e^+e^- HADRONIC CROSS SECTION AND THE MUON *g*-2

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SLAC

On Behalf of the *BaBar* Collaboration

QCD @ WORK 2014

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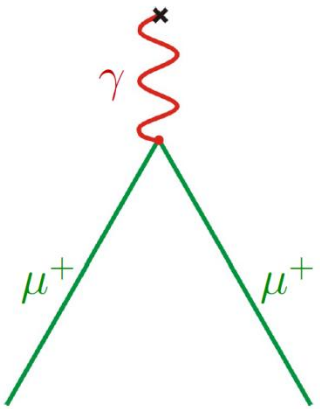
OVERVIEW

- The anomalous magnetic moment of the muon, a_μ
  - Motivation, experiment and theory
- g-2 Physics at the B-factories and more
  - Meson-photon form factors
  - Hadronic cross sections
- Summaries

MAGNETIC MOMENT OF THE MUON

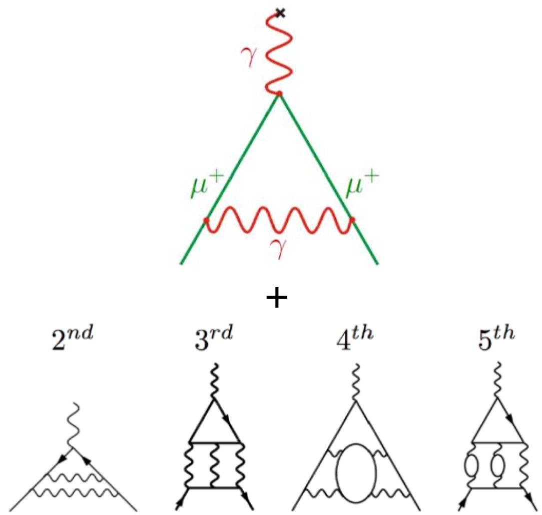
- Magnetic Moment: $\vec{\mu} = g\frac{e\hbar}{2mc}\vec{S}$
- Gyromagnetic factor g for
  - Dirac particles (point like fermions): $g=2$
  - Higher order contributions (QFT): $g\neq 2$



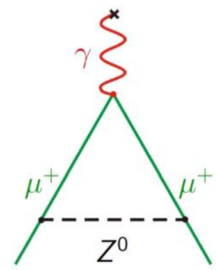
- Muon anomaly
  - $a_\mu=(g-2)_\mu/2$
  - $a_\mu^{theory(SM)}= a_\mu^{QED}+ a_\mu^{weak}+ a_\mu^{had}$

CONTRIBUTIONS TO THE MUON ANOMALY

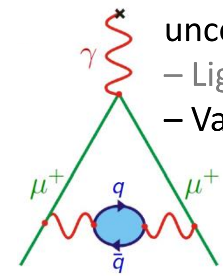
$$a_\mu^{\text{theory(SM)}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$



[T.Kinoshita *et al.*, PR L **109**, 111808 (2012)]



[A. Czarnecki *et al.*, PR D **67**, 073006 (2003)
Erratum-ibid. D**73**, 119901 (2006)]
[M. Knecht *et al.*, JHEP 0211, 003 (2002)]



[M. Davier *et al.*, EPJ **C** 71, 1515 (2011)]

Main source of theoretical uncertainty:
- Light-by-light (LbL)
- Vacuum polarization (VP)

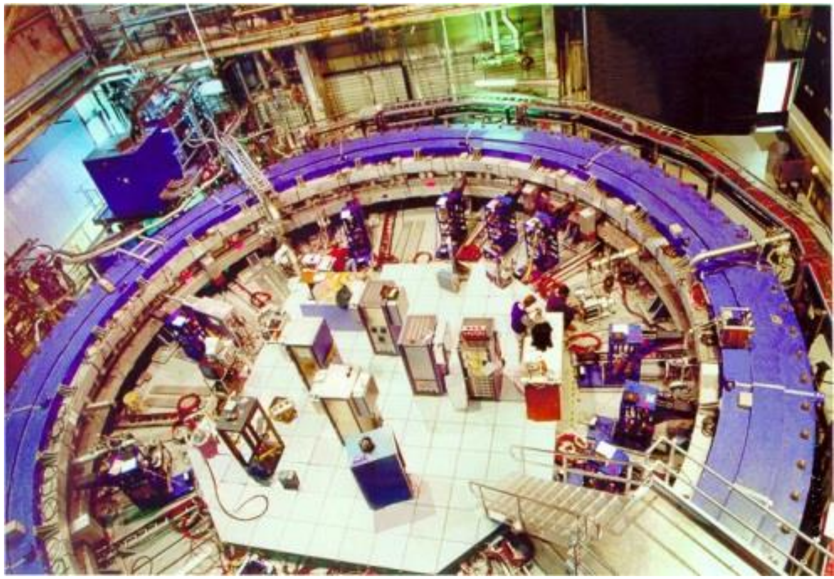
Historical note:

[Schwinger, PR 73,416 (1948)]
$a_e^{\text{QED,LO}} = \alpha/(2\pi) =$
$11\,620\,000 \times 10^{-10}$

Source	Value (10^{-10})	Uncertainty (10^{-10})
QED	11 658 471.895	0. 008
Weak	15.4	0.2
Hadronic	693.0	4.9

$(g\text{-}2)_\mu$ MEASUREMENT AT BNL

- E821 experiment at Brookhaven National Laboratory
  - [G.W. Bennett et al., PR D 73, 072003 (2006)]



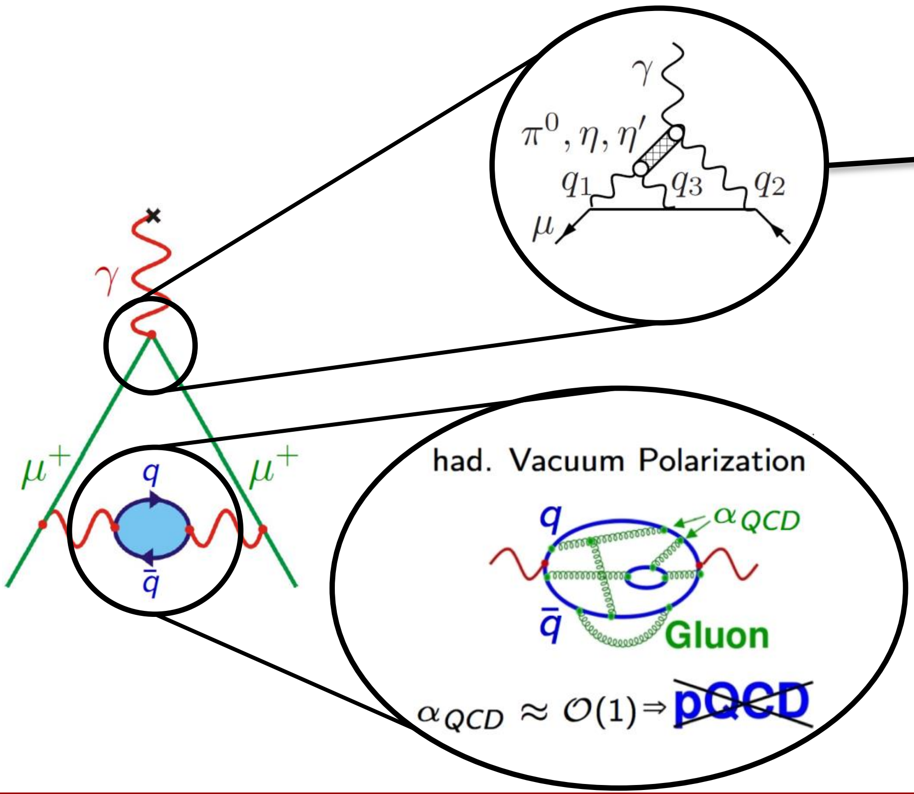
Source	Value (10^{-10})	Uncertainty (10^{-10})
QED	11 658 471.895	0. 008
Weak	15.4	0.2
Hadronic	693.0	4.9
BNL E821	11 659 208.9	6.4
BNL – SM Theory	28.7	8.0

- $a_\mu^{SM} - a_\mu^{EXP}$
  - 3.6 sigma apart
- New physics contribution to a_μ?

[M. Davier et al., EPJ C 71, 1515 (2011)]

Theoretical Prediction of a_μ^{had}

- Hadronic contributions dominate the uncertainty
  - Hadronic Light-by-Light (LbL)
  - Hadronic Vacuum Polarization (VP)



$$a_\mu^{had} = (693.0 \pm 4.9) \times 10^{-10}$$
$$a_\mu^{had,LbL} = (10.5 \pm 2.6) \times 10^{-10}$$

- Pseudoscalar meson exchange contribution important
- Photon-meson transition form factors (TFF) need to be measured

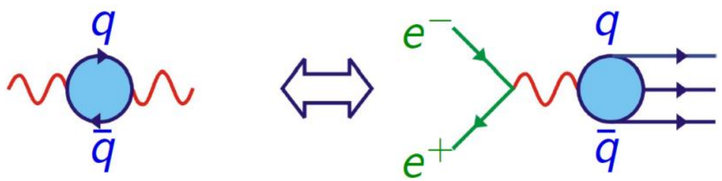
[J. Prades et al., arXiv:0901.0306 (2009)]

Main Hadronic Contribution

- Largest hadronic contribution to a_μ
- Largest absolute uncertainty of a_μ
- running of α_s →pQCD not applicable

$a_\mu^{had} = (693.0 \pm 4.9) \times 10^{-10}$
$a_\mu^{had,VP} = (692.3 \pm 4.2) \times 10^{-10}$

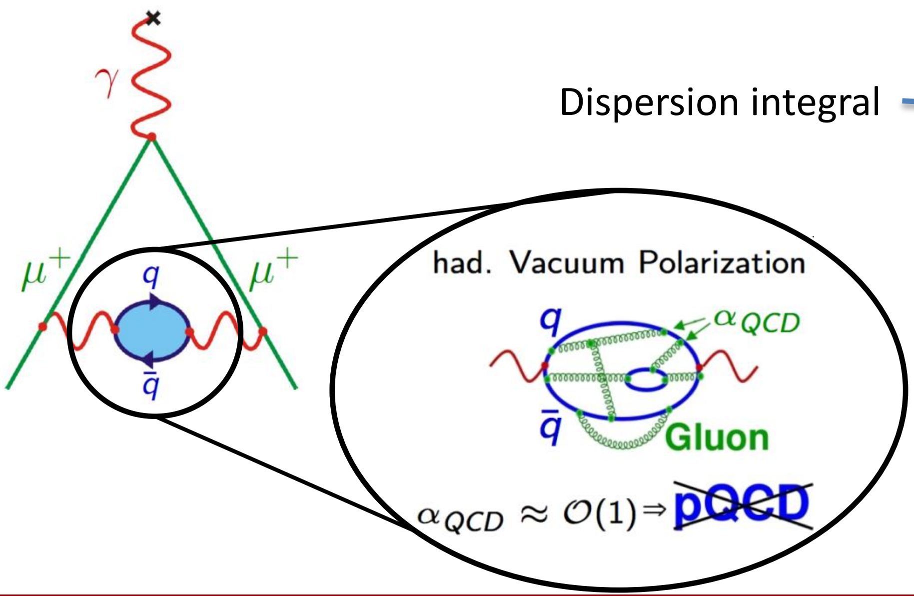
Optical theorem →



Dispersion integral →

$$a_{\mu,LO}^{had} = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\, \sigma_{had}(s)$$

$$\sim\frac{1}{s} \times \sim\frac{1}{s} = \sim\frac{1}{s^2}$$



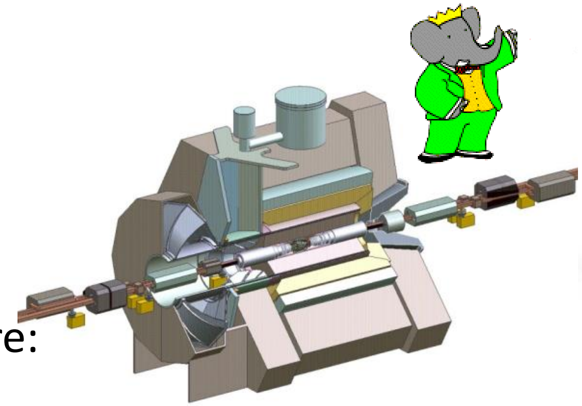
Low energy contributions important!
Needed: hadronic cross section σ_{had}

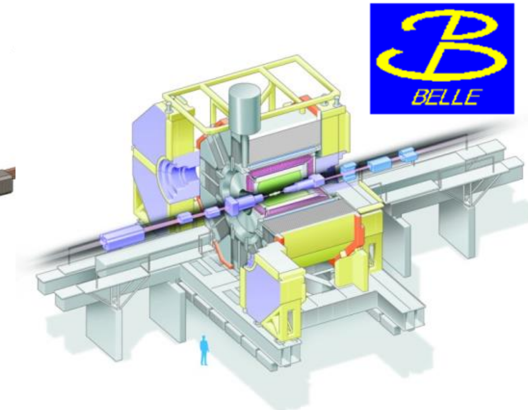
→Experimental Input!!!

[M. Davier et al., EPJ C 71, 1515 (2011)]

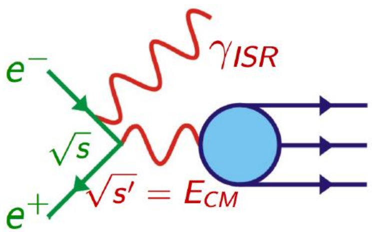
THE B-FACTORIES

- Asymmetric e^+e^- colliders
- High Luminosity
- At $B\bar{B}$ threshold
  - $\sqrt{s} = 10.58$ GeV → $\Upsilon(4S)$
  - Charm and tau factories and more:

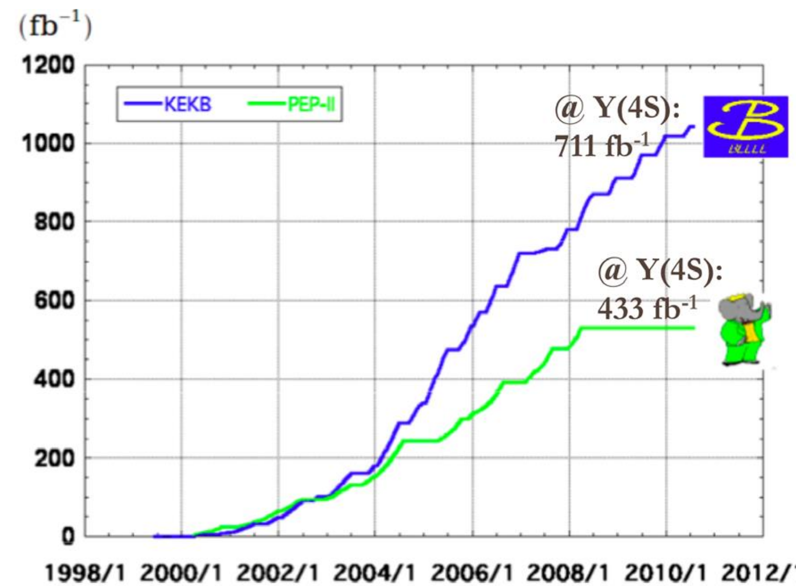




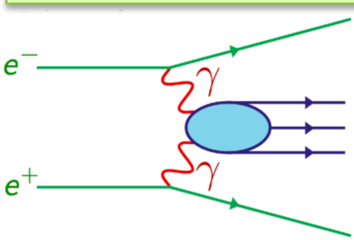
Initial State Radiation physics



High energy γ_{ISR} → low CM energy
Produces vectors, $J^{PC}=1^{--}$
Measure σ_{had}
→ Access to $a_\mu^{had,VP}$

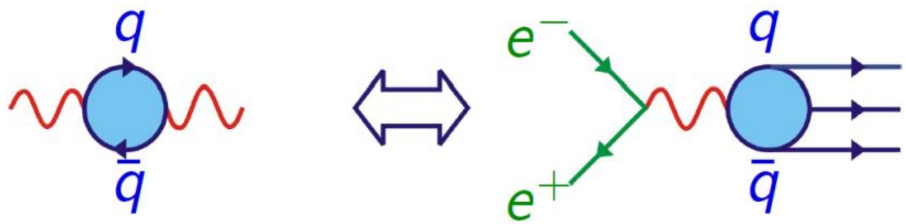


Photon-photon physics

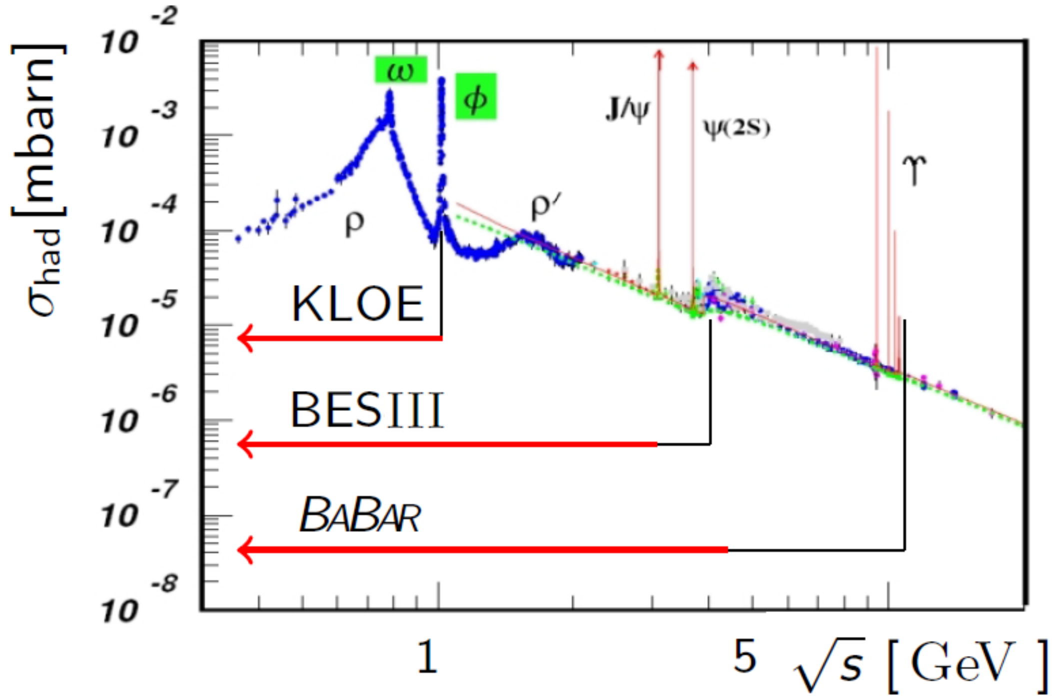


Produces pseudoscalars, $J^{PC}=0^{-+}$, …
Measures meson-photon transition form factors
→ Access to $a_\mu^{had,LbL}$

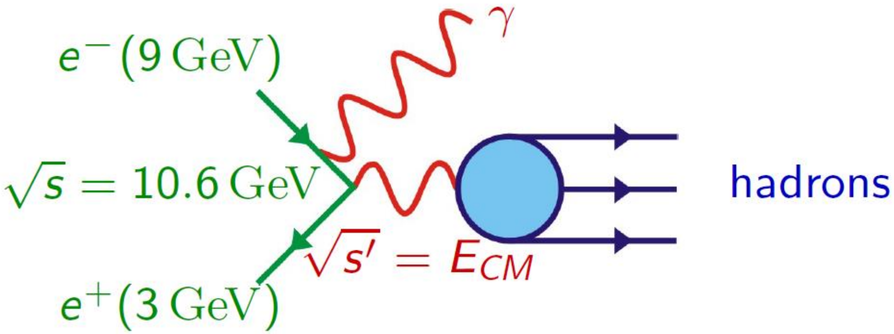
EXPERIMENTAL INPUT FOR a_μ^{had}

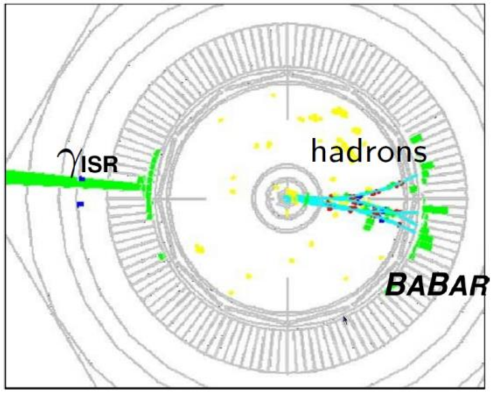


- Energy scan
  - CMD and SND at VEPP-2M and VEPP-2000 in Novosibirsk
  - BESIII at BEPCII in Beijing
- ISR
  - KLOE at DAΦNE in Frascati
  - *BABAR* at PEP-II in Stanford
  - BESIII at BEPCII in Beijing
  - Belle
    - Bhabha veto system of Belle trigger hinders low multiplicity light-hadron measurements in ISR
    - Different system to be used at Belle2



ISR PHYSICS AT BABAR





- Detect high energy photon: $E_\gamma > 3$ GeV
  - Defines E_{CM} and provides strong background rejection
- Event topology: ISR photon back-to-back to hadrons
  - High angular acceptance in hadronic cms
  - Efficiency >0 also at the hadron production threshold
- Kinematic fit including γ_{ISR}
  - Very good energy resolution (4 - 15MeV)
- Continuous measurement from threshold to 4.5 GeV
  - Provides common, consistent systematic uncertainties

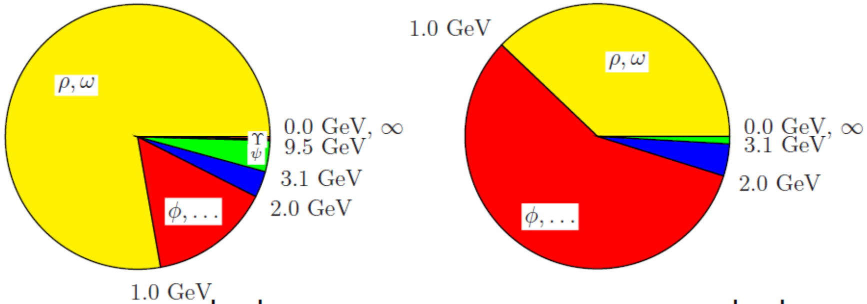
ISR ANALYSES AT BABAR

$e^+e^- \to \pi^+\pi^-$	PR D 86 (2012) 032013, PR L 103 (2009) 231801
$e^+e^- \to K^+K^-$	PR D 88, (2013) 032013
$e^+e^- \to \phi f_0(980)$	PR D 74 (2006) 091103, PR D 76 (2007) 012008
$e^+e^- \to \pi^+\pi^-\pi^0$	PR D 70 (2004) 072004
$e^+e^- \to K^+K^-\eta, K^+K^-\pi^0, K_S^0K^\pm\pi^\mp$	PR D 77 (2008) 092002, PR D 71 (2005) 052001
$e^+e^- \to 2(\pi^+\pi^-)$	PR D 85 (2012) 112009, PR D 76 (2007) 012008
$e^+e^- \to K^+K^-\pi^0\pi^0, K^+K^-\pi^+\pi^-, 2(K^+K^-)$	PR D 86 (2012) 012008, PR D 76 (2007) 012008
$e^+e^- \to K_S^0K_L^0, K_S^0K_L^0\pi^+\pi^-, K_S^0K_S^0\pi^+\pi^-, K_S^0K_S^0K^+K^-$	PR D 89 (2014) 092002
$e^+e^- \to 2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$	PR D 76 (2007) 092005
$e^+e^- \to 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-)K^+K^-$	PR D 73 (2006) 052003
$e^+e^- \to p\bar{p}$ (small $\sqrt{s}$)	PR D 87 (2013) 092005, PR D 73 (2006) 012005
$e^+e^- \to p\bar{p}$ (large $\sqrt{s}$)	PR D 88 (2013) 072009
$e^+e^- \to \Lambda\bar{\Lambda}, \Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$	PR D 76 (2007) 092006
$e^+e^- \to c\bar{c} \to \ldots$	... ...

Ongoing:

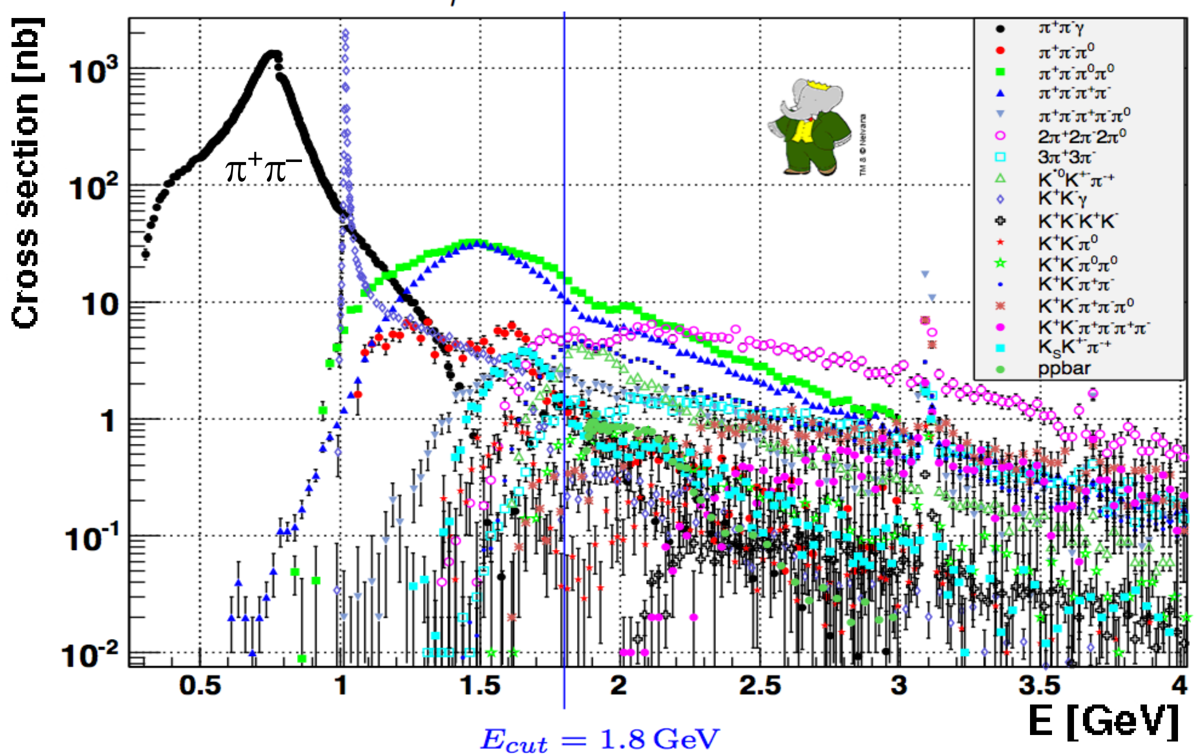
$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^0\pi^0\pi^0, K_S^0K^\pm\pi^\mp\pi^0/\eta$$

THE e^+e^- HADRONIC CROSS SECTION



Integral a_μ^{had} Uncertainty $\delta a_\mu^{\text{had}}$

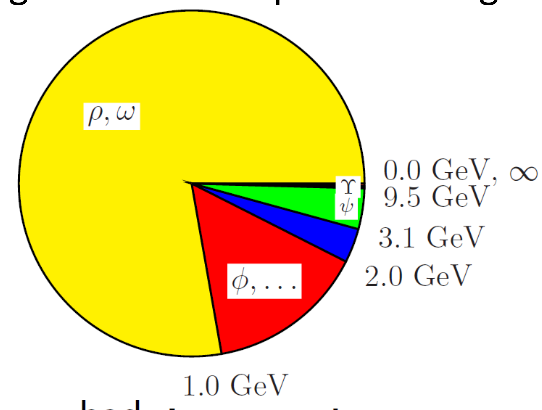
Contributions of different energy regions to the dispersion integral



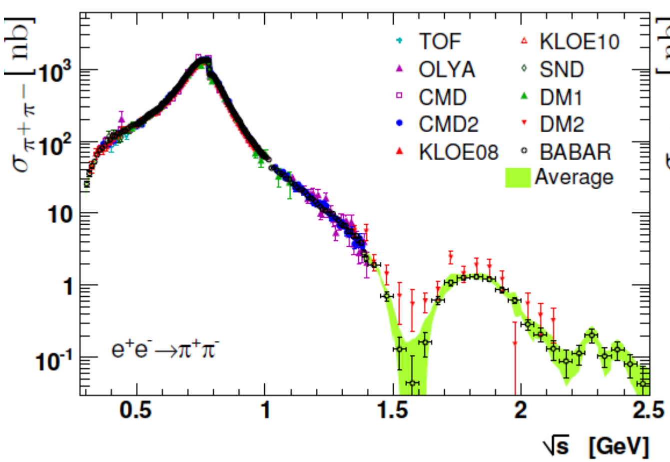
$e^+e^- \to \pi^+\pi^-$ Cross Section

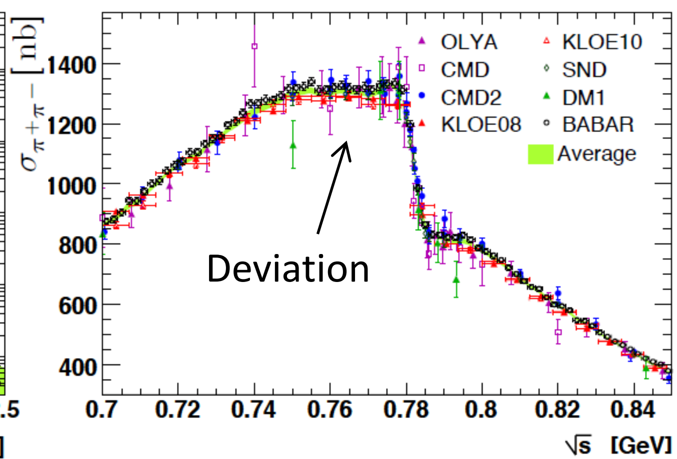
- Motivation: dominance of the E<1GeV region, accessed through $\pi^+\pi^-$
- Features of the cross section distribution
  - Includes possible FSR
  - Dominated by ρ(770) resonance
  - ρ-ω interference
  - Dip at 1.6GeV: interference between ρ' and ρ"
  - Dip at 2.2GeV: higher mass ρ state

Contributions of different energy regions to the dispersion integral



a_μ^{had} Integral





Systematic uncertainties at the ρ region

BaBar: 0.5%

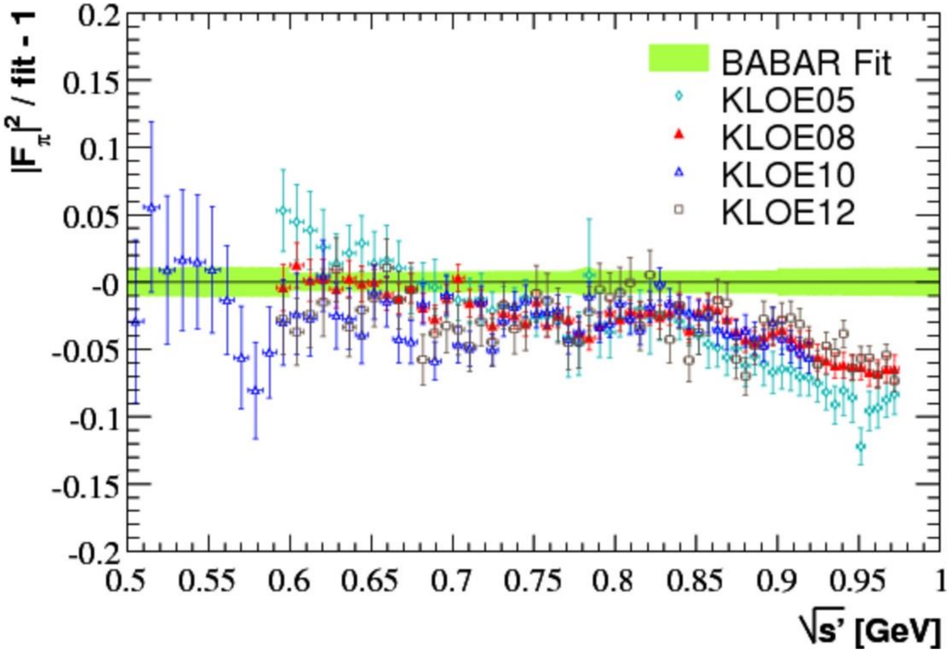
CMD2: 0.8%

SND: 1.5%

KLOE: 0.8%

[BaBar: PRD 86 (2012) 032013, PRL 103 (2009) 231801]

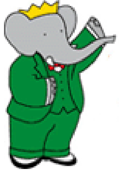
$e^+e^- \to \pi^+\pi^-$ Cross Section



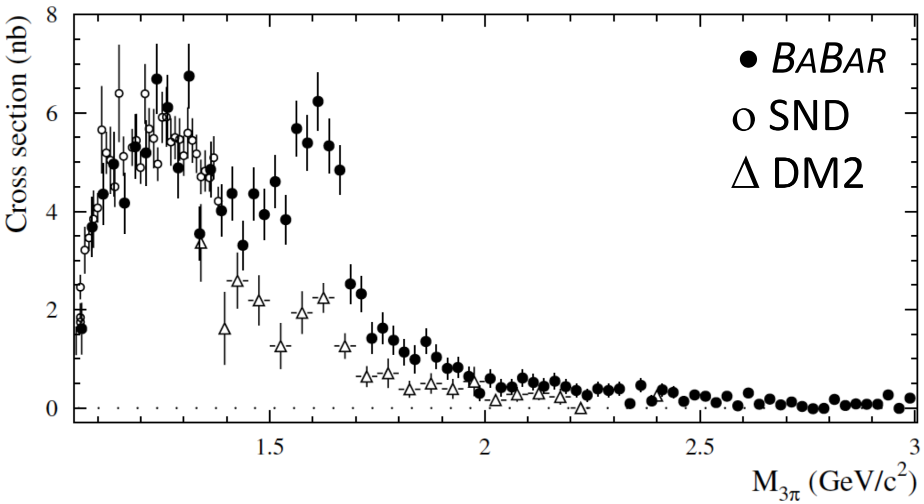
Pion form factor fitted with a Vector Dominance Model including ρ, ω, ρ′, ρ′′

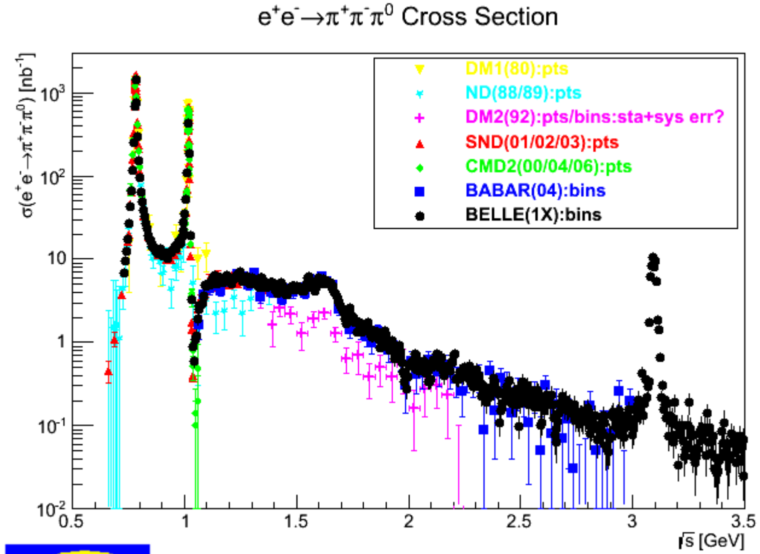
$$|F_\pi|^2(s') = \frac{3s'}{\pi\alpha^2(0)\beta_\pi^3}\sigma_{\pi\pi}(s')$$

- KLOE and *BaBar* dominate the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference, especially above ρ(770) peak
- Discrepancy leads to large uncertainty for a_μ^{had}



$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ Cross Section





BELLE

Belle Preliminary using Radiative Return
Talk by J. Crnkovic @Tau2012

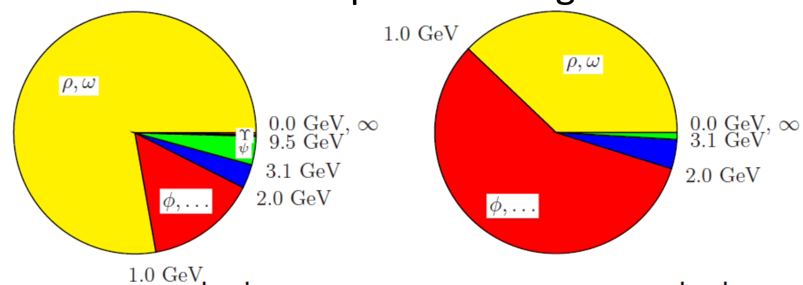
- Second largest contributor to a_μ^{had}
- Discrepancy in *ω'-ω"* region needs to be resolved
  - Preliminary Belle analysis using 527 fb^{-1} data sample agrees with *BABAR*

[BaBar: PRD 70 (2004) 072004

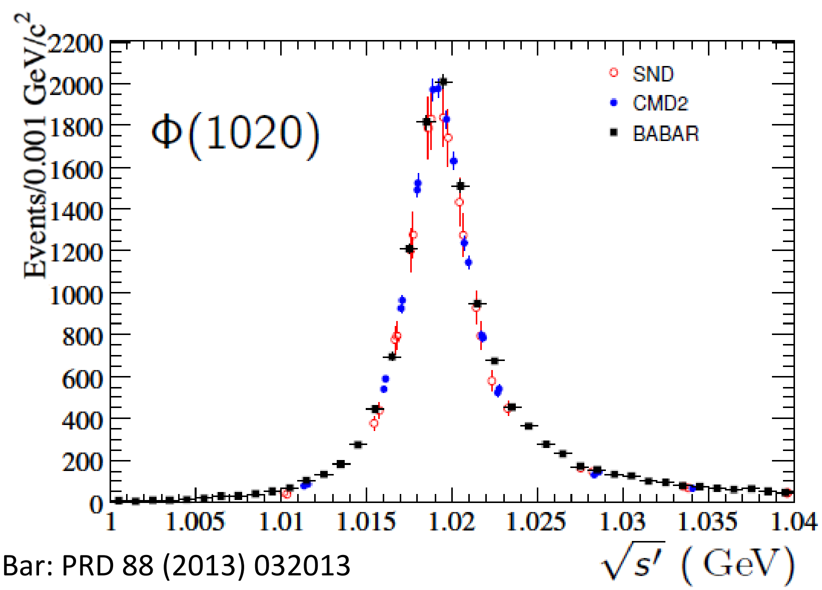
$e^+e^- \to K^+K^-$ CROSS SECTION

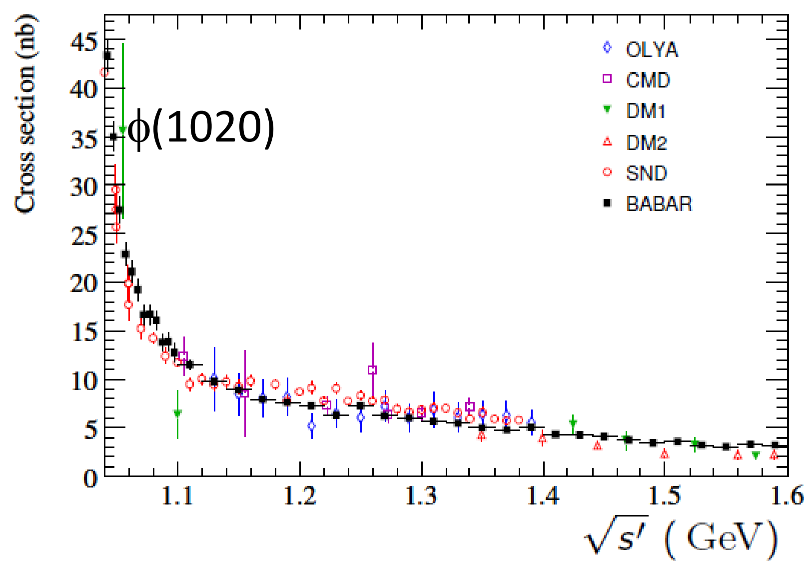
- Need precision measurements in the region 1GeV< E <2GeV
- Uncertainties at the ϕ peak
  - BaBar: 0.8%
  - CMD2: 2.2%

Contributions of different energy regions to the dispersion integral



Integral a_μ^{had} Uncertainty $\delta a_\mu^{\mathrm{had}}$

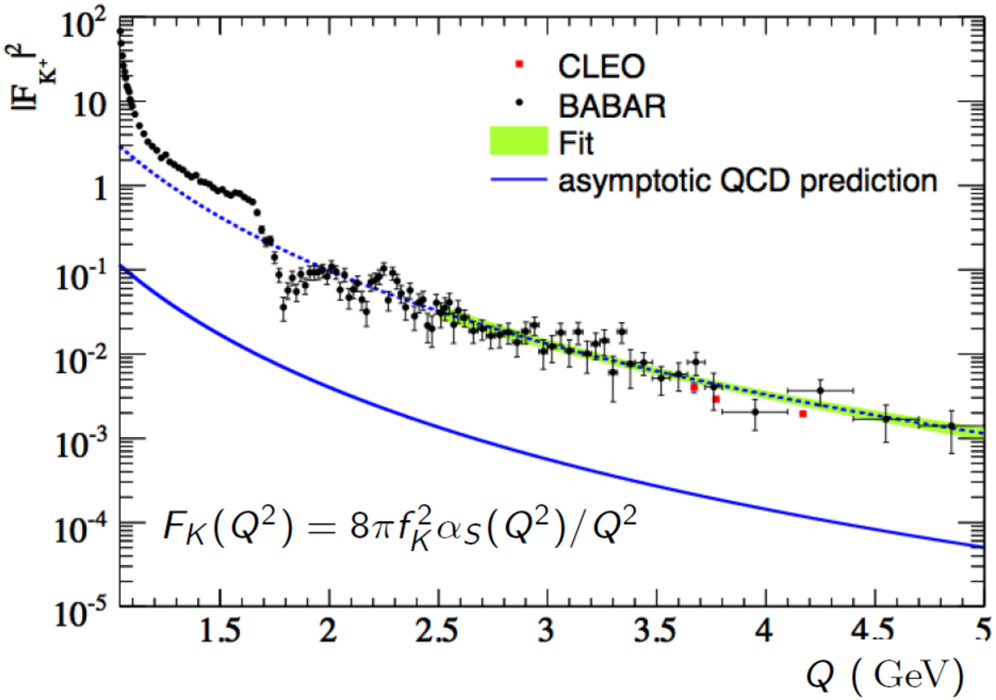




[BaBar: PRD 88 (2013) 032013

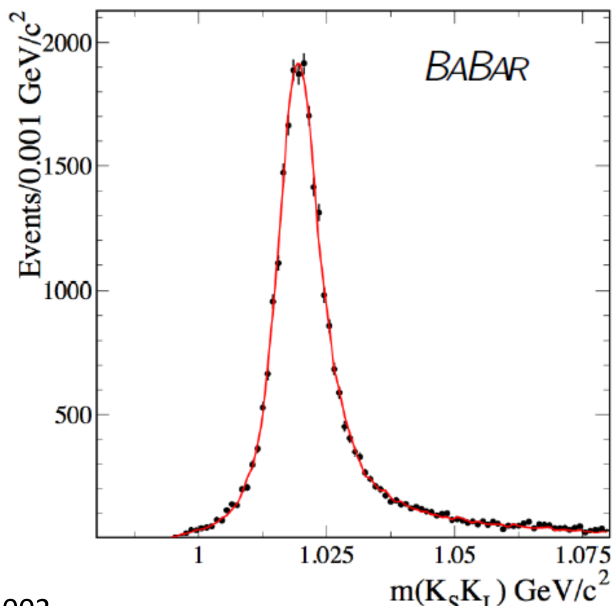
Charged Kaon Form Factor at Large Q^2

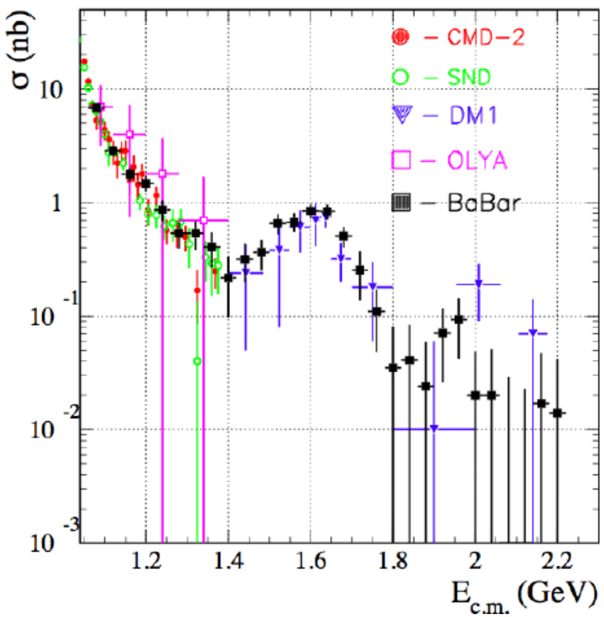
- Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)
- Power law: $F_K \propto \alpha_s (Q^2) Q^{-n}$ with n = 2
  - In good agreement with the data (2.5-5 GeV, $n = 2.10 \pm 0.23$)
- However data on $|F_K|^2$ are a factor $\approx$20 above prediction!
- No trend in data up to 25 GeV^2 for approaching the asymptotic QCD prediction



$e^+e^- \to K_S^0 K_L^0$ Cross Section

- Cross section measured on ϕ peak (1409±55nb) and in region above
- In good agreement with other experiments
  - Precision comparable to SND and CMD2
- Systematic uncertainties:
  - 2.9% on ϕ peak, dominated by trigger
  - ~10% above 0.5nb, ~30% below <0.5nb, dominated by background subtraction

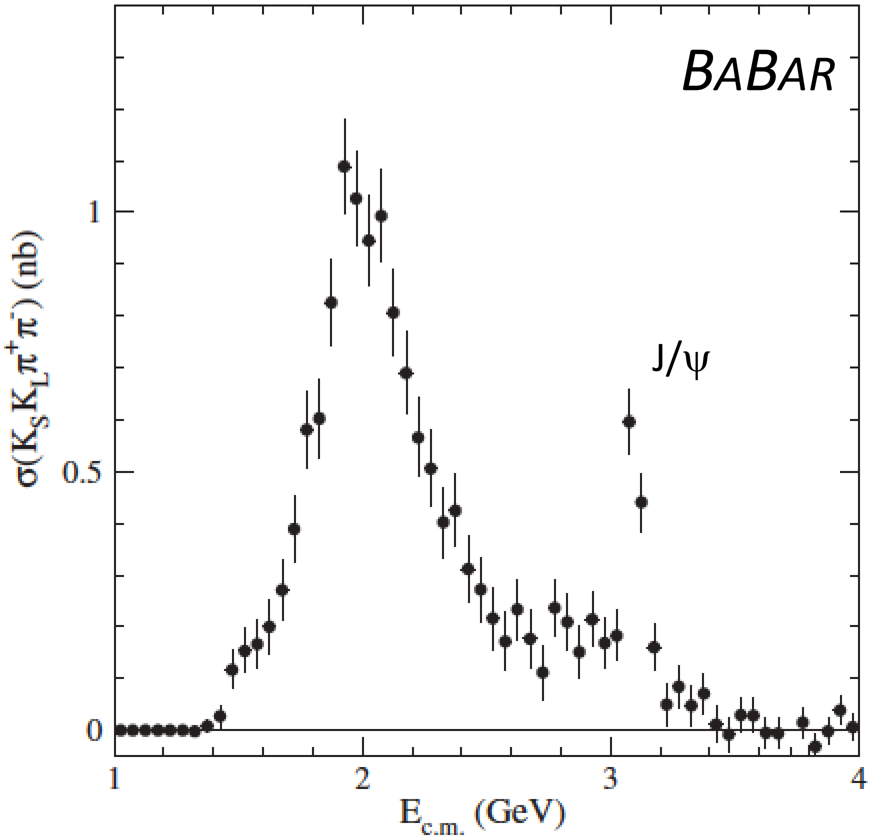




[BaBar: PRD 89(2014) 092002

$e^+e^- \to K_S^0 K_L^0 \pi^+\pi^-$ Cross Section

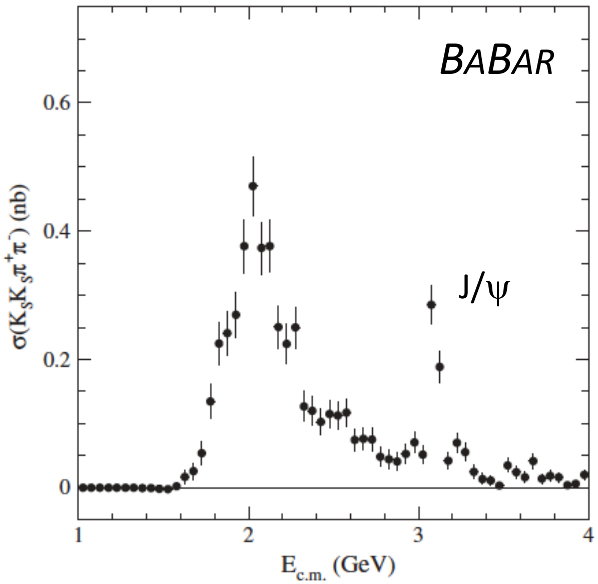
- 1^{st} measurement!
  - Previously only evaluated with isospin relations
- Systematic uncertainty dominated by background subtraction
  - ~10% at 2 GeV, ~30% at 1.5 – 3.0 GeV, 100% above 3.0 GeV
  - Dominated by $K^{*+} K^{*-}$



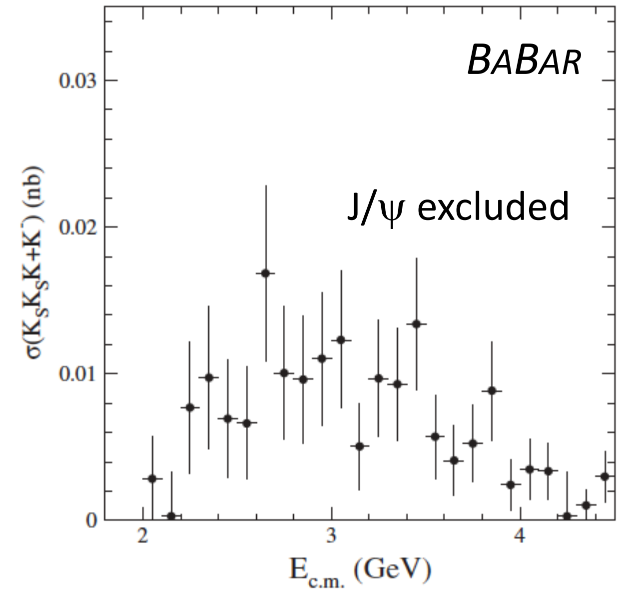
$B(J/\psi \to K_S^0 K_L^0 \pi^+\pi^-) = (3.7 \pm 0.6 \pm 0.4) \times 10^{-3}$

[BaBar: PRD 89(2014) 092002

$e^+e^- \to K_S^0 K_S^0 \pi^+\pi^-, K_S^0 K_S^0 K^+K^-$ Cross Sections



$B(J/\psi \to K_s^0 K_s^0 \pi^+\pi^-) = (1.68 \pm 0.16 \pm 0.08) \times 10^{-3}$



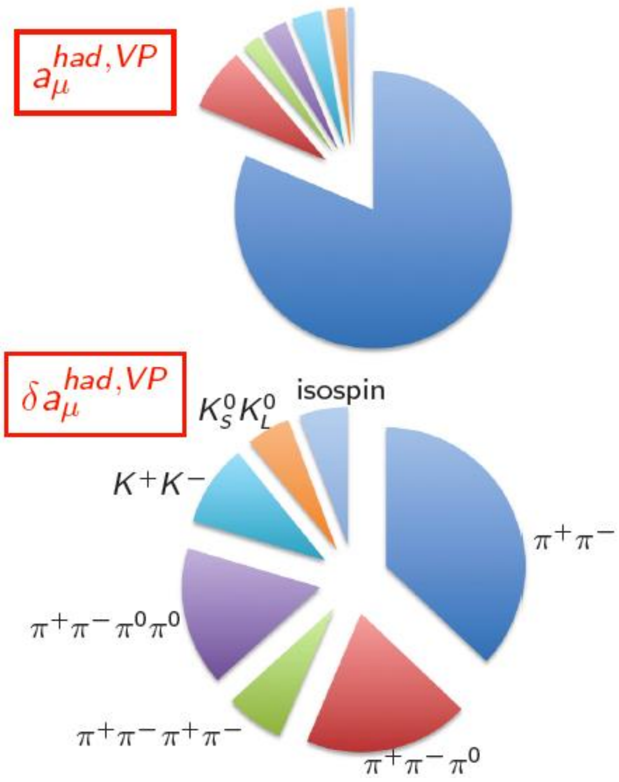
$B(J/\psi \to K_s^0 K_s^0 K^+K^-) = (0.42 \pm 0.08 \pm 0.02) \times 10^{-3}$

- 1st measurements!
  - Previously only evaluated with isospin relations
- Systematic uncertainties are ~5% in peak regions, ~20% in the range 1.5-3.0GeV, and 50-70% above 3 GeV
- The $K_s^0 K_s^0 K^+K^-$ mode is dominated by statistical uncertainties (~5%)

[BaBar: PRD 89(2014) 092002

SUMMARY ON (g-2)

Impact on $(g-2)_\mu$ from HVP



- channels estimated with isospin relations
  - largest contributions: $KK\pi$ and $KK\pi\pi$
- KK
  - *BABAR* reduces $\delta a_\mu^{had}(K^+K^-)$ by factor 2.7
  - $K_s^0 K_L^0$ not evaluated yet at *BABAR*
- $\pi^+\pi^-\pi^+\pi^-$
  - *BABAR* reduces $\delta a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-)$ by 40% w.r.t. previous *BABAR* analysis
    - Analysis not shown here
- $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^0\pi^0$
  - Future results from *BABAR*, BESIII, and CMD3
  - Belle 3π result also not evaluated yet

Data from M. Davier et al. EPJ C **71**,1515 (2011)
Plots compiled by A. Hafner – many thanks!

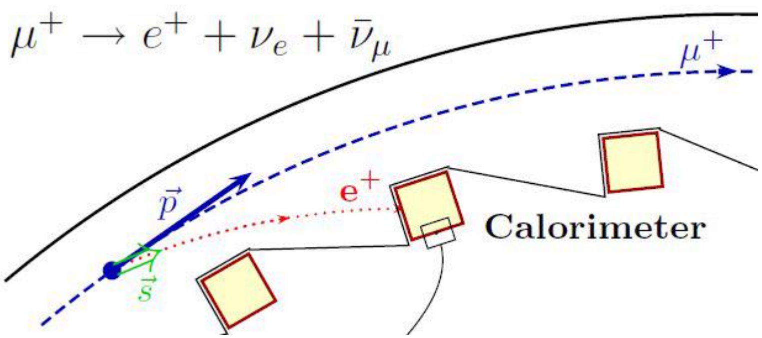
SUMMARY AND OUTLOOK

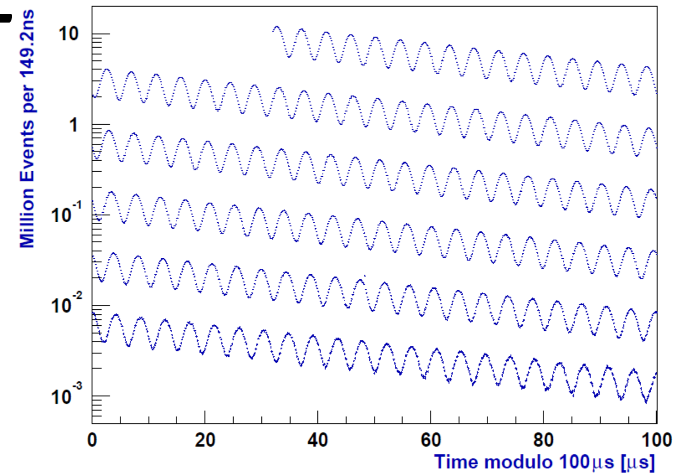
- The hadronic cross sections for $a_\mu^{had,VP}$ dominate the uncertainty of a_μ^{had} and a_μ^{theory}
  - $e^+e^- \to \pi^+\pi^-\pi^+\pi^-, K^+K^-, K_s^0 K_L^0$, are under control
  - $e^+e^- \to \pi^+\pi^-$ very precise measurements but not fully in agreement
    - Need more data
  - $e^+e^- \to \pi^+\pi^-\pi^0, \pi^+\pi^-\pi^0\pi^0$ need to be evaluated
    - *BABAR*, BESIII, SND, CMD3, and Belle
- Direct measurement of a_μ^{exp}
  - Fermilab g-2 collaboration (E989)
    - Nucl.Phys.Proc.Suppl. 225-227 (2012), 277-281
    - Aiming for precision of 0.14 ppm
  - J-PARC collaboration
    - Nucl.Phys.Proc.Suppl. 218 (2011) 242-246
    - Aiming for precision of 0.1 ppm

→ Overall improve precision by a factor 4

BACKUP

E821 AT BNL





- $\pi^+ \to \mu^+ \nu$ violates P → μ^+ longitudinally polarized
- μ^+ stored in a storage ring with constant $\vec{B}$
  - μ^+ rotating with cyclotron frequency: $\vec{\omega}_c$
  - μ^+ spin precessing frequency: $\vec{\omega}_s$
  - $\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] = -a_\mu \frac{q}{m_\mu}\vec{B}$

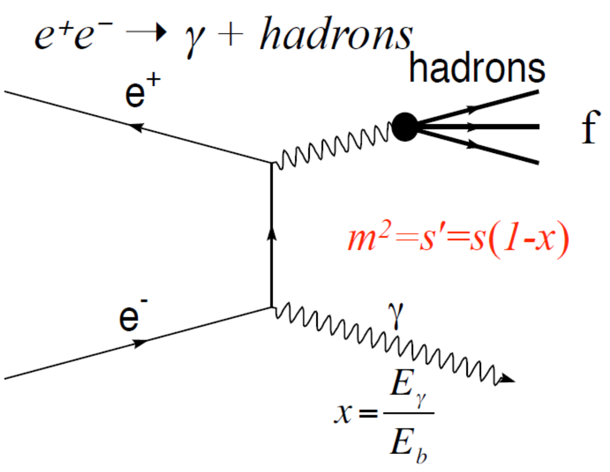
Magic energy:
E_μ=3.094GeV (γ=29.3)

- $\mu^+ \to e^+ \nu\nu$ violates P
- e^+ remembers μ^+ polarization
- Fraction of detected e^+ above an E-threshold is modulated with $\vec{\omega}_a$
- Measure $\vec{\omega}_a$ and $\vec{B}$

[G.W. Bennett et al., PR D 73, 072003 (2006)]

THE RADIATIVE RETURN

- Developed by KLOE and *BABAR*
- The emission of an ISR photon lowers the effective cm energy of the e^+e^- collision : $s \rightarrow s' = s(1-x)$
  - Study e^+e^- annihilations for a continuous and wide spectrum of energies $\sqrt{s'}$ below the nominal $\sqrt{s}$
  - No change of operating conditions of the collider
  - Optimal use of the available luminosity
- ISR studies at the $\Upsilon(4S)$ can yield the same observables as the low energy e^+e^- experiments!

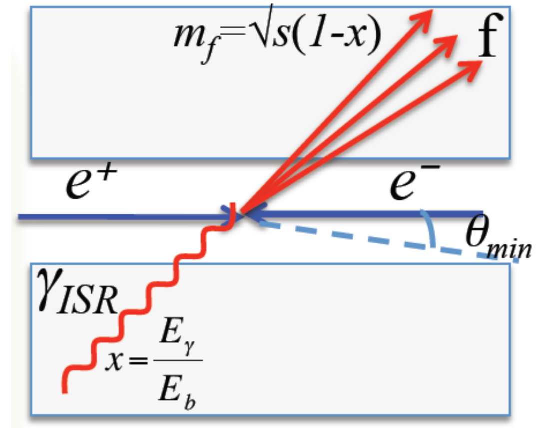


THE ISR METHOD

Born Approximation
$$\frac{d\sigma_{e^+e^-\to f\gamma}(s,m_f,\theta^*_\gamma)}{dm\, d\cos\theta^*_\gamma} = \frac{2m}{s} W(s,x,\theta^*_\gamma)\cdot\sigma_{e^+e^-\to f}(m_f)$$

$$x = \frac{E_\gamma}{E_b} \qquad m^2 = s' = s(1-x)$$

$\theta_\gamma{}^*$ ISR photon polar angle in the e^+e^- cm



Radiator function (at lowest order):
$$w_0(s,x,\theta^*) = \frac{\alpha}{\pi x}\left[\frac{(2-2x+x^2)\sin^2\theta^* - \frac{x^2}{2}\sin^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)^2} - \frac{4m_e^2}{s}\frac{(1-2x)\sin^2\theta^* - x^2\cos^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)}\right]$$

Cross section of $e^+e^- \to f$
$$\sigma_0(m_i) = \frac{\Delta N(m_i)}{\Delta m}\frac{1}{\varepsilon(s,m_i)(1+\delta_{rad})\mathrm{d}\mathcal{L}(m_i)/\mathrm{d}m}$$

Reconstruction efficiency

Radiative corrections

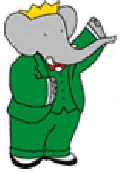
ISR luminosity

ISR differential luminosity
$$\frac{d\mathcal{L}}{dm} = \frac{2m}{s}\frac{\alpha}{\pi\cdot x}\cdot\left((2-2x+x^2)\log\frac{1+C}{1-C} - x^2C\right)L_{ee}$$

Luminosity integrated by the collider

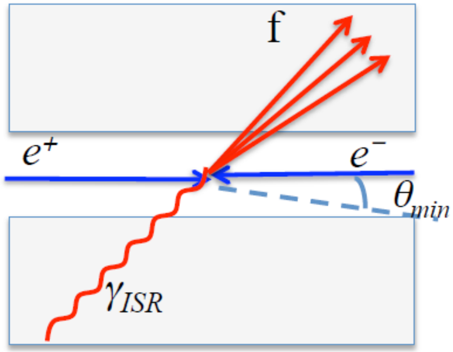
$C=\cos\theta^*_{min}$ Detector angular acceptance

- Obtained from integration of the radiator function over $\theta_\gamma{}^*$
- $20° < \theta_\gamma{}^* < 160°$: acceptance for the ISR photon is 15% in *BABAR*
- Known at <1% level



TAGGED/UNTAGGED ISR

- Tagged approach
  - fully reconstructed events ➞ great background reduction
  - ~90% signal loss
- Untagged approach
  - typically higher efficiency
  - higher background reduced by requiring the missing mass consistent with zero
- So, what is the more convenient approach?
  - It depends on experimental situation
  - At √s=10.58 GeV and for low m_f, (i.e. large x) the hadronic system has a large boost opposite to the photon direction
  - the efficiency is almost insensitive to tagging
- This is why at *BABAR*:
  - Light Quarks final states ⟺ Tagged analyses
  - Heavy Quarks final states ⟺ Untagged analyses



[Druzhinin et *al*, arXiv:1105.4975]