Parity violating asymmetry, giant resonances, and the neutron skin thickness

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INTRODUCTION

The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different nuclear interactions in the medium** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

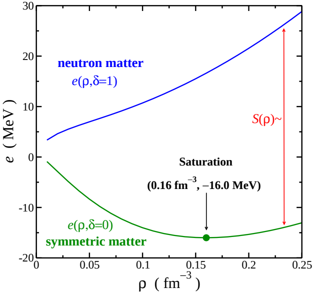
$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad + \bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\tau\Psi A^{(\rho)\mu} \\ &-e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

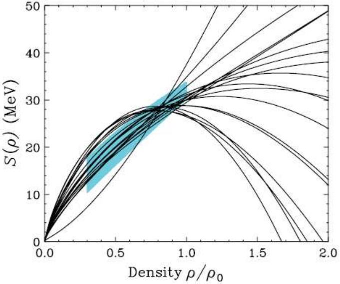
Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\rm eff}_{\rm Nucl} = V^{\rm long-range}_{\rm attractive} + V^{\rm short-range}_{\rm repulsive} + V_{\rm SO} + V_{\rm pair}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction**

The Nuclear Equation of State: Infinite System



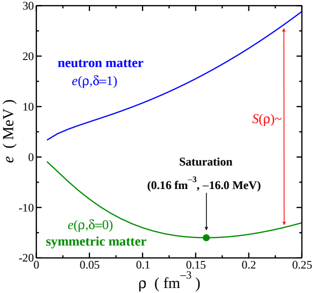


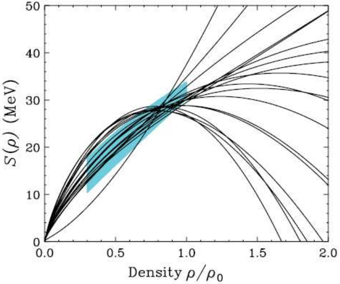
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



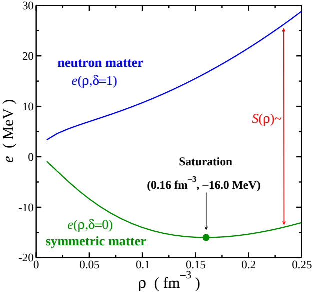


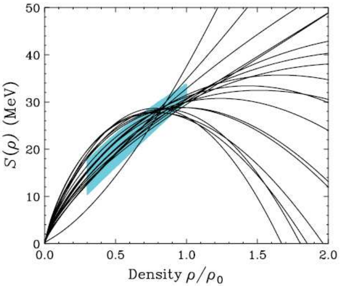
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



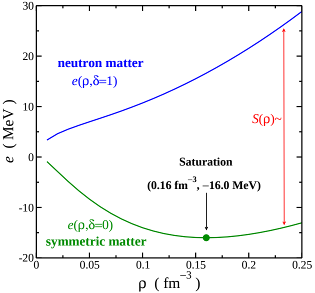


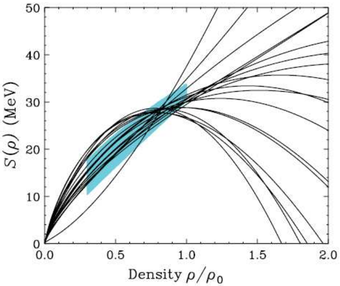
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter
- Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



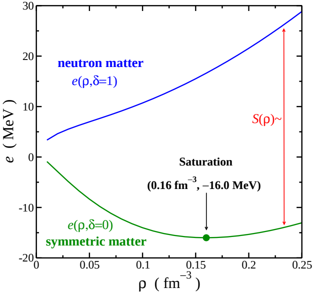


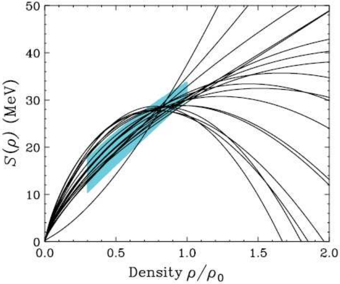
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho-\rho_0}{3\rho_0}\right]$$

The Nuclear Equation of State: Infinite System





$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + \beta^2\left(\boxed{J} + \boxed{L}\,x + \frac{1}{2}\boxed{K_{sym}}\,x^2 + \mathcal{O}(x^3)\right)$$

- $S(\rho_0) = J$
- $\left.\frac{d}{d\rho}S(\rho)\right|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2}$
- $\left.\frac{d^2}{d\rho^2}S(\rho)\right|_{\rho_0} = \frac{K_{sym}}{9\rho_0^2}$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0}\right]$$

Isovector properties in nuclei

- **In the past** (and also in the present), **neutron properties** in stable medium and heavy nuclei have been mainly measured by using **strongly interacting probes**.

  $\Downarrow$

  Limited knowledge of isovector properties

- **At present**,
  - the use of **rare ion beams** has opened the possibility of measuring properties of **exotic nuclei** $\Rightarrow$ **more info**
  - **parity violating elastic electron scattering** (PVES), a **model independent technique**, has allowed to estimate the **neutron radius** of a stable heavy nucleus like ^{208}Pb

  $\Downarrow$

  Promising perspectives for the near future

PARITY VIOLATING ASYMMETRY

Some basics ...

- **Electrons** interact by exchanging a γ or a Z_0 boson.
- While **protons** couple basically to γ, **neutrons** do it to Z_0.
- Electron motion governed by the Dirac equation:

  $$[\vec{\alpha} \cdot \vec{p} + \beta m_e + V(r)]\psi = E\psi$$

  where $V(r) = V_C(r) + \gamma^5 V_W(r)$
- Dirac equation for helicity states ($m_e \approx 0$)

  $$[\vec{\alpha} \cdot \vec{p} + (V_C(r) \pm V_W(r))]\psi_\pm = E\psi_\pm$$
- **Ultra-relativistic electrons, depending on their helicity,** will interact with the nucleus seeing a slightly different potential "αZ" $\pm$ "G_F".

Refs: Phys. Rev. C **57** 3430 (1998); Phys. Rev. C **63**, 025501 (2001); Phys. Rev. C **78**, 044332 (2008); Phys. Rev. C **82**, 054314 (2010); Phys. Rev. Lett. **106** 252501 (2011)

Some basics ...

- The **interference** between the DCS of electrons with + and − helicity states,

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}$$

- **Ultra-relativistic electrons** moving under the effect of $V_{\pm}$ where **Coulomb distortions** are important ⇒ solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).

  - Input for the calculation of $V_{\pm}$ are the ρ_n **and** ρ_p **(main uncertainty in ρ_n)** and **nucleon form factors** for the e-m and the weak neutral current.

Qualitative considerations ...

Within the Plane Wave Born Approximation,

$$A_{p\nu} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

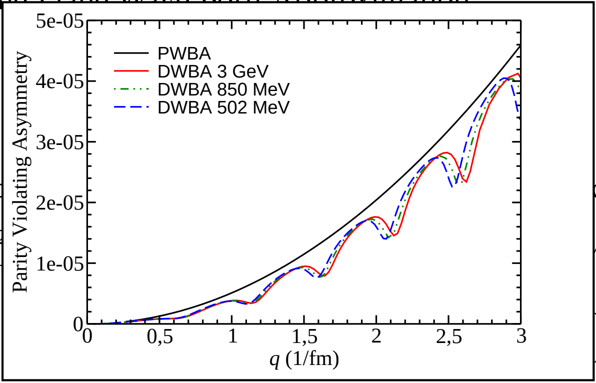
... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\Delta r_{np}(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2})\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\Delta r_{np} + \Delta r_{np}^2\right) \end{aligned}$$

variation of $A_{p\nu}$ at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Qualitative considerations ...

Within the Plane Wave Born Approximation



... which c... oximately,

$-\frac{q^2}{6} \ldots ^2)]$

variation of A_{pv} at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Qualitative considerations ...

Within the Plane Wave Born Approximation,

$$A_{p\nu} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

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variation of $A_{p\nu}$ at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

So, let us check DWBA results...

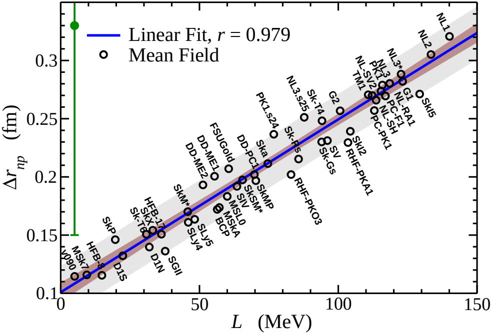
^{208}Pb: direct correlations

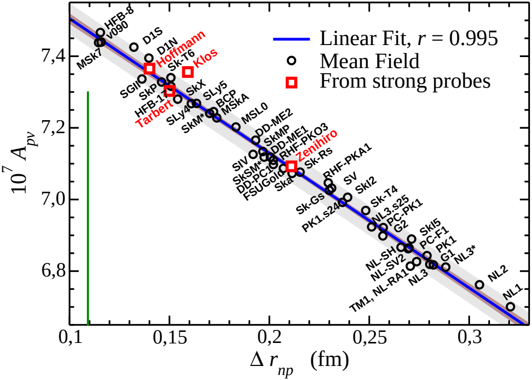
$\delta A_{pv} \sim$ **1%;**

$\delta\Delta r_{np} \sim 0.02$ **fm;**

δL **10 MeV**

X. Roca-Maza, *et al.*, PRL **106** 252501 (2011)





EDF correlations allows to determine Δr_{np} and L without direct assumpt. on ρ, JLab and Mainz forthcoming experiments

Different experiments on proton elastic scattering, antirpotonic atoms and pion-photoproduction agrees with the correlation

ISOVECTOR GIANT RESONANCES

Isovector Giant Resonances

- In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**
  e.g. within a classical picture: **"e-m interacting probes basically excite protons, protons drag neutrons thanks to the nuclear strong interaction, when neutrons approach too much to protons, they are pushed out"**
- **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"
- The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{S(\rho)}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Polarizability, Strength distribution and its moments

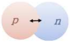
- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar/quadrupolar in our case) of the form ($Fe^{iwt} + F^{\dagger}e^{-iwt}$):

  $$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1, 2 \rightarrow \mathrm{Dipole}, \mathrm{Quadrupole})$$

- is proportional to the **static polarizability** for small oscillations

  $\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_{\nu})$

Isovector Giant Dipole Resonance:



Dipole polarizability: a macroscopic approach

electric polarizability measures tendency of the nuclear charge distribution to be distorted $\left(\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}\right)$

- The dielectric theorem establishes that the m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained ground state $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

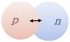
Adopting the Droplet Model:

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

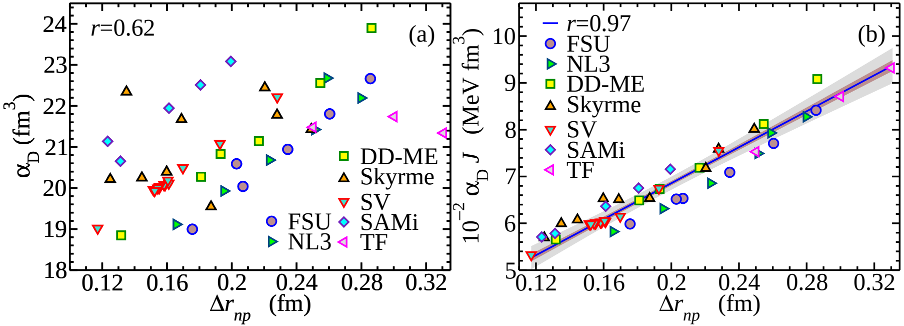
within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Isovector Giant Dipole Resonance in ^{208}Pb:



Dipole polarizability: microscopic results HF+RPA

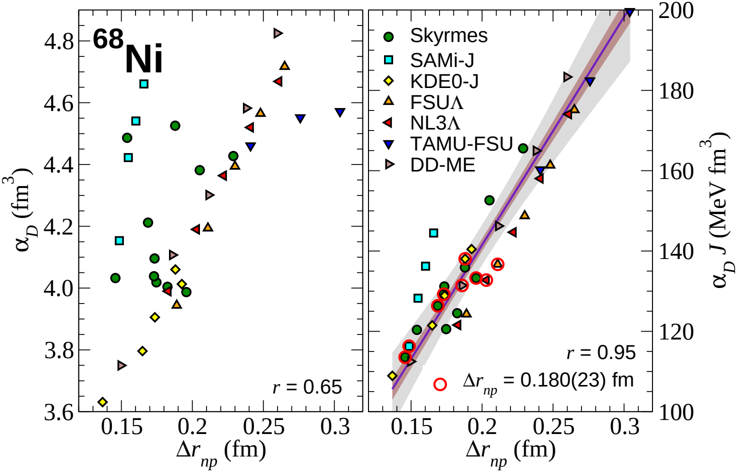


X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

Experimental dipole polarizability $\alpha_D = 20.1 \pm 0.6$ fm^3; A. Tamii *et al.*, PRL 107, 062502 (RCNP).

Isovector Giant Dipole Resonance in ^{68}Ni:

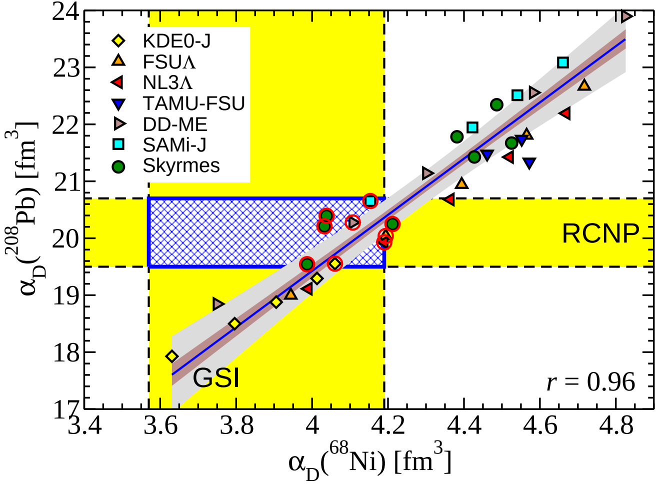
Dipole polarizability: microscopic results HF+RPA



Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm^3 D. M. Rossi *et al.*, PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm^3 “full” response D. M. Rossi, T. Aumann, and K. Boretzky.

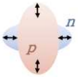
^{208}Pb vs ^{68}Ni:

Dipole polarizability: microscopic results HF+RPA



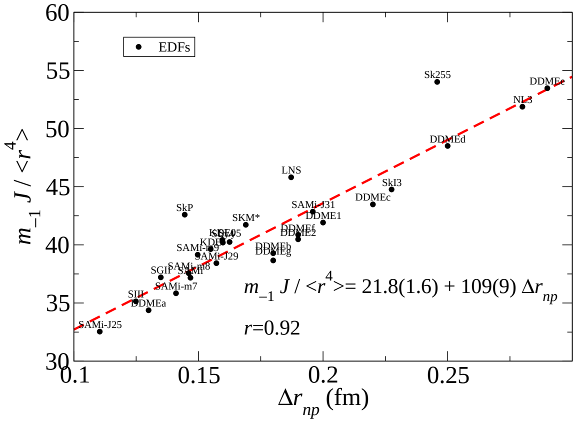
Just an indication: $\alpha_D(A=208)/\alpha_D(A=68) \sim (208/68)^{5/3}$;
Cercled models predict $\Delta r_{np}(^{208}\text{Pb}) = 0.17 \pm 0.03$ fm and $\Delta r_{np}(^{68}\text{Ni}) = 0.18 \pm 0.02$ fm; $J = 31 \pm 2$ MeV; $L = 43 \pm 19$ MeV.

Isovector Giant Quadrupole Resonance:



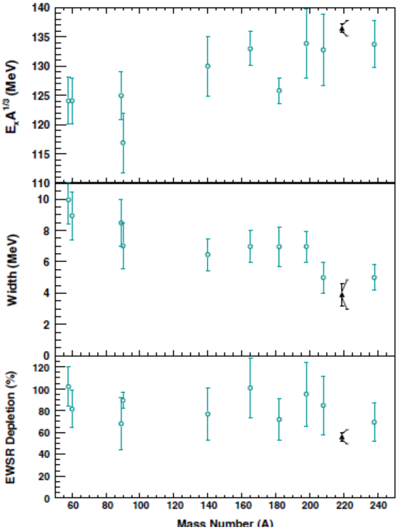
Quadrupole polarizability in ^{208}Pb

$$\alpha_Q \approx \frac{A\langle r^4\rangle}{16\pi J}\left[1+\frac{7}{2}\frac{\Delta r_{np}+\sqrt{\frac{3}{5}}\frac{e^2 Z}{70J}-\Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$



Giant Quadrupole Resonances

IVGQR: was experimentally known **[R. Pitthan, proceedings of Giant Multiple Resonance conference, Oak Ridge 1980]** but via a recent experimental technique the accuracy has been improved **[S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller PRL107 (2011)]**



$E_{excitation}$, **width and EWSR**

Giant Quadrupole Resonances

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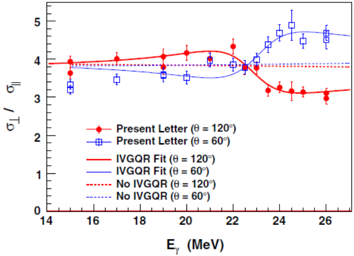
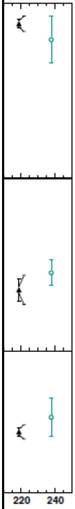
$$\frac{\sigma_{\parallel}}{\sigma_{\perp}} = \cos^2\theta + \frac{2|f_{E2}|\cos(\phi_{E2}-\phi_{E1})[\cos^3\theta-\cos\theta]}{|f_{E1}+D(E_\gamma,\theta)|}$$



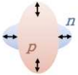


Key features in the new polarized Compton scattering experiment:

- almost **monoenergetic and polarized $\gamma-$ray beam**
- $\mathrm{E1}-\mathrm{E2}$ **interference** term has **opposite signs** in the forward and backward angles

[**S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller** PRL107 (2011)]

Isovector Giant Quadrupole Resonance:



Within the Quantum Harmonic Oscillator approach

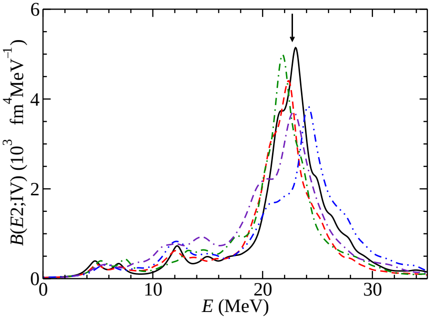
$$E_x^{IV} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{sym}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$$

and connecting V_{sym} with the Droplet model,

$V_{sym} \approx 8(a_{sym}(A) - a_{sym}^{kin})$

for non-relativistic models

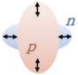
$a_{sym}^{kin} \sim \varepsilon_{F_0}/3$

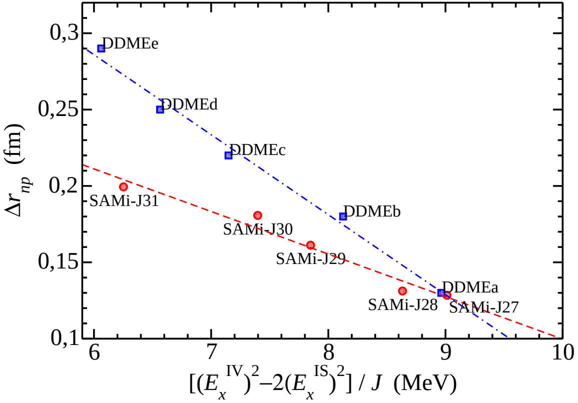


$$a_{sym}(A) \sim \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

Macroscopic and non-relativistic formula, estimate on $a_{sym}(A)$, difficult to assess systematic errors.

Isovector Giant Quadrupole Resonance:





X. Roca-Maza, *et al.* Phys. Rev. C 87, 034301 (2013).

$$\frac{\Delta r_{np} - \Delta r_{np}^{surf}}{\langle r^2\rangle^{1/2}} = \frac{2}{3}(I - I_C)\left\{1 - \frac{\varepsilon_{F_\infty}}{3J} - \frac{3}{7}\frac{I_C}{I - I_C} - \frac{A^{2/3}}{24\varepsilon_{F_\infty}}\left[\frac{\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2}{J}\right]\right\}$$

Mac-model predicted slope 0.025 MeV^{-1} fm; SAMi slope 0.027 MeV^{-1} fm; DD-ME slope 0.057 MeV^{-1} fm;

CONCLUSIONS

Conclusions:

- A precise and **model-independent** determination of Δr_{np} in ^{208}Pb via PVES experiments **probes** the **symmetry energy**.
- We demonstrate a close **linear correlation** between $A_{p\nu}$ and Δr_{np} within the same framework in which the Δr_{np} is correlated with L (expected to be better as heavier the nucleus).
- Other **experiments** fairly **agree** with the **correlation** between $A_{p\nu}$ and Δr_{np} in ^{208}Pb.
- EDFs show a linear correlation between $\alpha_{D,Q} J$ and Δr_{np}
- $A_{p\nu}$ and α_D are complementary **observables** that may set **tight constraints** on the **density dependence of the symmetry energy around saturation density, if precisely and/or systematically measured.**

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