

# Hadronic contributions to $(g - 2)_\mu$ : a new approach to light-by-light

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FOR FUNDAMENTAL PHYSICS

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# Outline

Introduction:  $(g - 2)_\mu$  and hadronic light-by-light

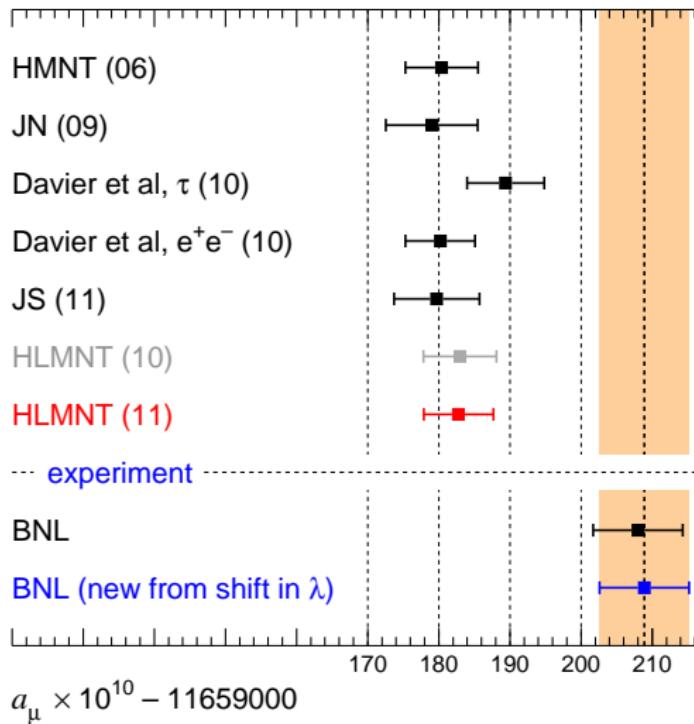
A dispersive approach to HLbL

Conclusions

arXiv:1402.7081

in collaboration with M. Hoferichter, M. Procura and P. Stoffer

# Status of $(g - 2)_\mu$ , experiment vs SM



# Status of $(g - 2)_\mu$ , experiment vs SM

Different contributions to the total SM result

	$a_\mu [10^{-11}]$	$\Delta a_\mu [10^{-11}]$
experiment	116 592 089.	63.
QED $\mathcal{O}(\alpha)$	116 140 973.21	0.03
QED $\mathcal{O}(\alpha^2)$	413 217.63	0.01
QED $\mathcal{O}(\alpha^3)$	30 141.90	0.00
QED $\mathcal{O}(\alpha^4)$	381.01	0.02
QED $\mathcal{O}(\alpha^5)$	5.09	0.01
QED total	116 584 718.95	0.04
electroweak, total	153.6	1.0
HVP (LO) <a href="#">[Hagiwara et al. 2011]</a>	6 949.	43.
HVP (HO) <a href="#">[Hagiwara et al. 2011]</a>	-98.	1.
HLbL <a href="#">[Jegerlehner-Nyffeler 2009]</a>	116.	40.
theory	116 591 839.	59.

# Hadronic light-by-light: irreducible uncertainty?

- ▶ Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) can be systematically improved  
(but going much below 1% is hard – dealing with radiative corrections poses serious problems)
- ▶ Hadronic light-by-light (HLbL) is more problematic:
  - ▶ “it *cannot* be expressed in terms of measurable quantities”
  - ▶ reliability of uncertainty estimate based more on consensus than on a systematic method
  - ▶ only first-principle method in sight: lattice QCD  
(when will it become competitive?)

# Different evaluations of HLbL

Jegerlehner Nyffeler 2009

**Table 13**

Summary of the most recent results for the various contributions to  $a_{\mu}^{\text{HLbL; had}} \times 10^{11}$ . The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
$\pi^0, \eta, \eta'$	$85 \pm 13$	$82.7 \pm 6.4$	$83 \pm 12$	$114 \pm 10$	-	$114 \pm 13$	$99 \pm 16$
$\pi, K$ loops	$-19 \pm 13$	$-4.5 \pm 8.1$	-	-	-	$-19 \pm 19$	$-19 \pm 13$
$\pi, K$ loops + other subleading in $N_c$	-	-	-	$0 \pm 10$	-	-	-
Axial vectors	$2.5 \pm 1.0$	$1.7 \pm 1.7$	-	$22 \pm 5$	-	$15 \pm 10$	$22 \pm 5$
Scalars	$-6.8 \pm 2.0$	-	-	-	-	$-7 \pm 7$	$-7 \pm 2$
Quark loops	$21 \pm 3$	$9.7 \pm 11.1$	-	-	-	$2.3 \pm$	$21 \pm 3$
Total	$83 \pm 32$	$89.6 \pm 15.4$	$80 \pm 40$	$136 \pm 25$	$110 \pm 40$	$105 \pm 26$	$116 \pm 39$

- ▶ large uncertainties (and differences among calculations) in individual contributions
- ▶ pseudoscalar pole contributions most important
- ▶ second most important: pion loop, *i.e.* two-pion cuts ( $K_s$  are subdominant)
- ▶ heavier single-particle poles decreasingly important (unless one models them to resum the high-energy tail)

# Approaches to Hadronic light-by-light

## ► Model calculations

- ▶ ENJL Bijnens, Pallante, Prades (95-96)
- ▶ NJL and hidden gauge Hayakawa, Kinoshita, Sanda (95-96)
- ▶ nonlocal  $\chi$ QM Dorokhov, Broniowski (08)
- ▶ AdS/CFT Cappiello, Cata, D'Ambrosio (10)
- ▶ Dyson-Schwinger Goecke, Fischer, Williams (11)
- ▶ constituent  $\chi$ QM Greynat, de Rafael (12)
- ▶ resonances in the narrow-width limit Pauk, Vanderhaeghen (14)

## ► Impact of rigorously derived constraints

- ▶ high-energy constraints taken into account in several models above  
addressed specifically by Knecht, Nyffeler (01)
- ▶ high-energy constraints related to the axial anomaly Melnikov, Vainshtein (04) and Nyffeler (09)
- ▶ sum rules for  $\gamma^* \gamma \rightarrow X$  Pascalutsa, Pauk, Vanderhaeghen (12)  
see also: workshop MesonNet (13)
- ▶ low-energy constraints–pion polarizabilities Engel, Ramsey-Musolf (13)

## ► Lattice

Blum et al. (05,12)

# Some notation

HLbL tensor:

$$\Pi^{\mu\nu\lambda\sigma} = i^3 \int dx \int dy \int dz e^{-i(x \cdot q_1 + y \cdot q_2 + z \cdot q_3)} \langle 0 | T \{ j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0) \} | 0 \rangle$$

where  $j^\mu(x) = \sum_i Q_i \bar{q}_i(x) \gamma^\mu q_i(x)$ ,  $i = u, d, s$

$$k = q_1 + q_2 + q_3 \quad k^2 = 0$$

## Helicity amplitudes

$$\begin{aligned} H_{\lambda_1\lambda_2,\lambda_3\lambda_4}(s, t, u) &\equiv \mathcal{M}(\gamma^*(q_1, \lambda_1)\gamma^*(q_2, \lambda_2) \rightarrow \gamma^*(-q_3, \lambda_3)\gamma(k, \lambda_4)) \\ &= \epsilon_\mu(\lambda_1, q_1)\epsilon_\nu(\lambda_2, q_2)\epsilon_\lambda^*(\lambda_3, -q_3)\epsilon_\sigma^*(\lambda_4, k)\Pi^{\mu\nu\lambda\sigma} \end{aligned}$$

with Mandelstam variables

$$s = (q_1 + q_2)^2 \quad t = (q_1 + q_3)^2 \quad u = (q_2 + q_3)^2$$

# Contribution to $a_\mu$

From gauge invariance:

$$\Pi_{\mu\nu\lambda\sigma}(q_1, q_2, k - q_1 - q_2) = -k^\rho \frac{\partial}{\partial k^\sigma} \Pi_{\mu\nu\lambda\rho}(q_1, q_2, k - q_1 - q_2).$$

Contribution to  $a_\mu$ :

$$a_\mu = \lim_{k \rightarrow 0} \text{Tr} \left\{ (\not{p} + m) \Lambda^\rho(p', p) (\not{p}' + m) \Gamma_\rho(p', p) \right\}$$

$$\Gamma_\rho = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 q_3^2} \frac{\gamma^\mu (\not{p}' + q_1 + m) \gamma^\lambda (\not{p} - q_2 + m) \gamma^\nu}{((p' + q_1)^2 - m^2)((p - q_2)^2 - m^2)} k^\sigma \partial_{k^\rho} \Pi_{\mu\nu\lambda\sigma}$$

with the projector

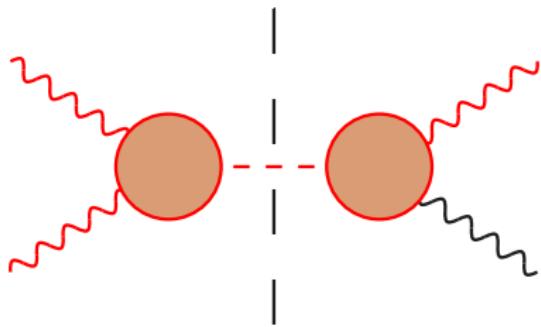
$$\Lambda^\rho(p', p) = \frac{m^2}{k^2(4m^2 - k^2)} \left\{ \gamma^\rho + \frac{k^2 + 2m^2}{m(k^2 - 4m^2)} (p + p')^\rho \right\}$$

$m$  denotes the mass of the muon,  $p$  and  $p' = p - k$  the momenta of the incoming and outgoing muon, respectively

# Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$



Pion pole: known

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$$F_\pi^V(q_1^2) F_\pi^V(q_2^2) F_\pi^V(q_3^2) \times \left[ \begin{array}{c} \text{Box diagram} \\ \text{Triangle diagram} \\ \text{Bulb diagram} \end{array} \right]$$

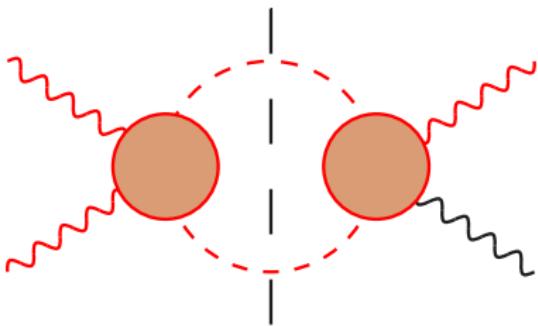
Contribution with two simultaneous cuts

- analytic properties like the box diagram in sQED
- triangle and bulb diagram required by gauge invariance
- multiplication with  $F_\pi^V$  gives the correct  $q^2$  dependence  
**it is not an approximation!**

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The “rest” with  $2\pi$  intermediate states has cuts only in one channel and is what will be calculated dispersively

# Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

Contributions of cuts with anything else other than one and two pions in intermediate states will be neglected

# Our dispersive representation of the HLbL tensor

$$\bar{\Pi}^{\mu\nu\lambda\sigma} = \sum_{i=1}^{15} \left( A_{i,s}^{\mu\nu\lambda\sigma} \Pi_i(s) + A_{i,t}^{\mu\nu\lambda\sigma} \Pi_i(t) + A_{i,u}^{\mu\nu\lambda\sigma} \Pi_i(u) \right)$$

- ▶ the  $\Pi_i(s)$  are single-variable functions having only a right-hand cut
- ▶ for the discontinuity we keep only the lowest partial wave
- ▶ the dispersive integral that gives the  $\Pi_i(s)$  in terms of its discontinuity has the required soft-photon zeros
- ▶ soft-photon zeros constrain the subtraction polynomial to vanish (unless one wanted to subtract more, which is unnecessary)

# Dispersion relations for the $\Pi_i(s)$

Imposing the same form of the soft-photon zeros as in the subamplitudes  $\gamma^*\gamma^* \rightarrow \pi\pi$  we obtain the following dispersion relations:

$$\Pi_1^s = \bar{h}_{++,++}^0(s) = \frac{s - q_3^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s' - q_3^2} \left( \frac{1}{s' - s} - \frac{s' - q_1^2 - q_2^2}{\lambda'_{12}} \right) \text{Im} \bar{h}_{++,++}^0(s')$$

$$y \Pi_2^s = \bar{h}_{00,++}^0(s) = \frac{s - q_3^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s' - q_3^2} \left( \frac{1}{s' - s} - \frac{s' - q_1^2 - q_2^2}{\lambda'_{12}} \right) \text{Im} \bar{h}_{00,++}^0(s')$$

with  $y = -\frac{q_1^2 q_2^2}{\xi_1 \xi_2}$  [and similarly for the others]

# Master formula

$$a_\mu^{\pi\pi} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{I^{\pi\pi}}{q_1^2 q_2^2 s((p+q_1)^2 - m^2)((p-q_2)^2 - m^2)},$$

$$I^{\pi\pi} = \sum_{i \in \{1,2,3,6,14\}} \left( T_{i,s} I_{i,s} + 2 T_{i,u} I_{i,u} \right) + 2 T_{9,s} I_{9,s} + 2 T_{9,u} I_{9,u} + 2 T_{12,u} I_{12,u}$$

with  $I_{i,(s,u)}$  dispersive integrals and  $T_{i,(s,u)}$  integration kernels

$$I_{1,s} = \frac{1}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{s' - s} \left( \frac{1}{s' - s} - \frac{s' - q_1^2 - q_2^2}{\lambda(s', q_1^2, q_2^2)} \right) \text{Im} \bar{h}_{++,++}^0(s'; q_1^2, q_2^2; s, 0),$$

$$T_{1,s} = \frac{16}{3} s \left\{ m^2 + \frac{8P_{21} p \cdot q_1}{\lambda_{12}} \right\}, \quad T_{1,u} = \frac{16}{3} \left\{ \frac{4P_{12}^2}{\lambda_{12}} - P_{12} - Z_u \right\},$$

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$$I_{6,s} = \frac{1}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{(s' - q_1^2 - q_2^2)(s' - s)^2} \text{Im} \bar{h}_{+-,+-}^{\textcolor{red}{2}}(s'; q_1^2, q_2^2; s, 0) \left( \frac{75}{8} \right)$$

Helicity amplitudes contribute up to  $J = 2$  ( $S$  and  $D$  waves)

# Master formula

The bars on the helicity amplitudes mean that we must subtract the FsQED contribution.

The unitarity relation for the barred imaginary parts read

$$\begin{aligned} \text{Im}_s \bar{h}_{J,ij}(s) &= \\ &= h_{J,i}^c(s; q_1^2, q_2^2) \left( h_{J,j}^c(s; q_3^2, 0) \right)^* - N_{J,i}(s; q_1^2, q_2^2) N_{J,j}(s; q_3^2, 0) \\ &\quad + \frac{1}{2} h_{J,i}^n(s; q_1^2, q_2^2) \left( h_{J,j}^n(s; q_3^2, 0) \right)^* \end{aligned}$$

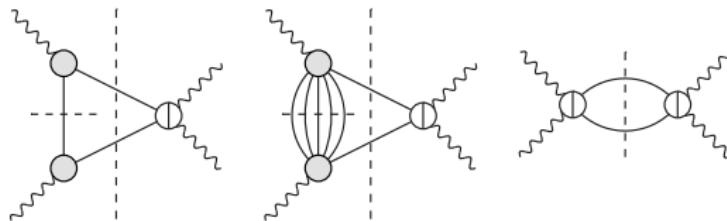
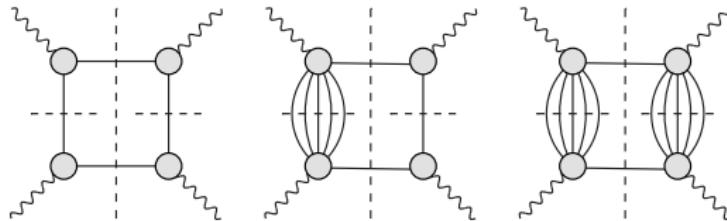
where:

$h_{J,i}^{c,n}$  = helicity amplitudes for  $\gamma^* \gamma^* \rightarrow \pi^+ \pi^-$  and  $\pi^0 \pi^0$  resp.

$N_{J,i}$  = partial-wave projection of the  $\gamma^* \gamma^* \rightarrow \pi^+ \pi^-$  Born term

# Master formula

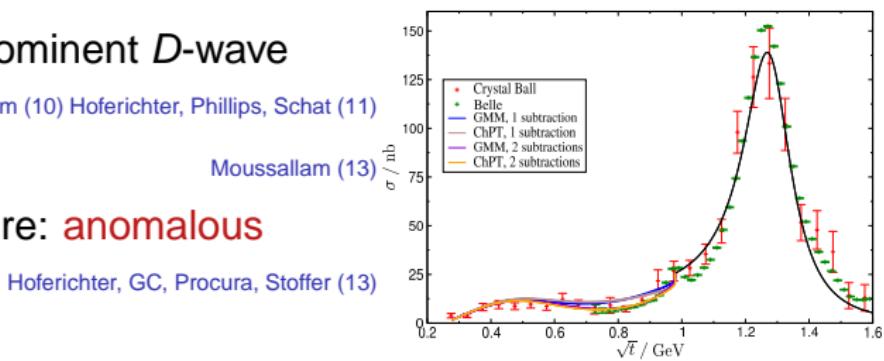
What contributions are included? How?



# Dispersion relations for $\gamma^*\gamma^* \rightarrow \pi\pi$

Roy-Steiner eqs. = Dispersion relations + partial-wave expansion  
+ crossing symmetry + unitarity + gauge invariance

- ▶ On-shell  $\gamma\gamma \rightarrow \pi\pi$ : prominent *D*-wave reson.  $f_2(1270)$  Moussallam (10) Hoferichter, Phillips, Schat (11)
- ▶  $\gamma^*\gamma \rightarrow \pi\pi$  Moussallam (13)
- ▶  $\gamma^*\gamma^* \rightarrow \pi\pi$ , new feature: anomalous thresholds Hoferichter, GC, Procura, Stoffer (13)



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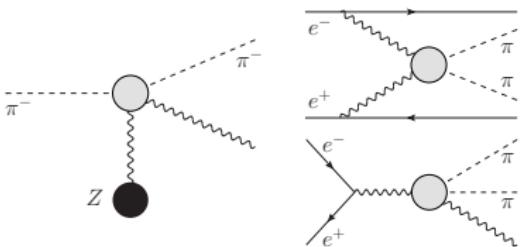
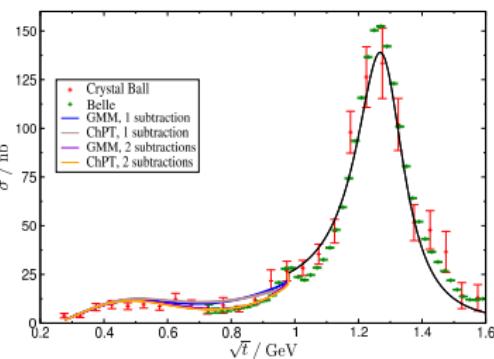
Moussallam (13)

- $\gamma^*\gamma^* \rightarrow \pi\pi$ , new feature: anomalous thresholds

Hoferichter, GC, Procura, Stoffer (13)

- Constraints

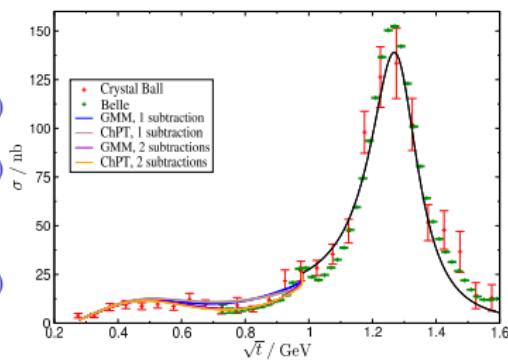
- Low energy: pion polar., ChPT
- Primakoff:  $\gamma\pi \rightarrow \gamma\pi$  at COMPASS, JLAB
- Scattering:  $e^+e^- \rightarrow e^+e^-\pi\pi$ ,  $e^+e^- \rightarrow \pi\pi\gamma$
- Decays:  $\omega, \phi \rightarrow \pi\pi\gamma$



# Dispersion relations for $\gamma^*\gamma^* \rightarrow \pi\pi$

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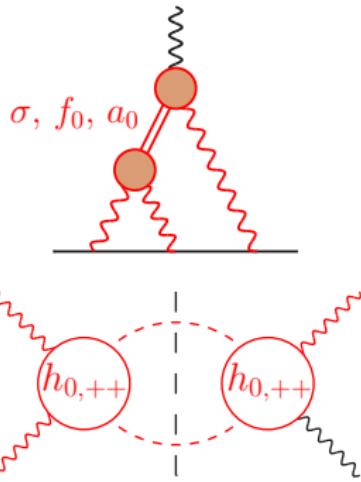
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Analysis of the Roy-Steiner equations for  $\gamma^*\gamma^* \rightarrow \pi\pi$  is in progress: any experimental input most welcome

# Physics of $\gamma^*\gamma^* \rightarrow \pi\pi$

- ▶  $\pi\pi$  rescattering  $\Leftrightarrow$  resonances, e.g.  $f_2(1270)$
- ▶ S-wave provides model-independent implementation of the  $\sigma$



# Physics of $\gamma^*\gamma^* \rightarrow \pi\pi$

- $\pi\pi$  rescattering  $\Leftrightarrow$  resonances, e.g.  $f_2(1270)$
- S-wave provides model-independent implementation of the  $\sigma$
- Analytic continuation with dispersion theory: resonance properties

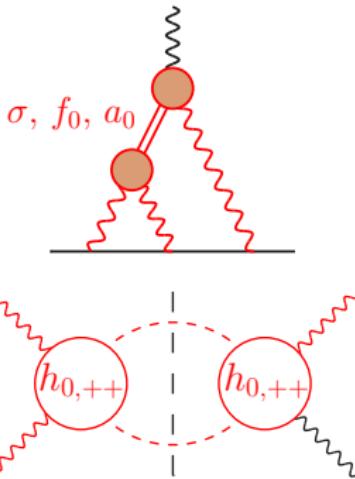
- Precise determination of  $\sigma$ -pole from  $\pi\pi$  scattering Caprini, GC, Leutwyler 2006

$$M_\sigma = 441^{+16}_{-8} \text{ MeV} \quad \Gamma_\sigma = 544^{+18}_{-25} \text{ MeV}$$

- Coupling  $\sigma \rightarrow \gamma\gamma$  from  $\gamma\gamma \rightarrow \pi\pi$   
Hoferichter, Phillips, Schat 2011

## $f_0(500)$ PARTIAL WIDTHS

$\Gamma(\gamma\gamma)$	DOCUMENT ID	TECN	COMMENT
<small>• • • We do not use the following data for averages, fits, limits, etc. • • •</small>			
1.7 ± 0.4	54 HOFERICHTER11	RVUE	Compilation
3.08 ± 0.82	55 MENNESSIER 11	RVUE	Compilation
2.08 ± 0.2 + 0.07 - 0.04	56 MOUSSALLAM11	RVUE	Compilation
2.08	57 MAO 09	RVUE	Compilation
1.2 ± 0.4	58 BERNABEU 08	RVUE	
3.9 ± 0.6	55 MENNESSIER 08	RVUE	$\gamma\gamma \rightarrow \pi^+\pi^-$ , $\pi^0\pi^0$
1.8 + 0.4	59 OLLFR 08	RVUE	Compilation

 $\Gamma_2$ 

$f_0(500)$  or  $\sigma$   
was  $f_0(600)$

$J^P(JPC) = 0^+(0^{++})$

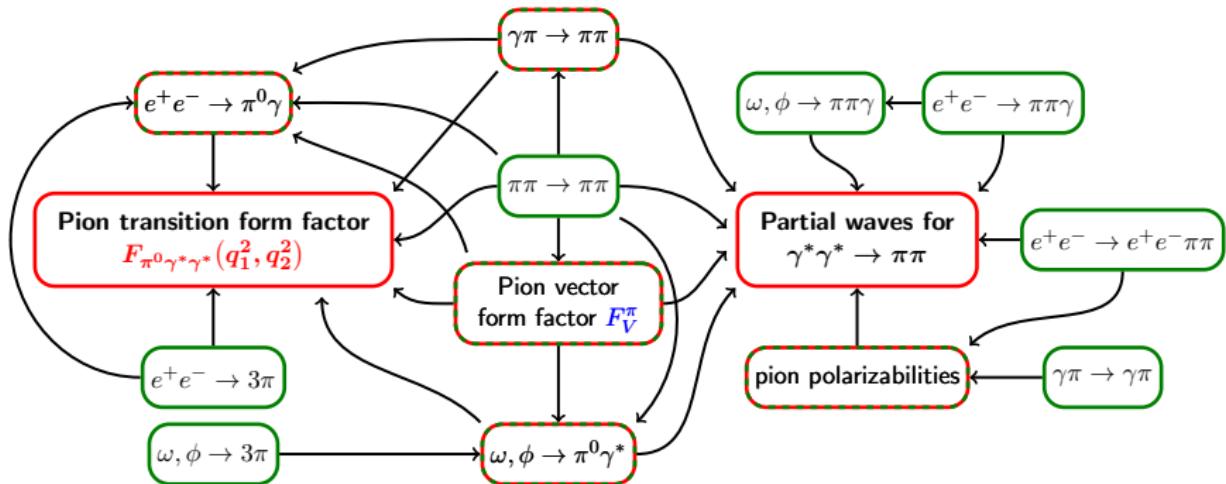
A REVIEW GOES HERE – Check our WWW List of Reviews

## $f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that  $\Gamma \approx 2 \text{ Im}(\sqrt{s}/\text{pole})$ .

VALUE (GeV)	DOCUMENT ID	TECN	COMMENT
<small>• • • We do not use the following data for averages, fits, limits, etc. • • •</small>			
(400–550) – i(200–350)	<b>OUR ESTIMATE</b>		
(445 ± 25) – i(278 ± 22)	1.2 GARCIA-MAR..11	RVUE	Compilation
(457 ± 14) – i(279 ± 17)	1.3 GARCIA-MAR..11	RVUE	Compilation
(442 ± 8) – i(274 ± 5)	4 MOUSSALLAM11	RVUE	Compilation
(452 ± 13) – i(259 ± 16)	5 MENNESSIER 10	RVUE	Compilation
(448 ± 43) – i(266 ± 43)	6 MENNESSIER 10	RVUE	Compilation
(455 ± 6 ± 31) – i(278 ± 6 ± 34)	7 CAPRINI 08	RVUE	Compilation

# Hadronic light-by-light: a roadmap



Artwork by M. Hoferichter

A reliable evaluation of the HLB L requires many different contributions by and a collaboration among theorists and experimentalists

# Outlook

- ▶ we have not considered so far non-diagonal (non-Cauchy) kernels – their complete derivation is in progress
- ▶ path to a numerical evaluation of the Master Formula:
  - ▶ take into account all experimental constraints on  
 $\gamma^{(*)}\gamma \rightarrow \pi\pi$
  - ▶ estimate the dependence on the  $q^2$  of the second photon  
(theoretically, there are no data yet on  $\gamma^*\gamma^* \rightarrow \pi\pi$ )
  - ▶ ⇒ solve the dispersion relation for  $\gamma^*\gamma^* \rightarrow \pi\pi$
- ▶ input the outcome into the (upgraded) master formula

# Conclusions

- ▶ I have presented a dispersive framework for the calculation of the HLbL contribution to  $a_\mu$
- ▶ which takes into account only single- and double-pion intermediate states  
[and all other 1-particle intermediate states ( $\eta, \eta', \dots$ )]
- ▶ we have derived a **master formula** which expresses the contribution of  $2\pi$  intermediate states to  $a_\mu$  in terms of (integrals over)  $\gamma^* \gamma^* \rightarrow \pi\pi$  helicity amplitudes
- ▶ this is a first step towards a **model-independent calculation** of the HLbL contribution to  $a_\mu$

# SM contributions to $(g - 2)_\mu$ : QED

$$a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfeld; Petermann; Suura&Wichmann '57; Elenz '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;  
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8796 (63) (\alpha/\pi)^4$$

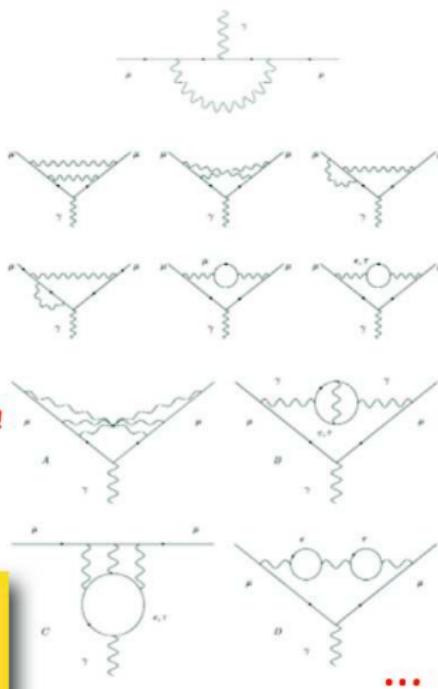
Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;  
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,  
Steinhauser et al. 2013 (analytic, in progress).

$$+ 753.29 (1.04) (\alpha/\pi)^5 \text{ COMPLETED!}$$

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,  
Karshenboim, ..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012

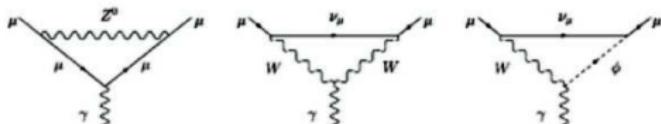
**Adding up, we get:**

$a_\mu^{\text{QED}} = 116584718.951 (22)(77) \times 10^{-11}$   
 from coeffs, mainly from 4-loop unc from  $\delta\alpha(\text{Rb})$   
 with  $\alpha=1/137.035999049(90)$  [0.66 ppb]



# SM contributions to $(g - 2)_\mu$ : electroweak

## ● One-loop term:



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} (1 - 4 \sin^2 \theta_W)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiw, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

## ● One-loop plus higher-order terms:

$$a_\mu^{\text{EW}} = 153.6 (1) \times 10^{-11}$$

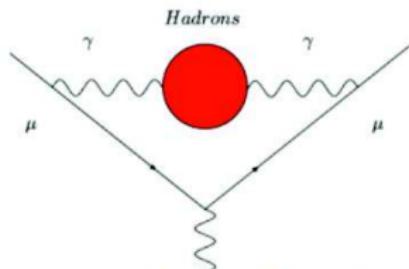
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.

with  $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

Hadronic loop uncertainties  
and 3-loop nonleading logs.



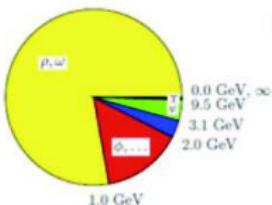
# SM contributions to $(g - 2)_\mu$ : HVP



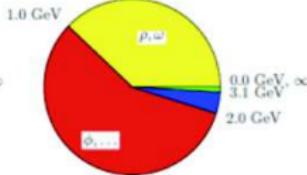
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

Central values



Errors<sup>2</sup>



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$a_\mu^{\text{HLO}} = 6903 (53)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$$= 6923 (42)_{\text{tot}} \times 10^{-11}$$

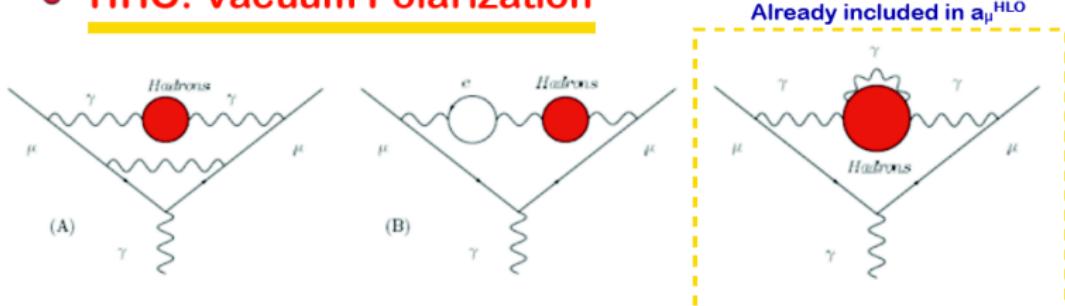
Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2r)

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al, JPG 38 (2011) 085003

# SM contributions to $(g - 2)_\mu$ : Higher-order HVP

- **HHO: Vacuum Polarization**



$\mathcal{O}(\alpha^3)$  contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{\text{HHO(vp)}} = -98 (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

Only tiny shifts if  $\tau$  data are used instead of the  $e^+e^-$  ones

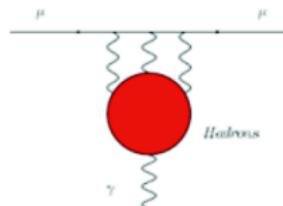
Davier & Marciano '04.

# SM contributions to $(g - 2)_\mu$ : hadronic light-by-light

## • HHO: Light-by-light contribution

Unlike the HLO term, for the hadronic l-b-l term we must rely on theoretical approaches.

This term had a troubled life! Latest values:



$$a_\mu^{\text{HHO}(\text{lbl})} = +80(40) \times 10^{-11} \quad \text{Knecht \& Nyffeler '02}$$

$$a_\mu^{\text{HHO}(\text{lbl})} = +136(25) \times 10^{-11} \quad \text{Melnikov \& Vainshtein '03}$$

$$a_\mu^{\text{HHO}(\text{lbl})} = +105(26) \times 10^{-11} \quad \text{Prades, de Rafael, Vainshtein '09}$$

$$a_\mu^{\text{HHO}(\text{lbl})} = +116(39) \times 10^{-11} \quad \text{Jegerlehner \& Nyffeler '09}$$

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- “Bound”  $a_\mu^{\text{HHO}(\text{lbl})} < \sim 160 \times 10^{-11}$  Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11
- Lattice? Very hard... in progress. M. Golterman @ PhiPsi 2013; T. Blum @ Lattice 2012
- Pion exch. in holographic QCD agrees. D.K.Hong, D.Kim '09; Cappiello, Catà, D'Ambrosio '11
- “By far not complete” calculation:  $188 \times 10^{-11}$  Fischer et al, PRD87(2013)034013
- Had lbl is likely to become the ultimate limitation of the SM prediction.

# SM contributions to $(g - 2)_\mu$ :

$$a_\mu^{\text{EXP}} = 116592089 (63) \times 10^{-11}$$

E821 – Final Report: PRD73  
(2006) 072 with latest value  
of  $\lambda = \mu_\mu / \mu_p$  from CODATA'06

$a_\mu^{\text{SM}} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$	$\sigma$
116 591 793 (66)	$296 (91) \times 10^{-11}$	3.2 [1]
116 591 813 (57)	$276 (85) \times 10^{-11}$	3.2 [2]
116 591 839 (58)	$250 (86) \times 10^{-11}$	2.9 [3]

with the “conservative”  $a_\mu^{\text{HHO}}(\text{lbf}) = 116 (39) \times 10^{-11}$  and the LO hadronic from:

- [1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
- [2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10  $2\pi$ )
- [3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10  $2\pi$ )