K constraint)

Small couplings to $\mu\bar{\mu}$, $\nu\bar{\nu}$

LR operators in ΔM_K help but fine-tuning

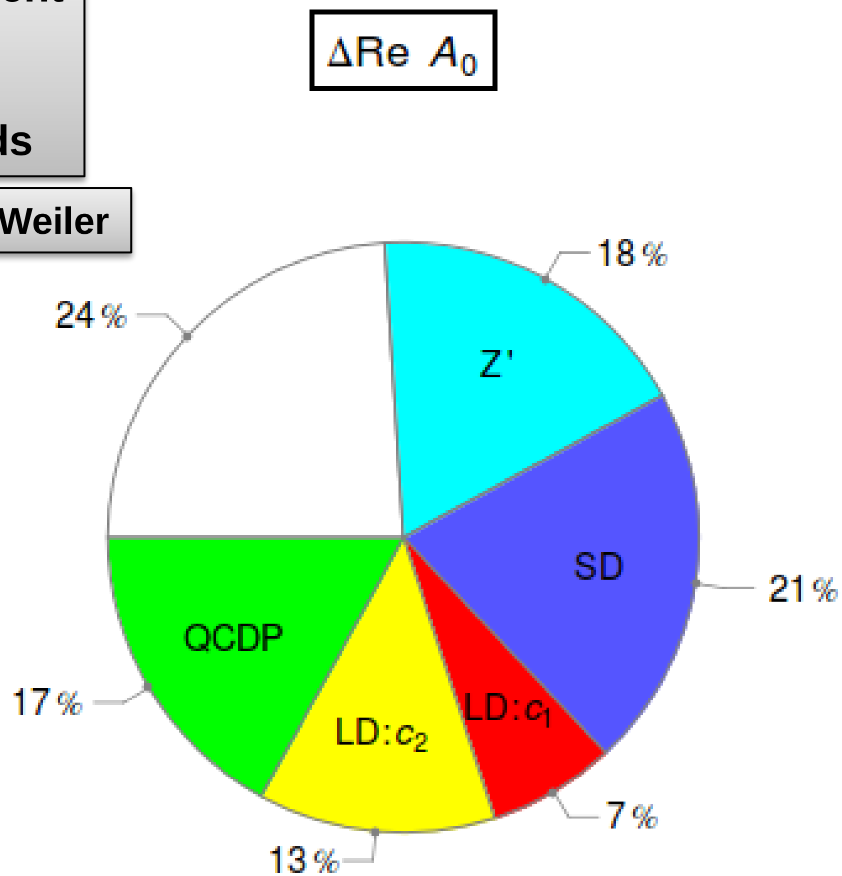
FCNC´s of Z-boson cannot help here: spoil Re A_2

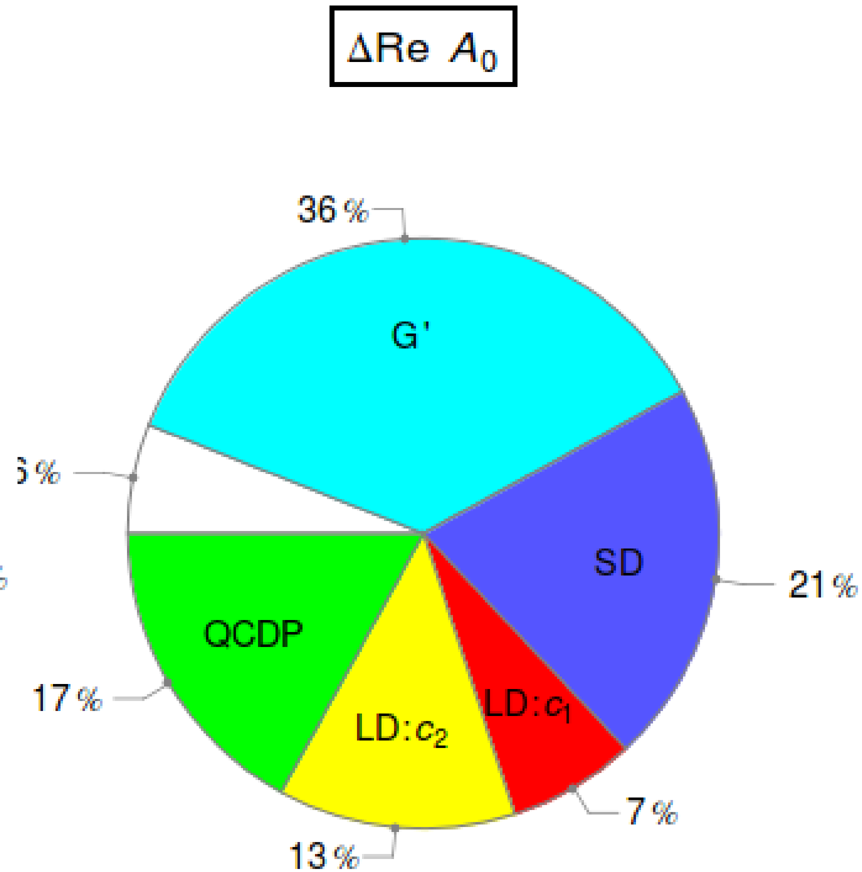
Z´and G´ with Tree Level FCNC´s enhancing QCD Penguins can do it

AJB, De Fazio, Girrbach 1404.3824

Consistent with LHC Bounds

de Vries + Weiler





R ≈ 18

R ≈ 21

Constraints from ε'/ε, ε_K, $K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$

1 ε'/ε and ε_K imply that in this NP-scenario

$$\boxed{|V_{ub}| \approx 3.9\cdot 10^{-3},\quad |V_{cb}| \approx 42.0\cdot 10^{-3}}$$

favoured (inclusive determinations)

2 $\boxed{0.75 \le B_6^{(1/2)} \le 1.0}$ favoured; $\boxed{\text{in SM } B_6^{(1/2)} \approx 1.0}$

3 $K_L \to \pi^0\nu\bar{\nu}$ (tiny effects)

$K^+ \to \pi^+\nu\bar{\nu}$ (can be sizably enhanced over SM)

Very non-MFV pattern

More Results with and without $\Delta I = 1/2$ Rule constraint in 1404.3824

Finale: Vivace !

New Physics beyond the SM must exist !!!

It is our duty to find it.
If not at the LHC then through high precision experiments.

Quark Flavour Physics
Lepton Flavour Violation
EDMs + $(g\text{-}2)_{\mu,e}$

Exciting Times are just ahead of us !!!

Towards Zeptouniverse In 12 Steps

1. CKM from Trees
2. Lattice
3. $\Delta F=2$ Observables
4. $B_{s,d} \to \mu^+\mu^-$, $B_{s,d} \to \tau^+\tau^-$
5. $B^+ \to \tau^+\nu_\tau$
6. $B \to X_s\gamma$, $B \to K^*\gamma$
7. $B \to X_s l^+l^-$, $B \to K^*(K) l^+l^-$
8. $K \to \pi\nu\bar{\nu}$
9. $B \to X_s\nu\bar{\nu}$, $B \to K^*(K)\nu\bar{\nu}$
10. ε'/ε
11. Charm, Top
12. LFV, EDMs, $(g-2)_{\mu,e}$

LFV, EDMs
$(g\text{-}2)_{\mu,e}$

Charm
Top

CKM from
Trees

12

11

1

ε'/ε

10

2

Lattice

$B \to X_s \nu\bar{\nu}$
$B \to K^*(K)\nu\bar{\nu}$

9

THANK YOU !

3

$\Delta F=2$
Observables

$K \to \pi\nu\bar{\nu}$

8

4

$B_{s,d} \to \mu^+\mu^-$
$B_{s,d} \to \tau^+\tau^-$

7

5

6

$B \to X_s l^+l^-$
$B \to K^*(K)l^+l^-$

$B \to X_s\gamma$
$B \to K^*\gamma$

$B^+ \to \tau^+\nu_\tau$

Backup

Pattern of Z´ Effects in 331 Models

(1311.6729)

$$\beta = -\frac{2}{\sqrt{3}}$$

$$\beta = -\frac{1}{\sqrt{3}}$$

Significant effects in $B_d \to K^* \mu^+\mu^-$

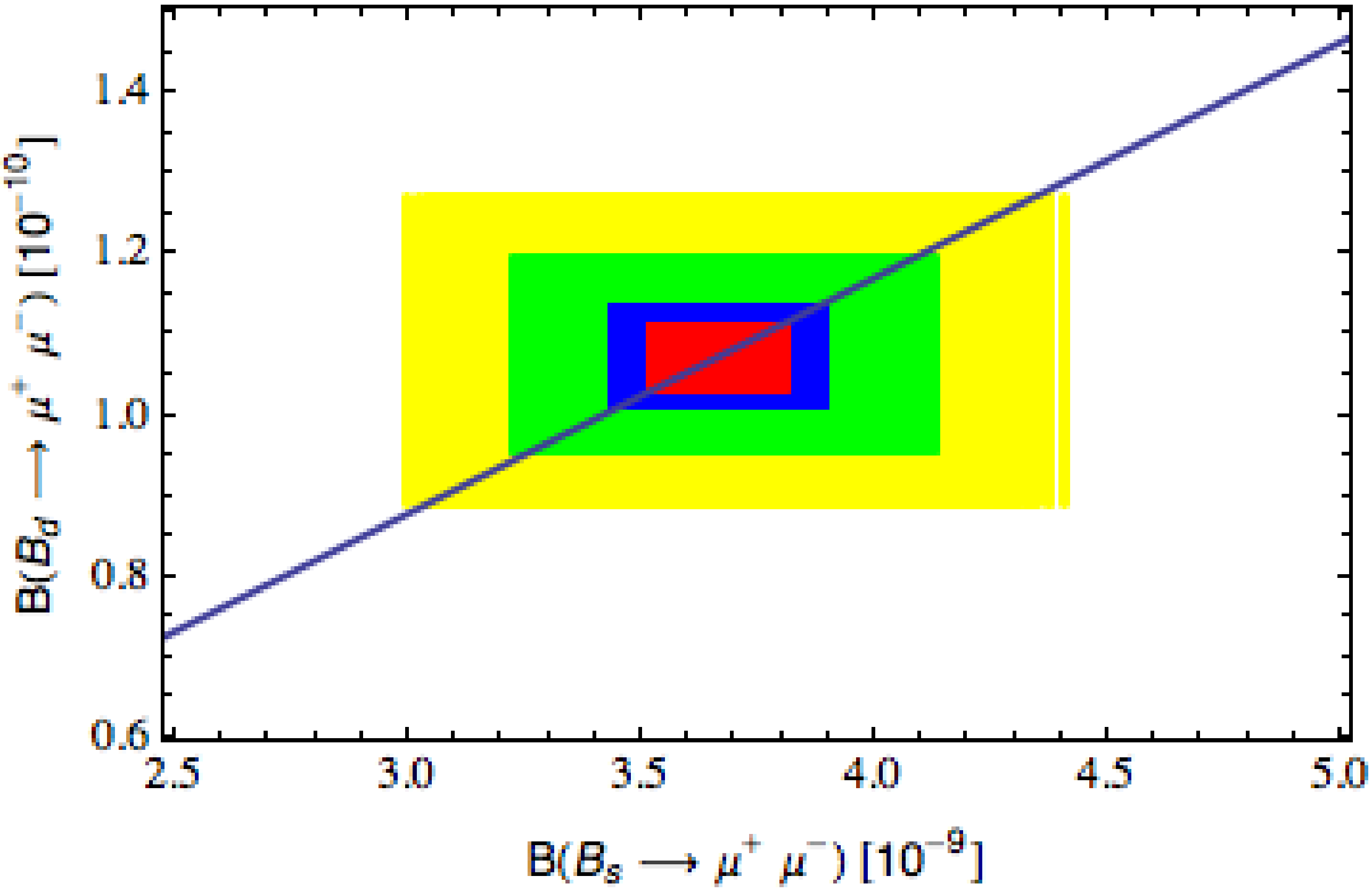
(but small in $B_{s,d} \to \mu^+\mu^-$**)**

$$\beta = \frac{1}{\sqrt{3}}$$

$$\beta = \frac{2}{\sqrt{3}}$$

Significant effects in $B_{s,d} \to \mu^+\mu^-$

(but small in $B_d \to K^* \mu^+\mu^-$**)**



1.4
1.2
1.0
0.8
0.6
2.5
3.0
3.5
4.0
4.5
5.0
$B(B_d \longrightarrow \mu^+ \mu^-)$ $[10^{-10}]$
$B(B_s \longrightarrow \mu^+ \mu^-)$ $[10^{-9}]$