

QCD@Work

International Workshop on Quantum Chromodynamics Theory and Experiment

VII edition

Giovinazzo (Bari – Italy), June 16 - 19, 2014



Low energy and nonperturbative QCD - Advances in perturbative QCD
Heavy quarks - Hot and dense QCD - Holographic methods for QCD

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Università del Salento - Lecce
Dipartimento Interateneo di Fisica - Bari
Dipartimento di Matematica e Fisica - Lecce

Magnetic susceptibility of the chiral condensate

Damir Becirevic

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Université Paris Sud et CNRS



Why bother?

- In many phenomenological applications:
 - ✓ Problem of $\Delta \rightarrow p\gamma$
 - ✓ $Z^*-\gamma-\gamma^*$ vertex for $(g-2)_\mu$
 - ✓ Magnetic moments of hadrons
 - ✓ Radiative decays of hadrons (soft photon physics)
 - ✓ Exclusive photo production of hard dijets
 - ✓ Neutron EDM (NP searches)
 - ✓ Modelling QGP, Holography, ...

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- Recent interest for QCD in an external magnetic field [cosmology, QGP in ext. field]

- QCD vacuum polarized - chiral symmetry is spontaneously broken; Index theorem
- Leading OPE power corrections to quark propagator - quark condensate
- GMOR - ChPT: $m_\pi^2 = \frac{4m_q}{f_\pi^2} \langle 0 | \bar{q}q | 0 \rangle + \mathcal{O}(m_q^2)$
- LQCD - compute the pion mass and the quark mass

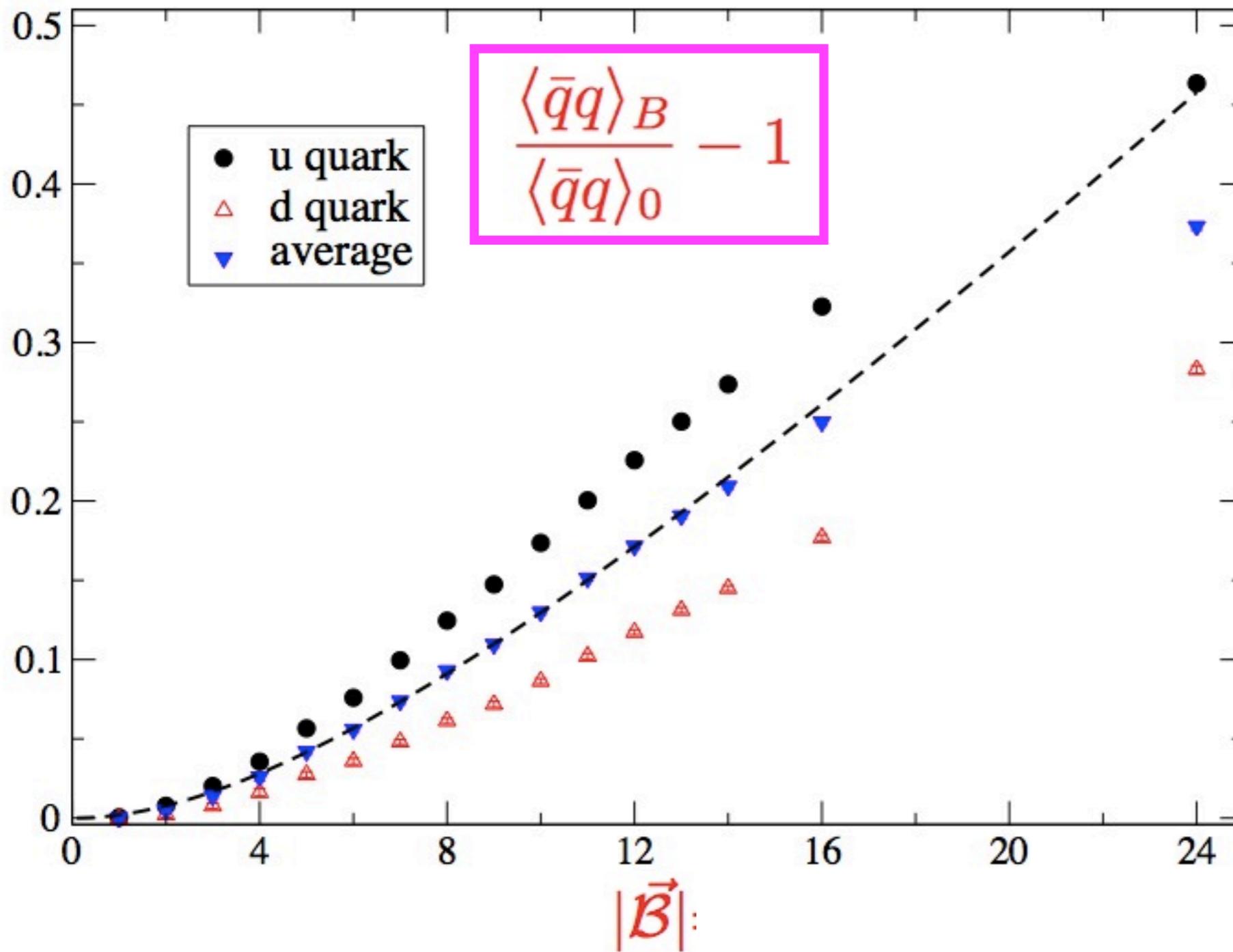
$$\implies \langle 0 | \bar{q}q | 0 \rangle \equiv \langle \bar{q}q \rangle^{\overline{\text{MS}}}(2 \text{ GeV}) = - (271 \pm 3 \text{ MeV})^3$$

$$D_\mu = \partial_\mu + igB_\mu^a(x)T^a + ieA_\mu(x)$$

- standard implementation
- net effect: modifies quark propagation

$$\langle S_i \rangle = \varepsilon_{ijk} \langle \bar{q} \sigma_{jk} q \rangle = \mathcal{B}_i \langle \bar{q} q \rangle \chi \quad \Rightarrow \quad \chi = \left. \frac{\partial \langle \vec{S} \rangle}{\partial \vec{\mathcal{B}}} \right|_{|\vec{\mathcal{B}}|=0}$$

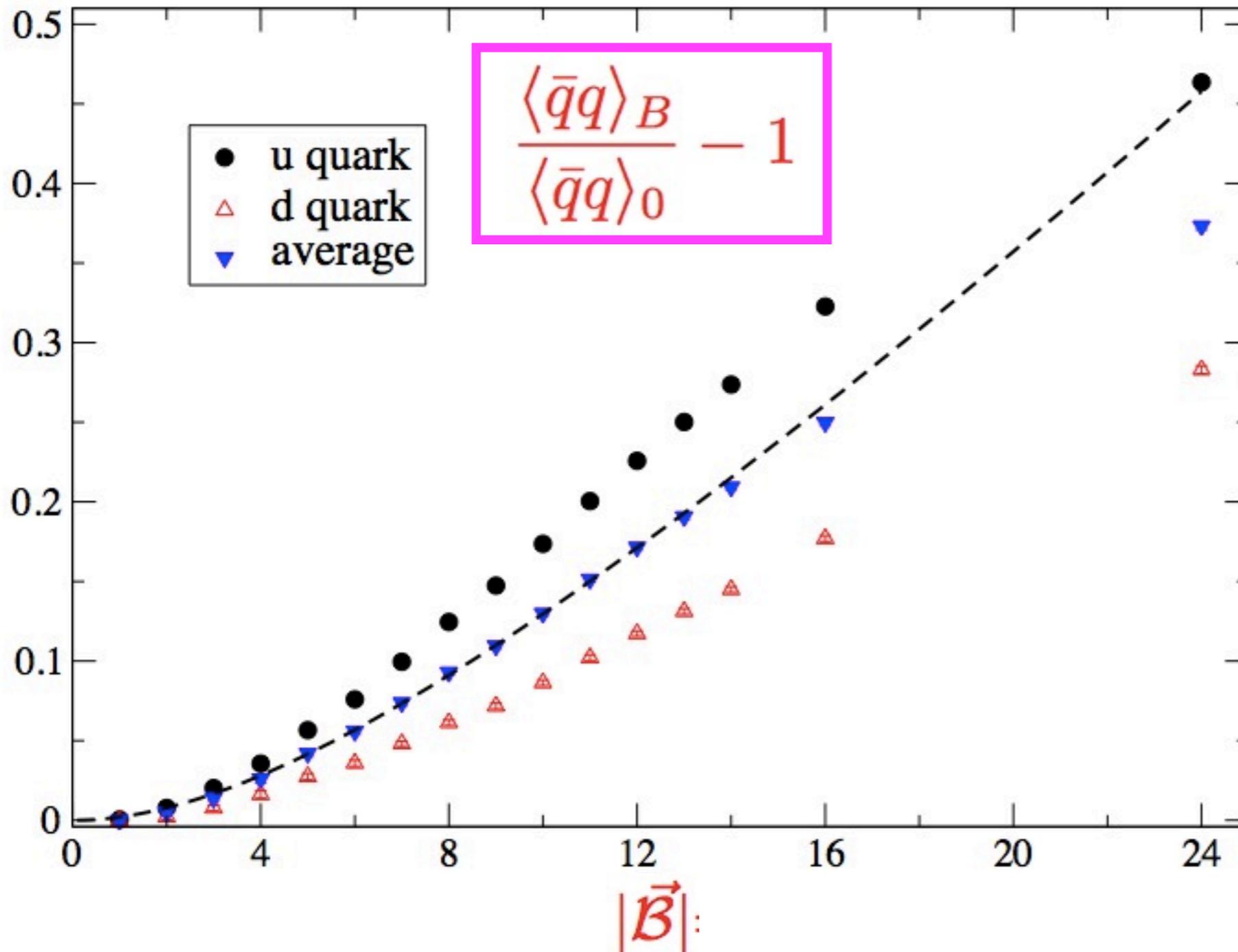
- gluonic stuff gets modified too: magnetic catalysis at $T=0$ is due to gauge field redistribution



Buividovich et al 2012

D'Elia & Negri 2011

Bali et al 2012



Buividovich et al 2012

D'Elia & Negri 2011

Bali et al 2012

ChPT OK

$$\frac{\langle \bar{q}q \rangle_B}{\langle \bar{q}q \rangle_0} = \frac{e\mathcal{B} \log 2}{8\pi^2 f_\pi^2} \mathcal{I}_B \left(\frac{m_\pi^2}{e|\mathcal{B}|} \right)$$

Shushpanov, Smilga 1996

Johnson, Kudla 2008

Dynamical quarks - 40% contribution to catalysis

“Obtaining susceptibility at zero magnetic field”

Ioffe, Smilga 1984
Balitsky et al 1985

Correlation function in a weak external EM field

$$F_{\mu\nu} = i \left(e_{\mu}^{(\lambda)} q_{\nu} - e_{\nu}^{(\lambda)} q_{\mu} \right) e^{iqx}$$

$$i \int d^4x e^{iqx} \langle 0 | \mathbf{T} [J_{\mu}(x) \tilde{J}(0)] | 0 \rangle_F$$

Soft photon processes:

$$D^* \rightarrow D\gamma, B^* \rightarrow B\gamma, B \rightarrow \ell\nu\ell\gamma, \Delta \rightarrow p\gamma, \dots$$

Response of the chiral condensate

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F = e_q \chi \langle 0 | \bar{q} q | 0 \rangle F_{\mu\nu}$$

Leading twist photon distribution amplitude needs

$$\langle 0 | \bar{q}(z) \sigma_{\mu\nu} [z, -z] q(-z) | \gamma^{(\lambda)}(q) \rangle = i f_\gamma^\perp (q_\mu e_\nu^{(\lambda)} - q_\nu e_\mu^{(\lambda)}) \int_0^1 du e^{i\xi q z} \phi_\gamma(u)$$

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | \gamma(p, \lambda) \rangle = i e_q f_\gamma^\perp(\mu) (e_\mu^\lambda p_\nu - e_\nu^\lambda p_\mu)$$

Photon coupling normalization

$$f_\gamma^\perp(\mu) = -\chi(\mu) \langle \bar{q} q \rangle(\mu)$$

$$\chi(\mu) = ?$$

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F = e_q \chi \langle 0 | \bar{q} q | 0 \rangle F_{\mu\nu}$$

$$\chi(\mu) = ?$$

VMD

$$\chi(\mu) = -\frac{f_\rho f_\rho^T(\mu)}{m_\rho \langle \bar{q} q \rangle} \approx 1.8 \text{ GeV}^{-2}$$

QCDSR

Ball et al 2003

$$\chi^{\overline{\text{MS}}}(2\text{GeV}) = -2.1(2) \text{ GeV}^{-2}$$

Narison 2008

$$\chi^{\overline{\text{MS}}}(2\text{GeV}) \approx \pm 5.6 \text{ GeV}^{-2}$$

GMOR-ish

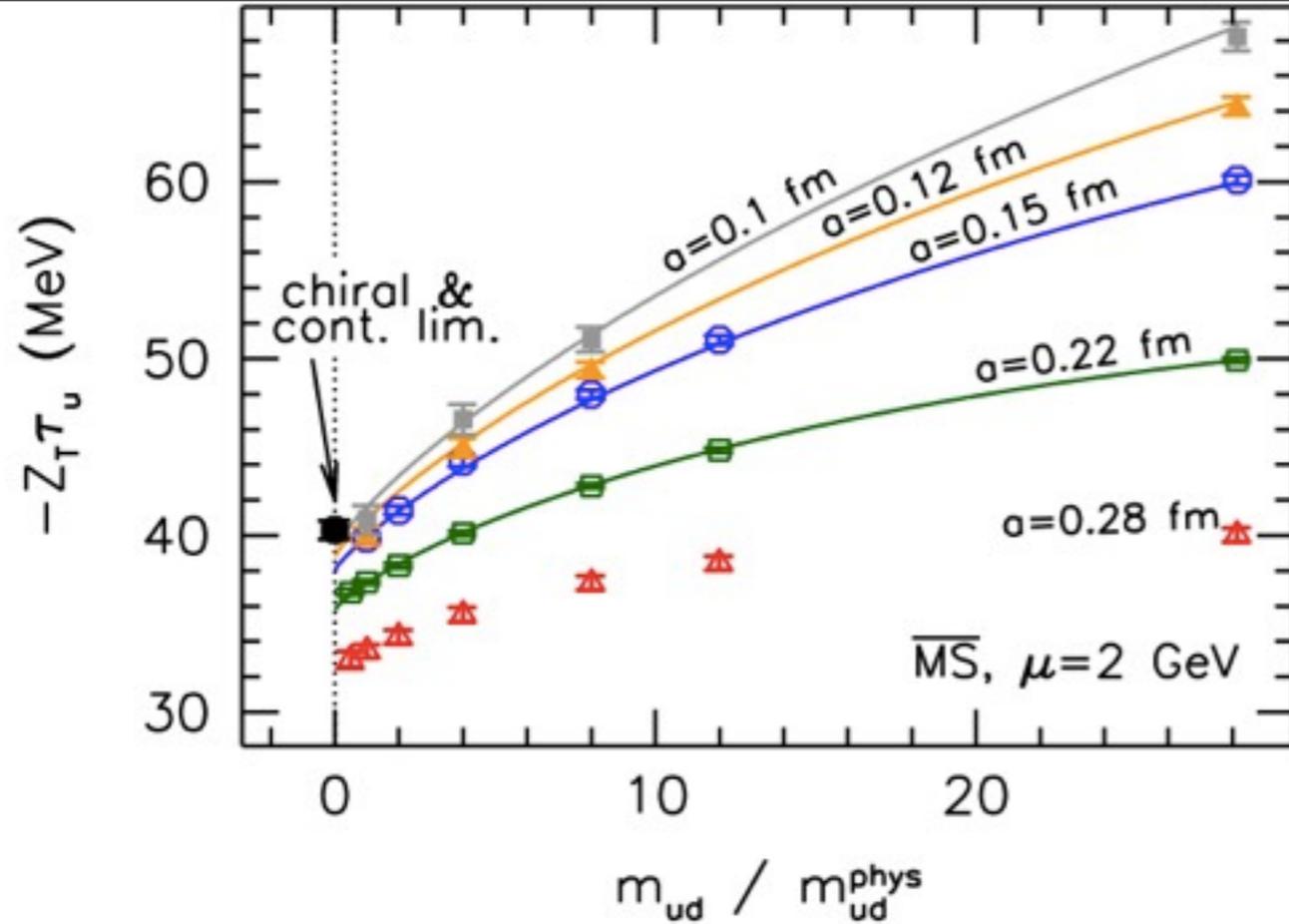
Vainshtein 2002

$$\chi = -\frac{N_c}{2\pi^2 f_\pi^2} \quad [\mu = ?]$$

Instanton
Model

$$\chi \approx -2.4 \text{ GeV}^{-2}$$

Petrov et al 1999



$$\tau = f_\gamma^\perp(2 \text{ GeV})$$

Bali et al 2012

- Recent unquenched Lattice QCD study
- Compute polarization tensor with a background magnetic field
- Take numerically the derivative

$$\chi_{u,d}^{\overline{\text{MS}}}(2 \text{ GeV}) = 2.06(9) \text{ GeV}^{-2}$$

How to get the susceptibility...

$$i \int dx e^{-ipx} \langle 0 | \mathbf{T} \{ V^\mu(x) T^{\alpha\beta}(0; \mu) \} | 0 \rangle = i (g^{\mu\alpha} p^\beta - g^{\mu\beta} p^\alpha) \underbrace{\chi(\mu, p^2) \langle \bar{q}q \rangle(\mu)}_{f(p^2, \mu)}$$

$\bar{q} \gamma^\mu q$
 $\bar{q} \sigma^{\alpha\beta} q$

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$\bar{q}\gamma^\mu q$ $\bar{q}\sigma^{\alpha\beta} q$

D.B., M.D'Elia, F. Sanfilippo 2014 + V.Braun

$$\Rightarrow \sum_{x_4=a\left(-\frac{N_T}{2}+1\right)}^{aN_T/2} x_4 C_{VT}(x_4)$$

$$C_{VT}(x_4) = \frac{1}{3} \sum_{x_i=a\left(-\frac{N_L}{2}+1\right)}^{aN_L/2} \langle 0 | \hat{V}_j(x) \hat{T}_{0j}(0; \mu) \rangle$$

$$\frac{1}{3} \sum_{j=1}^3 \sum_{\vec{x}} \langle \text{Tr}[\gamma_j S_q(x, 0) \gamma_0 \gamma_j S_q(0, x)] \rangle$$

$Z_V \bar{q}\gamma^\mu q$

$Z_T(\mu) \bar{q}\sigma^{\alpha\beta} q$

Gauge field configurations produced by ETMC

D.B., M.D'Elia, F. Sanfilippo 2014

- 13 gauge ensembles (dynamical u/d quarks included)
- Twisted Mass QCD with a Maximal Twist
- Non-perturbative renormalization in the RI-MOM scheme (3-loop conversion to MS)
- No contact term because it is a moment AND because of anti-periodicity of the correlation function

$$C_{VT}(N_T - x_4) = C_{V^t T^t}(x_4) = -C_{VT}(x_4)$$

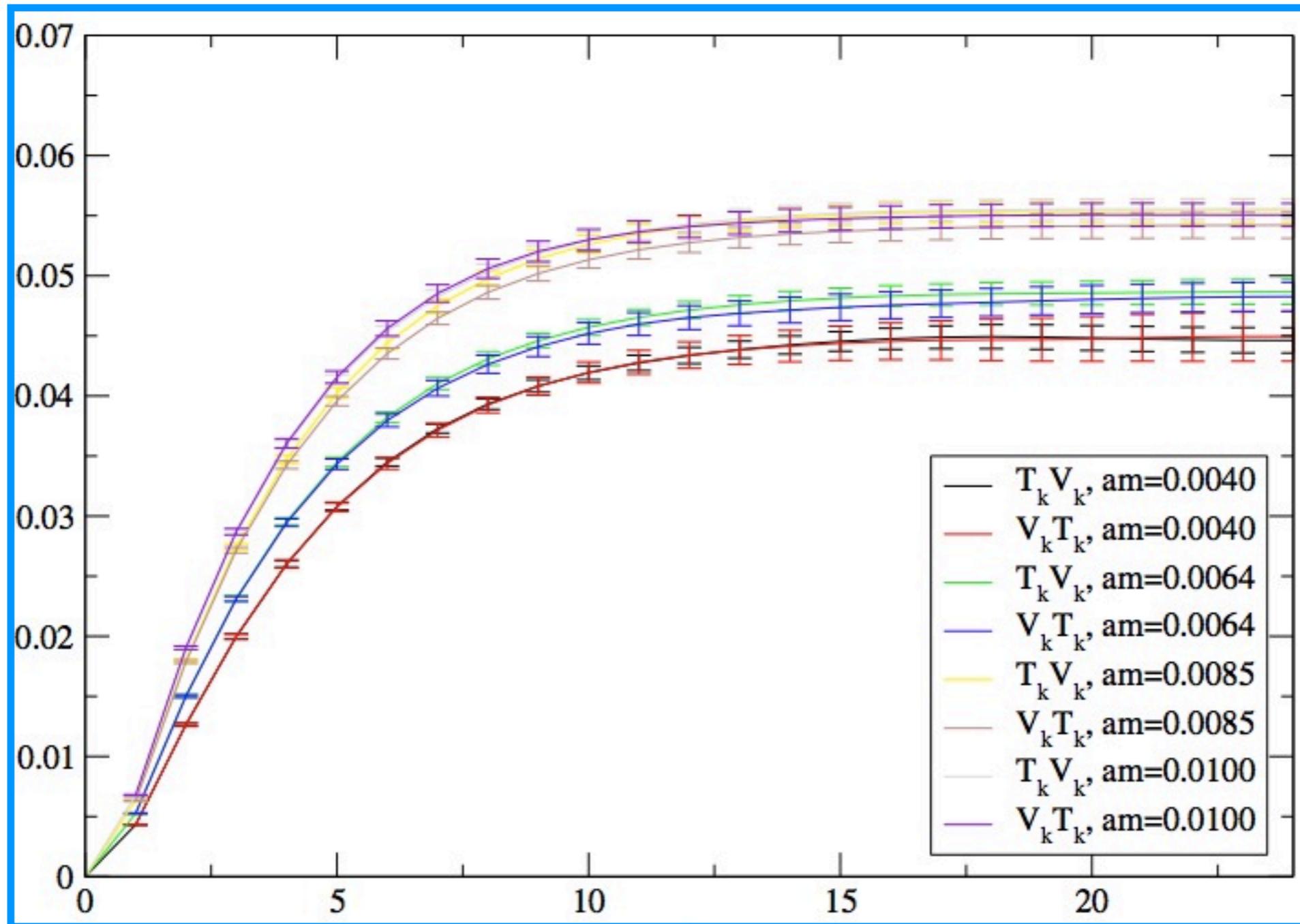
$$V^t : \gamma_0 \gamma_j^\dagger \gamma_0 = -\gamma_j$$

$$T^t : \gamma_0 (\gamma_0 \gamma_j)^\dagger \gamma_0 = +\gamma_0 \gamma_j$$

Integration

D.B., V.Braun, M.D'Elia, F. Sanfilippo 2014

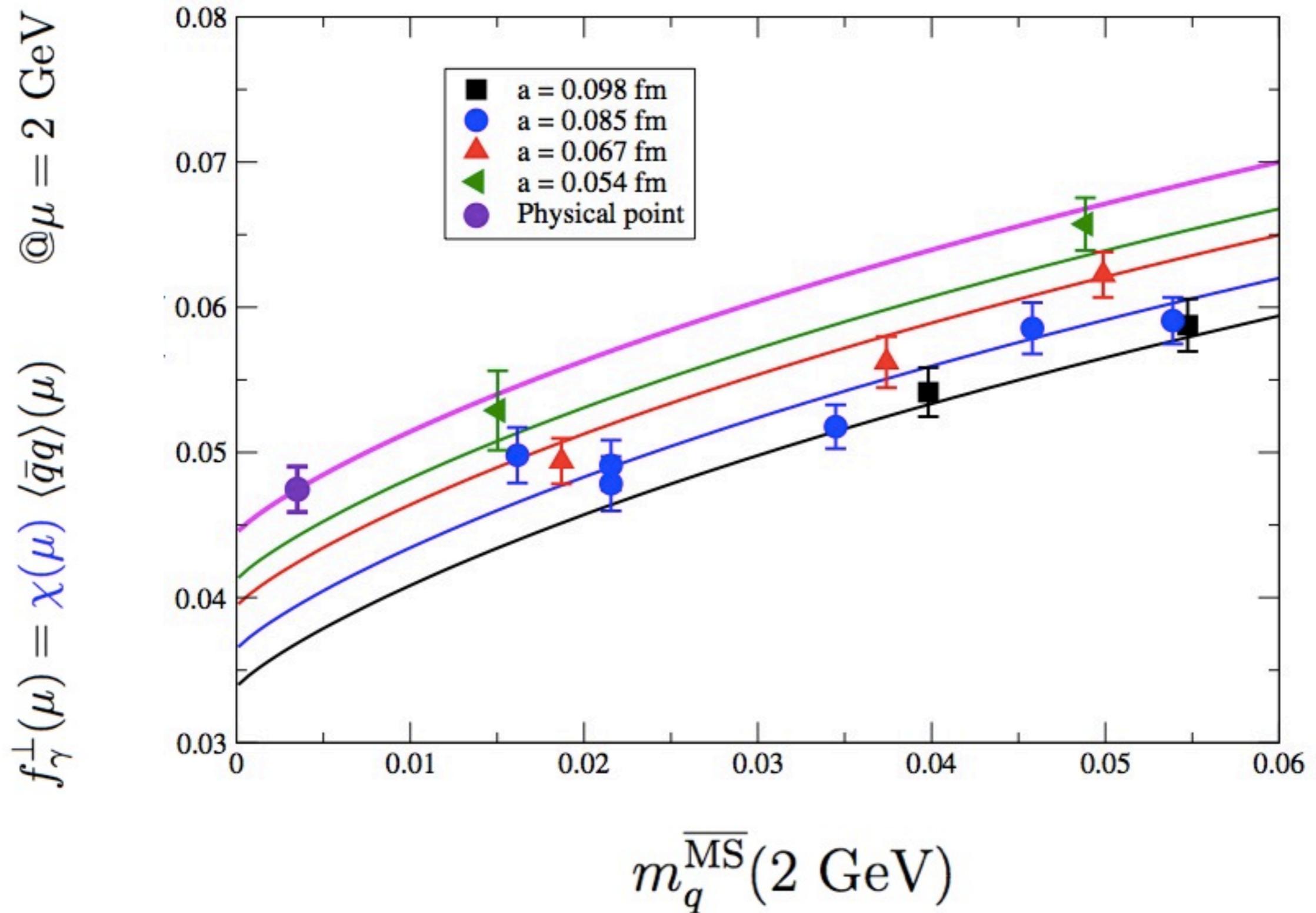
$$a N_T / 2 \sum_{x_4=a}^{N_T/2+1} x_4 C_{VT}(x_4)$$



x_4

Getting to Chiral Limit

D.B., V.Braun, M.D'Elia, F. Sanfilippo 2014

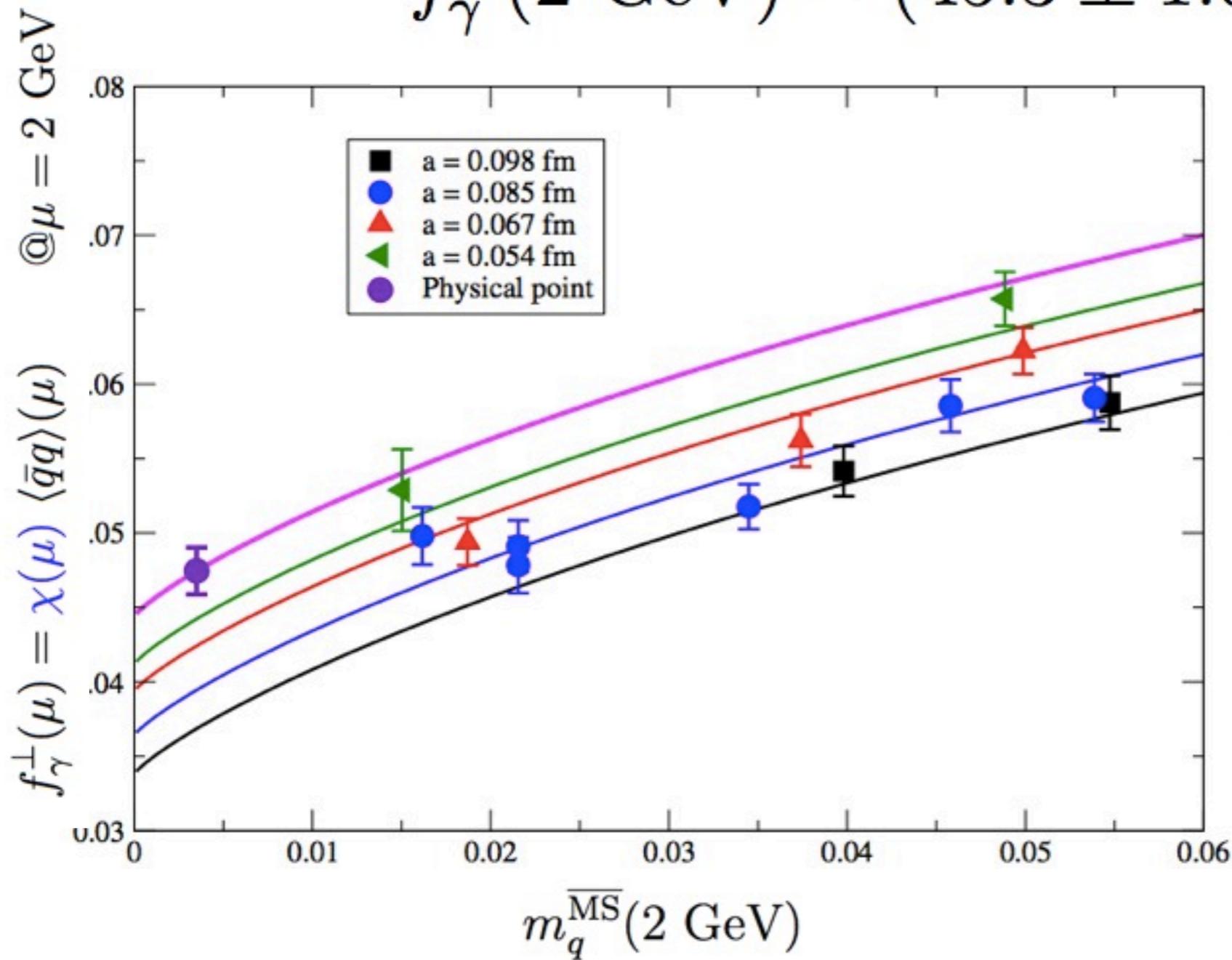


Cata, Mateu 2007: ChPT for tensor charges

Final Results (still preliminary)

D.B., V.Braun, M.D'Elia, F. Sanfilippo 2014

$$f_{\gamma}^{\perp}(2 \text{ GeV}) = (45.3 \pm 1.6) \text{ MeV}$$



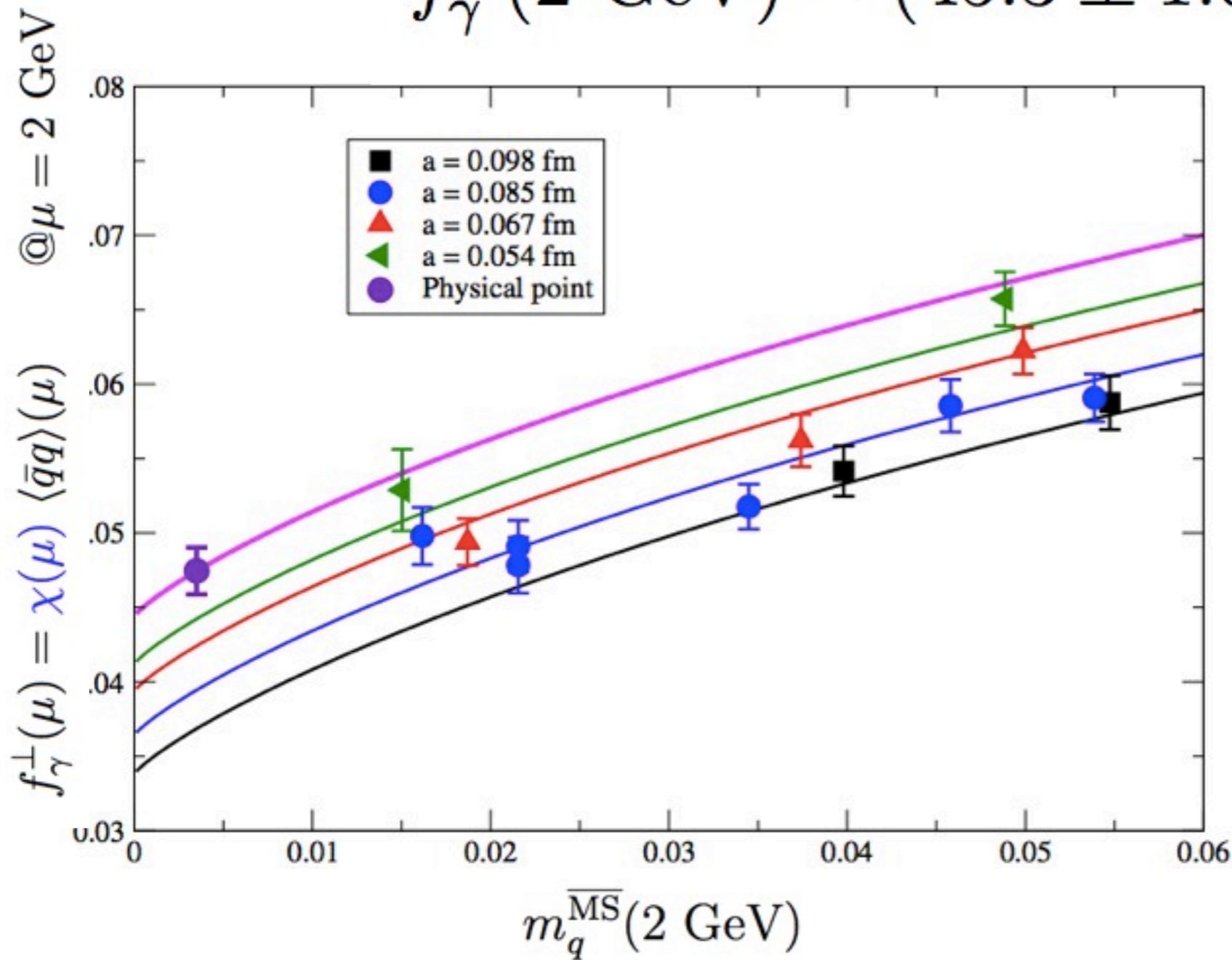
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$$\Rightarrow \chi^{\overline{\text{MS}}}(2 \text{ GeV}) = -2.28(12) \text{ GeV}^{-2}$$

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Lattice QCD I

Bali et al. 2012

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Lattice QCD II

This talk 2014

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