Bottom-quark fragmentation and impact on the top mass measurement

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2. QCD calculations and Monte Carlo codes for b-fragmentation in top events
3. Hadronization models and fits to LEP and SLD data
4. Estimate of systematic error on the top mass measurement
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Based on work by G.C., F.Mescia, V. Drollinger, A.D. Mitov, M. Cacciari, LEP, SLD and LHC top/heavy-quark working groups

Work in progress with F.Mescia and K.Tywoniuk (fragmentation) and M.L. Mangano (t-flavoured hadrons)

Reliable description of multiple radiation in top production and decay and of b-quark fragmentation is fundamental in the measurement of the top properties

Monte Carlo event generators (HERWIG/PYTHIA) widely used to simulate top production and decay and bottom-quark hadronization

LHC and Tevatron inclusive analyses (dilepton, lepton+jets and all-hadrons) propagate the uncertainty on b-fragmentation to the systematic error due to b-jet energy scale and b-tagging efficiency:

$\Delta m_t(\mathrm{bfrag}) \simeq 300$ MeV ; $\Delta m_t(\mathrm{syst}) \simeq 710$ MeV (Tevatron/LHC world average)

$J/\psi+$ lepton final states (10^3/year in high-luminosity phase)

$t \to bW$; $b \to B \to J/\psi\ X$; $J/\psi \to \mu^+\mu^-$; $W \to \ell\nu_\ell$

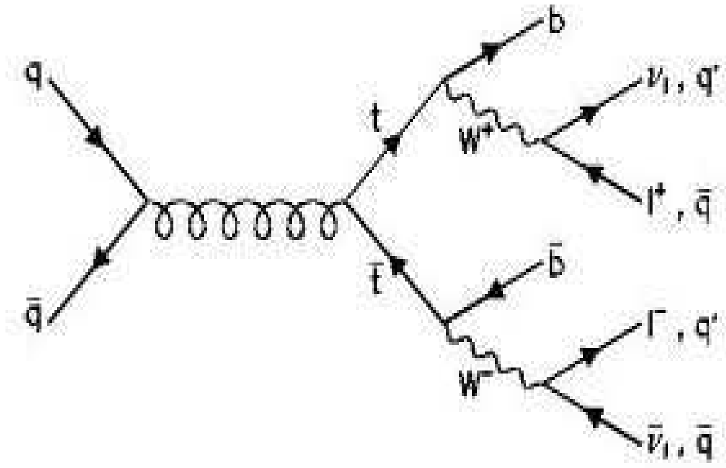
A. Kharchilava, PLB 476 (2000) 73, R. Chierici and A. Dierlamm, CMS Note 2006/058

$m_{3\ell}^{\max} = 0.56\ m_t - 25.3$ GeV Systematics (theo + exp): $\Delta m_t(\mathrm{syst}) \simeq 1.47$ GeV

b-fragmentation (PYTHIA+Peterson model): $\Delta m_t(\mathrm{frag}) \simeq 0.51$ GeV

Several calculations and tools are available for bottom fragmentation in top decays, but not unique strategy for the systematic error: comparing two tuned codes/computations, one program varying fragmentation parameters, etc.

Top production and decays at hadron colliders, e.g. in $q\bar{q}$ annihilation



Perturbative QCD allows one to calculate the parton-level (b-quark) spectrum

Phenomenological hadronization models are given in terms of non-perturbative fragmentation functions

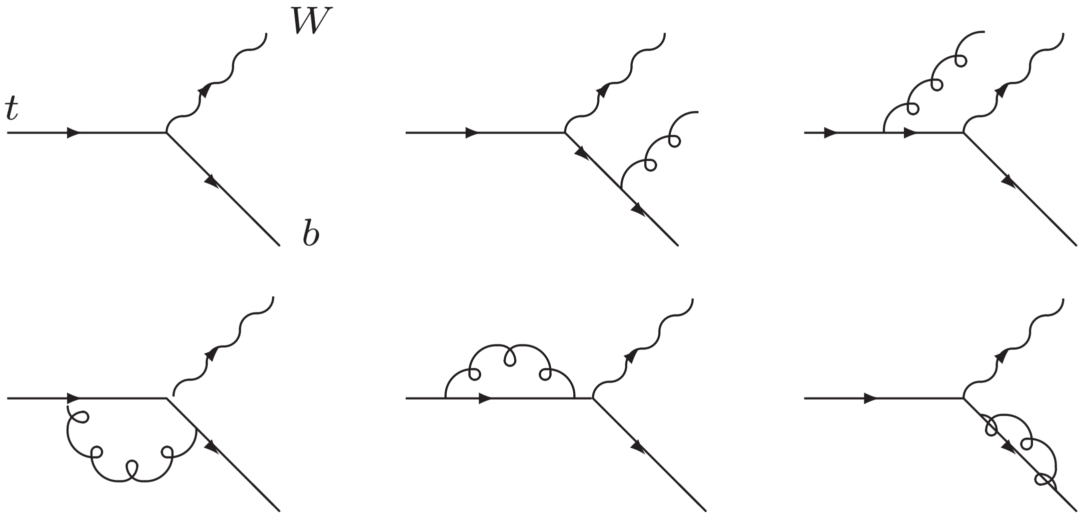
$$\sigma(t \to WB) = \sigma(t \to Wb) \otimes D_{np}(b \to B)$$

$D_{np}(b \to B)$ contains parameters to be fitted to experimental data

Narrow-width approximation (NWA):

$$\frac{d\sigma_{\rm had}}{dx_B}(t \to B) \simeq \frac{d\Gamma_{\rm had}}{dx_B}(t \to B) \quad ; \quad \frac{d\Gamma_{\rm had}}{dx_B}(t \to B) = \frac{d\Gamma_{\rm part}}{dx_b}(t \to b) \otimes D_{np}(b \to B)$$

Top decay at NLO:



$t(q) \to b(p_b)W(p_W)\ (g(p_g))$

$x_b = \frac{1}{1-m_W^2/m_t^2}\,\frac{2p_b\cdot p_t}{m_t^2}$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(x_b)\ln\frac{m_t^2}{m_b^2} + A(x_b)\right] + \mathcal{O}\left[\left(\frac{m_b}{m_t}\right)^p\right]$$

$$P_{qq}(x_b) = C_F\left(\frac{1+x_b^2}{1-x_b}\right)_+ \ ;\ \int_0^1 dx_b f(x_b)[g(x_b)]_+ = \int_0^1 dx_b[f(x_b)-f(1)]g(x_b)$$

Mass logarithms and large-x_b terms need resummation (soft/collinear radiation)

Calculations often carried out in the NWA, recently NLO with interference effects

Some relevant calculations for top decays

A.Czarnecki, PLB 252 (1990) 467: Total NLO top decay width

A.Czarnecki and K.Melnikov, PRD59 (1999) 014036: Total top decay NNLO width

G.C. and A.Mitov, NPB623 (2001) 247 b-quark energy spectrum, collinear resummation of $\ln(m_t/m_b)$ and some soft-enhanced logarithms in the NLL+NLO approximation. Hadron corrections from e^+e^- data

M. Cacciari, G.C. and A.Mitov,JHEP 0212 (2002) 015:
As above, but with complete soft NLL resummation

S.Biswas, K.Melnikov and M.Schulze, JHEP 1008 (2010) 048:
NLO distributions with collinear resummation; hadronization by the above fits

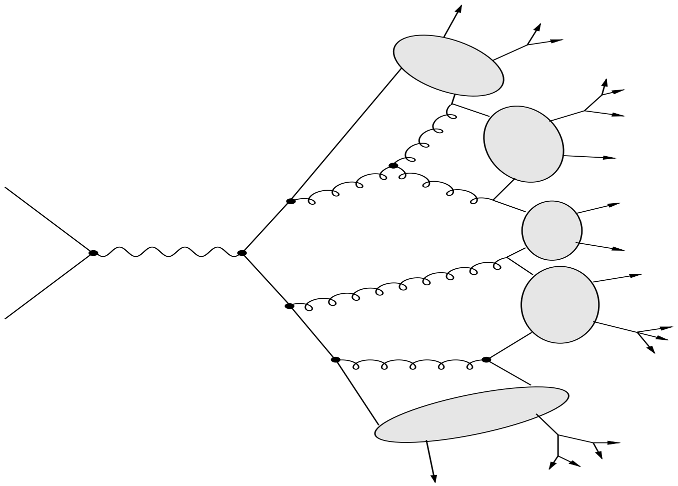
A.Denner, S.Dittmaier, S.Kallweit and S.Pozzorini, JHEP 1210 (2012) 110:
NLO for off-shell top production and decays, interface with showers and hadronization in progress

J. Gao, C.S. Li and H.X. Zhu (SCET), PRL110 (2013) 042001;
M. Brucherseifer, F. Caola and K. Melnikov, JHEP 04 (2013) 059:
NNLO distributions for top decays for massless b, not yet b-hadronization

Standard parton shower generators (PYTHIA, HERWIG): LO+LL plus some NLLs at large x $(\Lambda_{\overline{\rm MS}} \to \Lambda_{\rm MC} = \Lambda_{\overline{\rm MS}} \exp(4K\beta_0))$

Hadronization: NP fragmentation functions and Monte Carlo models

$$D_{\rm K}(x,\alpha) = (1+\alpha)(2+\alpha)x(1-x)^{\alpha} \quad ; \quad D_{\rm P}(x,\epsilon) = \frac{N_P}{x\,[1-1/x-\epsilon/(1-x)]}$$



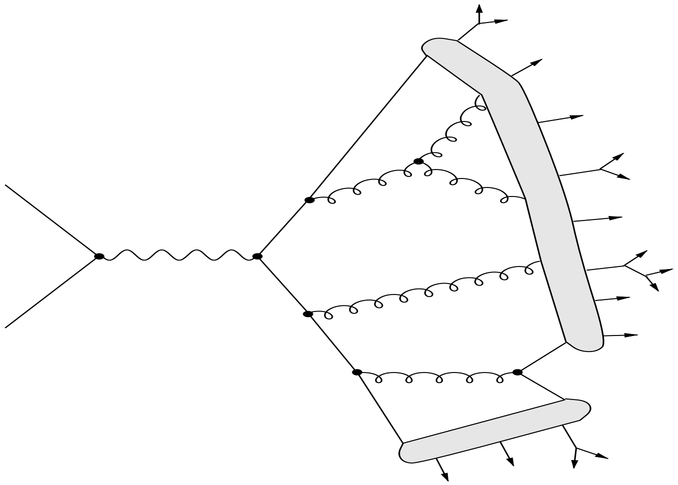
HERWIG: cluster model

Perturbative evolution ends at $Q^2 = Q_0^2$

Angular ordering $\Rightarrow$ colour preconfinement

Forced gluon splitting $(g \to q\bar{q})$

Colour-singlet clusters decay into the observed hadrons



PYTHIA: string model

q and $\bar{q}$ move in opposite directions

The colour field collapses into a string with uniform energy density

$q\bar{q}$ pairs are produced

The string breaks into the observed hadrons

Possible interface with NP fragmentation functions

Tuning involves hadronic and perturbative parameters: Q_0, $\Lambda_{\rm MC}$, m_g, etc. and relies on precise e^+e^- data (LHC data in future?)

G. C. and V. Drollinger, NPB (2005): weakly-decaying B-hadron data from OPAL (mesons and baryons), ALEPH (only mesons) and SLD (mesons and baryons)

HERWIG	PYTHIA
CLSMR(2) $=$ 0.3 (0.0)	PARJ(41) $=$ 0.85 (0.30)
DECWT $=$ 0.7 (1.0)	PARJ(42) $=$ 1.03 (0.58)
CLPOW $=$ 2.1 (2.0)	PARJ(46) $=$ 0.85 (1.00)
PSPLT(2) $=$ 0.33 (1.00)	
χ^2/dof $=$ 222.4/61 (739.4/61)	χ^2/dof $=$ 45.7/61 (467.9/61)

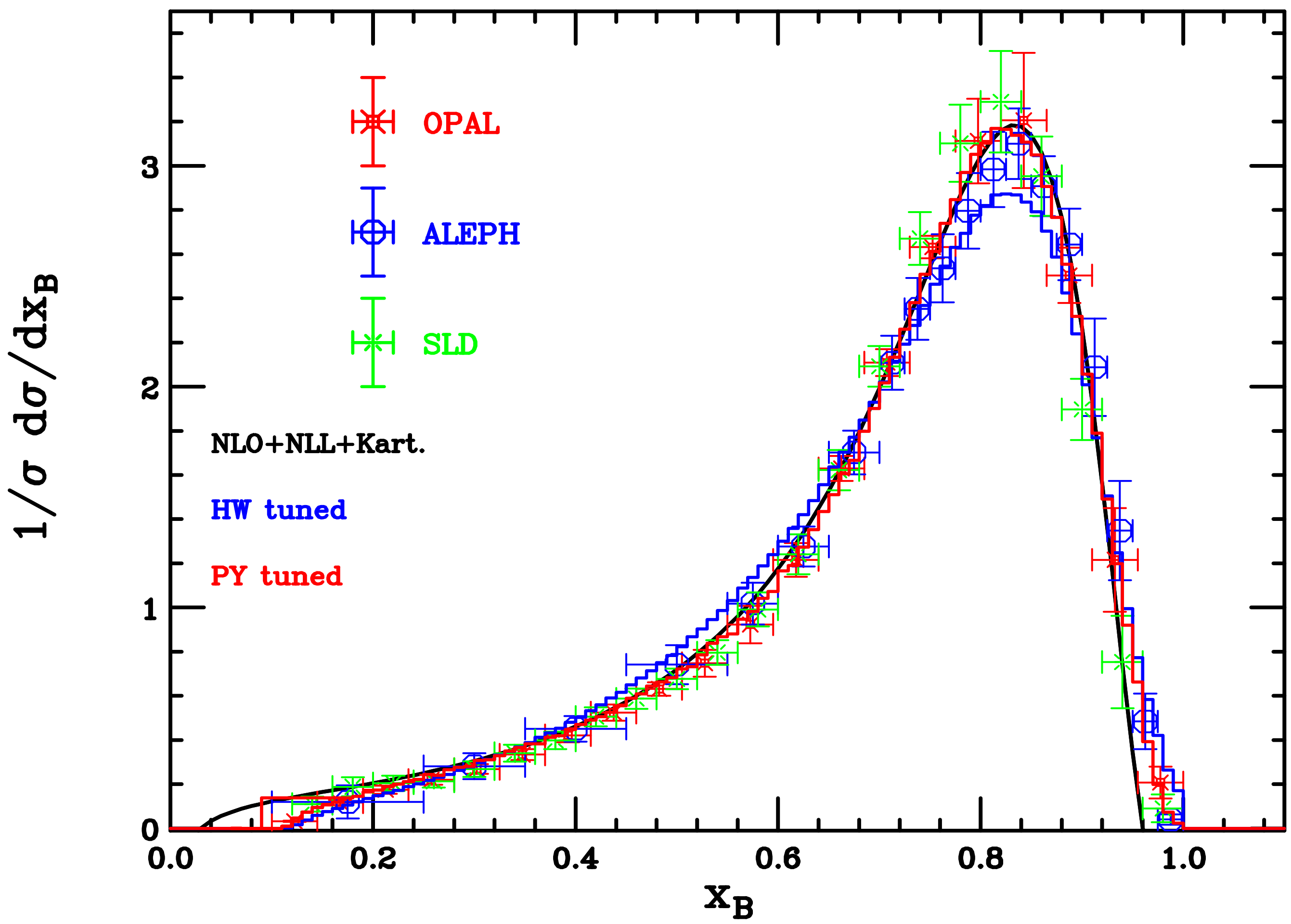
Lund/Bowler fragmentation function (PYTHIA):

$$f_B(z) \propto \frac{1}{z^{1+brm_b^2}}(1-z)^a \exp(-bm_T^2/z)$$

HERWIG tuned parameters describe hadron gaussian smearing (CLSMR), baryon/meson (CLPOW) and decuplet/octet (DECWT) ratios, mass spectrum of b-like clusters (PSPLT)

Our PYTHIA tuning in ATLAS jet-energy measurement (EPJ C73 (2013) 2304) and as a cross-check for top analyses

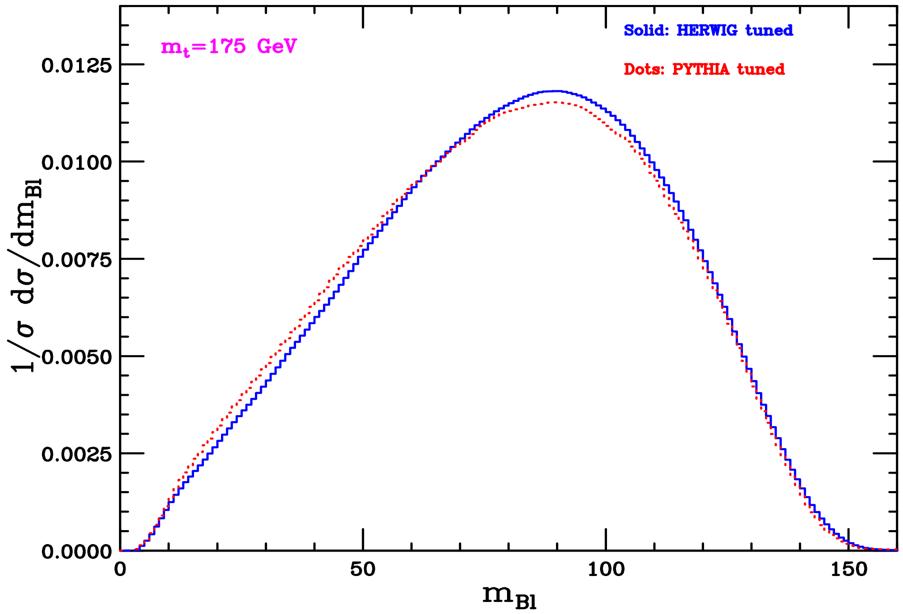
Comparing tuned HERWIG and PYTHIA and resummed calculations

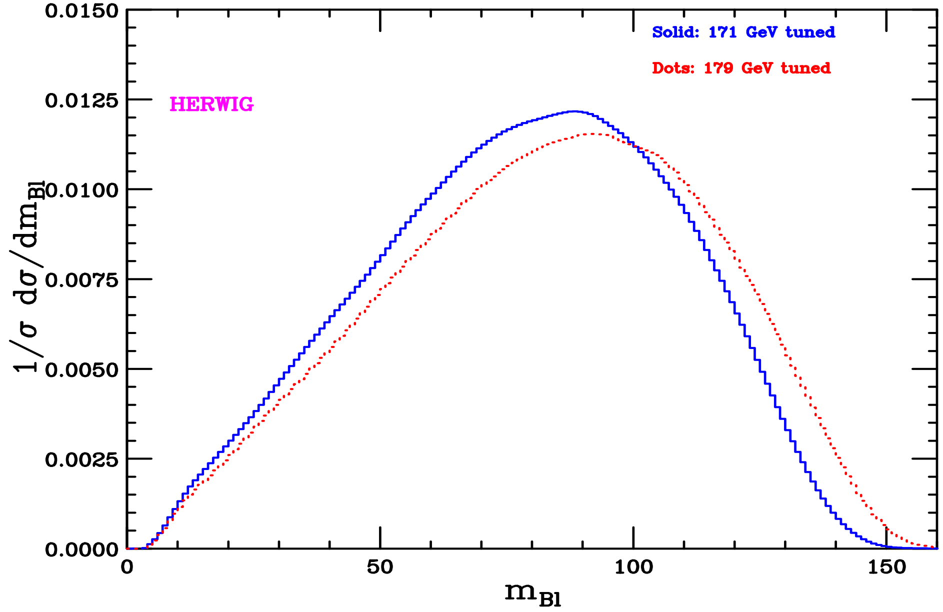


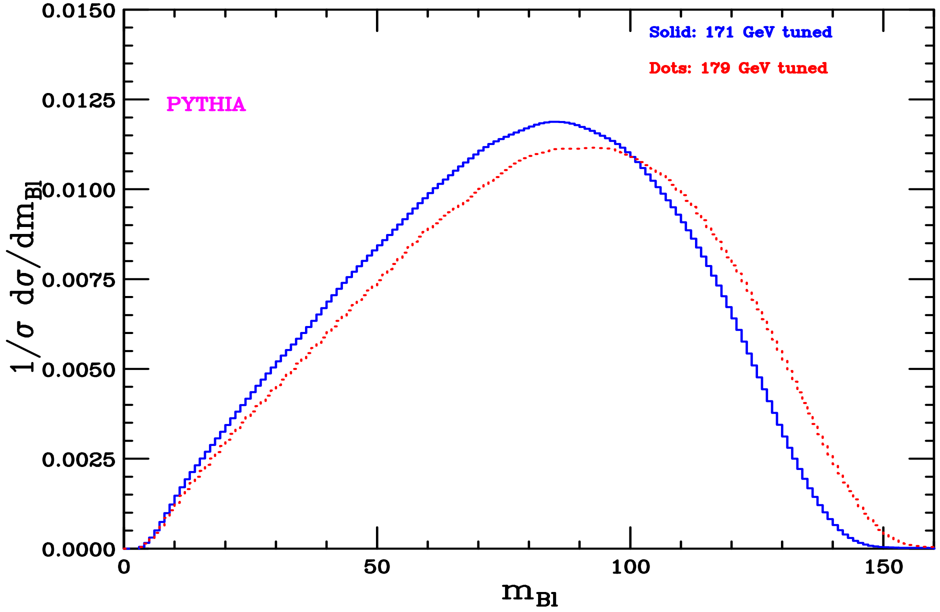
NLO+NLL: M.Cacciari and S.Catani, NPB617 (2001) 253-290

Best fit $(0.18 \leq x_B \leq 0.94)$: $\alpha = 17.178 \pm 0.303$, $\chi^2/\mathrm{dof} = 46.2/53$

B-lepton invariant mass according to tuned HERWIG and PYTHIA







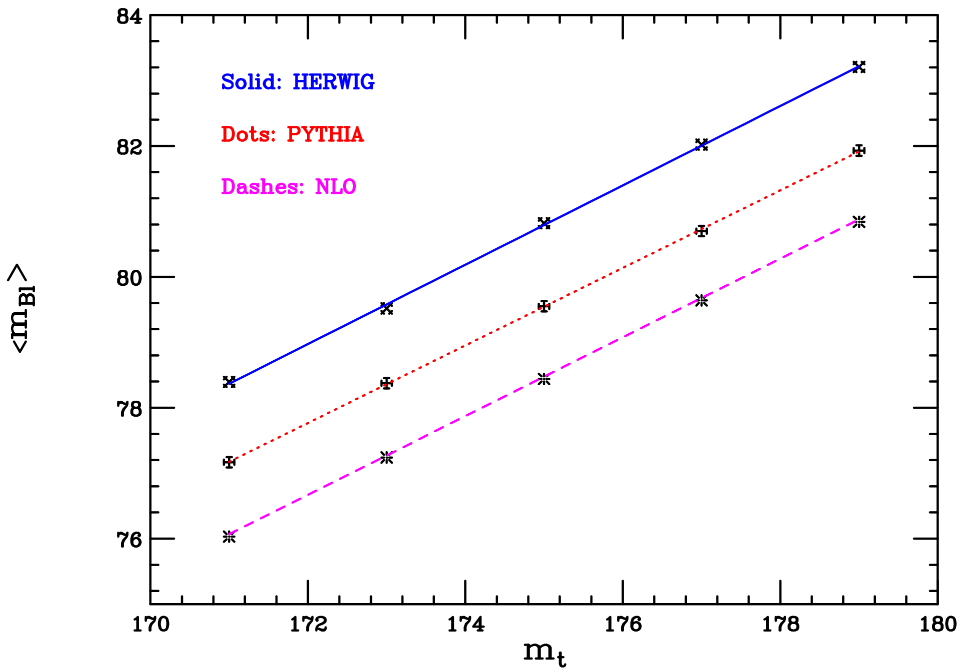
Linear fits to extract m_t from $m_{B\ell}$

HERWIG: $\langle m_{B\ell}\rangle_{\rm H} \simeq -25.31\ {\rm GeV} + 0.61\ m_t$; $\delta = 0.043$ GeV

PYTHIA: $\langle m_{B\ell}\rangle_{\rm P} \simeq -24.11\ {\rm GeV} + 0.59\ m_t$; $\delta = 0.022$ GeV

NLO: $\langle m_{B\ell}\rangle_{\rm NLO} \simeq -26.7\ {\rm GeV} + 0.60\ m_t$; $\delta = 0.004$ GeV

S.Biswas, K.Melnikov and M.Schulze, JHEP 1008 (2010) 048: $m_{B\ell}$ at NLO

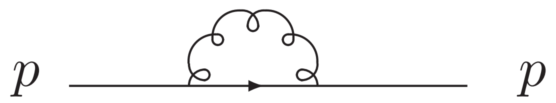


$\Delta\langle m_{B\ell}\rangle_{\rm H,P} \simeq 1.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm H,NLO} \simeq 2.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm P,NLO} \simeq 1.1$ GeV

NLO+showers for top decays or C++ codes may shed light on this discrepancy

Relating reconstructed top mass with theoretical definitions

Subtraction of the UV divergences in the self energy $\Sigma(p)$



Renormalized propagator: $S^{-1}(p) = -i[\not{p} - m_t^0 + \Sigma^R(p, m_t^0, \mu)]$

Mass is solution of equation $\not{p} - m_t + \Sigma^R(p, m_t, \mu) = 0$

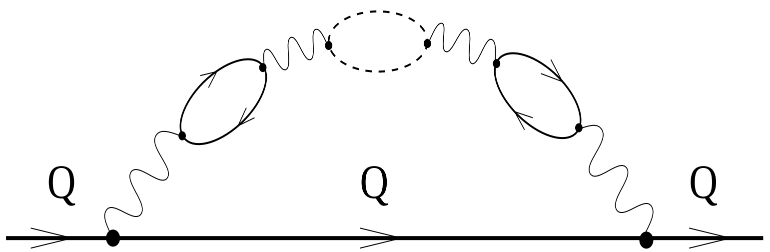
Pole mass:

$$\Sigma^R(p) = 0 \quad \text{and} \quad \frac{\partial \Sigma^R}{\partial \not{p}} = 0 \quad \text{for} \quad \not{p} = m$$

OK for electrons, but for quarks non-perturbative ambiguity: $\Delta m \sim \Lambda_{\rm QCD}$

Higher-order corrections lead to infrared renormalons:

$$\Sigma(m) \sim m \sum_n \alpha_S^{n+1} \, (2\beta_0)^n \, n!$$



$\overline{\rm MS}$ mass $\bar{m}_t(\mu)$– dimensional regularization $D = 4 - 2\epsilon$

$$\Sigma(p) = \frac{i\alpha_S C_F}{4\pi}\left\{\left[\frac{1}{\epsilon} - \gamma + \ln 4\pi + A(m_t^0, p, \mu)\right] \not{p} - \left[4\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi\right) + B(m_t^0, p, \mu)\right] m_t^0\right\}$$

Counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

Relation with the pole mass (coefficients c_i depending on $\ln[\mu^2/\bar{m}_t(\mu)^2]$)

$$m_t = \bar{m}_t(\mu)\left[1 + \alpha_S(\mu)c_1 + \alpha_S^2(\mu)c_2 + \dots\right]$$

Works well with off-shell quarks (e.g. $Z/H \to b\bar{b}$), but at threshold $\sim (\alpha_S/v^2)^k$

In order to make a statement on the nature of the reconstructed mass, one would need at least a NLO calculation, subtracting off the ultraviolet divergences

Typical experimental analyses (matrix-element, template methods) employ Monte Carlo parton showers, which are equivalent to LO+(N)LL calculations and miss width effects, higher-order corrections in the top self energy

Hadronization and non-perturbative effects play a role on hadron-level observables

One should try to relate the mass in the Monte Carlo codes to the mass definitions or, alternatively, use computations which are at least NLO

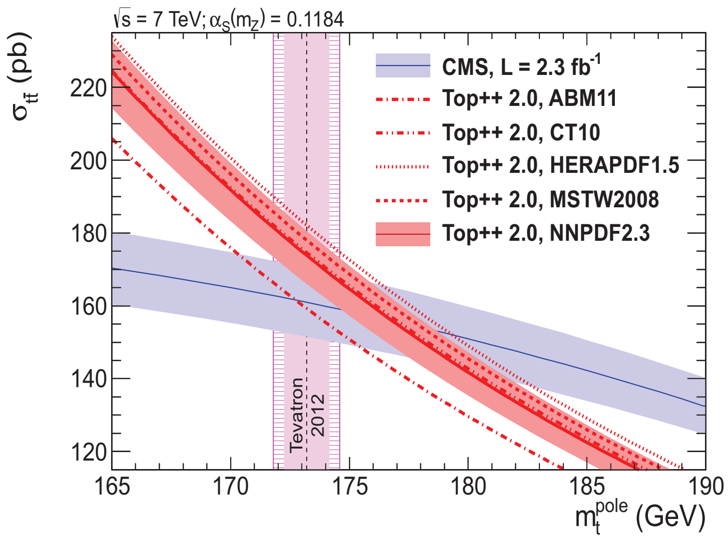
Total cross section for $t\bar{t}$ production recently computed at NNLO+NNLL:

$$\sigma_{\rm tot} = \sum_{i,j} \int d\beta\ \Phi_{ij}(\beta, \mu_F^2)\ \hat{\sigma}_{ij} \quad , \quad \beta = \sqrt{1 - 4m^2/\hat{s}} \quad , \quad \Phi_{ij} = \frac{2\beta}{1-\beta^2}\, x\, (f_i \otimes f_j)$$

At NNLO, for $\mu = \mu_F = \mu_R$ and $L = \ln(m^2/\mu^2)$ (Mitov, Fielder and Czakon, '13):

$$\hat{\sigma} = \frac{\alpha_S^2}{m_t^2} \left\{ \sigma^{(0)} + \alpha_S \left[\sigma_{ij}^{(1)} + L\sigma_{ij}^{(1,1)} \right] + \alpha_S^2 \left[\sigma_{ij}^{(2)} + L\sigma_{ij}^{(2,1)} + L^2 \sigma^{(2,2)} \right] \right\}$$

Threshold logarithms $\alpha_S^n[\ln^m(1-z)/(1-z)]_+$ $z = m_t^2/(x_i x_j \hat{s})$, $m \le 2n-1$



Scales: $\Delta\sigma \simeq 3\%$; pdfs: $\Delta\sigma \simeq 2.5\%$; α_S: $\Delta\sigma \simeq 1.5\%$, m_t: $\Delta\sigma \simeq 3\%$

Extracted pole mass exhibits large errors: $m_t^{\rm pole} = \left(176^{+3.8}_{-3.4}\right)$ GeV

World average (CDF, D0, ATLAS, CMS): $m_t = 173.34 \pm 0.27$ (stat) ± 0.71 (syst) GeV relies on Monte Carlo generators

Reconstructed mass $m_t^2 = (p_{b-\rm jet} + p_\nu + p_\ell)^2$ (with cuts on jets and leptons) with on-shell tops should be close to the top mass, up to widths and higher-order corrections

Attempts based on SCET have in fact shown that $m_t \simeq m_t^{\rm pole} + \mathcal{O}(\alpha_S \Gamma)$

A possible way out: run HERWIG with fictitious top-hadron states

Top quarks hadronize ($T^{\pm,0}$) and decay, e.g., through the spectator model

From a given observable R extract the Monte Carlo mass $m_T^{\rm MC}$

Study the same observable R with standard top samples, get $m_t^{\rm MC}$ and compare the extracted masses $m_T^{\rm MC} = m_t^{\rm MC} + \Delta m$

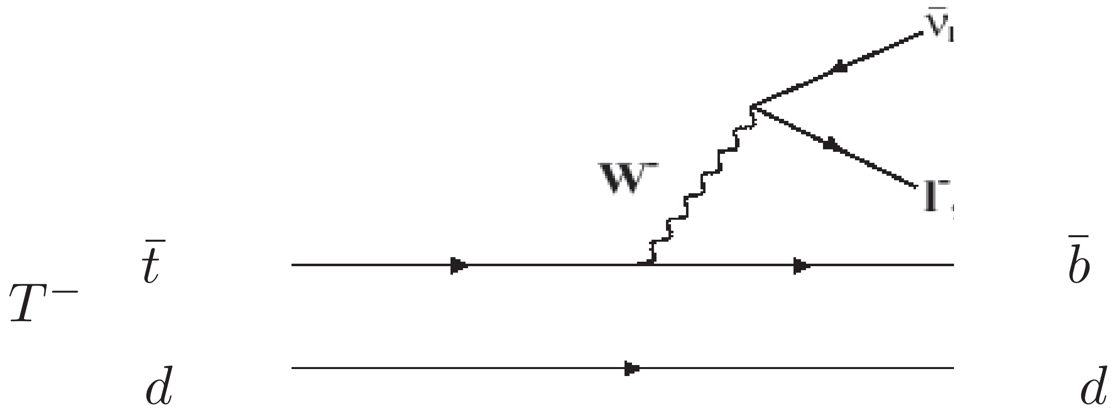
In the hadronized samples, the Monte Carlo mass can be related to the T-meson mass M_T and ultimately to the pole or $\overline{\rm MS}$ top-quark masses by using lattice, potential models, NRQCD, etc.

Connection between pole/$\overline{\rm MS}$ mass and the Monte Carlo mass

Investigate the dependence of the results on the specific analysis/observable and contributions to Δm (colour flow, gluon radiation, hadron decay models)

HERWIG for $e^+e^- \to t\bar{t}$ at $\sqrt{s} = 1$ TeV with top quarks hadronizing before decaying

t-flavoured mesons in the dilepton channel, i.e. $T^+ = (t\bar{d})$, $T^0 = (t\bar{u})$, $T^- = (\bar{t}d)$, etc.
Spectator model decays: $T^- \to (\bar{b}d)\ell^-\bar{\nu}_\ell + X \ldots \quad p_T^2 = (p_{\bar{b}} + p_W + p_q + p_X)^2$

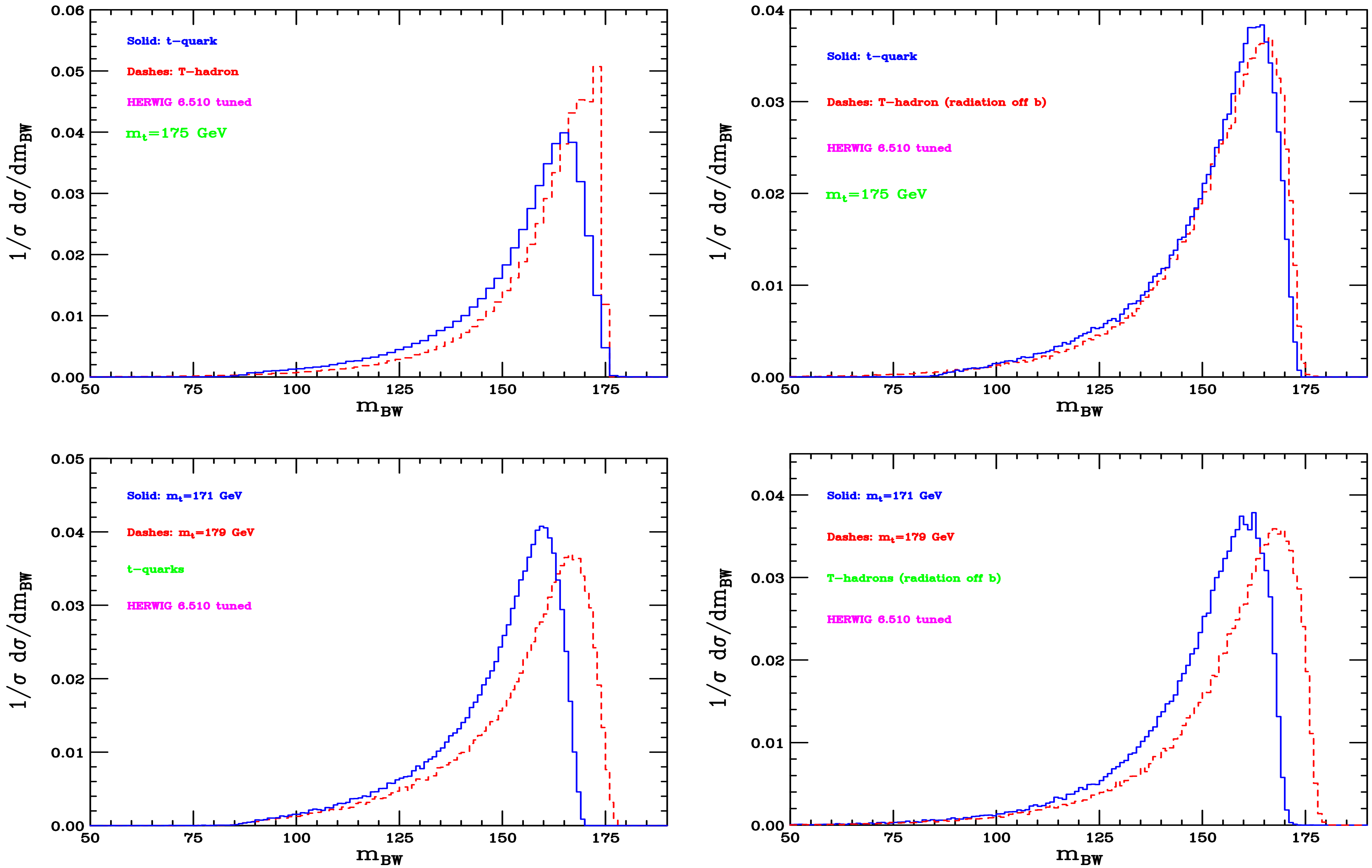


In a fraction of events, proportional to $\Delta_S(Q_b^2, Q_0^2)$, the b quarks in T decays do not radiate gluons: the $(\bar{b}q)$ cluster yields a B meson plus a soft hadron, e.g. pions

Spectator quarks likely do not radiate

In usual top decays before hadronization, the b-quark manages to form hard clusters decaying into B's and more energetic hadrons

Results with hadronized top quarks for BW invariant mass for fixed m_t^{MC} with and possibly without gluon radiation off the b (top plots) and varying m_t^{MC} (bottom)



Mellin moments - m_{BW} spectrum, allowing gluon emissions off the b quarks

T-hadrons:

m_t (GeV)	$\langle m_{BW} \rangle$ (GeV)	$\langle m_{BW}^2 \rangle$ (GeV^2)	$\langle m_{BW}^3 \rangle$ (GeV^3)	$\langle m_{BW}^4 \rangle$ (GeV^4)
171	148.76	2.24×10^4	3.41×10^6	5.24×10^8
173	150.44	2.29×10^4	3.53×10^6	5.48×10^8
175	152.18	2.35×10^4	3.66×10^6	5.74×10^8
177	153.80	2.40×10^4	3.77×10^6	5.99×10^8
179	155.61	2.45×10^4	3.91×10^6	6.28×10^8

t-quarks:

m_t (GeV)	$\langle m_{BW} \rangle$ (GeV)	$\langle m_{BW}^2 \rangle$ (GeV^2)	$\langle m_{BW}^3 \rangle$ (GeV^3)	$\langle m_{BW}^4 \rangle$ (GeV^4)
171	148.08	2.21×10^4	3.35×10^6	5.11×10^8
173	149.56	2.26×10^4	3.46×10^6	5.32×10^8
175	151.00	2.30×10^4	3.56×10^6	5.54×10^8
177	152.60	2.36×10^4	3.67×10^6	5.78×10^8
179	153.97	2.40×10^3	3.78×10^6	6.00×10^8

Conclusions and outlook

Bottom fragmentation in top decays is a source of uncertainty on the measurement of the top properties in inclusive (b-tagging and b-energy scale) and exclusive analyses

b-fragmentation relies on tuning hadronization models to e^+e^- data

Predictions for top decays yielded by the different codes exhibit some discrepancies, mostly driven by unsatisfactory tunings

Preliminary results on BW invariant mass from top-flavoured mesons

Perspectives:

Tuning PYTHIA 8 and HERWIG++ can be a valuable strategy to pursue

Using NLO+showers (POWHEG and aMC@NLO) and NNLO calculations

Tuning fragmentation parameters directly to LHC data ($t\bar{t}$, $b\bar{b}$, $Z/\gamma + b$)

Extending analysis with hadronized top quarks, e.g. b-jets vs. B-mesons, turning spectator-quark radiation on, studying dependence on shower cutoff, to shed light on current discrepancies and possibly make a statement on the nature of the reconstructed top mass