



**Hard Exclusive Reactions as Probes of  
Hadronic Deep Inelastic Structure**

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\* University of Iowa

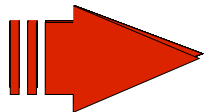
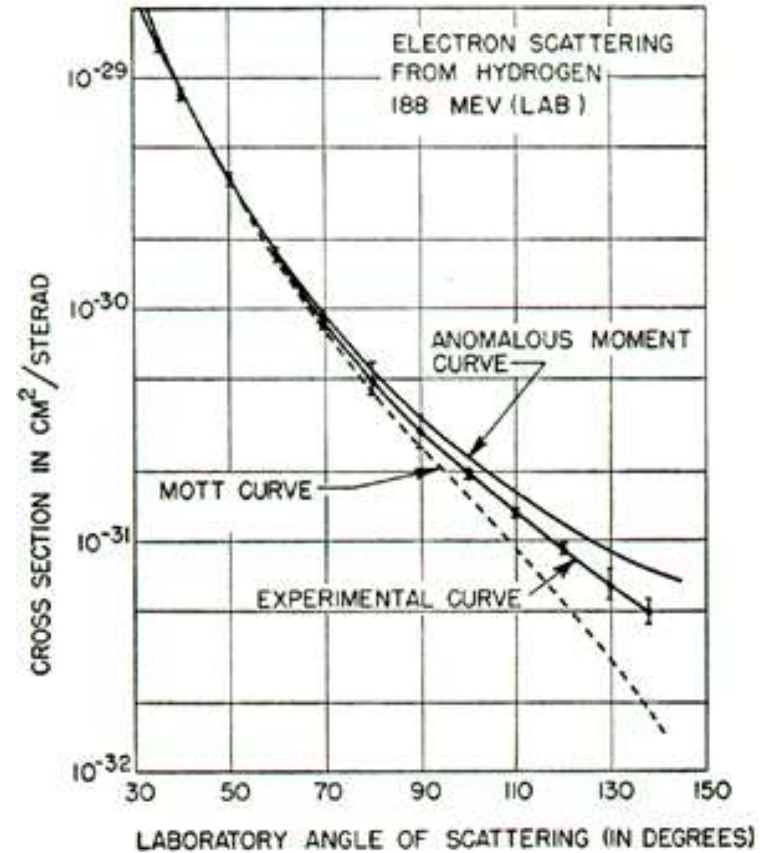
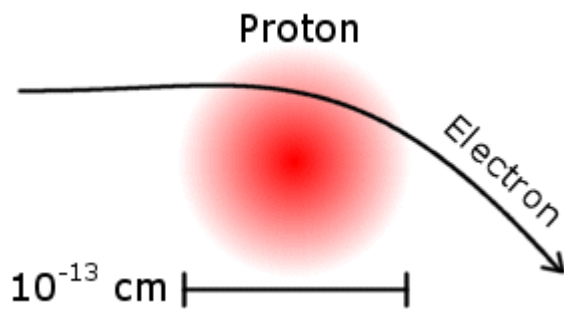
+ Stony Brook

# Outline

- An overview of the proton
- Outstanding Questions
- New experimental tools: Deeply virtual exclusive experiments and “femtoimages” (GPDs)
- Phenomenology:
  - DVCS Experiments on nucleons and nuclei
  - Exclusive  $\pi^0$  Electroproduction
- New computational methods: SelfOrganizing Maps?
- Conclusions

## What we know ...

- Charge radius:  
 $\approx 0.86$  fm

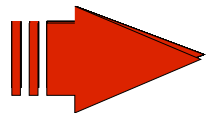
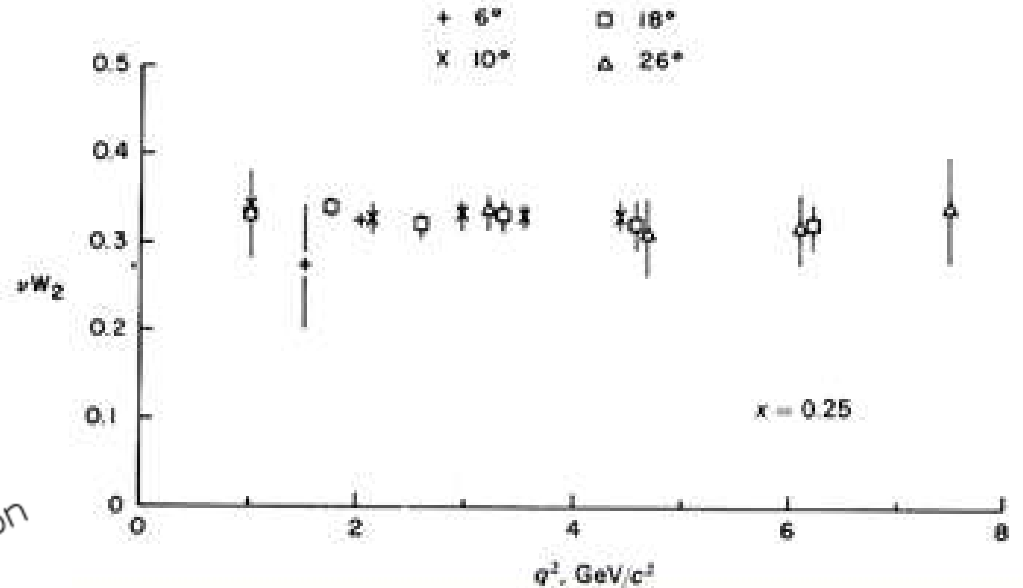
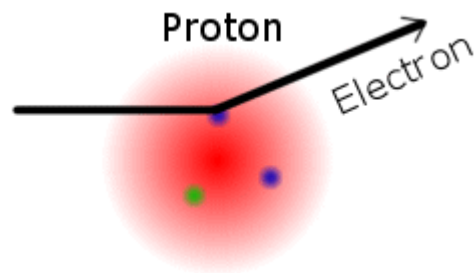


Measured by R. Hofstadter (1955)  
Inclusive, elastic ep scattering!

# What we know ...

● Proton is made of pointlike constituents:

➔ “partons”



Direct evidence observed by  
Friedman, Kendall and Taylor (1969):  
Inclusive, deep inelastic ep scattering!

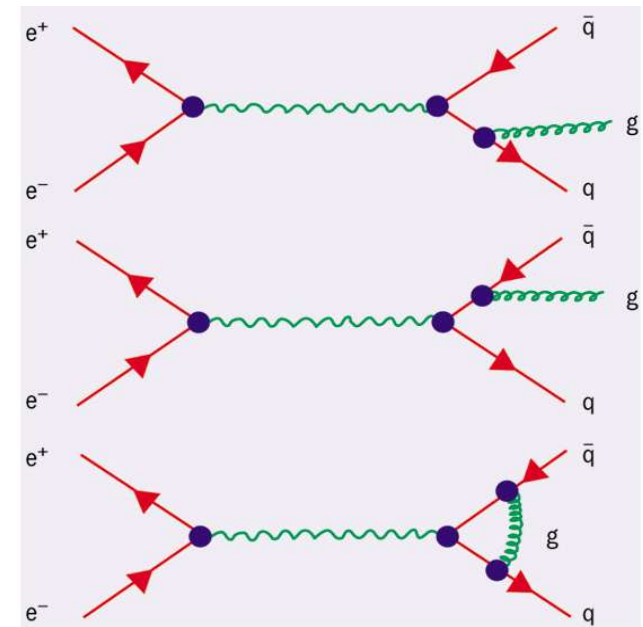
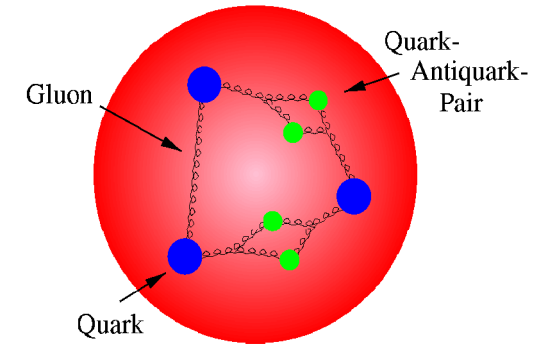
# Proton Dynamics: QCD

## Quarks:

u,d,s,c,...; 3 colors; spin 1/2;  $m_{u,d} \ll m_p$

## Gluons:

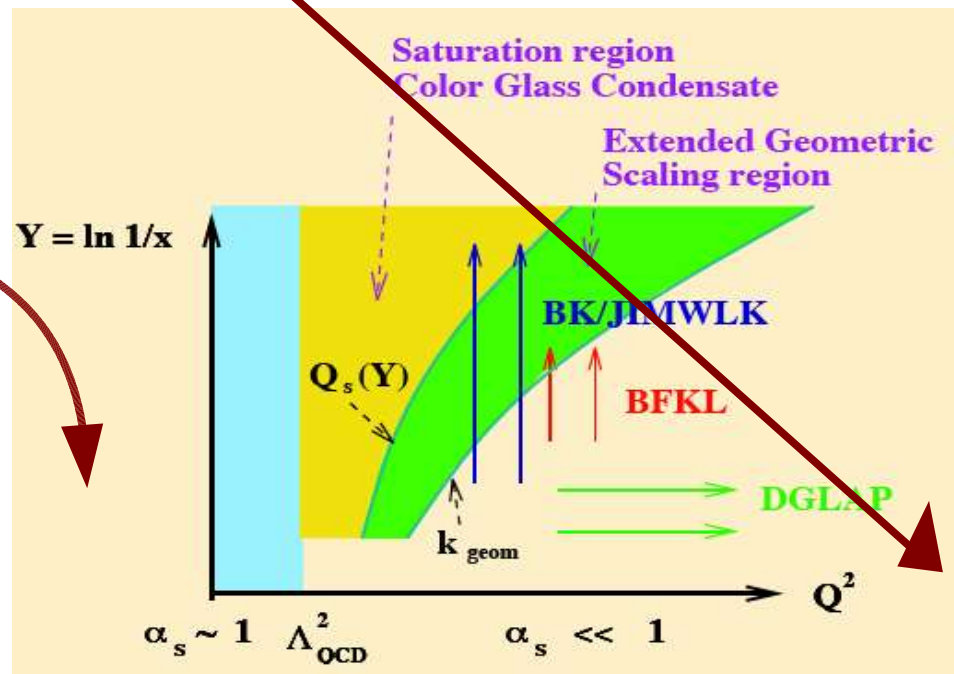
no charge; 8 "color types"; spin 1;  $m_g = 0$



G. Wolf, P. Söding, S.L. Wu, B. Wiik (1979)

# QCD: Asymptotic Freedom

- High energy/momentum transfer  $\leftrightarrow$  Probe small distances  
“strong” coupling is “weak”: perturbative QCD (DGLAP)
- Low momentum transfer  $\leftrightarrow$  Low spatial resolution  
**Relativistic, strongly coupled, many-body problem**



# How does one proceed? "A Flow Chart"

## Theoretical tools:

- ⇒ Lattice QCD
- ⇒ EFT (large  $N_c$ ,  $\chi$ PT, ...)
- ⇒ AdS/CFT



## Phenomenology

Devise observables and  
experimental probes



Experiments: measure and compare with theory

Validation

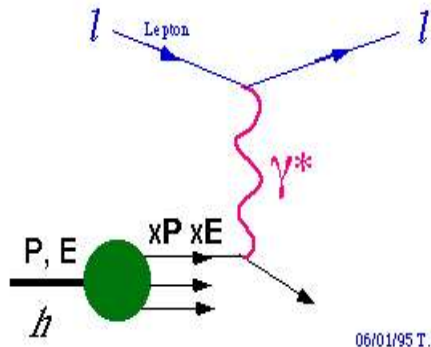
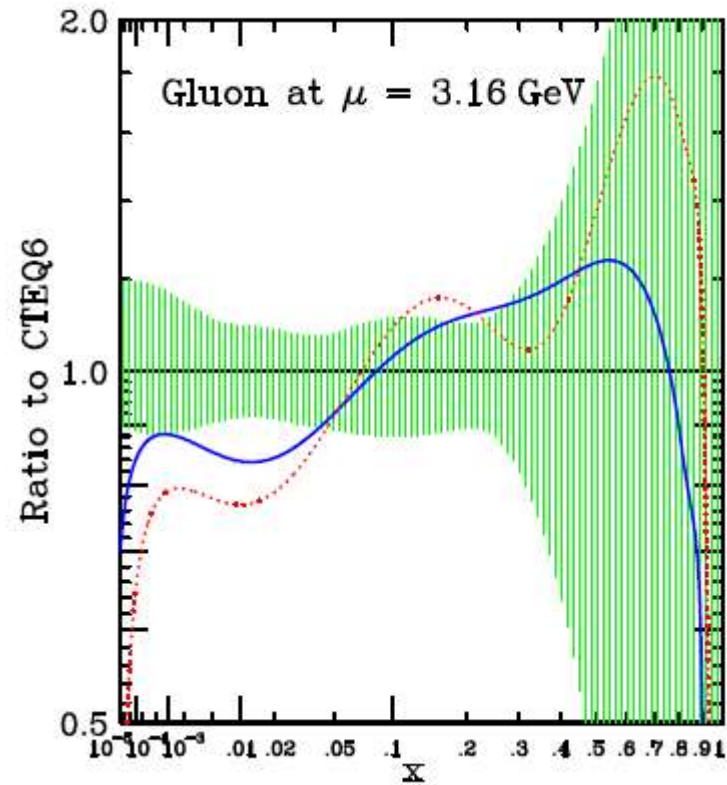
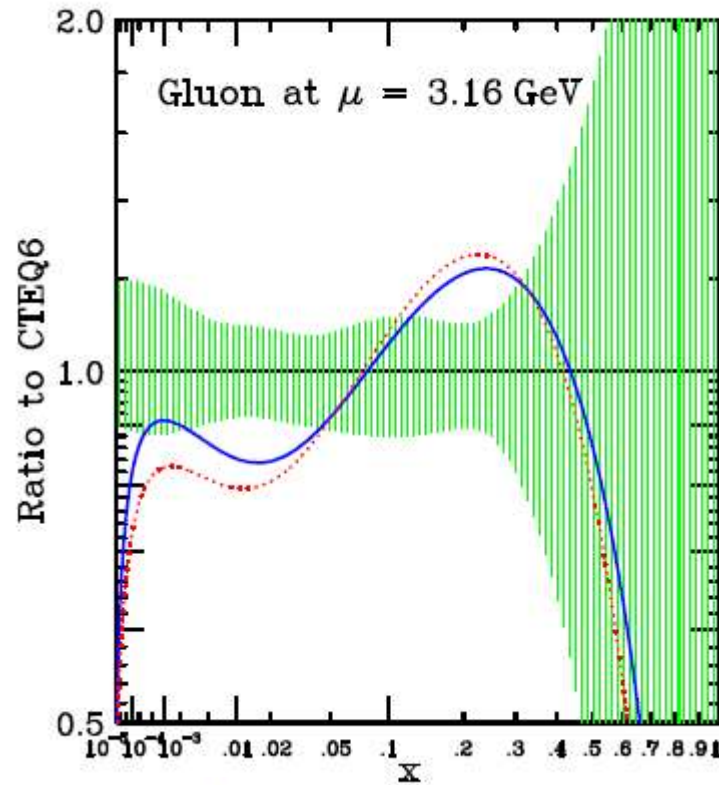


## 2. Outstanding Questions

## *Outstanding Questions arise through Validation*

- What are the momentum distributions of quarks, anti-quarks and gluons ?
- How is the flavor symmetry broken ?  $\bar{u} \neq \bar{d}$ ,  $s \neq \bar{s}$
- How do partons carry spin  $\frac{1}{2}$  of proton ?
- Longitudinal vs. transverse spin difference?
- Spatial distribution of quarks?
- Transition from partonic to hadronic d.o.f.: how are quarks and gluons correlated?
- How do protons and neutrons form atomic nuclei ?

# Example 1: momentum distributions



**Fundamental input for the LHC !**

# Example 2: Proton's Spin Structure

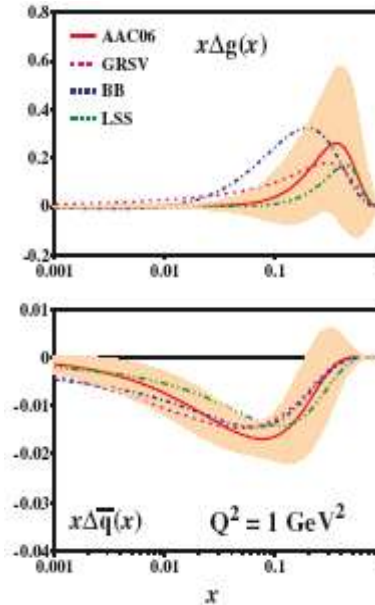
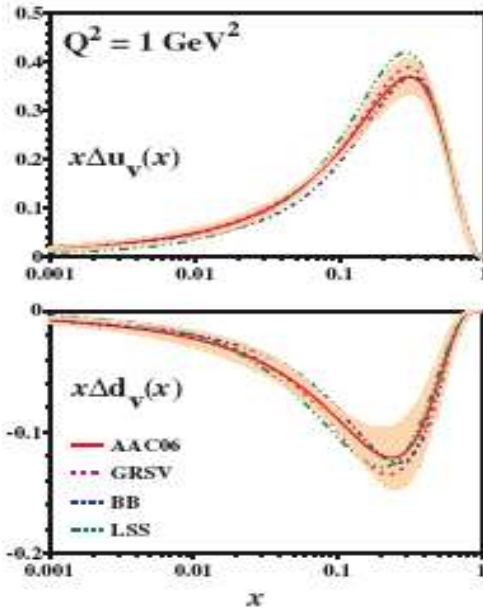
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

30 %

**QUARKS SPIN**

**GLUONS SPIN**

**Q & G  
ORBITAL ANGULAR MOM.**



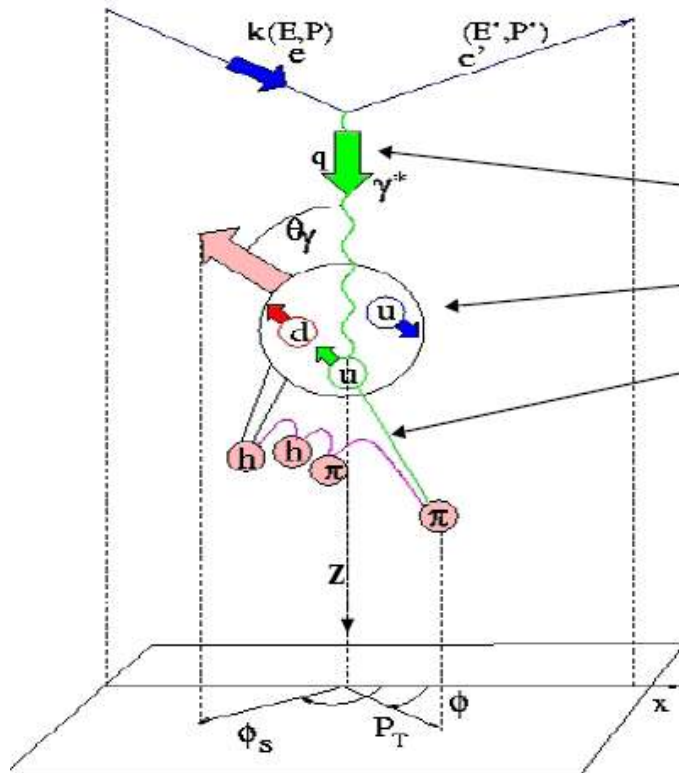
$\Delta g \approx 0$

?

# Proton Spin Crisis fostered searches for “different” observables

- Transversity (Goldstein & Moravscik, Jaffe, Ji, ...)
- Orbital angular momentum (Ji, ...)

“New type” of experiments: from inclusive to semi-inclusive ...



## Semi-inclusive DIS: Transversity

$$ep^\uparrow \rightarrow e'\pi X$$

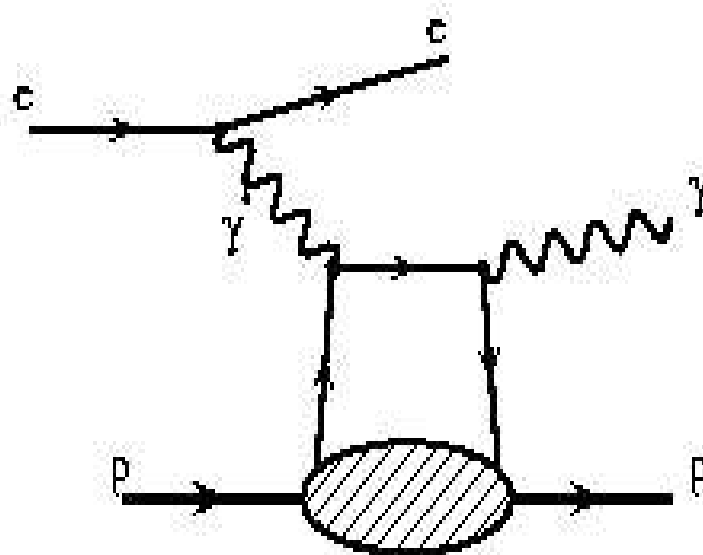
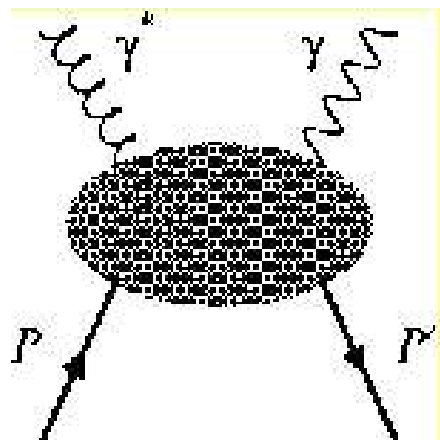
$$ep^\uparrow \rightarrow e'\Lambda' X$$

$$ep^\uparrow \rightarrow e'\pi\pi X$$

Observable =

azymuthal dist. of pions

... and from semi-inclusive to exclusive ...



Virtual Compton Scattering

Deeply Virtual Compton Scattering

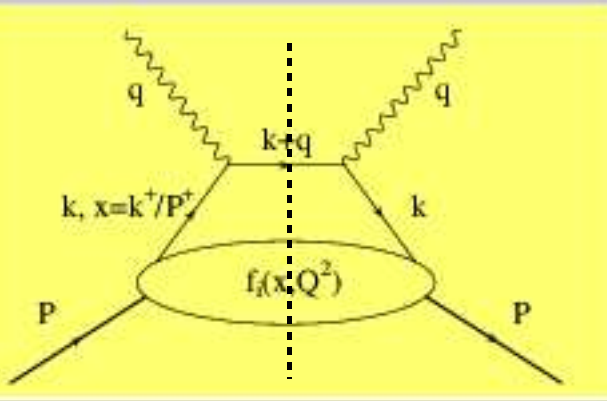
Bjorken limit



### 3. DVCS: new dimensions in proton studies

# DVCS and Generalized Parton Distributions 1

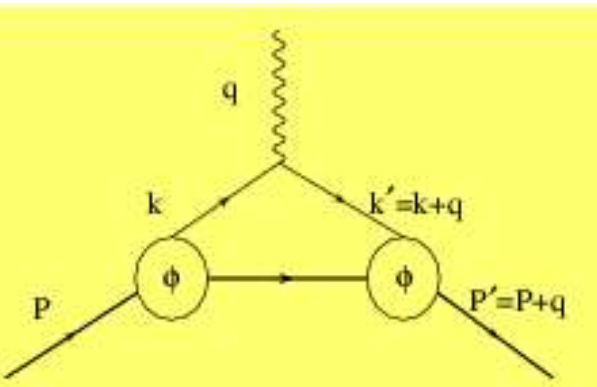
Deep inelastic Scattering



$$P^+ \int \frac{d\xi^-}{2\pi} e^{izP^+\xi^-} \langle P, S | \psi\left(-\frac{\xi^-}{2}\right) \gamma^+ \psi\left(\frac{\xi^-}{2}\right) | P, S \rangle = \bar{u}(P, S) \gamma^+ u(P, S) f(x)$$

Extract Parton Distribution

Elastic Scattering



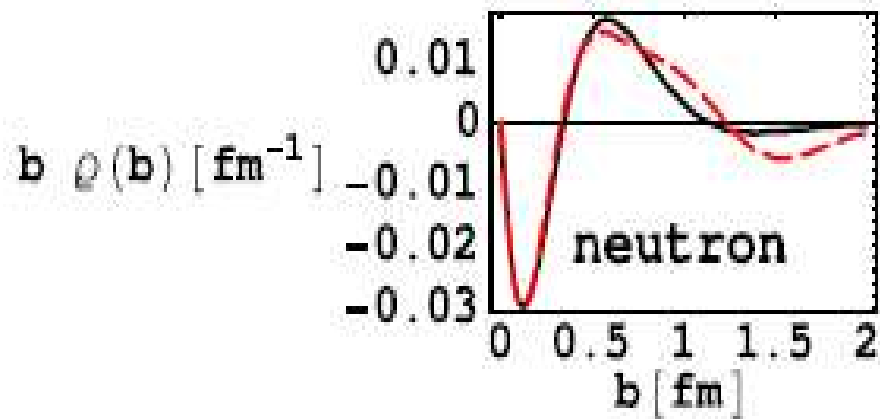
Extract Form Factors

$$\langle P', S' | \psi(0) \gamma^+ \psi(0) | P, S \rangle = \bar{u}(P', S') \left[ \gamma^+ F_1(Q^2) + \frac{i\sigma^{+\nu} q_\nu}{2M} F_2(Q^2) \right] u(P, S)$$

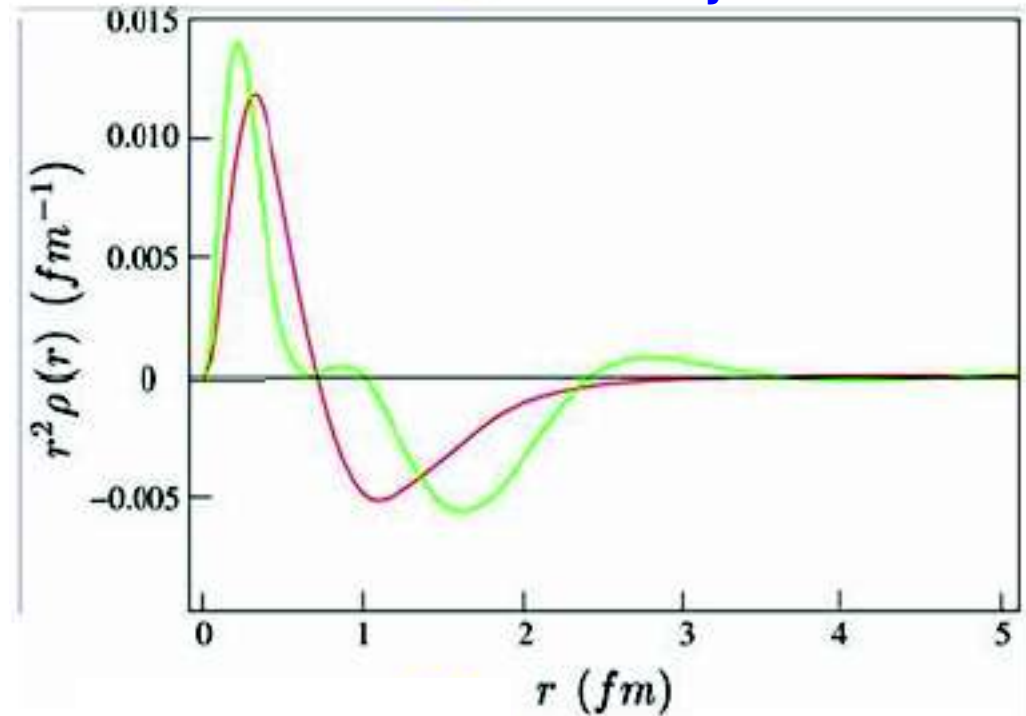


# DVCS and Generalized Parton Distributions 2

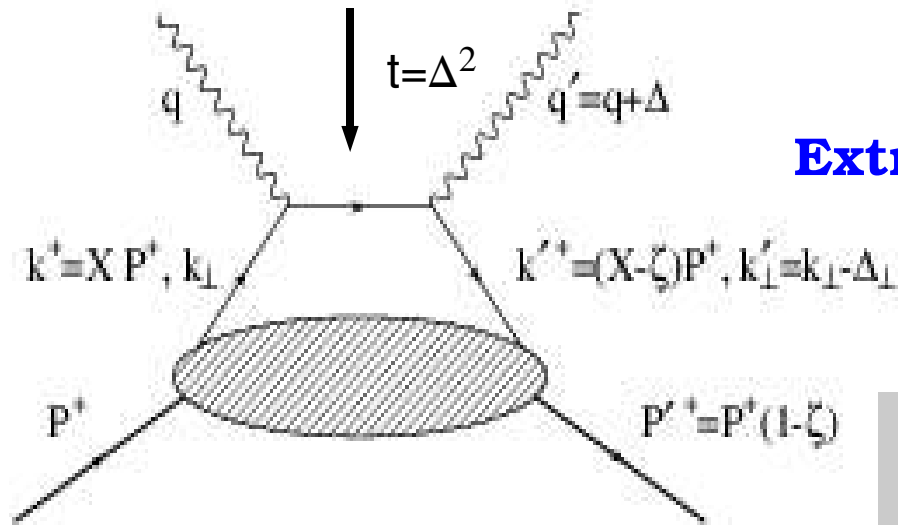
Breit Frame interpretation of the form factor (Sachs):



neutron density dist.



# DVCS and Generalized Parton Distributions 3

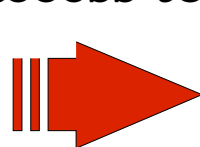


Extract “Generalized Parton Distributions”

$$\bar{p}^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P', S' | \psi \left( -\frac{\xi^-}{2} \right) \gamma^+ \psi \left( \frac{\xi^-}{2} \right) | P, S \rangle =$$

$$\bar{u}(P', S') \left[ \gamma^+ H(x, \xi, -\Delta^2) + \frac{i\sigma^{+\nu} q_{\nu}}{2M} E(x, \xi, -\Delta^2) \right] u(P, S)$$

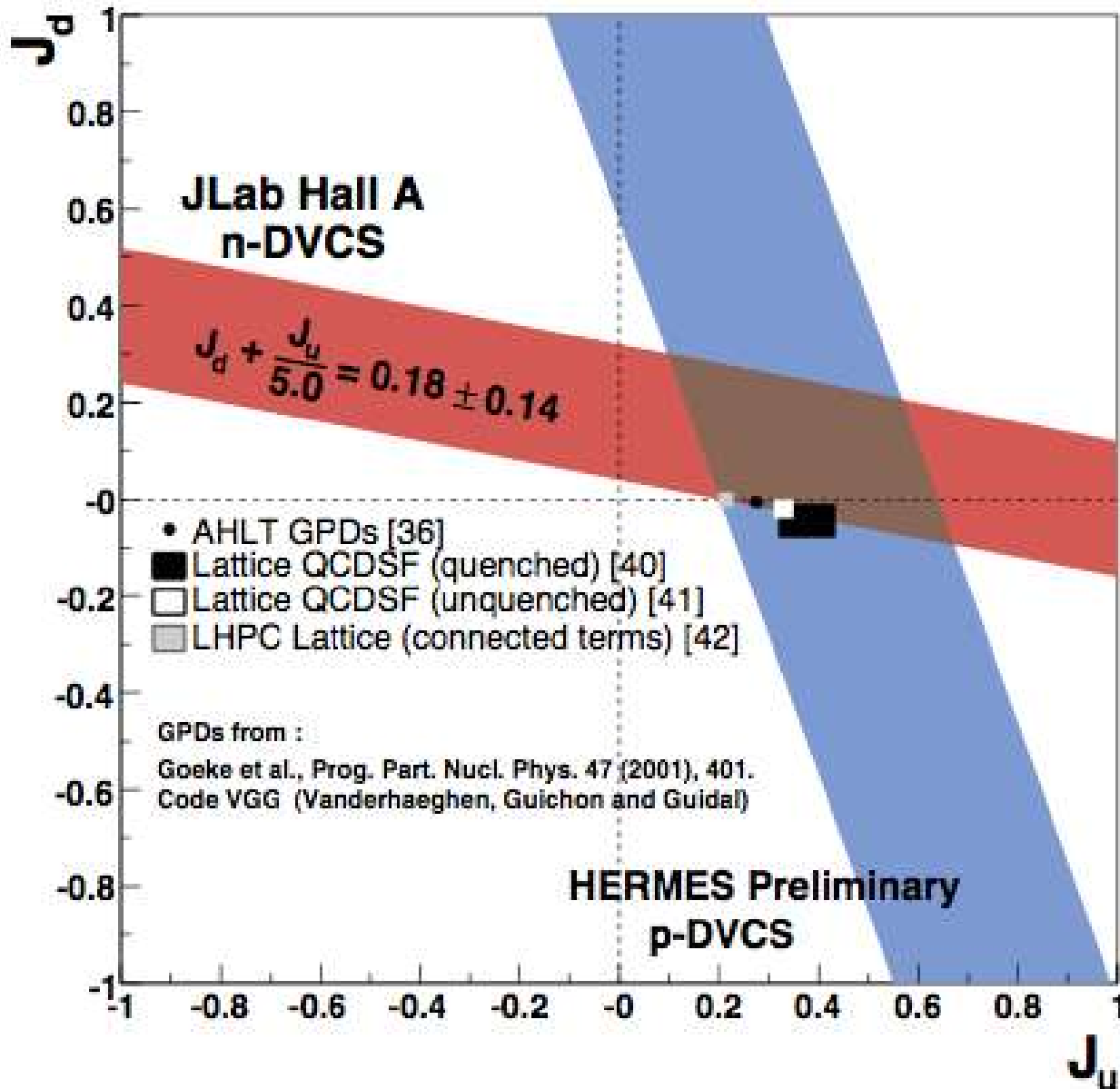
- GPDs are hybrids of PDFs and FFs: describe simultaneously  $x$  and  $t$ -dependences !
- GPDs give access to spatial d.o.f. of partons !
- GPDs give access to orbital angular momentum of partons!



$$\int dx H_q(x, \zeta, t) + E_q(x, \zeta, t) = 2J_q$$

X. Ji

# Orbital Angular Momentum (Camacho et al., PRL(2007))



AHLT includes only  
valence contribution!

$$J_q = (\kappa_q + 1) A_{20}(0)$$

# *DVCS and Generalized Parton Distributions 4: Optics*

Question #1: How do we interpret the spatial d.o.f. of partons?

## **Theoretical Ideas:**

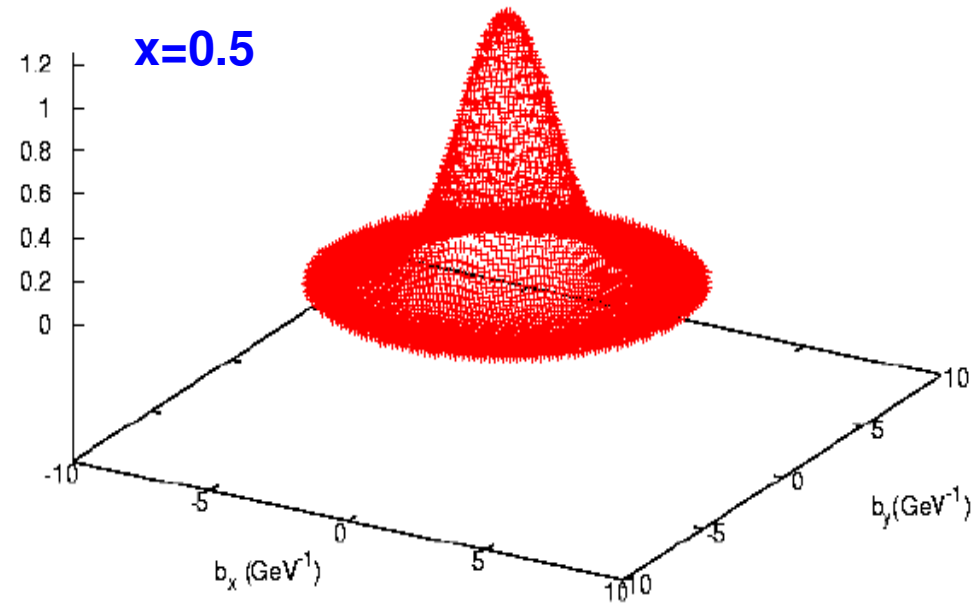
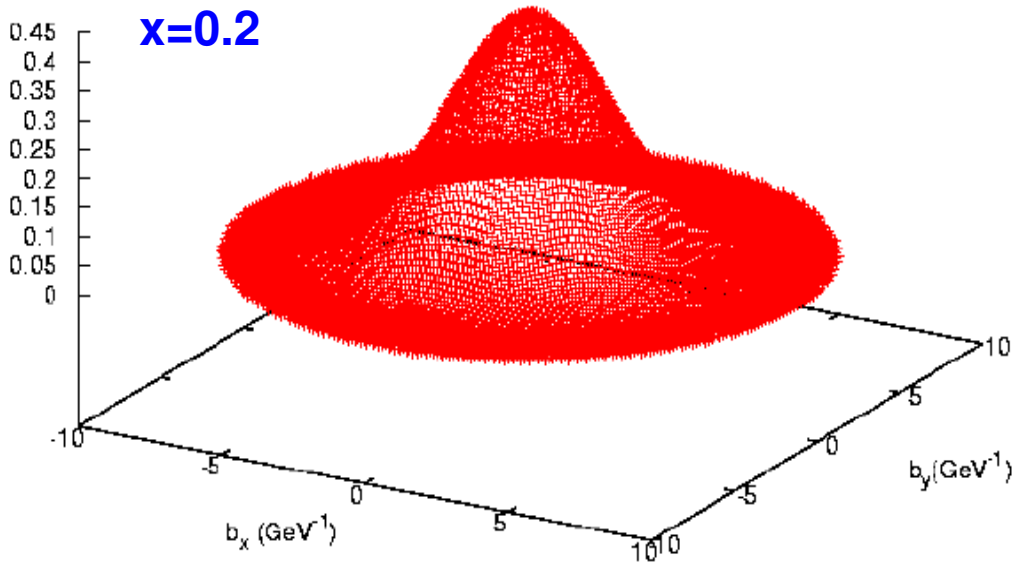
- Impact parameter dependent PDFs (M. Burkardt, 2000 ↔ D. Soper, 1977)
- Holography (Ralston and Pire, 2000)
- Interference patterns (Brodsky et al., 2006)
- Wigner Distributions (Belitsky, Ji, Yuan, 2004)

# DVCS and Generalized Parton Distributions 5: IPPDFs

$$\zeta = 0$$

$$q(x, \mathbf{b}) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i\mathbf{b} \cdot \Delta} H_q(x, 0, -\Delta^2)$$

$$\langle \mathbf{b}^2(x) \rangle = \mathcal{N}_b \int d^2 \mathbf{b} q(x, \mathbf{b}) \mathbf{b}^2$$



## DVCS and GPDs 6: Wigner Distributions

$$\zeta \neq 0$$

“Quantum Phase-Space” distributions (Wigner, 1932):  $f(\mathbf{p}, \mathbf{r})$

Not positive-definite because of uncertainty principle

Become positive in classical limit

Vast literature – observable (!) in atomic systems

**How do we generalize to relativistic systems/physics on the light-cone?**

Belitsky, Ji, Yuan: **Breit Frame**

$$\rho_+(\vec{r}, x) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} [H(x, \xi, t) - \tau E(x, \xi, t)]$$

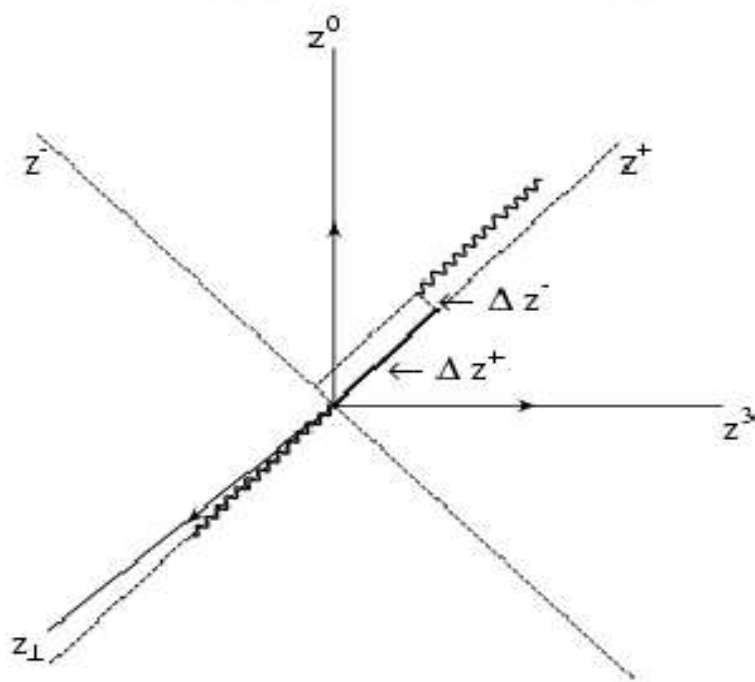
Phase-space Charge Density

$$j_+^z(\vec{r}, x) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} i[\vec{s} \times \vec{q}]^z \frac{1}{2M_N} [H(x, \xi, t) + E(x, \xi, t)]$$

Phase-space Convection Current

## But: ... “Ioffe time”

### Deeply Virtual Compton Scattering in Coordinate Space



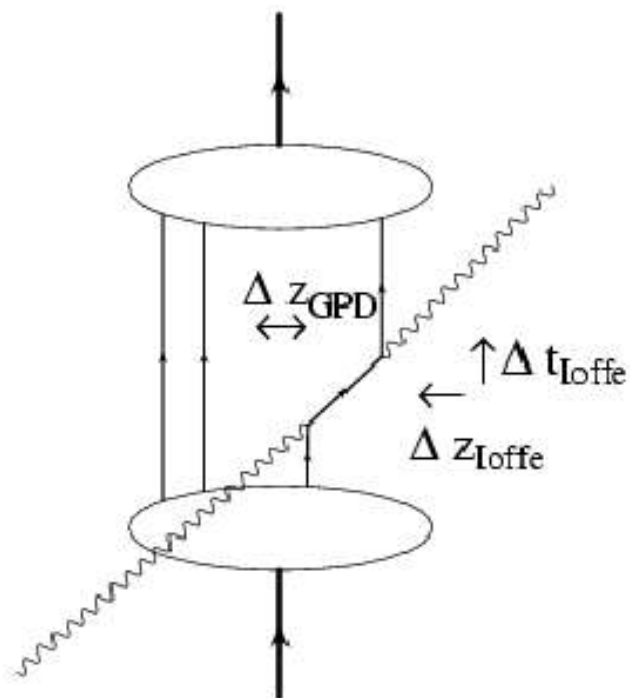
- Probe is local in  $\perp$  direction:

$$\Delta z^\perp \approx 1/\sqrt{Q^2}$$

- Non-local in  $\parallel$  direction:

$$\Delta z^+ \approx 1/M_N x_{Bj}$$

## Longitudinal variables



*quark's mobility* (P. Hoyer)

$$(\Delta z + \Delta t)_{\text{Ioffe}} \equiv \Delta z^+$$



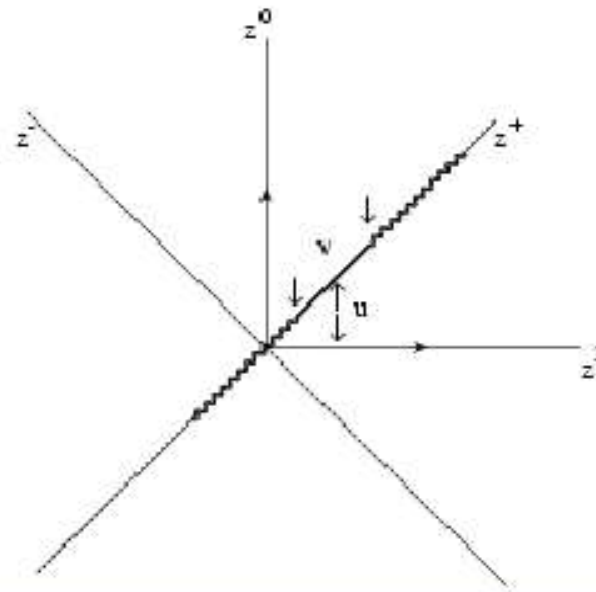
We are aware of large  $\Delta z^+$   
only through the observation  
of **nuclear shadowing**

⇒ Study the interplay between  $\Delta z^+$  and  $\Delta z_{\text{GPD}}$  in nuclei



## Correlation Function in Coordinate Space

*(work in progress...)*



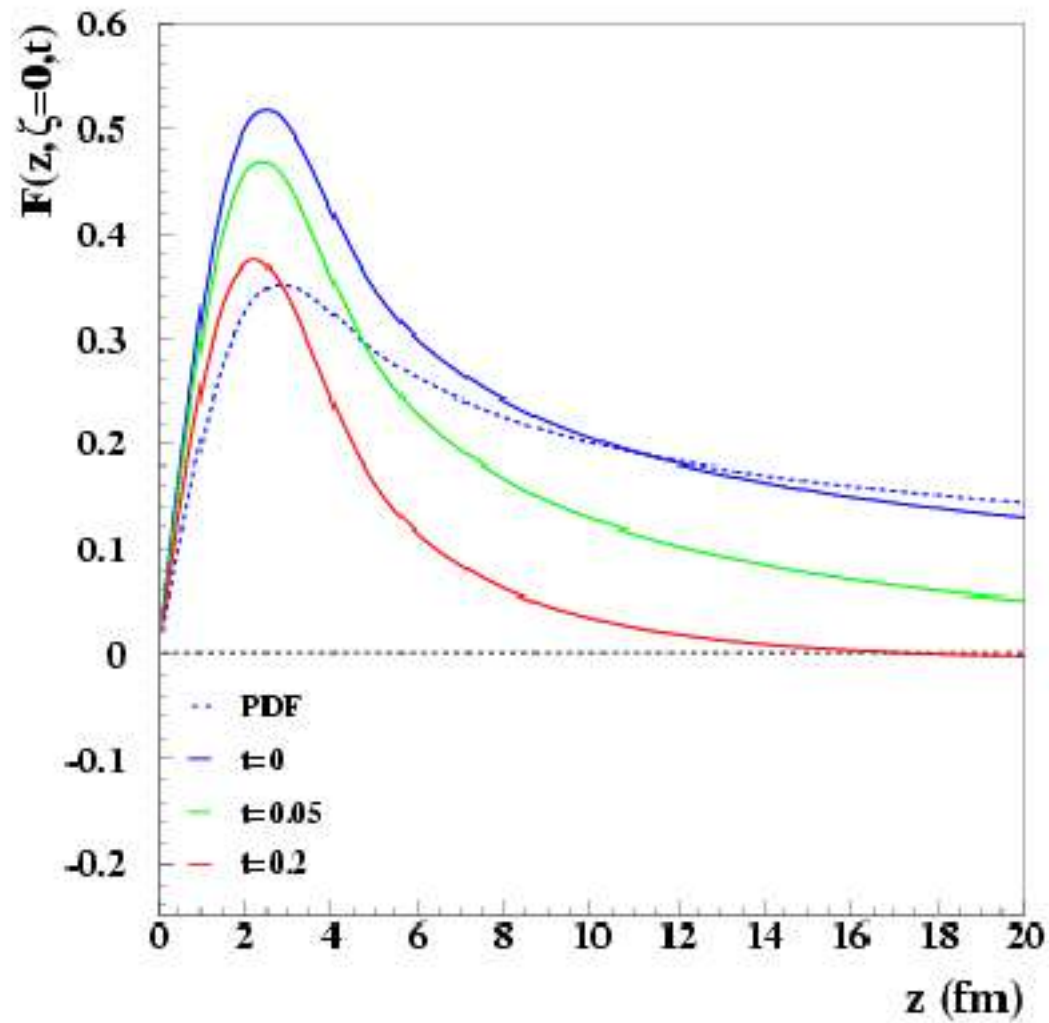
$$\langle P' | \bar{\Psi} \left( \frac{u+v}{2} z \right) \hat{z} \Psi \left( \frac{u-v}{2} z \right) | P \rangle_{z^2=0} = \bar{U}(P') \hat{z} U(P) \int d\zeta e^{iu\frac{\zeta}{2}(Pz)} F(vz, \zeta, t)$$

$$F(vz, \zeta, t) = \int dX H(X, \zeta, t) e^{ivX(Pz)}$$

$$\zeta = (\Delta z)/(Pz)$$

$$X = (kz)/(Pz)$$

Generalized Ioffe Time  
Distribution



S. Ahmad and S.L., preliminary

# DVCS Cross Section

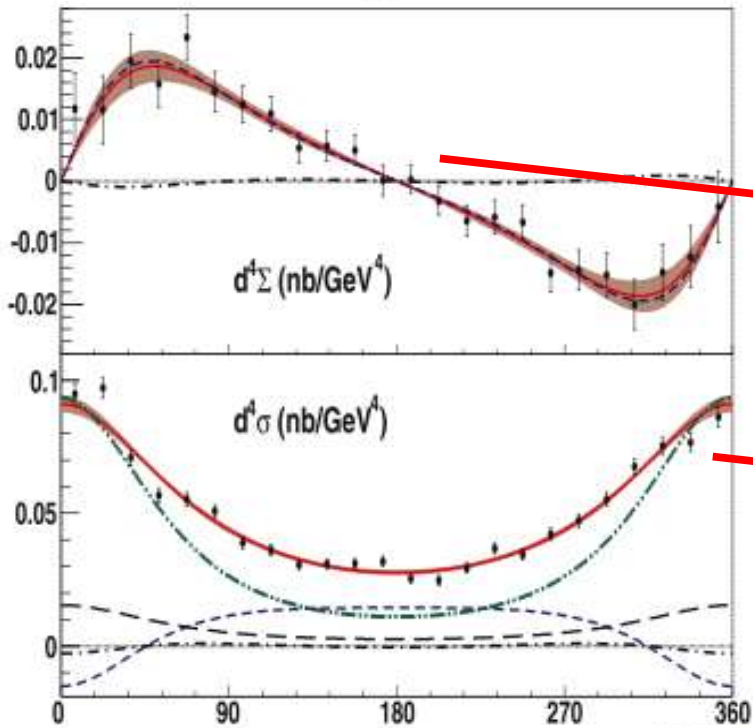
$$\begin{aligned} \frac{d^5\sigma(\lambda, \pm e)}{d^5\Phi} &= \frac{d\sigma_0}{dQ^2 dx_B} |\mathcal{T}^{BH}(\lambda) \pm \mathcal{T}^{DVCS}(\lambda)|^2 / |e|^6 \\ &= \frac{d\sigma_0}{dQ^2 dx_B} \left[ |\mathcal{T}^{BH}(\lambda)|^2 + |\mathcal{T}^{DVCS}(\lambda)|^2 \mp \mathcal{I}(\lambda) \right] \frac{1}{e^6} \end{aligned}$$

$$\frac{d^4\Sigma}{dQ^2 dx_{Bj} dt d\phi} \equiv \frac{d^4\sigma^+}{dQ^2 dx_{Bj} dt d\phi} - \frac{d^4\sigma^-}{dQ^2 dx_{Bj} dt d\phi} \propto \text{Im}\mathcal{H}$$

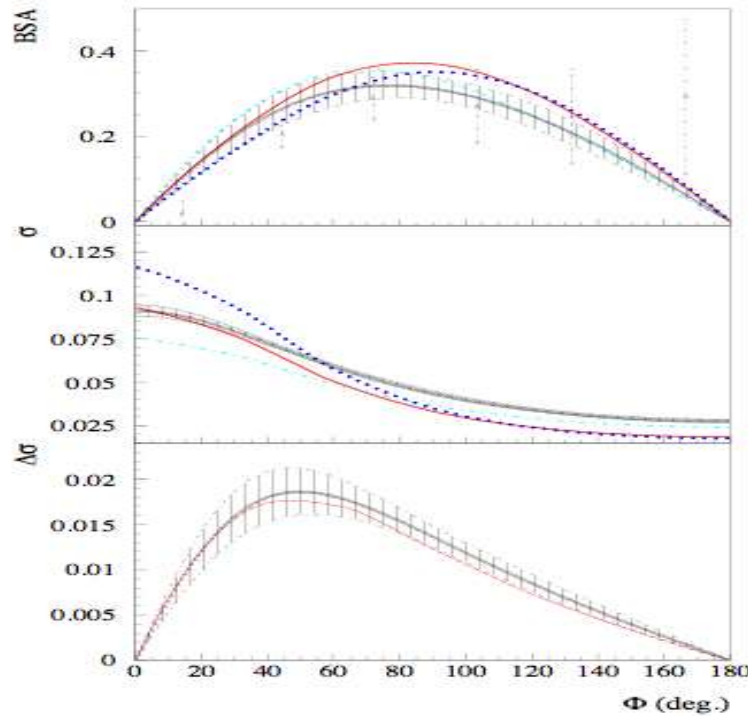
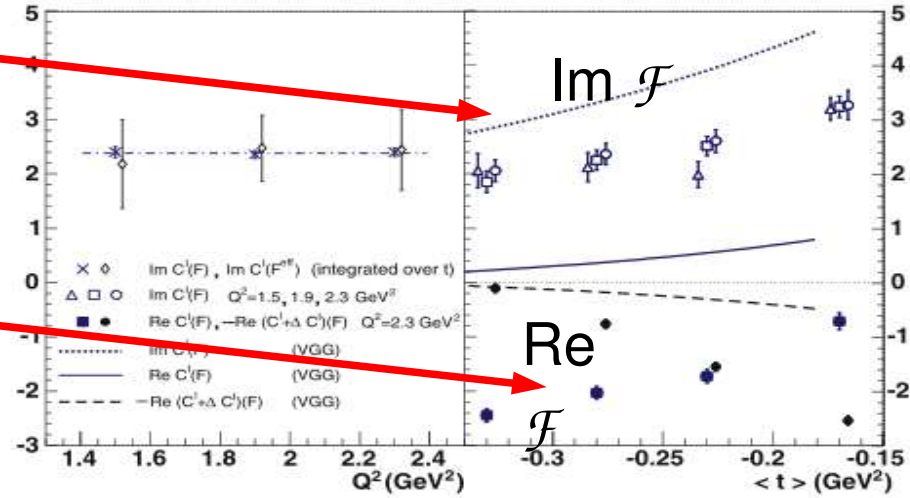
$$\frac{d^4\sigma}{dQ^2 dx_{Bj} dt d\phi} \equiv \frac{d^4\sigma^+}{dQ^2 dx_{Bj} dt d\phi} + \frac{d^4\sigma^-}{dQ^2 dx_{Bj} dt d\phi} \propto \text{Re}\mathcal{H}$$

$$\begin{aligned} \mathcal{F}(\zeta, t) &= -i\pi \sum_q e_q^2 \left[ \underline{F^q(\zeta, \zeta, t)} - F^q(-\zeta, \zeta, t) \right] + \\ &\quad \mathcal{P} \int_{1-\zeta}^1 dX \left( \frac{1}{X-\zeta} + \frac{1}{X} \right) F^q(X, \zeta, t). \end{aligned}$$

# VGG Crisis: Cannot reproduce both!



$x=0.36, Q^2=2.3 \text{ GeV}^2, t=-0.28 \text{ GeV}^2$



Guidal (2008)  
 Polyakov and Vanderhaeghen  
 (2008)

## What goes into a theoretically motivated parametrization...?

**The name of the game:** Devise a form combining essential dynamical elements with a flexible model that allows for a fully quantitative analysis constrained by the data

$$H_q(X,t) = R(X,t) G(X,t)$$

“Regge”

Quark-Diquark

Q<sup>2</sup> Evolution is an essential element!!

# Reaching a more advanced phase of extracting GPDs from data

*(a bit of summary from ECT\*, June'08)*

- ◆ No longer simple models (D. Muller)
- ◆ Include  $Q^2$  dependence (M. Diehl)
- ◆ Include all constraints from data DVCS, DVMP... (S.L.)
- ◆ Include new data as they become available... (S.L.)
- ◆ Use Lattice + Chiral Extrapolations (P. Hägler, A. Schaefer)
- ◆ Connect various experiments, separate valence from sea, flavors separation (T. Feldman)...
- ◆ **New!** Representation in terms of *dispersion relation* only necessary to measure imaginary part? Stronger polynomiality constraint (M. Diehl and Yu. Ivanov)

A similar program exists for TMDs (simpler partonic interpretation than GPDs) see M. Anselmino and collaborators

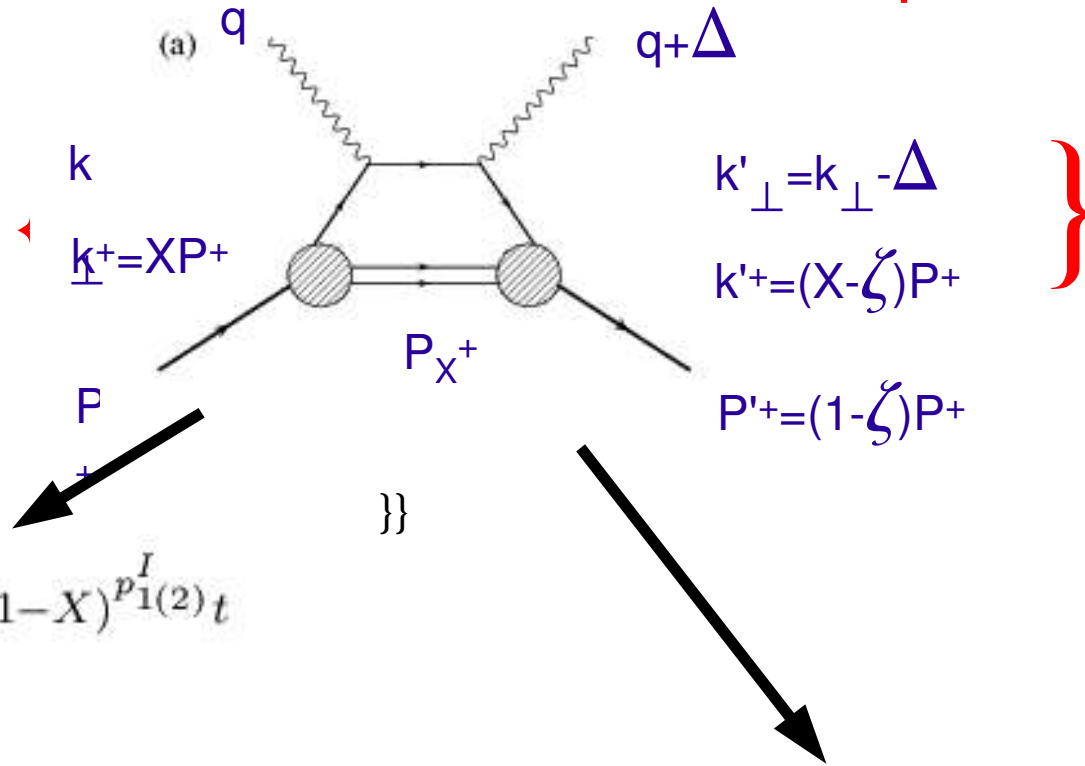
4. Extracting femtoimages requires  
new computational methods

# Proposed Strategy: Bottom-Up Approach

- Construct theoretically motivated parametrizations at a given *low* initial scale
- Merge data/information from:
  - Form factors  $\zeta=0$
  - PDFs  $\zeta=0$
  - Higher GPD moments (lattice calculations)  $\zeta\neq 0$
  - DVCS data  $\zeta\neq 0$
- Apply PQCD evolution to connect different sets of data



For  $\zeta = 0$  and in the DGLAP region  $\Rightarrow$  partonic picture



$$R_{1(2)}^I = X^{-\alpha^I - \beta_{1(2)}^I} (1-X)^{p_{1(2)}^I} t$$

$$G_{M_X}^\lambda(X, t) = \mathcal{N} \frac{X}{1-X} \int d^2\mathbf{k}_\perp \frac{\phi(k^2, \lambda)}{D(X, \mathbf{k}_\perp)} \frac{\phi(k'^2, \lambda)}{D(X, \mathbf{k}_\perp + (1-X)\mathbf{\Delta}_\perp)}$$

# AHLT Parameterization

$\zeta=0$

$$H^I(X, t) = G_{M_X^I}^{\lambda^I}(X, t) X^{-\alpha^I - \beta_1^I(1-X)^{p_1^I} t}$$

7 + 1 ( $Q_0$ ) parameters

v1

$$E^I(X, t) = \kappa G_{M_X^I}^{\lambda^I}(X, t) X^{-\alpha^I - \beta_2^I(1-X)^{p_2^I} t}$$

$$H^{II}(X, t) = G_{M_X^{II}}^{\lambda^{II}}(X, t) X^{-\alpha^{II} - \beta_1^{II}(1-X)^{p_1^{II} t}}$$

10 + 1 ( $Q_0$ ) parameters

v2

$$E^{II}(X, t) = G_{\tilde{M}_X^{II}}^{\tilde{\lambda}^{II}}(X, t) X^{-\tilde{\alpha}^{II} - \beta_2^{II}(1-X)^{p_2^{II} t}}$$

$\zeta \neq 0 \Rightarrow$  use v1 for DGLAP region ( $X \gg \zeta$ )

$$H^I(X, \zeta, t) = G_{M_X^I}^{\lambda^I}(X, \zeta, t) R_1^I(X, \zeta, t)$$

$$E^I(X, \zeta, t) = \kappa G_{M_X^I}^{\lambda^I}(X, \zeta, t) R_2^I(X, \zeta, t)$$

More details in AHLT, PRD 2007

# Summary of Constraints

## Constraints from Form Factors

$$\int_0^1 dX H^q(X, t) = F_1^q(t) \quad \text{Dirac}$$

$$\int_0^1 dX E^q(X, t) = F_2^q(t), \quad \text{Pauli}$$

## Constraints from Polynomiality

$$H_n^q(\xi, t) = \sum_{i=0}^{\frac{n-1}{2}} A_{n,2i}^q(t) \xi^{2i} + \text{mod}(n, 2) \xi^n C_n^q(t)$$

$$E_n^q(\zeta, t) = \sum_{i=0}^{\frac{n-1}{2}} B_{n,2i}^q(t) \xi^{2i} - \text{mod}(n, 2) \xi^n C_n^q(t).$$

## Constraints from PDFs

$$q(x) = H_q(x, 0, 0)$$

## Further Theoretical Constraints:

- Sensible prediction for hadron shape at  $x \rightarrow 1$
- Sensible prediction for  $k_T$  dependence (connection with TMDs!)

(SL and Taneja, 2004)

# GPDs from available data 2

## Parton Distribution Functions

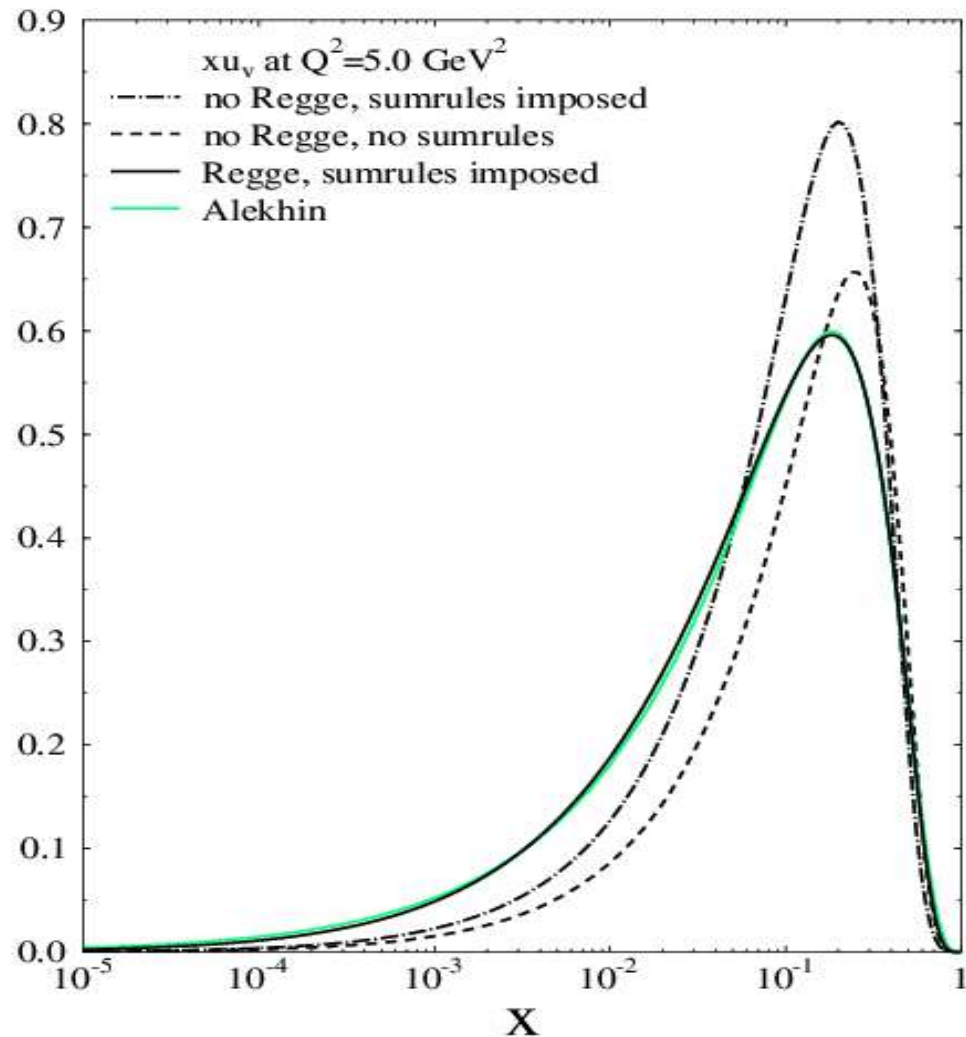
**Notice!** GPD parametric form is given at  $Q^2=\mu^2$  and evolved to  $Q^2$  of data.

**Notice!** We provide a parametrization for GPDs that simultaneously fits the PDFs:

$$H_q(X,t) = R(X,t) G(X,t)$$

Regge

Quark-Diquark

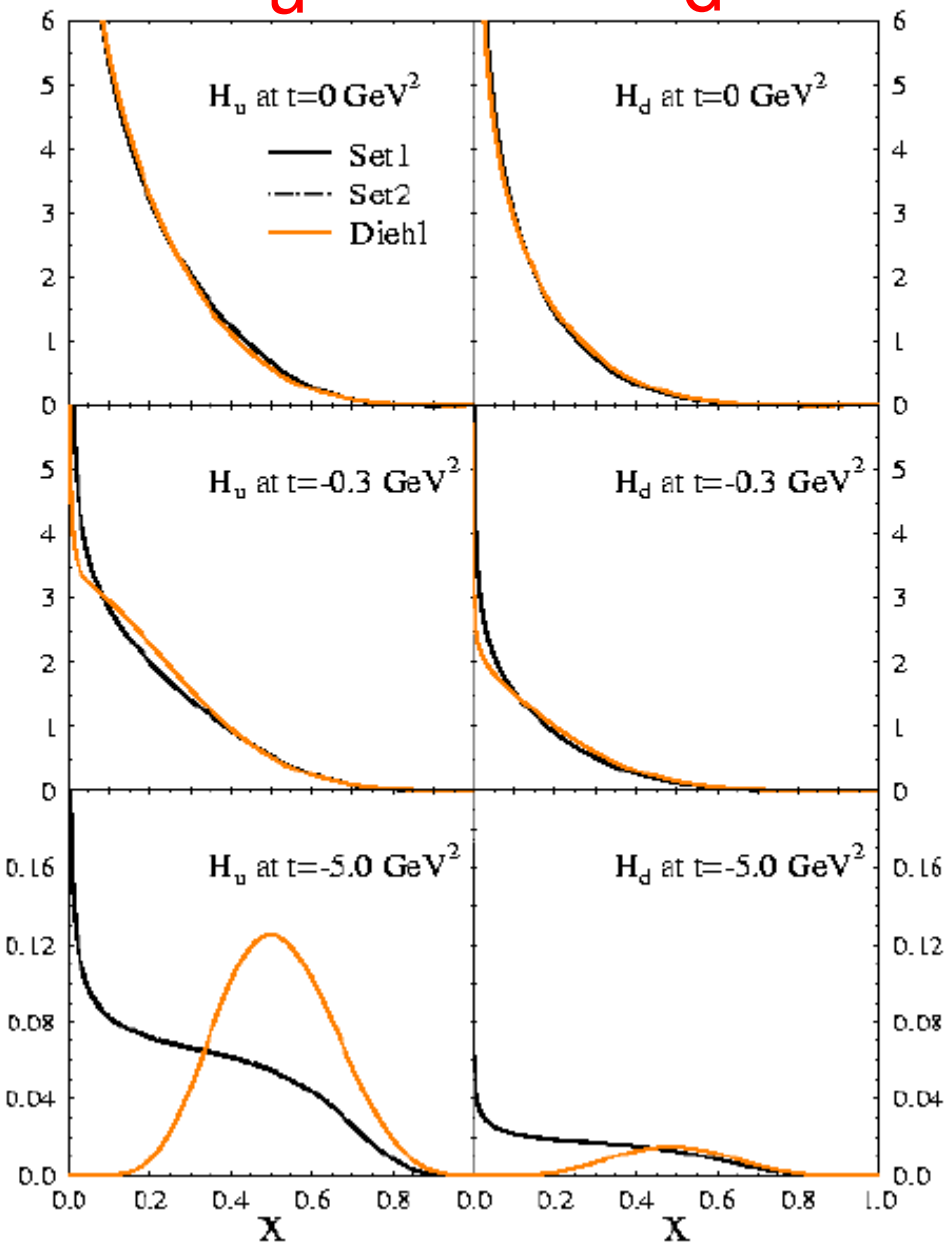


# Comparison with similar parametrizations at $\zeta=0$

$H_q(x,0,t)$

u

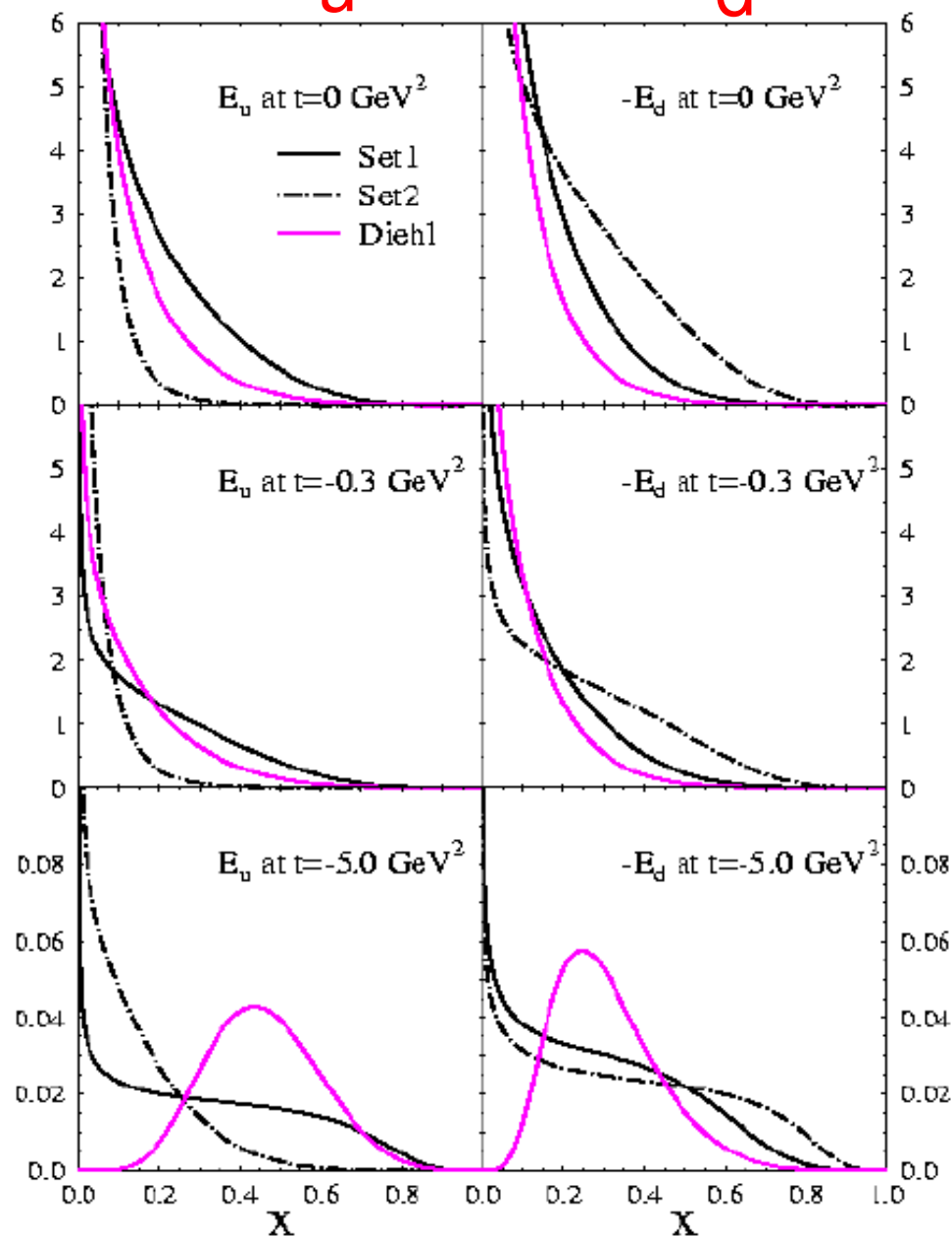
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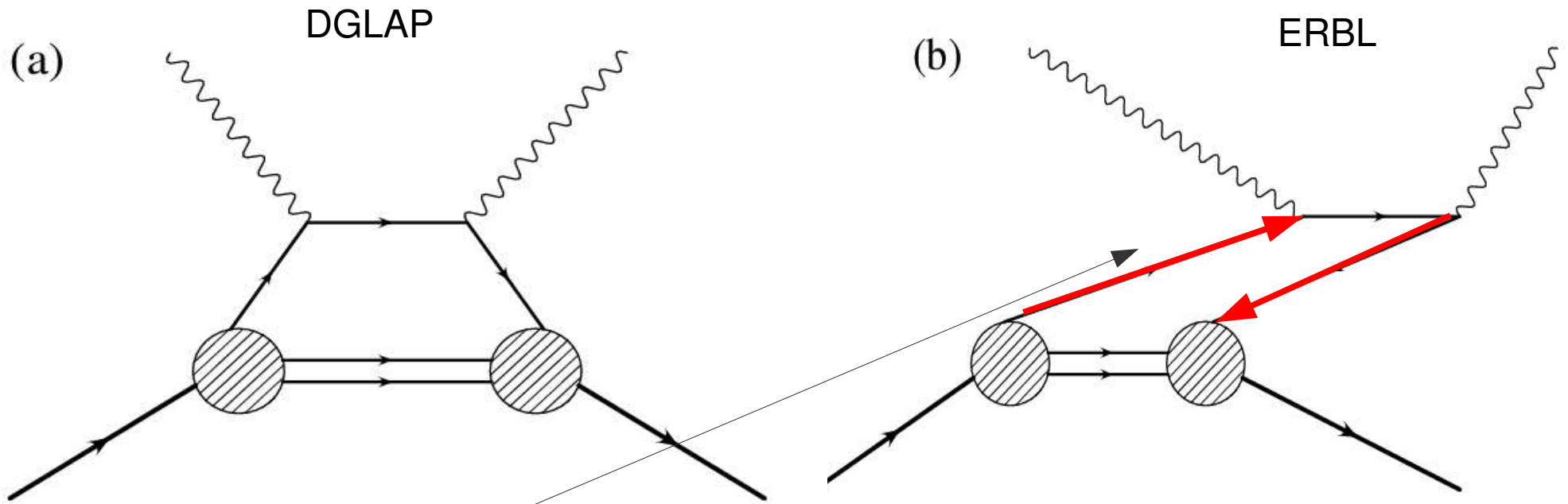
$E_q(x,0,t)$

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d

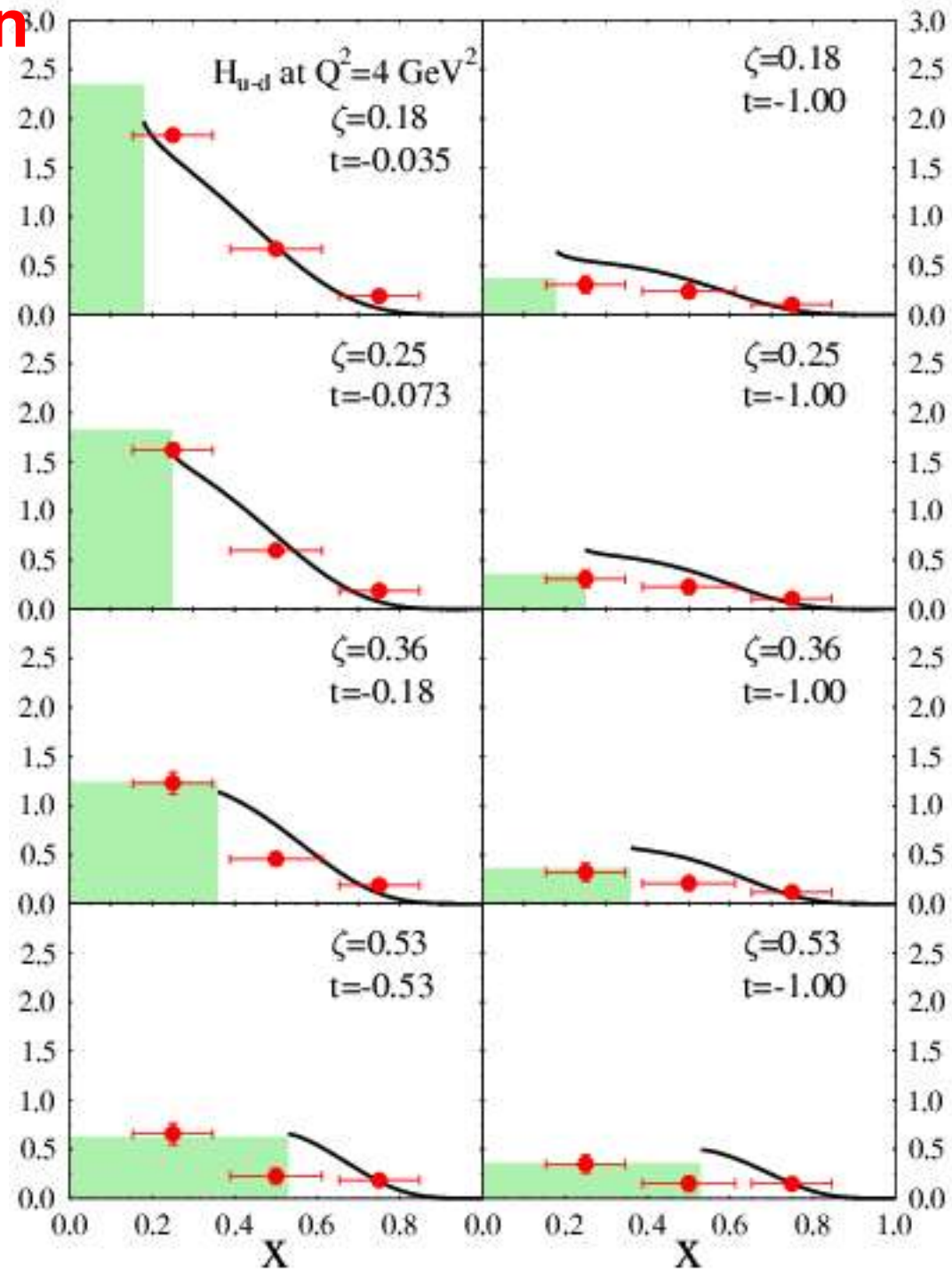


**Two different time orderings/pole structure!**



**Quark anti-quark** pair describes similar physics (**dual to**) Regge t-channel exchange!!

# ERBL Region



## GPDs from Bernstein moments

$$\bar{H}(X, \zeta, t) = \frac{(n+1)!}{k!} \sum_{l=0}^{n-k} \frac{(-1)^l}{l!(n-k-l)!} M_{l+k}$$

Mellin moments

$$\begin{aligned}\bar{H}_{02}(X_{02}) &= 3A_{10} - 6A_{20} + 3 \left[ A_{30} + \left( \frac{2\zeta}{2-\zeta} \right)^2 A_{32} \right], \\ \bar{H}_{12}(X_{12}) &= 6A_{20} - 6 \left[ A_{30} + \left( \frac{2\zeta}{2-\zeta} \right)^2 A_{32} \right], \\ \bar{H}_{22}(X_{22}) &= 3A_{30} + \left[ \left( \frac{2\zeta}{2-\zeta} \right)^2 A_{32} \right].\end{aligned}$$

First used for pdfs' in the '70s by Yndurain and collaborators



Weighted Average  $\Rightarrow$

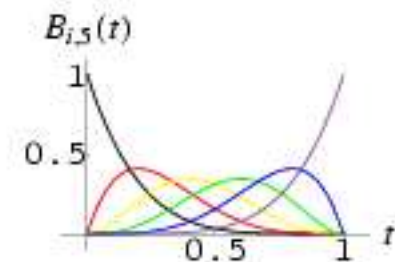
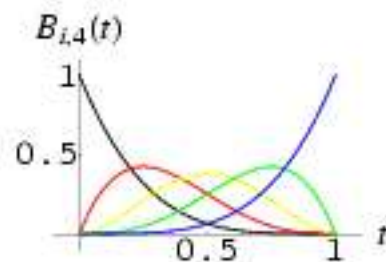
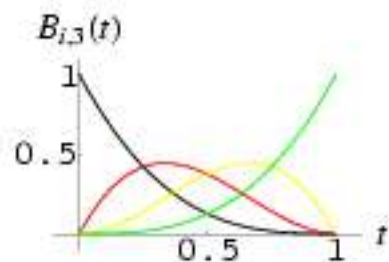
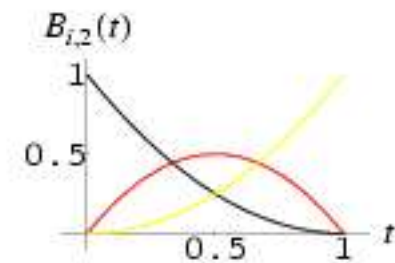
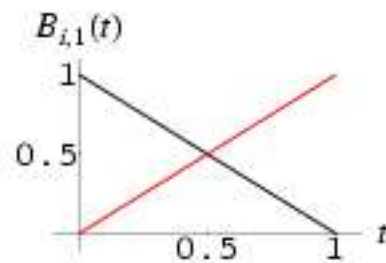
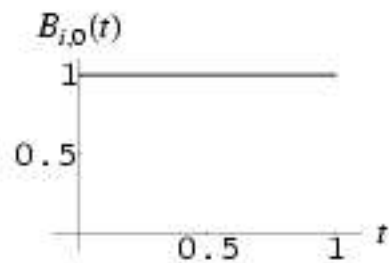
$$\bar{H}(X, \zeta, t) = \int_0^1 H(X, \zeta, t) b_{k,n}(X) dX \quad k = 1, \dots, n,$$

X-bin  $\Rightarrow$

$$\bar{X}_{k,n} = \int_0^1 X b_{k,n}(X) dX = \frac{k+1}{n+1},$$

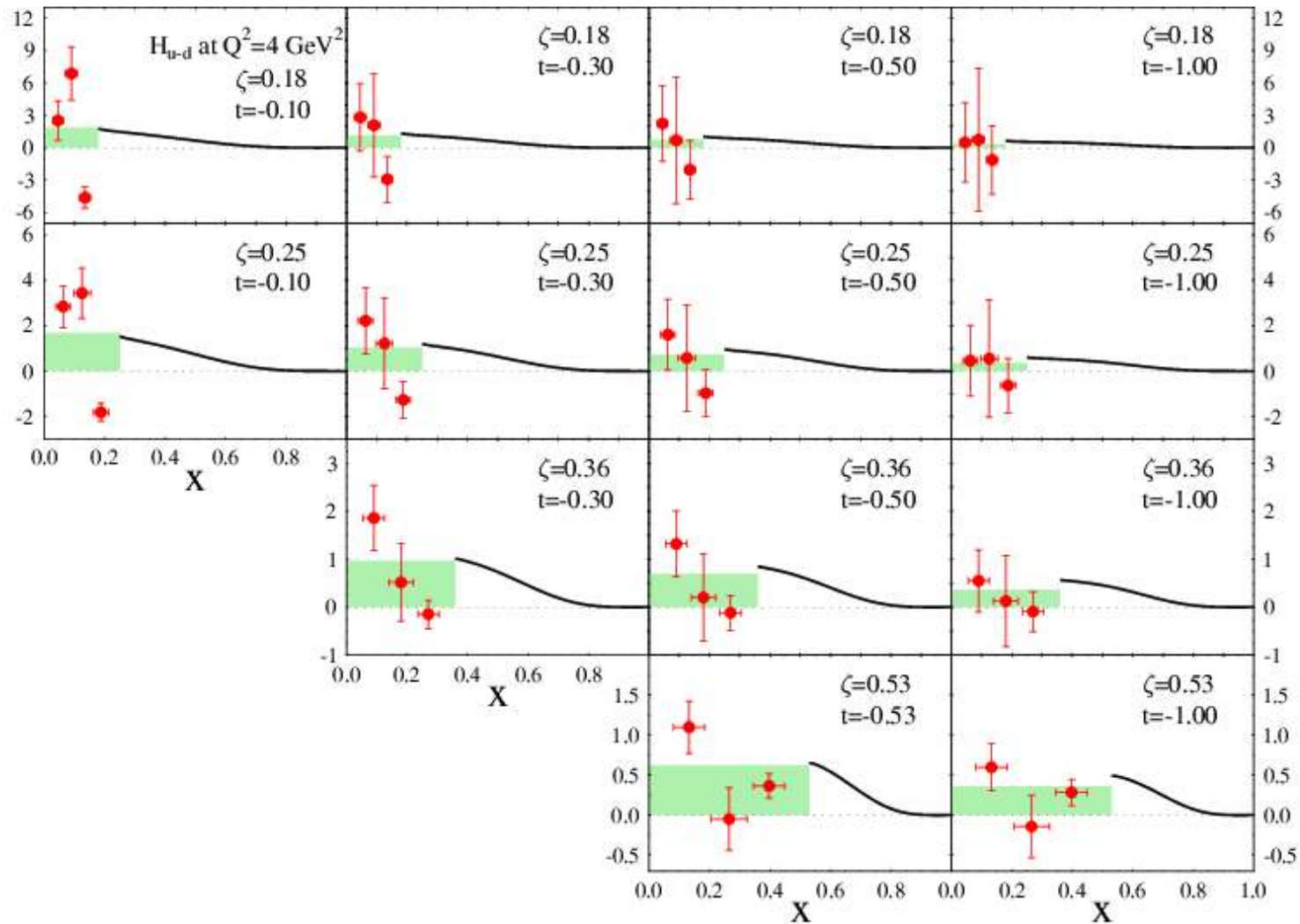
Dispersion  $\Rightarrow$

$$\Delta_{k,n} = \left( \overline{X^2}_{k,n} - \bar{X}_{k,n}^2 \right)^{1/2}$$

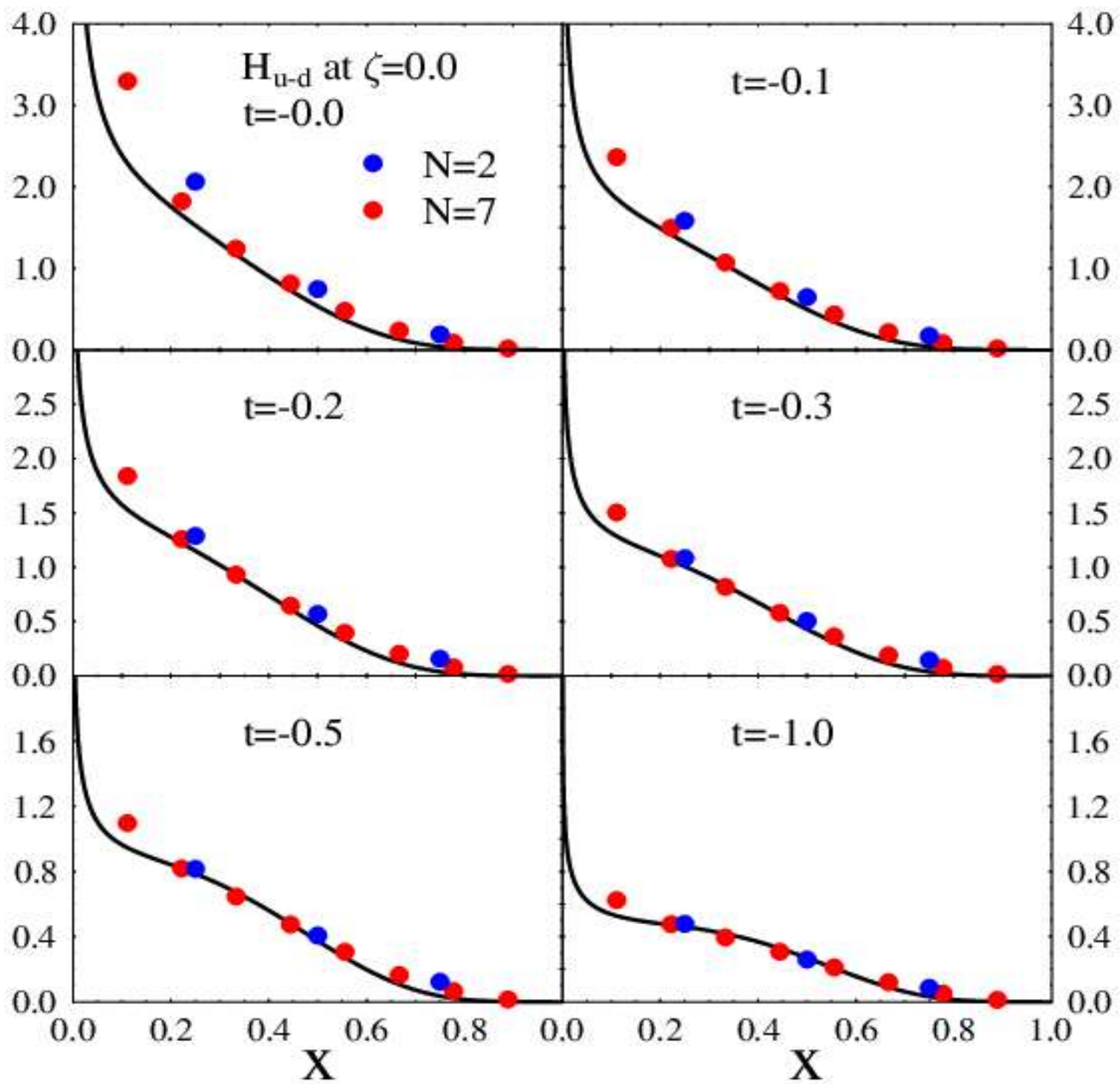


# ERBL Region

AHLT arXiv:0708.0268

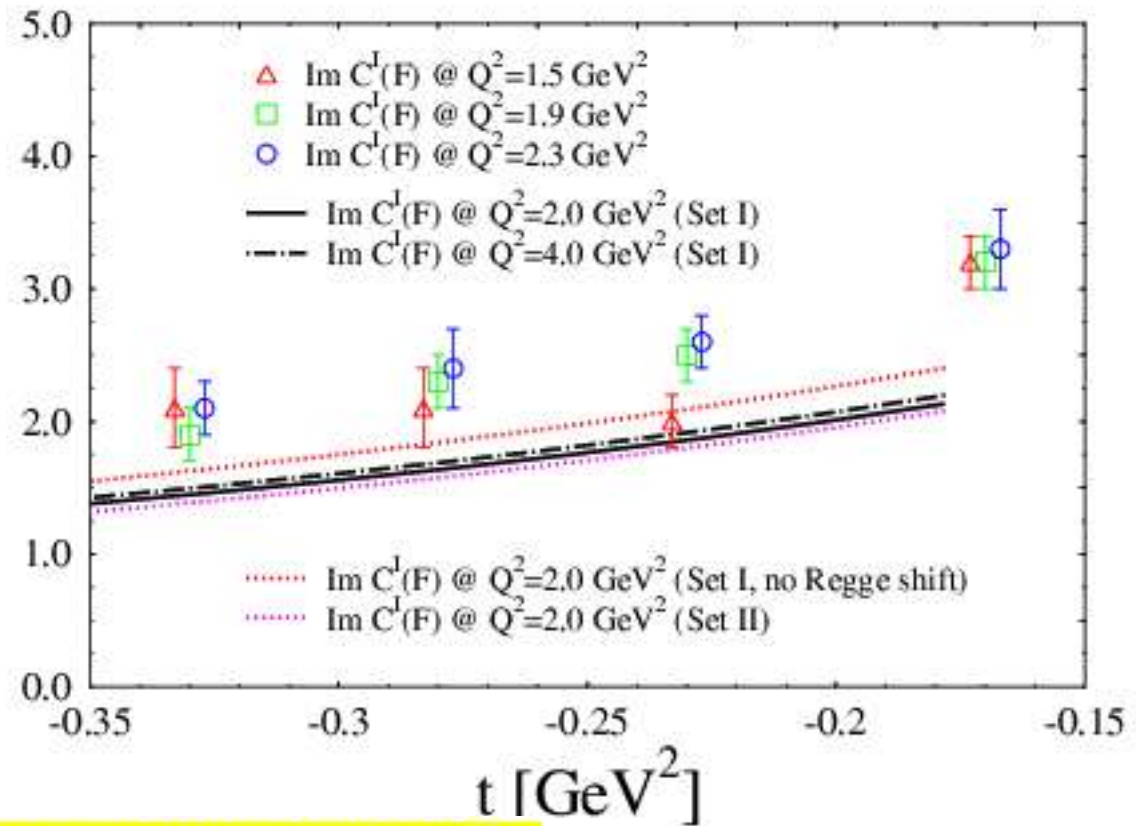
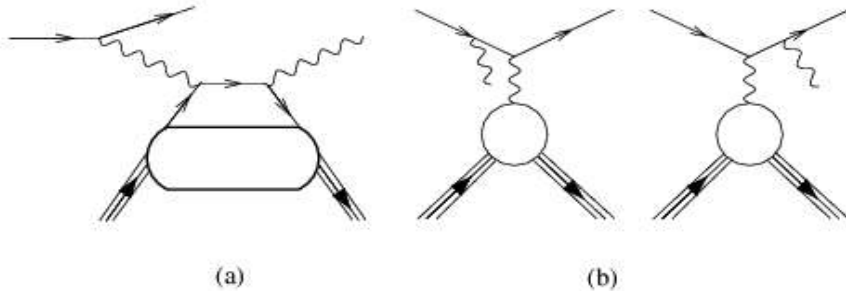


**Determined from lattice moments up to  $n=3$**



# Comparison with Jlab Hall A data (proton)

Munoz Camacho et al., (2006)



- Observable given by Interference Term between DVCS (a) and BH(b):

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \propto \sin\phi \left[ F_1(\Delta^2)\mathcal{H} + \frac{x}{2-x}(F_1 + F_2)\tilde{\mathcal{H}} + \frac{\Delta^z}{M^2}F_2(\Delta^2)\mathcal{E} \right]$$

$$\mathcal{H} = \sum_q e_q^2 (H(\xi, \xi, \Delta^2) - H(-\xi, \xi, \Delta^2))$$

**Note!!**

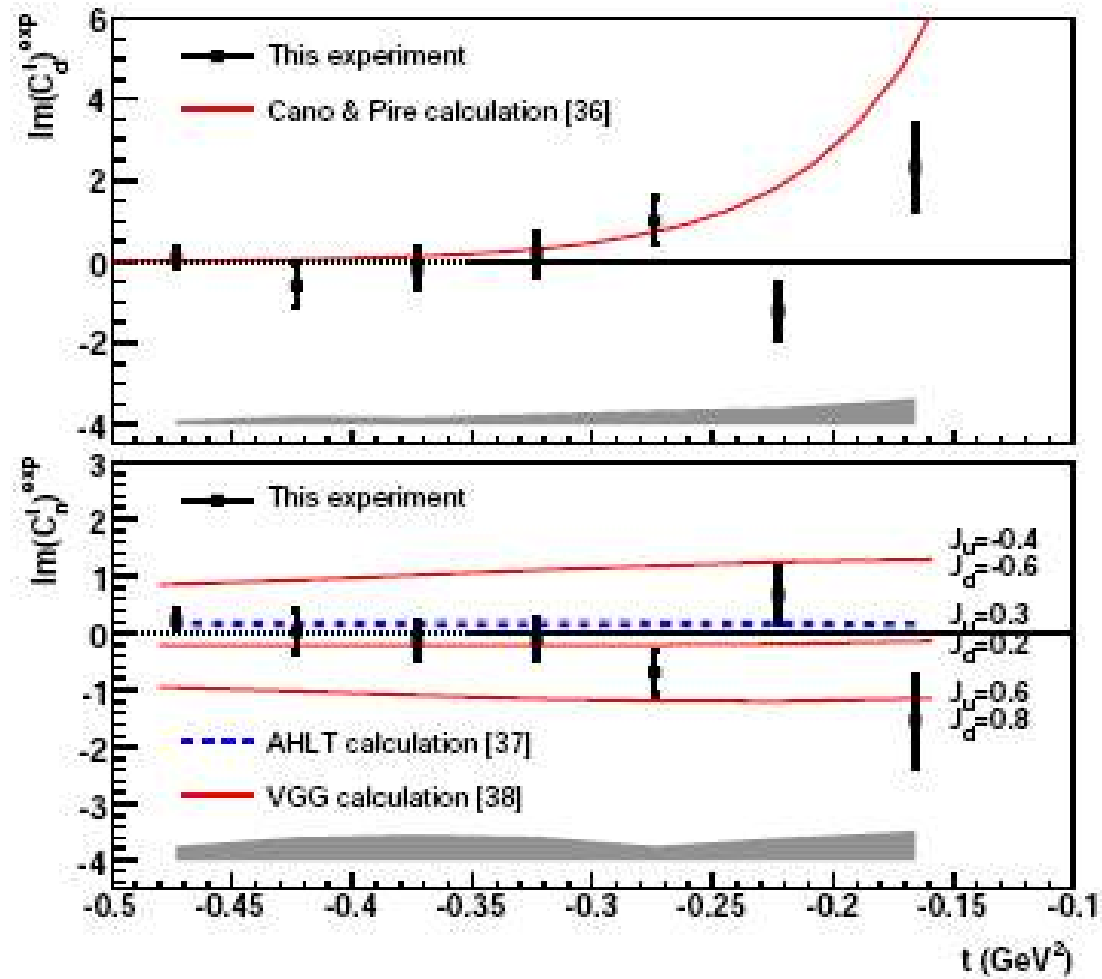
Im  $\mathcal{H}$  from asymmetry

Re  $\mathcal{H}$  from x-section

# Comparison with Jlab Hall A data

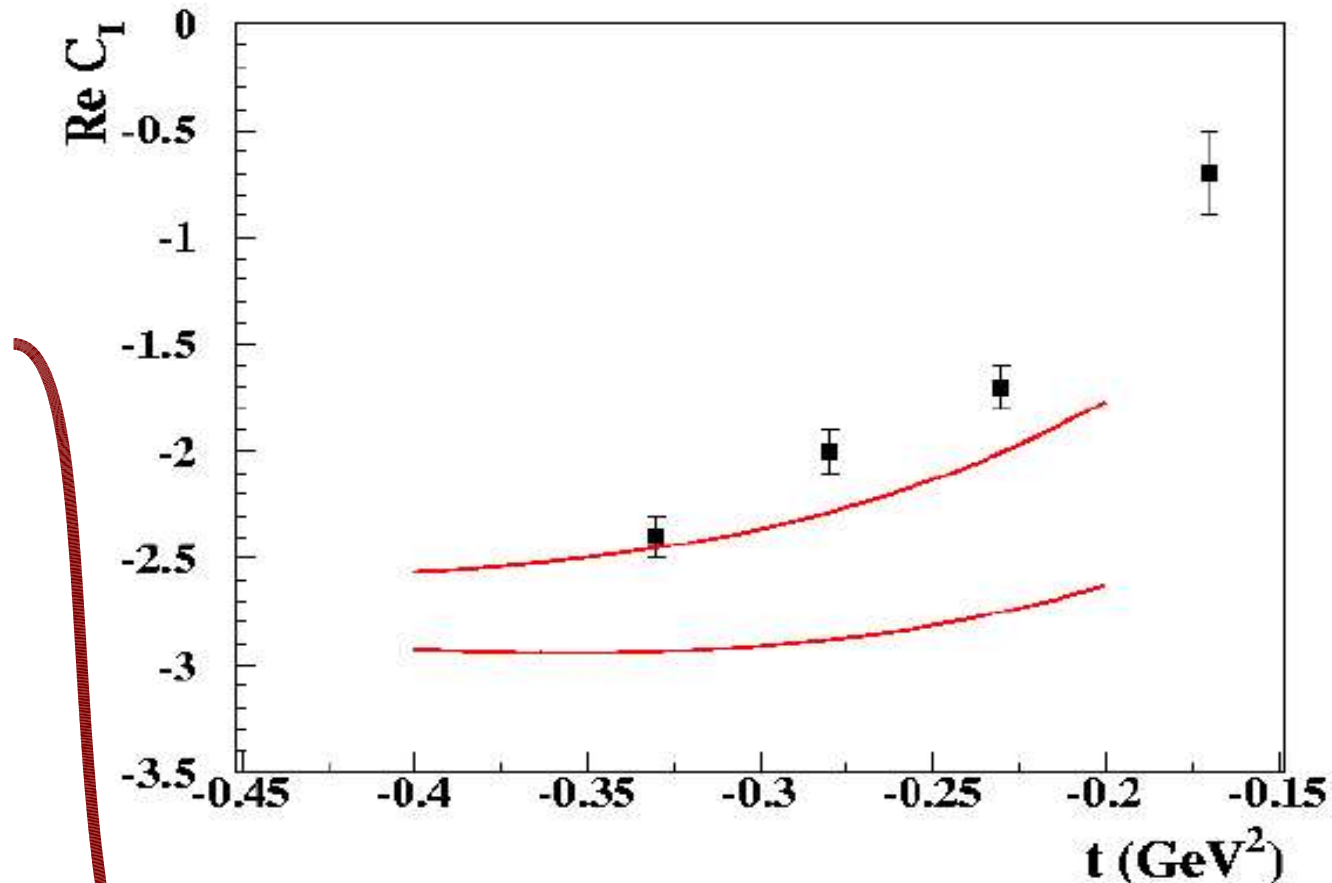
Mazouzet al., (2007)

(neutron)

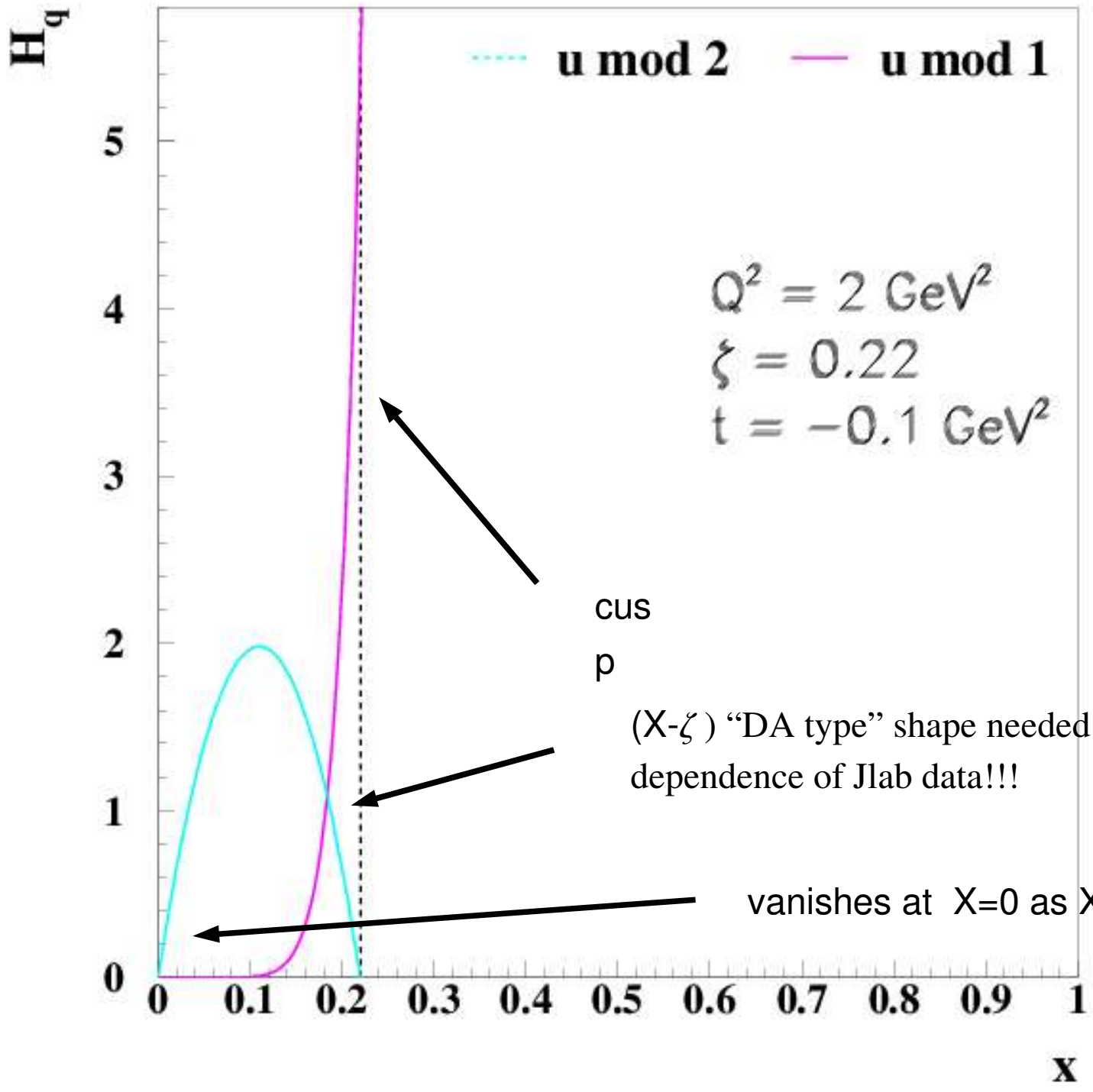


# Are the exclusive data “telling” us something?

Real Part (*S.Ahmad, S.L., preliminary*)

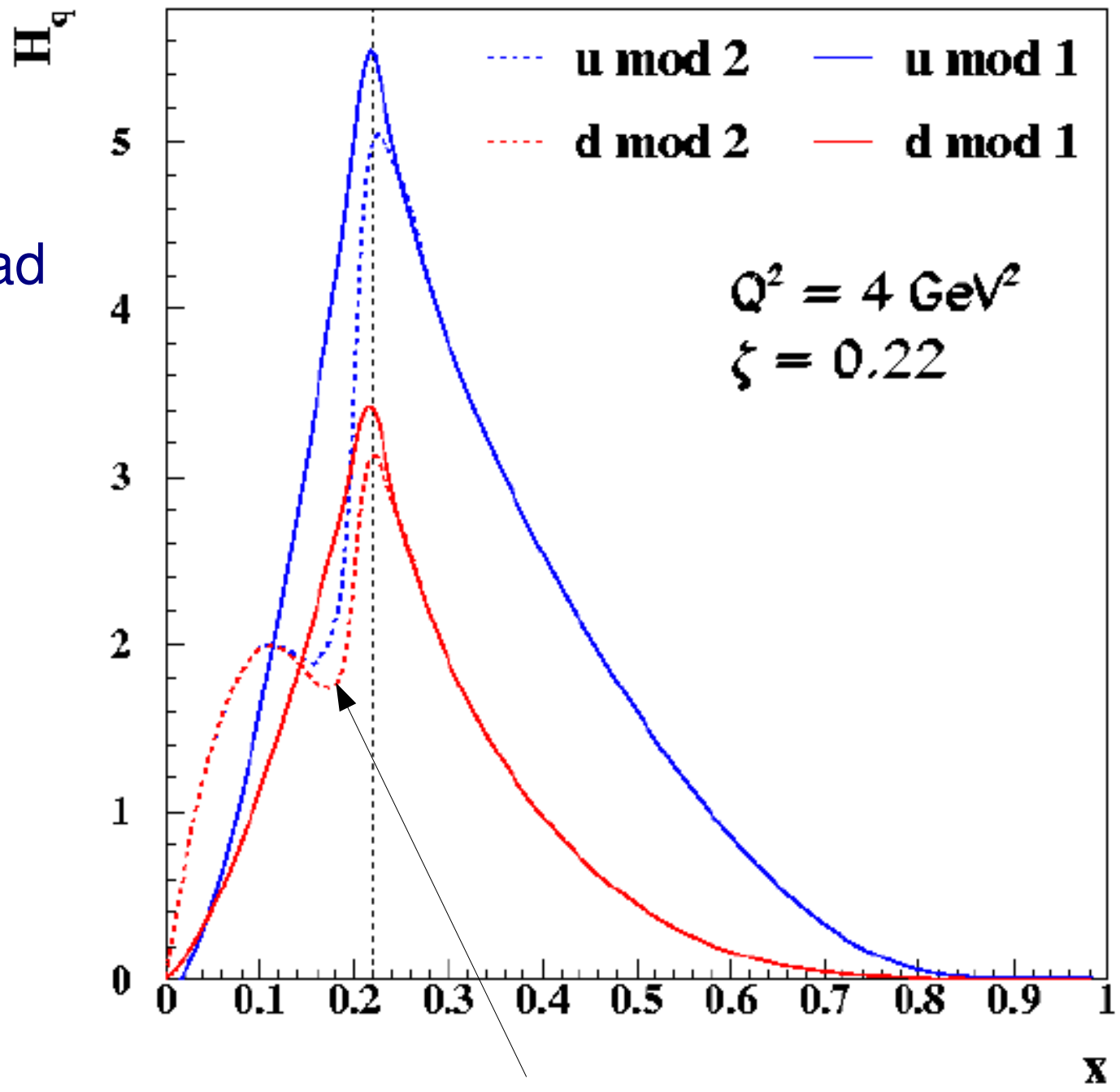


$$\mathcal{F}(\zeta, t) = -i\pi \sum_q e_q^2 [F^q(\zeta, \zeta, t) - F^q(-\zeta, \zeta, t)] + \mathcal{P} \int_{1-\zeta}^1 dX \left( \frac{1}{X-\zeta} + \frac{1}{X} \right) F^q(X, \zeta, t).$$



Fitted directly at Q of data

S. Ahmad



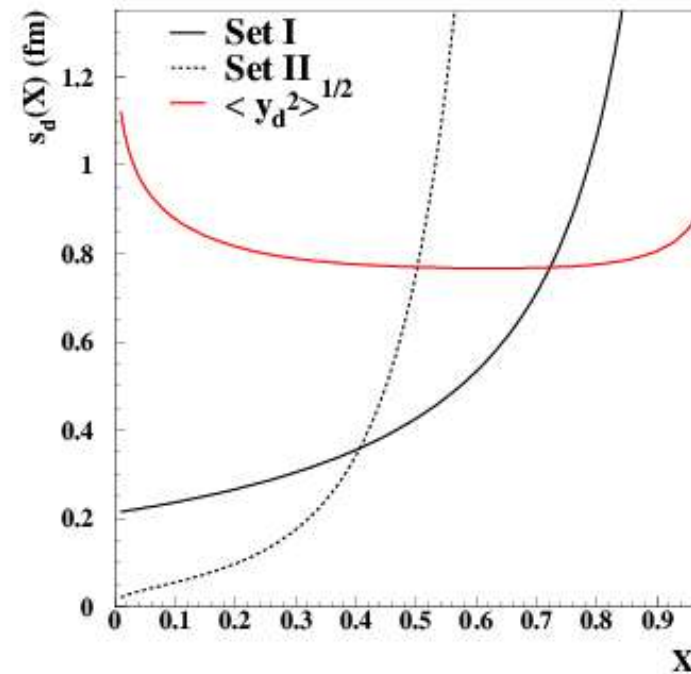
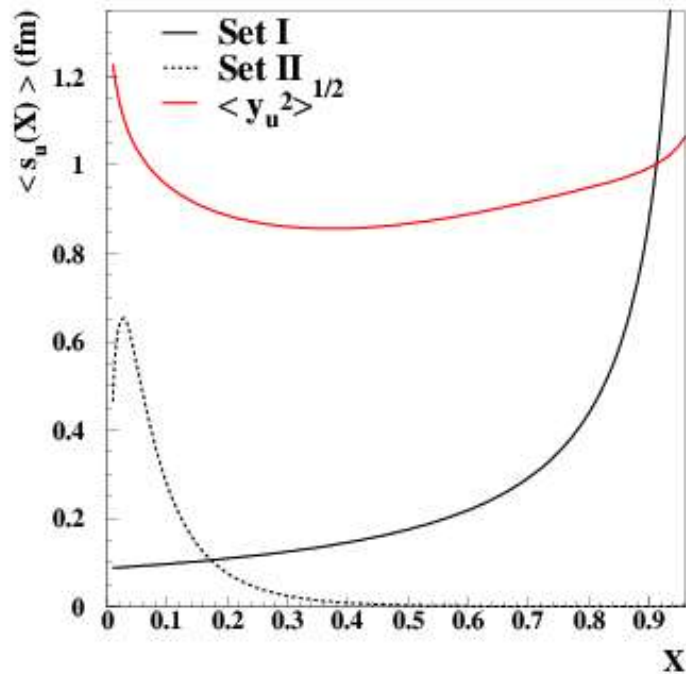
- Behavior determined by Jlab data on Real Part and  $Q^2$  dependence
- Consistent with lattice determination!



# Evaluation of interparton distances

$y$  is the average distance of quark  $q$  from the spectator quarks

$s$  is the average shift of quark along the  $y$ -axis when proton is (transversely) polarized along the  $x$ -axis



Sensitive to E!

# 3. Nuclei

# Deuteron: New sum rules

S I S.K.Taneja

$$\begin{aligned}
 & \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix(Pz)} \langle p' | \bar{q}(-\frac{1}{2}z) \not{\epsilon} q(\frac{1}{2}z) | p \rangle \Big|_{z=\lambda n_-} = -(\epsilon'^* \epsilon) \underline{H_1} \\
 & + \frac{(\epsilon n_-)(\epsilon'^* P) + (\epsilon'^* n_-)(\epsilon P)}{P n_-} \underline{H_2} - \frac{2(\epsilon P)(\epsilon'^* P)}{m^2} \underline{H_3} \\
 & + \frac{(\epsilon n_-)(\epsilon'^* P) - (\epsilon'^* n_-)(\epsilon P)}{P n_-} \underline{H_4} \\
 & + \left[ m^2 \frac{(\epsilon n_-)(\epsilon'^* n_-)}{(P n_-)^2} + \frac{1}{3}(\epsilon'^* \epsilon) \right] \underline{H_5},
 \end{aligned}$$

Cano and Pire (2001)

Form Factors  $\int_{-1}^1 dx H_i(x, \xi, t) = G_i(t) \quad (i = 1, 2, 3).$

$$\begin{aligned}
 G_C &= G_1 + \frac{2}{3} \eta G_Q, \\
 G_Q &= G_1 - G_2 + (1 + \eta) G_3, \\
 G_M &= G_2
 \end{aligned}$$

# Energy momentum tensor

$$\begin{aligned}
 \langle p' | \theta^{\mu\nu} | p \rangle = & - \frac{1}{2} \left[ P^\mu P^\nu - \frac{g^{\mu\nu}}{4} P^2 \right] (\epsilon'^* \epsilon) G_{1,2}(t) - \frac{1}{4} \left[ P^\mu P^\nu - \frac{g^{\mu\nu}}{4} P^2 \right] \frac{(\epsilon P)(\epsilon'^* P)}{M^2} G_{2,2}(t) \\
 & - \frac{1}{2} \left[ \Delta^\mu \Delta^\nu - \frac{g^{\mu\nu}}{4} \Delta^2 \right] (\epsilon'^* \epsilon) G_{3,2}(t) - \frac{1}{4} \left[ \Delta^\mu \Delta^\nu - \frac{g^{\mu\nu}}{4} \Delta^2 \right] \frac{(\epsilon P)(\epsilon'^* P)}{M^2} G_{4,2}(t) \\
 & + \frac{1}{4} \left[ (\epsilon'^* \mu (\epsilon P) + \epsilon^\mu (\epsilon'^* P)) P^\nu + \mu \leftrightarrow \nu - g^{\mu\nu} (\epsilon P)(\epsilon'^* P) \right] G_{5,2}(t) \\
 & + \left[ (\epsilon'^* \mu (\epsilon P) - \epsilon^\mu (\epsilon'^* P)) \Delta^\nu + \mu \leftrightarrow \nu + g^{\mu\nu} (\epsilon P)(\epsilon'^* P) - (\epsilon'^* \mu \epsilon^\nu + \epsilon'^* \nu \epsilon^\mu) \Delta^2 + \frac{g^{\mu\nu}}{2} (\epsilon'^* \epsilon) \Delta^2 \right] G_{6,2}(t)
 \end{aligned} \tag{2}$$

.. and relation with deuteron GPDs:

$$\begin{aligned}
 \int dx x H_1(x, \xi, t) - \frac{1}{3} \int dx x H_5(x, \xi, t) &= G_{1,2}(t) + \xi^2 G_{3,2}(t) \\
 \int dx x H_2(x, \xi, t) &= G_{5,2}(t) \\
 \int dx x H_3(x, \xi, t) &= G_{2,2}(t) + \xi^2 G_{4,2}(t) \\
 \frac{1}{4\xi} \int dx x H_4(x, \xi, t) &= \frac{M^2}{t} \int dx x H_5(x, \xi, t) = G_{6,2}(t)
 \end{aligned}$$

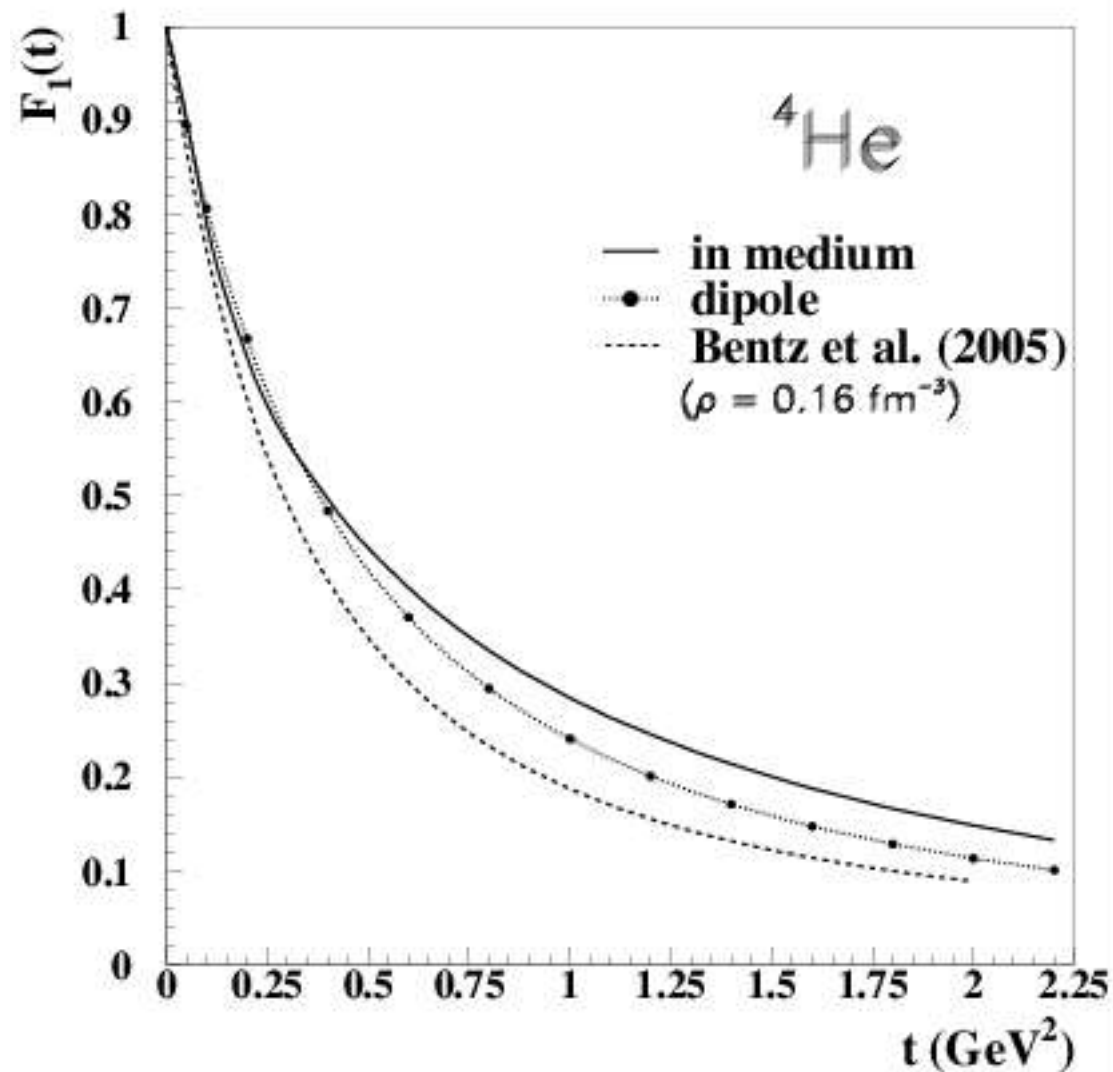
...inserting the energy momentum tensor i  $\langle p' | \int d^3x (\vec{x} \times \vec{T}_{q,g}^{0i})_z | p \rangle$



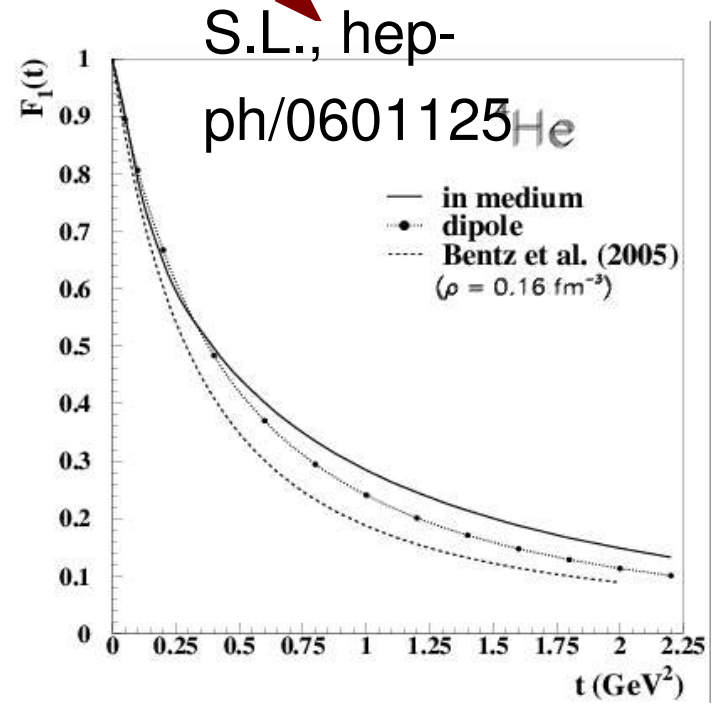
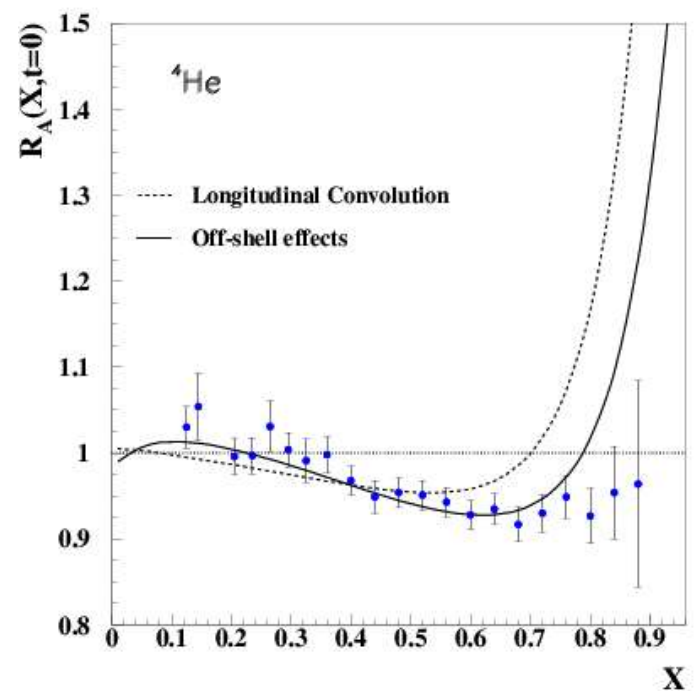
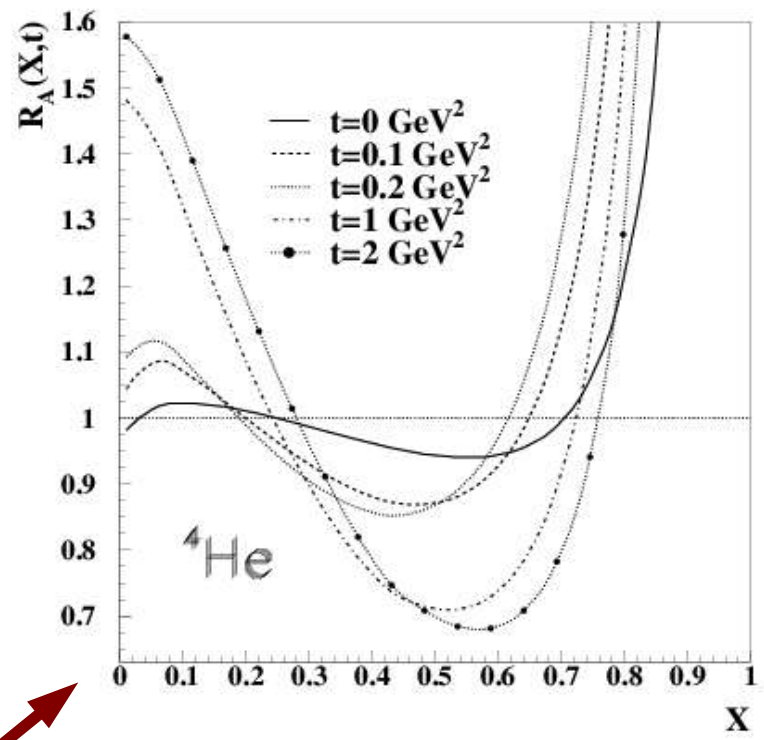
$$J_q = \frac{1}{2} \int dX X H_2^q(X, 0, 0) \equiv \frac{1}{2} G_{5,2}(0)$$

**An essential piece of information for extracting quarks angular momentum!**

# 4He: nuclear GPDs as tools to study in medium modifications

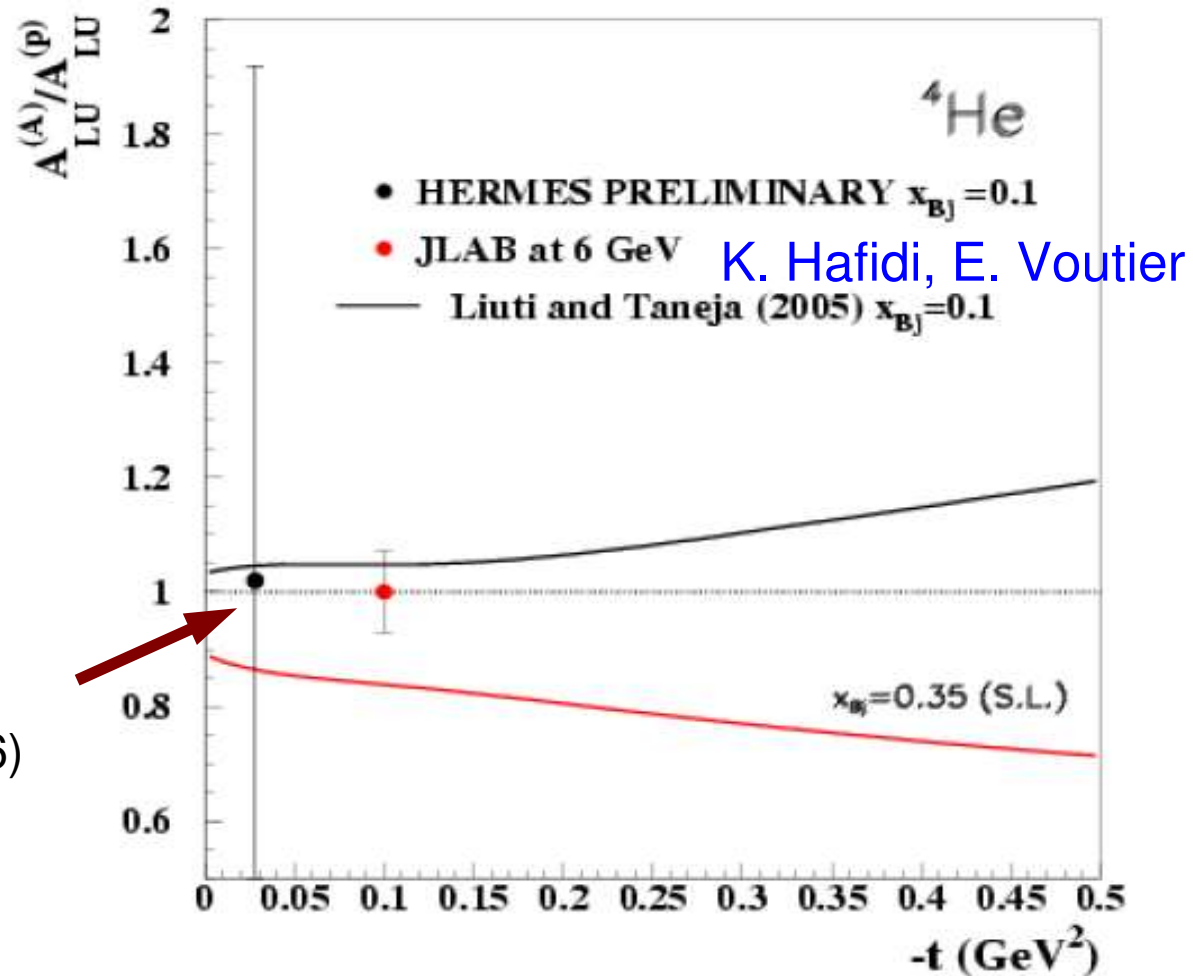


S.L., S.K. Taneja,  
 PRC72, 032201 (2005)



S.L., hep-  
 ph/0601125

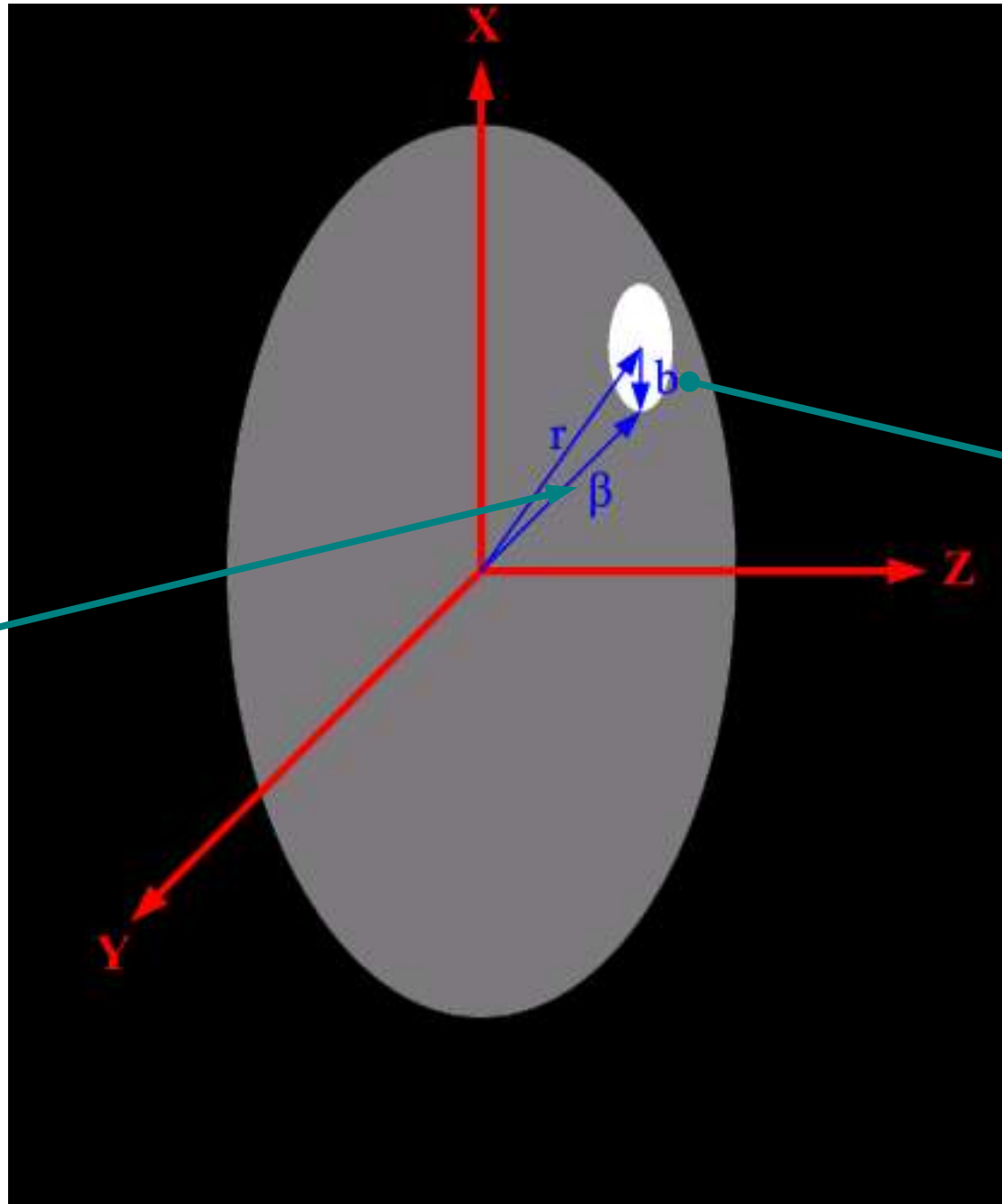
# Experiments on $^4\text{He}$ are feasible at Jlab:



HERMES (2006)



## Spatial structure of quarks and gluons in nuclei



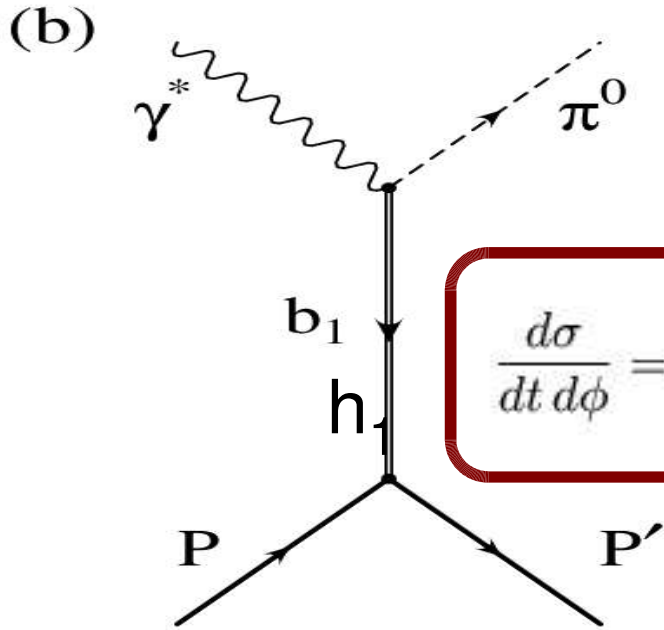
quark's position  
in nuclei

Burkardt-Soper  
impact parameter

# $\pi^0$ Electroproduction Observables and GPDs

# Exclusive $\pi^0$ electroproduction

$$ep \rightarrow e'p'\pi^0$$



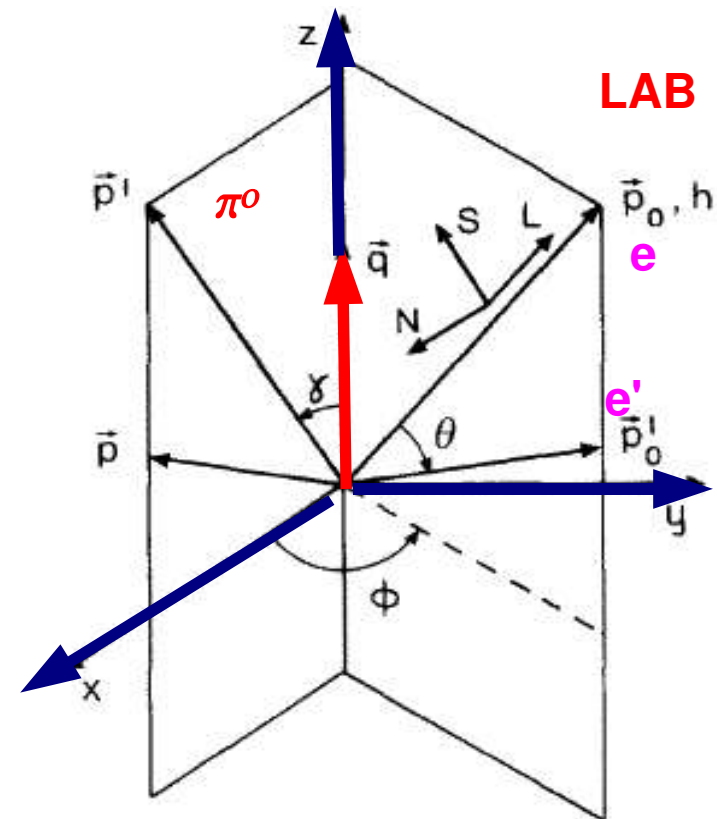
$$\frac{d\sigma}{dt d\phi} = \left( \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} \right) + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi + \sqrt{2\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{dt} \cos \phi$$

$$d\sigma \propto L_{\mu\nu}^{h=\pi^0} W_{\mu\nu}$$

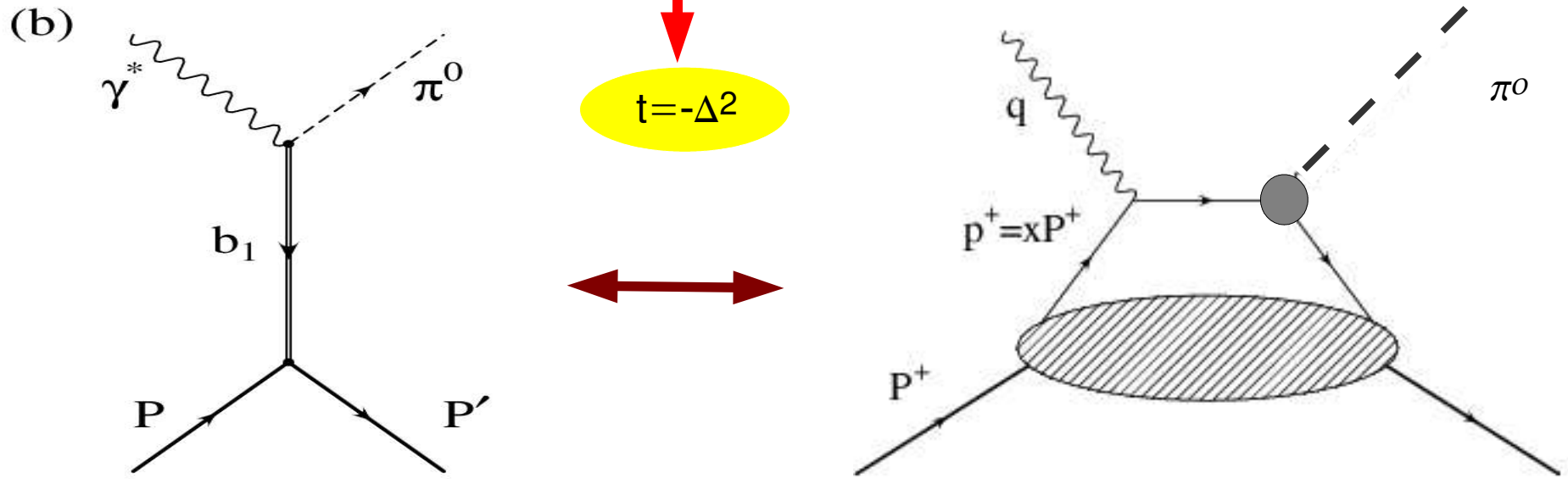
$L_{\mu\nu}^{h=\pi^0} \approx \gamma^*$  polarization density matrix

$W_{\mu\nu} = \sum J_\mu J_\nu^* \delta(E_i - E_f) =$  hadronic tensor

$$\frac{d\sigma_{TT}}{dt} = W_{yy} - W_{xx} \equiv 2\Re(J_1 J_{-1}^*)$$



# Dual Representation?

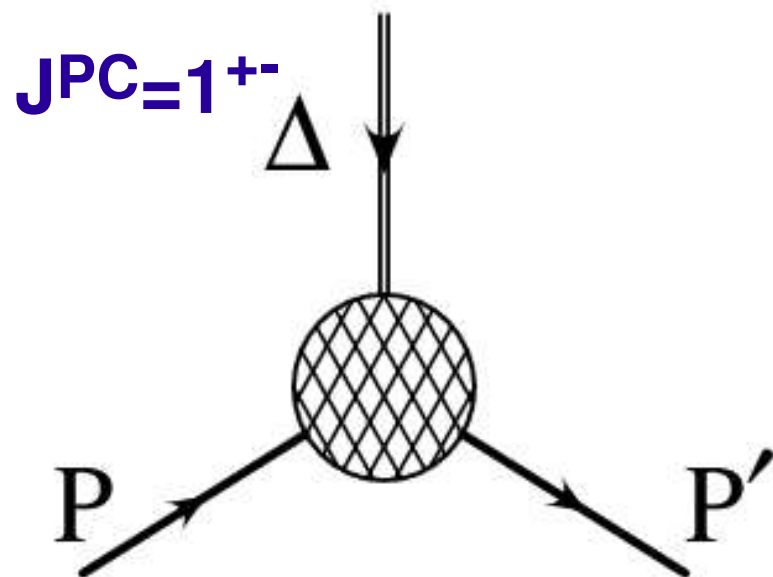
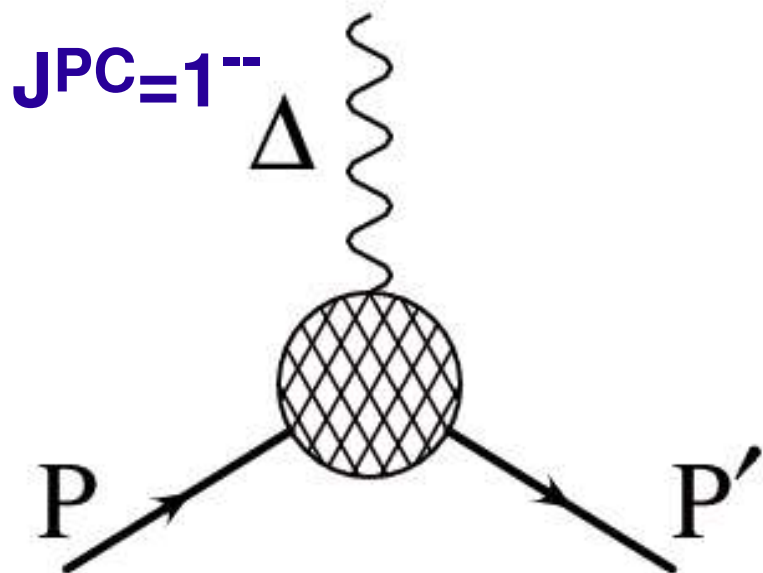


$$f_{1,\Lambda,0,\Lambda'} = \sum_{\lambda,\lambda'} g_{1,\lambda,0,\lambda} A_{\Lambda,\lambda';\Lambda,\lambda}$$

helicity amps.

“Quark-Hadron” Helicity Amplitudes  
(Marcus Diehl)

Only chiral-odd GPDs!!! 



$$i\sigma_{\mu\nu}\gamma_5$$

$\Leftrightarrow JPC=1^{--}, 1^{+-}, \dots \Leftrightarrow H_T, E_T, \dots$

$$\gamma_5$$

$\Leftrightarrow JPC=1^{++}, \dots$  ( $a_1$ -type exchange)  $\Leftrightarrow H, E, \dots$

# What goes into the quark-hadron amplitudes?

$$\mathcal{F}(\zeta, t) = -i\pi \sum_q e_q^2 [F^q(\zeta, \zeta, t) - F^q(-\zeta, \zeta, t)] + \mathcal{P} \int_{1-\zeta}^1 dX \left( \frac{1}{X-\zeta} + \frac{1}{X} \right) F^q(X, \zeta, t).$$

## Generalized Form Factors

$$\mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{H}}_T, \tilde{\mathcal{E}}_T$$

$$\mathcal{H}_T(X, 0, 0) = h_1(X) = \text{transversity}$$

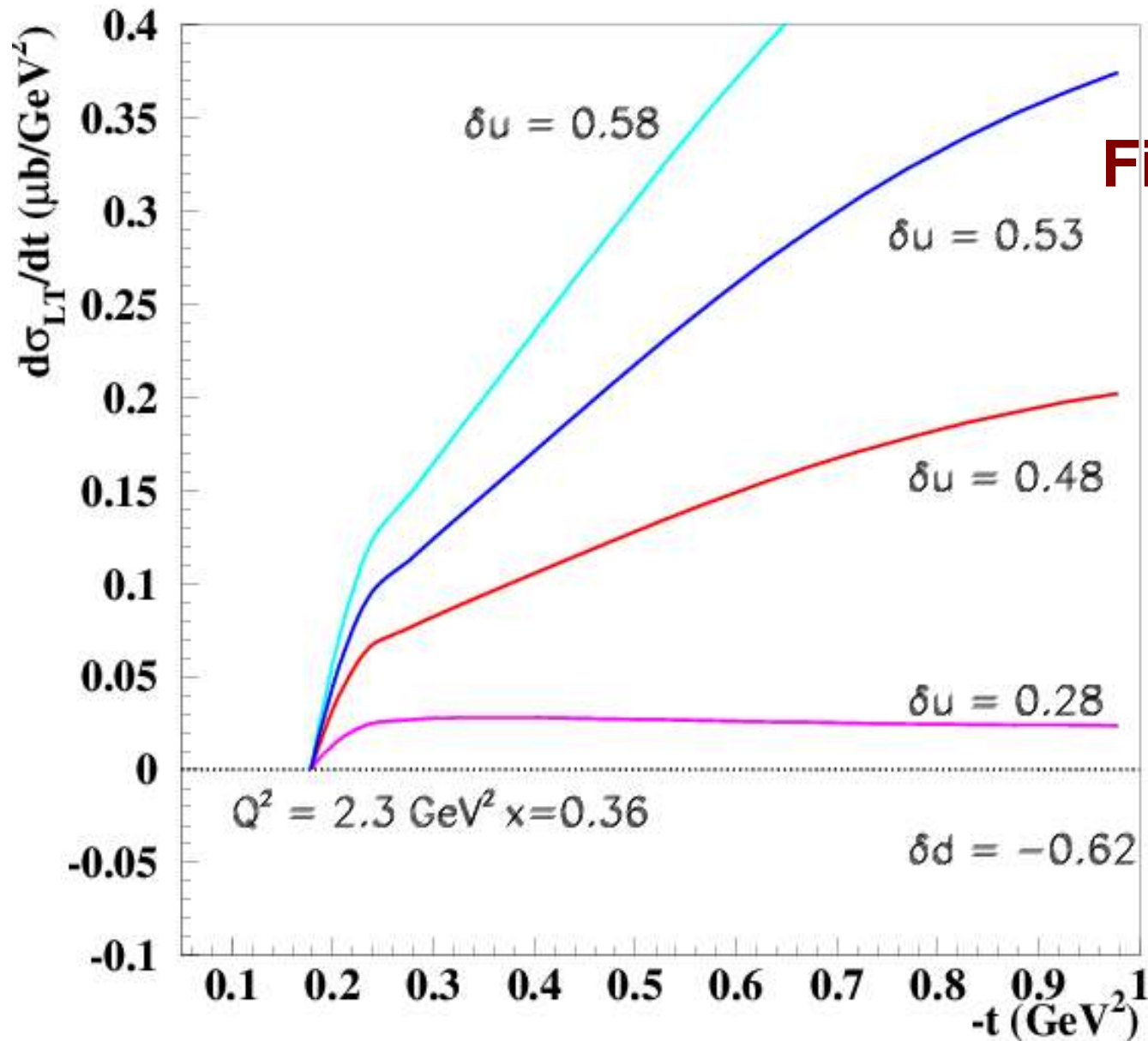
$$\int h_1(X, Q^2) dX = \delta q = \text{tensor charge}$$

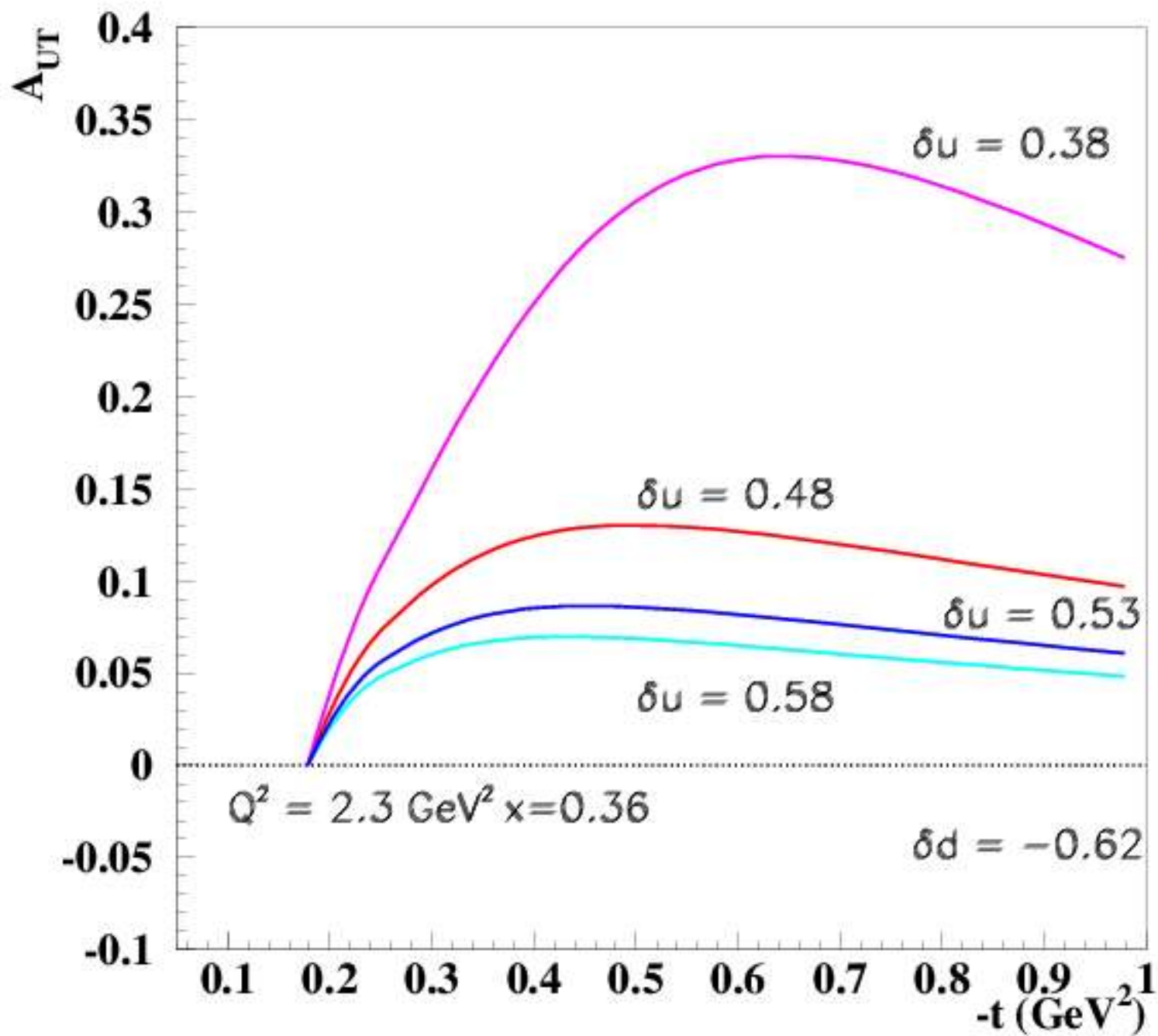
$$\tilde{\mathcal{E}}_2 = 2\tilde{\mathcal{H}}_T + \mathcal{E}_T$$

$$\int \mathcal{E}_2(X, 0, 0) dX = \kappa_T = \text{Burkardt's moment}$$

$$\int h_{1\perp}(X) dX d^2k_T \sim -\kappa_T \text{ (A.Metz)}$$

**Main Result:** Tensor Charge and Anomalous Transverse Moment treated as free parameters to be extracted from data  $\Rightarrow$  Show sensitivity!







# GPDs & hadron tensor for Spin 0 nuclear target (Liuti and Taneja, PRC 2005)

## $\pi^0$ production (with G. Goldstein)

- 1st consider nucleon target GPDs & Regge Cuts
  - Like  $\pi+N \rightarrow \pi+N$
  - 2 invariant amps (A,B)  $\rightarrow$  2 leading twist GPDs
    - Like  $\sim H(x,\zeta,t)$  &  $E(x,\zeta,t)$  or  $H_T(x,\zeta,t)$
  - $H(x,0,0) \sim A(x)$ ,  $H_T(x,0,0) \sim B(x)$  ( ) flips 1/2 particle helicity
    - $E_T$  more like Chiral-odd GPDs for nucleons than E
    - Carries over to  $4\text{He} \rightarrow q q' \rightarrow 4\text{He}'$
  - Can coherent  $e+{}^4\text{He} \rightarrow e'+\pi^0+{}^4\text{He}'$  be treated as pure spin 0 target?
    - Need 2 steps involving  ${}^4\text{He} \rightarrow N=N' \rightarrow {}^4\text{He}'$  &  $N \rightarrow q=q' \rightarrow N'$ 
      - and (PQCD?)  $\gamma^*+q \rightarrow \pi^0+q'$
    - 2 helicity amps longitudinal & transverse  $\gamma^*$
  - How can these distinguish H from chiral odd, C=parity odd structures?

# Conclusions and Outlook

- Comparison between GPD models and data is indeed possible...GPD extraction is possible!!!
- Approaching “Global Analysis”
- Interesting connections between TMDs and GPDs
- Proposed extraction of tensor charge and transverse anomalous moment from neutral pion production data
- Spatial structure of Nuclei