Rapidity resummation for B meson wave functions

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Talk based on:
Hsiang-nan Li, Yue-Long Shen, YMW, JHEP 02 (2013) 008.

QCD@Work, Giovinazzo, 18. 06. 2014

What are B meson wave functions?

- TMD wave function in k_T factorization (Collins, 2003):

$$\langle 0|\bar{q}(y)W_y(n)^\dagger I_{n;y,0}W_0(n)\Gamma h(0)|\bar{B}(v)\rangle$$
$$= -\frac{if_B m_B}{4}\mathrm{Tr}\left\{\frac{1+\not{v}}{2}\left[2\Phi_B^+(t,y^2)+\frac{\Phi_B^-(t,y^2)-\Phi_B^+(t,y^2)}{t}\not{y}\right]\gamma_5\Gamma\right\}.$$

  - Light-cone divergence regularized by the rapidity parameter $\zeta^2 = 4(v\cdot n)^2/n^2$.
  - Transverse gauge link $I_{n;y,0}$ to ensure a strict gauge invariance. Does not contribute in covariant gauge, but contributes in light-cone gauge (Belitsky, Ji and Yuan, 2003).

- Light-cone distribution amplitudes in collinear factorization (Grozin and Neubert, 97):

$$\langle 0|\bar{q}^\beta(z)\,[z,0]\,h_v^\alpha(0)|\bar{B}(v)\rangle$$
$$= -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\phi}_B^+(t)+\frac{\tilde{\phi}_B^-(t)-\tilde{\phi}_B^+(t)}{t}\not{z}\right\}\gamma_5\right]^{\alpha\beta}.$$

- LO (naive) QCD sum rule analysis:

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2}\,e^{-\omega/\omega_0}, \qquad \phi_B^-(\omega) = \frac{1}{\omega_0}\,e^{-\omega/\omega_0}.$$

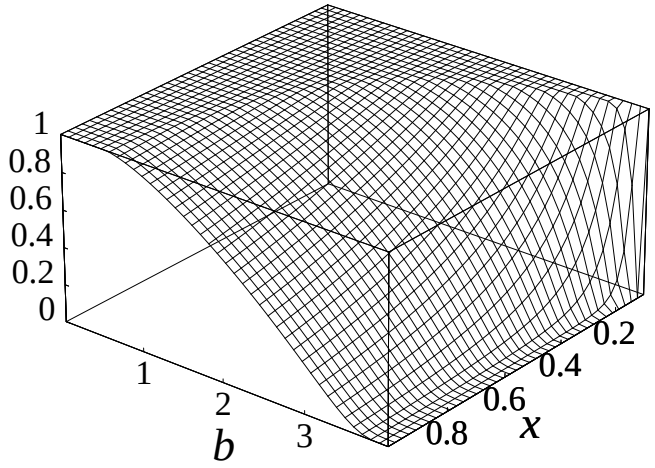
Why B meson wave functions?

- Universal nonperturbative quantities in exclusive B decays.

- NLO k_T factorization for the $B \to \pi$ form factor (Li, Shen, and Wang 2012):

$$F_i(Q^2) = \Phi_B \otimes \Phi_\pi \otimes H_i \otimes S \otimes J\,.$$

  - Soft contribution suppressed by the Sudakov mechanism (Botts and Sterman, 1989; Li and Sterman, 1992).
  - Transverse momentum dependence becomes important at the end-points.
  - Threshold resummation can suppress the end-point contribution further.



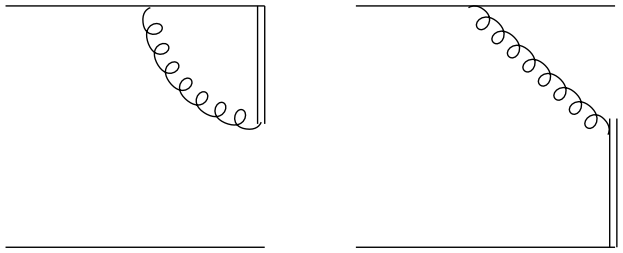
- Radiative leptonic $B \to \gamma \ell \nu$ decay as a benchmark channel to extract B meson wavefunctions (Charng and Li, 2005).
  [For B meson LCDAs, see Beneke and Rohrwild (2011), Braun and Khodjamirian (2013)]

Current status of B meson wave functions/LCDAs

NLO evolution kernel for ϕ_B^+	Lange and Neubert, 2003
NLO QCD sum rule	Braun, Ivanov and Korchemsky, 2005
NLO evolution kernel for ϕ_B^-	Bell and Feldmann, 2008
NLO evolution kernel beyond WW approximation	Descotes-Genon and Offen, 2009
OPE-based constraints in momentum space	Lee and Neubert, 2009
Diagonalize the Lange-Neubert kernel	Bell, Feldmann, Wang and Yip, 2013
Conformal symmetry of the Lange-Neubert kernel	Braun and Manashov, 2014
OPE-based constraints in dual momentum space	Feldmann, Lange and Wang, 2014
Rapidity resummation for B-meson wave functions	Li, Shen and Wang, 2013

Structure of B meson wavefunction at NLO

- Quark-Wilson-line vertex diagrams (Li, Shen and Wang, 2013):



(d) (e)

$$\Phi_{5d}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi} \ln \frac{\zeta_1^2}{m_B^2} \left(\frac{1}{\varepsilon} + \ln \frac{4\pi\mu_f^2}{m_g^2 e^{\gamma_E}} \right) H^{(0)},$$

$$\Phi_{5e}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \ln \frac{\zeta_1^2}{m_B^2} \left(\ln \frac{\zeta_1^2}{m_g^2} - \frac{1}{2} \ln \frac{\zeta_1^2}{m_B^2} + 2 \ln x_1 \right) H^{(0)}.$$

- The double rapidity logarithm $\ln^2 \zeta_1^2$ arises from the overlap of the collinear enhancement from a loop momentum l collinear to n and the soft enhancement.
- Mixed logarithm $\ln \mu_f \ln(\zeta^2/k_T^2)$ calls for simultaneous rapidity and scale evolutions.
- Similar to the joint resummation for Sudakov and threshold ecolutions (Li, 1998; Laenen, Sterman, Vogelsang, 2000, 2001).

How to resum the rapidity logarithms?

- Rapidity resummation of the light-meson wavefunctions (Collins, Soper, 1981; Li, 1998):

$$\zeta^2 \frac{d}{d\zeta^2}\Phi = -\frac{u^2}{n_- \cdot u}\frac{n_-^\alpha}{2}\frac{d}{du^\alpha}\Phi\,.$$

  - The collinear divergence arises from the region with a loop momentum collimated to the light meson momentum, and is insensitive to the Wilson line vector u.
  - The variation of u suppresses this collinear region, such that the differentiated gluon does not generate the collinear divergence.
  - The differentiated gluon can be factorized out of the light meson wave functions.

- What is different for B meson wave functions?
  - The collinear divergence arises from the region with a loop momentum collinear to the Wilson line vector n.
  - The differentiated gluon with respect to n can not be factorized out of the B meson wave functions.
  - Key insight: The collinear divergence is insensitive to the heavy-quark velocity v, the variation of v suppresses this collinear region such that the differentiated gluon can be factorized out.

Construction of evolution equation

- Velocity derivative under the constraint of EOM:

$$v^+ \frac{d}{dv^+}\Phi_B = \left(v^+ \frac{\partial}{\partial v^+} - v^- \frac{\partial}{\partial v^-}\right)\Phi_B \equiv \varepsilon_{\alpha\beta} v^\alpha \frac{\partial}{\partial v_\beta}\Phi_B \,.$$

  $\varepsilon_{\alpha\beta}$ is the anti-symmetric tensor: $\varepsilon_{+-} = -\varepsilon_{-+} = 1$.

- Rapidity derivative of the rescaled b quark interaction:

$$\zeta^2 \frac{d}{d\zeta^2}\Phi_B = \frac{v\cdot n}{2\varepsilon_{\alpha\beta} v^\alpha n^\beta} v^+ \frac{d}{dv^+}\Phi_B \,,$$

$$\frac{v\cdot n}{2\varepsilon_{\alpha\beta} v^\alpha n^\beta} v^+ \frac{d}{dv^+}\frac{v^\mu}{v\cdot l} = \frac{\hat{v}^\mu}{v\cdot l} \,,$$

$$\hat{v}^\mu \equiv \frac{v\cdot n}{2\varepsilon_{\alpha\beta} v^\alpha n^\beta} \varepsilon_{\rho\lambda} v^\rho \left(g^{\mu\lambda} - \frac{v^\mu l^\lambda}{v\cdot l}\right) .$$

- The rapidity evolution equation:

$$\zeta^2 \frac{d}{d\zeta^2}\Phi_B(x, k_T, \zeta^2, \mu_f) = \Gamma(x, k_T, \zeta^2) \otimes \Phi_B(x, k_T, \zeta^2, \mu_f) \,.$$

- The vertex $\hat{v}^\mu$ contracted to the vertex in Φ.
  - Suppression of collinear dynamics associated with the Wilson link.
  - No hard dynamics involved in the HQET B meson wave functions.
  - Only soft gluon radiations are relevant in the kernel $\Gamma(x, k_T, \zeta^2, \mu_f)$.

Rapidity evolution and UV renormaliztion

- Overlapping rapidity and UV divergence from b-quark-Wilson-line vertex correction:

$$-\frac{\alpha_s C_F}{4\pi}\ln\zeta^2\left(\frac{1}{\varepsilon}+\ln\frac{4\pi\mu_{\rm f}^2}{m_g^2 e^{\gamma_E}}\right).$$

- Rapidity evolution equation for un-renormalized B meson wave function:

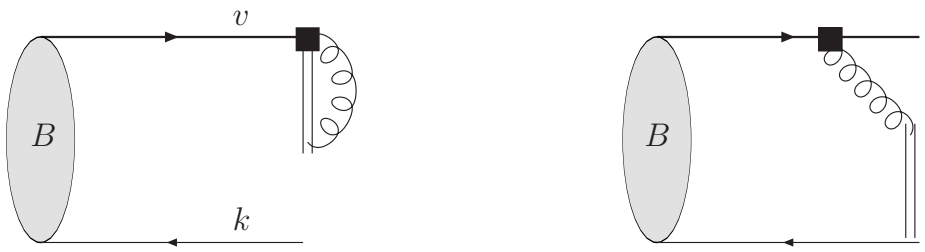
$$\begin{aligned}\frac{1}{Z_\Phi}\zeta^2\frac{d}{d\zeta^2}\Phi_B^{(b)}(x,k_T,\zeta^2,\mu_f) &= \frac{1}{Z_\Phi}K^{(b,1)}\otimes\Phi_B^{(b)}(x,k_T,\zeta^2,\mu_f)\\ &= K^{(b,1)}\otimes\Phi_B(x,k_T,\zeta^2,\mu_f)\,.\end{aligned}$$

- Rapidity evolution equation for renormalized B meson wave function:

$$\begin{aligned}\zeta^2\frac{d}{d\zeta^2}\Phi_B(x,k_T,\zeta^2,\mu_f) &= \frac{1}{Z_\Phi}\zeta^2\frac{d}{d\zeta^2}\Phi_B^{(b)}(x,k_T,\zeta^2,\mu_f)-\frac{1}{Z_\Phi}\left(\zeta^2\frac{d}{d\zeta^2}Z_\Phi\right)\Phi_B(x,k_T,\zeta^2,\mu_f)\\ &\equiv K^{(1)}\otimes\Phi_B(x,k_T,\zeta^2,\mu_f)\,.\end{aligned}$$

Soft function I

- Soft gluon radiations:



- The reducible diagram:

$$
\begin{aligned}
K_1^{(b,1)} &= -ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{v}\cdot n}{(v\cdot l + i\varepsilon)(l^2 + i\varepsilon)(n\cdot l + i\varepsilon)}\,, \\
&= -\frac{\alpha_s C_F}{4\pi}\Gamma(\varepsilon)\left(\frac{4\pi\mu_f^2}{\lambda^2}\right)^{\varepsilon}\left(\frac{v\cdot n}{\varepsilon_{\alpha\beta}v^\alpha n^\beta}\right)^2 .
\end{aligned}
$$

IR divergence regularized by the gluon mass λ.
Cancels between reducible and irreducible diagrams.

- Renormalized reducible kernel:

$$
\begin{aligned}
K_1^{(1)} &= K_1^{(b,1)} - \frac{1}{Z_\Phi}\left(\zeta^2 \frac{d}{d\zeta^2} Z_\Phi\right) \\
&= -\frac{\alpha_s C_F}{4\pi}\ln\left(\frac{\mu_f^2}{\lambda^2}\right)\left(\frac{v\cdot n}{\varepsilon_{\alpha\beta}v^\alpha n^\beta}\right)^2 .
\end{aligned}
$$

Soft function II

- The irreducible diagram:

$$K_2^{(1)} \otimes \Phi_B = ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{v}\cdot n}{(v\cdot l + i\varepsilon)(l^2 + i\varepsilon)(n\cdot l + i\varepsilon)} \times \Phi_B(x + l^+/P^+, k_T + l_T, \zeta^2, \mu_f)\,.$$

Fourier and Mellin transformations of $K_2 \otimes \Phi_B$:

$$\tilde{K}_2^{(1)}(N, b, \zeta^2) = \frac{\alpha_s C_F}{2\pi} \left(\frac{v\cdot n}{\varepsilon_{\alpha\beta} v^\alpha n^\beta} \right)^2 \left[K_0(\lambda b) - K_0\left(\sqrt{\zeta^2} \frac{m_B b}{N} \right) \right].$$

- Renormalized soft function:

$$\begin{aligned} \tilde{K}^{(1)}(N, b, \zeta^2, \mu_f) &= K_1^{(1)}(\mu_f) + \tilde{K}_2^{(1)}(N, b, \zeta^2) \\ &= -\frac{\alpha_s C_F}{2\pi} \left[\ln \frac{\mu_f b}{2} + \gamma_E + K_0\left(\sqrt{\zeta^2} \frac{m_B b}{N} \right) \right]. \end{aligned}$$

  - Confirm infrared finite soft kernel.
  - The soft function approaches $\ln b$ in the limit $\zeta m_B b \gg N$, and $\ln(N/\zeta)$ in the limit $N \gg \zeta m_B b$.

RG improved evolution kernel

- The ultraviolet and the light-cone divergences are from different kinematical region.
  - The scheme and scale evolutions are commutable:

$$\boxed{\mu_f \frac{d}{d\mu_f} \zeta^2 \frac{d}{d\zeta^2} \Phi_B = \zeta^2 \frac{d}{d\zeta^2} \mu_f \frac{d}{d\mu_f} \Phi_B \,.}$$

  - Independence of QCD evolution paths.

- RGE of the soft function:

$$\mu_f \frac{d}{d\mu_f} K^{(1)} = -\lambda_K = -\frac{\alpha_s C_F}{2\pi} \,.$$

- RG improvement:

$$\mathscr{K}^{(1)}(N, b, \zeta^2, \mu_f) = \tilde{K}^{(1)}(N, b, \zeta^2, \mu_c) - \int_{\mu_c}^{\mu_f} \frac{d\mu}{\mu} \lambda_K(\alpha_s(\mu))\, \theta(\mu_f - \mu_c) \,.$$

  - Choose $\mu_c = a\,\zeta\, m_B/N$ to diminish $\tilde{K}^{(1)}(N, b, \zeta^2, \mu_c)$ in the large N limit.
  - The small x suppression due to the resummation effect is independent of a.

Solution in Mellin and impact-parameter spaces

- Evolution equation in N and b spaces:

$$\zeta^2 \frac{d}{d\zeta^2} \tilde{\Phi}_B(N,b,\zeta^2,\mu_f) = \mathscr{K}^{(1)}(N,b,\zeta^2,\mu_f)\ \tilde{\Phi}_B(N,b,\zeta^2,\mu_f)\,.$$

- Solution:

$$\tilde{\Phi}_B(N,b,\zeta^2,\mu_f) = \exp\left[\int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \mathscr{K}^{(1)}(N,b,\tilde{\zeta}^2,\mu_f)\right] \tilde{\Phi}_B(N,b,\zeta_0^2,\mu_f)\,.$$

- RGE for μ_f evolution:

$$\mu_f \frac{d}{d\mu_f} \tilde{\Phi}_B(N,b,\zeta^2,\mu_f) = -\frac{\alpha_s C_F}{2\pi}\left(\ln\zeta^2 - 2\right) \tilde{\Phi}_B(N,b,\zeta^2,\mu_f)\,.$$

- Combined evolution:

$$\begin{aligned}\tilde{\Phi}_B(N,b) \quad &= \quad \exp\left[\int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \mathscr{K}^{(1)}(N,b,\tilde{\zeta}^2,\mu_f) - \int_{\mu_0}^{\mu_f} \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{2\pi} C_F \left(\ln\zeta_0^2 - 2\right)\right] \\ &\quad \times \tilde{\Phi}_B(N,b,\zeta_0^2,\mu_0)\,.\end{aligned}$$

Choose $\mu_f = a\,\zeta_0\,m_B$ and the upper rapidity bound needs to be replaced by $N^2\,\zeta_0^2$.

Resummation improved wave functions I

- Inverse Mellin transformation:

$$\Phi_B^{\pm}(x,k_T) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}(1-x)^{-N}\tilde{\Phi}_B^{\pm}(N,k_T)\,.$$

- Resummation with frozen α_s:

$$\tilde{\Phi}_B^{\pm}(N,k_T) = \exp\left[-\frac{\alpha_s C_F}{2\pi}\ln N\left(\ln a^2 + \ln N\right)\right]\tilde{\Phi}_B^{\pm}(N,k_T,\zeta_0^2)\,.$$

Factorized model for the initial condition:

$$\Phi_B^{\pm}(x,k_T,\zeta_0^2) = \phi_B^{\pm}(x,\zeta_0^2)\,\phi(k_T)\,.$$

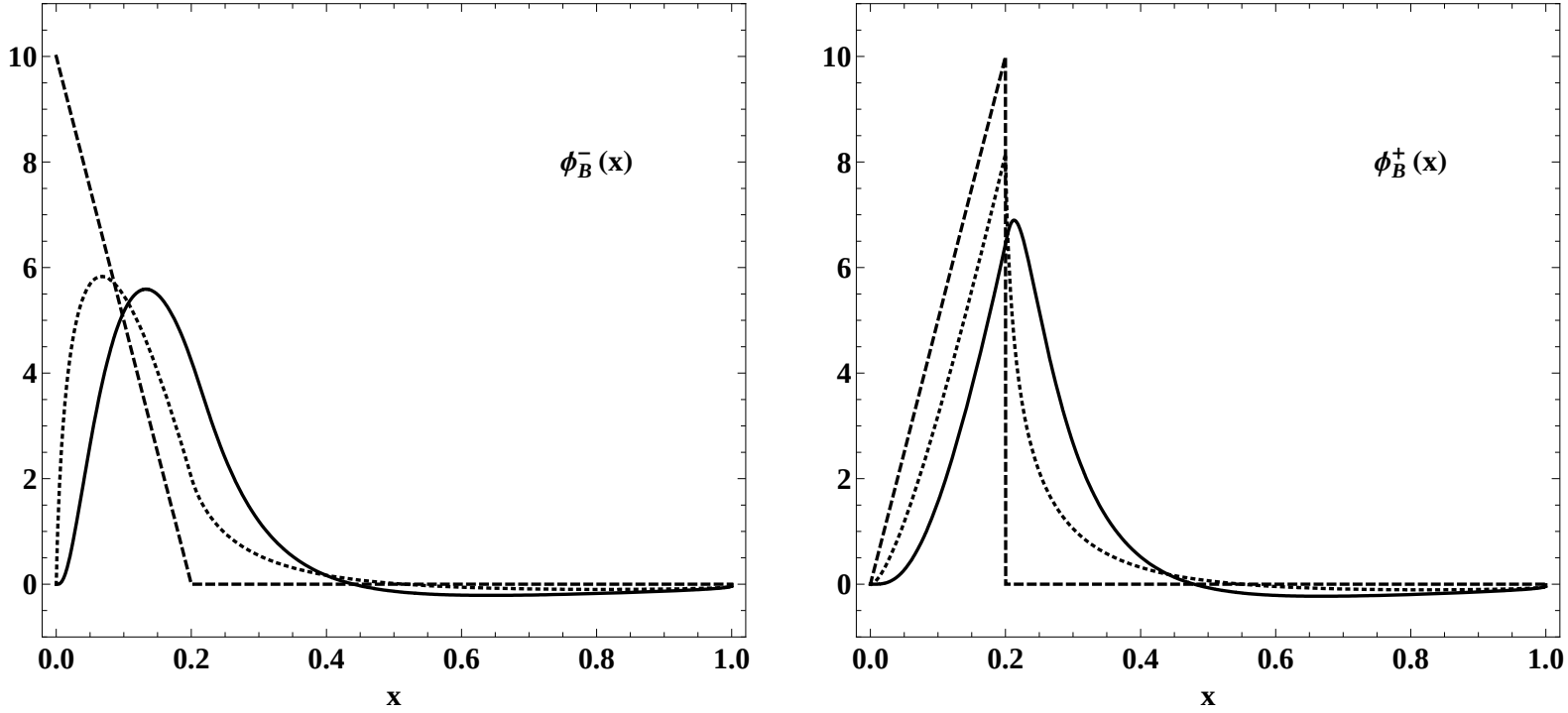
- Free-parton model for $\phi_B^{\pm}(x,\zeta_0^2)$:

$$\begin{aligned}
\phi_B^{-}(x,\zeta_0^2) &= \frac{2x_0 - x}{2x_0^2}\,\theta(2x_0 - x) \;\Rightarrow\; \frac{(1-2x_0)^{N+1} + 2x_0 N + 2x_0 - 1}{2x_0^2 N(N+1)}\,,\\
\tilde{\Phi}_B^{+}(N,k_T,\zeta_0^2) &= \frac{x}{2x_0^2}\,\theta(2x_0 - x) \;\Rightarrow\; \frac{1-(1-2x_0)^N(1+2x_0 N)}{2x_0^2 N(N+1)}\,.
\end{aligned}$$

Choose the contour to avoid the poles at $N=0$ and -1 as well as the branching cut on the negative real axis of N.

Resummation improved wave functions II

- Resummation effect in B meson wave functions:



  - Variation of a is not important because of $\ln^2 N \gg \ln a^2 \ln N$.
  - Smooth B meson wave functions after the resummation.
  - Strong suppression of small x region due to rapidity resummation.
  - Normalization conditions $\int_0^1 dx\, \phi_B^{\pm}(x) = 1$ are respected.

Resummation improved wave functions III

- Analytical parametrization (Solovtsov and Shirkov, 1999):

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0}\left[\frac{1}{\ln(\mu^2/\Lambda^2_{\rm QCD})} - \frac{\Lambda^2_{\rm QCD}}{\mu^2 - \Lambda^2_{\rm QCD}}\right].$$

- Resummation with running α_s:
  - Shift the peak positions towards large x a bit.
  - More effective suppression at small x.
  - Maintain the normalization conditions.

- Models of B meson wave functions in PQCD approach satisfy the above features:

$$\phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{m_B^2\, x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right].$$

- "Wandzura-Wilczek" relations of the two B meson wave functions do not hold after the rapidity resummation.

$B \to \pi$ form factors

- $B \to \pi$ form factors in QCD:

$$\begin{aligned}
\langle \pi(p')|\bar{q}\,\gamma^\mu b|\bar{B}(p)\rangle &= f_+(q^2)\left[p^\mu + p'^\mu - \frac{m_B^2 - m_\pi^2}{q^2}\,q^\mu\right] + f_0(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2}\,q^\mu\,, \\
\langle \pi(p')|\bar{q}\,\sigma^{\mu\nu} q_\nu b|\bar{B}(p)\rangle &= \frac{i f_T(q^2)}{m_B + m_\pi}\left[q^2(p^\mu + p'^\mu) - (m_B^2 - m_\pi^2)\,q^\mu\right].
\end{aligned}$$

- Non-lattice approaches to compute $B \to M$ form factors:
  - QCD factorization:

$$F_i(q^2) = C_i(E)\,\xi_a(E) + \Phi_B(\omega) \otimes T_i(E; \ln\omega, u) \otimes \Phi_M(u)\,.$$

  - SCET factorization:

$$\begin{aligned}
F_i(q^2) &= C_i(E)\,\xi_a(E) + C_i^{(B1)}(E,\tau) \otimes \Xi_a(\tau, E)\,, \\
\Xi_a(\tau, E) &= J_a(\tau; u, \omega) \otimes \Phi_B(M) \otimes \Phi_M(u)\,.
\end{aligned}$$

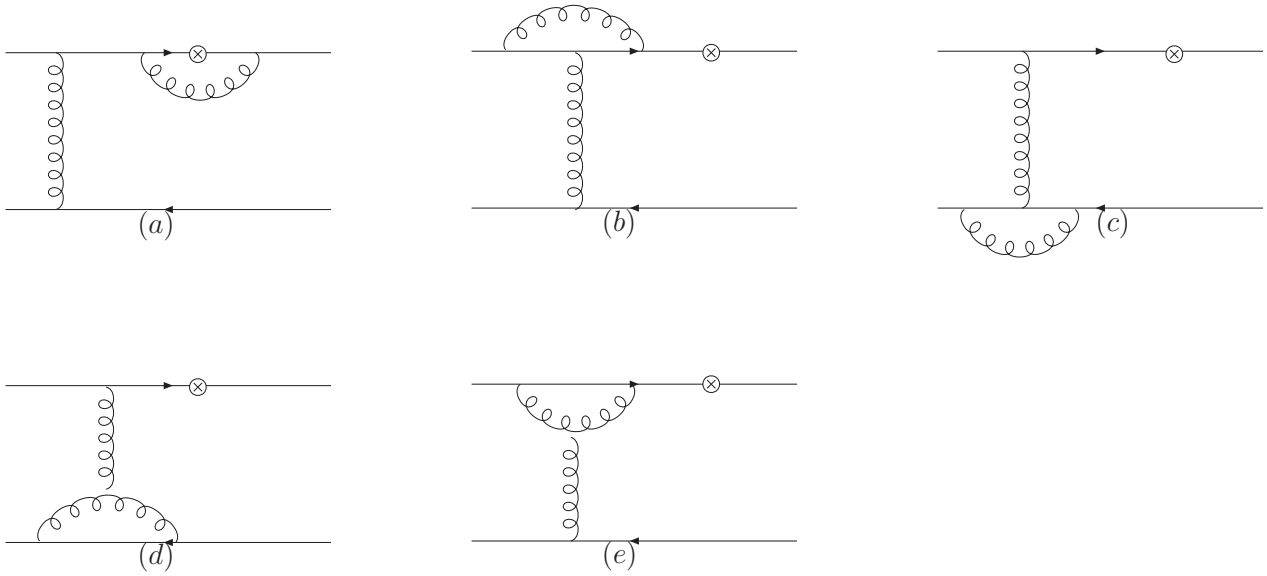
  - TMD (PQCD) factorization:

$$F_i(Q^2) = \Phi_B \otimes \Phi_M \otimes H_i \otimes S \otimes J\,.$$

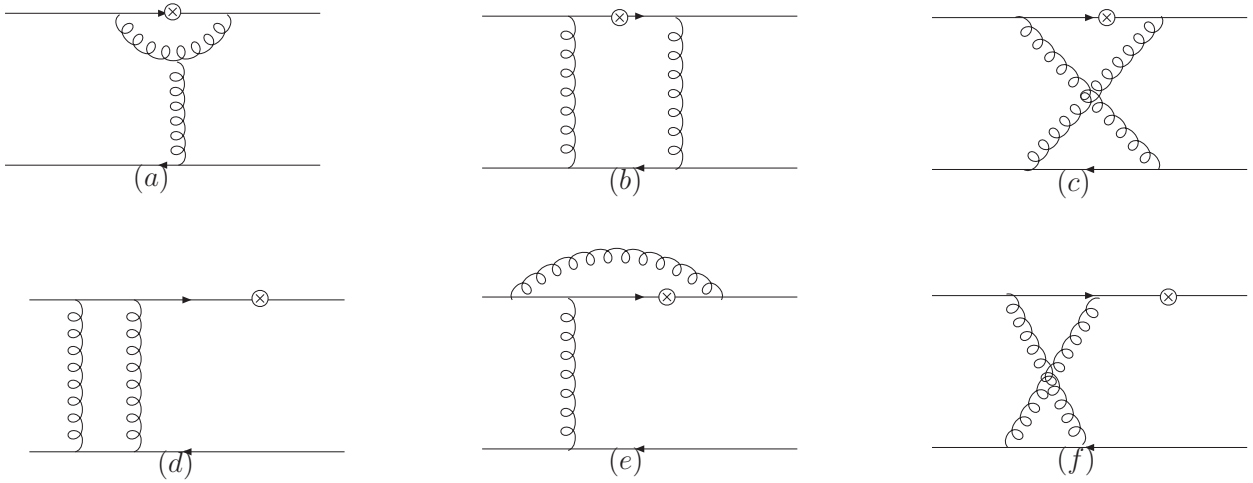
  - QCD and SCET sum rules: dispersion relation and quark-hadron duality.

NLO corrections in PQCD factorization

- Vertex corrections:

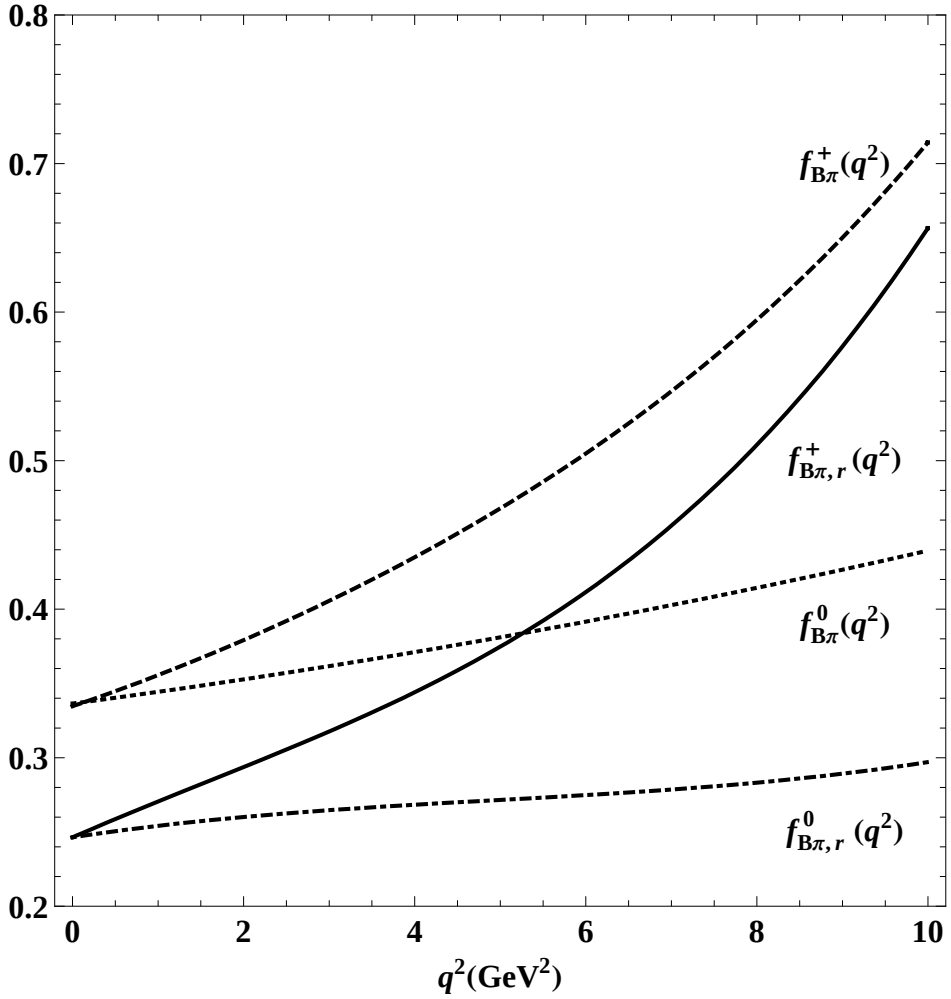


- Box and pentagon diagrams:



Rapidity resummation improved in $B \to \pi$ form factors

- Resummation effect in $B \to \pi$ form factors:



- Both form factors decreased by about 25% at $q^2 = 0$.
- Resummation effect on the shape is mild.
- $f^+_{B\pi}(0) = 0.24$ to be compared with LCSR prediction 0.28 ± 0.03.
- Improving the accuracy of $f^+_{B\pi}$ for the extraction of $|V_{ub}|$.

Conclusion and outlook

- Constructed an evolution equation to resum the double rapidity logarithm $\ln^2 \zeta$.
- Strong suppression at end points and the normalization conditions respected.
- Resummation effect results in 25% suppression for $B \to \pi$ form factors at $q^2 = 0$.
- More efforts are in demand on theory side:
  - Rapidity resummation for the hard scattering kernel.
  - OPE-based constraints on the B meson wave functions.
  - Relations between B meson wave functions and LCDAs.
  - Include the soft contribution in k_T factorization.
- Applications of rapidity resummation in $B \to \gamma \ell \nu$ decay and extraction of the shape parameters in B meson wavefunctions.